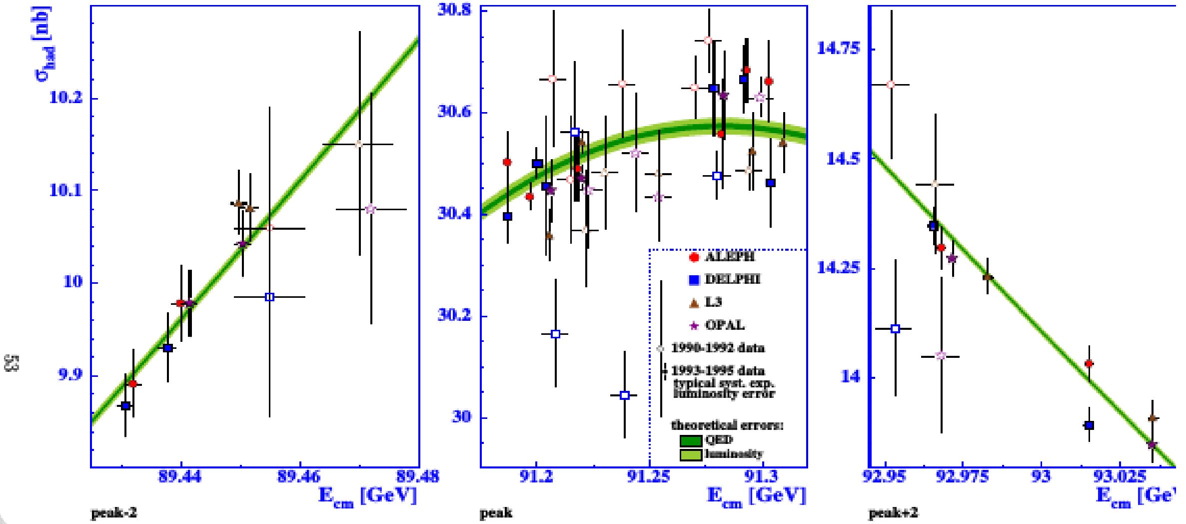
Karlsruhe Institute of Technology

Pseudo-Observables at LEP

Günter Quast **Workshop „Pseudo-observables: from LEP to LHC“** CERN, April 9th, 2015

Institut für Experimentelle Kernphysik



m_Z [GeV]	91.1876 ± 0.0021
Γ_Z [GeV]	2.4952 ± 0.0023
σ^0_{had} [nb]	41.541 ± 0.037
R^0_e	20.804 ± 0.050
R^0_μ	20.785 ± 0.033
R^0_τ	20.764 ± 0.045
$A^{0,e}_{FB}$	0.0145 ± 0.0025
$A^{0,\mu}_{FB}$	0.0169 ± 0.0013
$A^{0,\tau}_{FB}$	0.0188 ± 0.0017

Measurements in $e^+ e^-$ at LEP – goals

Precision measurements at the electroweak scale

- determination of parameters of the Z-Boson:
  mass, total width, partial widths, couplings to fermions
- WW-boson pair production and validation of gauge structure
  of ZWW and γWW vertices, search for “anomalous couplings”
- test of the quantum structure of the electroweak interaction
  effects from propagator and vertex loops

led to

The Nobel Prize in Physics 1999
Gerardus 't Hooft, Martinus J.G. Veltman

for “elucidating the quantum structure of the electroweak interaction”

and

prediction of the top-quark mass, and – together with top and W mass from Tevatron – prediction of Higgs boson mass from virtual corrections

The Menu of this Lecture

- Observables at LEP 1:

  ~500 measurements in total of

  * total cross sections

  * forward-backward asymmetries

  over 6 years by 4 experiments at ~20 “energy points”

- Choice of "Pseudo-Obervables"
  - 9 parameters, boils down to five with lepton-universality
  - model-independent with 16 parameters (S-matrix)

- several „electroweak libraries“ fast enough for fitting:

  BHM/MIZA, ZFITTER and TOPAZ0

  documented in "The Standard Model in the Making, Bardin/Passarino.

- Standard-Model fits: σ / A_{FB} vs. POs

- Final combination of LEP results is based on POs

  what are the uncertainties ?

- An application: the last fit by the LEP-EWWG to precision-POs

Measurements in $e^+ e^-$ at LEP – the four experiments

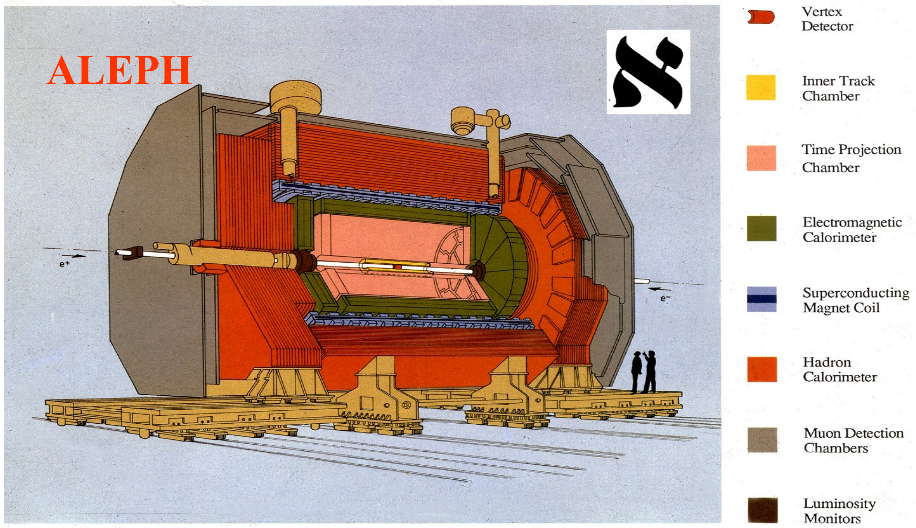
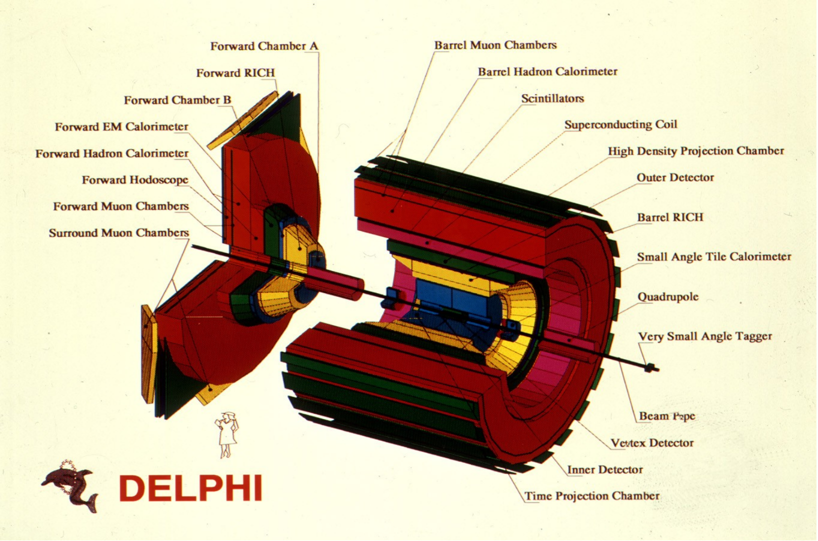
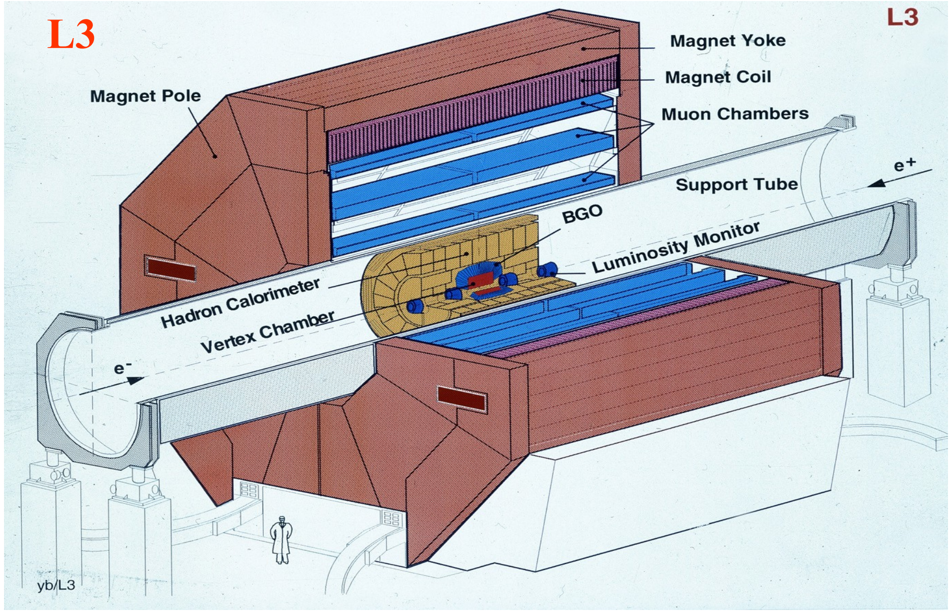
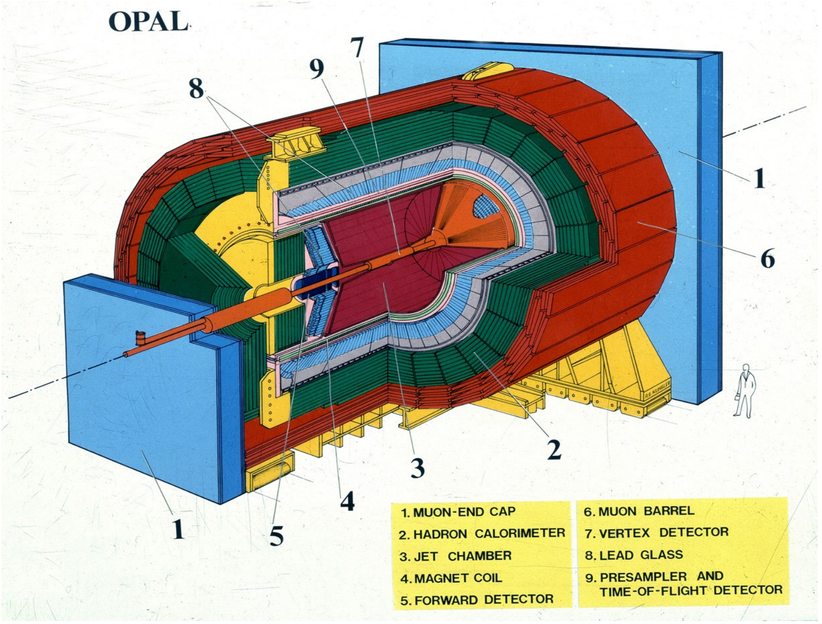


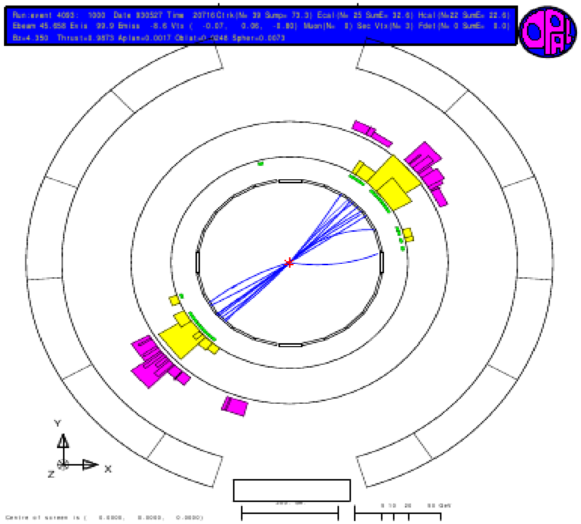
Fig. 1 - The ALEPH Detector

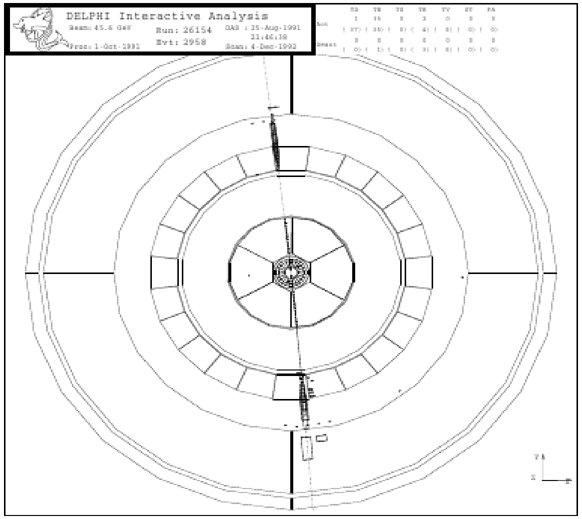




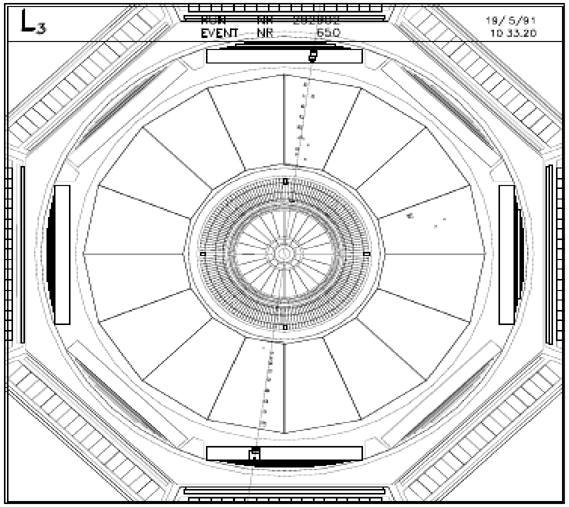


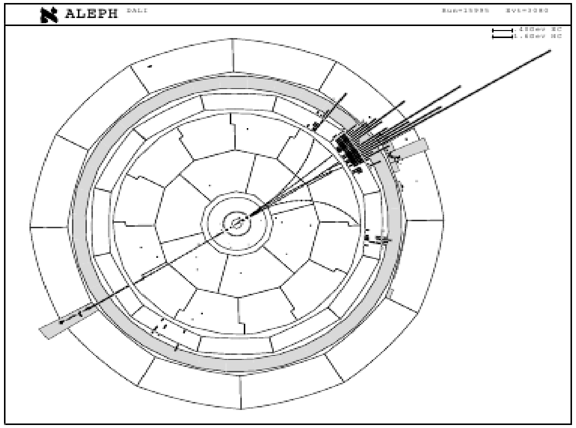
Event displays



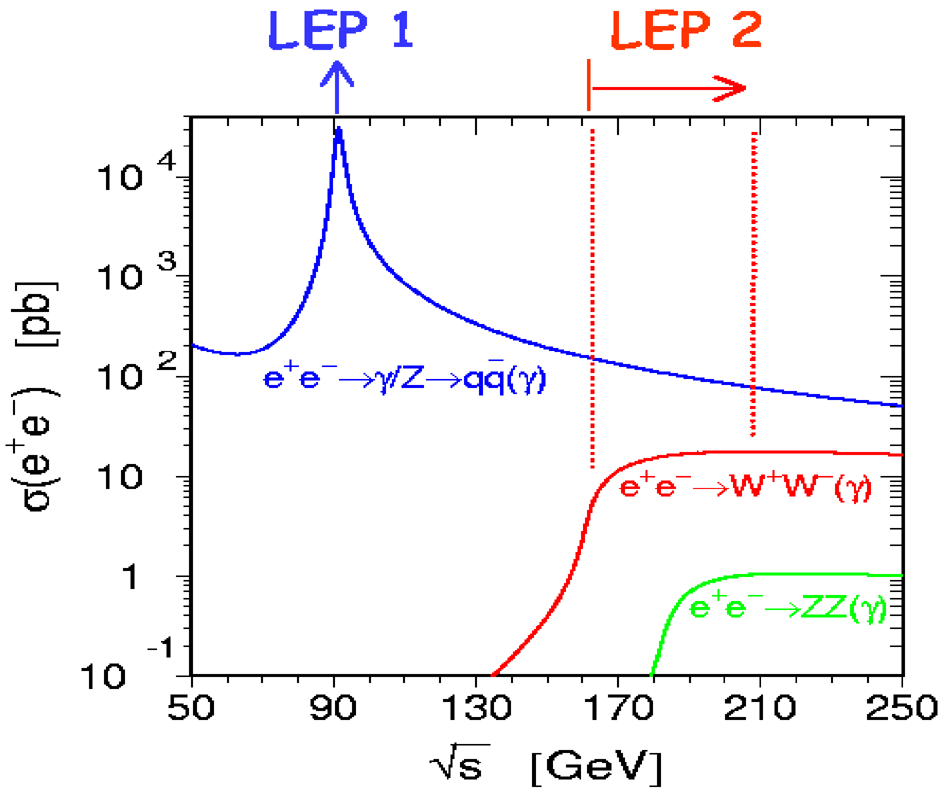


Event Displays from LEP experiments





LEP 1 and LEP 2



LEP 1: measurement of Z boson parameters (~16 million Z's)
LEP 2: measurement of W and Z boson pair production,
W boson Parameters

