The inclusion of fermions

Weyl spinors

$$(\tfrac{1}{2},0) \quad (0,\tfrac{1}{2})$$

$$\psi_L \qquad \psi_R$$

2-component spinors of SU(2)

Dirac spinor

$$\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$$

Rotations and Boosts

$$\psi_{L(R)} \to S_{L(R)}\, \psi_{L(R)}$$

$$S_{L(R)} = e^{i\frac{\boldsymbol{\sigma}}{2}.\boldsymbol{\omega}} \quad : \text{ Rotations}$$

$$S_{L(R)} = e^{\pm\frac{\boldsymbol{\sigma}}{2}.\boldsymbol{\nu}} \quad : \text{ Boosts}$$

Dirac Gamma matrices

$$\gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & \boldsymbol{\sigma}_i \\ -\boldsymbol{\sigma}_i & o \end{bmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

Weyl basis

$$\gamma_\mu \equiv (\gamma_0, \gamma_i) \text{ 4-vector}$$

$$\psi_{L(R)} = \tfrac{1}{2}(1 \mp \gamma_5)\psi$$

The Dirac equation

Fermions described by 4-cpt Dirac spinors ψ

- $\psi^{\dagger}\gamma^{0}\psi \equiv \bar{\psi}\psi$ Lorentz invariant
- New 4-vector γ_{μ}

The Lagrangian

$$L = i\bar{\psi}\,\gamma_{\mu}\partial^{\mu}\,\psi - m\bar{\psi}\,\psi$$

Dimension?

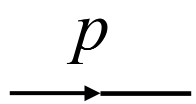
From Euler Lagrange equation obtain the Dirac equation

$$(i\gamma_{\mu}\partial^{\mu} - m)\psi = 0$$

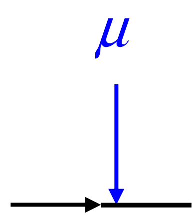
U(1) symmetry

$$\psi \to e^{i\alpha}\psi \quad \to \quad j^{\mu} = -e\bar{\psi}\,\gamma_{\mu}\,\psi$$

Feynman rules



$$\frac{i}{(\not{p} - m)}$$



$$ie\gamma_{\mu}$$

The Standard Model

$$SU(3)\otimes SU(2)\otimes U(1)$$

e.g. $SU(2)$ local gauge invariance

$$Q=\begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$

$$Q \rightarrow e^{ig_2\alpha(x).\frac{\sigma}{2}}Q$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$L = i\overline{Q}D_\mu\gamma^\mu Q$$

Need 3 gauge bosons

W^+, W^-, W^3

$$D_\mu = \partial_\mu + ig_2\frac{\sigma_i}{2}W_\mu^i$$

where

$$W_{\mu,i} \rightarrow W_{\mu,i} - \partial_\mu\alpha_i - g_2\varepsilon_{ijk}\alpha_j W_{\mu,k}$$

$$\left(\left[\frac{\sigma_i}{2},\frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}\right)$$

The strong interactions

QCD Quantum Chromodynamics

SU(3)

$$\begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_8 = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots$$

Symmetry :

Local conservation of
3 strong colour charges

$$\Psi_a \to \left(e^{ig_3 \boldsymbol{\alpha}(x).\boldsymbol{\lambda}}\right)^a_b \Psi_a$$

$$G^r_\mu \to G^r_\mu - \partial_\mu \alpha^r - g_3 f^{rst} \alpha^s G^t_\mu$$

$$\overline{\Psi}(\partial_\mu - ig_3 \lambda_r G^r_\mu) \gamma^\mu \Psi$$

QCD : a non-Abelian (SU(3)) local gauge field theory

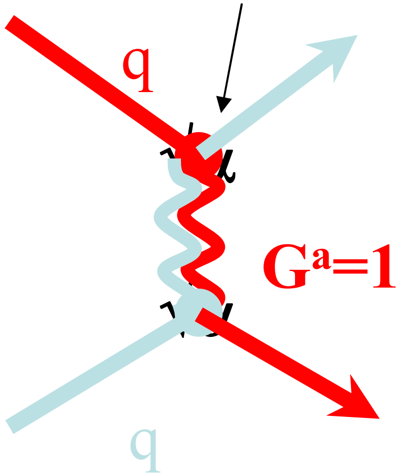
The strong interactions

QCD Quantum Chromodynamics

SU(3)

