Theoretical Concepts in Particle Physics (2)

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Yesterday...

- What is mechanics: $x(t) \Rightarrow L \Rightarrow$ symmetries
  - Q: the implication of $x_1 \to -3x_2$ and $x_2 \to -x_1/3$?
  - A: $q_1 = x_1 - 3x_2, \; q_2 = x_2 + 3x_1$ and we have $V(q_1)$
- What is field theory: $\phi(x, t)$ are the coordinates. x and t are parameters
- We can get the same axioms to field theories as for mechanics

Solving field theory

We also have an E-L equation for field theories

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi \right)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}$$

- We have a way to solve field theory
- Just like in Newtonian mechanics, we want to get $\mathcal{L}$ from symmetries!

Free field theory

- The "kinetic term" is promoted

$$T \propto \left(\frac{dx}{dt}\right)^2 \Rightarrow T \propto \left(\frac{d\phi}{dt}\right)^2 - \left(\frac{d\phi}{dx}\right)^2 \equiv (\partial_\mu \phi)^2$$

- Free particles fields have only kinetic terms

$$\mathcal{L} = (\partial_\mu \phi)^2 \Rightarrow \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial t^2}$$

  - An $\mathcal{L}$ of a free field gives a wave equation
  - As in Newtonian mechanics, what used to be the starting point, here is the final result
  - Why did we get the field equation as output?

Harmonic oscillator

The harmonic oscillator

Why do we care so much about harmonic oscillators?

- Because we really care about springs?
- Because we really care about pendulums?

The harmonic oscillator

Why do we care so much about harmonic oscillators?

- Because we really care about springs?
- Because we really care about pendulums?

Because almost any function around its minimum can be approximated as a harmonic function!

- Indeed, we usually expand the potential around one of its minima
- We identify a small parameter, and keep only few terms in a Taylor expansion

Classic harmonic oscillator

$$V = \frac{kx^2}{2}$$

We solve the E-L equation and get

$$x(t) = A\cos(\omega t) \qquad k = m\omega^2$$

- The period does not depend on the amplitude
- Energy is conserved

Which of the above two statements is a result of the approximation of keeping only the harmonic term in the expansion?

Coupled oscillators

Coupled oscillators

- There are normal modes
- The normal modes are not “local” as in the case of one oscillator
- The energy of each mode is conserved
- This is an approximation!
- Once we keep non-harmonic terms energy moves between modes

$$V(x,y) = \frac{k_1 x^2}{2} + \frac{k_2 y^2}{2} + \alpha x^2 y$$

- What determines the rate of energy transfer?

Things to think about

- Relations between harmonic oscillators and fields
- When can we treat an oscillator as harmonic?

Quantum mechanics

What is QM?

- Many ways to formulate QM
- For example, we promote $x \to \hat{x}$
- We solve QM if we know the wave function $\psi(x,t)$
- How many wave function describe a system?
- The wave function is mathematically a field

The quantum SHO

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \qquad E_n = (n + 1/2)\hbar\omega$$

- We also like to use

$$H = (a^\dagger a + 1/2)\hbar\omega \qquad a, a^\dagger \sim x \pm ip \qquad x \sim a + a^\dagger$$

- We call $a^\dagger$ and a creation and annihilation operators

$$a|n\rangle \propto |n-1\rangle \qquad a^\dagger|n\rangle \propto |n+1\rangle$$

- So far this is abstract. What can we do with it?

Couple oscillators and Fields

- With many DOFs, $a \to a_i \to a(k)$
- And the states

$$|n\rangle \to |n_i\rangle \to |n(k)\rangle$$

- And the energy

$$(n + 1/2)\hbar\omega \to \sum (n_i + 1/2)\hbar\omega_i \to \int [n(k) + 1/2]\hbar\omega dk$$

- Just like in mechanics, we expand around the minimum of the fields, and to leading order we have SHOs
- In QFT fields are operators while x and t are not

SHO and photons

I have two questions:

- What is the energy that it takes to excite a SHO by one level?
- What is the energy of the photon?

SHO and photons

I have two questions:

- What is the energy that it takes to excite a SHO by one level?
- What is the energy of the photon?

Same answer

$$\hbar\omega$$

- Why the answer to both question is the same? Can we learn anything from it?

What is a particle?

Excitations of SHOs are particles

More on QFT

What about masses?

- A SHO give a “free” Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2$$

- We can add “potential” terms (without derivatives)

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + m^2\phi^2$$

- Here m is the mass of the particle. Still free particle
- (HW) Show that it is a mass by showing that $\omega^2 = k^2 + m^2$

What about other temrs?

- We can add terms. How do we choose what to add?
- Must be invariant under the symmetries
- We keep some leading terms (usually, up to ϕ^4)
- Lets add $\lambda\phi^4$. We get the non-linear wave equation

$$\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial t^2} = 4\lambda\phi^3$$

- We do not knwo how to solve it classically, nor in QM

What about fermions?

- We see how photons are related to SHO
- We can construct a fermion SHO

$$[a, a^\dagger] = 1 \quad \rightarrow \quad \{b, b^\dagger\} = 1$$

- No classical analogue since $b^2 = 0$
- We can then think of fermionic fields. They can generate only one particle in a given state

A short summary

- Particles are excitations of fields
- The fundeumental Lagrangian is giving in term of fields
- Our aim is to find L
- We can only solve the linear case, that is, the equivalent of the SHO.

Perturbation theory

Perturbation theory

$$H = H_0 + H_1 \qquad H_1 \ll H_0$$

- In many cases pertubation theory is a mathematical tool
- There are cases, however, that PT is a better way to describe the physics
- Many times we prefer to work with EV of H (why?)
- Yet, at times it is better to work with EV of H_0 (why?)

PT for 2 SHOs

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- Classically α moves energy between the two modes
- How it goes in QM?
- Recall the Fermi golden rule

$$P \propto |\mathcal{A}|^2 \times \text{P.S.} \qquad \mathcal{A} \sim \langle f|\alpha x^2 y|i\rangle$$

- If the initial state is $|0,1\rangle$ what transitions are alowed?

$$x \sim a_x + a_x^\dagger \qquad y \sim a_y + a_y^\dagger$$

PT for 2 SHOs again

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- since the perturbation is $x^2 y$ we see that $f = |2,0\rangle$

$$\mathcal{A} \sim \langle 2,0|\alpha x^2 y|0,1\rangle \sim \langle 2,0|(a_x + a_x^\dagger)(a_x + a_x^\dagger)(a_y + a_y^\dagger)|0,1\rangle$$

- a_y in y annihilates the y "particle" and $(a_x^\dagger)^2$ in x^2 creates two x "particles"
- It is a decay of a particle y into two x particles with lifetime of $\tau \propto \alpha^2$

Even More PT

$$V' = \alpha x^2 z + \beta xyz \qquad \omega_z = 10,\ \omega_y = 3,\ \omega_x = 1$$

- Calculate $y \to 3x$ using 2nd order PT

$$\mathcal{A} \sim \langle 3,0,0|\mathcal{O}|0,1,0\rangle \qquad \mathcal{O} \sim \sum \frac{\langle 3,0,0|V'|n\rangle\langle n|V'|0,1,0\rangle}{E_n - E_{0,1,0}}$$

- What intermediate states contribute? $|1,0,1\rangle$ and $|2,1,1\rangle$

$$|0,1,0\rangle \to |1,0,1\rangle \to |3,0,0\rangle + |0,1,0\rangle \to |2,1,1\rangle \to |3,0,0\rangle$$

What is the meaning of the $1/\Delta E$?