(main) Achievements at LEP

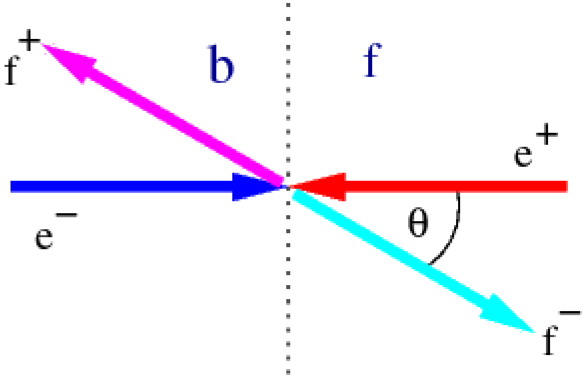
LEP 1: 17 Million Z-boson decays recorded by the four experiments

- precise determination of Z-boson parameters,
- determination of the number of light neutrino-species
- precise measurements of the weak mixing-angle
- prediction of top-quark and W-boson masses from radiative corrections

LEP 2: integrated Luminosity of 3/fb recorded by the four experiments
at centre-of-mass energies between 130 and 209 GeV

- measurements of W-pair production (from ~40'000 W-pairs in total)
- W-boson mass and width
- studies of fermion- and photon-pair production
- studies of four-fermion processes
- self-couplings of electroweak gauge bosons
- searches: Higgs-Boson, supersymmetry …
- *together with top- and W-boson mass from Tevatron:*
  prediction of Higgs-boson mass from radiative corrections

The observables at LEP 1: differential cross sections



total cross section

$$\boxed{\sigma_{tot} \equiv \int_{-1}^{1} \frac{d\sigma}{d\cos\theta} d\cos\theta}$$

depends on ($1+\cos^2\Theta$) terms only

Forward-backward asymmetry

$$\boxed{A_{FB} \equiv \frac{\int_0^1 \frac{d\sigma}{d\cos\theta}\, d\cos\theta - \int_{-1}^{0} \frac{d\sigma}{d\cos\theta}\, d\cos\theta}{\sigma_{tot}}}$$

depends on $\cos\Theta$ terms only

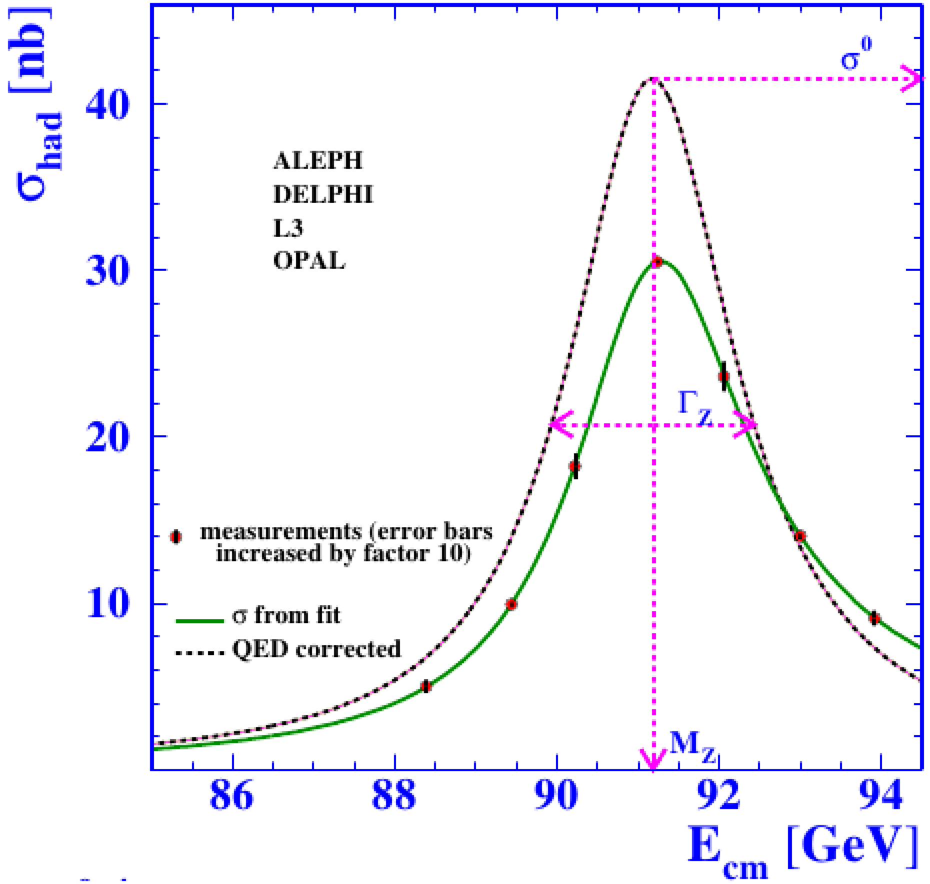
Principle:

Extract combinations of Z-couplings to fermions form measurements of σ_{tot} and A_{FB} in different channels and at different energies

Fermion-pair cross-section around Z resonance

experimentally:

$$\sigma(E_i) = \frac{N^{\mathrm{cand}}_{\mathrm{f\bar{f}}}(E_i) - N^{\mathrm{bkg}}_{\mathrm{f\bar{f}}}(E_i)}{\varepsilon_{\mathrm{ac}}(E_i)} \, \frac{1}{\int L\,(E_i)}\,,$$



Breit-Wigner resonance
only recovered after application
of large (photonic) corrections

Interference with photon assumed
as in Standard Model

Fundamental parameters of interest are **„hidden“.**

Need theoretical corrections to access them.

M_Z: „LEP definition“ vs. „pole mass“

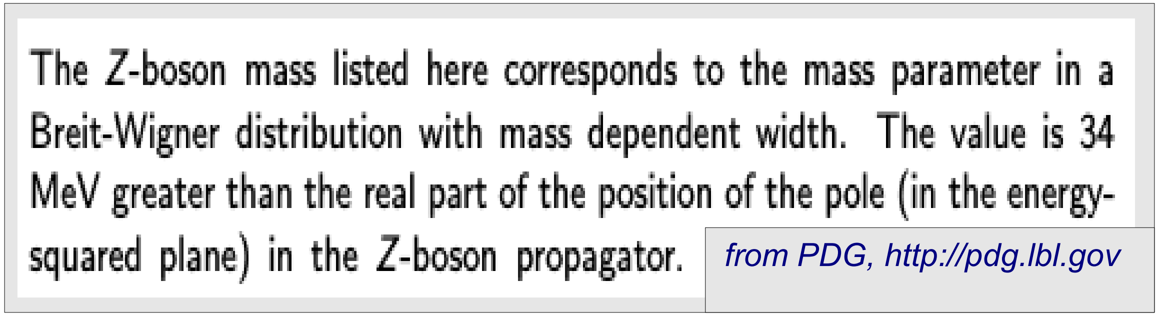
Yellow Report „Z Physics at LEP 1“ (CERN-89-08) suggested to use a „*Breit-Wigner with s-dependent width*“ to parametrise the Z resonance:

$$\sigma_{Z\to f\bar{f}}(s) \;=\; \sigma_f^{\text{peak}} \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

this differs from the usual „pole mass“ by a factor $\sqrt{1+{\Gamma_Z}^2/{M_Z}^2}$

(corresponding to 34 MeV)

and is the origin of the remark on the Z-boson mass in the PDG table:



The Z-boson mass listed here corresponds to the mass parameter in a Breit-Wigner distribution with mass dependent width. The value is 34 MeV greater than the real part of the position of the pole (in the energy-squared plane) in the Z-boson propagator.

from PDG, http://pdg.lbl.gov

btw.: the same remark holds for M_W

Message: while “observables” (like cross sections) are rather unambiguous, the exact definition of “pseudo-observables” does matter !

The improved Born-approximation

can cast differential cross-section

into a **Born-type structure** with complex effective couplings:

$$\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\sigma_{\mathrm{ew}}}{d\cos\theta}(\mathrm{e}^+\mathrm{e}^- \to \mathrm{f}\bar{\mathrm{f}}) \;=$$

$$\underbrace{|\alpha(s)Q_{\mathrm{f}}|^2\,(1+\cos^2\theta)}_{\gamma}$$

$$\underbrace{-8\Re\left\{\alpha^*(s)Q_{\mathrm{f}}\,\chi(s)\left[\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Vf}}(1+\cos^2\theta)+2\mathcal{G}_{\mathrm{Ae}}G_{\mathrm{Af}}\cos\theta\right]\right\}}_{\gamma-\mathrm{Z\ interference}}$$

$$\underbrace{\begin{gathered}+16|\chi(s)|^2\left[\;(|\mathcal{G}_{\mathrm{Ve}}|^2+|\mathcal{G}_{\mathrm{Ae}}|^2)(|\mathcal{G}_{\mathrm{Vf}}|^2+|G_{\mathrm{Af}}|^2)(1+\cos^2\theta)\right.\\ \left.+8\Re\{\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Ae}}^*\}\Re\{\mathcal{G}_{\mathrm{Vf}}G_{\mathrm{Af}}^*\}\;\;\cos\theta\;\right]\end{gathered}}_{\mathrm{Z}}$$

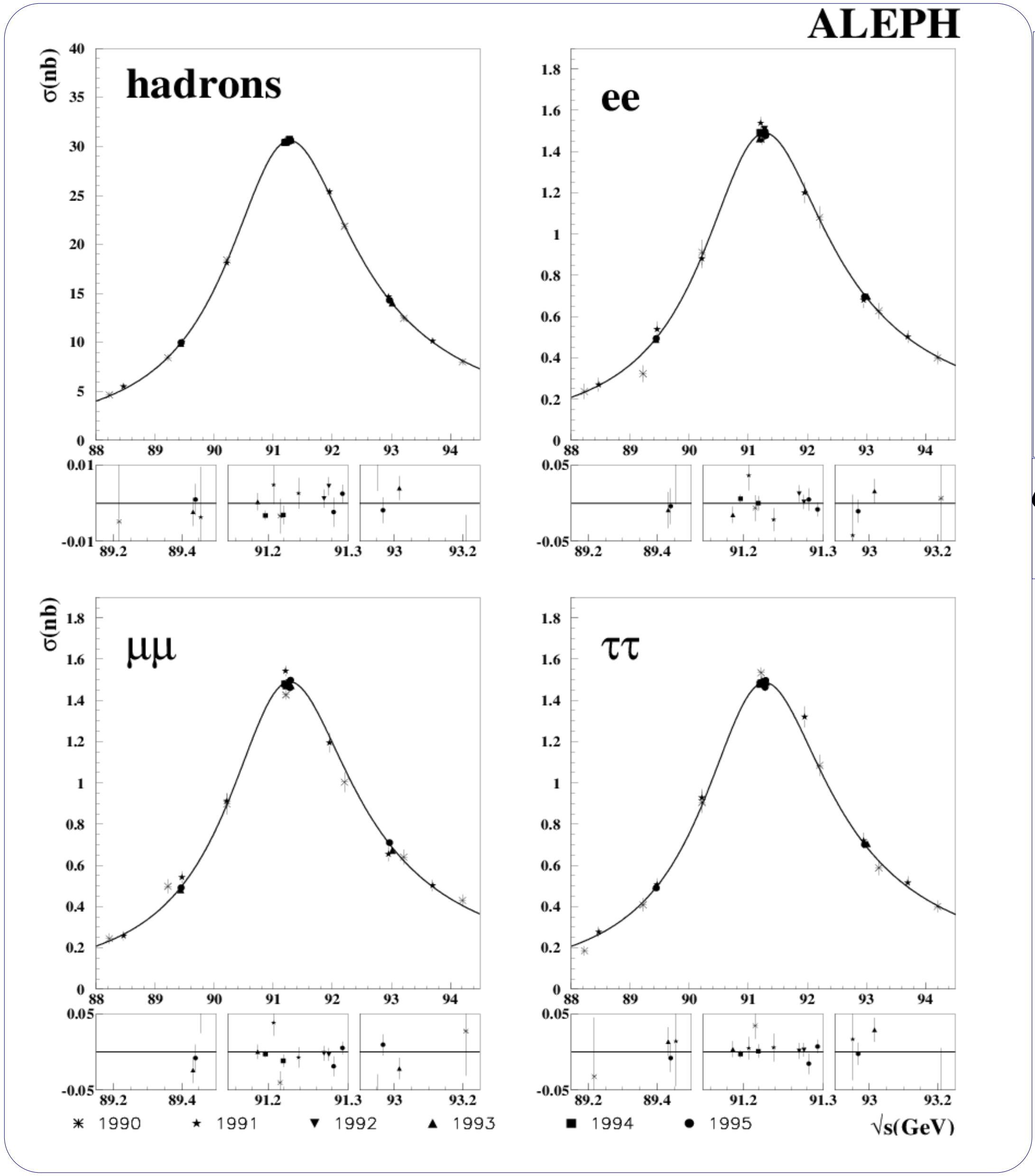
with

$$\text{with}\quad \chi(s)=\frac{m_{\mathrm{Z}}^2}{8\pi\sqrt{2}}\frac{s}{s-m_{\mathrm{Z}}^2+i\Gamma_{\mathrm{Z}}m_{\mathrm{Z}}}$$

Improved Born Approximation: Remarks

- This parametrisation describes the main features of the measurements around the Z resonance
- parameters are not “realistic observables”, but rather “pseudo-observables” which receive significant theoretical corrections;
  however, their definition is <u>close</u> to experimental observables
- very fortunate: QED-effects depend in a model-independent manner on the resonance properties
- QED-deconvoluted pseudo-observables absorb electroweak corrections
- There are, however, small non-factorizable (complex-valued) corrections, so-called “remnants”, wich are included in the complex couplings
  These effects are small in the SM
  (e.g. amounting to 0.05% for σ^0, which is negligible compared to the experimental errors)
  but may not be so small in other (arbitrary exotic) theories !
- the parametrisation is “model-independent” in all cases where predicted remnants are small

Example: final ALEPH results, hadrons and leptons (publ. 1999)



Oberservables:

measured cross-sections
@ LEP 1 by one experiment:

80 individual measurements:
4 final-states @
20 different “energy points”

$$\sigma_i^f(E_i), \quad i = 1, \ldots 20, \; f = \mathrm{q}, \mathrm{e}, \mu, \tau$$

well described by 5 POs:

$$m_\mathrm{Z}$$

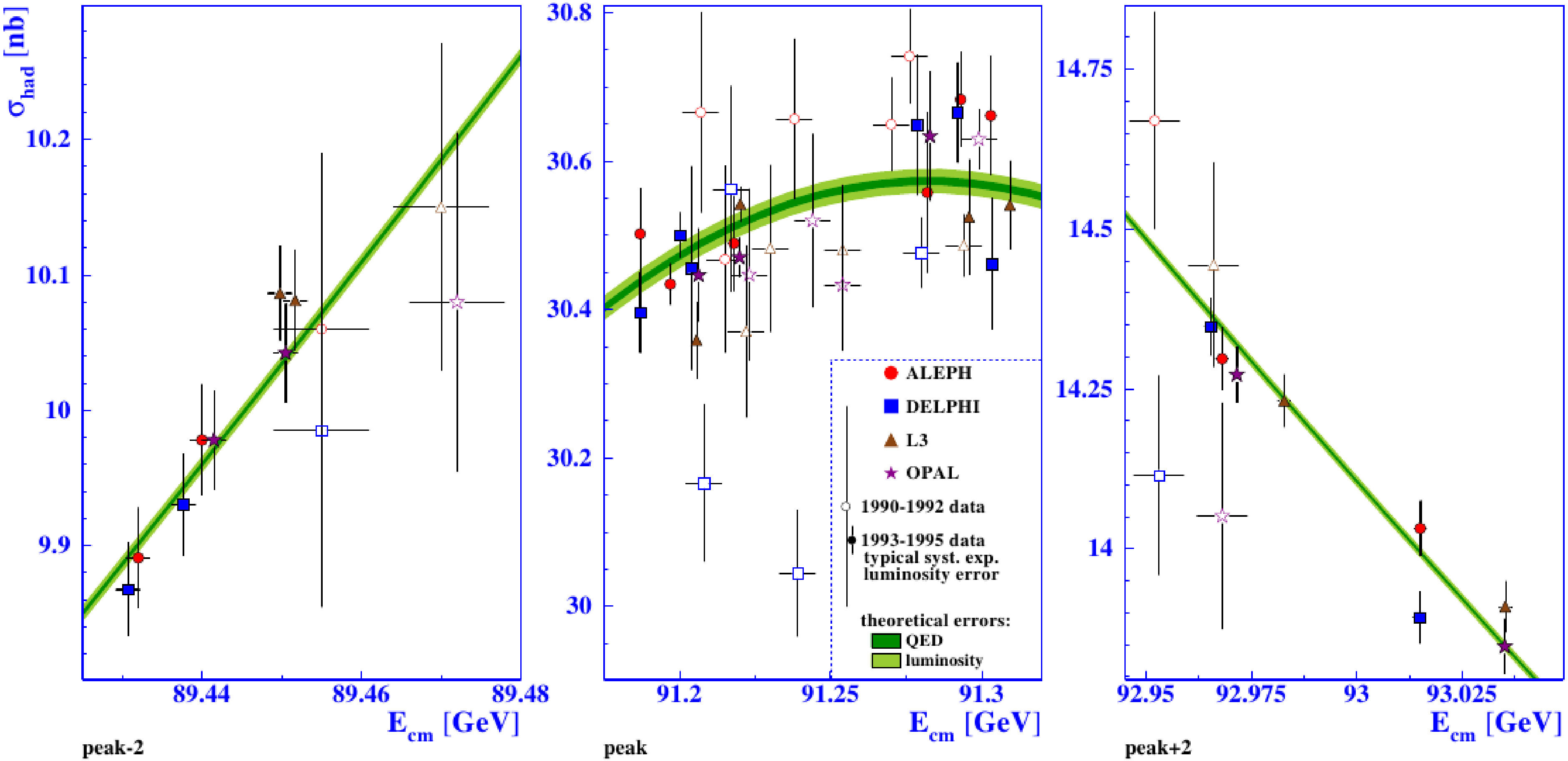
$$\sigma^0_\mathrm{had} = \frac{12\pi}{m_\mathrm{Z}^2} \frac{\Gamma_\mathrm{e}\Gamma_\mathrm{had}}{\Gamma_\mathrm{Z}^2}$$

$$R_\mathrm{e} = \Gamma_\mathrm{e}/\Gamma_\mathrm{had}$$

$$R_\mu = \Gamma_\mu/\Gamma_\mathrm{had}$$

$$R_\tau = \Gamma_\tau/\Gamma_\mathrm{had}$$

Observables: Hadronic cross-sections by the four experiments

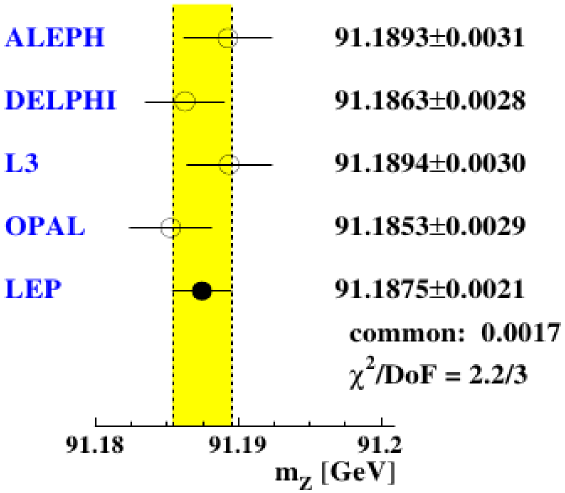


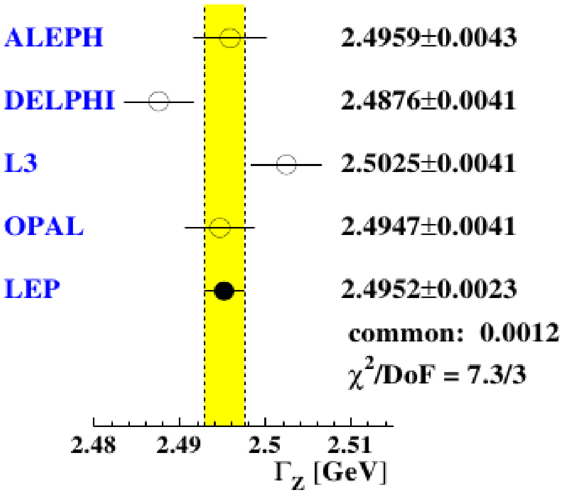
The challenge: ~300 individual measurements
(different channels, CM energies and data taking periods) to obtain

M_Z, Γ_Z and pole production cross sections of q, e, μ and τ

Combining four sets of "Pseudo-Observables"

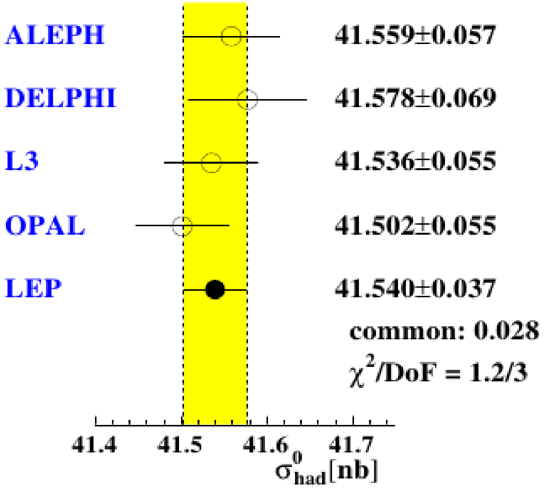
Parametrization of differential cross-section using **"pseudo-observables"**

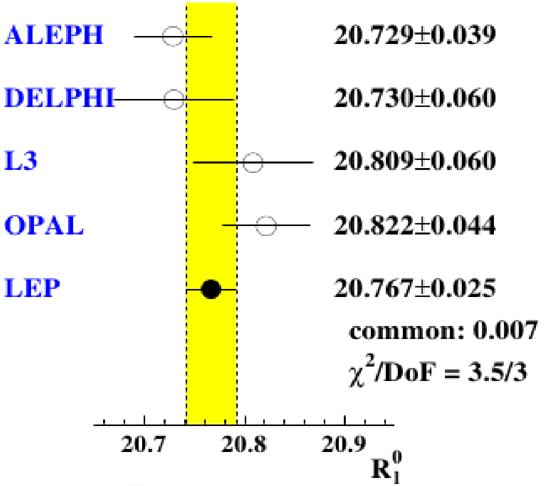




Choice of parameters such that correlated experimental errors are minimized:

- m_Z
- Γ_Z
- $\sigma^o_{\mathrm{had}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{\mathrm{had}}}{\Gamma_Z^2}$
- $R_e = \Gamma_{\mathrm{had}} / \Gamma_{ee}$
- $R_\mu = \Gamma_{\mathrm{had}} / \Gamma_{\mu\mu}$
- $R_\tau = \Gamma_{\mathrm{had}} / \Gamma_{\tau\tau}$





partial decay width

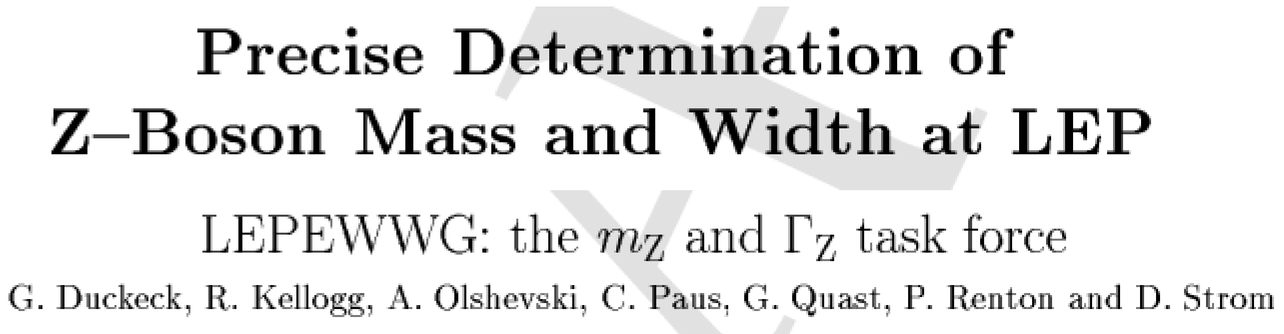
$$\Gamma_{ff} \propto \left(g_{Vf}{}^2 + g_{Af}{}^2\right) \text{ for f=e, } \mu, \tau$$

R_ℓ is a common parameter for a massless, universal lepton

Ok to combine experiments at PO-level only ?

??? Is it ok to use pseudo-observables ?
Or, must the measurements be combined at cross-section level ?

Check performed by LEP-EWWG



Precise Determination of
Z–Boson Mass and Width at LEP

LEPEWWG: the m_Z and Γ_Z task force

G. Duckeck, R. Kellogg, A. Olshevski, C. Paus, G. Quast, P. Renton and D. Strom

internal note LEPEWWG/LS 98-01

Tested with 4×7 precisely measured hadronic cross sections

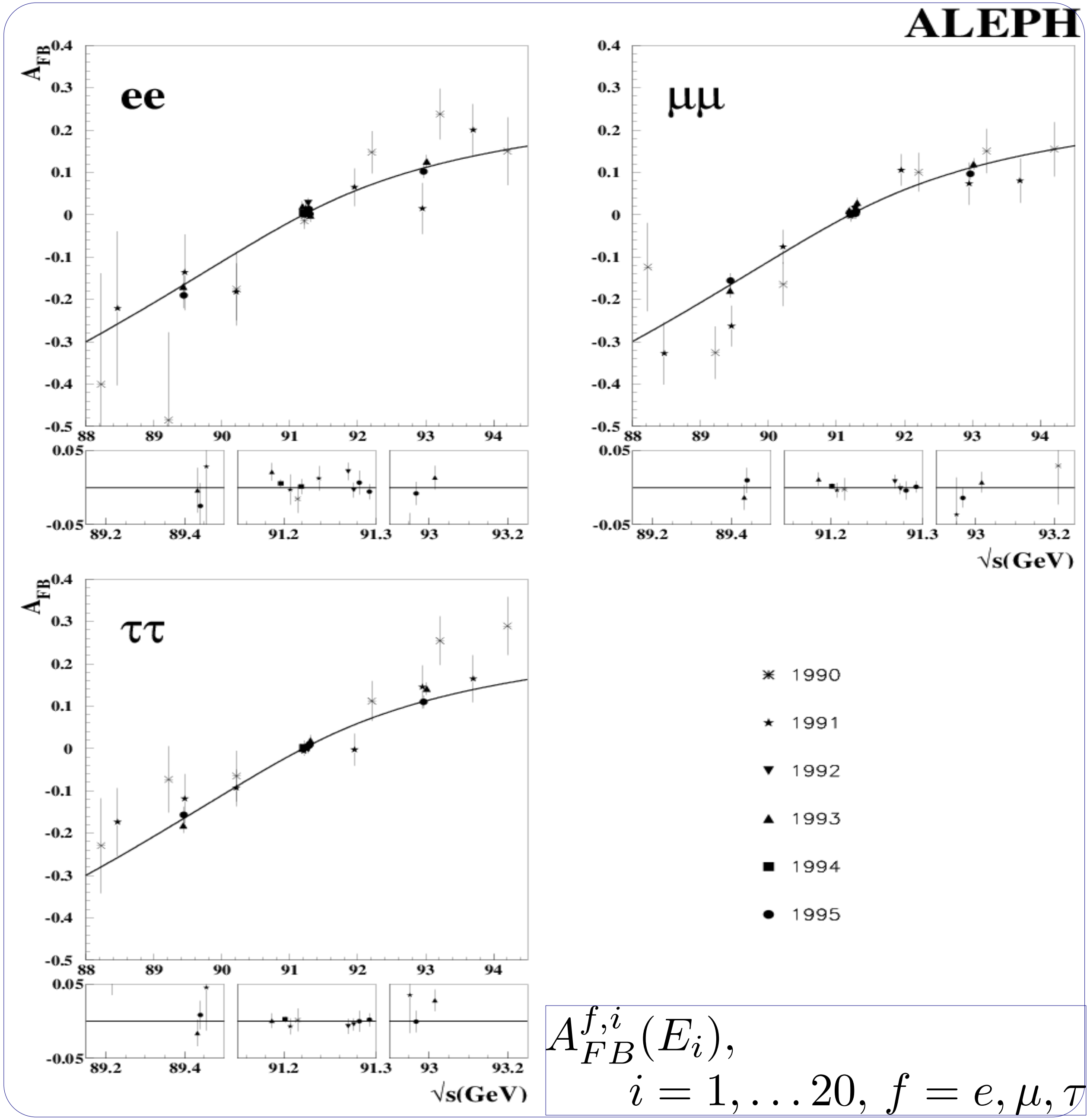
1. combine four sets of (three) parameters
2. determine parameters by combining 28 cross-section measurements

Results indicated only a small experimental problem
(resulting form treatment of energy errors on the '93 and '95 values of the Z mass)

but no „theoretical“ problem

→ final combination of all LEP I results was based on POs

Example: letpton forward-backward asymmetries



$$A_{FB}^{f,i}(E_i),$$
$$i = 1, \ldots 20,\ f = e, \mu, \tau$$

Parametrization requires three more POs:

$$A_{FB}^{0,f} \equiv \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{f}},\ \mathrm{f} = \mathrm{e}, \mu, \tau$$

with

$$\mathcal{A}_{\mathrm{f}} = \frac{2g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1+(g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}$$

n.b. :

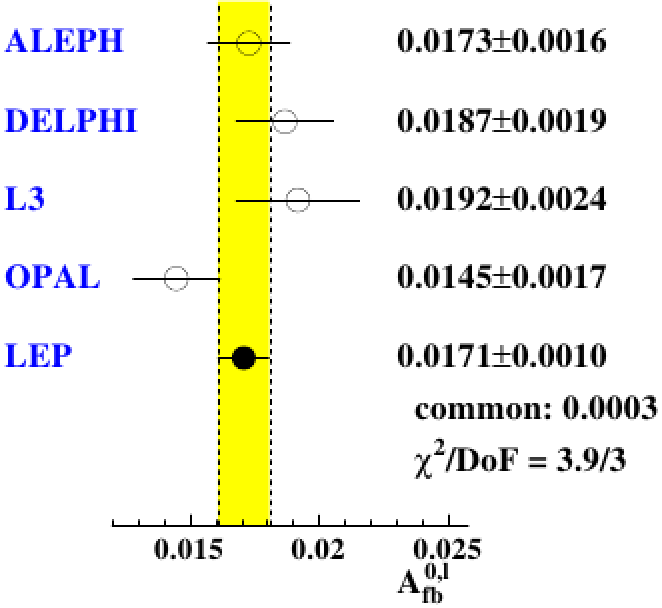
the description of all ALEPH measurements by the 9 parameters is good, with χ^2/d.o.f.= 169/176

