

Nuclear Astrophysics

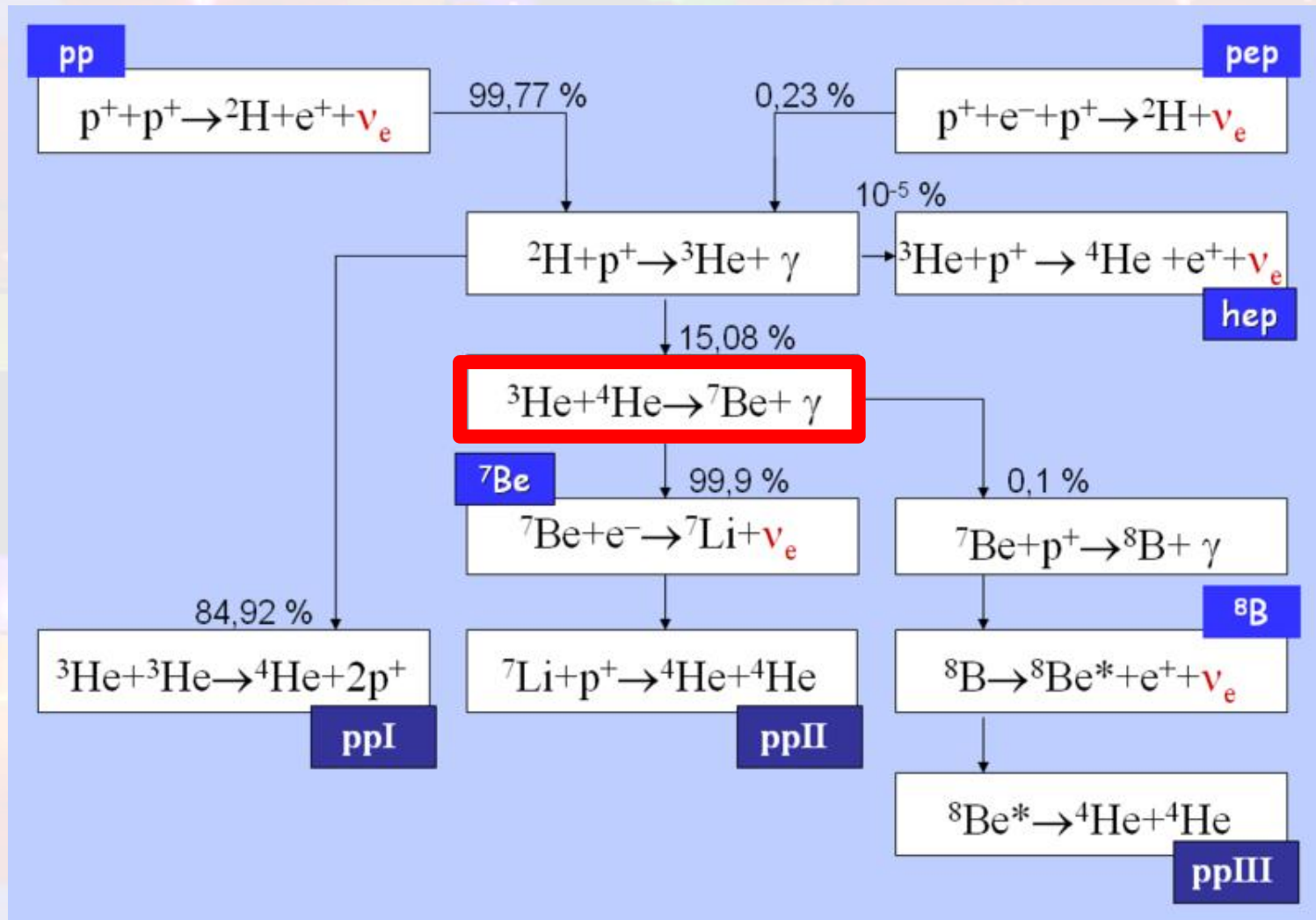
An Introduction

Lecture N°3

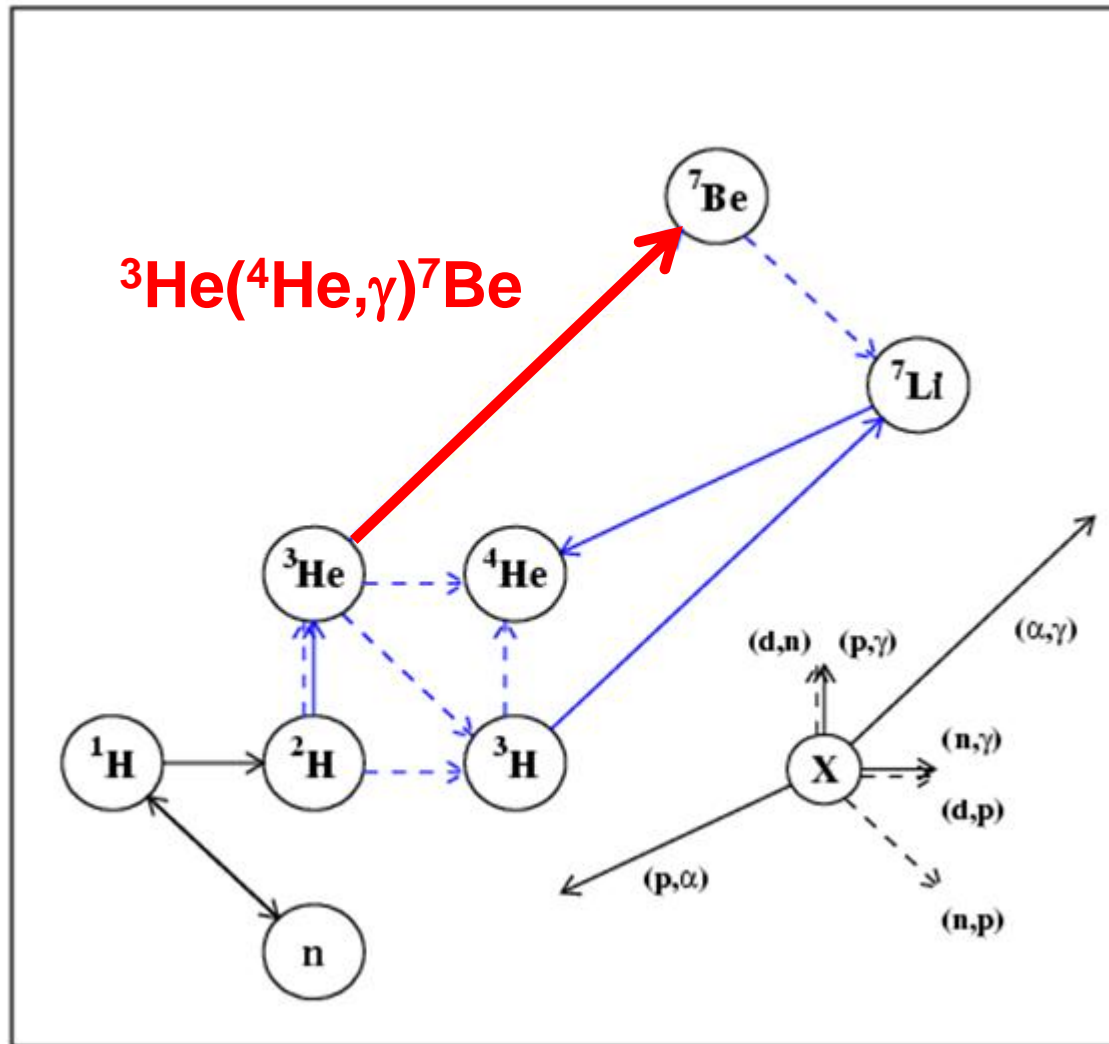
F. De Oliveira Santos

Example 5

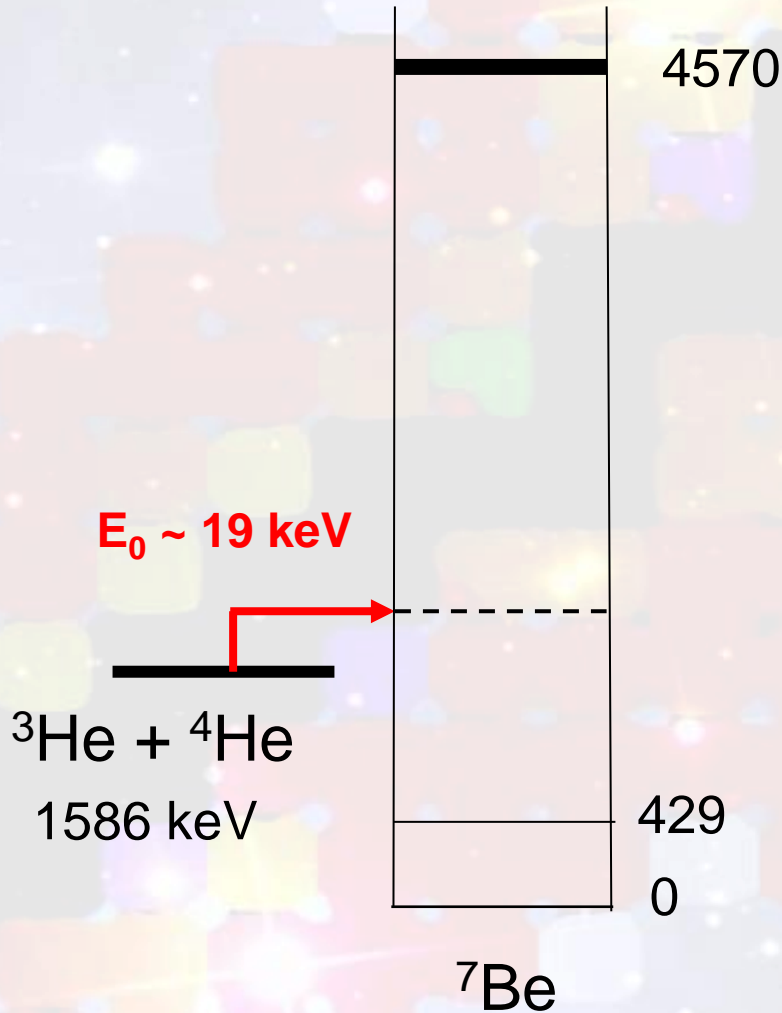
Motivation: The solar neutrinos problem



Another motivation: Primordial abundance of ${}^7\text{Li}$



"Theoretical" investigations of ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$



Gamow Peak

$T_6 = 15$ $E_0 \sim 19$ keV

Nuclear Structure ${}^7_4\text{Be}$

$\Delta: 15769.55$ $S_n: 10676.5$ $S_p: 5605.799$
 $Q_{EC}: 861.81518$

Levels and γ -ray branchings:

$0, 3/2^-, 53.297$ d, [ACDGHJK], $T=1/2$,
 $\%EC=100$

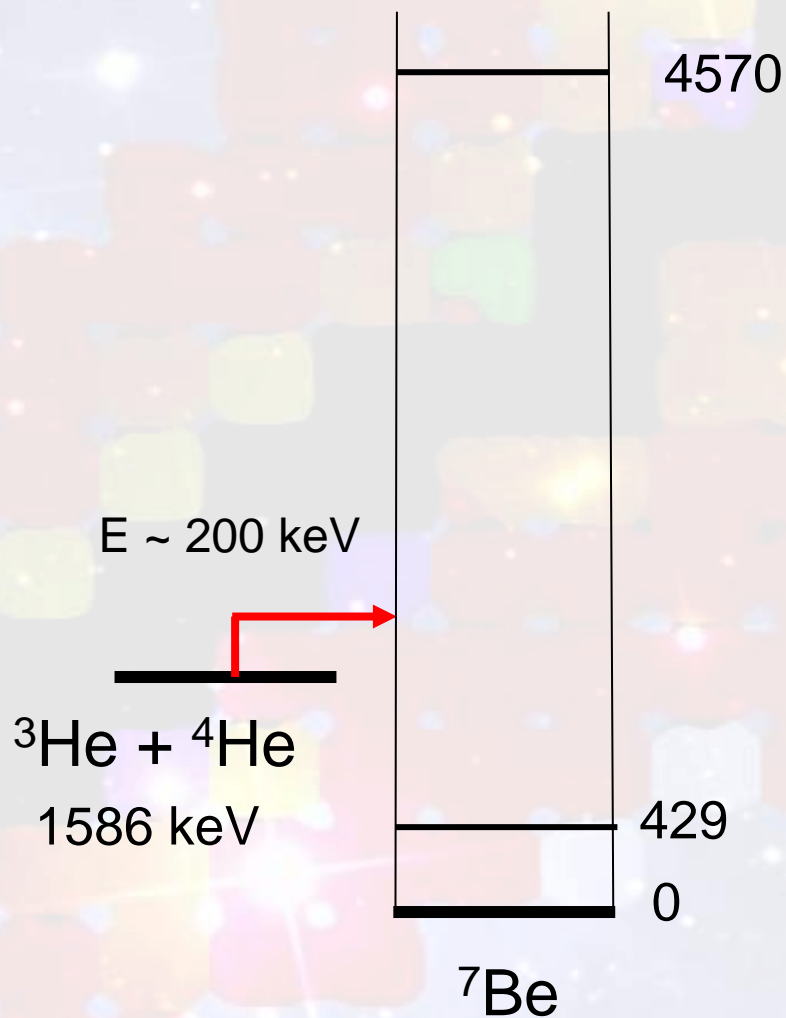
429.08 10, $1/2^-, 133.17$ fs, [ACDGHJK],
 $T=1/2$

$\gamma_0 429.07$ ($\dagger, 100$) M1

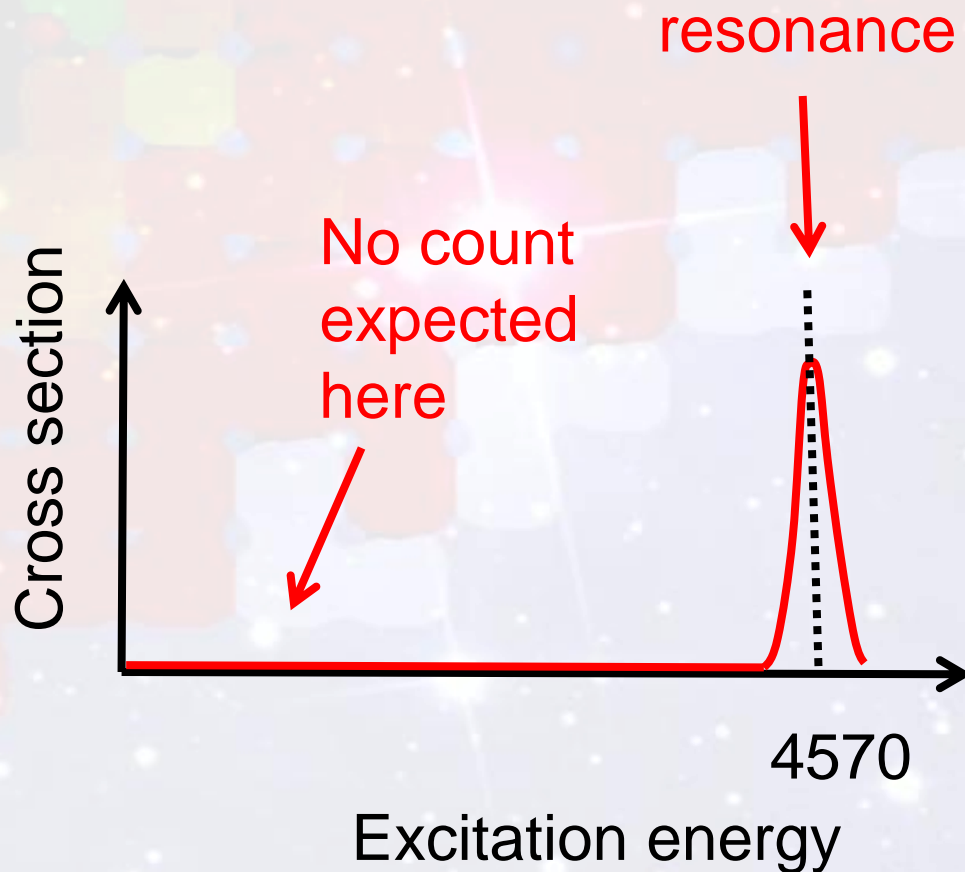
4570 50, $7/2^-, \Gamma=175.7$ keV, [BDHIJK],
 $T=1/2$

- No resonance predicted
- Tail of the 4570 keV resonance?

