Implementation of K-matrix formalism in the $D^0 \rightarrow K_S\pi^+\pi^-$ amplitude model

By Rudin Petrossian-Byrne

Supervisors: ***Dr. Marco Gersabeck & Stefanie Reichert***

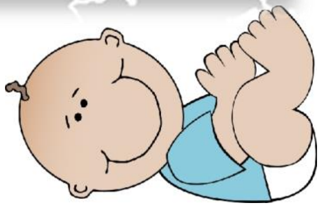
CERN Summer Student Programme 2014

Before The Physics…. Hi!

My name is Rudin

from Dublin

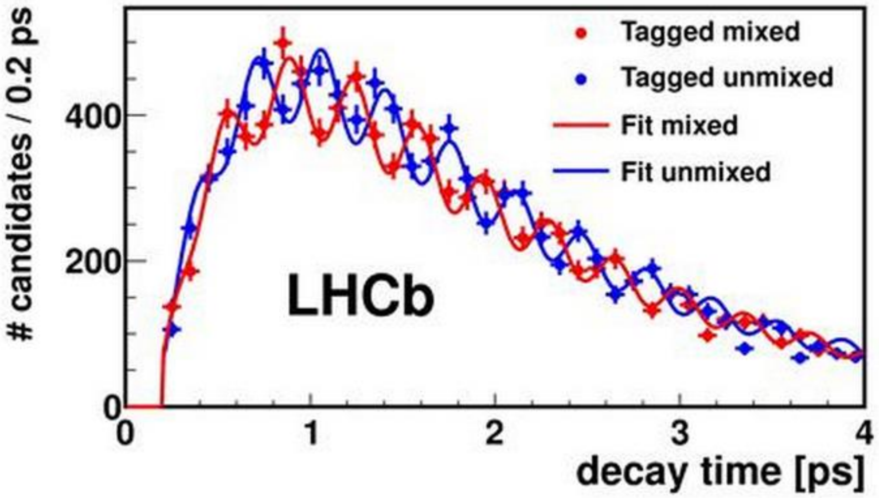
studying Physics (Undergraduate)



The big picture

Mixing and CPV

$$D^0 \rightarrow \overline{D^0} \rightarrow D^0 \rightarrow \overline{D^0} \rightarrow D^0$$ **Mixing**



B_s^0 mixing

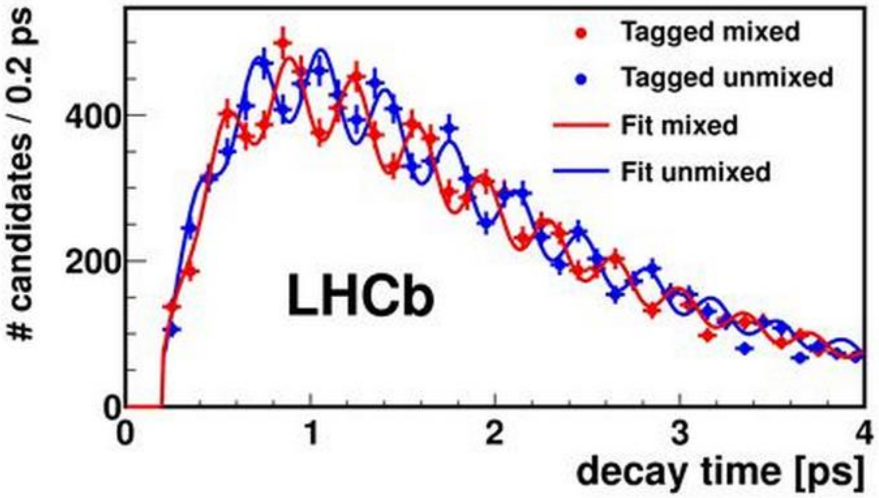
K^0 – strange quark

B^0 – bottom quark

The big picture

Mixing and CPV

$D^0 \to \overline{D^0} \to D^0 \to \overline{D^0} \to D^0$ **Mixing**



B_s^0 mixing

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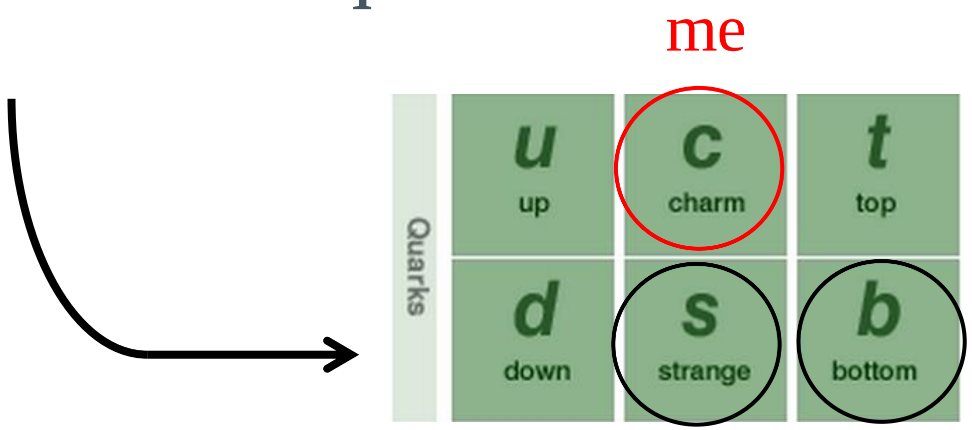


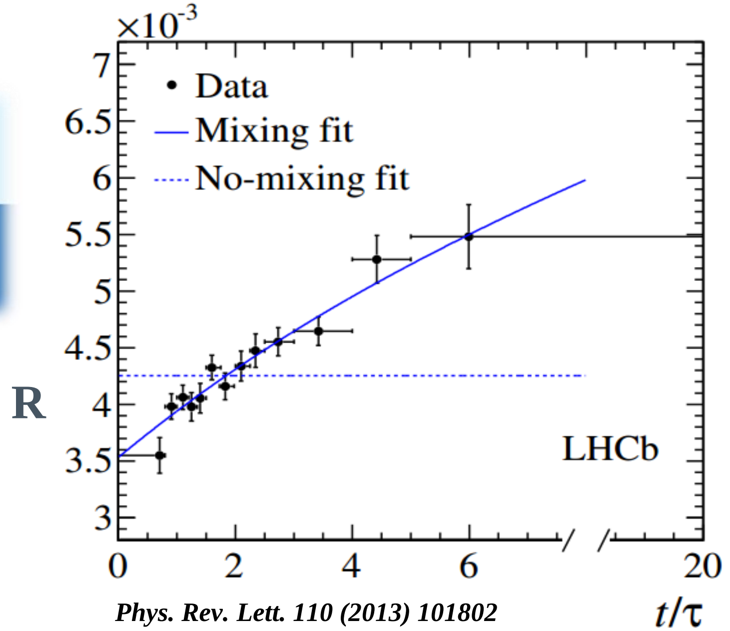
The big picture

Mixing and CPV with Charm

Charm Quark

- *2007 – First Evidence from **BaBar** and **Belle** for **mixing** in charmed neutral mesons*
- *2012 – **LHCb** find > **5σ** evidence for **mixing** in a single measurement*
  - **Mixing parameters very small**

$$\boldsymbol{x} = \frac{M_1 - M_2}{\Gamma} \qquad \boldsymbol{y} = \frac{\Gamma_1 - \Gamma_2}{2\Gamma} \qquad \propto (10^{-2})$$



Phys. Rev. Lett. 110 (2013) 101802

- ***No significant evidence for CPV in charm to date***
  - **SM 'predicts' <u>small</u> CPV**

  ↪ **Observation at current sensitivity would imply *<u>New Physics</u>***

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$$D^0 \rightarrow K_s \pi^+ \pi^-$$