$$\begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

Strong coupling, α_3



G^a=1..8

Gauge boson
(J=1)
"Gluons"

Symmetry :

Local conservation of
3 strong colour charges

$$\Psi_a \to \left(e^{i\boldsymbol{\alpha}(x).\boldsymbol{\lambda}}\right)_b^a \Psi_a$$

$$G_\mu^r \to G_\mu^r - \tfrac{1}{g_3}\partial_\mu \alpha^r - f^{rst}\alpha^s G_\mu^t$$

$$\overline{\Psi}(\partial_\mu - ig_3\lambda_r G_\mu^r)\gamma^\mu \Psi$$

QCD : a non-Abelian (SU(3))
local gauge field theory

Weak Interactions

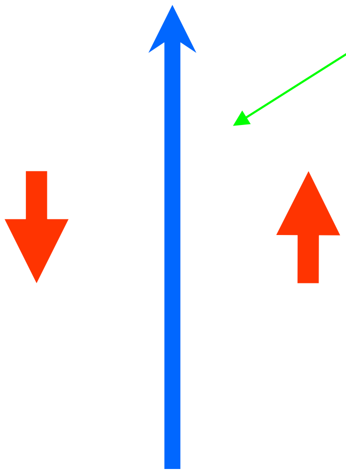
Fermi theory of β decay $\quad n \to p e \bar{\nu}_e$

$$\frac{G_F}{\sqrt{2}}\left[\bar{u}_n \gamma^\sigma (1-\gamma_5) u_p\right]\left[\bar{u}_{\nu_e} \gamma_\sigma \underbrace{(1-\gamma_5) u_e}\right]$$