"Theoretical" investigations of ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$

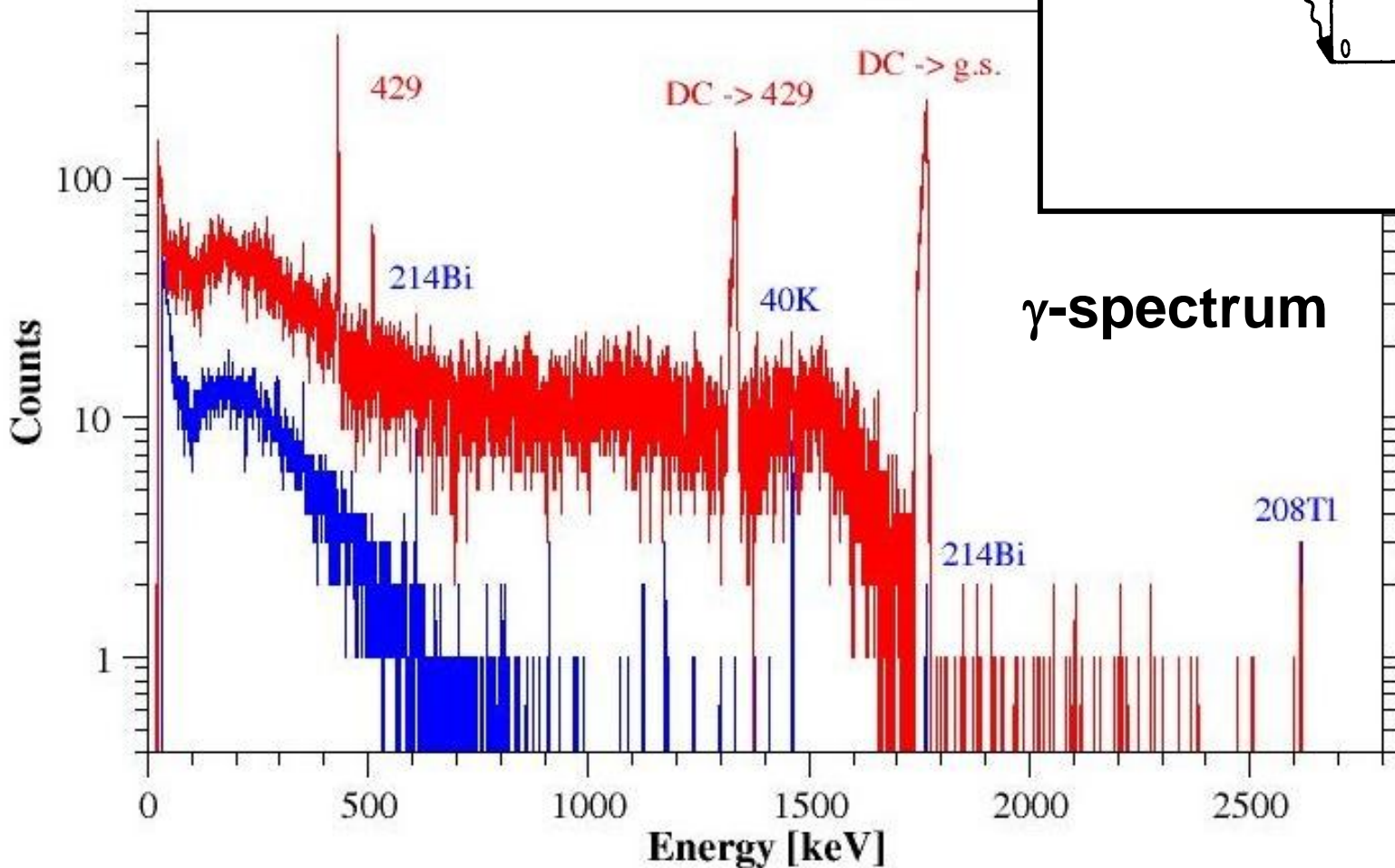
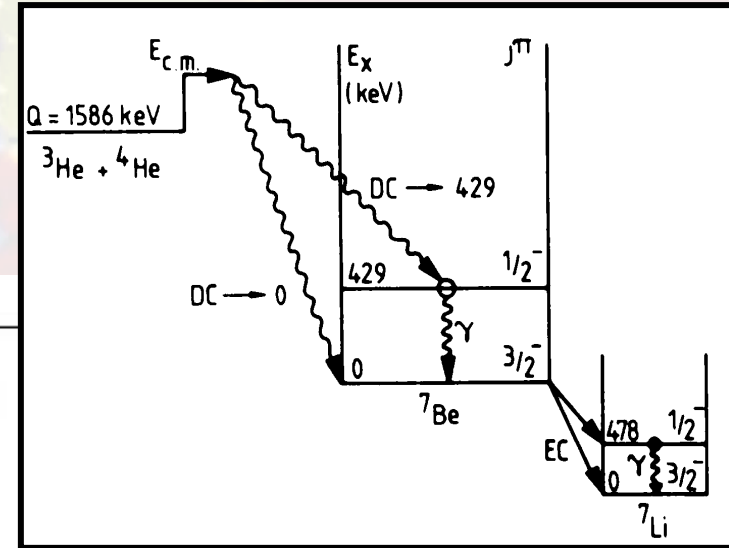


Compound nucleus model



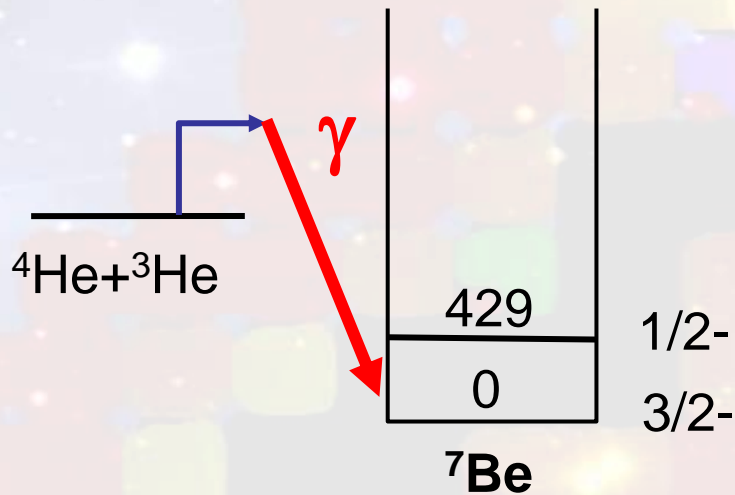
But γ -rays are measured!

$$E_{\gamma} = (E_{cm} + Q) - E_i$$



Direct (non-resonant) radiative capture reaction

(not to be confused with Direct Measurement)



$$\sigma_{\text{DC}}(E) \propto \left| \left\langle {}^7\text{Be} \left| H_{\gamma} \right| {}^4\text{He}+{}^3\text{He} \right\rangle \right|^2$$

Final bound
state wave
function

Initial continuum
wave function

Electromagnetic
operator describing
the transition

Can occur at all projectile energies.

Smooth energy dependence of cross section.

Multipolarity	Electric Transition Rate (s^{-1})	Magnetic Transition Rate (s^{-1})
1	$1.587 \times 10^{15} E_\gamma^3 B(E1)$	$1.779 \times 10^{13} E_\gamma^3 B(M1)$
2	$1.223 \times 10^9 E_\gamma^5 B(E2)$	$1.371 \times 10^7 E_\gamma^5 B(M2)$
3	$5.689 \times 10^2 E_\gamma^7 B(E3)$	$6.387 \times 10^0 E_\gamma^7 B(M3)$
4	$1.649 \times 10^{-4} E_\gamma^9 B(E4)$	$1.889 \times 10^{-6} E_\gamma^9 B(M4)$
5	$3.451 \times 10^{-11} E_\gamma^{11} B(E5)$	$3.868 \times 10^{-13} E_\gamma^{11} B(M5)$

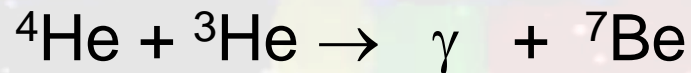
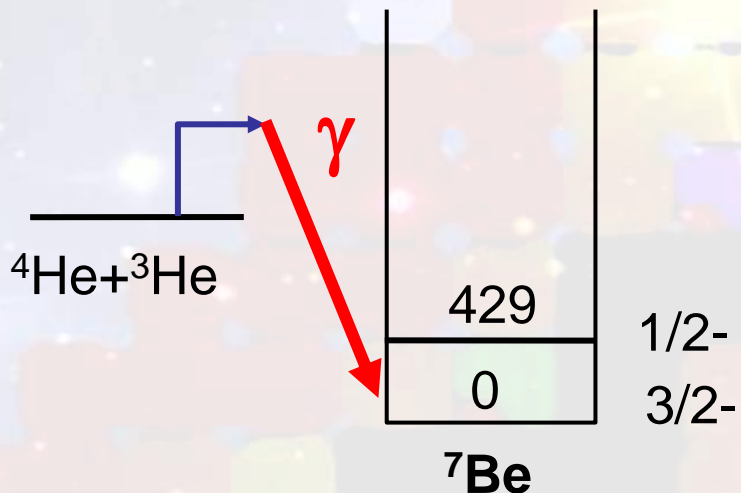
Table 2.2: Transition probabilities $T(s^{-1})$ expressed by $B(EL)$ in $(e^2(fm)^{2L})$ and $B(ML)$ in $(\frac{e\hbar}{2mc}(fm)^{2L-2})$. E_γ is the γ -ray energy, in MeV. (Taken from ref [69]).

$$E_\gamma = 1 \text{ MeV}$$

E1	10^{-14} s
E2	$1.3 \times 10^{-8} \text{ s}$
E3	$2.9 \times 10^{-2} \text{ s}$

M1	$3.2 \times 10^{-14} \text{ s}$
M2	$4.5 \times 10^{-8} \text{ s}$
M3	10^{-1} s

Angular momentum matching



Exit channel ($\gamma + {}^7\text{Be}$)

	γ	$\gamma+3/2-$	$\gamma+1/2-$
E1	1-	1/2+ , 3/2+, 5/2+	1/2+ , 3/2+
M1	1+	1/2-, 3/2-, 5/2-	1/2-, 3/2-
E2	2+	1/2-, 3/2-, 5/2-, 7/2-	3/2-, 5/2-

Entrance channel (${}^3\text{He} + {}^4\text{He}$)

		${}^3\text{He}$	${}^4\text{He}$	Total
s-wave	0+	1/2+	0+	1/2+
p-wave	1-	1/2+	0+	1/2-, 3/2-

For the two final states, the most intense contribution is a s-wave capture coupled to an E1 γ -transition.

Direct non-resonant radiative capture reaction

Expanding the electrostatic potential in spherical harmonics

$$\varphi(\mathbf{r}) = \frac{Z_p e}{|\mathbf{r} - \mathbf{r}_p|} = \sum_{\lambda\mu} \frac{4\pi Z_p e}{2\lambda + 1} Y_{\lambda\mu}^*(\hat{\mathbf{r}}_p) Y_{\lambda\mu}(\hat{\mathbf{r}}) \begin{cases} r_p^{-\lambda-1} r^\lambda & r_p > r \\ r_p^\lambda r^{-\lambda-1} & r_p < r \end{cases}$$

Bohr & Mottelson vol I

Operators for electric transitions of multipolarity λ

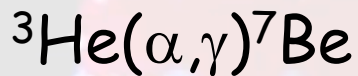
$$\mathcal{O}_{E\lambda\mu} = e_\lambda r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}})$$

$$\sigma_{\text{DC}}^{\text{E1}}(\mathbf{E}) \propto \frac{1}{E} \omega \mathbf{S} \cdot \mathbf{E}_\gamma^3 \left| \int \varphi_{n_f l_f}(r) r \chi(\mathbf{E}, r) dr \right|^2$$

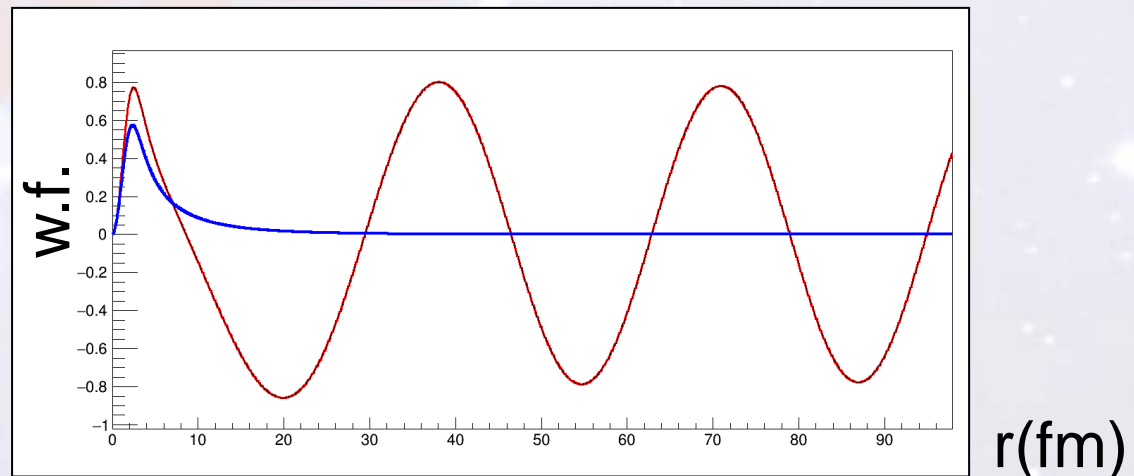
Spectroscopic factor

Final bound state wave function

Initial continuum wave function



$$S = \left| \left\langle {}^7\text{Be} \left| {}^4\text{He} + {}^3\text{He} \right. \right\rangle \right|^2$$



RADCAP



Available online at www.sciencedirect.com



Computer Physics Communications 156 (2003) 123–141

Computer Physics
Communications

www.elsevier.com/locate/cpc

RADCAP: A potential model tool for direct capture reactions [☆]

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Abstract

A computer program is presented aiming at the calculation of bound and continuum states, reduced transition probabilities, phase-shifts, photo-disintegration cross sections, radiative capture cross sections, and astrophysical S-factors, for a two-body nuclear system. The code is based on a potential model of a Woods–Saxon, a Gaussian, or a M3Y, type. It can be used to calculate nuclear reaction rates in numerous astrophysical scenarios.

Codes:

TEDCA

<https://nucastro.org/codes.html#TEDCA>

DIRCAD

RADCAP

http://cpc.cs.qub.ac.uk/summaries/ADSH_v1_0.html

```
deoliveira@GANP014 ~/radcap
$ ./a.exe
Enter:
1 for M3Y Potential
2 for energy and wavefunction bound states
3 for reduced transition probab. between bound states
4 for phase-shifts and wavefunctions of continuum stat.
5 for S-factors, response functions, etc.
```

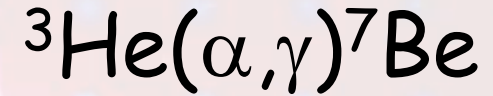
It requires, at least, to know the spectroscopic factors

$$S = \left| \left\langle {}^7\text{Be} \left| {}^4\text{He} + {}^3\text{He} \right. \right\rangle \right|^2$$

Estimation of the cross section / counts

Table 2.3 Classification of the main reactions involved in nuclear astrophysics.

Process		Examples	S(0) (MeV-b)
Nuclear	Non - resonant	${}^6\text{Li}(p,\alpha){}^3\text{He}$	≈ 3
	Resonant $\left\{ \begin{array}{l} \ell_R = \ell_{min} \\ \ell_R > \ell_{min} \end{array} \right.$	${}^3\text{He}(d,p)\alpha$ ${}^{11}\text{B}(p,\alpha){}^8\text{Be}$	≈ 6 ≈ 300
	Multiresonance	${}^{22}\text{Ne}(\alpha,n){}^{25}\text{Mg}$	$\approx 10^8$
	Subthreshold state	${}^{13}\text{C}(\alpha,n){}^{16}\text{O}$	$\approx 10^7$
Electromagnetic	Non - resonant	${}^6\text{Li}(p,\gamma){}^7\text{Be}$	$\approx 10^{-4}$
	Resonant $\left\{ \begin{array}{l} \ell_R = \ell_{min} \\ \ell_R > \ell_{min} \end{array} \right.$	${}^{12}\text{C}(p,\gamma){}^{13}\text{N}$ ${}^7\text{Be}(p,\gamma){}^8\text{B}$	$\approx 10^{-3}$ $\approx 2 \times 10^{-5}$
	Multiresonance	${}^{22}\text{Ne}(\alpha,\gamma){}^{26}\text{Mg}$	$\approx 2 \times 10^3$
	Subthreshold state	${}^{12}\text{C}(\alpha,\gamma){}^{16}\text{O}$	≈ 0.5
Weak	Non-resonant	$p(p,e^+\nu)d$	$\approx 4 \times 10^{-25}$
		${}^3\text{He}(p,e^+\nu){}^4\text{He}$	$\approx 10^{-22}$



S(0) ~ 0.1 keV barn



Theoretical Models for Nuclear Astrophysics - P. Descouvemont

$$\sigma(E) \equiv \frac{S(E)}{E} \exp(-2\pi\eta)$$

$$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E} \right)^{1/2} \quad (E \text{ en keV})$$

$$\sigma(17 \text{ keV}) = 2 \times 10^{-19} \text{ b}$$

$$1 \text{ mAe} \quad N_{\text{inc}}({}^3\text{He}) = 3 \times 10^{15} \text{ pps}$$

$$N_{\text{reactions}} = N_{\text{inc}} N_{\text{target}} \sigma(E)$$

$$N_{\text{target}} \sim 10^{20} \text{ at/cm}^2$$

$$N_{\text{reactions}} = 2 / \text{year}$$

Not possible to measure directly at E0

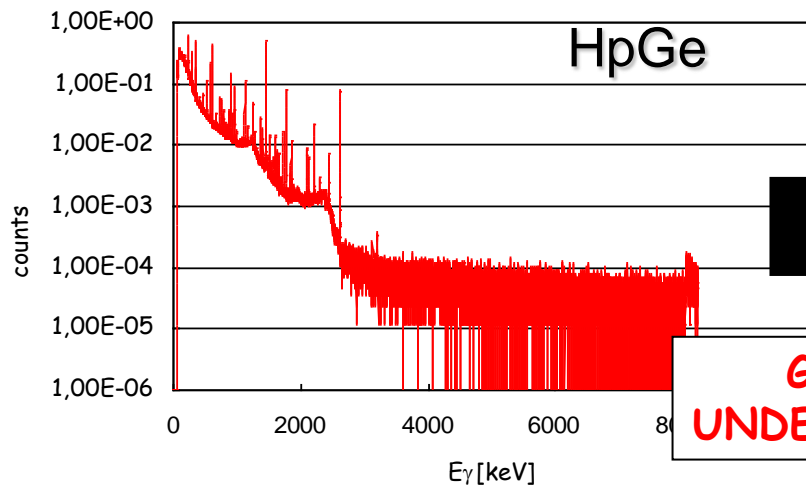
Cross section measurement requirements

$$R_{\text{lab}} > B_{\text{cosm}} + B_{\text{env}} + B_{\text{beam induced}}$$

Environmental radioactivity has to be considered (shielding)
+ intrinsic detector bck