Allows measuring **x** *and* **y** *parameters directly*

D^0 $\left\{ \begin{array}{l} c \\ \bar{u} \end{array} \right.$

c ——— 

$\bar{u}$ ——— 

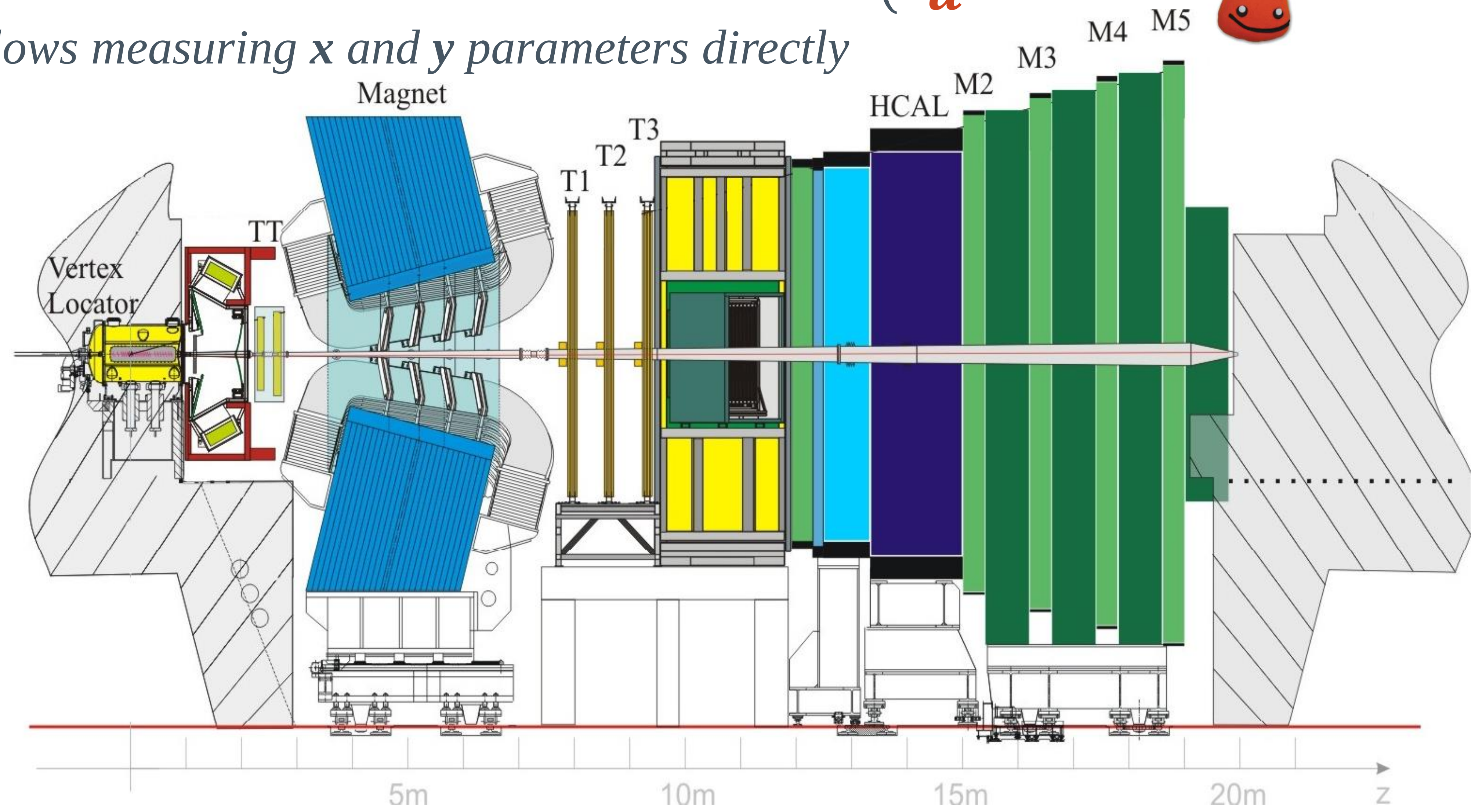
$\begin{pmatrix} modern \\ notation \end{pmatrix}$

$$D^0 \rightarrow K_s \pi^+ \pi^-$$

$$D^0 \begin{cases} c \\ \bar{u} \end{cases}$$

$\left(\begin{matrix} modern \\ notation \end{matrix}\right)$

*Allows measuring **x** and **y** parameters directly*



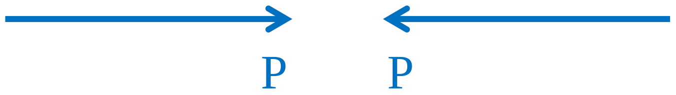
$D^0 \to K_s \pi^+ \pi^-$

D^0 { c — $\bar{u}$ }

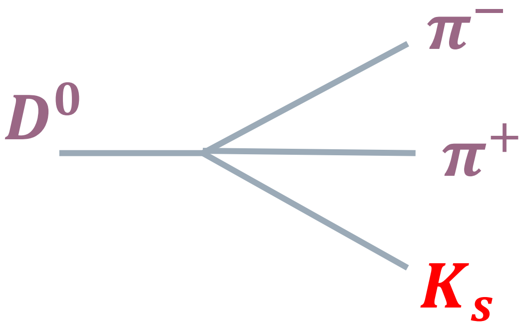
$\left(\text{modern notation}\right)$

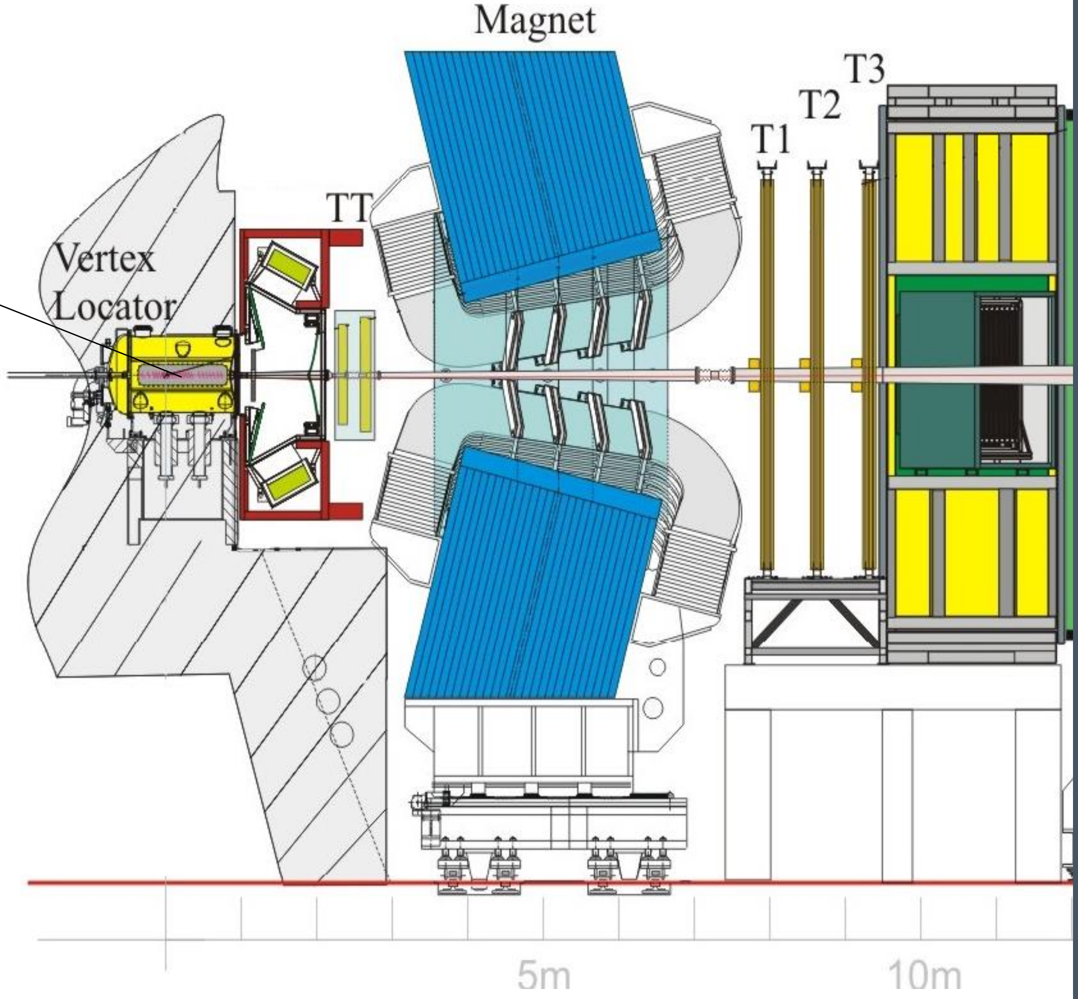
*Allows measuring **x** and **y** parameters directly*

- Protons collide in the VELO



- Very short life-time of **charm quark**
  D^0 travels ~2 mm, then decays





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$D^0 \rightarrow K_s \pi^+ \pi^-$

