Nuclear Astrophysics

An Introduction

Lecture 1

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Introduction

Motivations

Origin of the chemical elements

Energy generation of stellar events

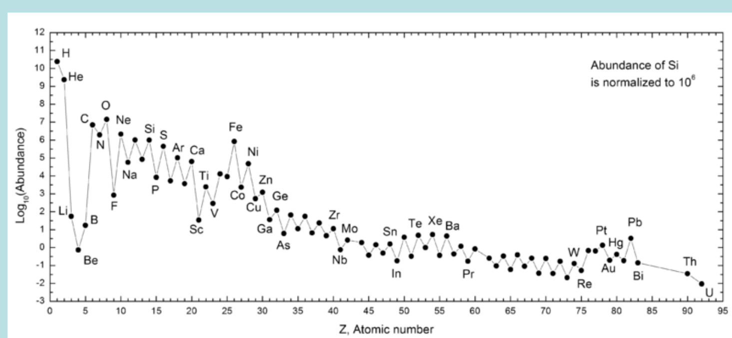
Time scales of the Universe

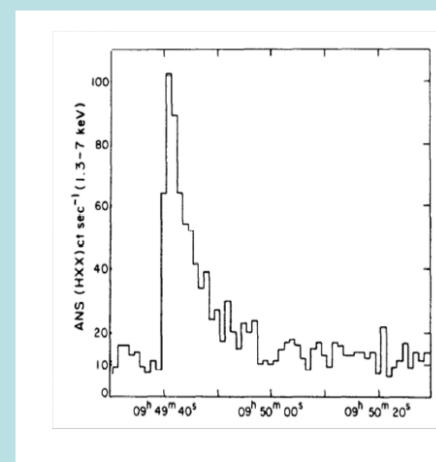
Observables

Abundances of the elements

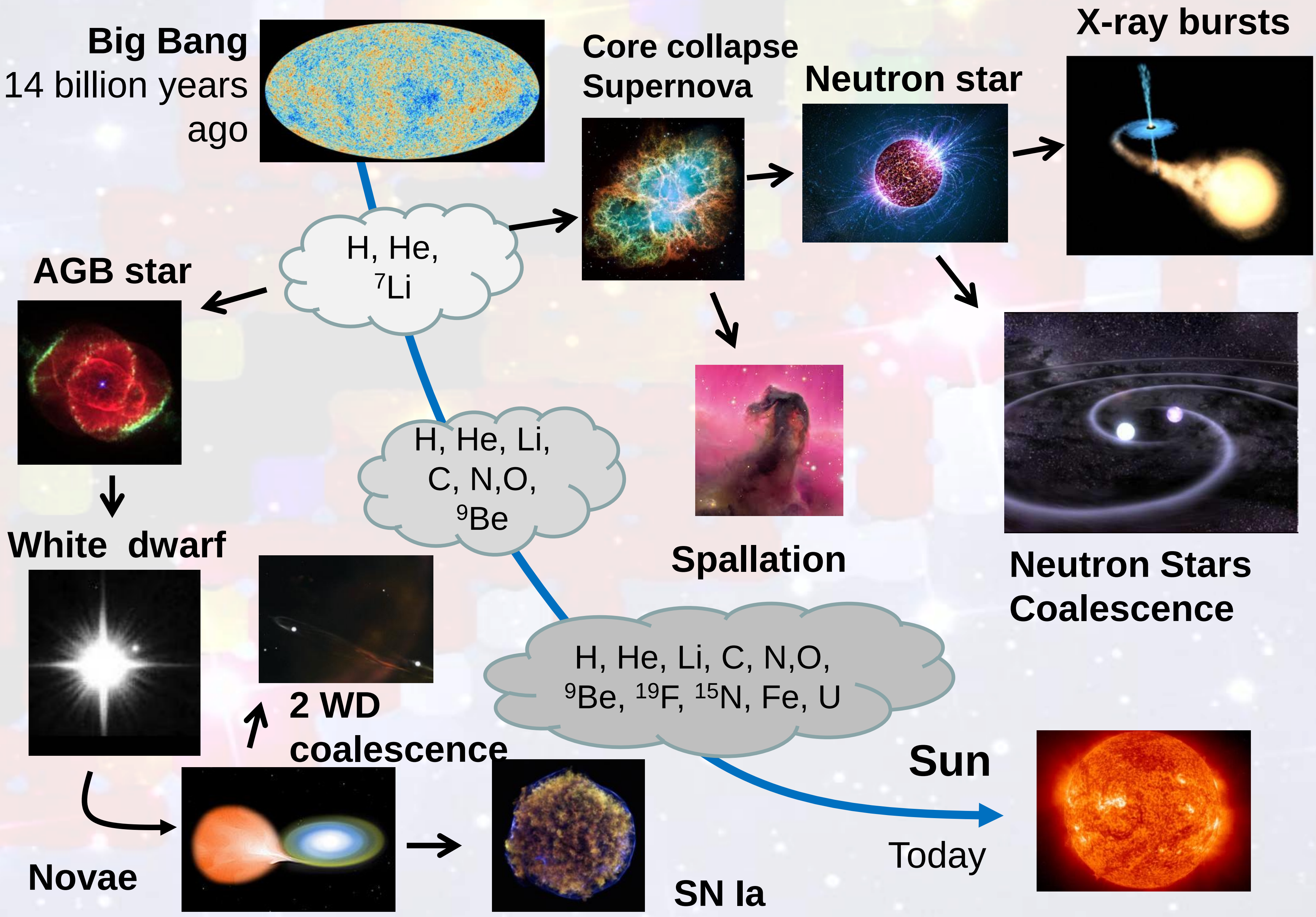
Luminosities

Light profile





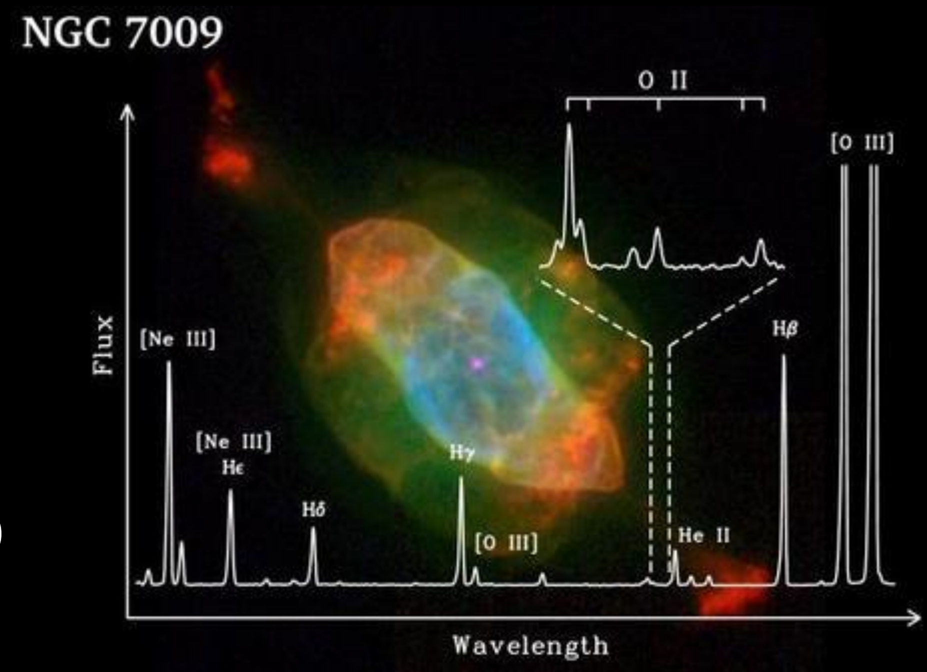
Nucleosynthesis



Outline of the Lectures

- An introduction - Basics
- A historical approach
- Mostly light / neutron deficient nuclei
- With exercises (check solutions in the afternoon)
- Several examples of experiments
- pp rate calculated for the first time

Abundance of the chemical elements



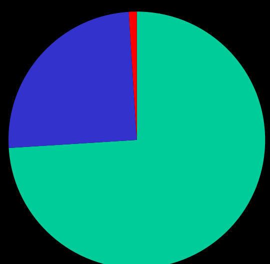
https://www.ucl.ac.uk/star/research/stars_galaxies/PNe

1814 **Discovery of dark absorption lines in the Sun's spectrum** (Fraünhofer)

Nebula lines

Red : Hydrogen (H_α à 656 nm)

Green: Oxygen (500 nm [OIII])



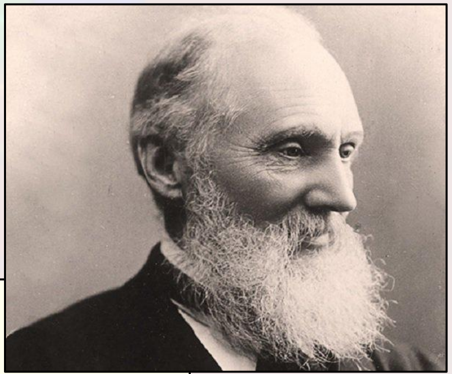
1868

Discovered in the Sun by Jules Jansen & N. Lockyer →

Element	In atoms	In mass
H	94 %	74 %
He	5.9 %	25 %
O	0.06 %	0.8 %

Origin of Sun's heat?

