Nuclear Astrophysics:
The origin of heavy elements
Lecture 1: Evolution massive stars

Gabriel Martínez Pinedo
GSI
TECHNISCHE UNIVERSITÄT DARMSTADT

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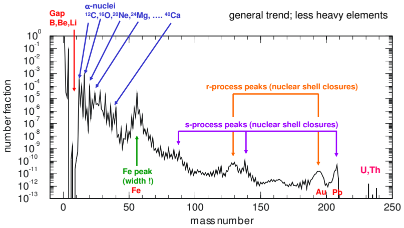
Outline

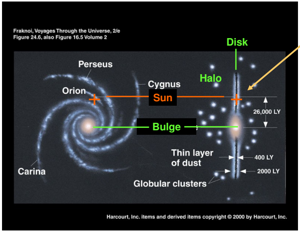
What is Nuclear Astrophysics?

- Nuclear astrophysics aims at understanding the nuclear processes that take place in the universe.
- These nuclear processes generate energy in stars and contribute to the nucleosynthesis of the elements and the evolution of the galaxy.

Hydrogen mass fraction	X = 0.71
Helium mass fraction	Y = 0.28
Metallicity (mass fraction of everything else)	Z = 0.019
Heavy Elements (beyond Nickel) mass fraction	4E-6



3. The solar abundance distribution

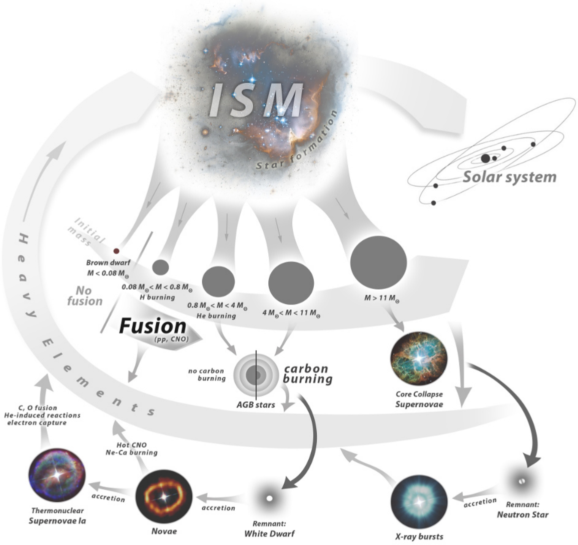


solar abundances:

Elemental (and isotopic) composition of Galaxy at location of solar system at the time of it's formation

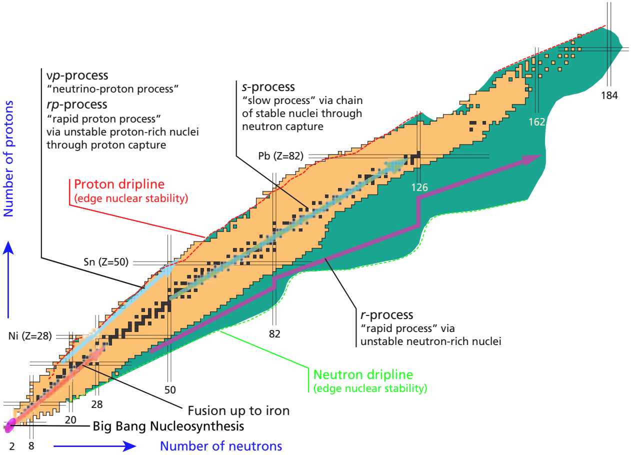
K. Lodders, Astrophys. J. **591**, 1220-1247 (2003)

Cosmic Cycle

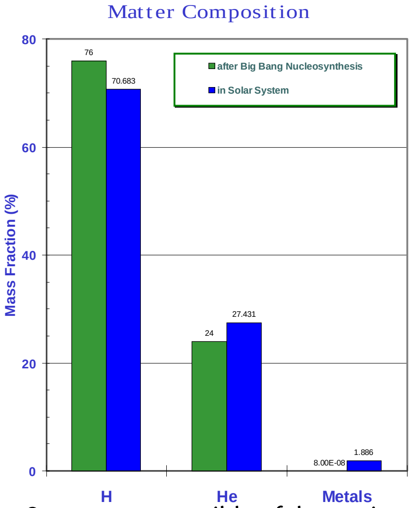


Nucleosynthesis processes

In 1957 Burbidge, Burbidge, Fowler and Hoyle and independently Cameron, suggested several nucleosynthesis processes to explain the origin of the elements.



Composition of the Universe after Big Bang

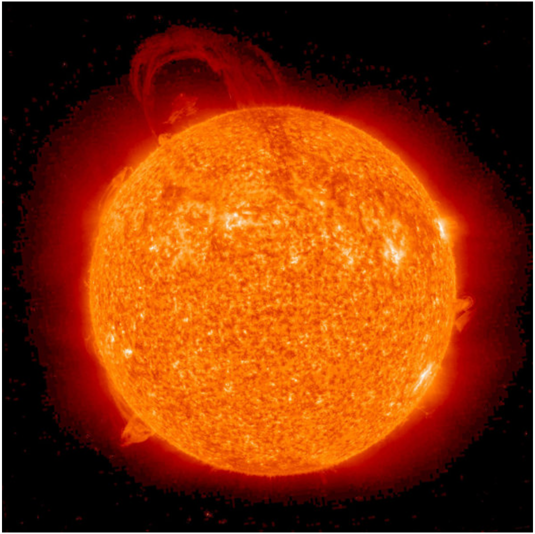


Stars are responsible of destroying Hydrogen and producing "metals". This requires weak interaction processes to convert protons into neutrons.

Nuclear Alchemy: How to make Gold in Nature?

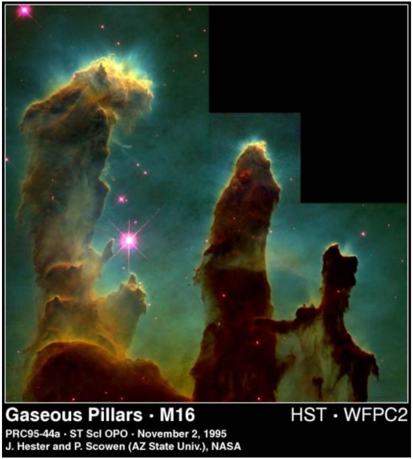
Pieter Bruegel (The Elder): The Alchemist

Nuclear Alchemy: How to make Gold in Nature?