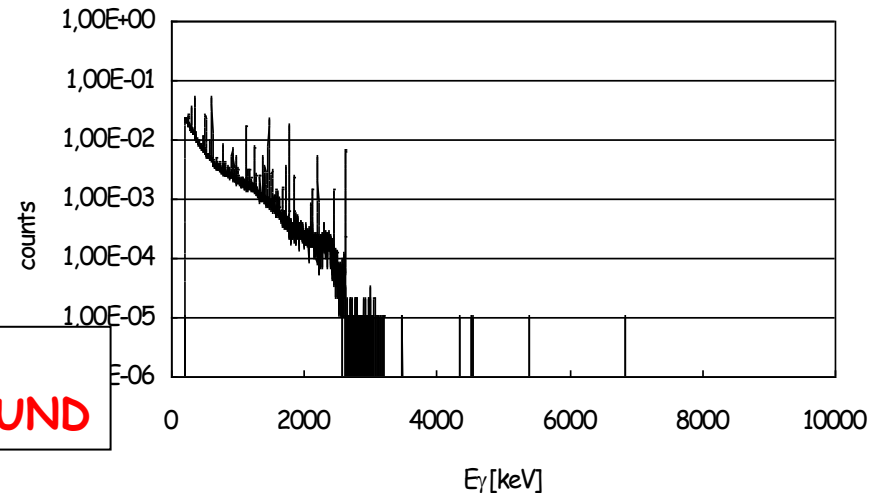
Beam induced bck from impurities in beam & targets → high purity and detector techniques (coincidence)

$3\text{MeV} < E_{\gamma} < 8\text{MeV}$: **0.5 Counts/s**



GOING UNDERGROUND

0.0002 Counts/s



Exercise: Lowest accessible energy

If $R_{\text{lab}} > 0.0002 \text{ Counts/s}$

Lowest accessible energy? ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$

0.25 mAe of ${}^4\text{He} \Rightarrow N_{\text{inc}}({}^3\text{He}) = 0.75 \times 10^{15} \text{ pps}$

$N_{\text{target}} \sim 10^{18} \text{ at/cm}^2$

$S(E=0) \sim 0.1 \text{ keV barn}$

γ Detection efficiency ~ 0.004

Solution: Lowest accessible energy

$$0.0002 = 0.004 \times N_{\text{inc}} \times N_{\text{target}} \times \sigma$$



$$\sigma = 6.7 \times 10^{-11} \text{ barns}$$



$$\sigma(E) = \frac{0.1}{E} \exp(-2\pi\eta) = 6.7 \times 10^{-11}$$

$$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E} \right)^{1/2} = 163.9 \times E^{-0.5}$$



$$E \sim 100 \text{ keV}$$



Gran Sasso

Direct measurement at LUNA

Laboratory for Underground Nuclear Astrophysics

(shielding \equiv 4000 m water equivalent)

LUNA 1
(1992-2001) ●
50 kV

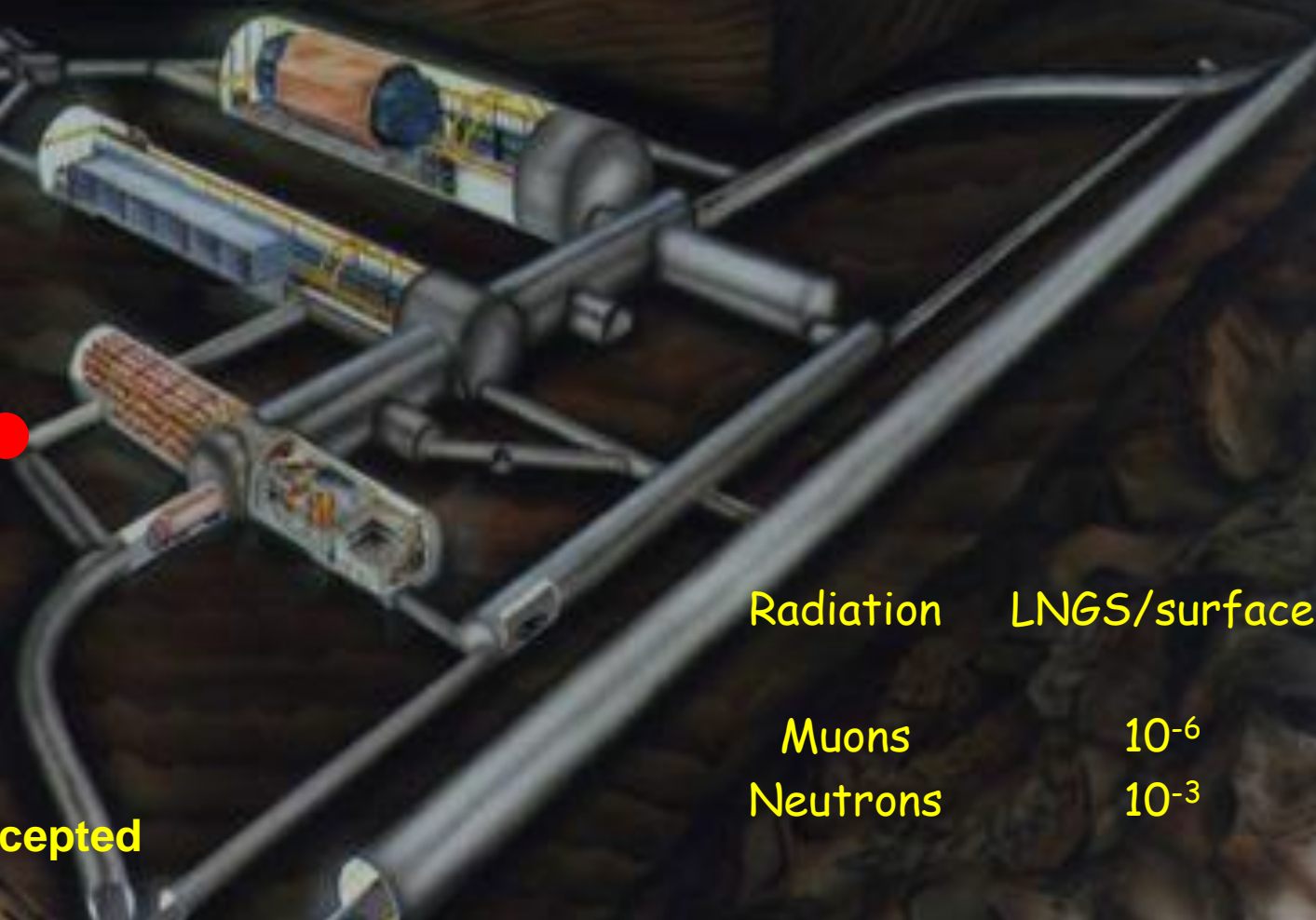
LUNA 2 ●
(2000→...)
400 kV

(2012) LUNA-MV accepted

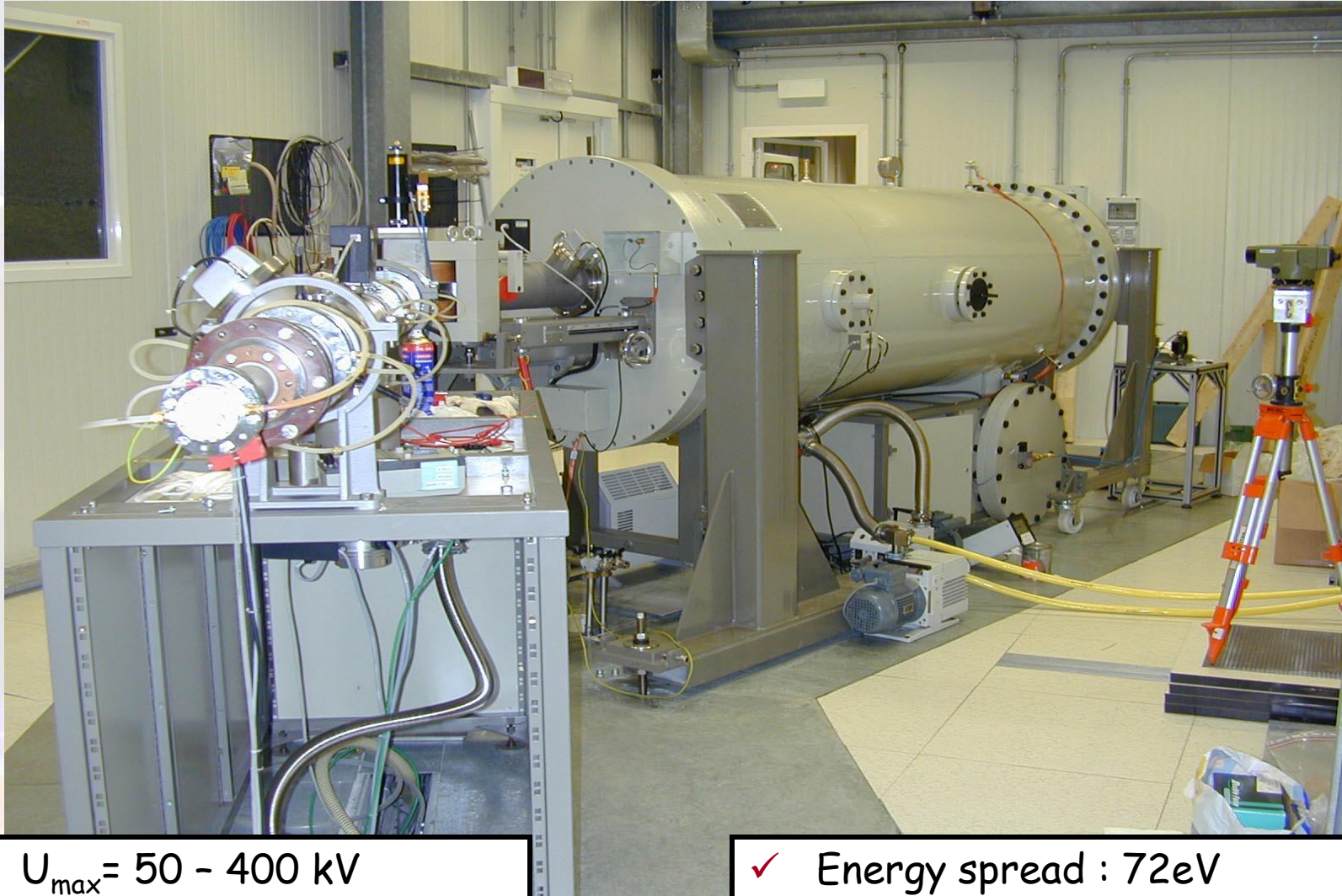
Radiation LNGS/surface

Muons 10^{-6}

Neutrons 10^{-3}



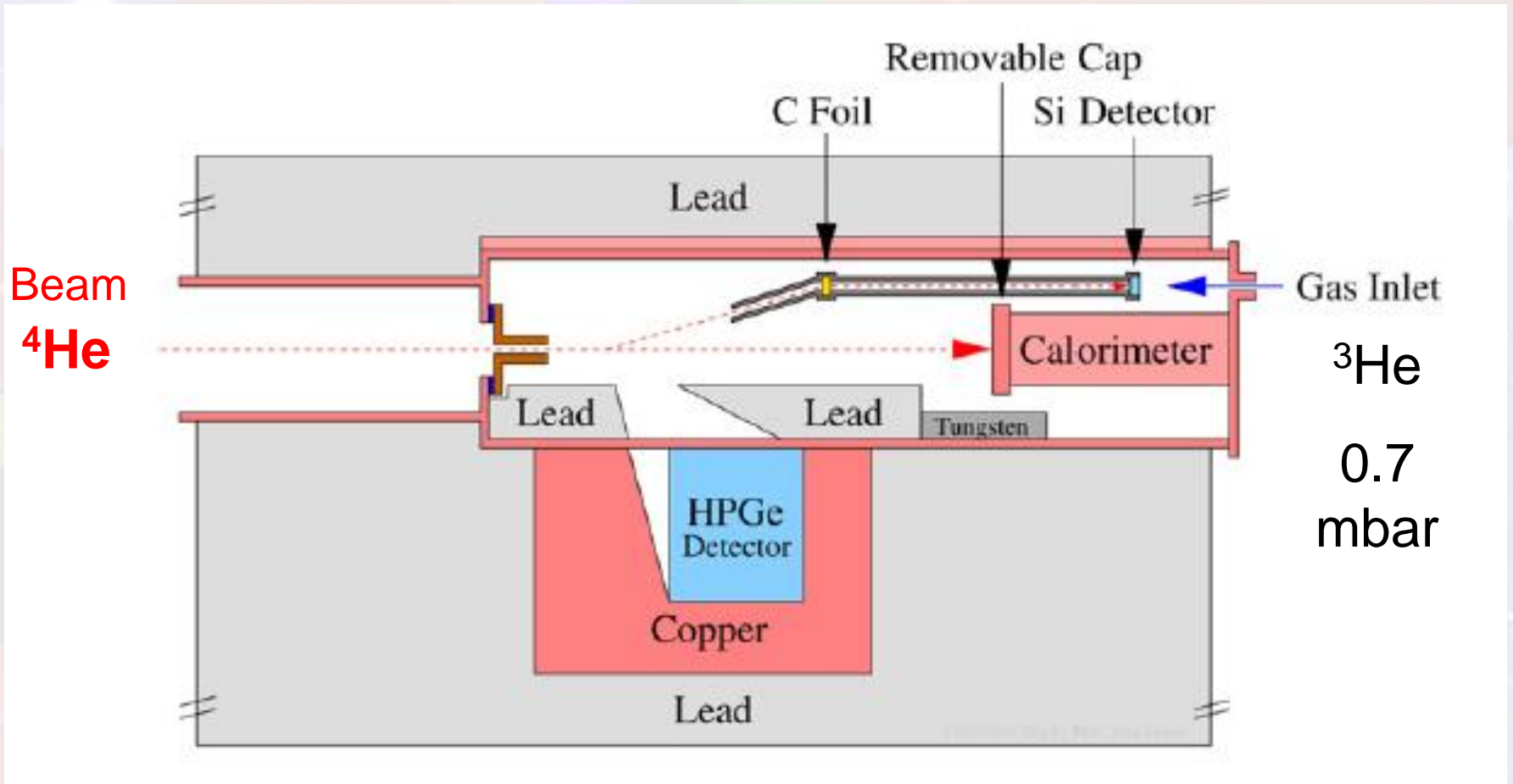
LUNAI I

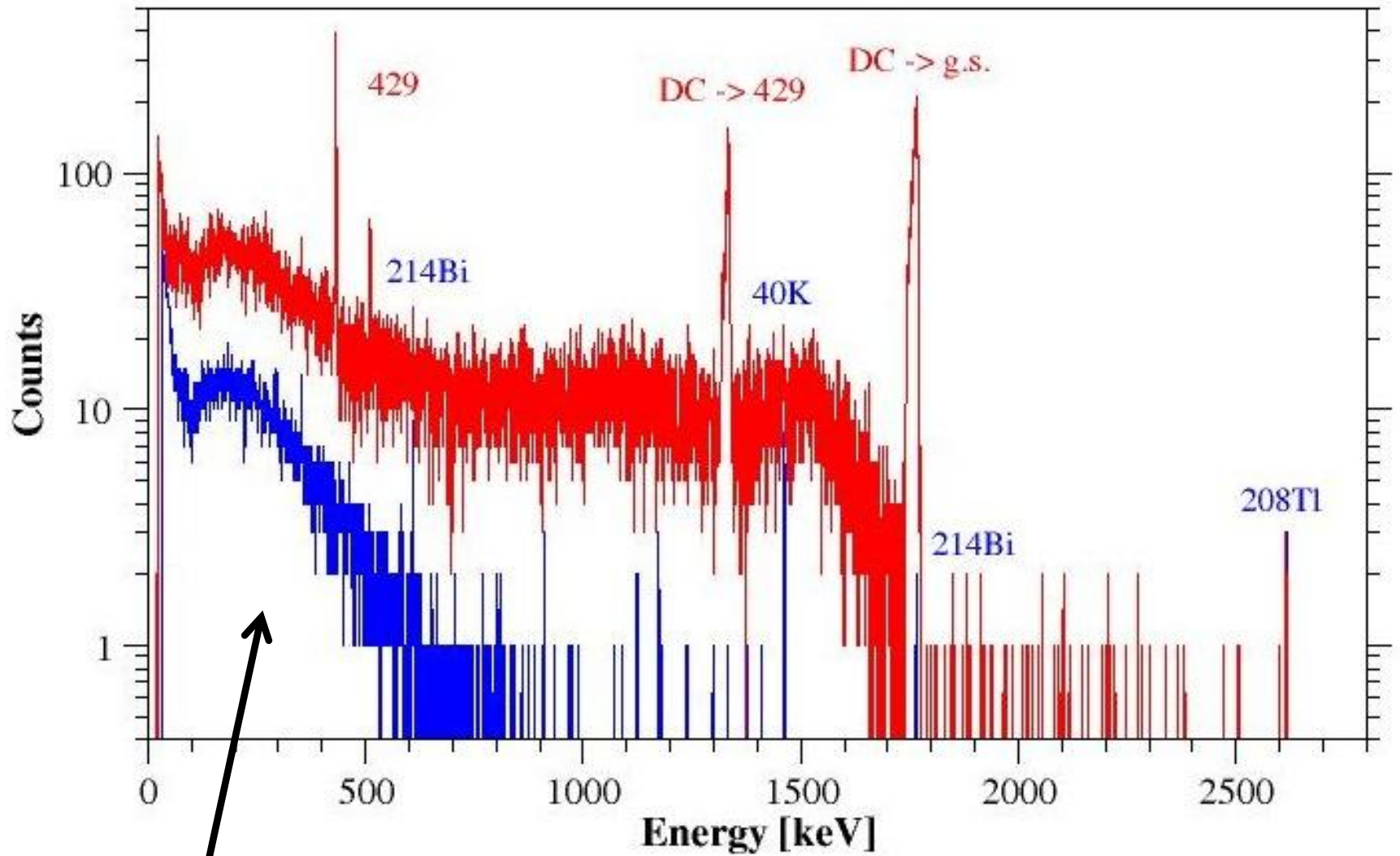


- ✓ $U_{\max} = 50 - 400 \text{ kV}$
- ✓ $I \sim 500 \mu\text{A}$ for protons
 $I \sim 250 \mu\text{A}$ for alphas