$$u_{e^-_L} \equiv u^c_{e^+_R}$$

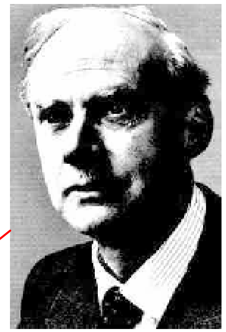
velocity

Left-handed



m=0

Right-handed

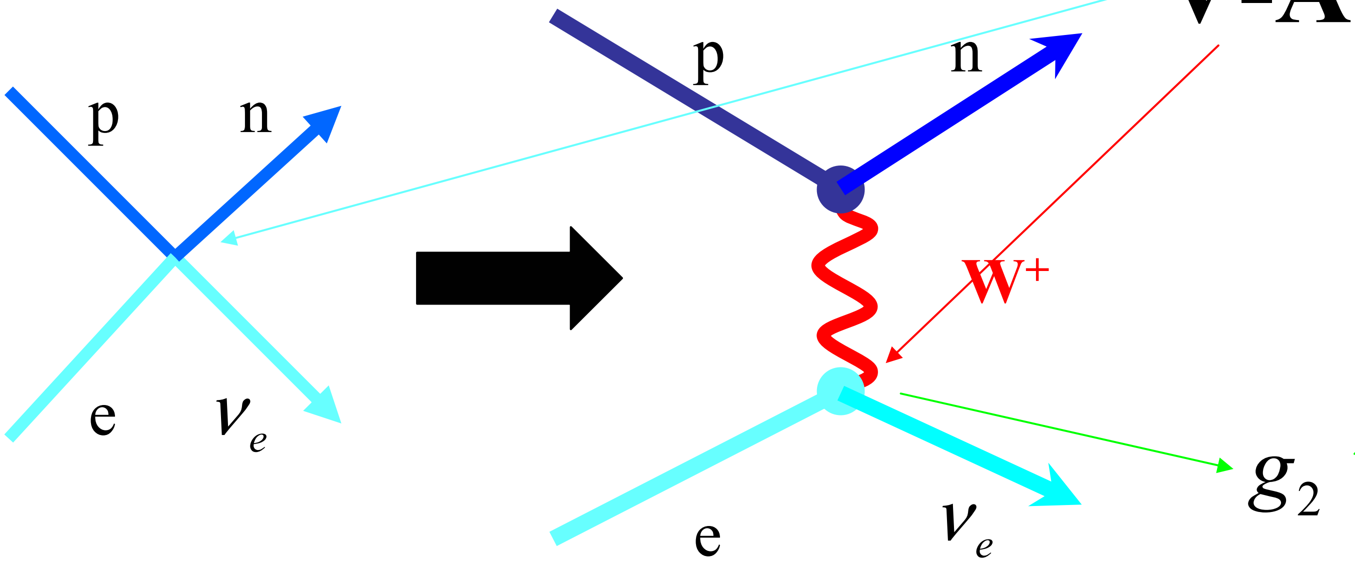


Weak Interactions

Fermi theory of β decay $\quad n \rightarrow p e \bar{\nu}_e$

$$L = \frac{G_F}{\sqrt{2}}\left[\bar{u}_n \gamma^\sigma (1-\gamma_5) u_p\right]\left[\bar{u}_{\nu_e} \gamma_\sigma (1-\gamma_5) u_e\right]$$

V-A $\quad \not{P}, \not{C}$



$$\frac{G_F}{\sqrt{2}} = \frac{g_2^{\,2}}{8 M_W^{\,2}}$$

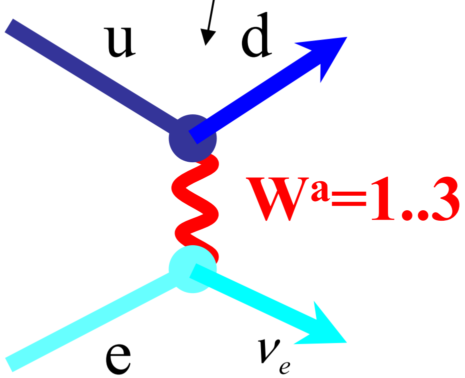
Weak Interactions

SU(2) local gauge theory

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$u_R, d_R, e_R$$

Weak coupling, α_2



Gauge boson (J=1)

$W^{\pm}, Z$

Neutral currents

Symmetry :

Local conservation of 2 weak isospin charges

$$\Psi_a \to \left(e^{ig_2 \boldsymbol{\alpha}(x).\tau}\right)_b^a \Psi_a$$

$$W_\mu^r \to W_\mu^r - \partial_\mu \alpha^r - f^{rst} \alpha^s W_\mu^t$$

$$\overline{\Psi}(\partial_\mu - ig_2 \lambda_r W_\mu^r)\gamma^\mu \Psi$$

A non-Abelian (SU(2)) local gauge field theory

Massive vector propagator (W, Z bosons)

$$\left(g^{\nu\mu}(\partial^2 + M^2) - \partial^\nu \partial_\mu\right) B^\mu = j^\nu$$

$$(-g^{\mu\nu}(-p^2 + M^2) + p_\mu p_\nu)^{-1} = \frac{i(-g^{\mu\nu} + p^\mu p^\nu / M^2)}{p^2 - M^2}$$

$$B_\mu = \varepsilon_\mu e^{ip.x}$$

Free particle solution

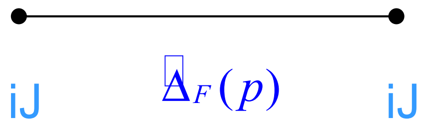
$$\varepsilon^{(\lambda=\pm 1)} = \mp(0, 1, \pm i, 0)/\sqrt{2}$$