Stars are the cauldrons where elements are synthesized

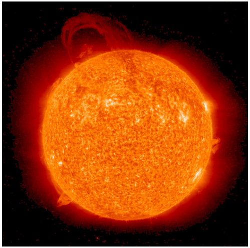
Star formation

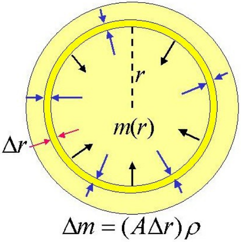


- Stars are formed from the contraction of molecular clouds due to their own gravity.
- Contraction increases temperature and eventually nuclear fusion reactions begin. A star is born.
- Contraction time depends on mass: 10 millions years for a star with the mass of the Sun; 100,000 years for a star 11 times the mass of the Sun.

The evolution of a Star is governed by gravity

What is a star?





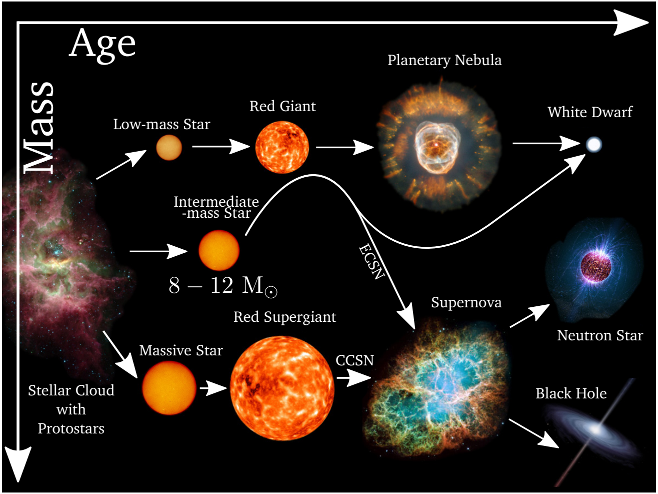
- A star is a self-luminous gaseous sphere.
- Stars produce energy by nuclear fusion reactions. A star is a self-regulated nuclear reactor.
- Gravitational collapse is balanced by pressure gradient: hydrostatic equilibrium.

$$dF_{\text{grav}} = -G\frac{mdm}{r^2} = [P(r+dr) - P(r)]dA = dF_{\text{pres}}$$
$$dm = 4\pi r^2 \rho dr$$
$$-G\frac{m\rho}{r^2} = \frac{dP}{dr}$$

- Further equations needed to describe the transport of energy from the core to the surface, and the change of composition (nuclear reactions). Supplemented by an EoS: $P(\rho, T)$.

Stellar Evolution

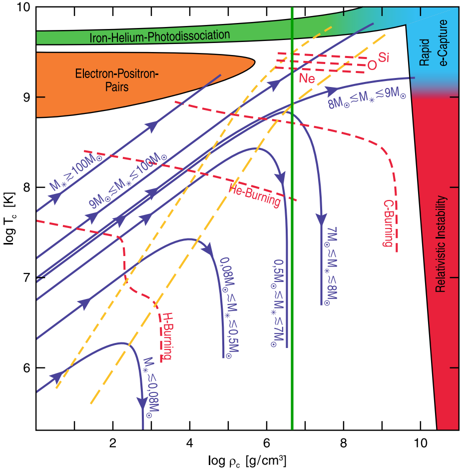
Star evolution, lifetime and death depends on mass?



Why such distinct outcomes?

Core evolution

Hydrostatic equilibrium together with equation of state determines the evolution of the star core.



Green: transition from relativistic to non-relativistic electrons.

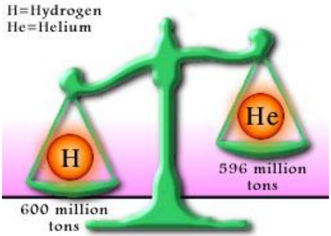
from H.-T. Janka, Annu. Rev. Nucl. Part. Sci. **62**, 407 (2012).

Where does the energy come from?

Energy comes from nuclear reactions in the core.

$$4\,^{1}\mathrm{H} \rightarrow \,^{4}\mathrm{He} + 2e^{+} + 2\nu_e + 26.7\ \mathrm{MeV}$$

$$E = mc^2$$

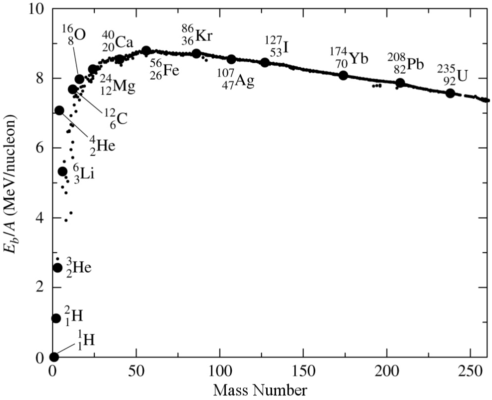


The Sun converts 600 million tons of hydrogen into 596 million tons of helium every second. The difference in mass is converted into energy. The Sun will continue burning hydrogen during 5 billions years.

Energy released by H-burning:
6.45×10^{18} erg g^{-1} $= 6.7$ MeV/nuc
Solar Luminosity: 3.85×10^{33} erg s^{-1}

Nuclear Binding Energy

Liberated energy is due to the gain in nuclear binding energy.



- Rest mass energy is converted into kinetic energy of the particles. This translates into heat that raises the temperature unless energy transported away.
- During hydrostatic burning phases stars maintain an equilibrium: produced energy is transported and later emitted at the surface.