$D^0 \begin{cases} c \\ \bar{u} \end{cases}$

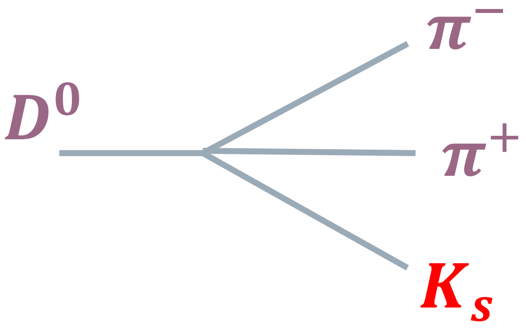
(modern notation)

*Allows measuring **x** and **y** parameters directly*

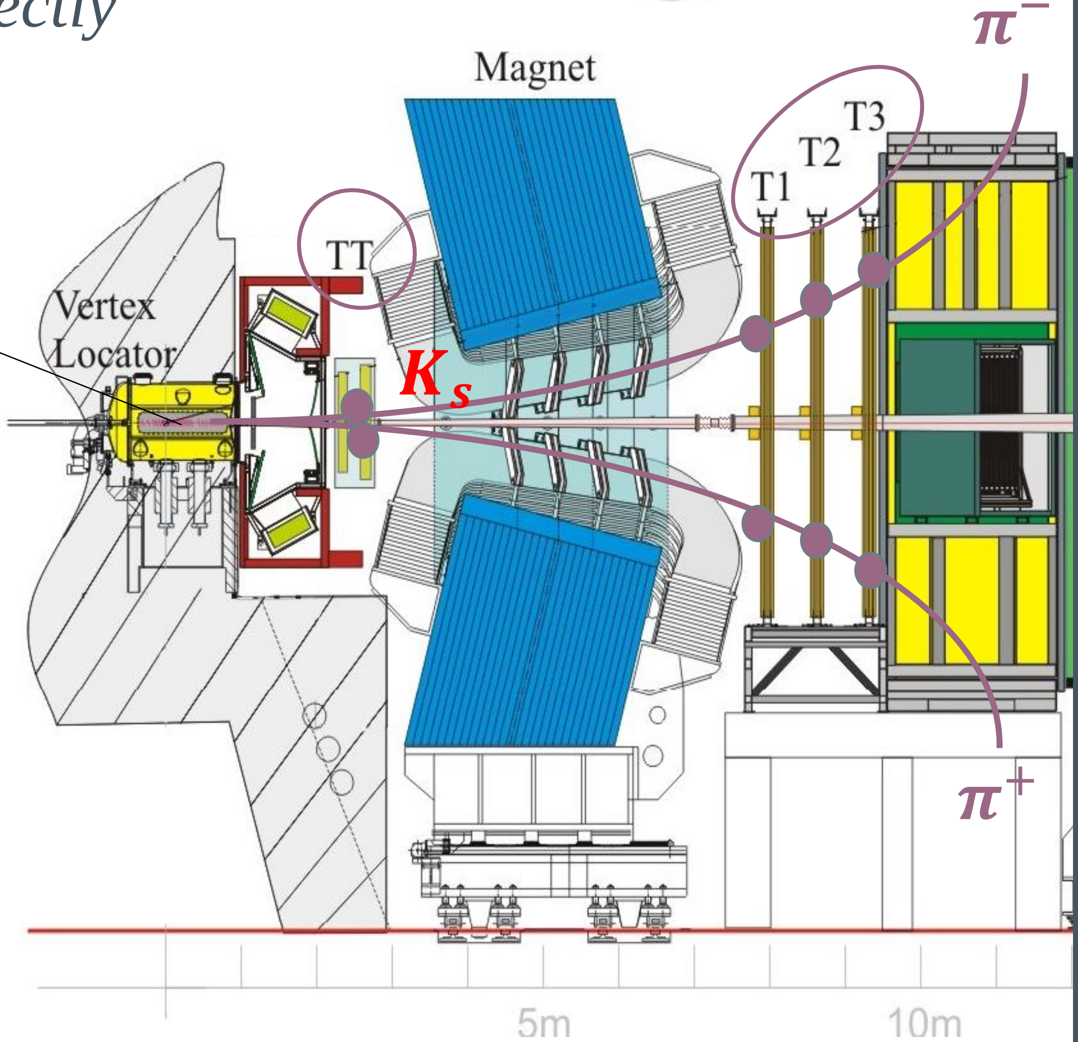
- Protons collide in the VELO

P → ← P

- Very short life-time of **charm quark**
  D^0 travels ~2 mm, then decays



detected by TT tracker.
bend through magnet.
detected by T1, T2, T3.



$D^0 \to K_s \pi^+ \pi^-$

D^0 { c  , $\bar{u}$  } (modern notation)

Allows measuring ***x*** *and* ***y*** *parameters directly*

- Protons collide in the VELO

  → P P ←

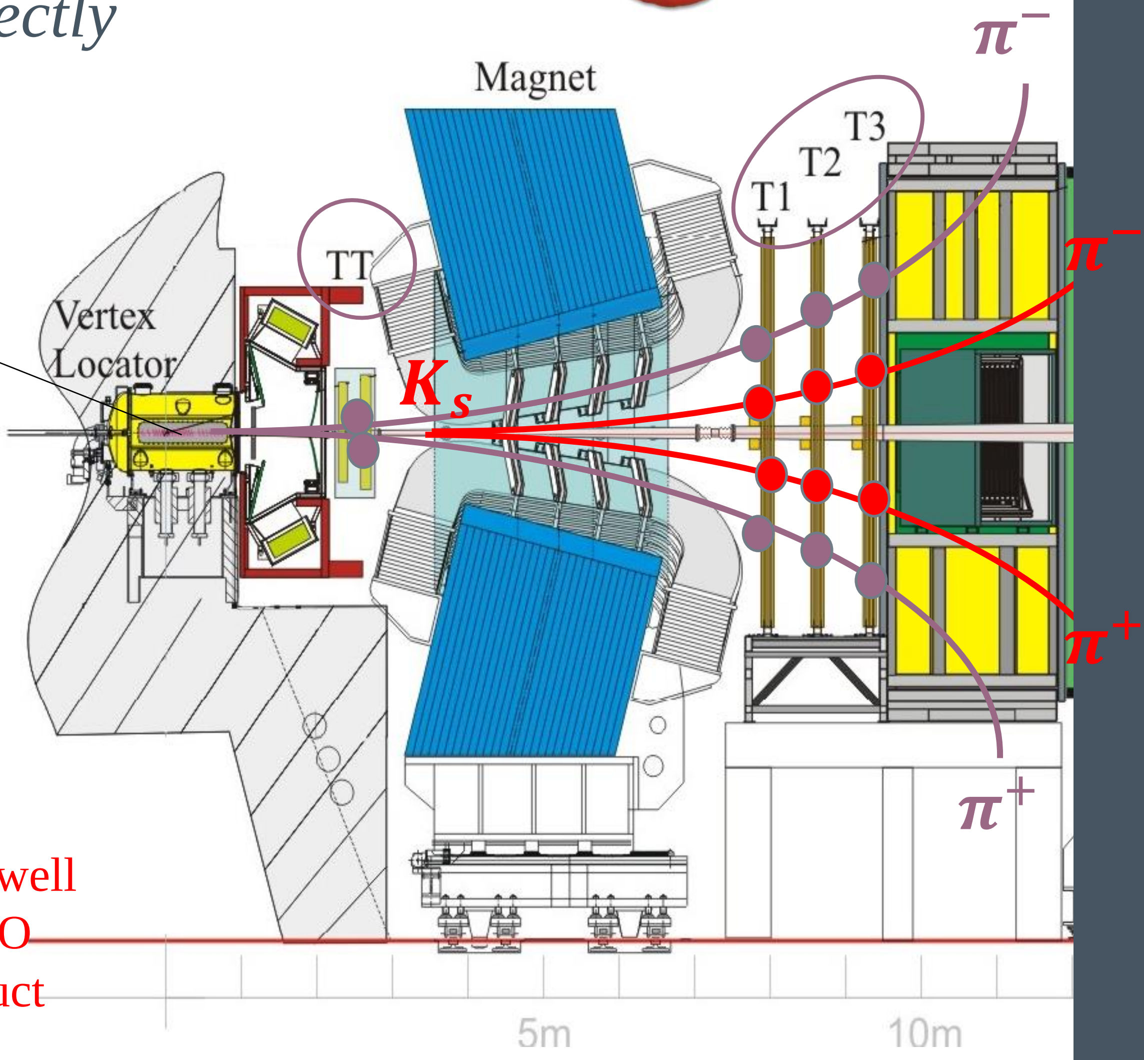
- Very short life-time of **charm quark**
  D^0 travels ~2 mm, then decays

D^0 → π^-, π^+, K_s

π^-, π^+ } detected by TT tracker. bend through magnet. detected by T1, T2, T3.