Power of Sun = 4×10^{26} Watts



1862

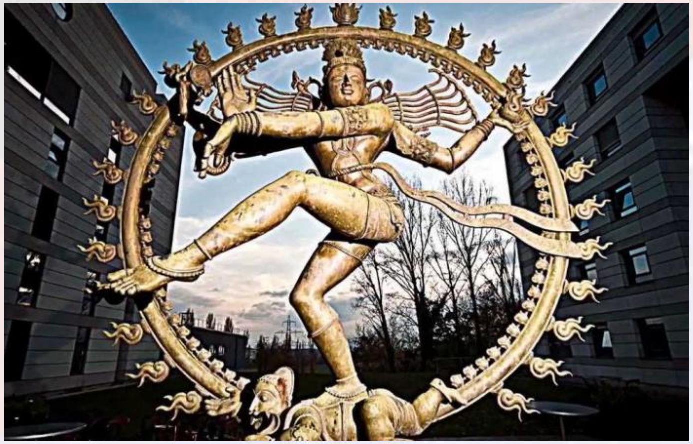
On the Age of the Sun's Heat

By Sir William Thomson (Lord Kelvin)

Macmillan's Magazine, vol. 5 (March 5, 1862), pp. 388-393.
From reprint in *Popular Lectures and Addresses*, vol. 1, 2nd edition, pp. 356-375.

The second great law of thermodynamics involves a certain principle of *irreversible action in Nature*. It is thus shown that, although mechanical energy is *indestructible*, there is a universal tendency to its dissipation, which produces gradual augmentation and diffusion of heat, cessation of motion, and exhaustion of potential energy through the material universe. [1] The result would inevitably be a state of universal rest and death, if the universe were finite and left to obey existing laws. But it is

Several ideas…



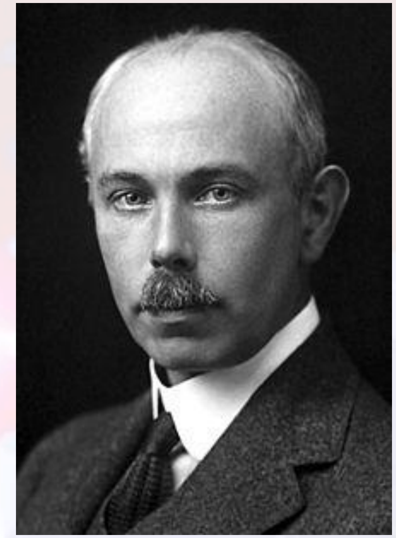
Conversion of gravitational energy into heat
Meteorites falling on the Sun
Maximum age ~ 10^7 years

Chemical reactions?
Mass of the Sun = 2×10^{30} kg
~10 eV/ reaction $\Rightarrow$ age ~ 10^4 years

Radioactivity?
6 nW / T (Earth) $\Rightarrow$ 10^{19} W

Precise mass measurements

1919



Francis William Aston

LIX. *The Mass-Spectra of Chemical Elements.* *By* F. W. Aston, *M.A., D.Sc., Clerk Maxwell Student of the University of Cambridge* *.

[Plate XV.]

THE following paper is an account of some results obtained by the analyses of gases by means of the Positive Ray Spectrograph or, as it may be more conveniently termed, Mass-Spectrograph. The principle of the method by which a focussed spectrum is obtained depending solely on the ratio of mass to charge has already been described †, but for the sake of others experimenting in this field it is now proposed to give an account of the actual apparatus in some detail.

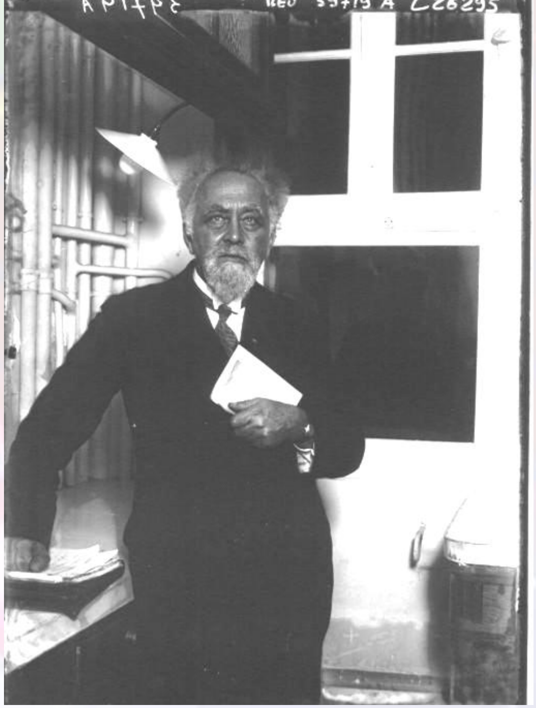
* Communicated by the Author.
† F. W. A., Phil. Mag. xxxviii. Dec. 1919, p. 707.

Aston, F. W. (1920). LIX. The mass-spectra of chemical elements. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 39(233), 611-625.

1919 Jean Perrin

proposed that the Sun and other stars are powered by **nuclear reactions**.

Origine de la chaleur solaire Annales de Phys., 1919
Revue du Mois, 1920
L'Astronomie, 1922



Notice sur les travaux scientifiques *J. Perrin*
Académie des Sciences *1923*

La condensation d'hydrogène en hélium se fera peut-être en deux ou trois étapes, avec formation intermédiaire de nébulium ; en définitive, par addition des équations de transformation correspondantes, elle pourra s'écrire, selon notre schéma :

$$W + H\mathrm{Sn} + 4\mathrm{H} \longrightarrow \mathrm{He} + H\mathrm{Sn}'$$

en appelant W les énergies cinétiques (peut-être notables) avant rencontre, n les fréquences des radiations absorbées (s'il y en a), et n' les fréquences des rayons ultra X émis à chaque transformation.

Or les mesures précises classiques des poids atomiques de l'hydrogène et de l'hélium donnent, pour 4H, 32 milligrammes de plus que pour He ; perte de masse impliquant une diminution d'énergie interne égale à 0,032 c^2 d'après la formule d'Einstein, soit 3.10^{19} ergs. L'énergie rayonnée HSn' comprend outre ces 3.10^{19} ergs, l'énergie inconnue W + HSn.

Nuclear reaction
$4H \rightarrow {}^4He$

$E = mc^2$
26.7 MeV/reaction

Exercise

Power of Sun = 4×10^{26} Watts

26 MeV/reaction

?????   T/s of H is transformed into ^{4}He

Mass of the Sun = 2×10^{30} kg

94% of H

It could provide ?????   years of burning

Solution

Power of Sun = 4×10^{26} Watts

26 MeV/reaction

620×10^{6} T/s of H

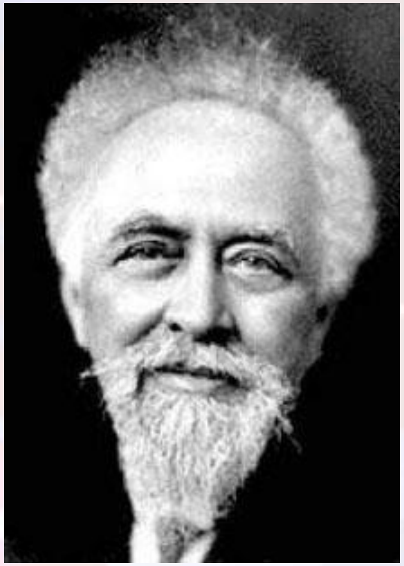
Mass of the Sun = 2×10^{30} kg

94% of H

~ 10^{11} years

1923

Notice sur les travaux scientifiques *J. Perrin*
Académie des Sciences 1923



Après l'hélium apparaîtront successivement les éléments plus lourds, et pour beaucoup d'entre eux sans émission d'énergie comparable à celle qui accompagne la formation de l'hélium. On sait en effet que par exemple le poids atomique du carbone est, au dix-millième près, 3 fois celui de l'hélium ; il ne se perd donc pas dans cette union plus que 1 milligramme (ce qui permet encore une «chaleur de réaction », inaccessible à nos balances actuelles, mais pourtant de quelques milliards de calories).

L'équation d'une telle réaction s'écrirait

$$\mathrm{W} + 3\,\mathrm{He} \longrightarrow \mathrm{C} + H\mathrm{n}'$$

$$3\ {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$$

(known today as the Triple Alpha reaction)

Is this guess confirmed by theory?