Types of reactions

Nuclei in the astrophysical environment can suffer different reactions:

- Decay

$$^{56}\mathrm{Ni} \rightarrow {}^{56}\mathrm{Co} + e^{+} + \nu_e$$
$$^{15}\mathrm{O} + \gamma \rightarrow {}^{14}\mathrm{N} + p$$

$$\frac{dn_a}{dt} = -\lambda_a n_a$$

In order to dissentangle changes in the density (hydrodynamics) from changes in the composition (nuclear dynamics), the abundance is introduced:

$$Y_a = \frac{n_a}{n}, \quad n \approx \frac{\rho}{m_u} = \text{Number density of nucleons (constant)}$$

$$\frac{dY_a}{dt} = -\lambda_a Y_a$$

Rate can depend on temperature and density

Types of reactions

Nuclei in the astrophysical environment can suffer different reactions:

- Capture processes

$$a + b \to c + d$$

$$\frac{dn_a}{dt} = -n_a n_b \langle \sigma v \rangle$$

$$\frac{dY_a}{dt} = -\frac{\rho}{m_u} Y_a Y_b \langle \sigma v \rangle$$

decay rate: $\lambda_a = \rho Y_b \langle \sigma v \rangle / m_u$

- 3-body reactions:

$$3\,^4\mathrm{He} \to {}^{12}\mathrm{C} + \gamma$$

$$\frac{dY_\alpha}{dt} = -\frac{\rho^2}{2m_u^2} Y_\alpha^3 \langle \alpha\alpha\alpha \rangle$$

decay rate: $\lambda_\alpha = Y_\alpha^2 \rho^2 \langle \alpha\alpha\alpha \rangle / (2m_u^2)$

Reaction rates

Consider n_a and n_b particles per cubic centimeter of species a and b. The rate of nuclear reactions

$$a + b \rightarrow c + d$$

is given by:

$$r_{ab} = n_a n_b \sigma(v) v, \quad v = \text{relative velocity}$$

In stellar environment the velocity (energy) of particles follows a thermal distribution that depends of the type of particles.

- Nuclei (Maxwell-Boltzmann)

$$N(v)dv = N 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) dv = N\phi(v)dv$$

- Electrons, Neutrinos (if thermal) (Fermi)

$$N(p)dp = \frac{g}{(2\pi\hbar)^3} \frac{4\pi p^2}{e^{(E(p)-\mu)/kT} + 1} dp$$

- photons (Bose)

$$N(p)dp = \frac{2}{(2\pi\hbar)^3} \frac{4\pi p^2}{e^{pc/kT} - 1} dp$$

Stellar reaction rate

The product σv has to be averaged over the velocity distribution $\phi(v)$ (Maxwell-Boltzmann)

$$\langle \sigma v \rangle = \int_0^\infty \phi(v)\sigma(v) v dv$$

that gives:

$$\langle \sigma v \rangle = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 \sigma(v) \exp\left(-\frac{mv^2}{2kT} \right) dv, \quad m = \frac{m_1 m_2}{m_1 + m_2}$$

or using $E = mv^2/2$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT} \right) dE$$

Inverse reactions

Let's have the reaction

$$a + b \to c + \gamma \quad Q = m_a + m_b - m_c$$

We are interested in the inverse reaction. One can use detailed-balance to determine the inverse rate. Simpler using the concept of chemical equilibrium.

$$\frac{dN_{ab}}{dt} = \frac{dN_c}{dt} = 0$$

$$N_a N_b \langle \sigma v \rangle = N_c \lambda_c \qquad \frac{N_a N_b}{N_c} = \frac{\lambda_c}{\langle \sigma v \rangle}$$

Using equilibrium condition for chemical potentials: $\mu_a + \mu_b = \mu_c$

$$\mu(Z, A) = m(Z, A)c^2 + kT \ln\left[\frac{n(Z, A)}{G(Z, A)} \left(\frac{2\pi\hbar^2}{m(Z, A)kT}\right)^{3/2}\right]$$

Inverse reactions

One obtains:

$$\frac{N_a N_b}{N_c} = \frac{g_a g_b}{g_c}\left(\frac{m_a m_b}{m_c}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} e^{-Q/kT}$$

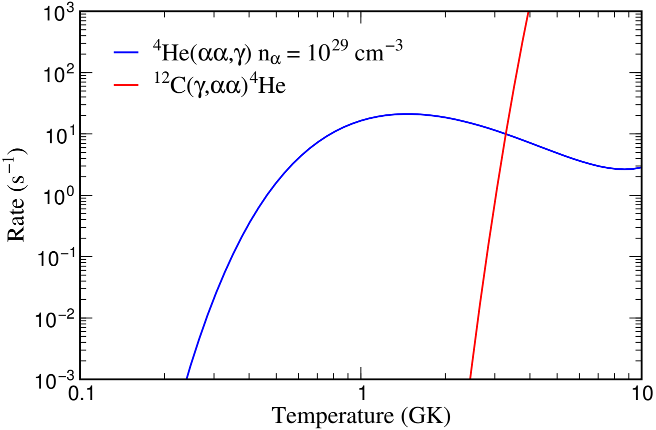
Finally, we obtain:

$$\lambda_c = \frac{g_a g_b}{g_c}\left(\frac{m_a m_b}{m_c}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} e^{-Q/kT}\langle\sigma v\rangle$$

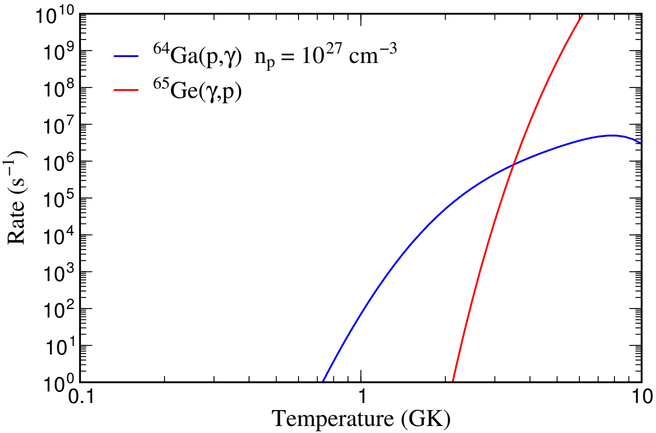
For a reaction $a + b \to c + d$ ($Q = m_a + m_b - m_c - m_d$):

$$\langle\sigma v\rangle_{cd} = \frac{g_a g_b}{g_c g_d}\left(\frac{\mu_{ab}}{\mu_{cd}}\right)^{3/2} e^{-Q/kT}\langle\sigma v\rangle_{ab}$$