K_s → π^+ π^-

K_s } 33% – decay in VELO as well
66% – decay outside VELO
→ harder to reconstruct



$D^0 \to K_s \pi^+ \pi^-$

$D^0 \begin{cases} c \\ \bar{u} \end{cases}$

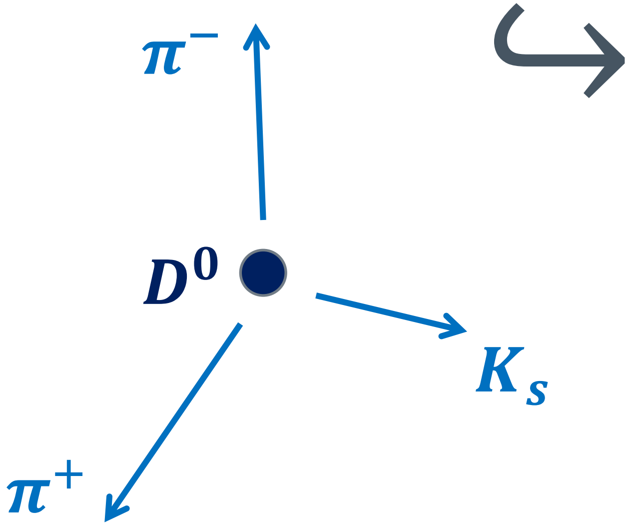
$\left(\begin{matrix} modern \\ notation \end{matrix}\right)$

Allows measuring ***x*** *and* ***y*** *parameters directly*

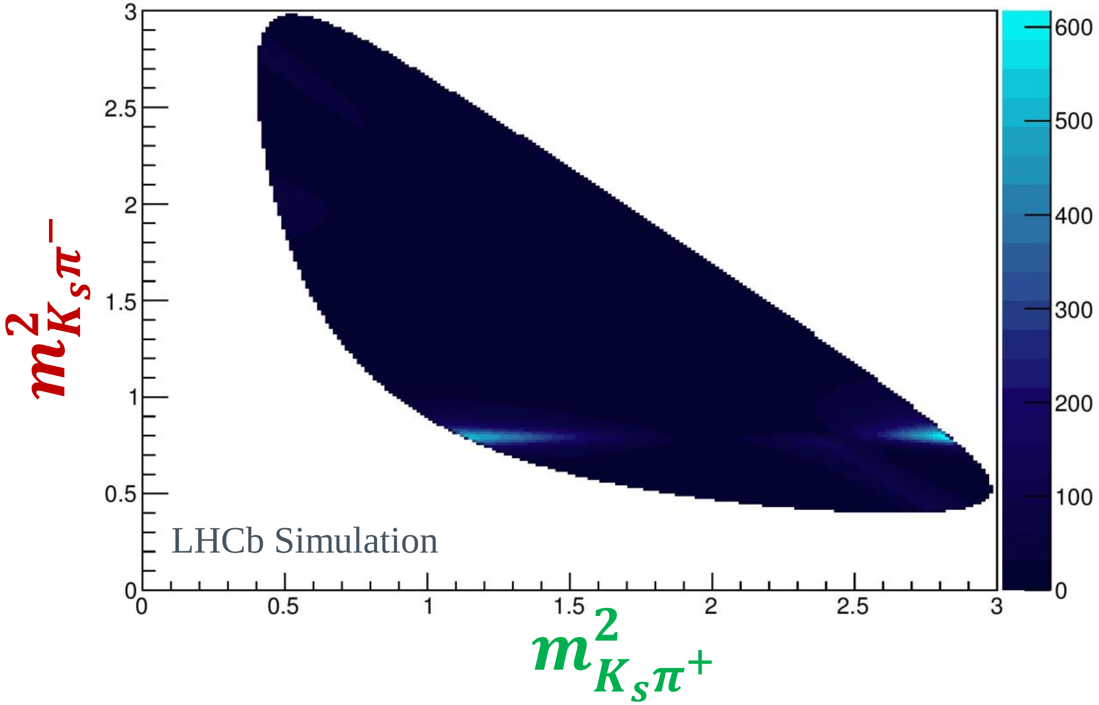
➢ 3-Body Decay

Unlike 2-body decays, energy of daughter particles not well-defined

Range of possible values.



Dalitz plot
graphical
representation



$D^0 \rightarrow K_s\pi^+\pi^-$

$D^0 \begin{cases} c \\ \bar{u} \end{cases}$

(modern notation)

*Allows measuring **x** and **y** parameters directly*

- **3-Body Decay**

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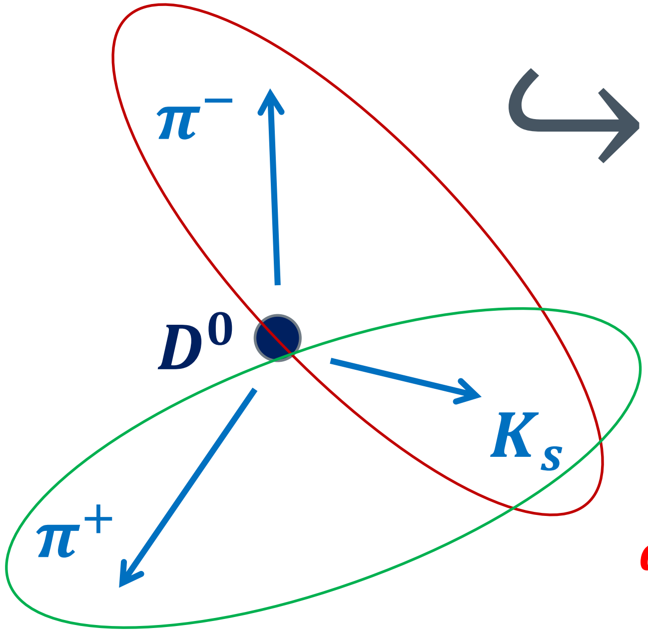
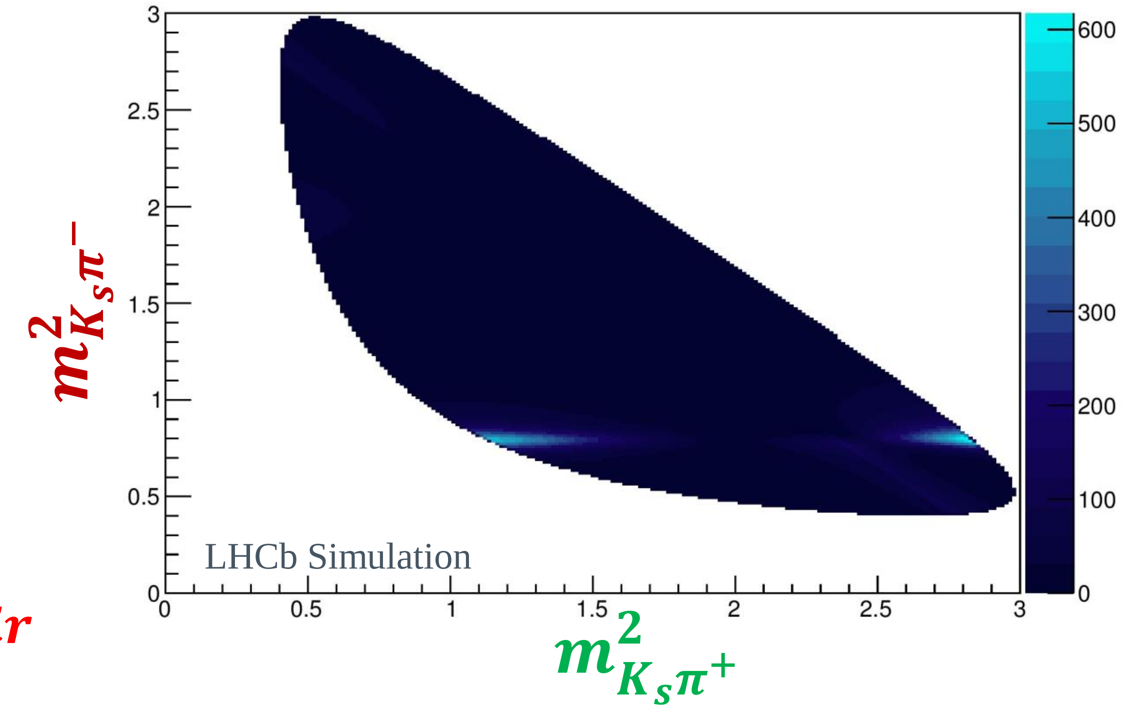