The Reaction Rate

$$1 + 2 \rightarrow 3$$

$$dN_3 = N_1 \; N_2 \; v \; \sigma(v) \; \varphi(v) \; dv \; dt$$

- dN_3: Number of particles 3 produced per cm³
- $N_1 \; N_2$: Densities particles/cm³
- v: Velocity
- $\sigma(v)$: Cross Section
- $\varphi(v) \; dv$: Probability to get the velocity between v and v+dv

$$dN_3 = N_1 N_2 \int \varphi(v) \, v \, \sigma(v) \, dv \; dt$$

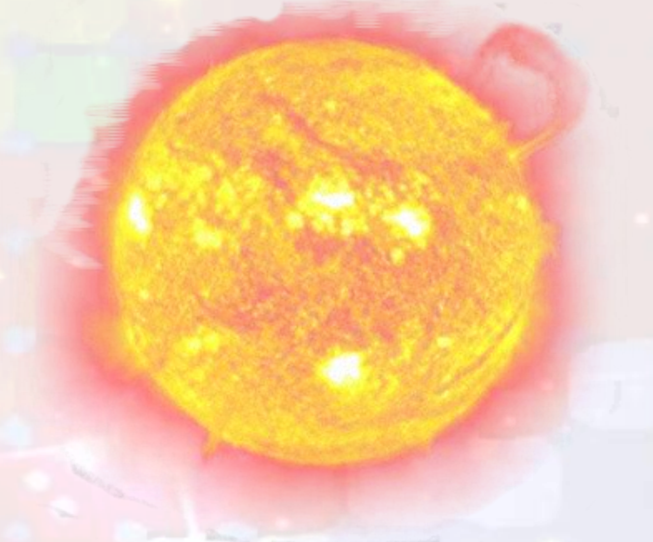
Reaction Rate cm³ s⁻¹

Need:

a) Velocity distribution $\varphi(v)$

b) Cross section $\sigma(v)$

Velocity distribution



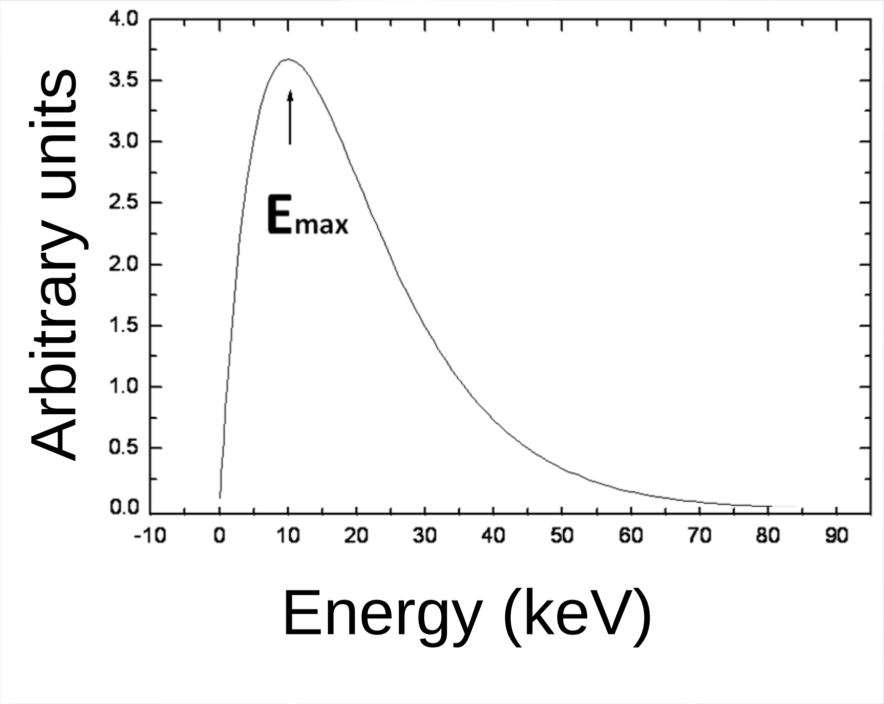
Hydrostatic equilibrium

$\Rightarrow$ T ~ 15×10^6 K

Interacting nuclei in plasma are in thermal equilibrium at temperature T $\Rightarrow$

Maxwell – Boltzmann Velocity Distribution

$$\varphi(v) = 4\pi \left(\frac{\mu}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT} \right)$$



E_{max} = k T = 0.086 T6 (keV)

Sun E_{max} ~ 0.86 keV

Cross section

From quantum mechanical, we expect:

$$\sigma(E) \propto \pi\lambda^2 \propto \frac{1}{E}$$

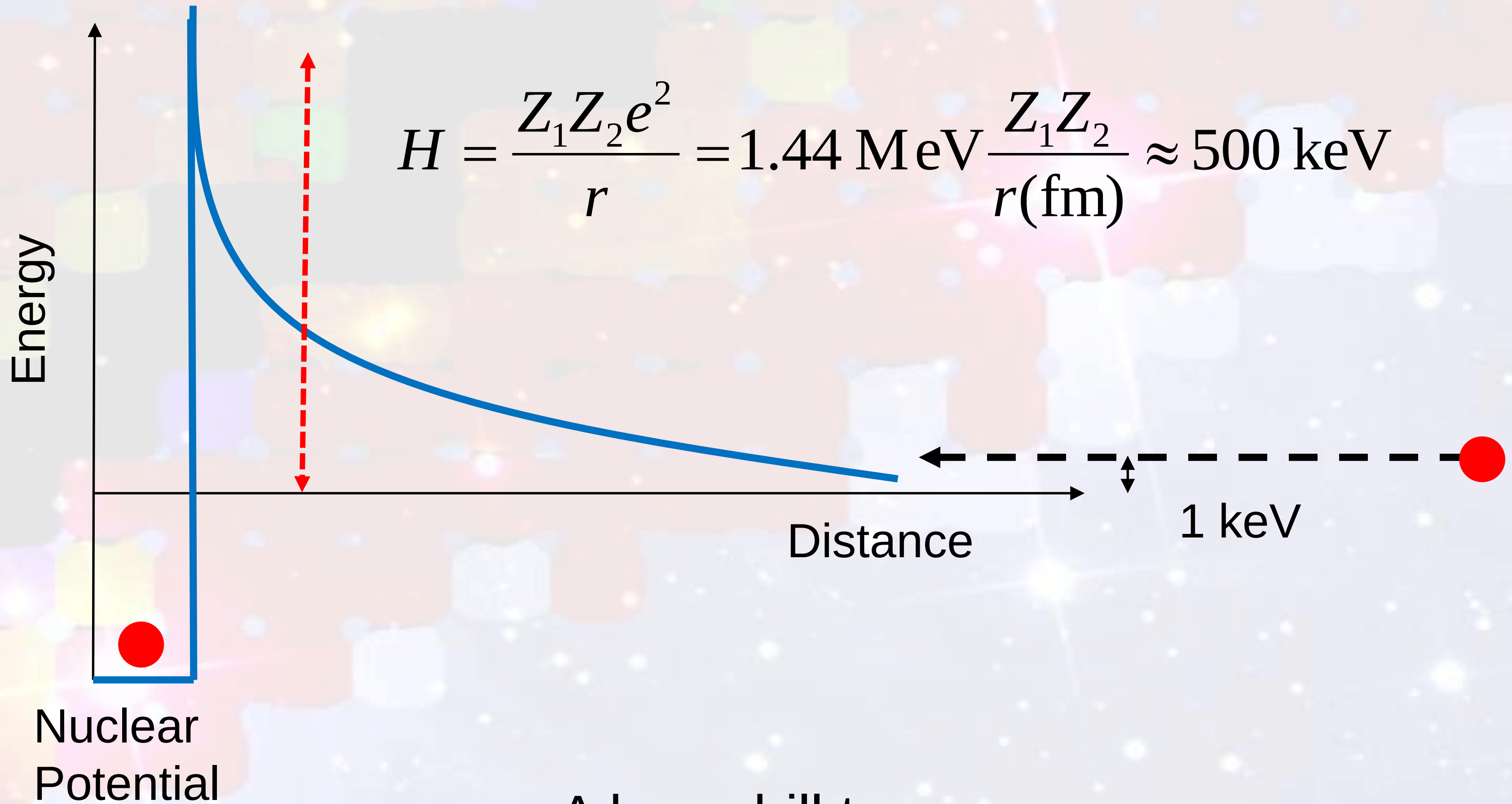
$$\lambda : \text{de Broglie wave-lenght}$$

But not only….