$$\varepsilon^{(\lambda=0)} = (|\mathbf{p}|, 0, 0, E)/M$$

Helicity polarisation vectors

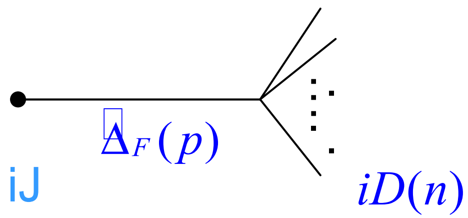
$$\sum_\lambda \varepsilon_\mu^{(\lambda)*} \varepsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}$$

Propagation of unstable scalar particle



$$= -iJ^2 \tilde{\Delta}_F(p)$$

No decay



$$= -iJ\tilde{\Delta}_F(p)D(n)$$

Particle decays into final state n

Optical theorem – conservation of probability, time evolution is unitary

$$S^\dagger S = SS^\dagger = 1$$

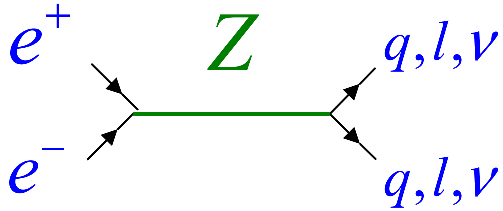
$$S_{fi} = \delta_{fi} + iT_{fi}$$

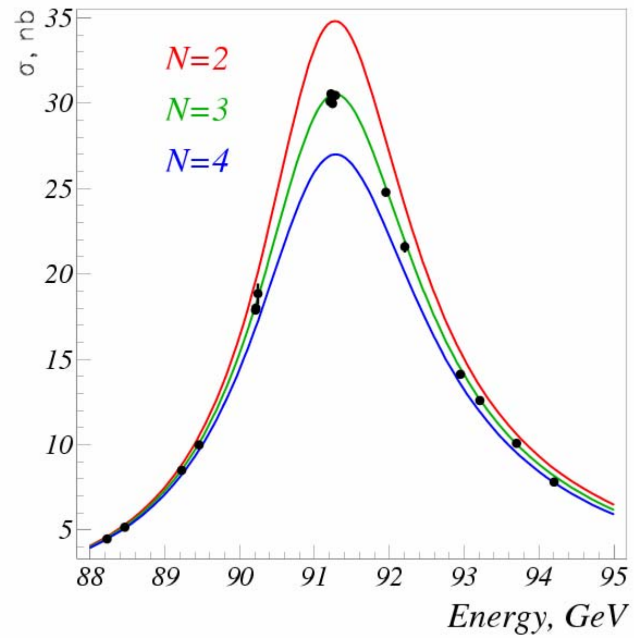
$$\mathrm{Im}(T_{kk}) = \tfrac{1}{2}\sum_n |T_{nk}|^2$$

$$-J^2\,\mathrm{Im}(\tilde{\Delta}_F(p)) = \tfrac{1}{2}\sum_n \left|-iJ\tilde{\Delta}_F(p)D\right|^2 = J^2\left|\tilde{\Delta}_F(p)\right|^2 \underbrace{\int \tfrac{1}{2}\left|D(n)\right|^2 dQ}_{m\Gamma_{tot}}$$

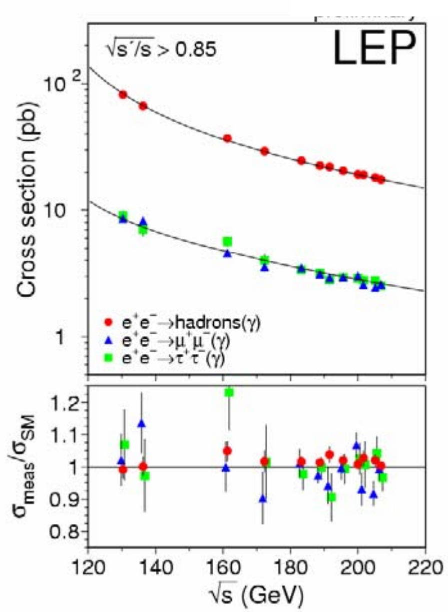
$$\tilde{\Delta}_F(p) = \frac{1}{p^2 - m^2 + im\Gamma_{tot}}$$