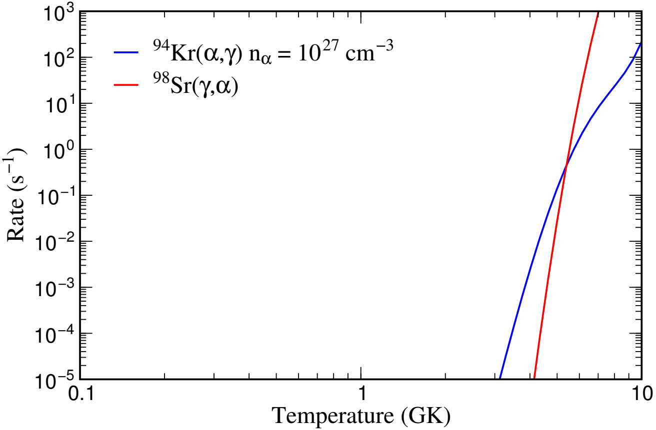
Rate Examples: ^{4}He$(\alpha\alpha, \gamma)$



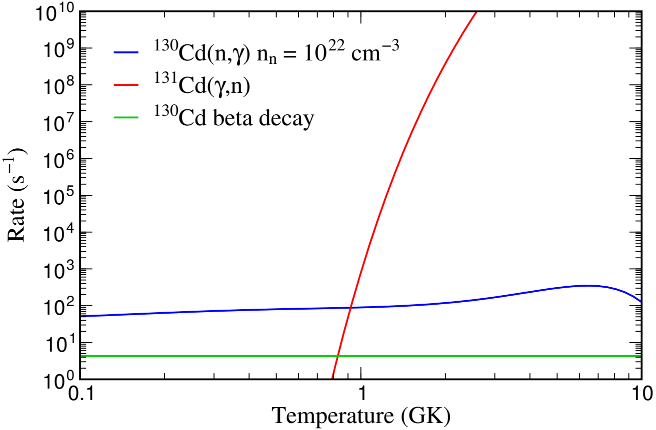
Rate Examples: (p, γ)



Rate Examples: (α, γ)



Rate examples: (n, γ)



Summary (reaction rates)

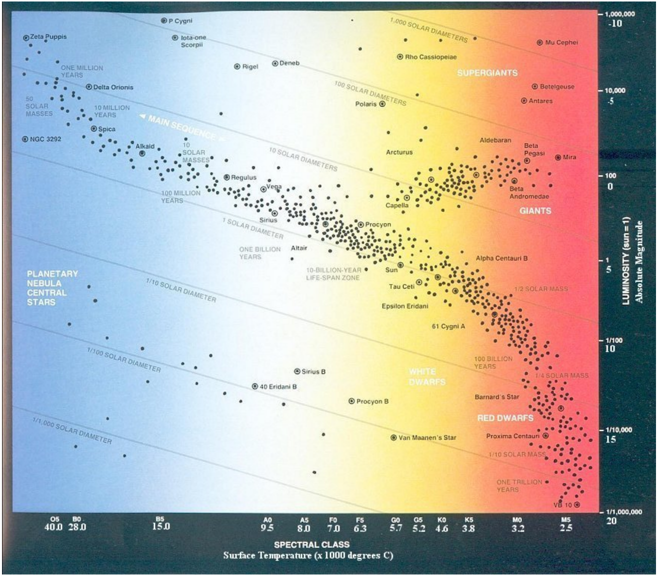
General features reaction rates:

- Reactions involving neutral particles, neutrons, are almost independent of the temperature of the environment.
- Charged particles reactions depend very strongly on temperature (tunneling coulomb barrier).
- At high temperatures inverse reactions (γ, n), (γ, p), (γ, α) become important, $n_\gamma \propto T^3$

We can distinguish two different regimes:

- Nuclear reactions are **slower** than dynamical time scales (expansion, contraction,...) of the system: Hydrodynamical burning phases.
- Nuclear reactions are **faster** than dynamical time scales: Explosive Nucleosynthesis.

Hertzspung-Russell diagram



Hydrogen burning: ppI-chain

Step 1: $p + p \rightarrow {}^2\mathrm{He}$ (not possible)

$p + p \rightarrow d + e^+ + \nu_e$

Step 2: $d + p \rightarrow {}^3\mathrm{He}$

$d + d \rightarrow {}^4\mathrm{He}$ (d abundance too low)

Step 3: ${}^3\mathrm{He} + p \rightarrow {}^4\mathrm{Li}$ (^{4}Li is unbound)

${}^3\mathrm{He} + d \rightarrow {}^4\mathrm{He} + n$ (d abundance too low)

${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p$

$d + d$ not going because Y_d is small and $d + p$ leads to rapid destruction.
${}^3\mathrm{He} + {}^3\mathrm{He}$ goes because $Y_{3\mathrm{He}}$ gets large as nothing destroys it.

The relevant S-factors

$p(p, e^+\nu_e)d$: $S_{11}(0) = (4.00 \pm 0.05) \times 10^{25}$ MeV b
calculated

$p(d,\gamma)^3\mathrm{He}$: $S_{12}(0) = 2.5 \times 10^{-7}$ MeV b
measured at LUNA

$^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$: $S_{33}(0) = 5.4$ MeV b
measured at LUNA

Laboratory Underground for Nuclear Astrophysics (Gran Sasso).

Reaction Network ppI-chain

$$\frac{dY_p}{dt} = -Y_p^2 \frac{\rho}{m_u} \langle \sigma v \rangle_{pp} - Y_d Y_p \frac{\rho}{m_u} \langle \sigma v \rangle_{pd} + Y_3^2 \frac{\rho}{m_u} \langle \sigma v \rangle_{33}$$

$$\frac{dY_d}{dt} = \frac{Y_p^2}{2} \frac{\rho}{m_u} \langle \sigma v \rangle_{pp} - Y_d Y_p \frac{\rho}{m_u} \langle \sigma v \rangle_{pd}$$

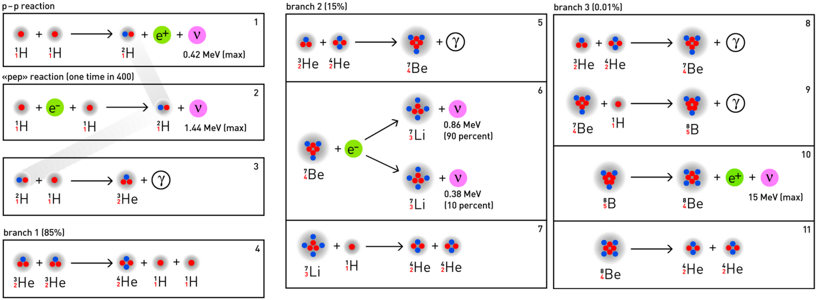
$$\frac{dY_3}{dt} = Y_d Y_p \frac{\rho}{m_u} \langle \sigma v \rangle_{pd} - Y_3^2 \frac{\rho}{m_u} \langle \sigma v \rangle_{33}$$

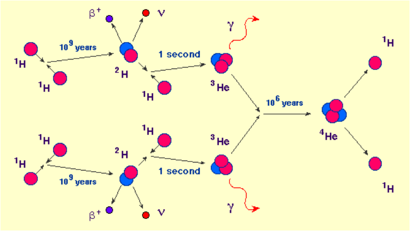
$$\frac{dY_4}{dt} = \frac{Y_3^2}{2} \frac{\rho}{m_u} \langle \sigma v \rangle_{33}$$

Stiff system of coupled differential equations.