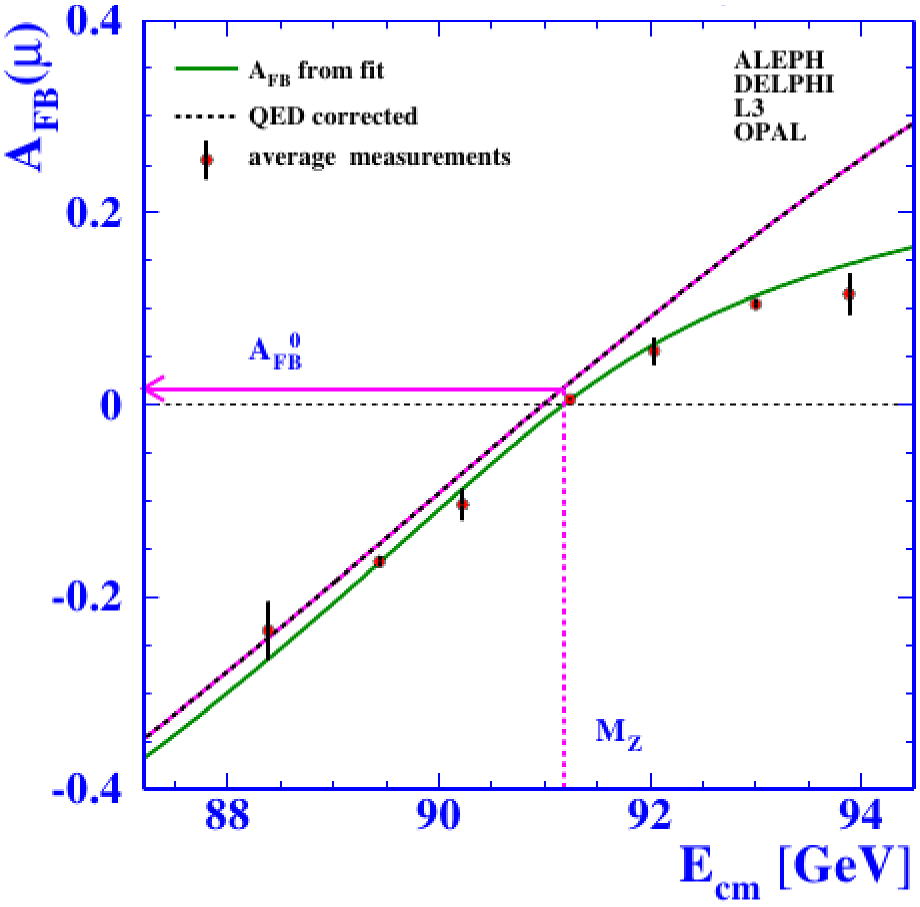
Forward-Backward Asymmetry

$$A_{FB} = \frac{N_{forw} - N_{back}}{N_{tot}} = \frac{\int_0^1 \frac{d\sigma}{d\cos\theta}\, d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta}\, d\cos\theta}{\sigma_{tot}}$$

pseudo-observable:

$$A_{FB}^{0,f} \equiv \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \quad \text{with} \quad \mathcal{A}_f = \frac{2\, g_{Vf}/g_{Af}}{1 + \left(g_{Vf}/g_{Af}\right)^2}$$





LEP I: combined line-shape results

Set of combined, well-understood pseudo-observables and their correlated errors represents a very effective way to preserve "legacy results":

Without lepton universality		Correlations								
$\chi^2/\mathrm{dof} = 32.6/27$		m_Z	Γ_Z	σ^0_{had}	R^0_e	R^0_μ	R^0_τ	$A^{0,e}_{\mathrm{FB}}$	$A^{0,\mu}_{\mathrm{FB}}$	$A^{0,\tau}_{\mathrm{FB}}$
m_Z [GeV]	91.1876 ± 0.0021	1.000								
Γ_Z [GeV]	2.4952 ± 0.0023	−0.024	1.000							
σ^0_{had} [nb]	41.541 ± 0.037	−0.044	−0.297	1.000						
R^0_e	20.804 ± 0.050	0.078	−0.011	0.105	1.000					
R^0_μ	20.785 ± 0.033	0.000	0.008	0.131	0.069	1.000				
R^0_τ	20.764 ± 0.045	0.002	0.006	0.092	0.046	0.069	1.000			
$A^{0,e}_{\mathrm{FB}}$	0.0145 ± 0.0025	−0.014	0.007	0.001	−0.371	0.001	0.003	1.000		
$A^{0,\mu}_{\mathrm{FB}}$	0.0169 ± 0.0013	0.046	0.002	0.003	0.020	0.012	0.001	−0.024	1.000	
$A^{0,\tau}_{\mathrm{FB}}$	0.0188 ± 0.0017	0.035	0.001	0.002	0.013	−0.003	0.009	−0.020	0.046	1.000

With lepton universality		Correlations				
$\chi^2/\mathrm{dof} = 36.5/31$		m_Z	Γ_Z	σ^0_{had}	R^0_ℓ	$A^{0,\ell}_{\mathrm{FB}}$
m_Z [GeV]	91.1875 ± 0.0021	1.000				
Γ_Z [GeV]	2.4952 ± 0.0023	−0.023	1.000			
σ^0_{had} [nb]	41.540 ± 0.037	−0.045	−0.297	1.000		
R^0_ℓ	20.767 ± 0.025	0.033	0.004	0.183	1.000	
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	0.055	0.003	0.006	−0.056	1.000

Possible Alternative: combination at cross-section level

We could also have produced combined cross-sections and asymmetries,

$$\sigma_i^f(E_i),\ i=1,\ldots 20,\ f=\mathrm{q},\mathrm{e},\mu,\tau,\quad A_{\mathrm{FB}}^{f,i}(E_i),\ i=1,\ldots 20,\ f=\mathrm{e},\mu,\tau$$

but these would have been very complicated to handle:

- correlated errors on all σ_i^f
- not measured at a fixed value of centre-of-mass energy, E_i
  (there is a spread in energy due to the fill-to-fill reproducibility of the beam-energy and the natural beam-energy spread of the accelerator)
  energy-spread correction depends on the line shape !
- energy errors are correlated

So, one would have needed a large set of numbers to describe the measurements:

$\sigma_i^f(E_i),\ \Delta\sigma_i^f,\ A_{\mathrm{FB}}^{f,i},\ \Delta A_{\mathrm{FB}}^{f,i}, \Delta E_i,\ \delta_{E_i}^{\mathrm{spread}}$

and the correlation matrices C_σ, C_A, C_E

for a total of ~80 combined cross-sections and 60 combined asymmetries around 20 different energy points

Such sets of numbers have been produced by each experiment individually, however, no LEP combination at this level has been attempted.

S-Matrix Approach: a more general set of parameters

The S-Matrix Ansatz describes the differential cross-section assuming one massless and one massive vector boson:

$$\sigma^0_{\mathrm{tot,f}}(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{g_{\mathrm{f}}^{tot}}{s} + \frac{j_{\mathrm{f}}^{tot}(s-\overline{m}_{\mathrm{Z}}^2) + r_{\mathrm{f}}^{tot}\, s}{(s-\overline{m}_{\mathrm{Z}}^2)^2 + \overline{m}_{\mathrm{Z}}^2 \overline{\Gamma}_{\mathrm{Z}}^2} \right] \quad \text{with } \mathrm{f} = \mathrm{had}, \mathrm{e}, \mu, \tau$$

$$A^0_{\mathrm{fb,f}}(s) = \pi\alpha^2 \left[\frac{g_{\mathrm{f}}^{fb}}{s} + \frac{j_{\mathrm{f}}^{fb}(s-\overline{m}_{\mathrm{Z}}^2) + r_{\mathrm{f}}^{fb}\, s}{(s-\overline{m}_{\mathrm{Z}}^2)^2 + \overline{m}_{\mathrm{Z}}^2 \overline{\Gamma}_{\mathrm{Z}}^2} \right] / \sigma^0_{\mathrm{tot,f}}(s)$$

note the different definition of m_Z & Γ_Z

The r and j parameters scale the Z exchange and γ/Z interference contributions,

i.e. couplings and interference terms are treated as free parameters

S-Matrix parameters can be related to SM-parameters:

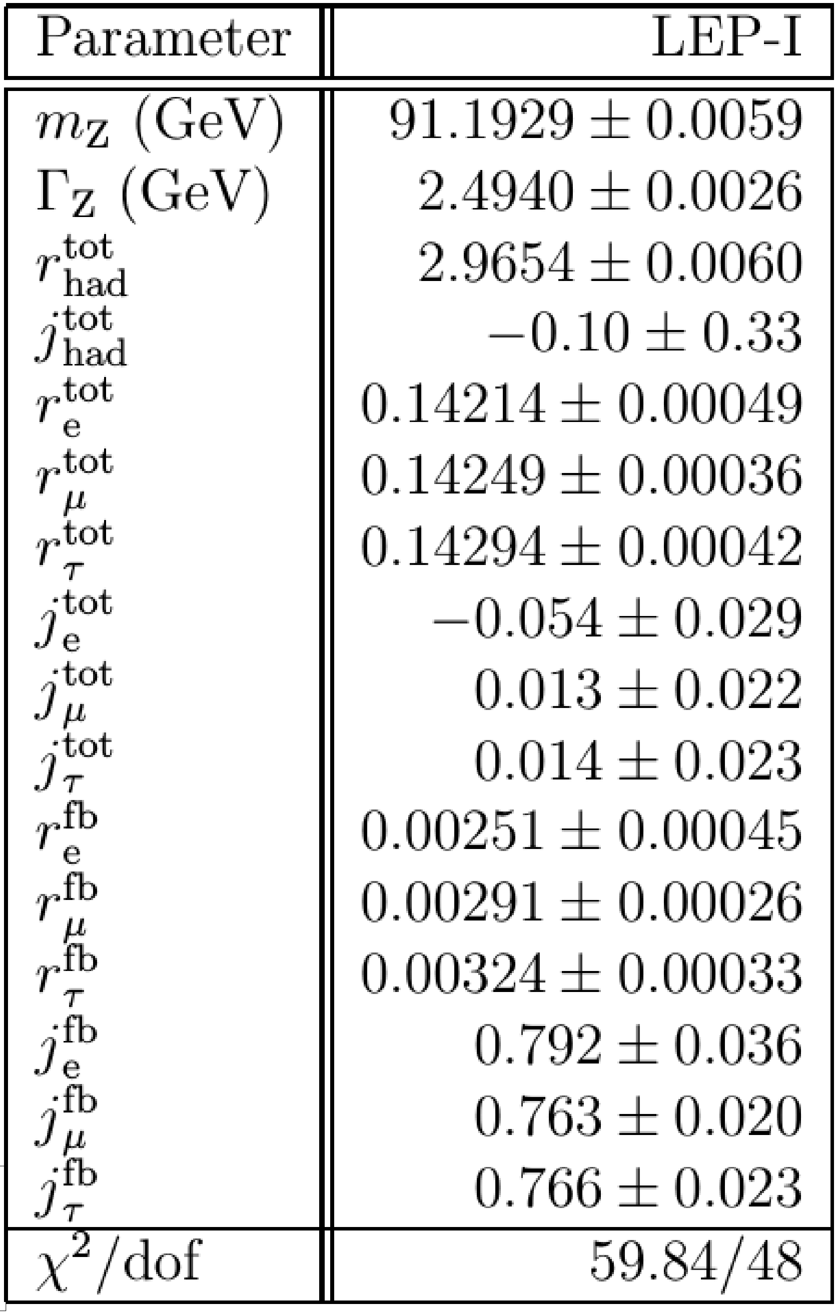
$$\begin{aligned}
r_{\mathrm{f}}^{\mathrm{tot}} &= \kappa^2 \left[g_{\mathrm{Ae}}^2 + g_{\mathrm{Ve}}^2 \right] \left[g_{\mathrm{Af}}^2 + g_{\mathrm{Vf}}^2 \right] - 2\kappa\, g_{\mathrm{Ve}}\, g_{\mathrm{Vf}} C_{Im} \\
j_{\mathrm{f}}^{\mathrm{tot}} &= 2\kappa\, g_{\mathrm{Ve}}\, g_{\mathrm{Vf}} \left(C_{Re} + C_{Im} \right) \\
g_{\mathrm{f}}^{\mathrm{tot}} &= Q_{\mathrm{e}}^2 Q_{\mathrm{f}}^2 \left| F_A(m_{\mathrm{Z}}) \right|^2 \\
r_{\mathrm{f}}^{\mathrm{fb}} &= 4\kappa^2 g_{\mathrm{Ae}}\, g_{\mathrm{Ve}}\, g_{\mathrm{Af}}\, g_{\mathrm{Vf}} - 2\kappa\, g_{\mathrm{Ae}}\, g_{\mathrm{Af}} C_{Im} \\
j_{\mathrm{f}}^{\mathrm{fb}} &= 2\kappa\, g_{\mathrm{Ae}}\, g_{\mathrm{Af}} \left(C_{Re} + C_{Im} \right) \\
g_{\mathrm{f}}^{\mathrm{fb}} &= 0\,,
\end{aligned}$$

with

$$\begin{aligned}
\kappa &= \frac{G_F m_{\mathrm{Z}}^2}{2\sqrt{2}\,\pi\alpha} \approx 1.50 \\
C_{Im} &= \frac{\Gamma_{\mathrm{Z}}}{m_{\mathrm{Z}}} Q_{\mathrm{e}} Q_{\mathrm{f}}\, \mathrm{Im}\left\{ F_A(m_{\mathrm{Z}}) \right\} \\
C_{Re} &= Q_{\mathrm{e}} Q_{\mathrm{f}}\, \mathrm{Re}\left\{ F_A(m_{\mathrm{Z}}) \right\} \\
F_A(m_{\mathrm{Z}}) &= \frac{\alpha(m_{\mathrm{Z}})}{\alpha}\,,
\end{aligned}$$

S-Matrix Approach: combined results

without assuming lepton universality,
this results in 16 parameters:



Parameter	LEP-I
m_Z (GeV)	91.1929 ± 0.0059
Γ_Z (GeV)	2.4940 ± 0.0026
$r_{\mathrm{had}}^{\mathrm{tot}}$	2.9654 ± 0.0060
$j_{\mathrm{had}}^{\mathrm{tot}}$	-0.10 ± 0.33
$r_{\mathrm{e}}^{\mathrm{tot}}$	0.14214 ± 0.00049
r_{μ}^{tot}	0.14249 ± 0.00036
r_{τ}^{tot}	0.14294 ± 0.00042
$j_{\mathrm{e}}^{\mathrm{tot}}$	-0.054 ± 0.029
j_{μ}^{tot}	0.013 ± 0.022
j_{τ}^{tot}	0.014 ± 0.023
$r_{\mathrm{e}}^{\mathrm{fb}}$	0.00251 ± 0.00045
r_{μ}^{fb}	0.00291 ± 0.00026
r_{τ}^{fb}	0.00324 ± 0.00033
$j_{\mathrm{e}}^{\mathrm{fb}}$	0.792 ± 0.036
j_{μ}^{fb}	0.763 ± 0.020
j_{τ}^{fb}	0.766 ± 0.023
χ^2/dof	$59.84/48$

main difference to standard procedure:

error on m_Z is larger due to free interference term, but well consistent

combined result of the four LEP experiments on S-Matrix parameters, LEP EWWG (2013)

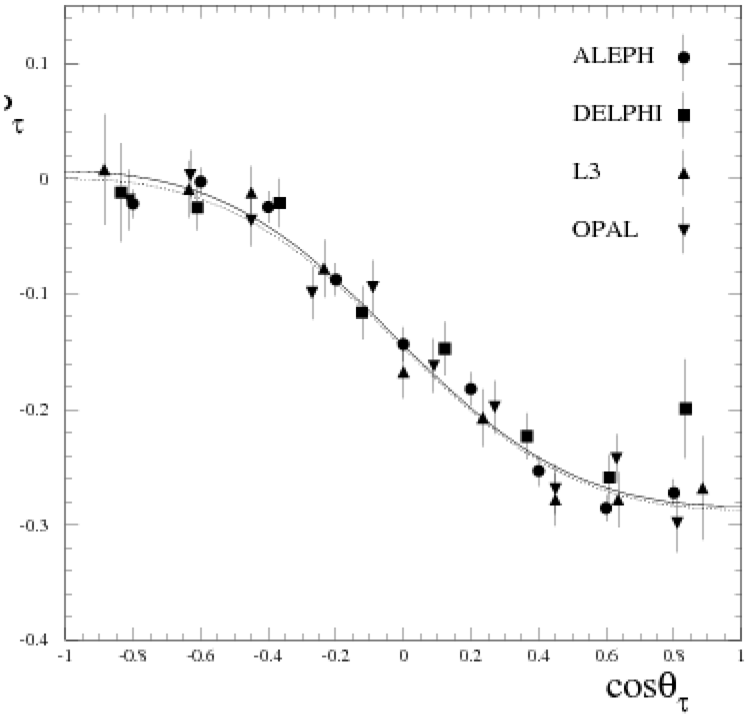
τ lepton polarization

in $e^+e^- \rightarrow \tau^+\tau^-$:

Spin in final state can be measured assuming V-A structure in τ decay

average τ polarization depends on e and τ couplings

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau(1+\cos^2\theta)+2\mathcal{A}_e\cos\theta}{1+\cos^2\theta+2\mathcal{A}_e\mathcal{A}_\tau\cos\theta}$$


Measured P_τ vs $\cos\theta_\tau$

allows a precise measurement of vector and axial vector couplings of the Z-boson to τ leptons

SLDs main contribution

Measurements at SLAC linear collider:

polarized **e^-** colliding with unpolarized **e^+** at $\sqrt{s}=M_Z$

measurements analogous to LEP, but

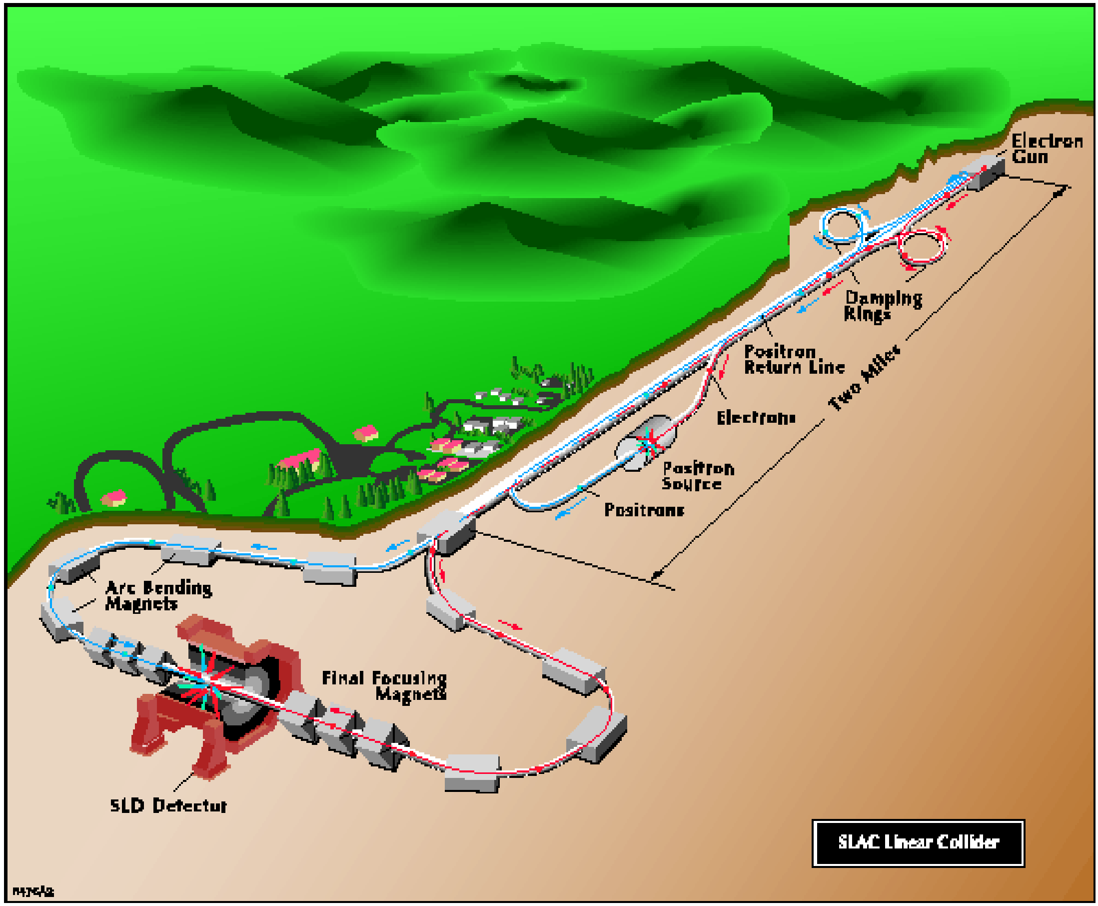
can determine σ and A_{FB} for left- and right-handed **e^-**

POs from From SLD:

$$A_{LR} = \frac{1}{\mathcal{P}_e}\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \propto \mathcal{A}_e$$

$$A_{fb,LR} = \frac{1}{\mathcal{P}_e}\left(A_{fb,L} - A_{fb,R}\right) \propto \mathcal{A}_f$$

A_{LR} is most sensitive single measurement to $\sin^2\theta_W^{eff}$



Z-couplings

IBA-parametrisation gives access to the vector and axial- vector couplings of the Z boson to fermions;

small imaginary parts of the couplings taken from Standard Model

(so-called „Standard Model remnants“)

Effective couplings are functions of Standard-Model parameters

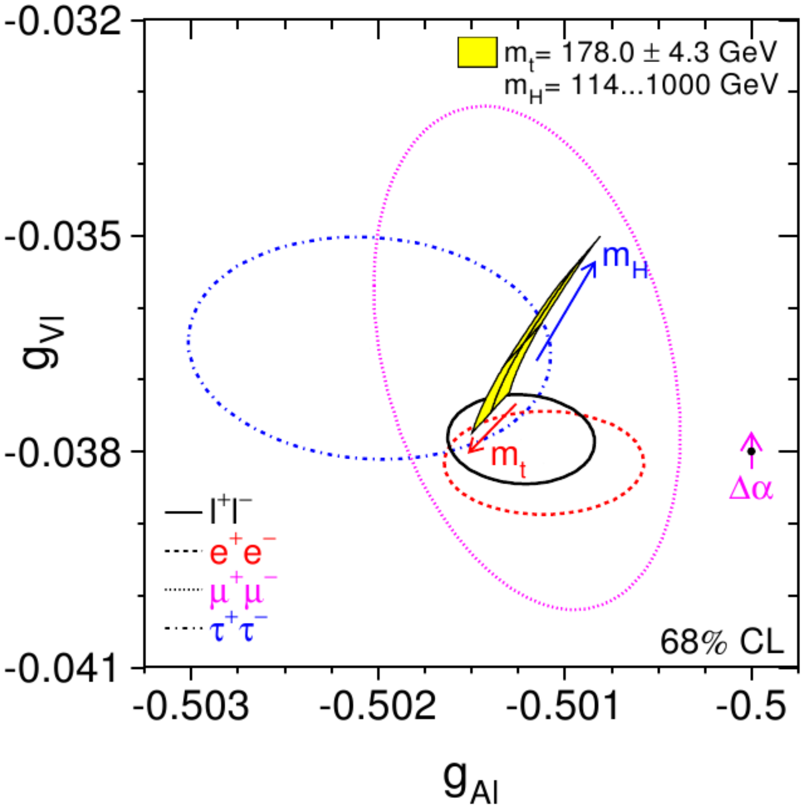
$$\mathcal{G}_{\mathrm{V,A\,f}} = \mathcal{G}_{\mathrm{V,A\,f}}(\alpha_{\mathrm{em}}, G_F, M_{\mathrm{Z}}, m_{\mathrm{top}}, M_{\mathrm{Higgs}}, \ldots)$$

due to virtual corrections.

Can use (real parts of) vector and axial-vector couplings as fit parameters

$$g_{\mathrm{Vf}} = \Re(\mathcal{G}_{\mathrm{Vf}})$$

$$g_{\mathrm{Af}} = \Re(\mathcal{G}_{\mathrm{Af}})$$



effective ρ-parameter and effective $\sin^2\theta_w$

further away from measurements, but closer to the structure of the ew corrections

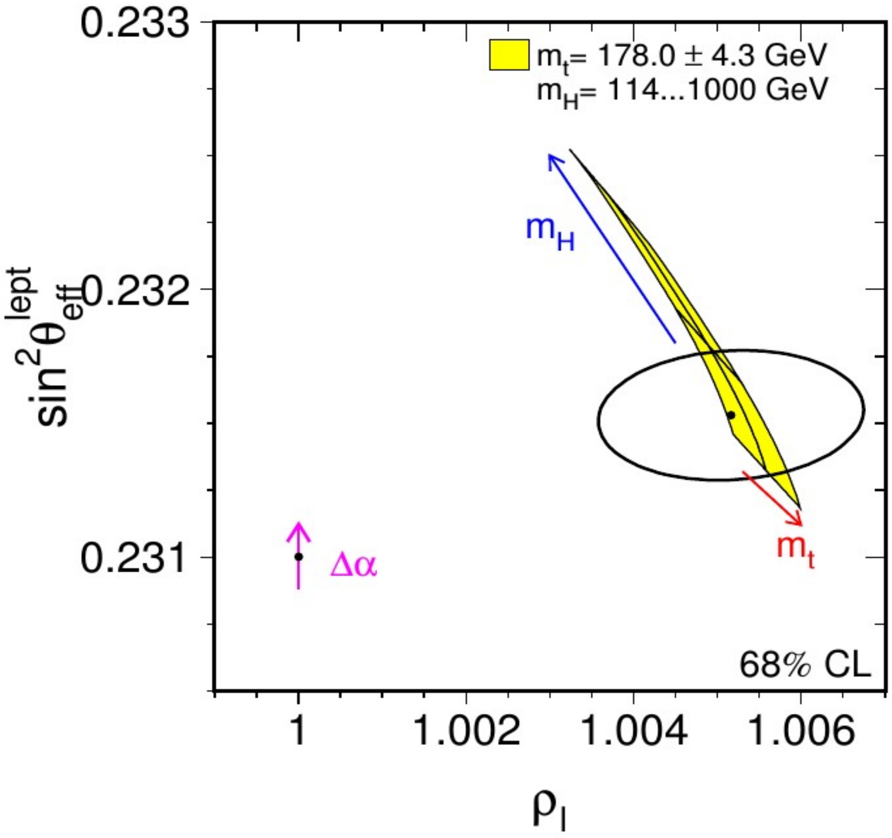
$$\Delta\rho \text{ and } \sin^2\Theta_w^{\text{eff}}$$

tree-level relation

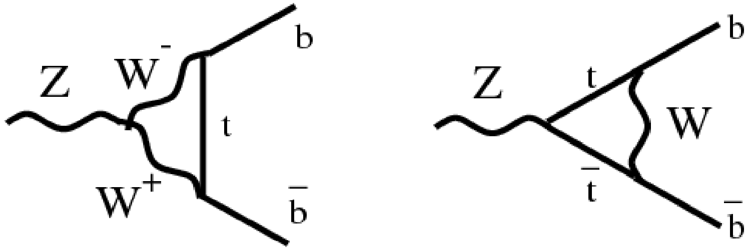
$$\begin{aligned} g_V^{\text{tree}} &\equiv g_L^{\text{tree}} + g_R^{\text{tree}} = \sqrt{\rho_0}\,(T_3^f - 2Q_f \sin^2\theta_W^{\text{tree}}) \\ g_A^{\text{tree}} &\equiv g_L^{\text{tree}} - g_R^{\text{tree}} = \sqrt{\rho_0}\,T_3^f\,. \end{aligned}$$

becomes relation of „effective“ parameters

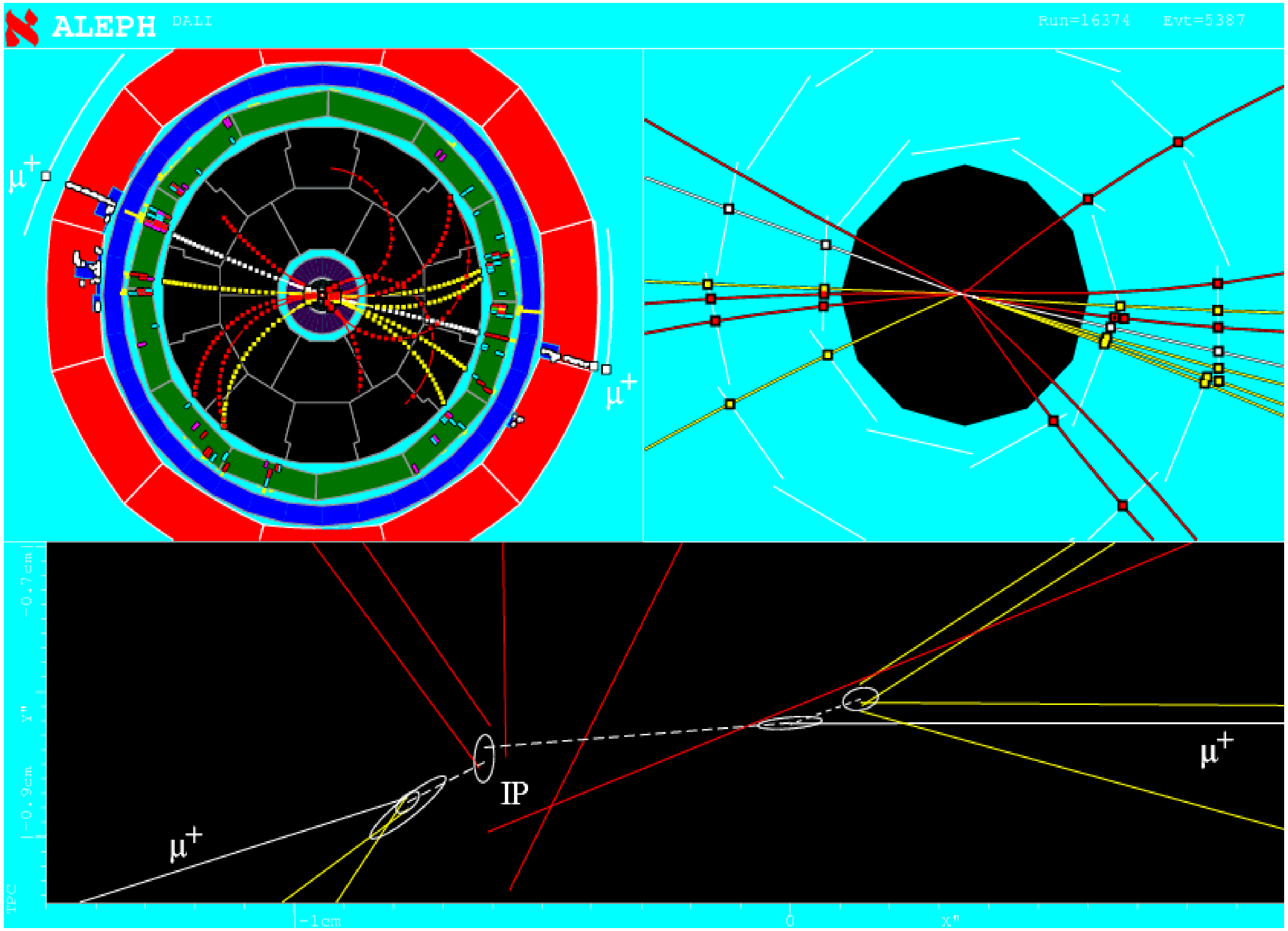
$$\begin{aligned} g_{Vf} &\equiv \sqrt{\rho_f}\,(T_3^f - 2Q_f \sin^2\theta_{\text{eff}}^f) \\ g_{Af} &\equiv \sqrt{\rho_f}\,T_3^f\,, \end{aligned}$$