- ✓ Energy spread : 72 eV
- ✓ Total uncertainty is $\pm 300 \text{ eV}$
between $E_p = 100 \div 400 \text{ keV}$

Direct Measurement of ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$

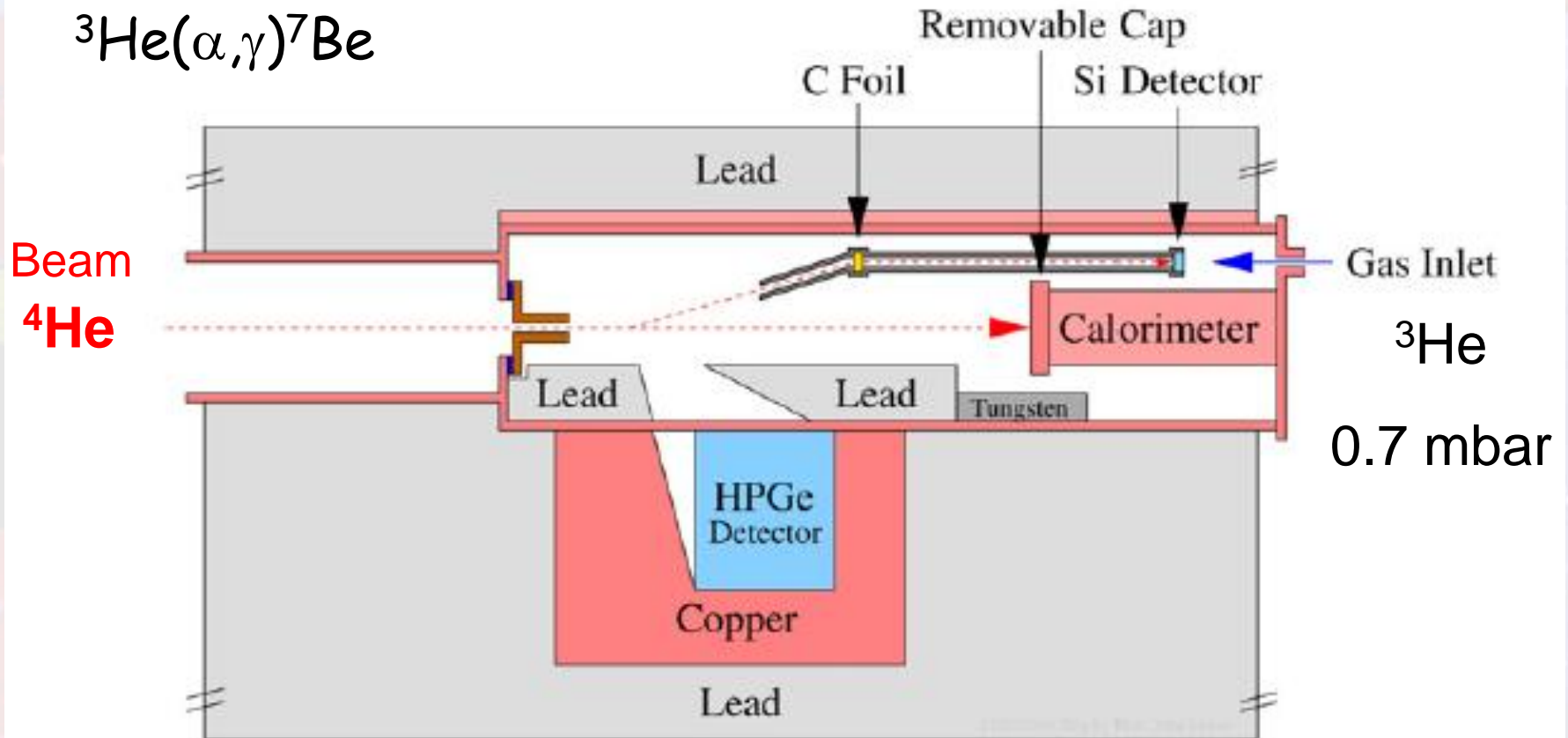




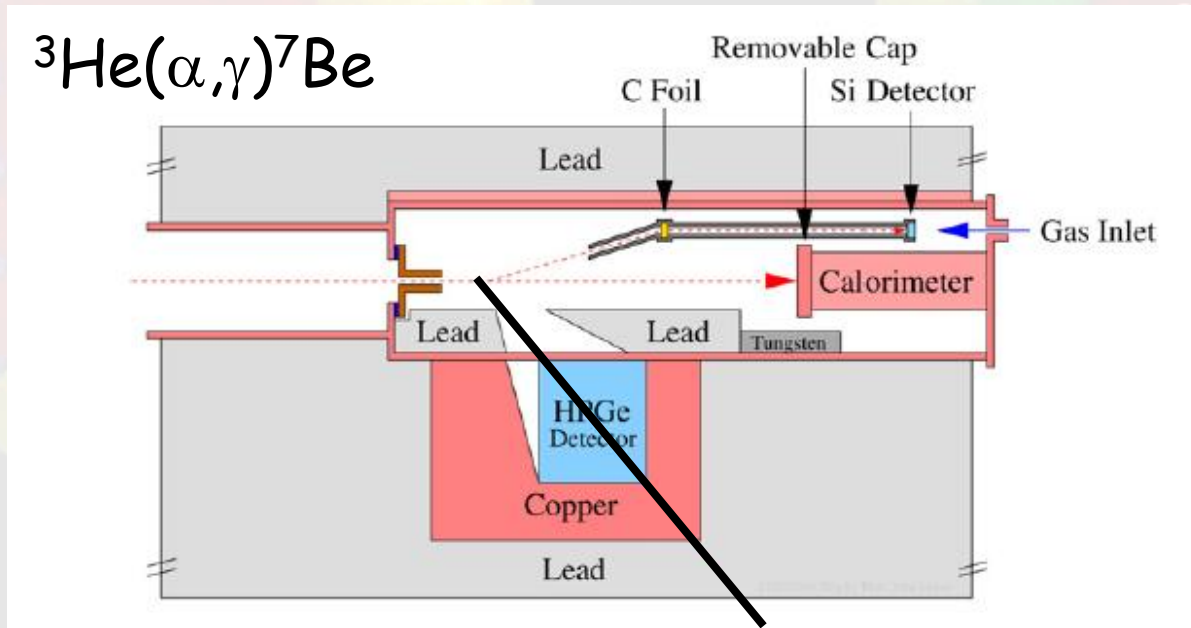
Background

Is there anything missing here?

$$\sigma = \frac{N_{\text{detected}}}{N_{\text{inc}} N_{\text{target}} \times \text{efficiency}}$$



What about the angular distribution?



Measured only
around $\theta = 55^\circ$

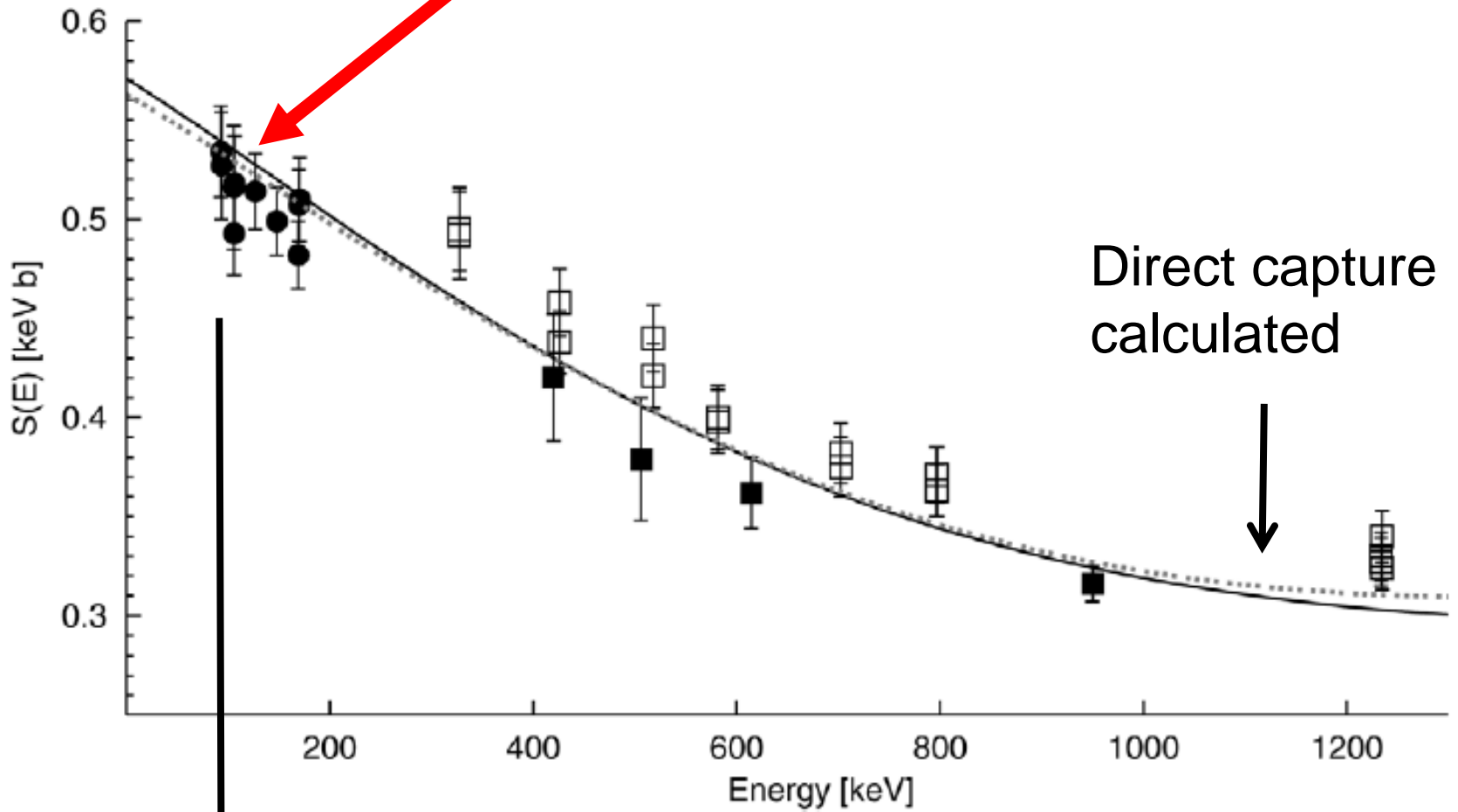
Total cross section **based on theoretical predictions** for the angular distribution + extrapolations

$$W(\theta) = 1 + a_1 P_1(\theta) + a_2 P_2(\theta) + \dots,$$

where a_1 and a_2 are the coefficients of the Legendre polynomials $P_1(\theta)$ and $P_2(\theta)$.

Resulted in 2.5 % uncertainties...

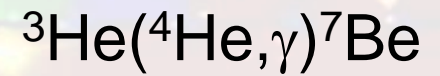
New results of Luna



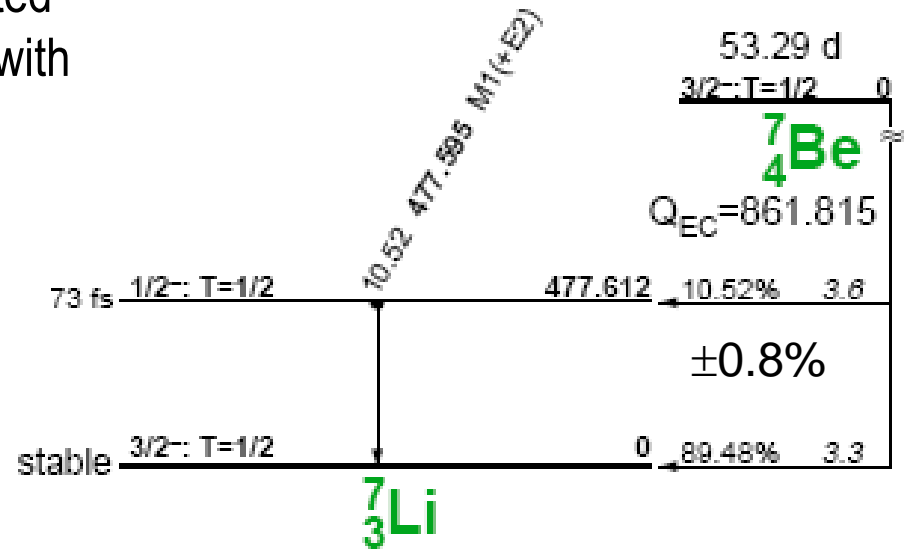
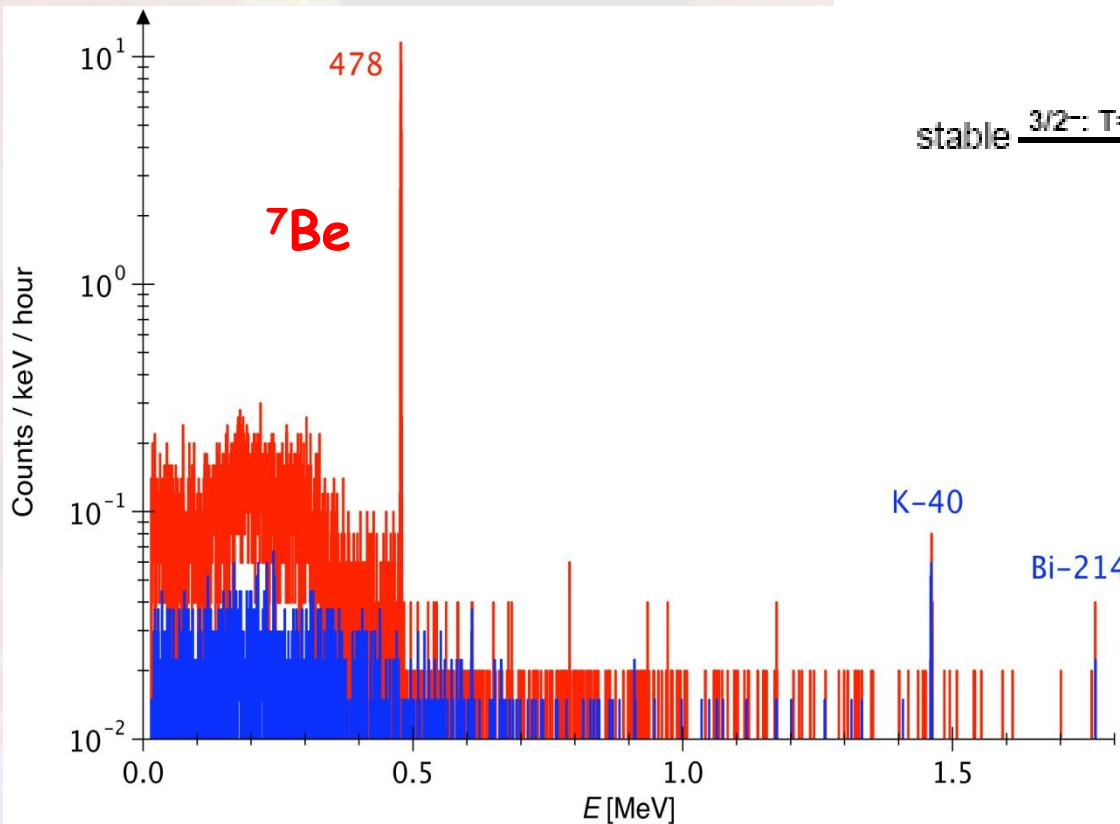
Measured down to 93 keV