The Coulomb barrier

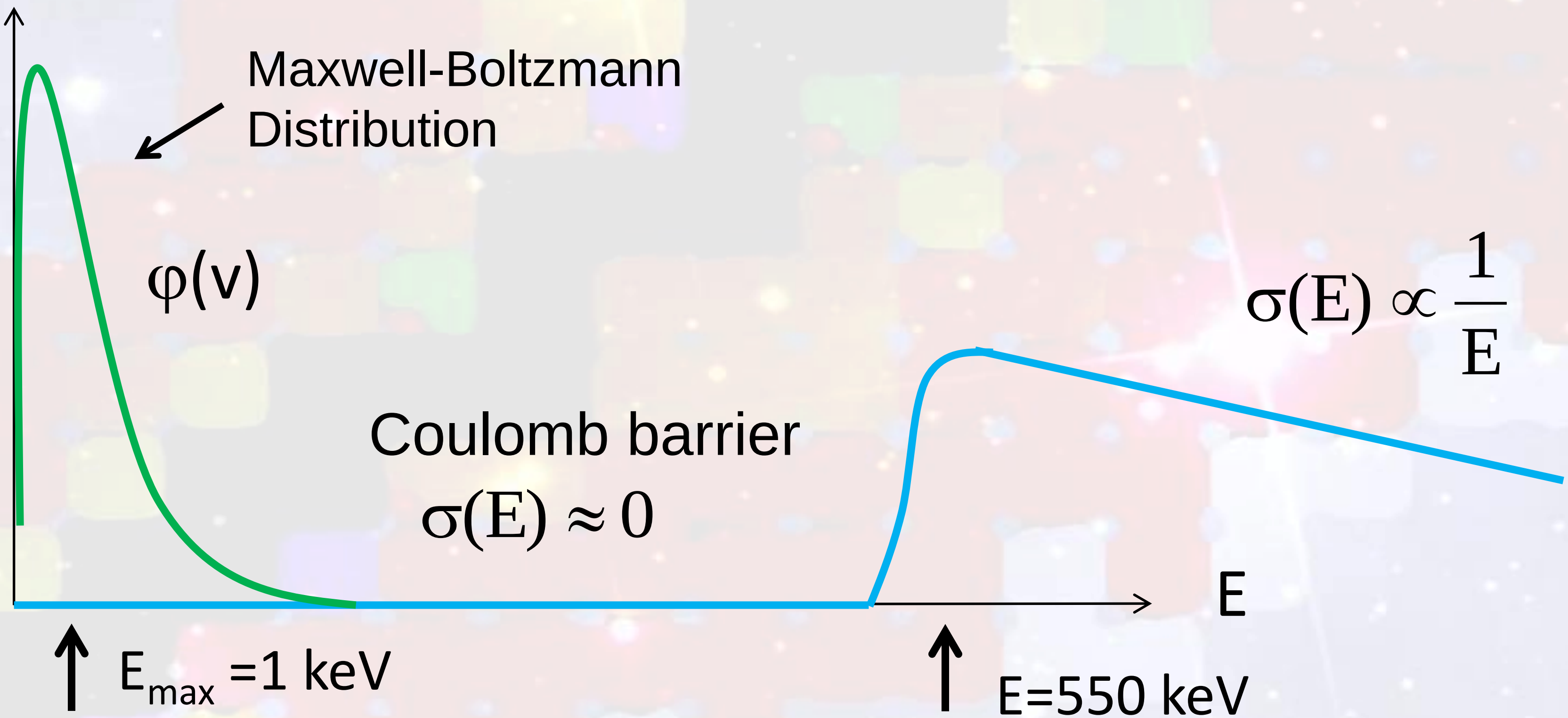
In the case of p+p

$$H = \frac{Z_1 Z_2 e^2}{r} = 1.44\,\mathrm{MeV}\frac{Z_1 Z_2}{r(\mathrm{fm})} \approx 500\,\mathrm{keV}$$



A huge hill to overcome

Is the temperature high enough?
(to induce nuclear reactions)



Probability $$\frac{\varphi(550keV)}{\varphi(1keV)} = 10^{-275}$$

Reaction Rate

$$\int v\,\sigma(v)\,\varphi(v)\,dv = 0$$

In the Sun ~ 10^{57} protons

No reaction!

1926

Second Series *December, 1926* *Vol. 28, No. 6*

THE

PHYSICAL REVIEW

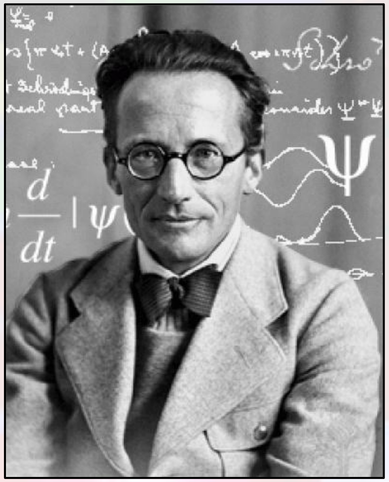
AN UNDULATORY THEORY OF THE MECHANICS OF ATOMS AND MOLECULES

By E. Schrödinger

Abstract

The paper gives an account of the author's work on a new form of quantum theory. §1. The Hamiltonian analogy between mechanics and optics. §2. The analogy is to be extended to include real "physical" or "undulatory" mechanics instead of mere geometrical mechanics. §3. The significance of wave-length; macro-mechanical and micro-mechanical problems. §4. The wave-equation and its application to the hydrogen atom. §5. The intrinsic reason for the appearance of discrete characteristic frequencies. §6. Other problems; intensity of emitted light. §7. The wave-equation derived from a Hamiltonian variation-principle; generalization to an arbitrary conservative system. §8. The wave-function physically means and determines a continuous distribution of electricity in space, the fluctuations of which determine the radiation by the laws of ordinary electrodynamics. §9. Non-conservative systems. Theory of dispersion and scattering and of the "transitions" between the "stationary states." §10. The question of relativity and the action of a magnetic field. Incompleteness of that part of the theory.

1. The theory which is reported in the following pages is based on the very interesting and fundamental researches of L. de Broglie[1] on what he called "phase-waves" ("ondes de phase") and thought to be associated with the motion of material points, especially with the motion of an electron or proton. The point of view taken here, which was first



$$\frac{\hat{\vec{\mathbf{p}}}^{2}}{2m}|\Psi(t)\rangle + V\left(\hat{\vec{\mathbf{r}}},t\right)|\Psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle$$

1928

G. Gamow

Alpha particle was considered
a preformed cluster, moving
around the core and penetrating
quantum mechanically
the Coulomb barrier

About the quantum theory of the atomic nucleus

Zur Quantentheorie des Atomkernes.

Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Es wird der Versuch gemacht, die Prozesse der α-Ausstrahlung auf Grund der Wellenmechanik näher zu untersuchen und den experimentell festgestellten Zusammenhang zwischen Zerfallskonstante und Energie der α-Partikel theoretisch zu erhalten.

§ 1. Es ist schon öfters* die Vermutung ausgesprochen worden, daß im Atomkern die nichtcoulombschen Anziehungskräfte eine sehr wichtige Rolle spielen. Über die Natur dieser Kräfte können wir viele Hypothesen machen.

Es können die Anziehungen zwischen den magnetischen Momenten der einzelnen Kernbauelemente oder die von elektrischer und magnetischer Polarisation herrührenden Kräfte sein.

Jedenfalls nehmen diese Kräfte mit wachsender Entfernung vom Kern sehr schnell ab, und nur in unmittelbarer Nähe des Kernes überwiegen sie den Einfluß der Coulombschen Kraft.

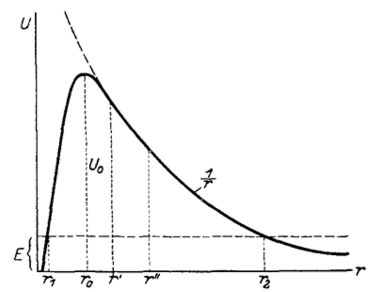


Fig. 1.

Aus Experimenten über Zerstreuung der α-Strahlen können wir schließen, daß, für schwere Elemente, die Anziehungskräfte bis zu einer Entfernung $\sim 10^{-12}$ cm noch nicht merklich sind. So können wir das auf Fig. 1 gezeichnete Bild für den Verlauf der potentiellen Energie annehmen.

Hier bedeutet r'' die Entfernung, bis zu welcher experimentell nachgewiesen ist, daß Coulombsche Anziehung allein existiert. Von r' beginnen die Abweichungen (r' ist unbekannt und vielleicht viel kleiner als r'') und bei r_0 hat die U-Kurve ein Maximum. Für $r < r_0$ herrschen schon die Anziehungskräfte vor, in diesem Gebiet würde das Teilchen den Kernrest wie ein Satellit umkreisen.

* J. Frenkel, ZS. f. Phys. **37**, 243, 1926; E. Rutherford, Phil. Mag. **4**, 580, 1927; D. Enskog, ZS. f. Phys. **45**, 852, 1927.

Zeitschrift für Physik, Volume 51, Issue 3-4, pp. 204-212

Quantum tunneling

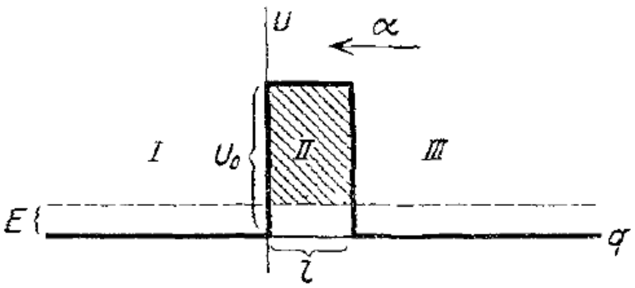


Fig. 2.

208 G. Gamow,

So sehen wir, daß die Amplitude der durchgegangenen Welle um so kleiner ist, je kleiner die Gesamtenergie E ist, und zwar spielt der Faktor:

$$e^{-lk'} = e^{-\frac{2\pi \cdot \sqrt{2m}}{h}\sqrt{U_0 - E}\,\cdot\, l}$$

in dieser Abhängigkeit die wichtigste Rolle.