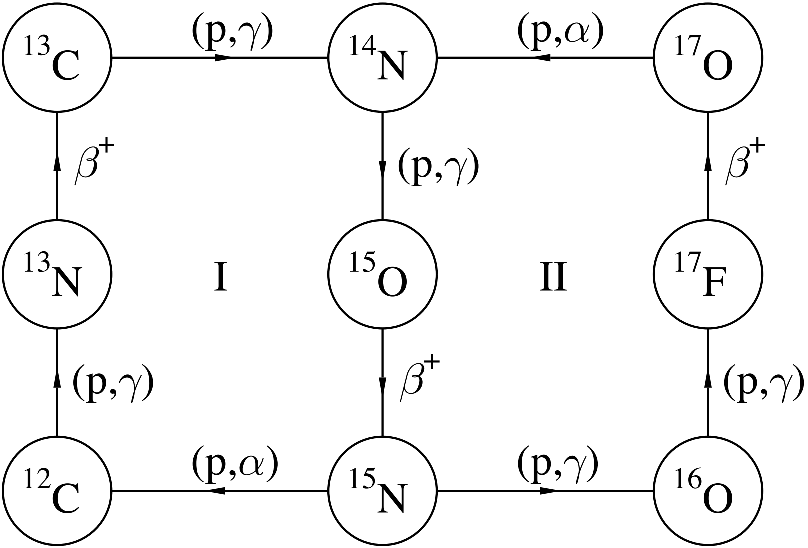
pp chains

Once ^{4}He is produced can act as catalyst initializing the ppII and ppIII chains.



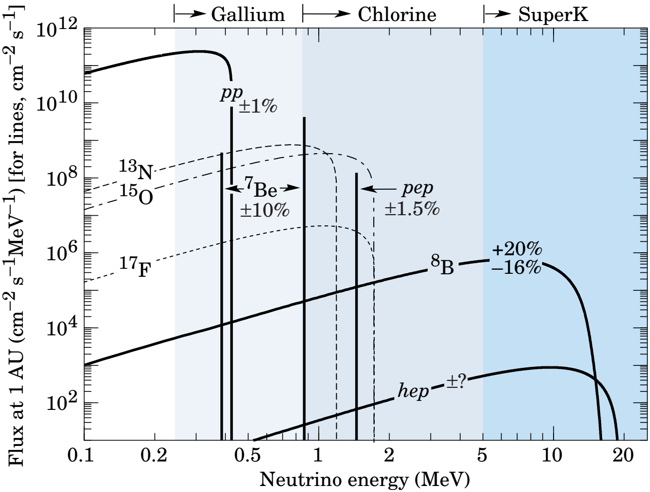


The other hydrogen burning: CNO bicycle

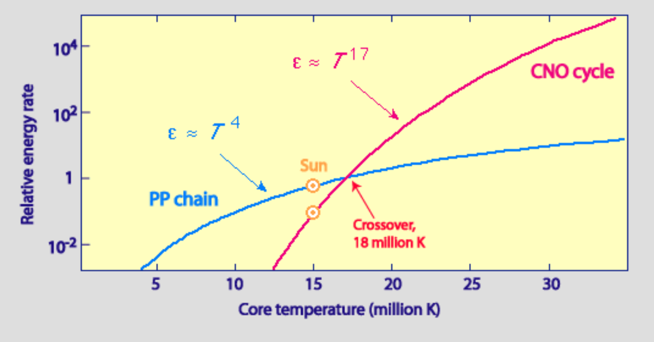


Neutrino spectrum (Sun)

This is the predicted neutrino spectrum



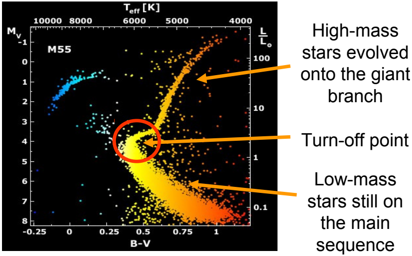
Energy generation: CNO cycle vs pp-chains



Consequences

- Stars slightly heavier than the Sun burn hydrogen via CNO cycle.
- CNO cycle goes significantly faster. Such stars have much shorter lifetimes

Mass ($M_\odot$)	lifetime (yr)
0.8	1.4×10^{10}
1.0	1×10^{10}
1.7	2.7×10^{9}
3.0	2.2×10^{8}
5.0	6×10^{7}
9.0	2×10^{7}
16.0	1×10^{7}
25.0	7×10^{6}
40.0	1×10^{6}



Helium Burning

- Once hydrogen is exhausted the stellar core is made mainly of helium. Hydrogen burning continues in a shell surrounding the core.
- $^4\mathrm{He} + p$ produces $^5\mathrm{Li}$ that decays in 10^{-22} s.
- Helium survives in the core till the temperature become large enough ($T \approx 10^8$ K) to overcome the coulomb barrier for $^4\mathrm{He} + {}^4\mathrm{He}$. The produced $^8\mathrm{Be}$ decays in 10^{-16} s. However, the lifetime is large enough to allow the capture of another $^4\mathrm{He}$:

$$3\,{}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$$

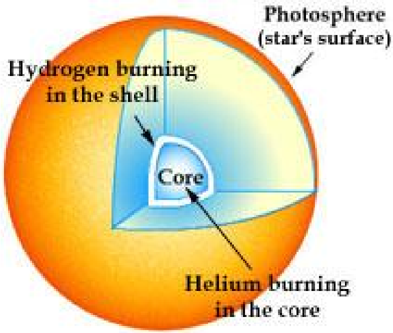
- Hoyle suggested that in order to account for the large abundance of Carbon and Oxygen, there should be a resonance in $^{12}\mathrm{C}$ that speeds up the production.
- $^{12}\mathrm{C}$ can react with another $^4\mathrm{He}$ producing $^{16}\mathrm{O}$