Example 6

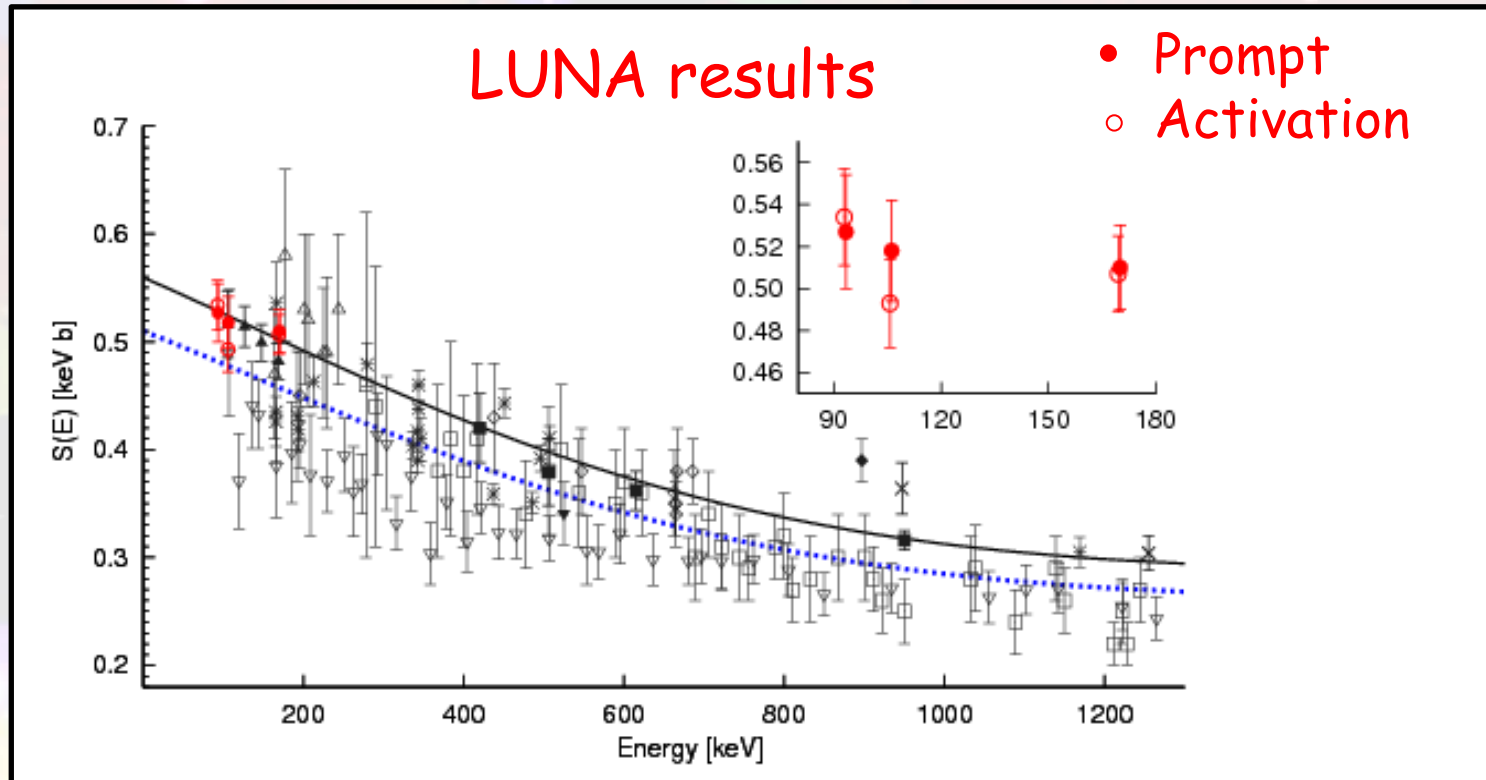
Activation measurement



After the irradiation, the catcher was dismantled and counted in close geometry subsequently with two 120% relative efficiency HPGe detectors.



Prompt/ Activation Results



Example 7

Indirect measurement The case of $^{15}\text{N}(\alpha, \gamma)^{19}\text{F}$

The main reaction producing ^{19}F in the AGB stars

Rate not well known (quasi) Impossible to measure directly!

Température T_6	E_α^{cm} (MeV)	(MeV)	J^π
370	0,536 $l = 3$	4,550	5/2+
207	0,364 $l = 3$	4,377	7/2+
2,3	0,018 $l = 4$	4,032	9/2-
<hr/> $^{15}\text{N} + \alpha$ 4,0138		<hr/> ^{19}F	

prediction

$$\Gamma_\alpha \sim 9 \times 10^{-8} \text{ eV}$$

Impossible to
measure directly!?

$$\Gamma_\alpha = S_\alpha \frac{\hbar^2 s}{\mu} |R_L^{DW}(s)|^2 P_L(Q, s).$$

Spectroscopic Factor

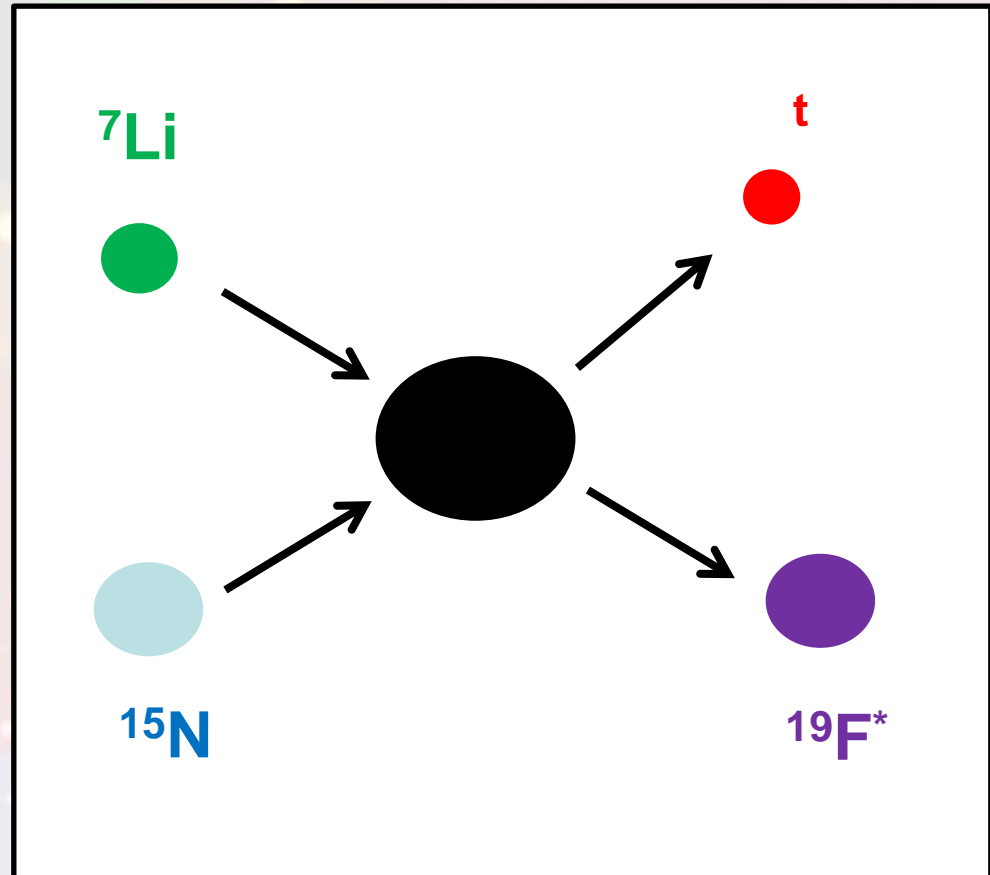
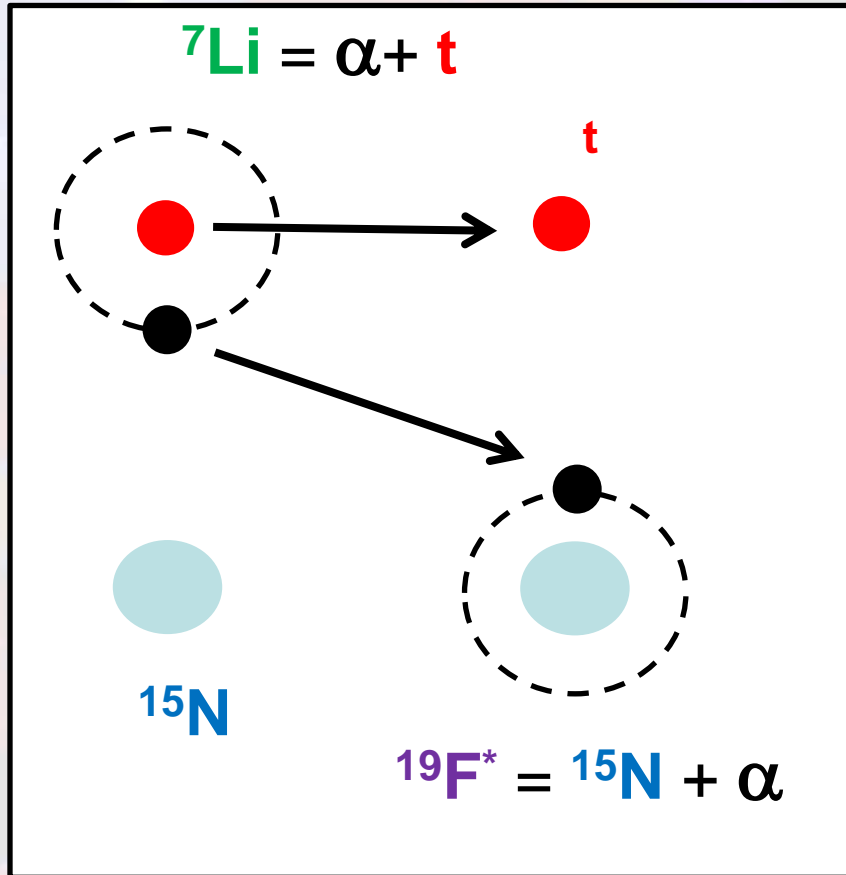
$$S_\alpha = \left| \left\langle ^{19}\text{F}^*(4.377 \text{ MeV}) \left| ^{15}\text{N} + ^4\text{He} \right. \right\rangle \right|^2$$

Measurement of $^{15}\text{N}(^7\text{Li},t)^{19}\text{F}$

Two reaction mechanisms

Direct Transfer Reaction

Compound Nucleus



$$\frac{d\sigma}{d\Omega}^{\text{exp}} = SN \left(\frac{d\sigma}{d\Omega} \right)_{DWBA} + C \left(\frac{d\sigma}{d\Omega} \right)_{HSFB}$$

Statistical model

High density of states

Calculate the Compound Nucleus with the Hauser-Feshbach model

$$\sigma_i = \frac{\pi}{k^2} (2\ell + 1) T_i$$

Transmission functions

$$\sigma_{i,f} = \frac{\pi}{k^2} (2\ell + 1) \frac{T_i T_f}{\sum_{\gamma} T_{\gamma}}$$

Branching to final state "f"

$$\sigma_{jk}(E) = \pi \lambda_j^2 \frac{1}{(2J_I + 1)(2J_j + 1)} \sum_{J^{\pi}} (2J + 1) \frac{T_j(E, J^{\pi}) T_k(E, J^{\pi})}{T_{tot}^{\mu}(E)}$$

Ingredients

- Nuclear Level Density
- Optical potentials
- γ -ray strength functions
- ...

Codes

SMOKER, NON-SMOKER, THALYS, MOST, SMARAGD, HSFB, PACE

<http://www.astro.ulb.ac.be/pmwiki/Brusslib/Talys>

<http://nucastro.org/web smoker.html>

Database: KADONIS

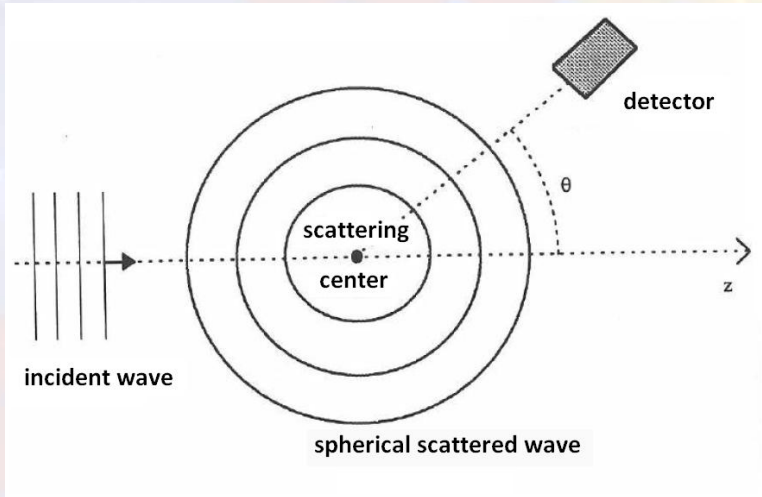
...

Calculate the Direct transfer reaction with the DWBA model

Distorted Wave Born Approximation

Reaction $A(a,b)B$

$a = b+x$ & $B = A + x$



$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi \hbar^2)^2} \frac{k_b}{k_a} \frac{1}{(2J_A + 1)(2S_a + 1)} \sum_{M_A M_B m_a m_b} |T|^2$$

$$T_{DWBA}(\Theta, \Phi) = \int \underbrace{\chi_{\beta}^{(-)}(\vec{k}_{\beta}, \vec{r}_{\beta})^*}_{B(b,b)B} \underbrace{\langle B, b | W | A, a \rangle}_{\text{Form factor}} \underbrace{\chi_{\alpha}^{(+)}(\vec{k}_{\alpha}, \vec{r}_{\alpha})}_{\text{Elastic scattering wave function } A(a,a)A} d^3\vec{r}_{\alpha} d^3\vec{r}_{\beta}$$

$B(b,b)B$

Form factor

Elastic scattering
wave function
 $A(a,a)A$

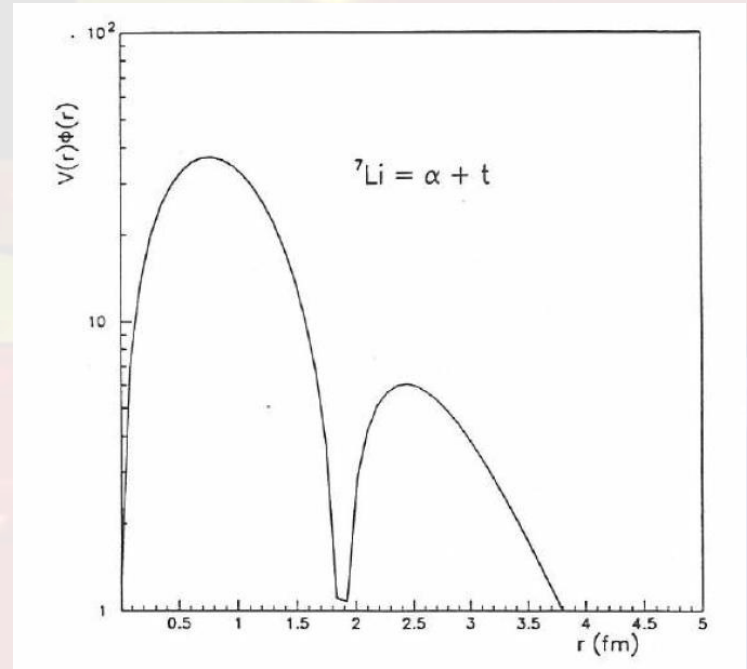
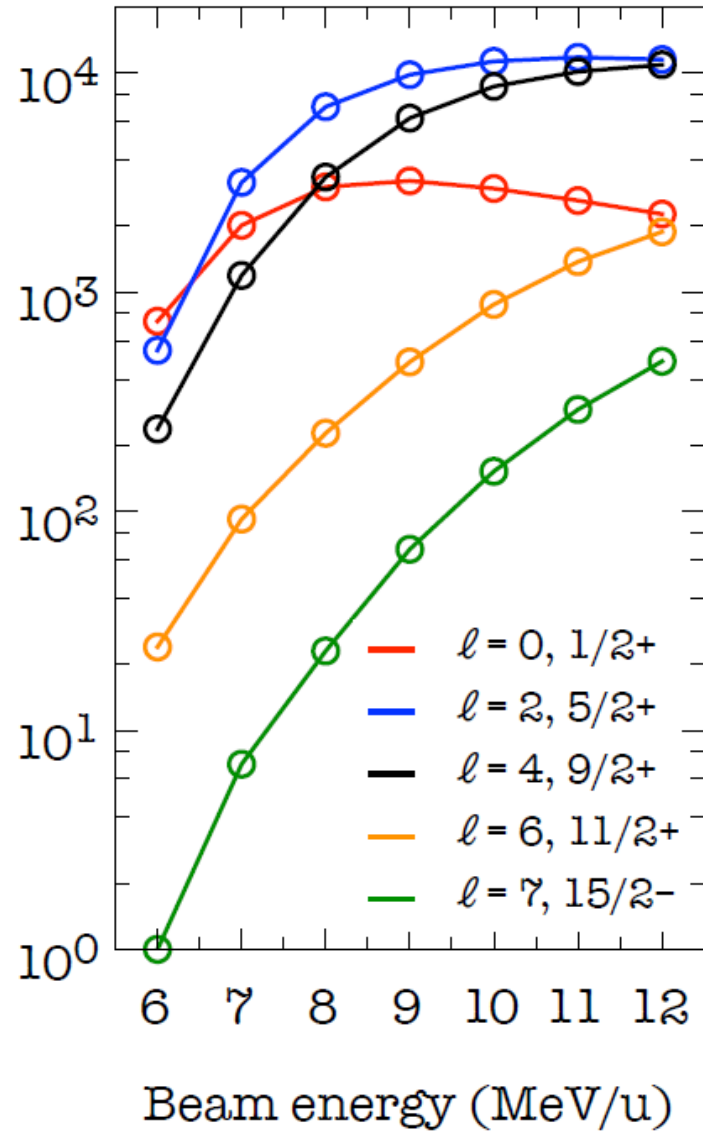
$$\langle B, b | W | A, a \rangle = \int \underbrace{\Psi_B^* \Psi_b^* W \Psi_A \Psi_a}_{\text{Internal wave functions}} d\xi$$

Internal wave functions

$$W = V_{bx}$$

Spin matching

Cross section (a.u.)