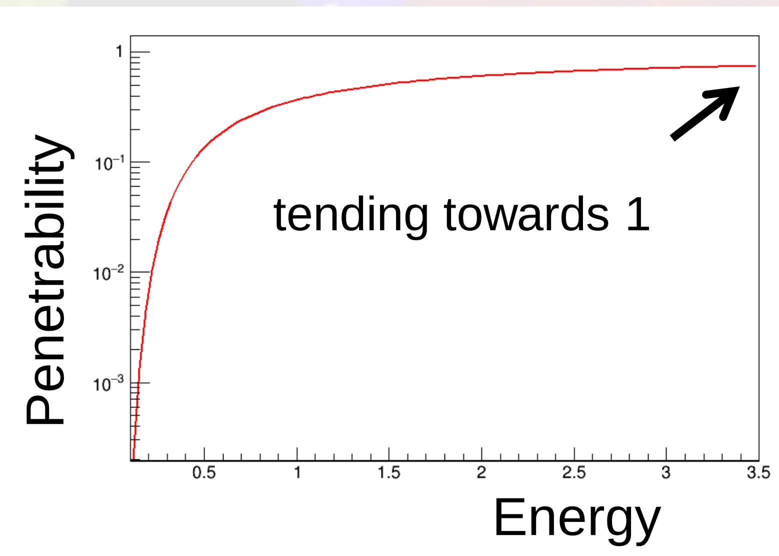
Quantum-mechanical tunneling probability
Gamow factor
Penetrability

$$P(E) = \exp(-2\pi\,\eta)$$

$$\eta = \frac{Z_1 Z_2}{\hbar\, v}\,\frac{q^2}{4\pi\varepsilon_0}$$

$$P(E) = \exp\left(-\frac{b}{\sqrt{E}}\right)$$

$$\sigma(E) \propto \lambda^2 \quad P(E)$$



Penetrability

- The Gamow expression of Penetrability is an approximation

- Angular momentum ℓ should be taken into account

$$V_{\text{eff}}(r) = V(r) + \frac{\hbar^2 l(l+1)}{2m_0 r^2}$$

Effective potential with the centrifugal barrier term

More accurate expression: $$P_\ell(E,R) = \frac{1}{F_\ell^2(E,R) + G_\ell^2(E,R)}$$

with regular and irregular Coulomb function

Be careful, in the literature there are different definitions of “P”, the penetrability,

$$P_\ell(E,R) = \frac{1}{F_\ell^2(E,R) + G_\ell^2(E,R)}$$

$$P_\ell(E,R) = \frac{\mathrm{Rho}}{F_\ell^2(E,R) + G_\ell^2(E,R)}$$

With Rho $\propto$ $E^{1/2}$

One program for penetrability

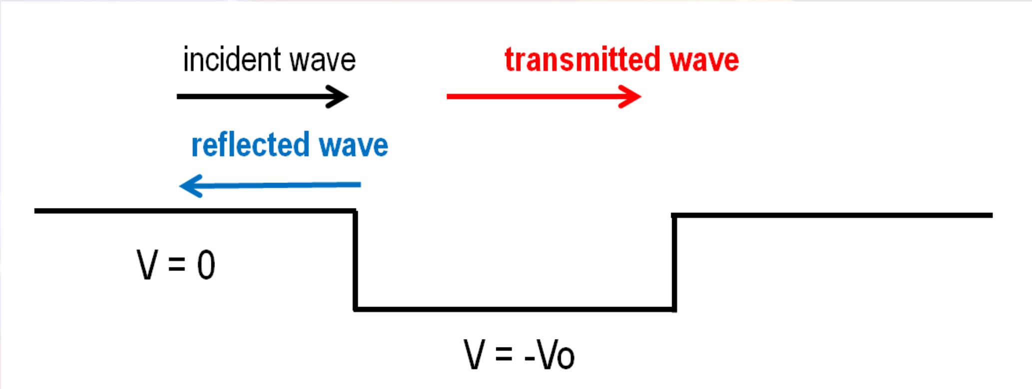
```
essai.c
#include <stdio.h>
#include <gsl/gsl_sf_coulomb.h>
#include <math.h>


int main (void)
{
  double R0 = 1.25;
  double AC=4.0;
  double ZC=2.0;
  double AP=23.0;
  double ZP=12.0;
  double R=R0*(pow(AC,1./3.)+pow(AP,1./3.));
  double el=4.6;
  double elab=(AC+AP)*el/AC;
  double ETA=.1575*ZC*ZP*sqrt(AP/elab);
  double XMU=AC*AP/(AC+AP);
  double RHO= .2187*XMU*R*sqrt(elab/AP);
  double L_min=0;
  int kmax=6;
  double fc_array[10] = {0.};
  double gc_array[10] = {0.};
  double F_exponent;
  double G_exponent;
  int i=0;
  double P,G;

  gsl_sf_coulomb_wave_FG_array (L_min, kmax, ETA, RHO, fc_array, gc_array, &F_exponent, &G_exponent);

  do
    {
      printf ("l=%d   F=%e  G=%e  \n", i,fc_array[i],gc_array[i]);
      P=RHO/(fc_array[i]*fc_array[i]+gc_array[i]*gc_array[i]);
      G=3.0*pow(197.33,2)/XMU/931.5/pow(R,2)*P*1.e6;
      printf ("P=%e  G=%e eV \n",P,G);
      i++;
    }
  while(i<4);

  return 0;
}
```

Compiled with

gcc –Wall – I/usr/include –c essai.c
gcc –L/lib essai.o –lgsl –lgslcblas –lm

Reactions with neutrons

Penetrability



Discontinuity in potential gives rise to partial reflection of incident wave

$$P_\ell(E) \propto E^{1/2+\ell}$$

$E \rightarrow 0$

Cross section

$$\sigma(E) \propto E^{\ell-1/2}$$

Consequence:

s-wave neutron capture usually dominates at low energies

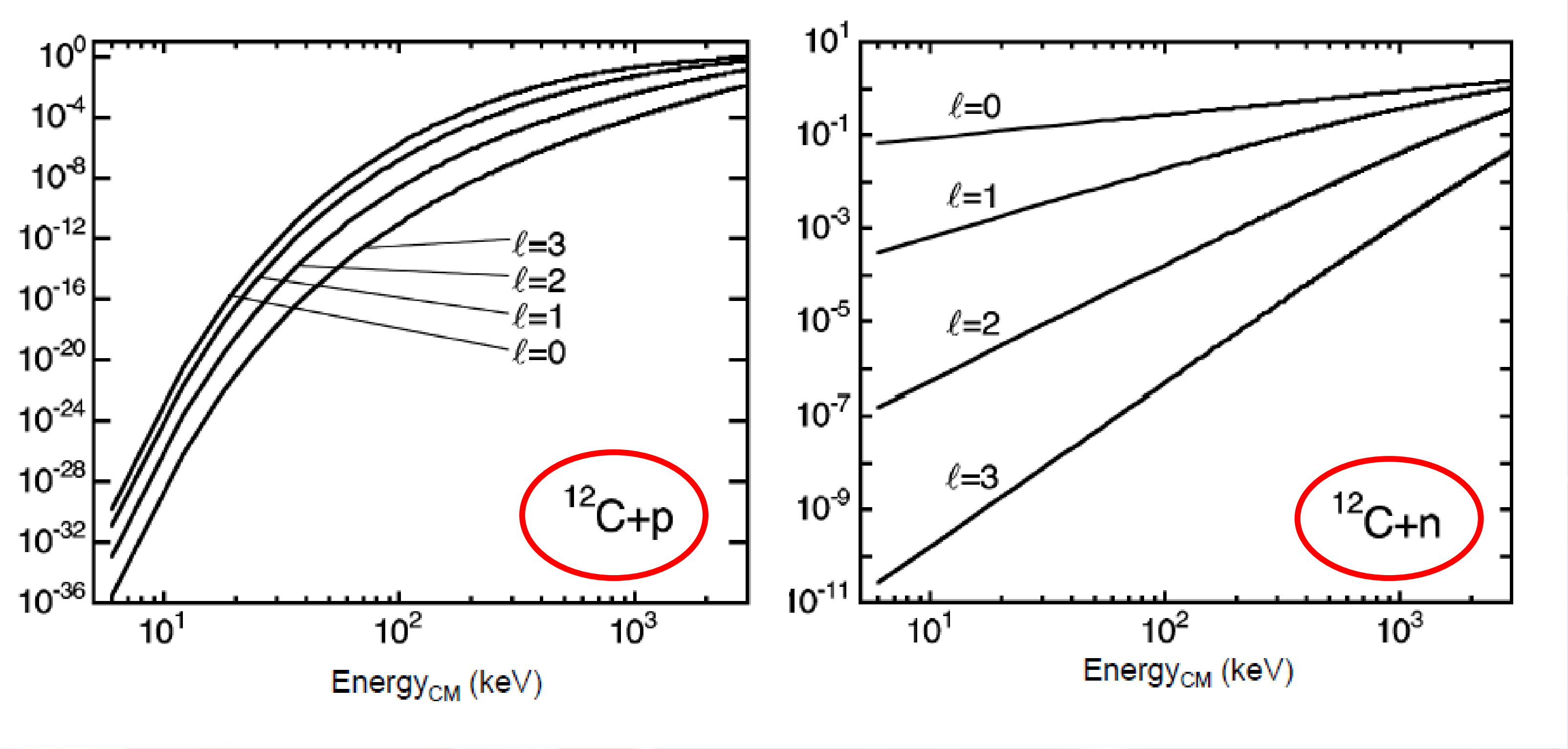
for $\ell = 0$ $\quad \sigma(E) \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$