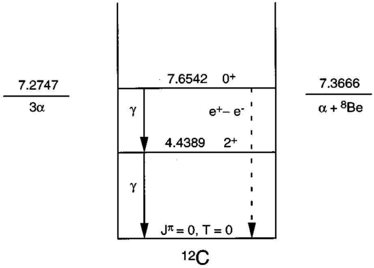
$$^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O} + \gamma$$

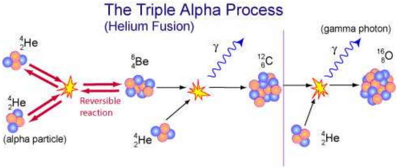
- These are the two main reactions during helium burning.

Hoyle State and tripple α reaction

Red giant structure







End of helium burning

Nucleosynthesis yields from stars depend on mass:

stars with $M \lesssim 8\ \mathrm{M}_\odot$ These stars eject their envelopes during helium shell burning producing planetary nebula and white dwarfs. Constitute the site for the s process.

stars with $M \gtrsim 12\ \mathrm{M}_\odot$ These stars will ignite carbon burning under non-degenerate conditions. The subsequent evolution proceeds in most cases to core collapse. These stars make the bulk of newly processed matter that is returned to the interstellar medium.

stars with $8\ \mathrm{M}_\odot \lesssim M \lesssim 12\ \mathrm{M}_\odot$ The end products of these stars is still under discussion and may depend on metallicity. Potentially they can produce ONe white-dwarfs (planetary nebula), thermonuclear supernova, and/or electron capture supernova.

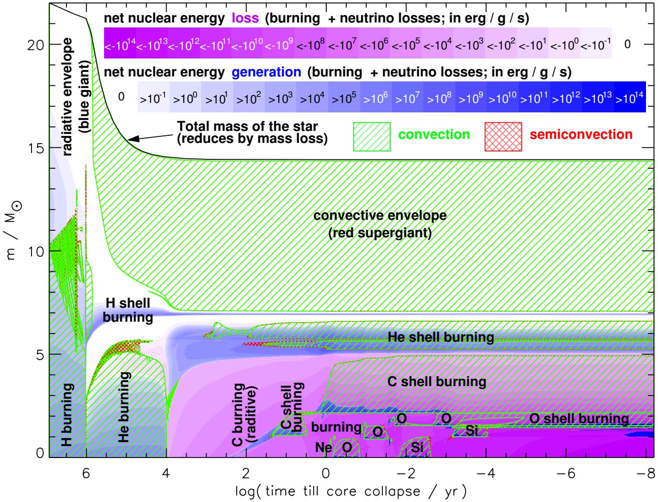
Stellar life

Nuclear burning stages

(e.g., 20 solar mass star)

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	$4\,H \xrightarrow{CNO} {}^4He$
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$
C	Ne, Mg	Na	0.8	10^3	$^{12}C + {}^{12}C$
Ne	O, Mg	Al, P	1.5	3	$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	$^{16}O + {}^{16}O$
Si	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	$^{28}Si(\gamma,\alpha)\ldots$

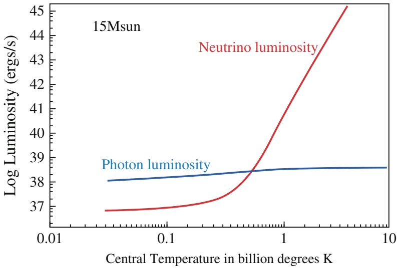
Energy generation vs energy loss



22 $M_\odot$ evolution, from Heger and Woosley 2001.

Photon vs neutrino luminosities

Neutrino luminosity is larger than photon luminosity. Weak interaction processes start to determine the evolution.



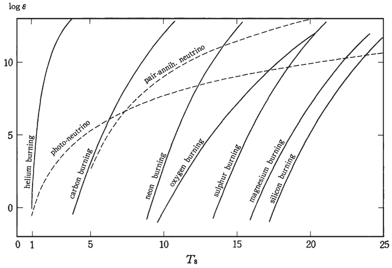


Fig. 9.2 A comparison between the neutrino losses and various nuclear energy generation reactions according to Hayashi et al. (1962). The rates are given in erg/g/s as a function of temperature in units of 10^8 K. The density is 10^4 g/cm^3. Note that several reactions shown in the figure are no longer considered relevant to massive stars. The neutrino rates have been updated since 1962, but the general picture of neutrino losses taking over energy generation remains today

From G. Shaviv, *The Synthesis of the Elements*

Carbon Burning

Burning conditions:

for stars > 8 M_o (solar masses) (ZAMS)

T~ 600-700 Mio
ρ ~ 10^5-10^6 g/cm^3

Major reaction sequences:

$$^{12}\mathrm{C}+{}^{12}\mathrm{C}\rightarrow {}^{24}\mathrm{Mg}^{*}\rightarrow {}^{23}\mathrm{Mg}+n-2.62\ \mathrm{MeV}$$
$$\rightarrow {}^{20}\mathrm{Ne}+\alpha+4.62\ \mathrm{MeV}$$
$$\rightarrow {}^{23}\mathrm{Na}+p+2.24\ \mathrm{MeV}.$$

dominates by far

of course p's, n's, and a's are recaptured … ^{23}Mg can b-decay into ^{23}Na

Composition at the end of burning:

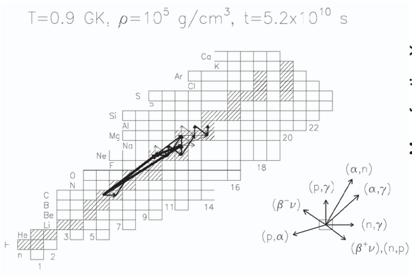
mainly ^{20}Ne, ^{24}Mg, with some ^{21,22}Ne, ^{23}Na, ^{24,25,26}Mg, ^{26,27}Al