Programs:

DWUCK
PTOLEMY
FRESCO

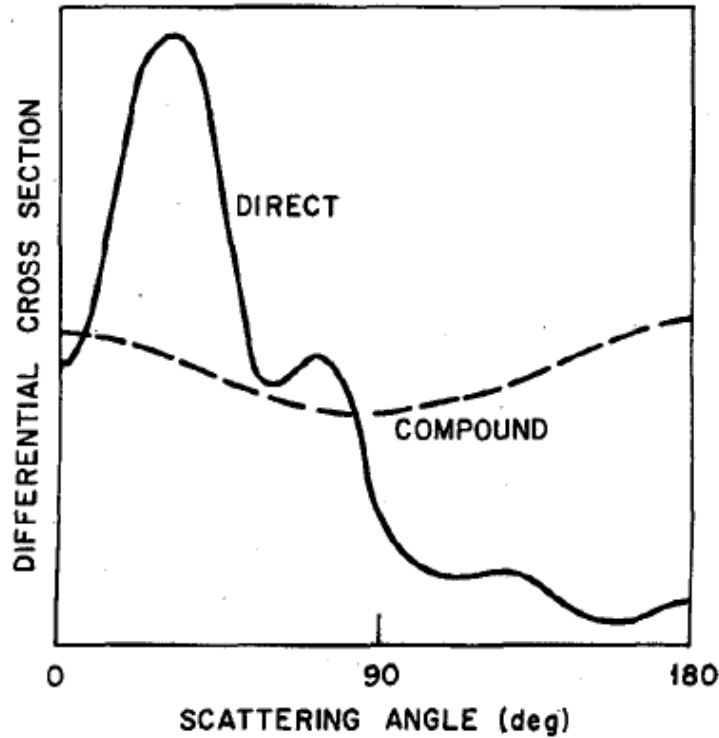
<http://www.fresco.org.uk/>

...

Angular distributions

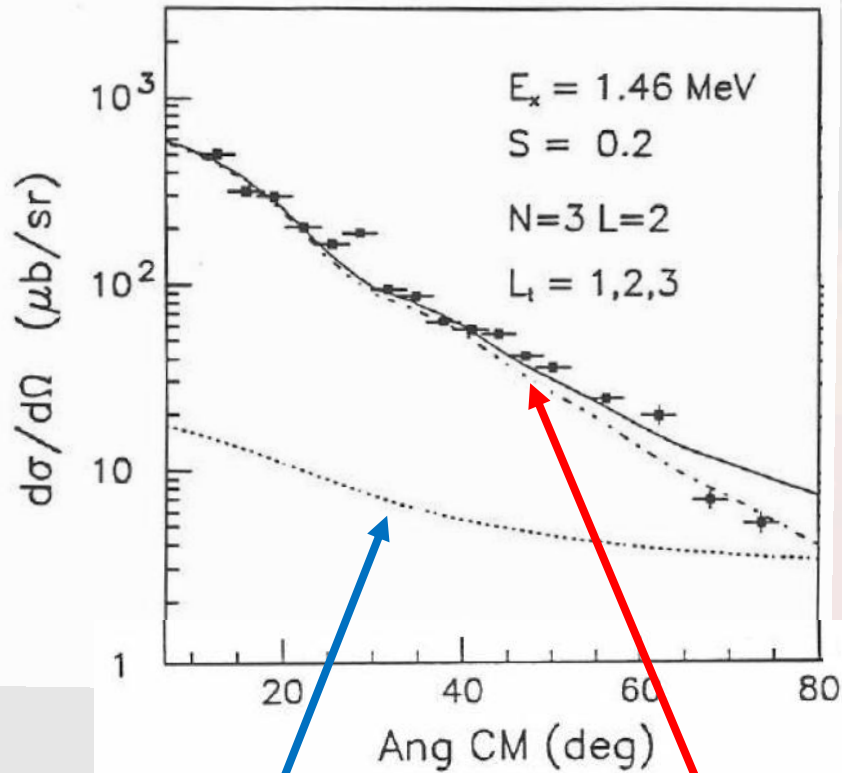


Compound nucleus, no preference for left or right, symmetric around $\theta = 90^\circ$.



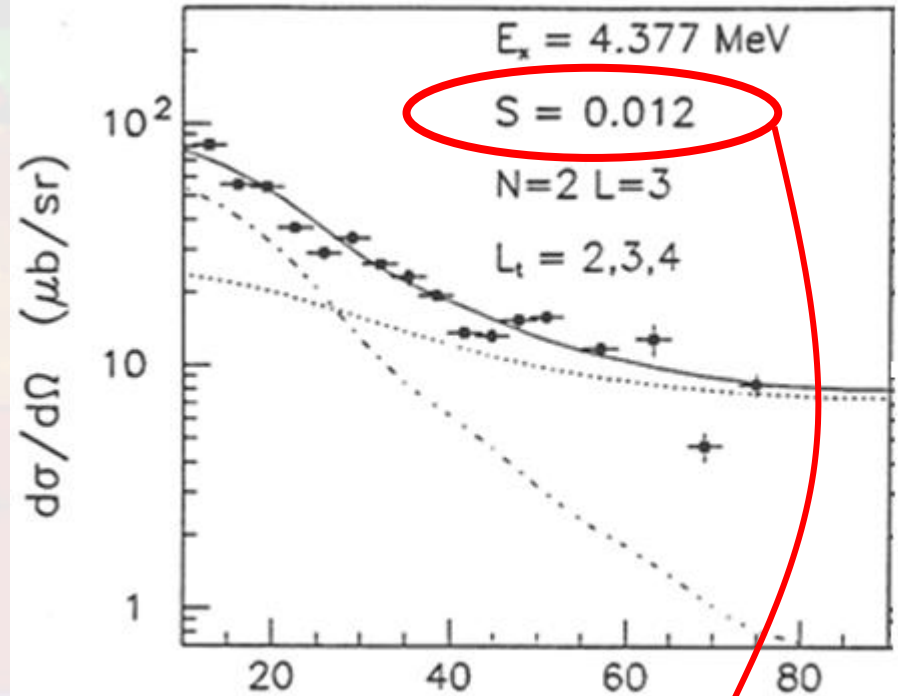
That is not the case for **direct reactions**. Often peaked at 0° .

Results



Hauser-Feshbach

DWBA



$$\Gamma_\alpha = (1.5^{+1.5}_{-0.8}) \times 10^{-9} \text{ eV}$$

(Orsay - 28 MeV)

Ref. 'Determination of alpha widths in ^{19}F relevant to fluorine nucleosynthesis'

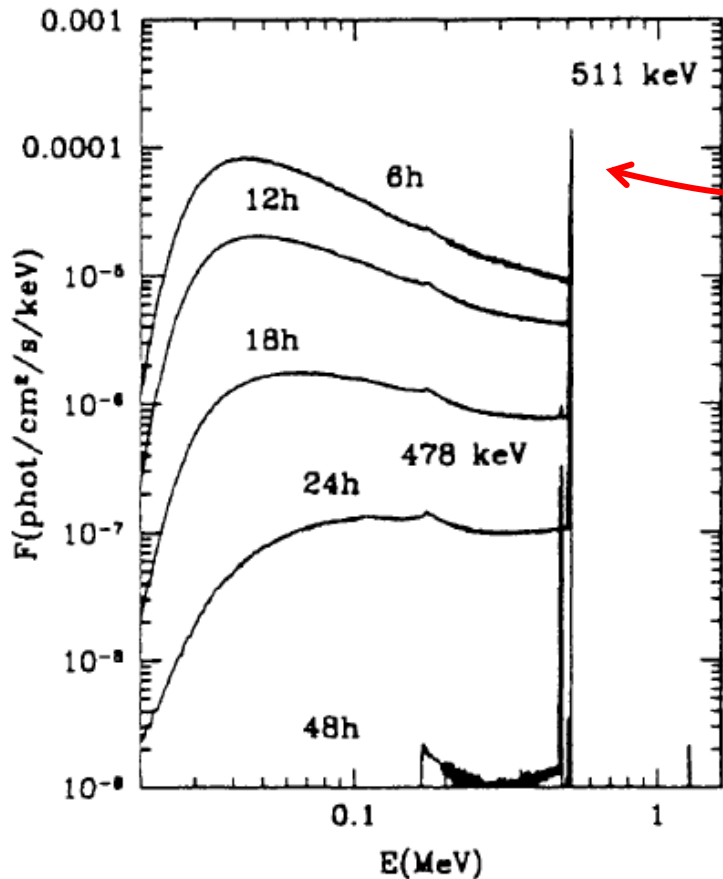
F. de Oliveira, A. Coc, et al.

Nuclear Physics A 597 (1996) 231-252

Example 8

Novae explosions

Predicted Gamma-ray flux

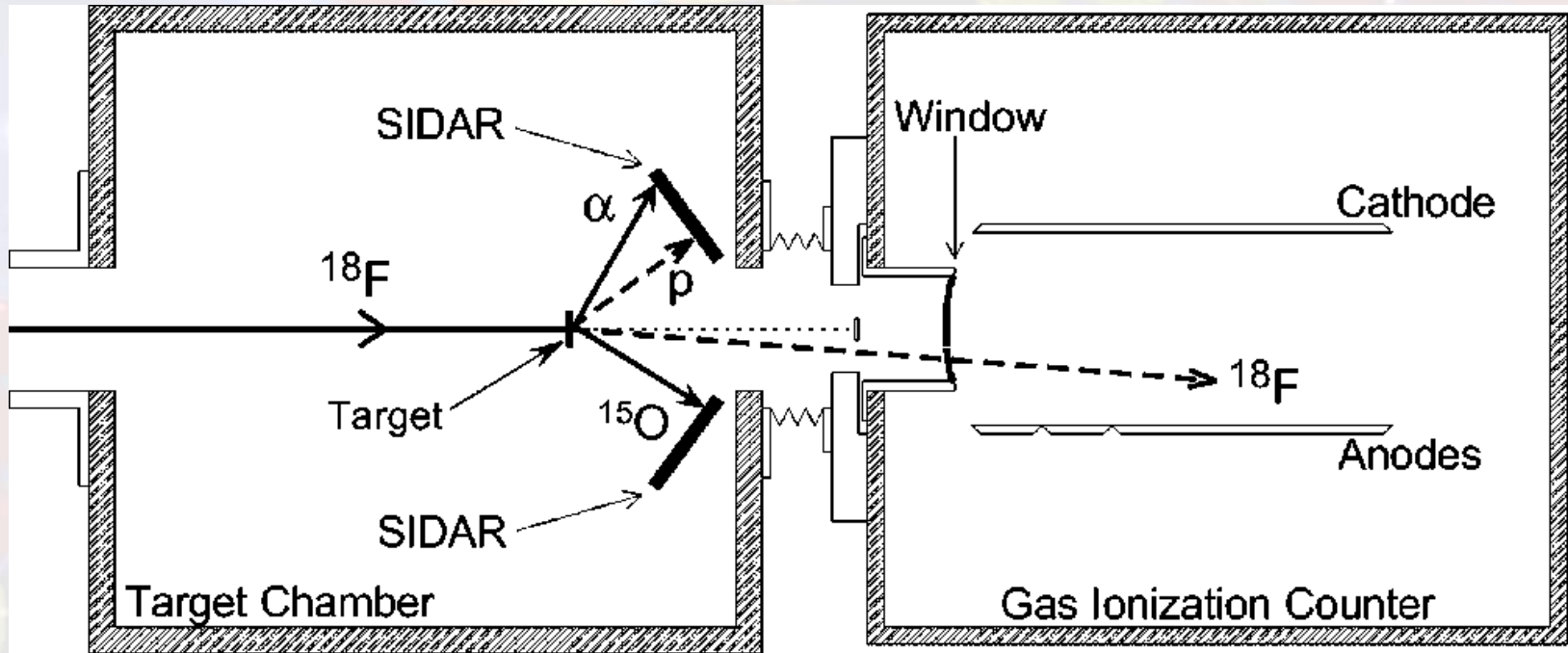


- Large Uncertainties remain, especially in $^{18}\text{F}(p,\alpha)^{15}\text{O}$
- Reaction rate is determined by resonant contributions from states in the compound nucleus ^{19}Ne

M. Hernanz, J. Jose, A. Coc, J. Gomez-Gomar, and J. Isern, *Astrophys. J.* 526, L97 (1999).

A. Coc, M. Hernanz, J. Jose, and J.-P. Thibaud, *Astron. Astrophys.* 357, 561 (2000).

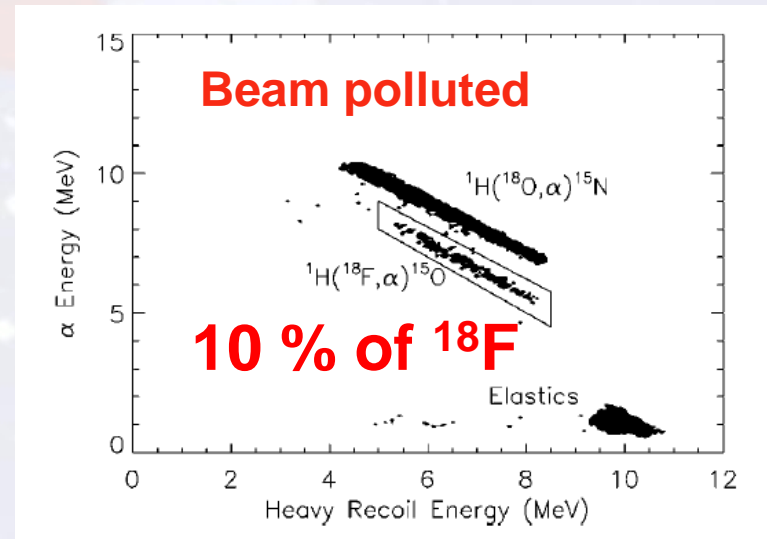
ORNL - Holifield Radioactive Ion Beam Facility



Hydrogen rich target

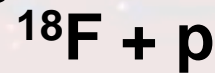
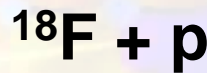