$$(\Delta_F(x) \propto e^{-m\Gamma_{tot}t})$$

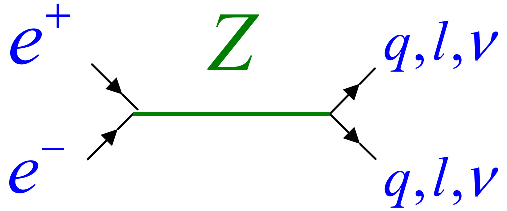


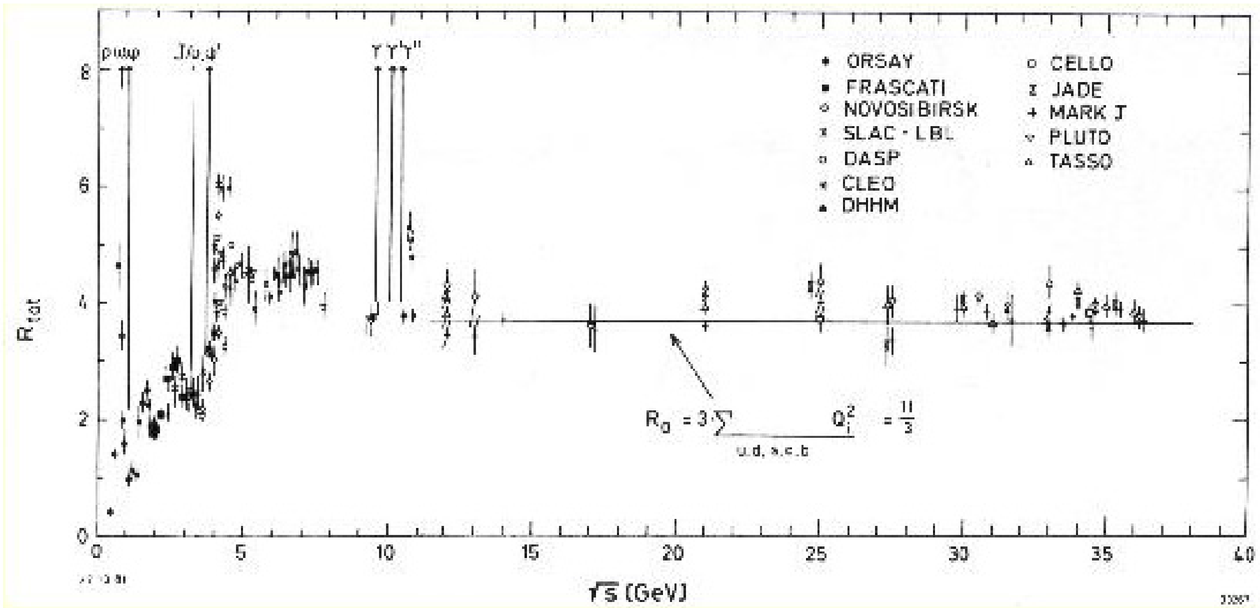


(N is no. of light - wrt M_Z - neutrinos)



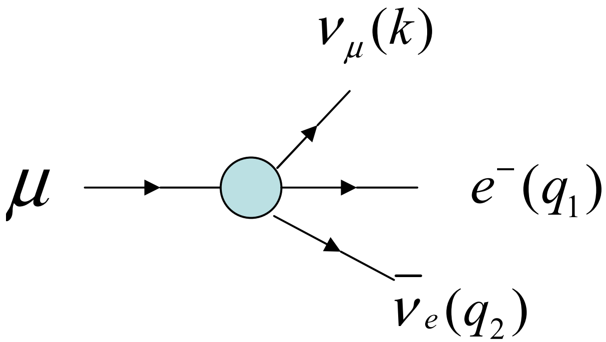
$$\sigma(e^+e^- \to hadrons) = \frac{12\pi\,\Gamma(Z \to ee)\Gamma(Z \to hadrons)}{(E^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$





$$\frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3\sum_{f=1}^{n_f} Q_f^2$$