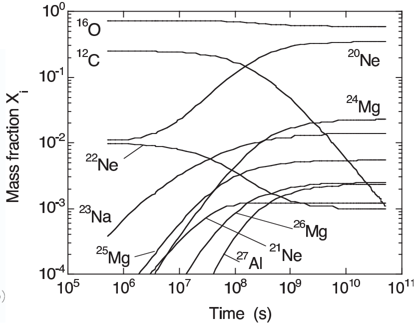
of course ^{16}O is still present in quantities comparable with ^{20}Ne (not burning … yet)

Carbon fusion

- As the α, proton and neutron (at $T > 1.1$ GK) channels are open in ^{12}C + ^{12}C fusion, the reaction proceeds dominantly by the strong interaction.
- Each fusion reaction produces a light nuclide and a heavier fragment.
- At the high temperatures light particles react much faster than the basic ^{12}C + ^{12}C fusion reaction.
- They produce other nuclides, eg. α particles react with ^{22}Ne (produced by α reactions on ^{14}N) via the $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$.
- Neutrons react very fast producing nuclei with neutron excess like ^{23}Na, ^{25,26}Mg, ^{29,30}Si.
- α particle reactions lead to an increase in the abundance of ^{20}Ne, ^{24}Mg, and a small extend of ^{28}Si.
- Carbon gets depleted and finally is less abundant than ^{16}O.

Core C burning evolution (constant temperature)





Christian Iliadis, *Nuclear Physics of Stars*.

Neon Burning

Burning conditions:

for stars > 12 M_o (solar masses) (ZAMS)

T~ 1.3-1.7 Bio K
$\rho \sim 10^6$ g/cm^3

Why would neon burn before oxygen ???

Answer:

Temperatures are sufficiently high to initiate **photodisintegration** of ^{20}Ne

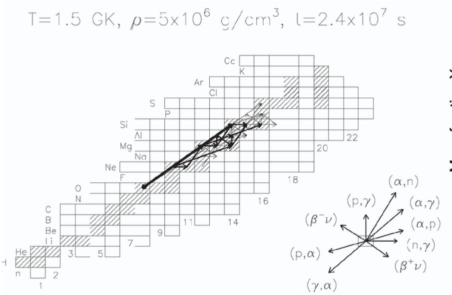
$$\left.\begin{array}{l} ^{20}\mathrm{Ne}+\gamma \rightarrow {}^{16}\mathrm{O} + \alpha \\ ^{16}\mathrm{O}+\alpha \rightarrow {}^{20}\mathrm{Ne} + \gamma \end{array}\right\}$$ equilibrium is established

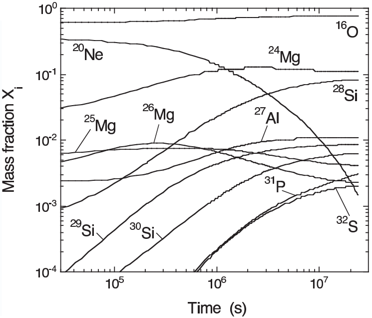
this is followed by (using the liberated helium)

$$^{20}\mathrm{Ne}+\alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

so net effect: $$2\ {}^{20}\mathrm{Ne} \rightarrow {}^{16}\mathrm{O} + {}^{24}\mathrm{Mg} + 4.59\ \mathrm{MeV}.$$

Core Ne burning evolution (constant temperature)





Christian Iliadis, *Nuclear Physics of Stars*.

Oxygen Burning

Burning conditions:

T~ 2 Bio
ρ ~ 10^7 g/cm^3

Major reaction sequences:

$$^{16}\mathrm{O}+{}^{16}\mathrm{O}\rightarrow{}^{32}\mathrm{S}^*\rightarrow{}^{31}\mathrm{S}+n+1.45\ \mathrm{MeV} \quad (5\%)$$
$$\rightarrow{}^{31}\mathrm{P}+p+7.68\ \mathrm{MeV} \quad (56\%)$$
$$\rightarrow{}^{30}\mathrm{P}+d-2.41\ \mathrm{MeV} \quad (5\%)$$
$$\rightarrow{}^{28}\mathrm{Si}+\alpha+9.59\ \mathrm{MeV}. \quad (34\%)$$

plus recapture of n,p,d,α

Main products:

^{28}Si,^{32}S (90%) and some ^{33,34}S,^{35,37}Cl,$^{36.38}$Ar, ^{39,41}K, ^{40,42}Ca

Oxygen burning and quasi-equilibrium

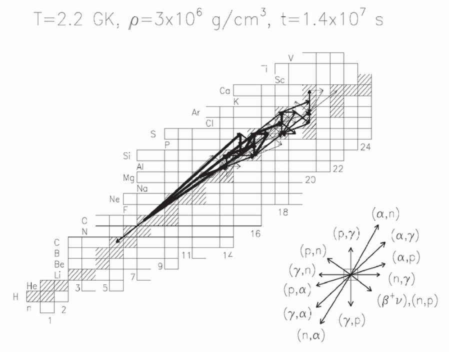
- Oxygen (and Silicon burning) show quasi-equilibrium behavior.
- Some reactions are fast enough to keep abundances of nuclei in relative equilibrium.
- Specific reactions connecting the different regions are not fast enough to be in equilibrium.

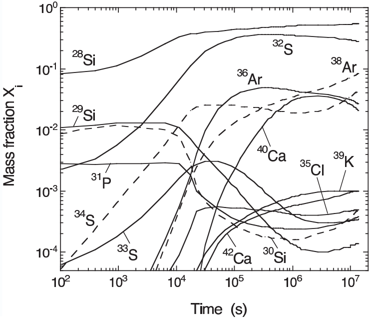
Nuclei not in equilibrium are ^{16}O and ^{24}Mg:

$$
\begin{aligned}
^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{28}\mathrm{Si} + \alpha \\
^{24}\mathrm{Mg} + \alpha &\rightarrow {}^{28}\mathrm{Si} + \gamma
\end{aligned}
$$

Two ^{16}O and one ^{24}Mg are converted into two ^{28}Si; additional α capture on ^{28}Si produces ^{32}S, ^{36}Ar, ^{38}Ar, and ^{40}Ca (quasi-equilibrium region)

Core O burning evolution (constant temperature)





Christian Iliadis, *Nuclear Physics of Stars*.