Using a thin target and changing the beam energy by small steps



Rutherford

Direct Coulomb Scattering

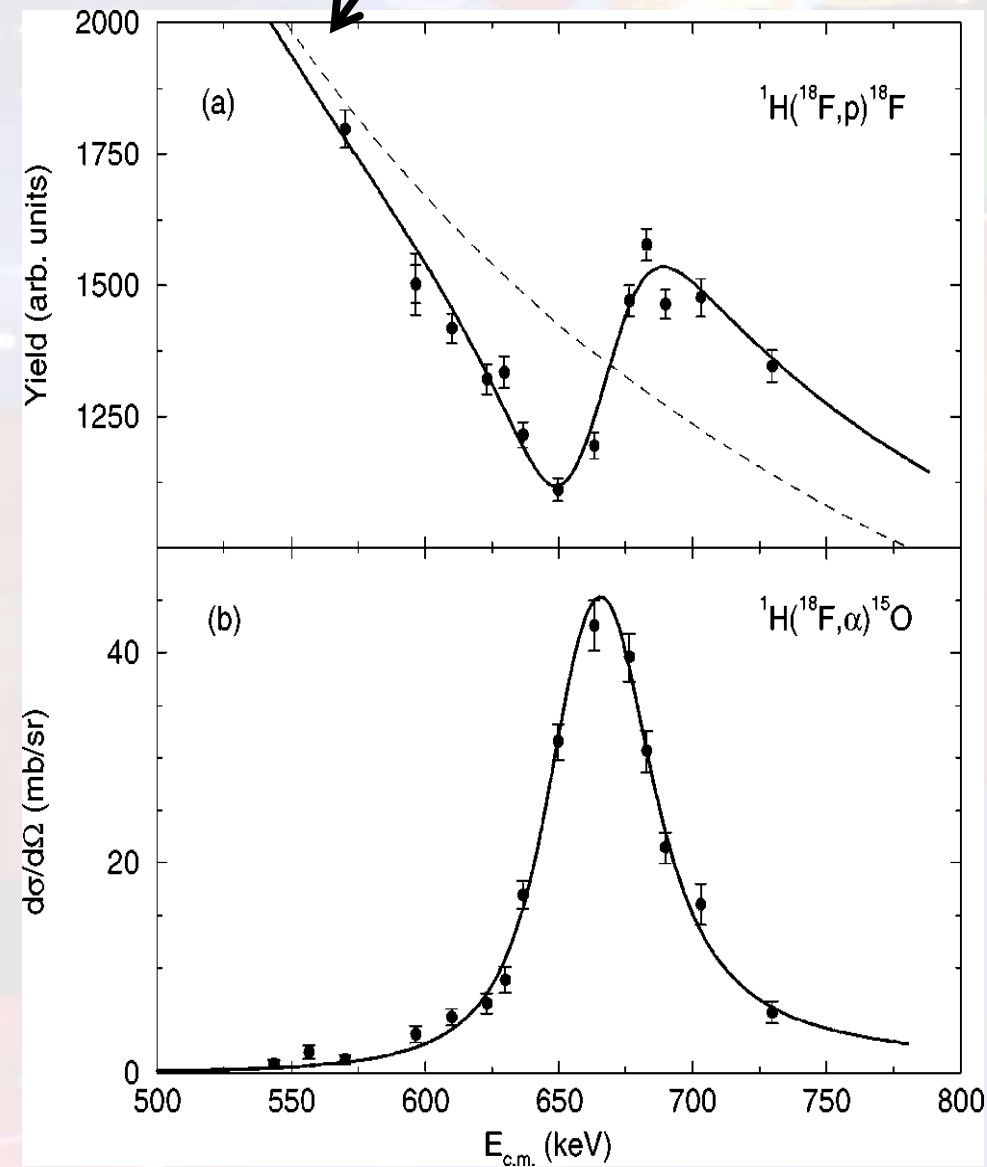


Compound Nucleus

Position $\Rightarrow E_x$

Width of the peak $\Rightarrow \Gamma$

Analysis of the spectra through the R-Matrix formalism



R-matrix formalism

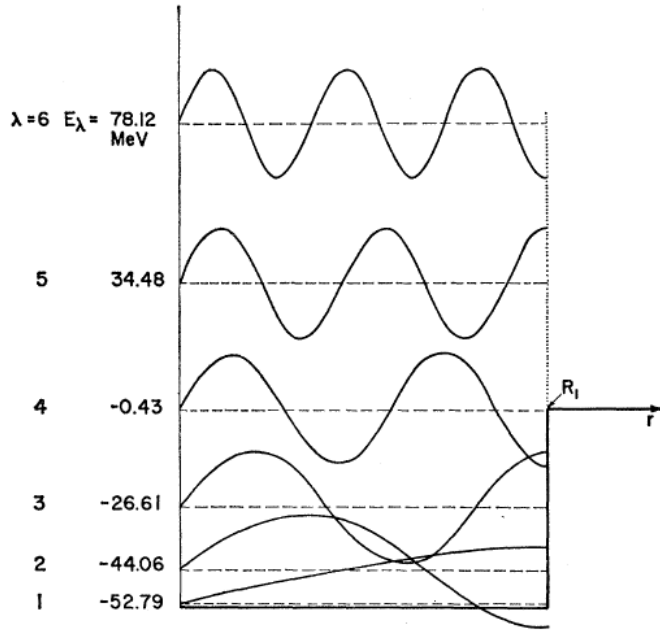


FIG. 2. The first six standing waves of the square well, constructed with a boundary condition number ($b = 0$) appropriate to low-energy scattering.

Read

[Vogt, Erich Rev. Mod. Phys. 34 p72](#)

Charity Eur. Phys. J. Plus (2016) 131:63

Because the X_λ form a complete orthonormal set of functions, the actual wave function ϕ may be expanded in terms of them

$$\phi = \sum_{\lambda} C_{\lambda} X_{\lambda}$$

The scattering of s-wave neutrons by a square well :

$$\sigma = \frac{\pi}{k^2} \left| \underbrace{2 \sin kR_1 e^{ikR_1}}_{\text{Scattering due to a hard sphere of radius } R_1} - \frac{\Gamma_{\lambda}}{(E_{\lambda} - E + \Delta_{\lambda}) - \frac{1}{2}i\Gamma_{\lambda}} \right|^2$$

where

$$\Gamma_{\lambda} \equiv 2kR_1\gamma_{\lambda}^2$$

$$\Delta \equiv b\gamma_{\lambda}^2$$

$$\gamma_{\lambda}^2 \equiv (\hbar^2/2mR_1)|X_{\lambda}(R_1)|^2 = \text{reduced width}$$

Scattering
due to a
hard sphere
of radius R_1

The multichannel R-matrix code AZURE

Only for Relatively low density of states

PHYSICAL REVIEW C **81**, 045805 (2010)

AZURE: An *R*-matrix code for nuclear astrophysics

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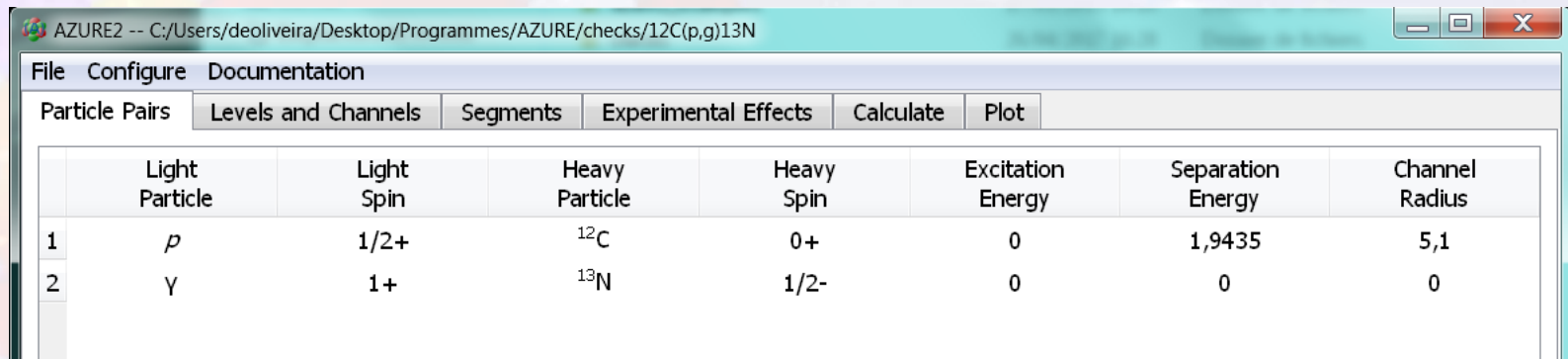
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AZURE2 -- C:/Users/deoliveira/Desktop/Programmes/AZURE/checks/12C(p,g)13N

File Configure Documentation

Particle Pairs Levels and Channels Segments Experimental Effects Calculate Plot

	Light Particle	Light Spin	Heavy Particle	Heavy Spin	Excitation Energy	Separation Energy	Channel Radius
1	p	1/2+	^{12}C	0+	0	1,9435	5,1
2	γ	1+	^{13}N	1/2-	0	0	0

<https://azure.nd.edu/downloads.php>

File Configure Documentation

- Particle Pairs
- Levels and Channels
- Segments
- Experimental Effects
- Calculate
- Plot

Compound Nucleus Levels

Include	Fix?	Level Spin	Energy [MeV]
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	1/2+	6,08
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	3/2+	7,07
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	1/2+	7,9

+ -

Channels In Selected Level

Fix?	Channel Pair	s	l
<input checked="" type="checkbox"/>	$^{18}\text{F} + p$ [0.000 MeV]	1/2	2
<input checked="" type="checkbox"/>	$^{18}\text{F} + p$ [0.000 MeV]	3/2	0
<input checked="" type="checkbox"/>	$^{18}\text{F} + p$ [0.000 MeV]	3/2	2
<input checked="" type="checkbox"/>	$^{15}\text{O} + \alpha$ [0.000 MeV]	1/2	1

Channel Configuration

- 8 Maximum Orbital Momentum
- 1 Maximum Gamma Multipolarity
- 1 Maximum Gamma Multipolarities Per Decay

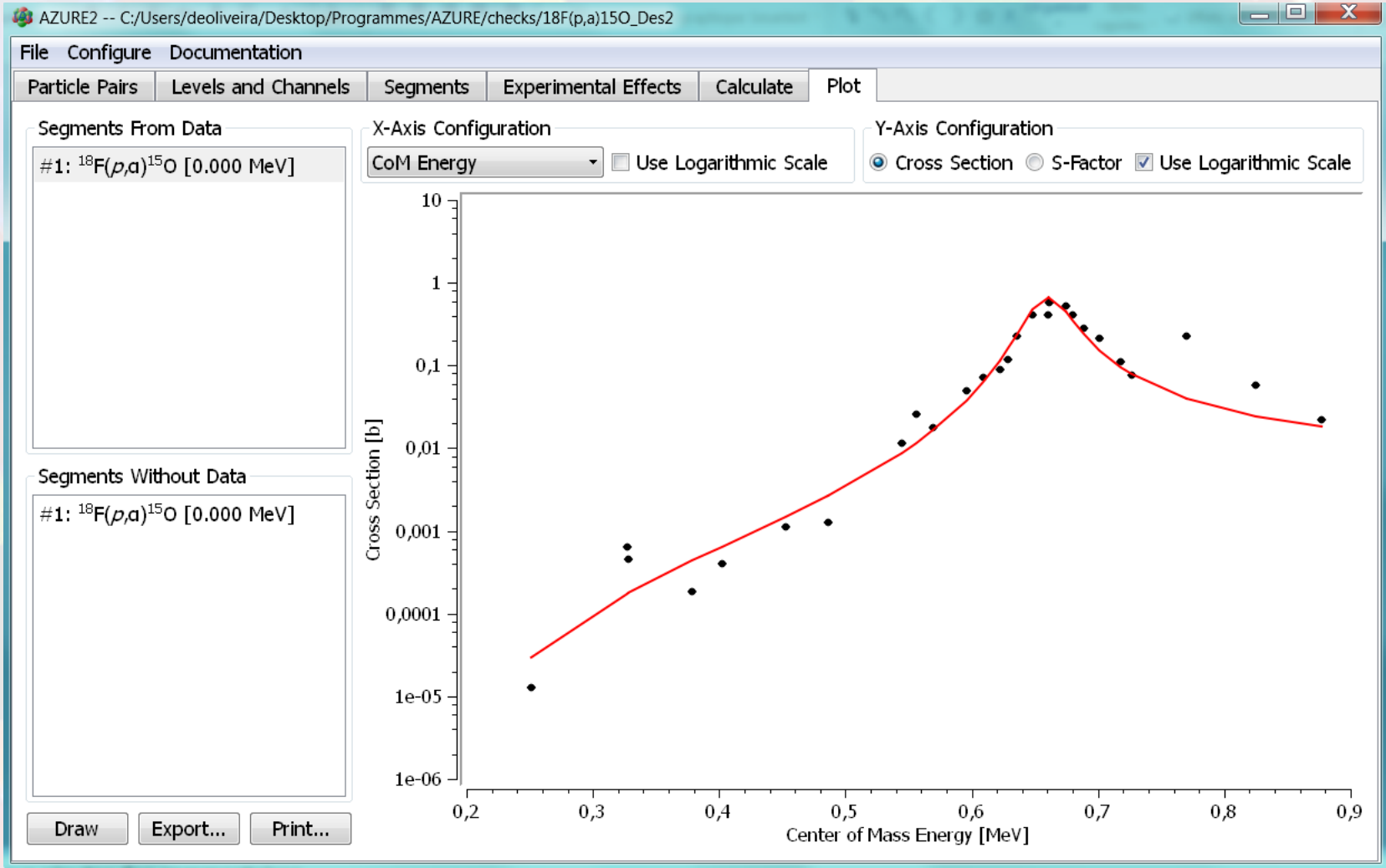
Channel Details (select from list to view):

7.07 MeV level with spin 3/2+
 transitioning via pair key #1
 Channel configuration is
 $s = 3/2, l = 0$

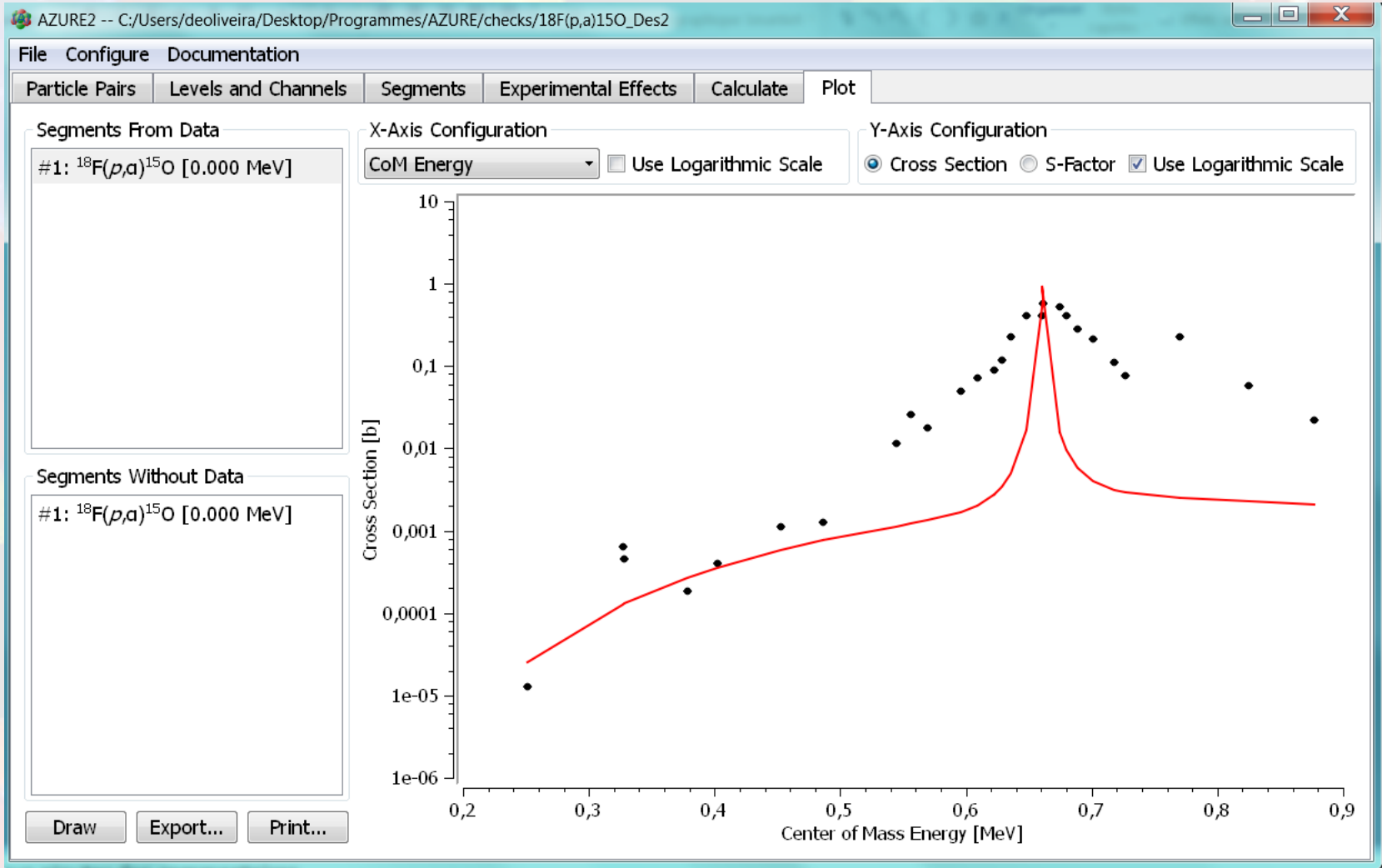
Light Particle Spin: 1/2+
 Light Particle Z: 1
 Light Particle M: 1
 Light Particle G: 0
 Heavy Particle Spin: 1+
 Heavy Particle Z: 9
 Heavy Particle M: 18
 Heavy Particle G: 0
 Excitation Energy: 0
 Separation Energy: 6.41
 Channel Radius: 5.1