Reaction rate

$$< \sigma v > = < \frac{1}{v} v > = cte \int \varphi(v) dv$$

"exact" formula for the Penetrability

$P_\ell(E)$



From C. Iliadis "Nuclear Physics of Stars"

Exercise

$$\text{Probability} \propto \exp(-2\pi\,\eta)$$

$$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}$$

with E in keV

μ in u.m.a

Probability ???

p+p (1keV)

Solution

$$\text{Probability} \propto \exp(-2\pi\,\eta)$$

$$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}$$

with E in keV

μ in u.m.a

Prob = 10^{-10}

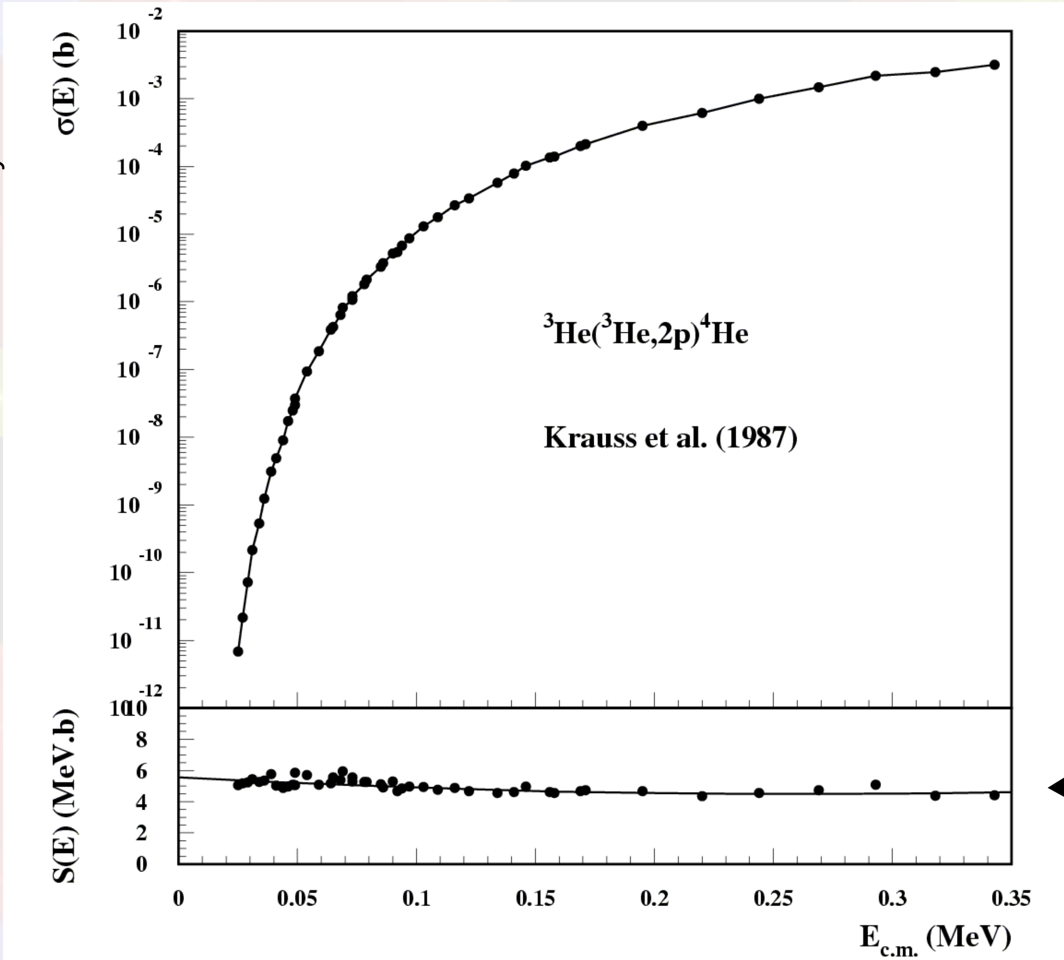
p+p (1keV)

The Astrophysical Factor

$$\sigma(E) \equiv \frac{S(E)}{E} \exp\left(-2\pi\eta\right)$$

Tunnel Effect

Cauldrons in the Cosmos C. Rolfs & W. Rodney



Astrophysical Factor

S(E) ~ constant

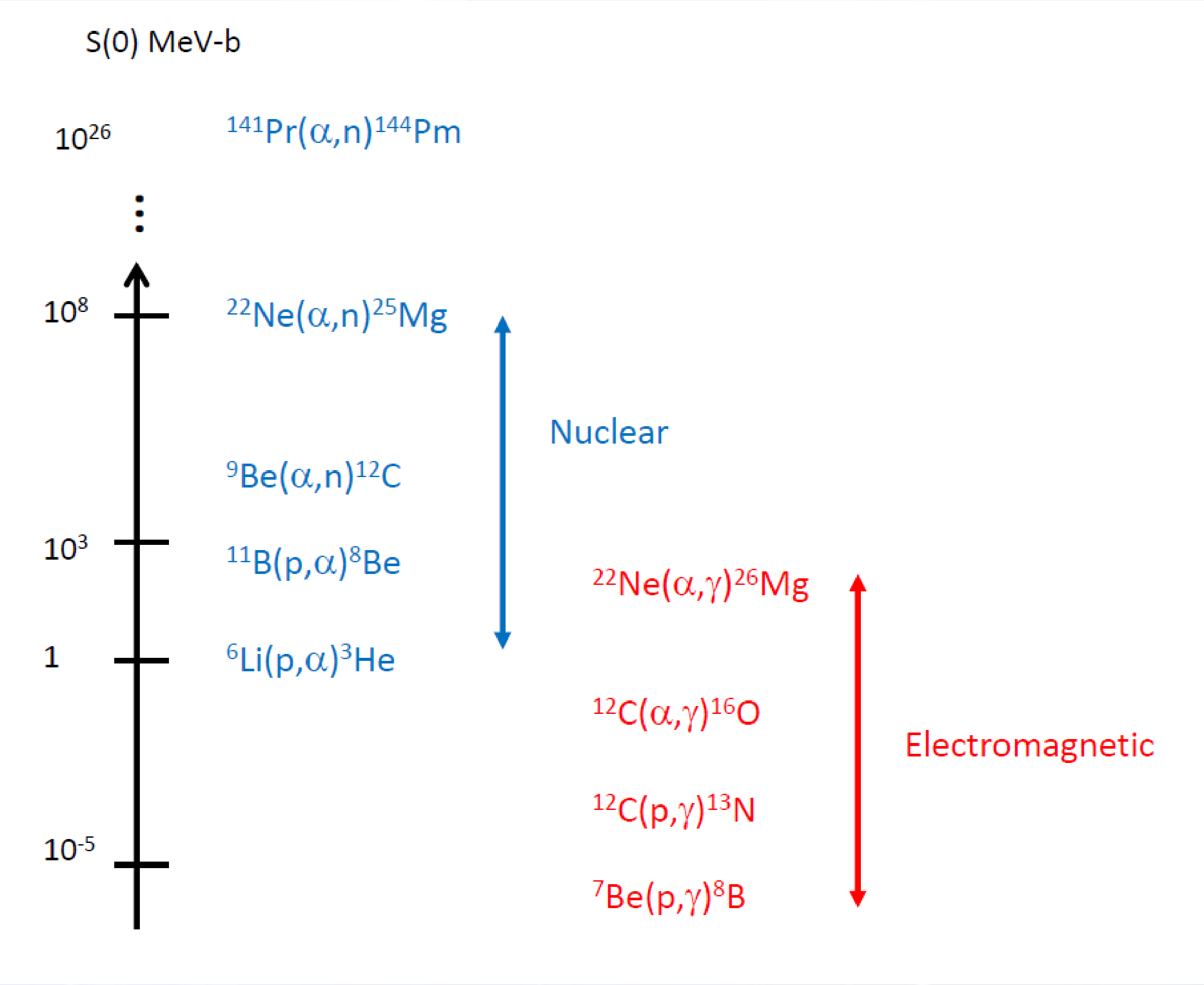
$$S(E) \sim \left|\langle f \left| H_{nuclear} \right| i \rangle\right|^2$$