Dalitz plot
graphical representation

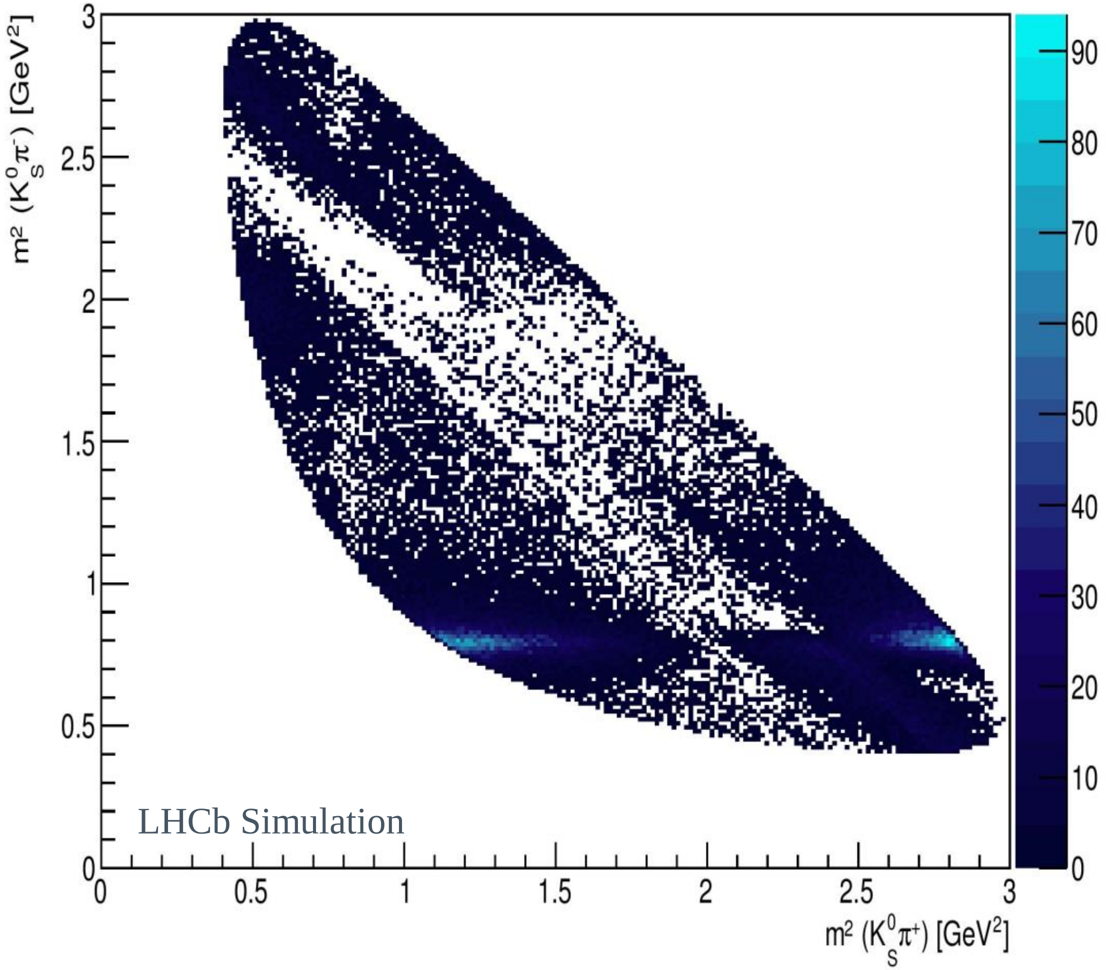
Plot invariant mass of one pair against other pair



$D^0 \rightarrow K_s\pi^+\pi^-$

Probability $|M|^2 \neq const$

Not homogenously populated with events
Some daughter energies more probable



$D^0 \to K_s \pi^+ \pi^-$

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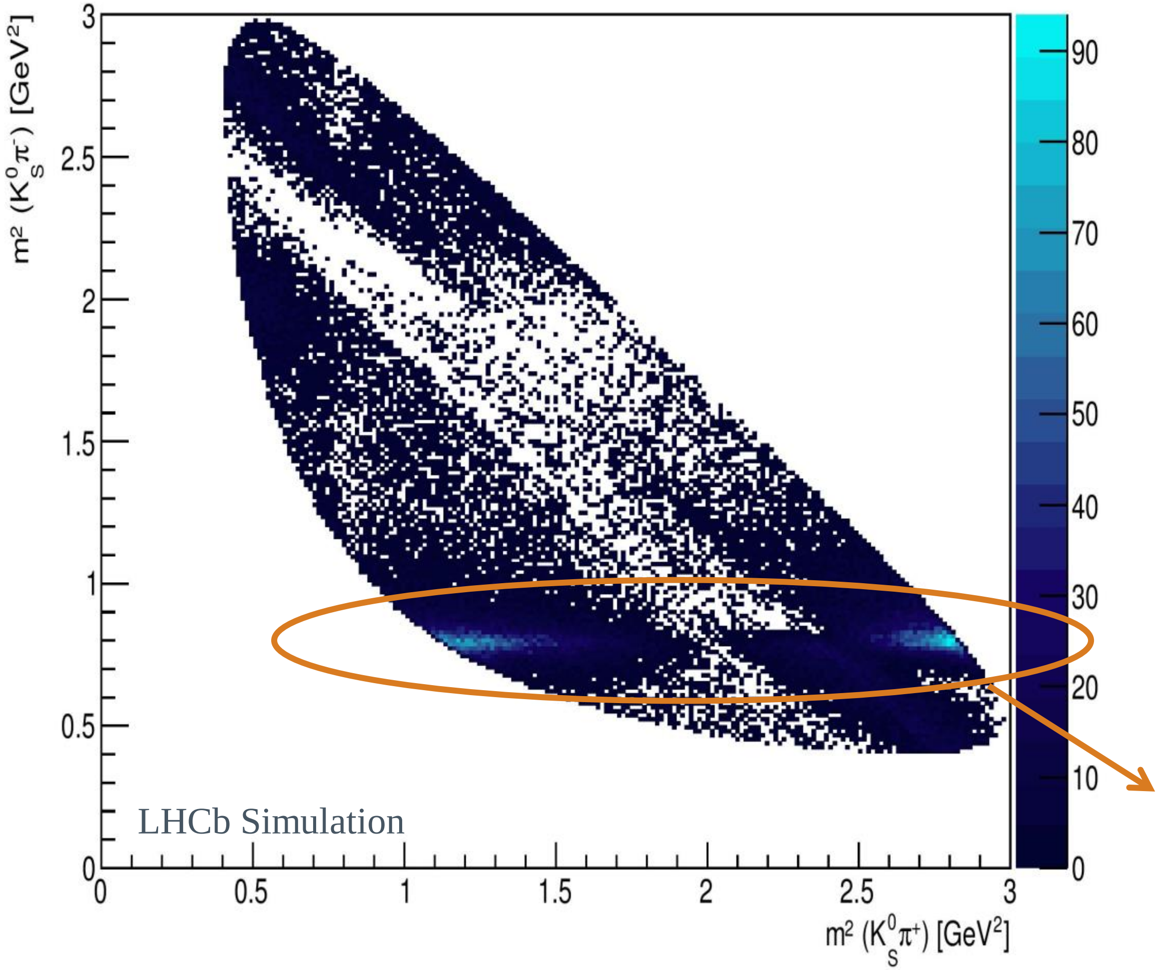

What we observe:
Dalitz plot is characterised by
Resonances
+ non-resonant background

Resonances (bands of high $|M|^2$)

Corresponds to

$D^0 \to r\, K_s \to K_s\, \pi^+\, \pi^-$

↑

Intermediate particle, e.g. ρ^0

$D^0 \to r\, \pi^+ \to K_s\, \pi^+\, \pi^-$

$D^0 \to r\, \pi^- \to K_s\, \pi^+\, \pi^-$

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$|M|^2$ is clearly affected by resonances