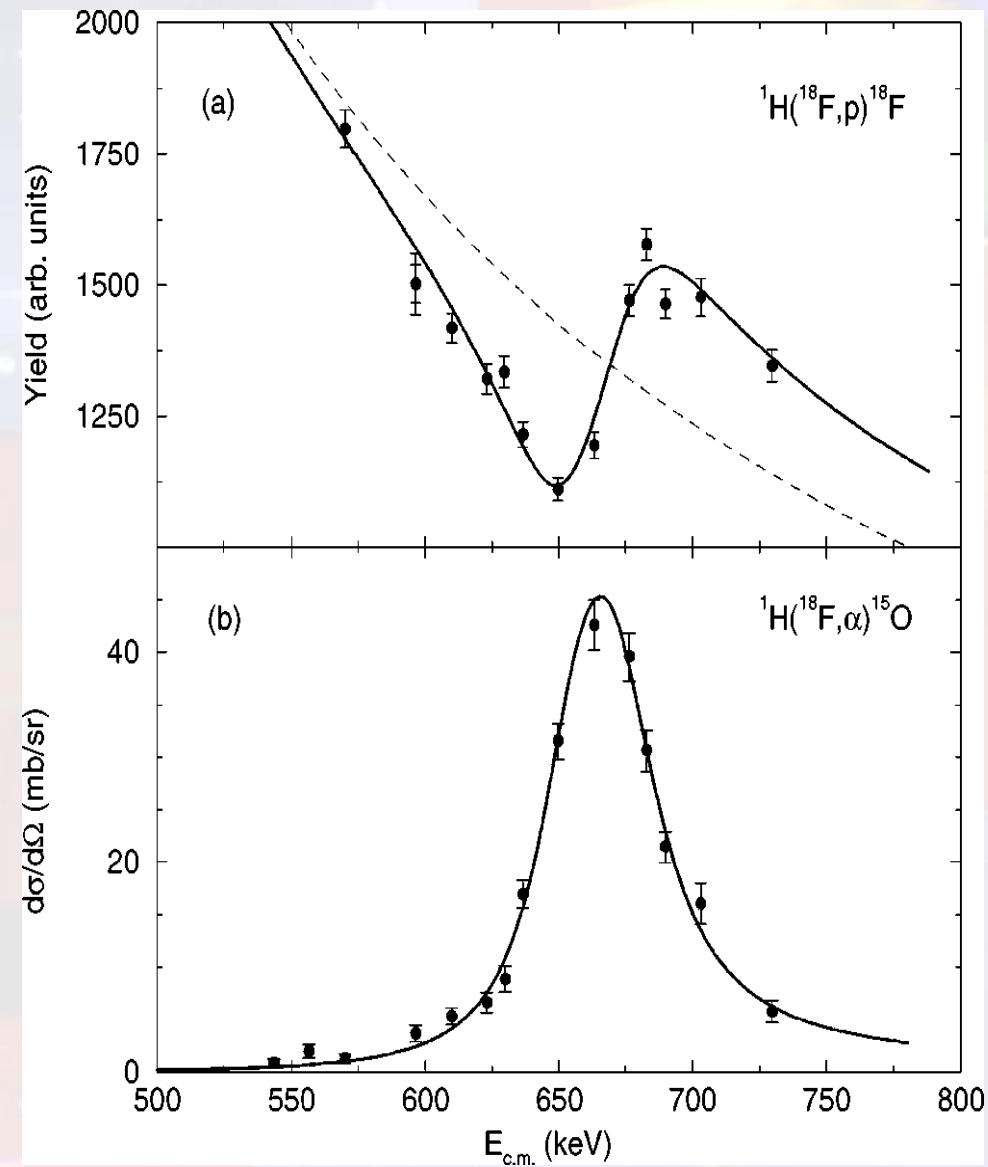
Partial Width: 15200 eV

$$J=3/2+$$



$$J=5/2+$$





$$E_x = 7076 \pm 2 \text{ keV}$$

$$l = 0 \quad 1/2^+ \text{ or } 3/2^+$$

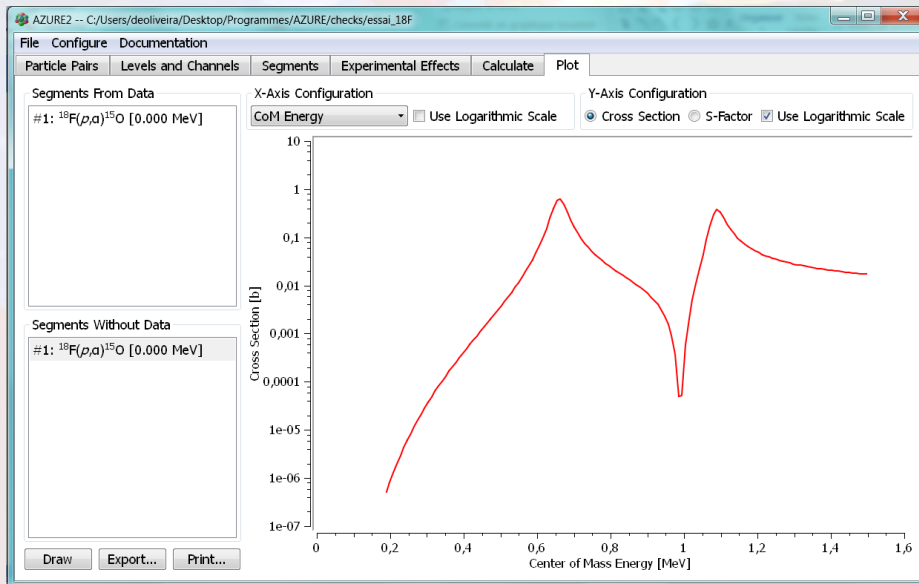
$$\Gamma = 39.0 \pm 1.6 \text{ keV}$$

$$\Gamma_p/\Gamma = 0.39 \pm 0.02$$

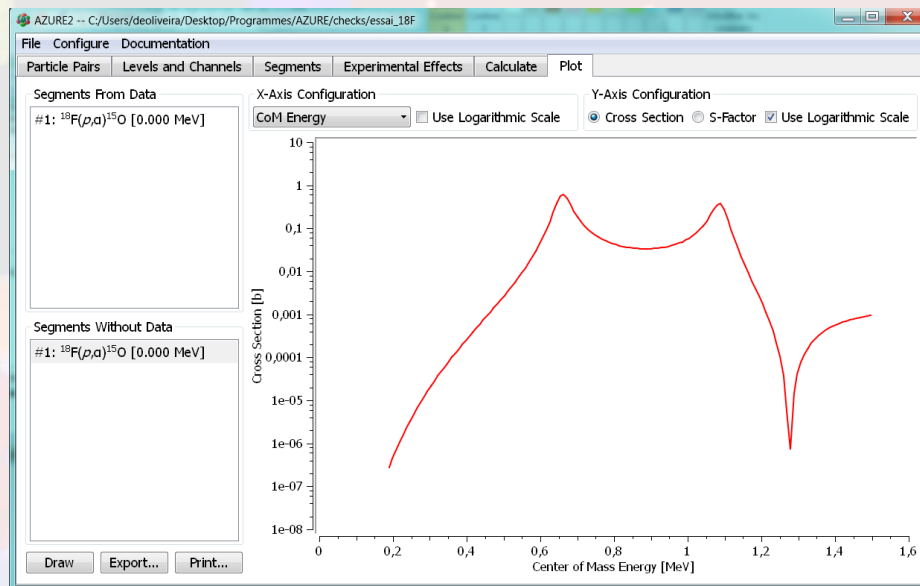


$$J^\pi = \frac{3}{2}^+$$

Interferences between two states



Two hypothetical
3/2+ states in
 ^{19}Ne interfering



The sign of
the width
matters

Subthreshold resonance

The screenshot shows the AZURE2 software interface with the following components:

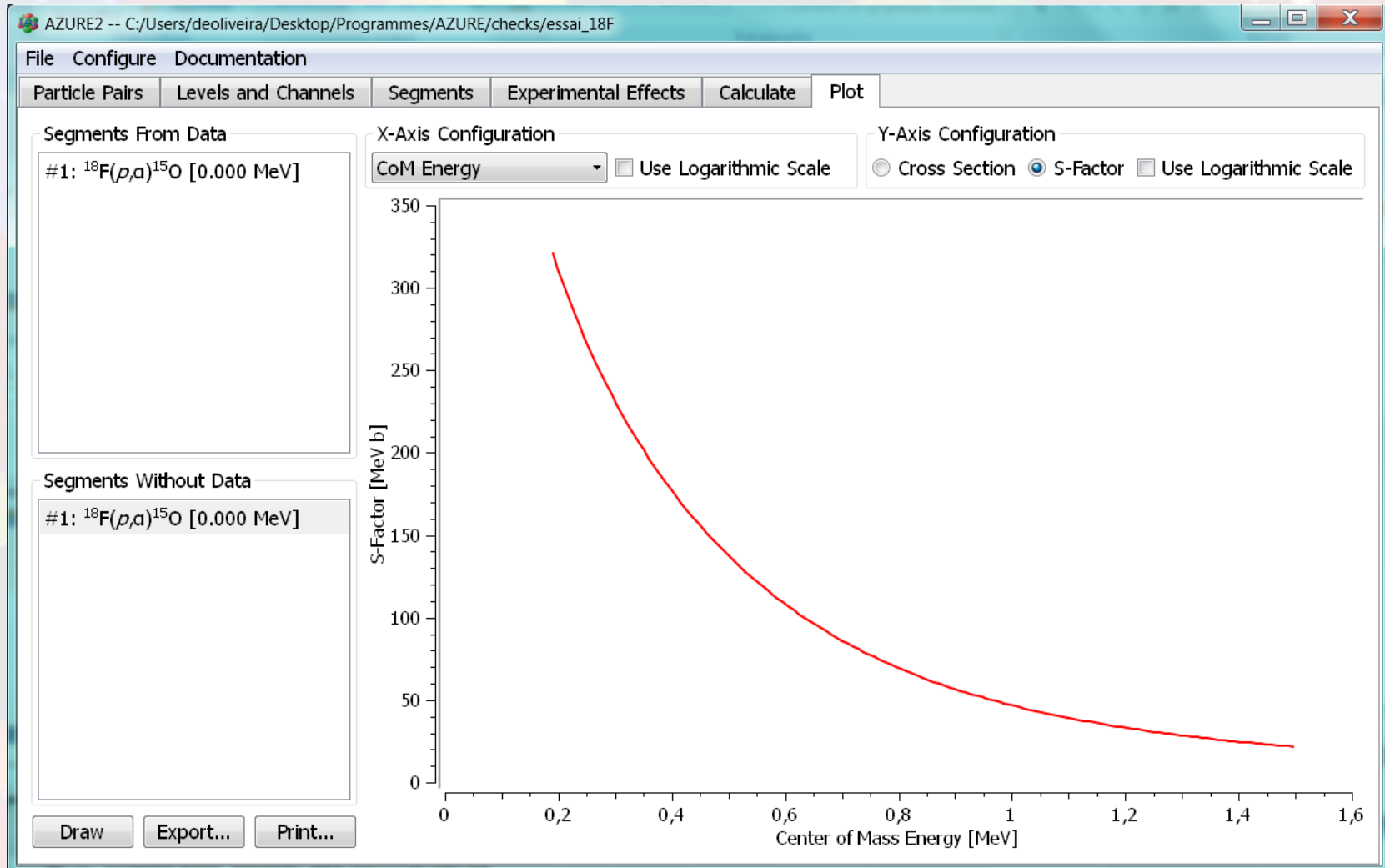
- Compound Nucleus Levels:** A table with columns 'Include', 'Fix?', 'Level Spin', and 'Energy [MeV]'. The first row (1/2+ at 6.08 MeV) is circled in red.
- Channels In Selected Level:** A table with columns 'Fix?', 'Channel Pair', 's', and 'l'. The first row (18F+p [0.000 MeV] with s=1/2, l=0) is highlighted in blue.
- Channel Configuration:** A panel with three spin values (8, 1, 1) and their corresponding labels: 'Maximum Orbital Momentum', 'Maximum Gamma Multipolarity', and 'Maximum Gamma Multiplicities Per Decay'.
- Channel Details:** A text area showing parameters for the 6.08 MeV level, including spin (1/2+), Z (1), M (1), G (0), separation energy (6.41 MeV), and channel radius (5.1 fm).
- ANC:** A field containing the value '25' followed by the unit 'fm^{-1/2}'.

A red arrow points from the circled level in the 'Compound Nucleus Levels' table to the text 'Sp = 6.410 MeV Subthreshold state' below the screenshot.

Sp = 6.410 MeV
Subthreshold state

Asymptotic normalization
coefficient

Subthreshold resonance

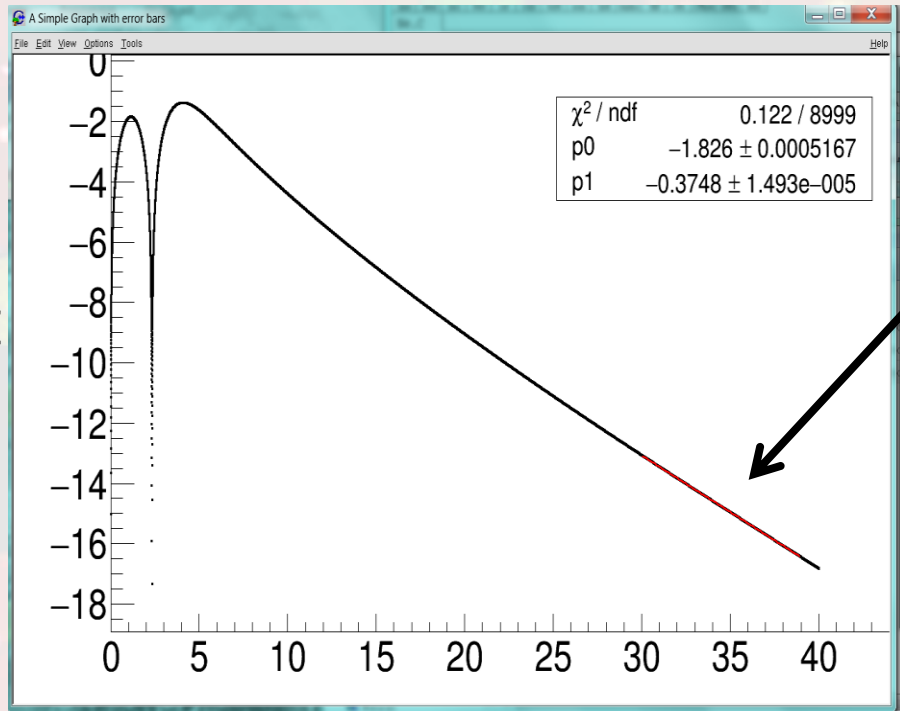


Subthreshold resonance

$$\text{ANC}^{\text{exp}} = S \times \text{ANC}^{\text{single particle}}$$

S spectroscopic factor

Wave function can be calculated with code dwu or RADCAP



$$u_\ell(r) = \text{ANC}^{\text{s.p.}} \times W_{-\eta, \ell+1/2}(-2k_p r)$$

Fitted with the
Whittaker function
for $r \rightarrow \infty$

Single-particle wave function r (fm)

Single-particle wave function $^{19}\text{Ne} = ^{18}\text{F} + p \quad 2s_{1/2}$

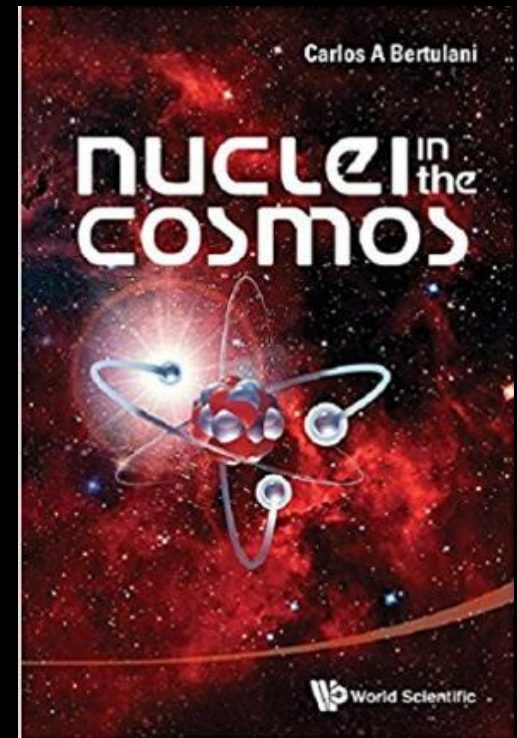
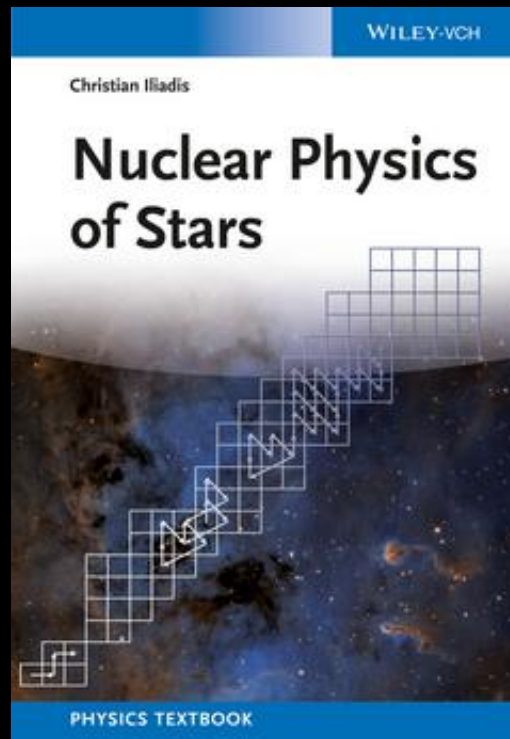
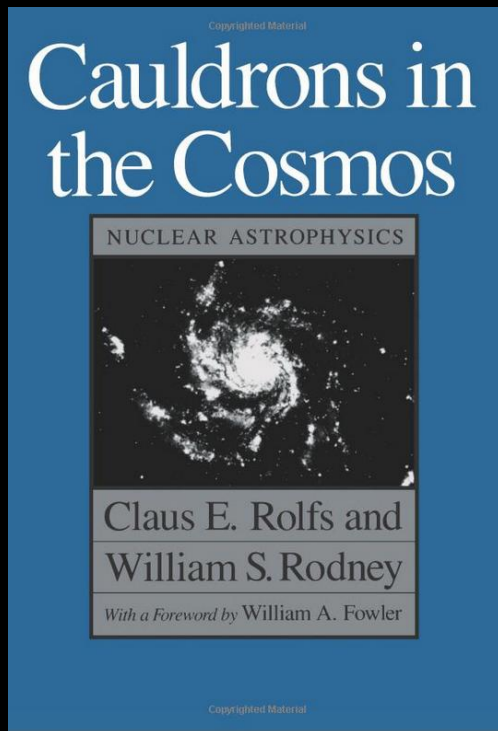
Conclusion

This was just a brief introduction!
There are many more aspects to discover

<http://cococubed.asu.edu/index.html>

<https://nucastro.org/Welcome.html>

<http://mesa.sourceforge.net/index.html>





The end

Thank you