Classification of the Reactions

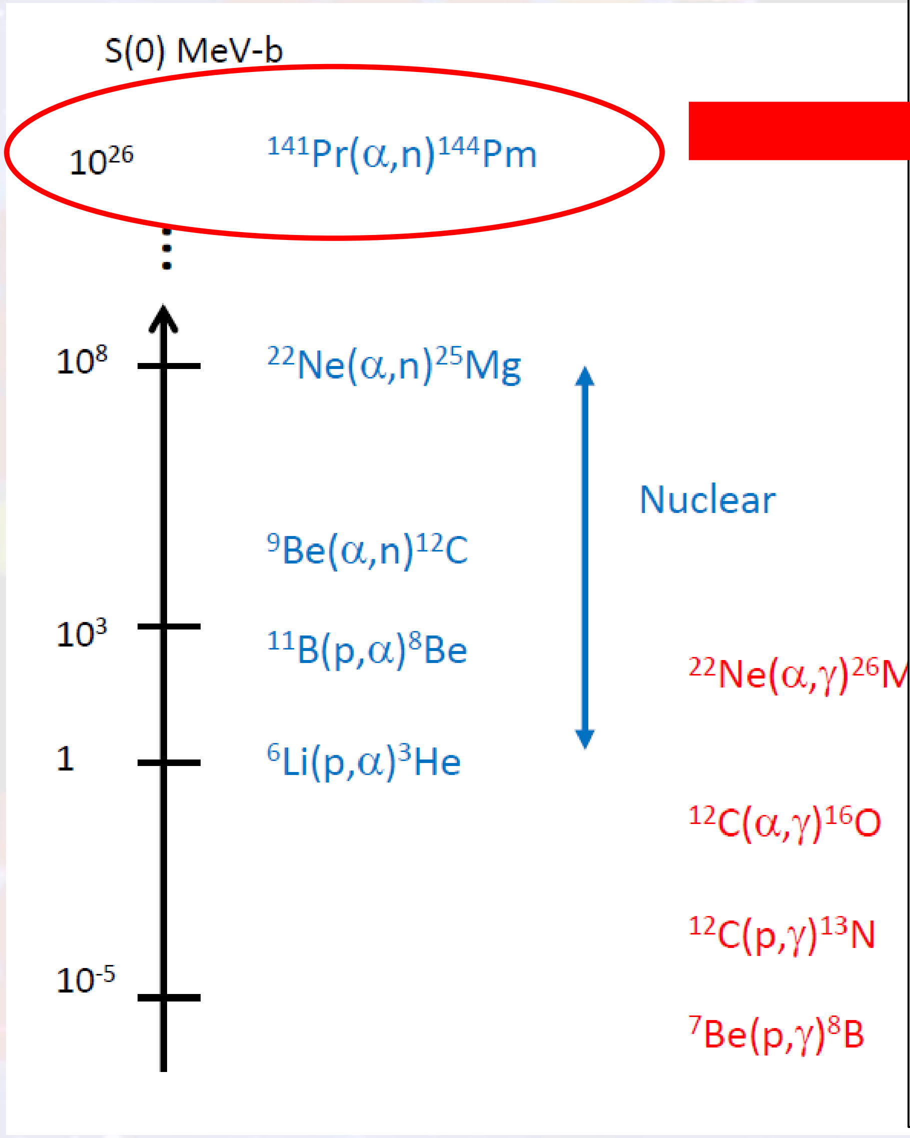
Table 2.3 *Classification of the main reactions involved in nuclear astrophysics.*

Process		Examples	$S(0)$ (MeV-b)
Nuclear	Non – resonant	$^{6}Li(p,\alpha)^{3}He$	≈ 3
	Resonant: $\ell_R = \ell_{min}$	$^{3}He(d,p)\alpha$	≈ 6
	Resonant: $\ell_R > \ell_{min}$	$^{11}B(p,\alpha)^{8}Be$	≈ 300
	Resonant: multiresonance	$^{22}Ne(\alpha,n)^{25}Mg$	$\approx 10^{8}$
	Subthreshold state	$^{13}C(\alpha,n)^{16}O$	$\approx 10^{7}$
Electromagnetic	Non – resonant	$^{6}Li(p,\gamma)^{7}Be$	$\approx 10^{-4}$
	Resonant: $\ell_R = \ell_{min}$	$^{12}C(p,\gamma)^{13}N$	$\approx 10^{-3}$
	Resonant: $\ell_R > \ell_{min}$	$^{7}Be(p,\gamma)^{8}B$	$\approx 2 \times 10^{-5}$
	Resonant: multiresonance	$^{22}Ne(\alpha,\gamma)^{26}Mg$	$\approx 2 \times 10^{3}$
	Subthreshold state	$^{12}C(\alpha,\gamma)^{16}O$	≈ 0.5
Weak	Non-resonant	$p(p,e^{+}\nu)d$	$\approx 4 \times 10^{-25}$
		$^{3}He(p,e^{+}\nu)^{4}He$	$\approx 10^{-22}$

Classification of the Reactions



This is very counter-intuitive! Why is it so?



$\sigma_{measured}$ (11 MeV) ~ 1 μbarn

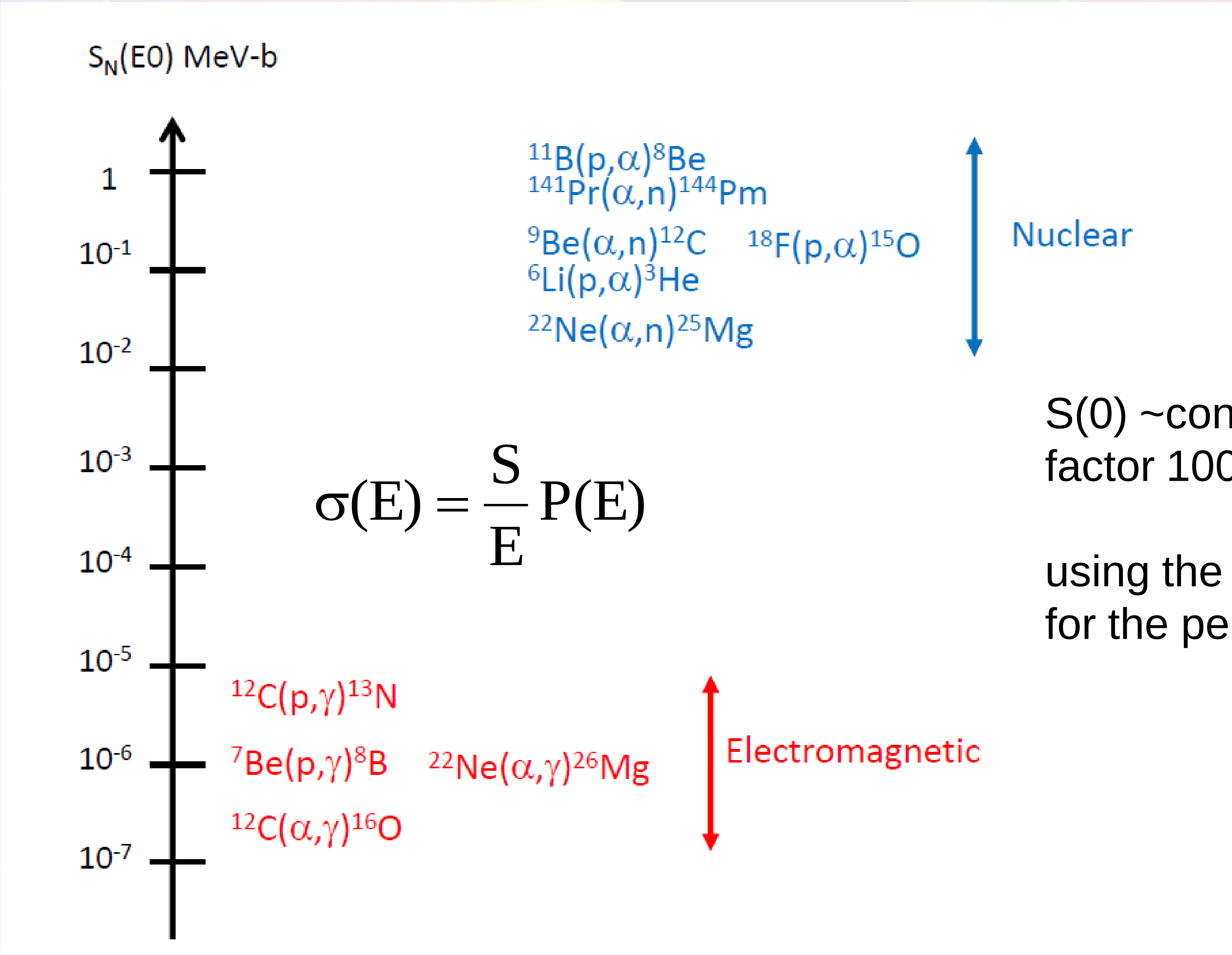
$$\sigma(E) = \frac{S}{E} \exp(-2\pi\eta)$$

Z=59 $\Rightarrow$ $\exp(-2\pi\eta) = 7\ 10^{-31}$

$\Rightarrow$ $S = 1.25 \times 10^{25}$

The extremely low penetrability seems to be compensated by the extremely high nuclear astrophysical factor

The Gamow factor is an approximation



$$\sigma(E) = \frac{S}{E} P(E)$$

S(0) ~constant within a factor 100

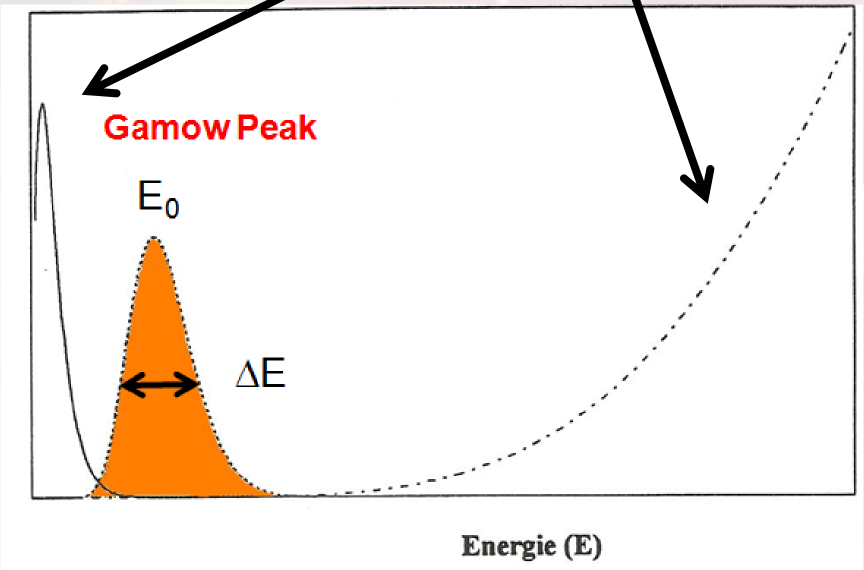
using the exact formula for the penetrability