$D^0 \rightarrow K_s\pi^+\pi^-$

THE ISOBAR MODEL

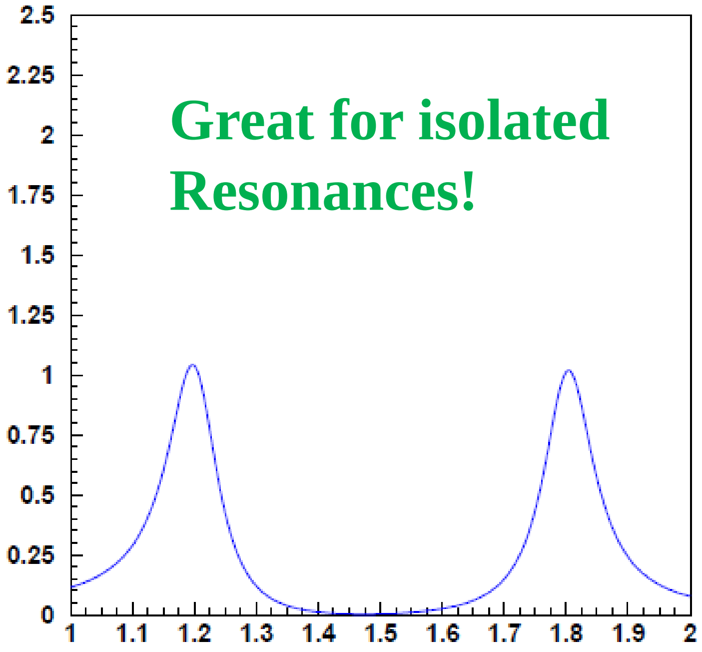
- Resonant decay treated as a superposition of 2-body decays
  e.g. $D \rightarrow rK_s$ and $r \rightarrow \pi^+\pi^-$

- Each resonance has a probability amplitude M_r

$$M = \sum_r a_r M_r$$

$$M_r = \langle \pi^+\pi^- | r \rangle \, \Delta_r(m_{\pi^+\pi^-}) \, \langle K_s r | D^0 \rangle$$

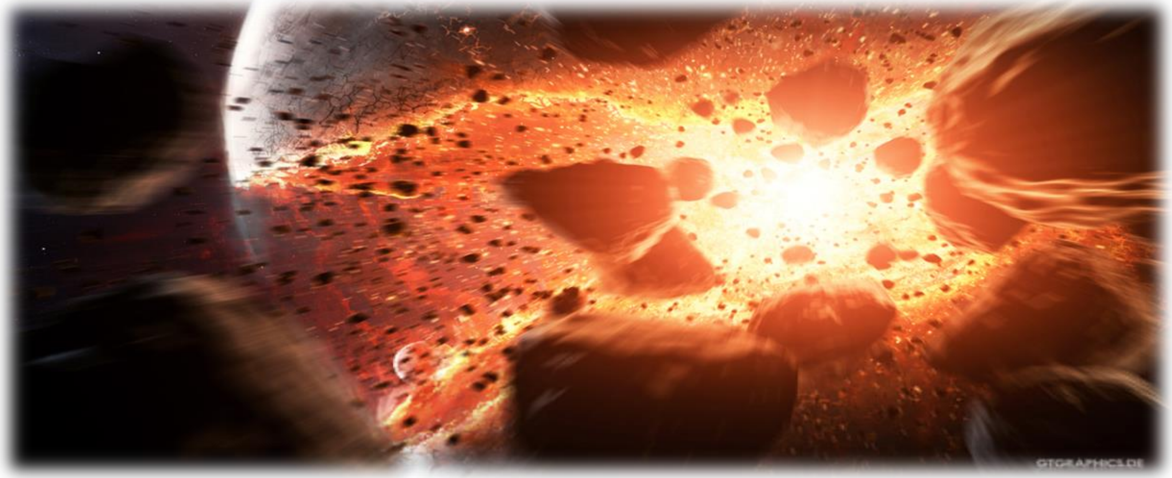
- Approximate by a **relativistic Breit–Wigner** probability distribution function



PROBLEM

If Resonances overlap

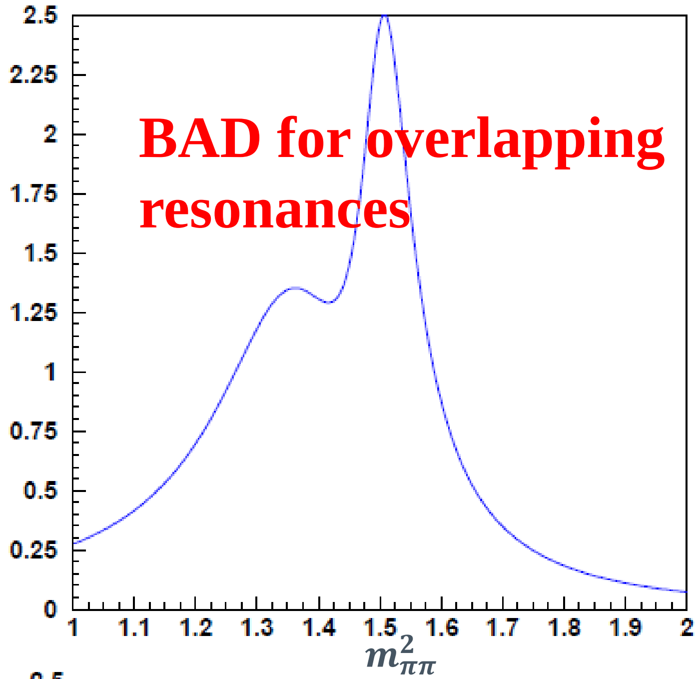
Total probability > 1

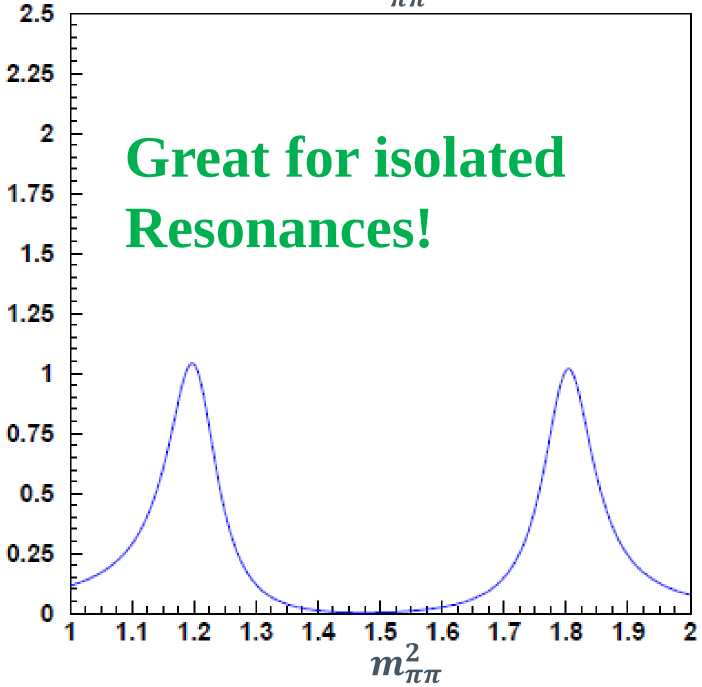


↪ Not good

↪ Theorists are unhappy

↪ Should probably do something about that…



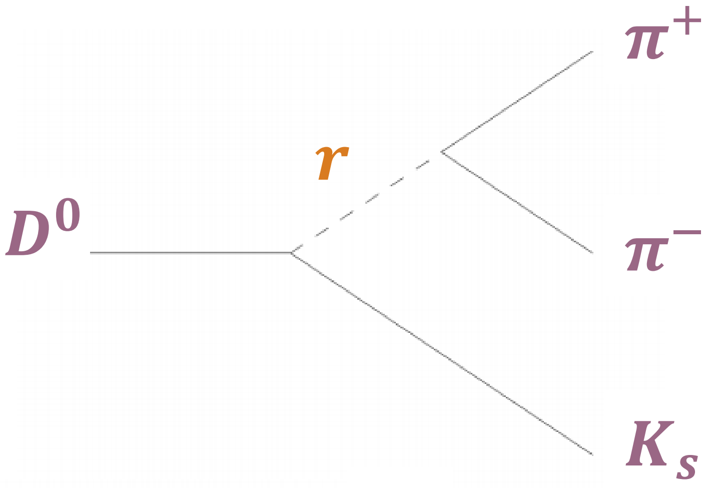


Curtis A. Meyer, Carnegie Mellon University, ‘A k-matrix tutorial’ (2008)