More measurements: b-quarks



special diagrams involving
top-quarks

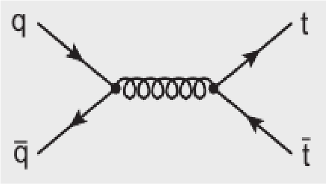


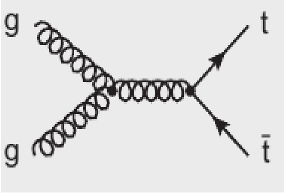
Two more POs:

$$R_b = \frac{\Gamma_{b\bar{b}}}{\Gamma_{\mathrm{had}}}$$

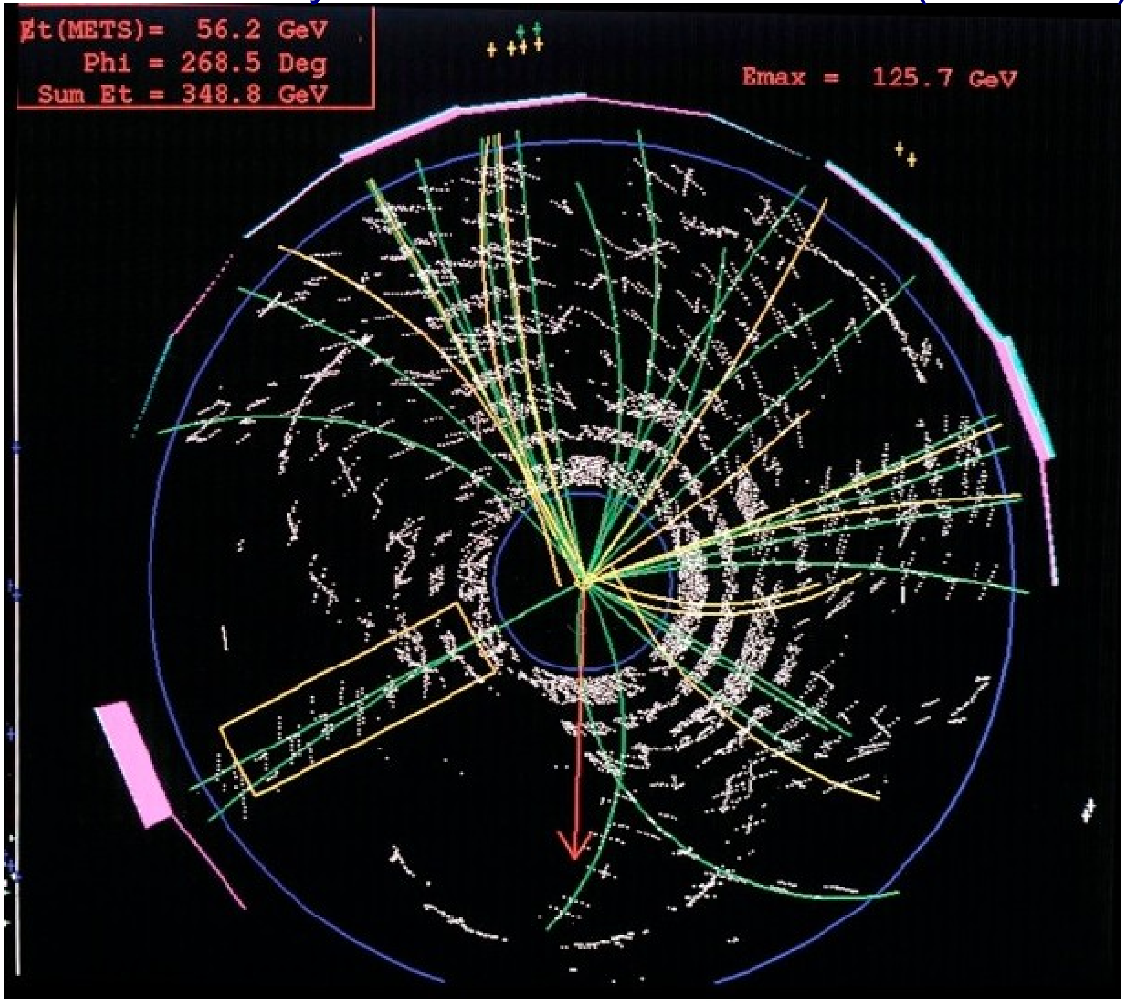
$$A_{\mathrm{FB}}^{0,b} = \frac{4}{3}\mathcal{A}_e\mathcal{A}_b$$

Top-Quark mass





Early $t\bar{t}$ candidate event in CDF (09/24/92)



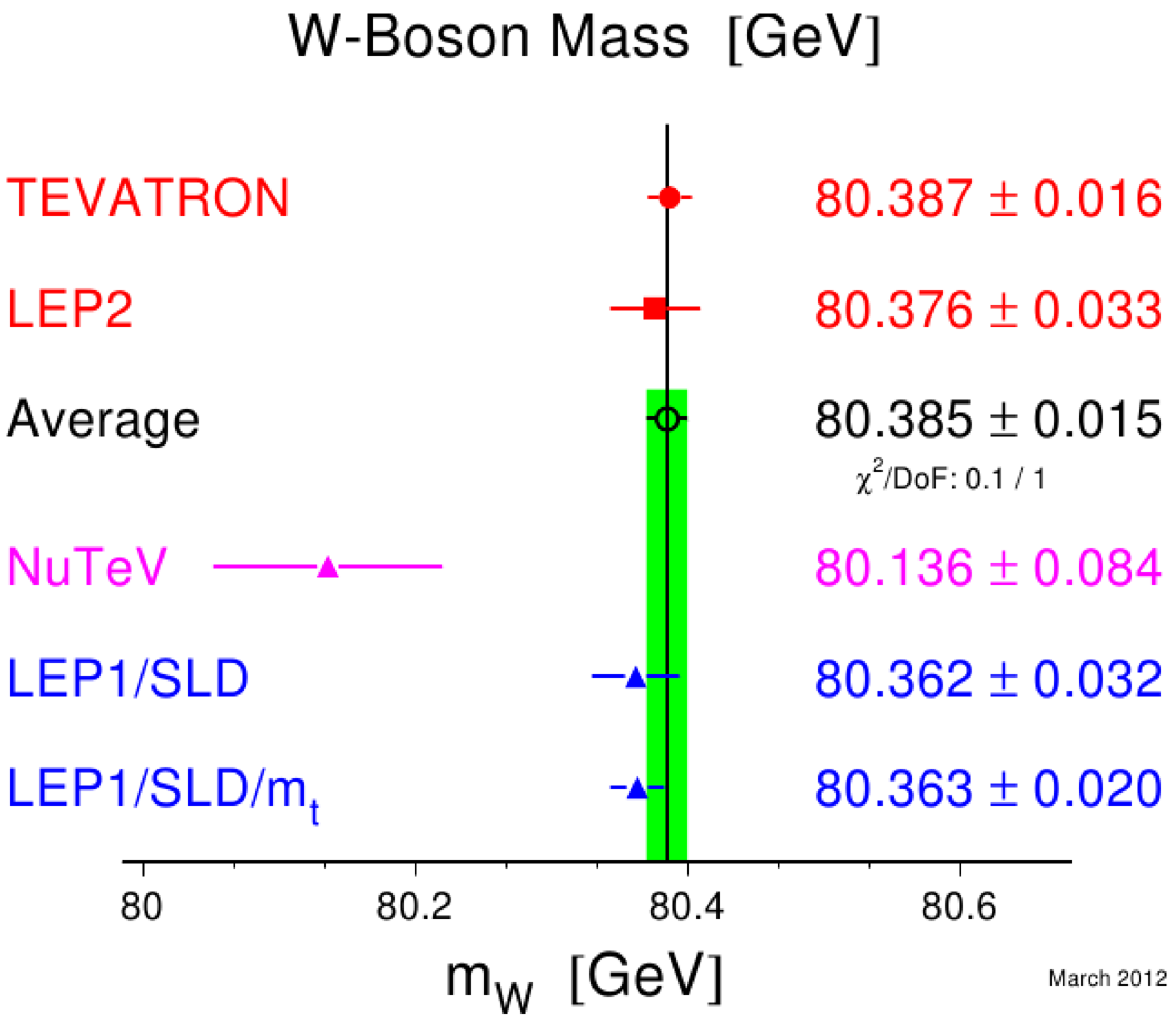
large corrections from virtual top quarks,

$$\propto G_F m_{\rm top}^2\,,$$

require precise measurements of the top-quark mass

sensitivity to Higgs-boson mass only after precise value of $m_{\rm top}$ became available

W-boson mass



Together with m_Z, m_W fixes the on-shell weak mixing angle !

Electroweak libraries

Theoretical calculations by **many theorists**
are incorporated in computer codes,
the **“electroweak libraries”**

EW libraries compared during “Precision calculation WS”, 1995

BHM / MIZA	Burgers, Hollik, Martinez, Teubert
LEPTOP	ITEP Moscow group: Novikov, Okun, Rozanov, Vysotsky
TOPAZ0	Torino-Pavia group: Montagna, Nicrosini, Passarino, Piccinini, Pittau
WHO	Beenakker, Burgers, Hollik
ZFITTER	Dubna-Zeuthen group: Bardin, Bilenky, Chizov, Olchevsky, S. Riemann, T. Riemann, Sachwitz, Sazonov, Sedykh, Sheer

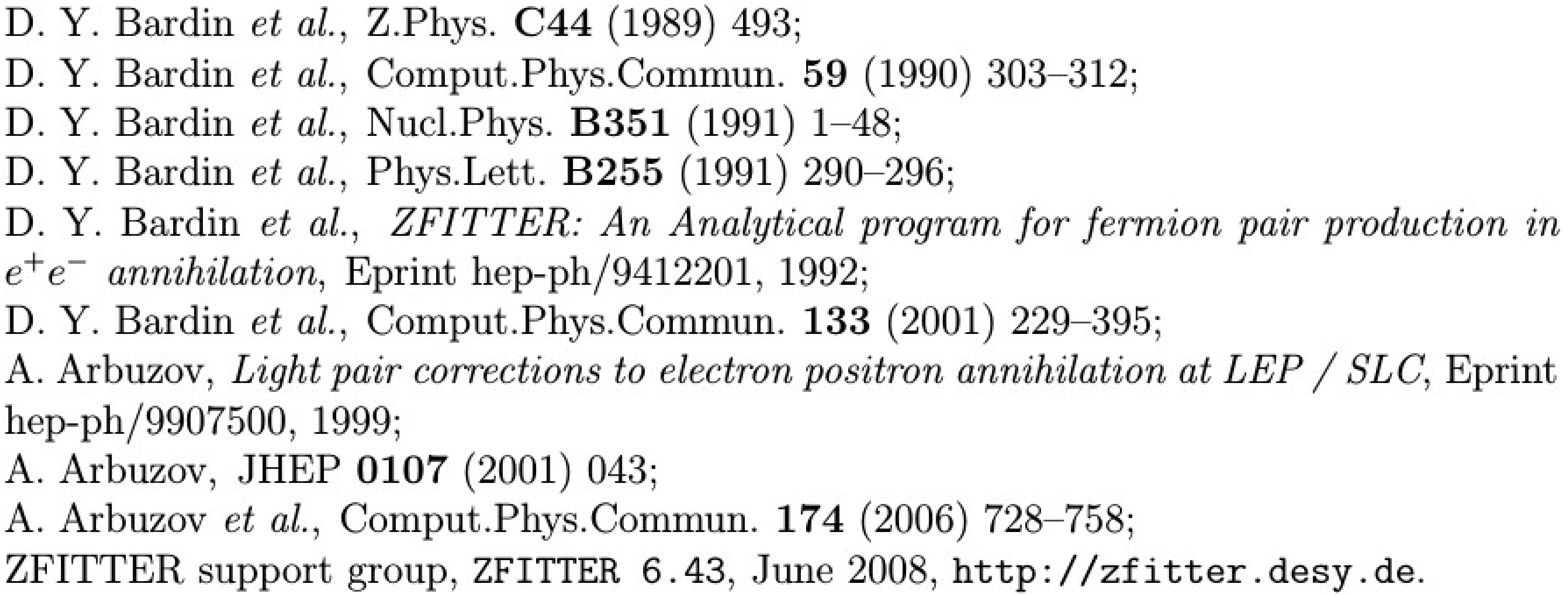
TOPAZ0 ***and*** **ZFFITER** ***continued to be compared/developed into the LEP 2 era***

- KK2f *(fermion-pair production LEP2)* Jadach, Ward, Was

ZFITTER references

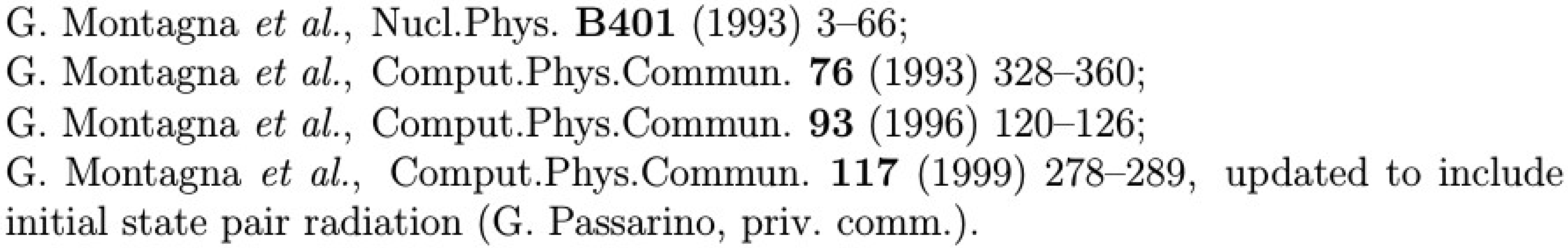
Two very complete program packages,
thoroughly compared and used for final results and combinations:

ZFITTER references:



D. Y. Bardin *et al.*, Z.Phys. **C44** (1989) 493;
D. Y. Bardin *et al.*, Comput.Phys.Commun. **59** (1990) 303–312;
D. Y. Bardin *et al.*, Nucl.Phys. **B351** (1991) 1–48;
D. Y. Bardin *et al.*, Phys.Lett. **B255** (1991) 290–296;
D. Y. Bardin *et al.*, *ZFITTER: An Analytical program for fermion pair production in e^+e^- annihilation*, Eprint hep-ph/9412201, 1992;
D. Y. Bardin *et al.*, Comput.Phys.Commun. **133** (2001) 229–395;
A. Arbuzov, *Light pair corrections to electron positron annihilation at LEP / SLC*, Eprint hep-ph/9907500, 1999;
A. Arbuzov, JHEP **0107** (2001) 043;
A. Arbuzov *et al.*, Comput.Phys.Commun. **174** (2006) 728–758;
ZFITTER support group, `ZFITTER 6.43`, June 2008, `http://zfitter.desy.de`.