Gamow energy and window

$$\langle\sigma v\rangle = \int \sigma(v)\phi(v) v\, dv = \int S(E)\exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) dE$$

Maximum reaction rate at E_0

$$\frac{d}{dE}\left[\exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right)\right] = 0$$



Gamow energy

$$E_0 = 1.22\,(Z_1^2 Z_2^2\, \mu\, T_6^2)^{\frac{1}{3}} \quad \text{keV}$$

E_0 = 5.9 keV (p+p)

Gamow window

$$\Delta E = 0.749\left(Z_1^2 Z_2^2 \mu T_6^5\right)^{1/6} \quad \text{keV}$$

ΔE = 6.4 keV

Exercise

$$p + p \Rightarrow {}^{2}He \Rightarrow p + p$$

$\sigma(E_0)$?

we suppose S(E) = constant = 1 MeV barn

Rate of reaction ?

at T6=15

How many times per second a proton is transformed into ${}^{2}He$?

at density=150 g/cm3

$$\int_0^\infty \exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) dE \approx \pi^{1/2} \frac{\Delta}{2} \exp\left(-\frac{3E_0}{kT}\right)$$

Solution: $\sigma(E_0)$

$$E_0 = 1.22\,(Z_1^2 Z_2^2\,\mu\,T_6^2)^{\frac{1}{3}}\ \text{keV}$$

$$\sigma(E) = \frac{S}{E}\exp(-2\pi\,\eta)$$

$$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}$$

with E in keV and μ in amu

$E_0 = 5.9$ keV

$\sigma(E_0) = 19$ mb

Solution: Reaction Rate

$$< \sigma v > = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{S}{(kT)^{3/2}} \underbrace{\int_0^\infty \exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) dE}$$

$$\approx \quad \pi^{1/2} \frac{\Delta}{2} \exp\left(-\frac{3E_0}{kT}\right)$$

$$< \sigma v > = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{S}{(kT)^{3/2}} \pi^{1/2} \frac{\Delta}{2} \exp\left(-\frac{3E_0}{kT}\right)$$

$$E_0 = 1.22\,(Z_1^2 Z_2^2\, \mu\, T_6^2)^{\frac{1}{3}} \quad \text{keV}$$

$$\Delta E = 0.749\left(Z_1^2 Z_2^2 \mu T_6^5\right)^{1/6} \quad \text{keV}$$

$$< \sigma v > = 1.30\text{x}10^{-12} S(\text{MeV.barn}) Z_1^{1/3} Z_2^{1/3} \mu^{-1/3} T_6^{-2/3} \exp(-42.43 (Z_1^2 Z_2^2\, \mu\, T_6^{-1})^{\frac{1}{3}})$$

k_B = $1.38\text{x}10^{-23}$ JK^{-1}
1 u = 931.5 MeV/c^2

Reaction rate at T_6=15

$$< \sigma v > = 3\text{x}10^{-19} \text{ cm}^3\text{s}^{-1}$$

Solution: rate of proton capture

$$p + p \Leftrightarrow {}^2He$$

$$dN_{2He} = + N_p N_p \int \varphi(v)\, v\, \sigma(v)\, dv \;\; dt$$

Reaction Rate $cm^3\ s^{-1}$

$$dN_p = -2\,\frac{1}{2}\,N_p^2 < \sigma v > dt$$

$$dN_p = -\lambda\, N_p dt$$

Abundance = 74% in mass

$$\lambda = 20 \times 10^6\ s^{-1}$$

$$\tau = \frac{1}{\lambda} = \frac{1}{N_p < \sigma v >}$$

$$\tau = 5 \times 10^{-8}\ s$$

Exercise

$$p + {}^{16}O \Rightarrow {}^{17}F + \gamma \qquad Q=+600\ \text{keV}$$

$\sigma(E_0)$?

Rate of reaction ?

Lifetime of proton ?

Solution

$$p + {}^{16}O \Rightarrow {}^{17}F + \gamma$$

$E_0 = 29$ keV

We suppose S(E) = constant = 10^{-3} MeV barn

$\sigma(E_0) = 9.0 \times 10^{-22}$ b

$<\sigma v> = 2.2 \times 10^{-45}$ cm^3 s^{-1}

$N_{16O} = 0.8\%$ (in mass) x 150 g/cm^3

$N_p \approx$ constant

$dN_{16O} = - <\sigma v> N_p N_{16O} dt = - \lambda N_{16O} dt$

Approximate reaction time

$\tau = 6.8 \times 10^{18}$ s $= 2 \times 10^{11}$ years

1929

Zur Frage der Aufbaumöglichkeit der Elemente in Sternen.

Von R. d'E. Atkinson und F. G. Houtermans in Berlin-Charlottenburg.

(Eingegangen am 19. März 1929.)

The first ones to apply quantum mechanics to astrophysics

Tabelle 1.

Element	Eindringungswahrscheinlichkeit $\bar{W}_1$ pro Stoß	Lebensdauer bezüglich Protoneneindringung τ
2 He 4	$2{,}7 \cdot 10^{-8}$	8 sec
3 Li 6	$9{,}7 \cdot 10^{-11}$	37 min
3 Li 7	$10{,}6 \cdot 10^{-11}$	34 min
(4 Be 6	$4{,}1 \cdot 10^{-13}$	6,3 Tage)
(4 Be 7	$5{,}1 \cdot 10^{-13}$	4,9 „)
(4 Be 8	$5{,}8 \cdot 10^{-13}$	4,3 „)
4 Be 9	$6{,}5 \cdot 10^{-13}$	3,9 „
(4 Be 10	$7{,}1 \cdot 10^{-13}$	3,6 „)
(4 Be 11	$7{,}6 \cdot 10^{-13}$	3,5 „)
5 B 10	$5{,}0 \cdot 10^{-15}$	1,4 Jahre
5 B 11	$5{,}4 \cdot 10^{-15}$	1,3 „
6 C 12	$6{,}2 \cdot 10^{-17}$	110 „
7 N 14	$8{,}4 \cdot 10^{-19}$	8 200 „
8 O 16	$1{,}5 \cdot 10^{-20}$	470 000 „
9 F 19	$3{,}1 \cdot 10^{-22}$	$2{,}3 \cdot 10^{7}$ „
10 Ne 20	$6{,}7 \cdot 10^{-24}$	$1{,}0 \cdot 10^{9}$ „

~Penetrability

~Effective lifetime

Zeitschrift für Physik A 54(9), 656-665 (1929)

Exercise

$$p + {}^{16}O \Rightarrow {}^{17}F + \gamma$$

We have seen:

$<\sigma v> = 2.2 \times 10^{-45}\ cm^3\ s^{-1}$

$N_{16O} = 0.8\%$ (in mass) $\times$ 150 g/cm^3

$N_p \approx$ constant

$dN_{16O} = - <\sigma v>\ N_p\ N_{16O}\ dt$

But only ~ 1% of the Sun's mass has T6=15 and density~150g cm^{-3}

Is this reaction responsible for the Sun's heat?

Solution

Rate (T6=15) $\langle \sigma v \rangle = 2.2 \times 10^{-45}\ cm^3\ s^{-1} = 1.32 \times 10^{-21}\ mole^{-1}\ cm^3\ s^{-1}$

$dN_{reactions} = \langle \sigma v \rangle\ N_p\ N_{16O}\ dt$

Energy production rate:

$$P(W\,cm^{-3}) = Q\,N_A\,\frac{dN_{reactions}}{dt}$$

Q=0.6 MeV/reaction
Avogadro number $N_A = 6.02 \times 10^{23}$
$N_p = 150\ g/cm^3 \times 74\% = 111\ moles\ cm^{-3}$
$N_p = 150\ g/cm^3 \times 0.8\ \% = 1.2\ moles\ cm^{-3}$

→ $P = 10^{-8}\ W\ cm^{-3}$

1 % of Mass of the Sun (2×10^{30} kg) = $1.3 \times 10^{29}\ cm^3$
P(16O+p)= 1.3×10^{21} W
Power of Sun P = 4×10^{26} Watts

→ 3.3×10^{-6} of the Sun’s heat

<σv> is the KEY quantity

Energy production | Timing | Change in abundances

To be determined from experiments and theoretical considerations

As star evolves

$\Rightarrow$ Temperature changes
$\Rightarrow$ Measure / Evaluate $\sigma(E)$ for each energy

The end

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