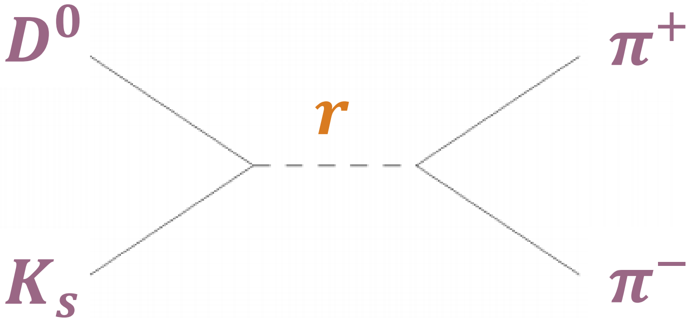
ALTERNATIVE MODEL

Makes use of **K-Matrix formalism** to describe resonances
$D^0 \rightarrow K_s r \rightarrow K_s \pi^+ \pi^-$ with $spin(r) = 0$ $(\pi\pi - S\ wave)$

Assumption:
$D^0 \rightarrow K_s \pi^+ \pi^-$ equivalent dynamics to K_s scattering off $\pi^+ \pi^-$



Need to consider all possible decays of r in the analysis
$r \rightarrow \pi\pi$
$r \rightarrow KK$
$r \rightarrow \pi\pi\pi\pi$
$r \rightarrow \eta\eta$
$r \rightarrow \eta\eta'$



K-Matrix Formulation

$$F = (I - iK\rho)^{-1}\mathbf{P}$$

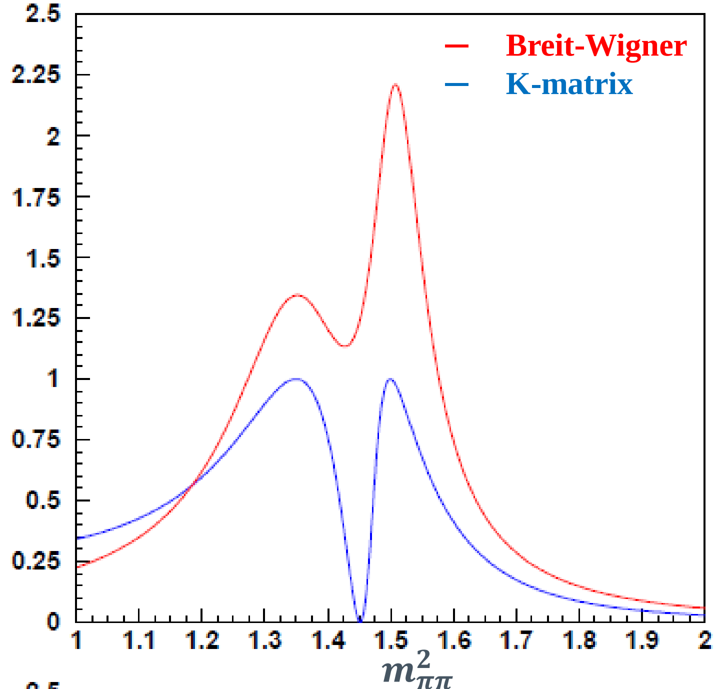
Pseudo-propagator
Comes from scattering data.

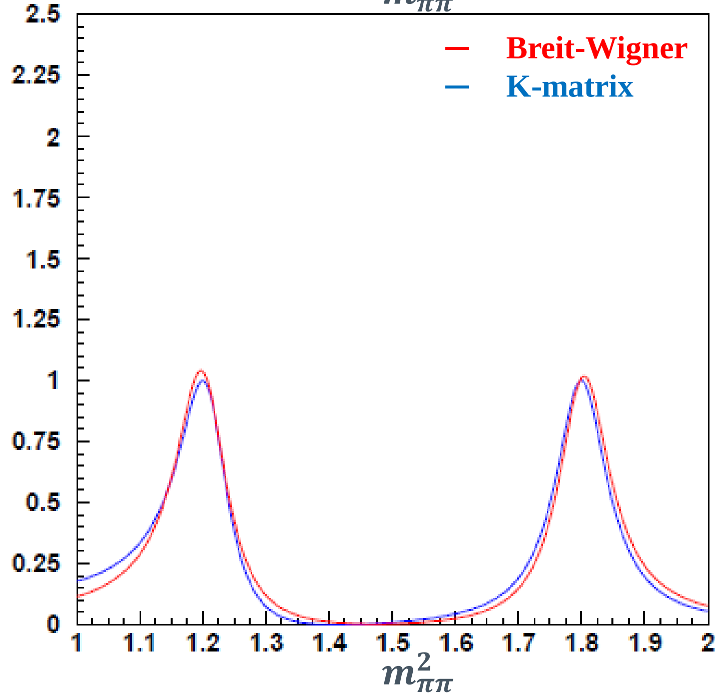
Production Vector
Describes Couplings of resonances to D^0

$\boldsymbol{K}$ = *K - Matrix*
$\boldsymbol{\rho}$ = *phase-space factor*

$$\boldsymbol{M_r} = \langle \pi^+\pi^- | r \rangle \; \boldsymbol{F} \, \langle K_s r | D^0 \rangle$$

Upholds unitarity by construction!





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Implementation: CODING

My Job:
Implement **K-matrix** description in **fitting** code for
$D^0 \rightarrow K_s\pi^+\pi^-$

implement $F = (I - iK\rho)^{-1}\mathbf{P} \rightarrow$ fit to data

Programming language: CUDA ($\sim$ *C*++)

Running on a GPU (Graphical Processing Unit)

– *extremely fast parallelisation*

What are we fitting to data? $F = (I - iK\rho)^{-1}\mathbf{P}$

K – K-matrix: *comes entirely from scattering data*

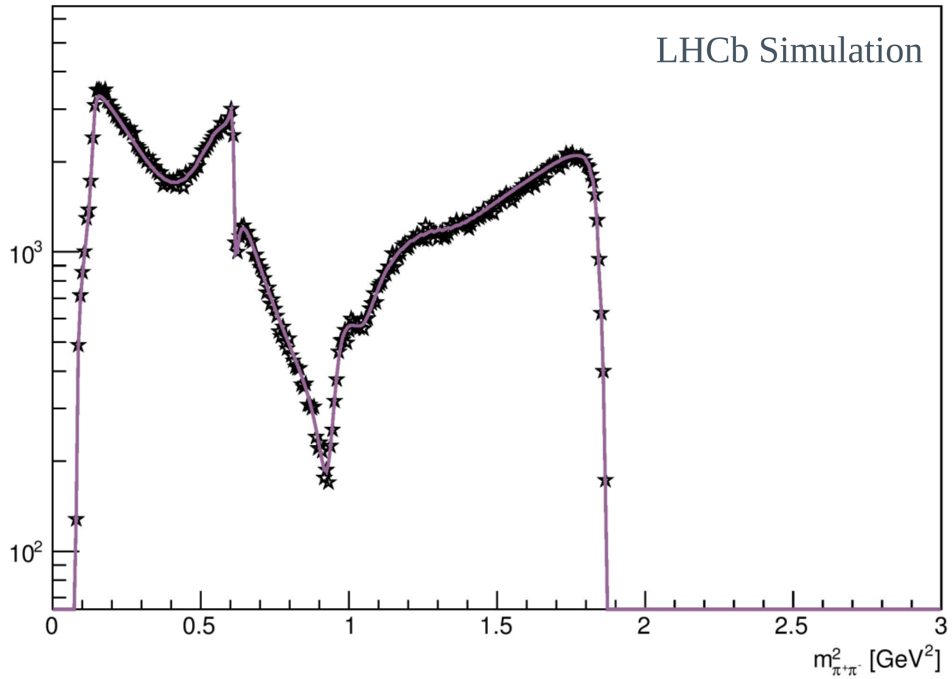
ρ – phase space matrix: *depends only on masses*

P – production vector: describes coupling to resonances ← Need to fit

$$P_j = \sum_{\alpha} \frac{\beta_{\alpha}^{0} g_{\alpha j}^{0}}{m_{\alpha}^{2} - m_{\pi\pi}^{2}} + f_{\pi\pi j}^{pr} \frac{1 - s_0^{pr}}{m_{\pi\pi}^{2} - s_0^{pr}}$$