μ *decay*



Fermi theory ('40s)

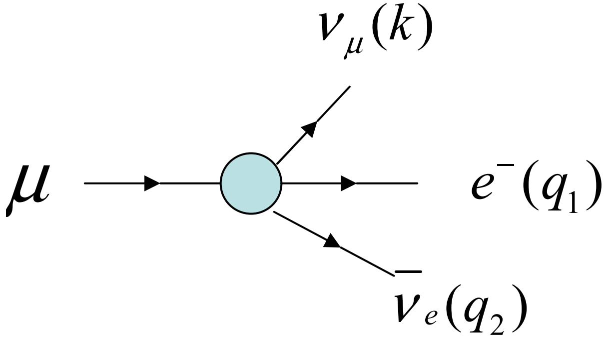
$$M = \frac{G_F}{\sqrt{2}} \bar{u}(k)\gamma^\mu(1-\gamma_5)u(p)\, \bar{u}(q\ \ )\gamma_\mu(1-\gamma_5)v(p)$$

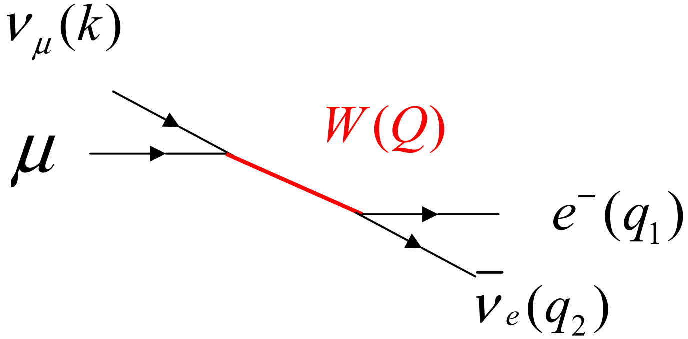
$$\Gamma_{tot} = \frac{1}{192\pi^3} m_\mu^5 G_F^2$$

The hard part!

$$\tau_\mu^{\exp t} = \frac{1}{\Gamma} = 2.19703(4)\, 10^{-6} \text{ sec} \quad \Rightarrow \quad G_F = 1.16637(1)\, 10^{-5}\ GeV^{-2}$$

μ *decay*





$$\frac{Q^\mu Q^\nu}{M_W^2} \sim \frac{m_\mu m_e}{M_W^2} \sim 0$$

$$M = i g_W \,\bar{u}(k)\gamma^\mu(1-\gamma_5)u(p) \quad \frac{g_{\mu\nu} - \frac{Q_\mu Q_\nu}{M_W^2}}{Q^2 - M_W^2 + i\varepsilon} \quad g_W \,\bar{u}(q\;\;)\gamma_\mu(1-\gamma_5)v(p)$$

In μ decay $\quad Q^2 \le O(m_\mu^2) << M_W^2$

$$\Rightarrow \quad \frac{g_W^2}{Q^2 - M_W^2} \sim \frac{-g_W^2}{M_W^2} \quad \Rightarrow \quad \frac{g_W^2}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

Fundamental principles of particle physics

- Introduction - Fundamental particles and interactions
- Symmetries I - Relativity
- Quantum field theory - Quantum Mechanics + relativity
- Theory confronts experiment - Cross sections and decay rates
- Symmetries II – Gauge symmetries, the Standard Model
- Fermions and the weak interactions

⇨ The Standard Model and Beyond

Have Fun!