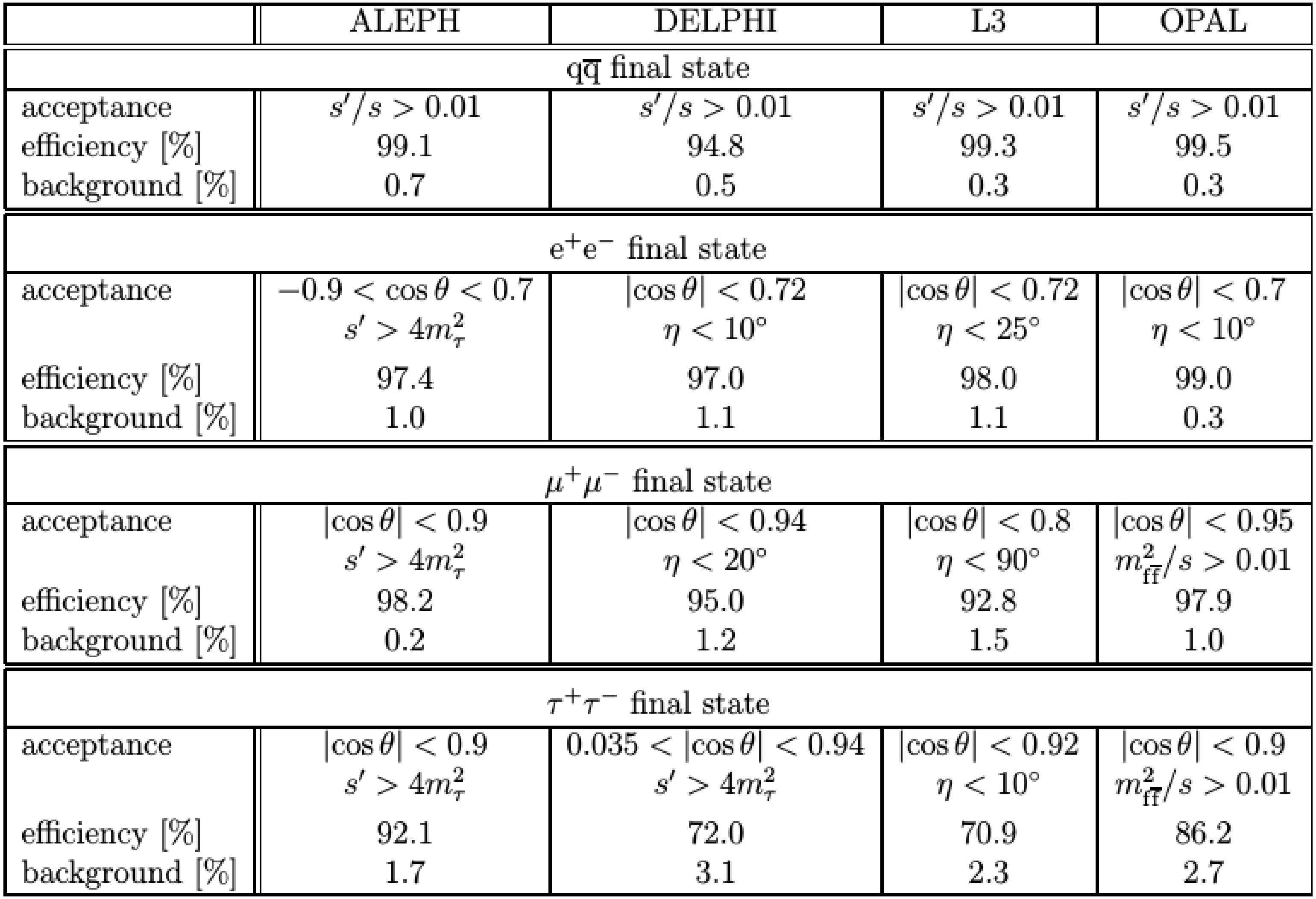
TOPAZ0 references:



G. Montagna *et al.*, Nucl.Phys. **B401** (1993) 3–66;
G. Montagna *et al.*, Comput.Phys.Commun. **76** (1993) 328–360;
G. Montagna *et al.*, Comput.Phys.Commun. **93** (1996) 120–126;
G. Montagna *et al.*, Comput.Phys.Commun. **117** (1999) 278–289, updated to include initial state pair radiation (G. Passarino, priv. comm.).

Detailed procedures: input to ew libraries

Measuremens of total cross-sections and forward-backward asymmetries were corrected to an „ideal“ acceptance that can be handled by the ew libraries



	ALEPH	DELPHI	L3	OPAL
$q\bar{q}$ final state				
acceptance	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$
efficiency [%]	99.1	94.8	99.3	99.5
background [%]	0.7	0.5	0.3	0.3
e^+e^- final state				
acceptance	$-0.9 < \cos\theta < 0.7$ $s' > 4m_\tau^2$	$\|\cos\theta\| < 0.72$ $\eta < 10°$	$\|\cos\theta\| < 0.72$ $\eta < 25°$	$\|\cos\theta\| < 0.7$ $\eta < 10°$
efficiency [%]	97.4	97.0	98.0	99.0
background [%]	1.0	1.1	1.1	0.3
$\mu^+\mu^-$ final state				
acceptance	$\|\cos\theta\| < 0.9$ $s' > 4m_\tau^2$	$\|\cos\theta\| < 0.94$ $\eta < 20°$	$\|\cos\theta\| < 0.8$ $\eta < 90°$	$\|\cos\theta\| < 0.95$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	98.2	95.0	92.8	97.9
background [%]	0.2	1.2	1.5	1.0
$\tau^+\tau^-$ final state				
acceptance	$\|\cos\theta\| < 0.9$ $s' > 4m_\tau^2$	$0.035 < \|\cos\theta\| < 0.94$ $s' > 4m_\tau^2$	$\|\cos\theta\| < 0.92$ $\eta < 10°$	$\|\cos\theta\| < 0.9$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	92.1	72.0	70.9	86.2
background [%]	1.7	3.1	2.3	2.7

Interfaces to ew libraries

Different „interfaces“ in the codes allow to calculate

- **cross-sections and asymmetries** (within ideal acceptance) as functions of
  - Standard-Model Parameters $(\alpha_{\rm em}, G_F, \alpha_s, M_{\rm Z}, m_{\rm top}, M_{\rm H}, \text{light fermion masses})$
  - POs $(M_{\rm Z}, \Gamma_{\rm Z}, \sigma^0_{\rm had}, R_{\rm e}, R_\mu, R_\tau, A^{0,\rm e}_{\rm FB}, A^{0,\mu}_{\rm FB}, A^{0,\tau}_{\rm FB})$
    or $(M_{\rm Z}, \Gamma_{\rm Z}, \Gamma_{\rm had}, \Gamma_{\rm e}, \Gamma_\mu, \Gamma_\tau, A^{0,\rm e}_{\rm FB}, A^{0,\mu}_{\rm FB}, A^{0,\tau}_{\rm FB})$
  - couplings $(M_{\rm Z}, \Gamma_{\rm Z}, \sigma^0_{\rm had}, g_{\rm Vf}, g_{\rm Af})$

- **POs** from Standard-Model-Parameters $(\alpha_{\rm em}, G_F, \alpha_s, M_{\rm Z}, m_{\rm top}, M_{\rm H}, \text{light fermion masses})$

remark: other „interfaces“, like
the „ε-parameters“ *(Altarelli, Barbierei, Jadach, Caravaglios)* or
“STU-parameters” *(Peskin, Takeuchi)* ,
were also implemented

SM fit to σ / A_{FB} vs. fit to POs

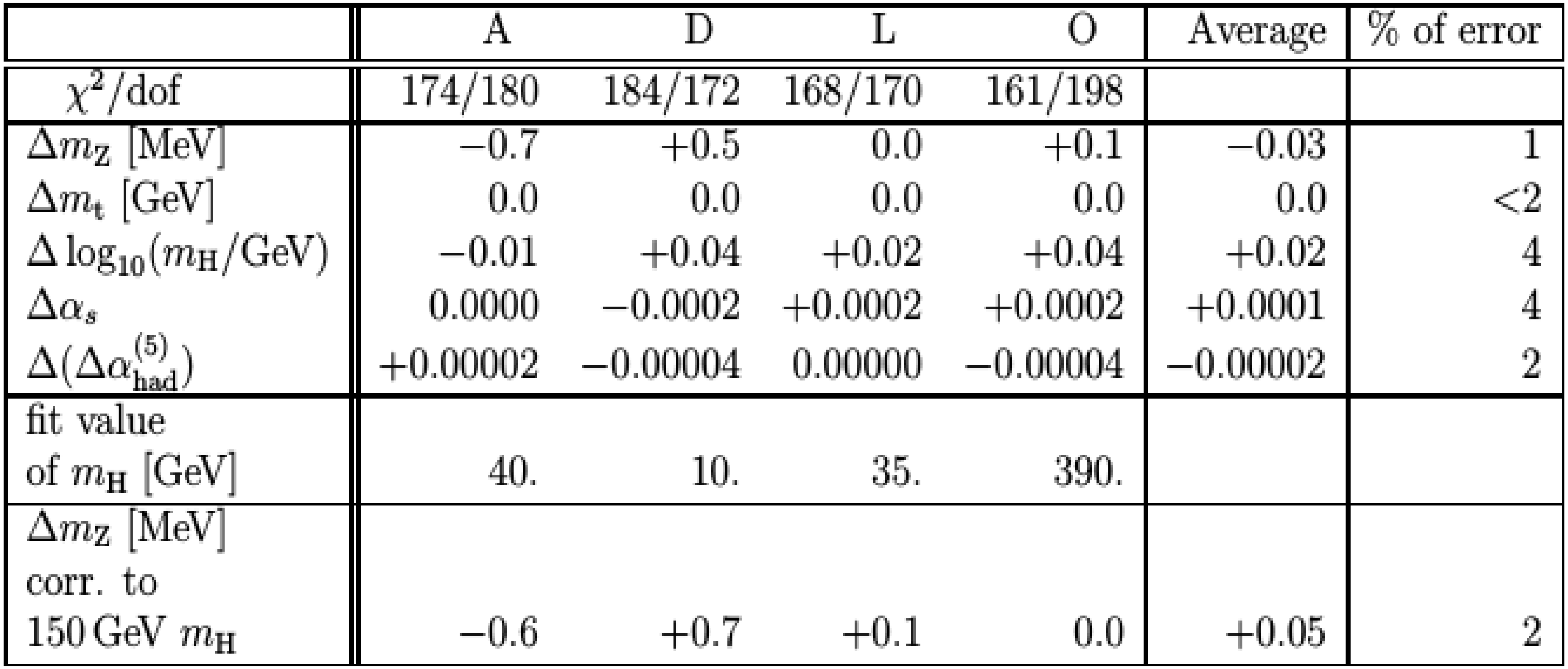
direct SM-fit to σ / A_{FB}

vs.

fit to **POs**

Difference in extracted SM-Parameters:

Table from
“Z-pole report”, Phys. Rept. 427 (2006)



	A	D	L	O	Average	% of error
χ^2/dof	174/180	184/172	168/170	161/198		
Δm_Z [MeV]	-0.7	$+0.5$	0.0	$+0.1$	-0.03	1
Δm_t [GeV]	0.0	0.0	0.0	0.0	0.0	<2
$\Delta \log_{10}(m_H/\mathrm{GeV})$	-0.01	$+0.04$	$+0.02$	$+0.04$	$+0.02$	4
$\Delta\alpha_s$	0.0000	-0.0002	$+0.0002$	$+0.0002$	$+0.0001$	4
$\Delta(\Delta\alpha^{(5)}_{\mathrm{had}})$	$+0.00002$	-0.00004	0.00000	-0.00004	-0.00002	2
fit value of m_H [GeV]	40.	10.	35.	390.		
Δm_Z [MeV] corr. to 150 GeV m_H	-0.6	$+0.7$	$+0.1$	0.0	$+0.05$	2

Dominant effect is on M_Z, but small compared to total error of 2.1 MeV

POs are an excellent representation of the experimental measurements

Theoretical uncertainties on POs

Theoretical uncertainties arise from:

- QED radiative corrections (known to full 2nd order and 3rd order LL)
- residual Standard-Model dependencies
  - * parameric uncertainties from (at the time) unknown Higg-boson mass, top-quark mass and $\alpha_{em}(s)$.
    (α_s self-consistently fitted from hadronic cross section)
  - * genuine theoretical uncertainties from missing higher orders and detailed treatment in codes
- "ambiguities" in the parametrisation of the differential cross-section near the Z-resonance in terms of the POs

Detailed comparisons of the three available codes and their different "options"

BHM / MIZA TOPAZ0 ZFITTER

allowed to constrain theoretical uncertainties:

dominated by QED corrections:

$\pm$0.3 MeV on M_Z (~15% of total error)

$\pm$0.2 MeV on Γ_Z (<10% of total error)

dominated by "choice of parametrisation":

$\pm$0.004 on R_l (~15% of total error)

Z-Pole Legacy Results

Legacy results of precision measurements around the Z-Pole
are represented by a set of Pseudo-observables

$$\begin{array}{lll}
m_{\mathrm{Z}} & & \\
\Gamma_{\mathrm{Z}} & \sigma^0_{\mathrm{had}} & R_\ell \\
A^{0,\ell}_{\mathrm{FB}} & A_\ell(P_\tau) & R^0_{\mathrm{b}} \\
R^0_{\mathrm{c}} & A^{0,\mathrm{b}}_{\mathrm{FB}} & A^{0,\mathrm{c}}_{\mathrm{FB}} \\
A_\ell(SLD) & A_{\mathrm{b}}(SLD) & A_{\mathrm{c}}(SLD)
\end{array}$$

These can be expressed as functions of more fundamental parameters

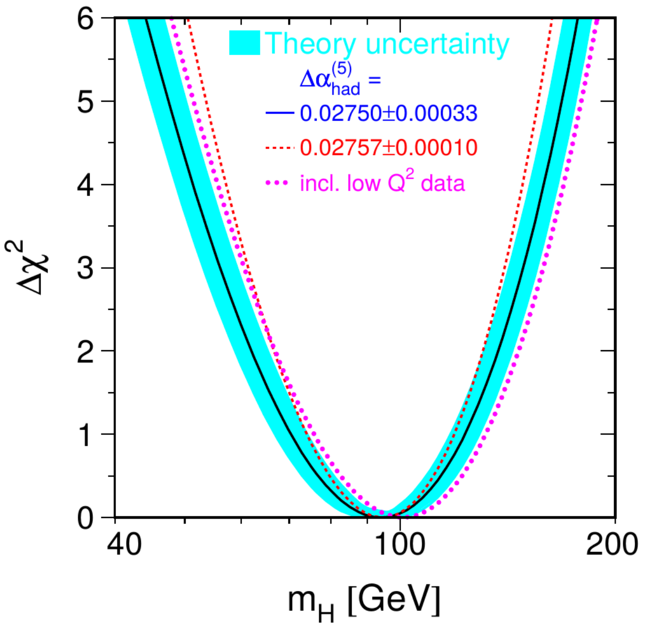
$$(\alpha_{\mathrm{em}}, G_F, \alpha_s, M_{\mathrm{Z}}, m_{\mathrm{top}}, M_{\mathrm{H}}, \text{light fermion masses})$$

and serve to

- constrain Standard Model parameters
- test alternative theories

Standard-Model fit to precision pseudo-observables

Pseudo-Observables defined and measured at LEP 1,2 are important members of a longer list of precision „observables“

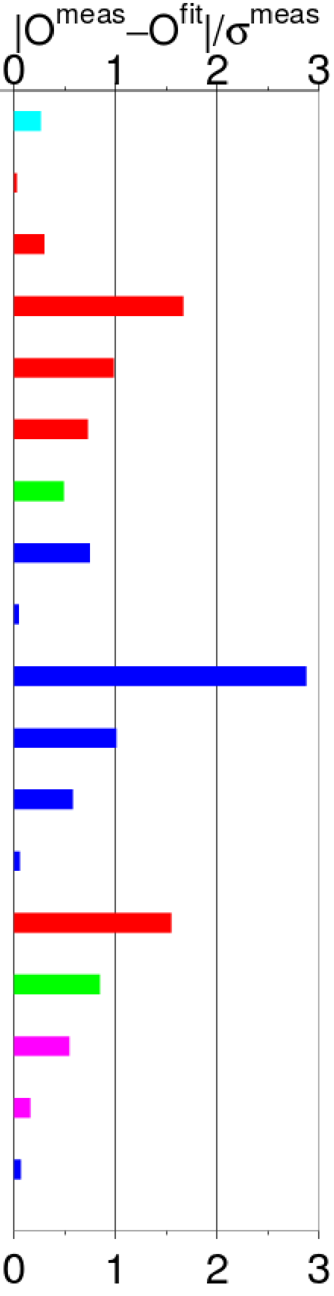


Best-Fit Value of Higgs mass

$$M_{\mathrm{H}} = 94^{+29}_{-24}\, GeV$$

$$\chi^2_{\mathrm{prob}} = 21\,\%$$

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



LEP EWWG, March 2012
SM fit with free Higgs mass

proof-of-principle: top-mass prediction 1993/94

Status

Moriond conference, March 1994:

direct search for top at Tevatron:

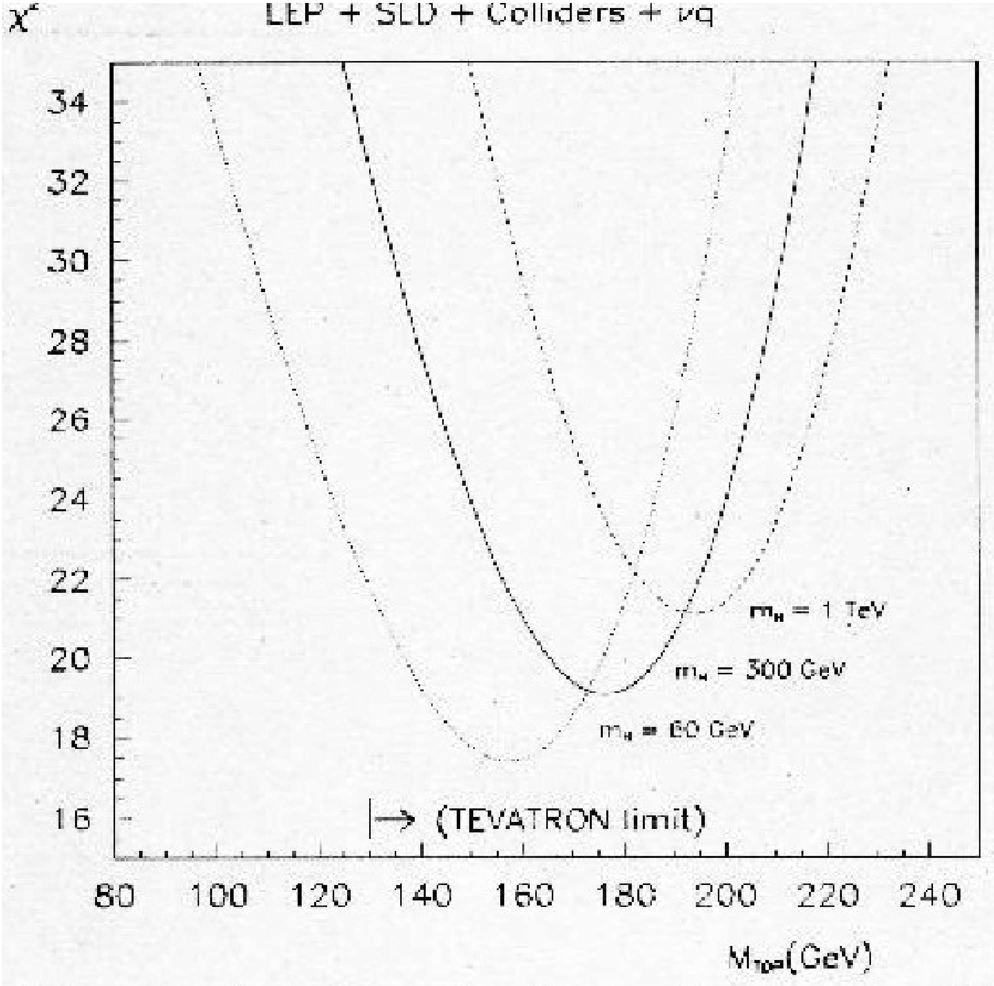
$$m_\mathrm{t} > 130\,\mathrm{GeV}/c^2$$

from **radiative corrections**:

$$m_\mathrm{t} = 177 \pm 11\,{}^{+18}_{-19}{}_\mathrm{Higgs}\;\mathrm{GeV}/c^2$$

a little later in summer, direct **observation** of top quark (CDF)**:**

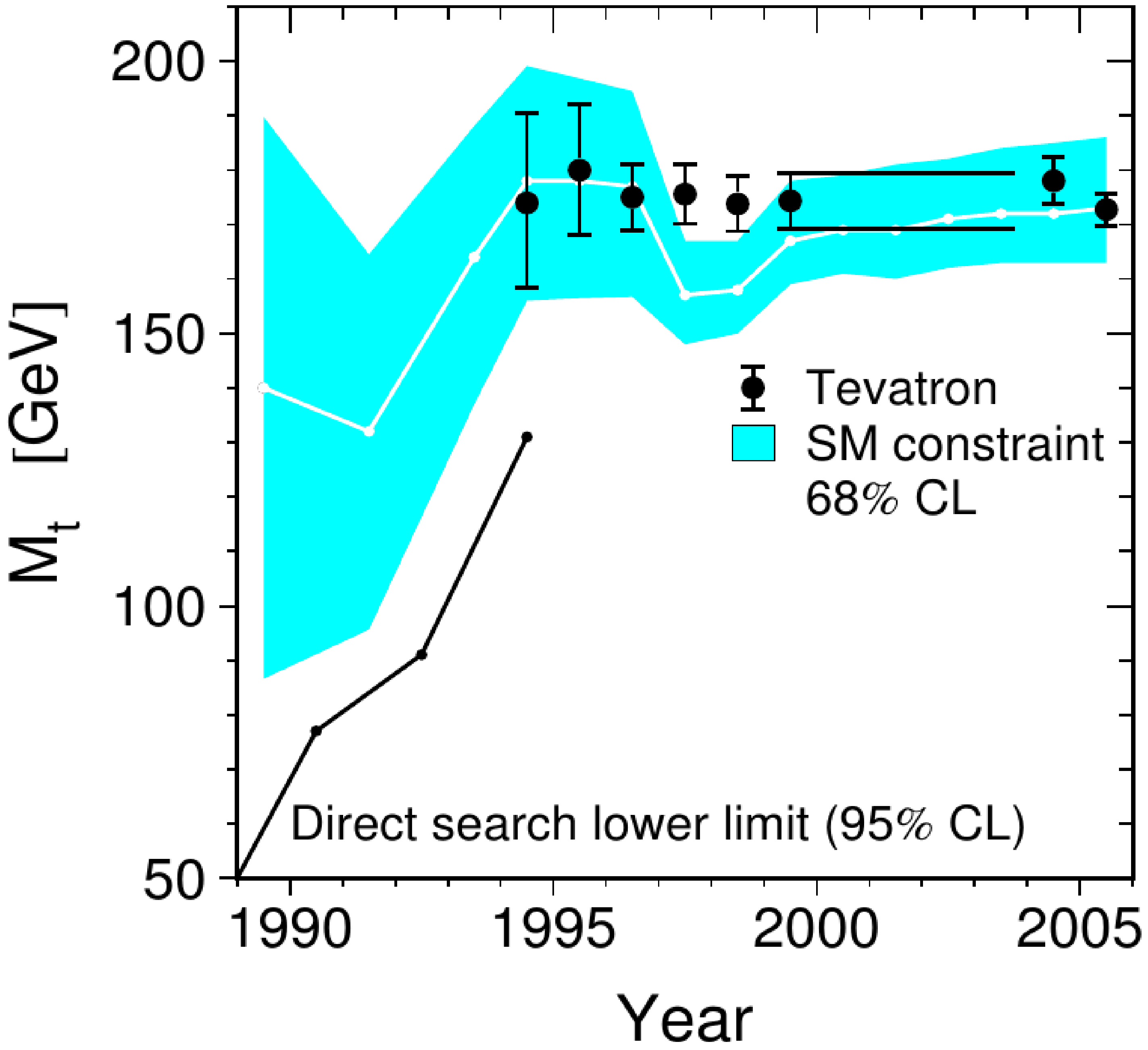
$$m_\mathrm{t} = 174 \pm 10\,{}^{+13}_{-12}{}_\mathrm{syst}\;\mathrm{GeV}/c^2$$



Excellent agreement between

top from loops and from direct measurement !

indirect vs. direct top-quark mass

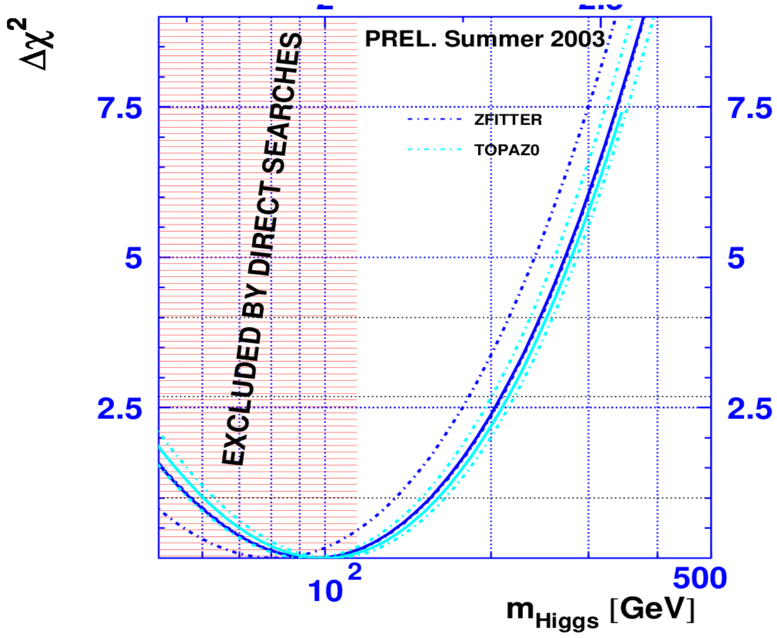


publication of "Z-pole report"

Estimating Theoretical Uncertainties

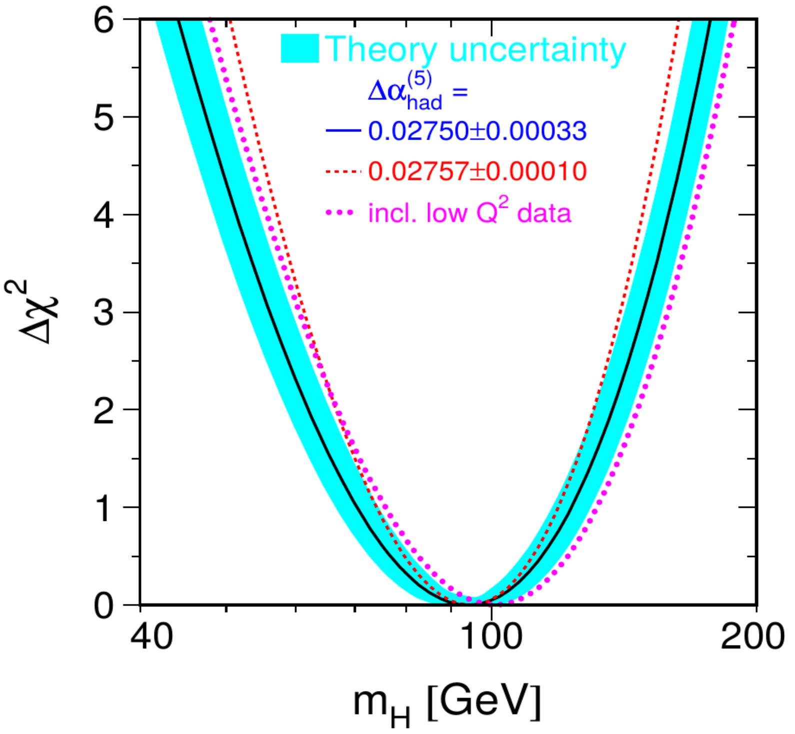
Uncertainties evaluated by WG "Precision calculations for the Z resonance" (1997) comparing different, but (to present knowledge) equivalent treatments of

- *re-summation techniques*
- *factorisation schemes*
- *choice of scales for vertex corrections* *etc.*



example of SM-fit to precision observables with different codes and "options"

→



"Blue-Band-Plot" (2012)

Into the Future

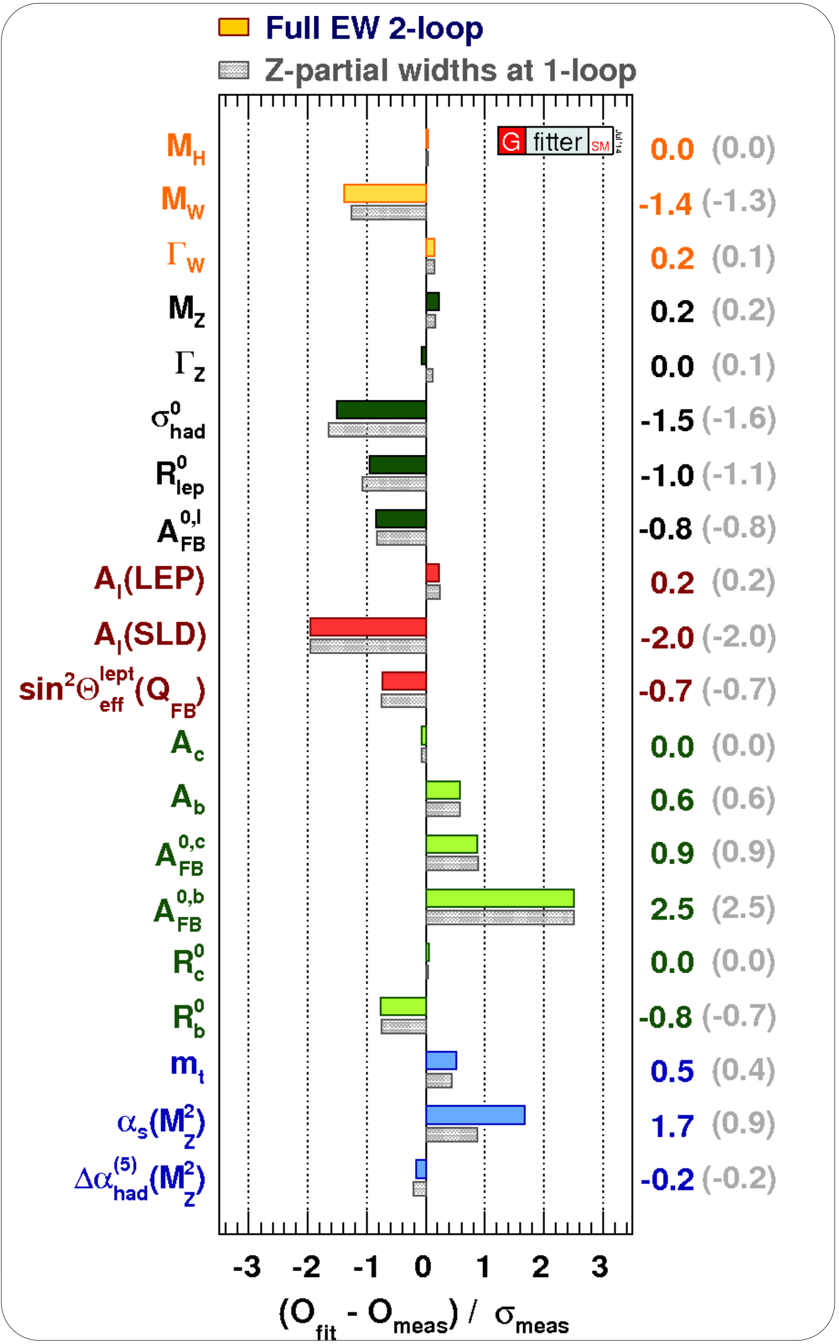
LEP Electroweak Working Group
finished its mission with the publication of the “LEP 2 report” *Phys. Rept. 532 (2013)*

POs continue to play their role, and calculations are still being improved, e.g. full EW 2-loop calculation of Z partial widths (A. Freitas, 2014), included in a new-generation fitting tool

global ew fit by GFITTER team
`http://project-gfitter.web.cern.ch`

Higgs-boson mass:

$$M_{\mathrm{H}} = 93^{+25}_{-21}\,GeV$$



Concluding remarks

Lessons to learn for future endeavours

(personal view, and thanks to Martin Grünewald for some input):

- reaching agreement on a set of Pseudo-Observables (”POs”) is tedious and takes much time
  - consensus of theoretical and experimental communities is essential, both should be involved from the very beginning
  - need at least two “tools” supporting the common set of POs
  - interface to experimental fitting-tools must be well designed to support complex use cases
- an established set of POs, derived from a well-defined set of input measurements, is much easier to handle than a long list of (raw) experimental measurements, or even worse, limits and constraints on a variety of parameters derived from sub-sets of measurements
- experiments are free to use other approaches – but results based on common agreements should always be included

Change agreements when something truly better comes along ...

Thanks for your attention

(some) selected literature

- CERN Yellow Report „Z Physics at LEP 1“ (CERN-89-08)
- CERN Yellow Report “Precision calculations on the Z resonance (CERN-95-03)
- D. Bardin, G. Passarino, „The Standard Model in the Making“, Oxford University Press
- ZFITTER: D. Bardin et al., Z: Phys C44 (1989), and later documents
- TOPAZ0: G. Montagna et al., Nucl.Phys. B401 (1993), and later documents
- The ALEPH, DELPHI, L3 OPAL, SLD Collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, „Precision Electroweak Measurements on the Z Resonance“, Phys. Rept. 427 (2006)
- The ALEPH, DELPHI, L3 OPAL Collaborations, the LEP Electroweak Working Group, „Electroweak Measurements in Electron-Positron Collisions at W-Boson-Pair Energies at LEP“, Phys. Rept. 532 (2013)
- D. Bardin, M. Grünewald, G. Passarino, “Precision Calculation Project Report”, arXiv:hep-ph/9902452