Silicon Burning

Burning conditions:

T~ 3-4 Bio
$\rho \sim 10^9$ g/cm^3

Reaction sequences:

- Silicon burning is fundamentally different to all other burning stages.
- **Complex network of fast (γ,n), (γ,p), (γ,a), (n,γ), (p,γ), and (a,γ) reactions**
- The net effect of Si burning is: 2 ^{28}Si --> ^{56}Ni,

need new concept to describe burning:

Nuclear Statistical Equilibrium (NSE)

Quasi Statistical Equilibrium (QSE)

Photodissociation reactions

Important role of photodissociation reactions

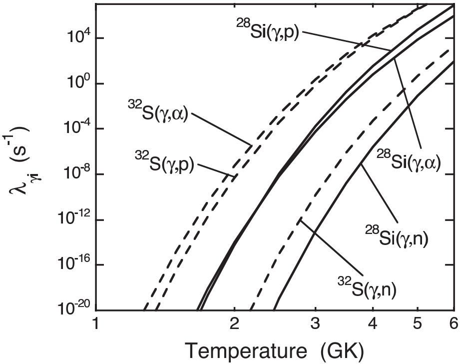
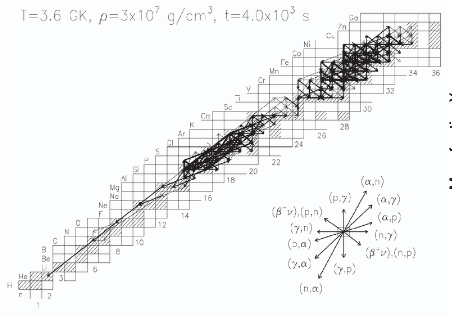
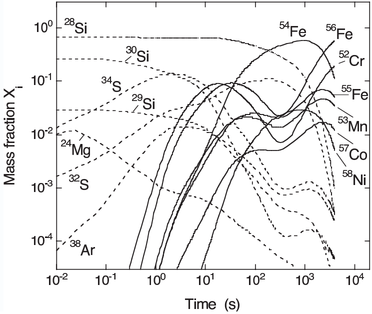


Fig. 5.51 Decay constants for the photodisintegrations of ^{28}Si (solid lines) and ^{32}S (dashed lines) versus temperature. The curves are calculated from the rates of the corresponding forward reactions.

Christian Iliadis, *Nuclear Physics of Stars*.

Core Si burning evolution (constant temperature)





Christian Iliadis, *Nuclear Physics of Stars*.

Nuclear Statistical Equilibrium

At high temperatures compositium can be aproximated by Nuclear Statistical Equilibrium.

- Composition is given by a minimum of the Free Energy: $F = U - TS$. Under the constrains of conservation of number of nucleons and charge neutrality.
- It is assumed that all nuclear reactions operate in a time scale much shorter than any other timescale in the system.

Nuclear Statistical Equilibrium favors free nucleons at high temperatures and iron group nuclei at low temperatures. Production of nuclei heavier than iron requires that nuclear statistical equilibrium breaks at some point.

Nuclear Statistical Equilibrium

The minimum of the free energy is obtained when:

$$\mu(Z,A) = (A - Z)\mu_n + Z\mu_p$$

implies that there is an equilibrium between the processes responsible for the creation and destruction of nuclei:

$$A(Z,N) \rightleftarrows Zp + Nn + \gamma$$

Processes mediated by the strong and electromagnetic interactions proceed in a time scale much sorter than any other evolutionary time scale of the system.

Nuclear abundances in NSE

Nuclei follow Boltzmann statistics:

$$\mu(Z,A) = m(Z,A)c^2 + kT \ln\left[\frac{n(Z,A)}{G_{Z,A}(T)}\left(\frac{2\pi\hbar^2}{m(Z,A)kT}\right)^{3/2}\right]$$

with $G_{Z,A}(T)$ the partition function:

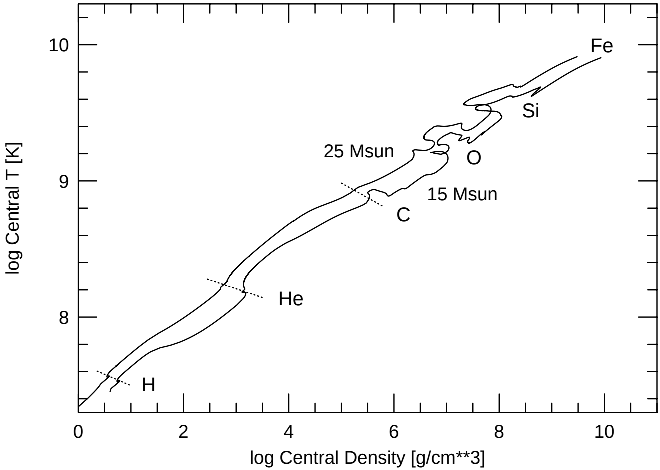
$$G_{Z,A}(T) = \sum_i (2J_i+1)e^{-E_i(Z,A)/kT} \approx \frac{\pi}{6akT}\exp(akT)\ (a \sim A/9\ \text{MeV})$$

Results in Saha equation:

$$Y(Z,A) = \frac{G_{Z,A}(T)A^{3/2}}{2^A}\left(\frac{\rho}{m_u}\right)^{A-1} Y_p^Z Y_n^{A-Z}\left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3(A-1)/2} e^{B(Z,A)/kT}$$

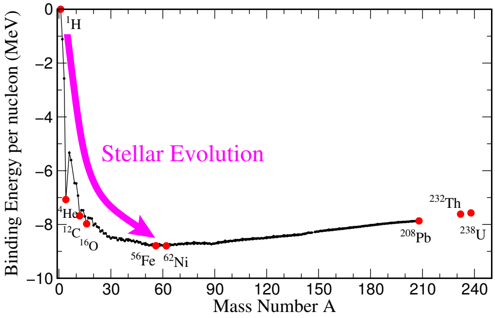
Composition depends on two parameters: Y_p, Y_n

Core evolution (Summary)



From Woosley, Heger & Weaver, Rev. Mod. Phys. **74**, 1015 (2002)

Stellar Evolution (Summary)



At the end of evolution the composition stellar core is given by Nuclear Statistical Equilibrium (NSE).

- Nuclear reactions operate in a time scale much shorter than the dynamical timescale in which the system evolves.
- Composition is given by a minimum of the Free Energy: $F = U - TS$. Under the constrains of conservation of number of nucleons and charge neutrality.

Nuclear Statistical Equilibrium favors free nucleons at high temperatures and iron group nuclei at low temperatures for constant entropy.