21 floating variables

β_{α}^{0} and $f_{\pi\pi j}^{pr}$ are complex



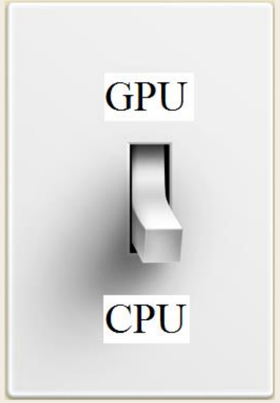
STATUS

- **Wrestling with errors**
  - *Observing things (printing to screen) changes variables.*

  → memory issues probably to blame

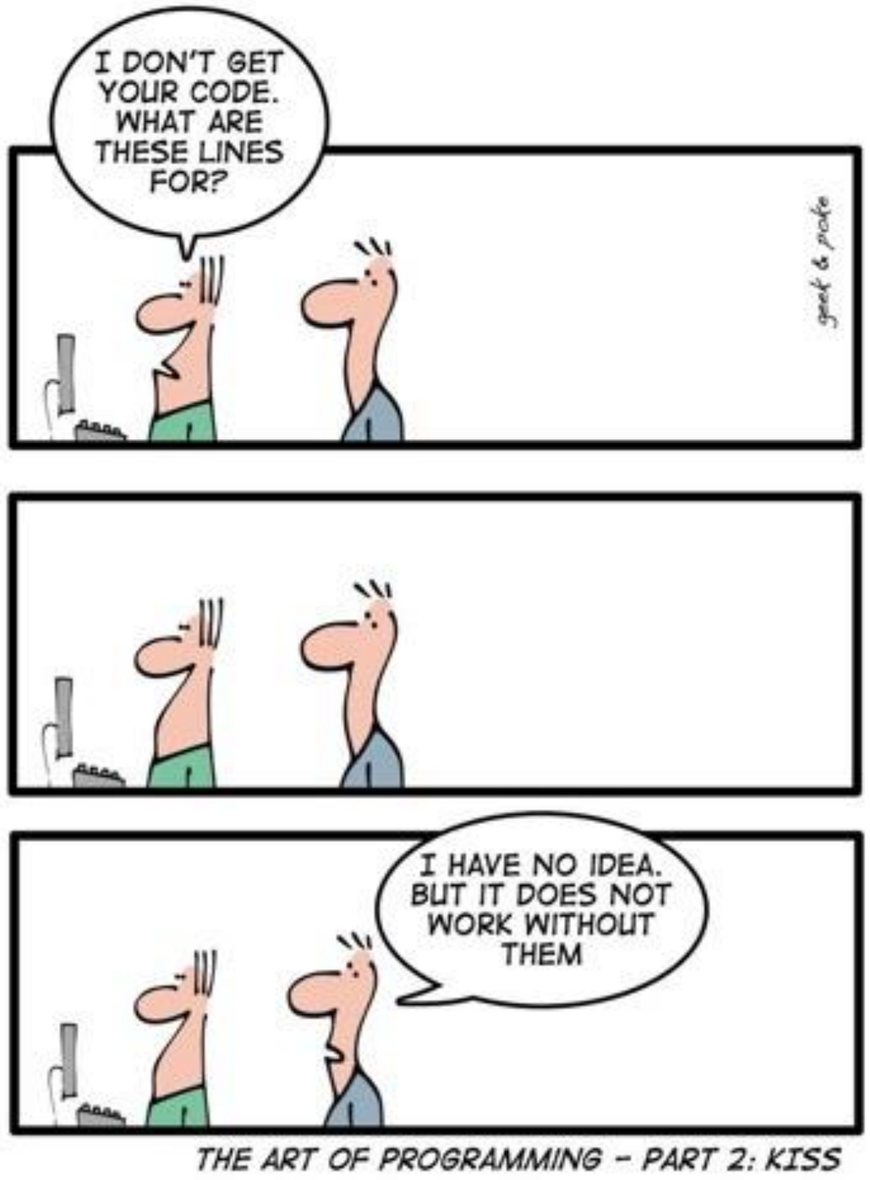
- *This Week:* ***Code Working on a CPU!***
  - *Positive cross-checks (e.g. D^0 decay time τ)*

  → *Implementation complete*

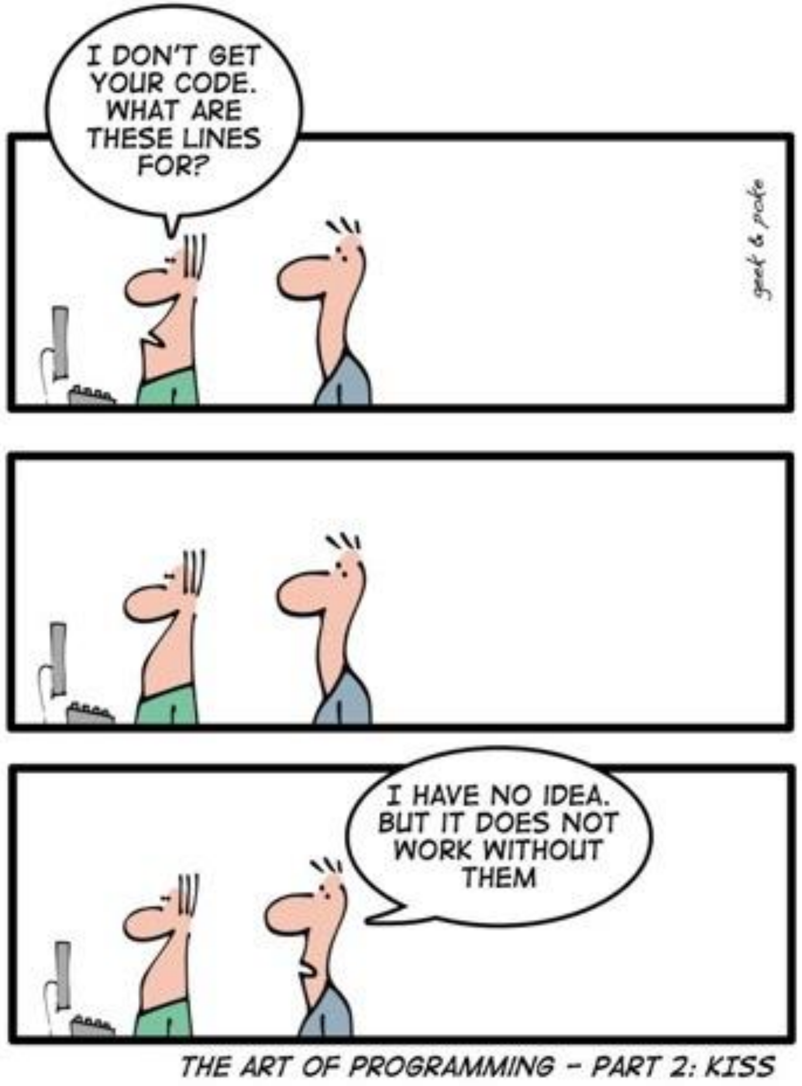


- ***GPU memory issues currently being addressed***

THANK YOU


I DON'T GET YOUR CODE. WHAT ARE THESE LINES FOR?
geek & poke
I HAVE NO IDEA. BUT IT DOES NOT WORK WITHOUT THEM
THE ART OF PROGRAMMING – PART 2: KISS

Questions?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THE ART OF PROGRAMMING - PART 2: KISS

Spare Slides

What are we fitting to data?

K-Matrix

$$K_{ij} = \left(\sum_{\alpha} \frac{g^0_{\alpha i} g^0_{\alpha j}}{m^2_\alpha - m^2_{\pi\pi}} + f^{sc}_{ij} \frac{1 - s^{sc}_0}{m^2_{\pi\pi} - s^{sc}_0}\right)\left[\frac{1 - s^A_0}{m^2_{\pi\pi} - s^A_0}\left(m^2_{\pi\pi} - \frac{a_A m^2_\pi}{2}\right)\right]$$

Production vector

$$P_j = \sum_{\alpha} \frac{\beta^0_\alpha g^0_{\alpha j}}{m^2_\alpha - m^2_{\pi\pi}} + f^{pr}_{\pi\pi j} \frac{1 - s^{pr}_0}{m^2_{\pi\pi} - s^{pr}_0}$$

Sum over poles   Non resonant term   Correction term

21 floating variables

β^0_α and $f^{pr}_{\pi\pi j}$ are complex → both real and imaginary parts nee

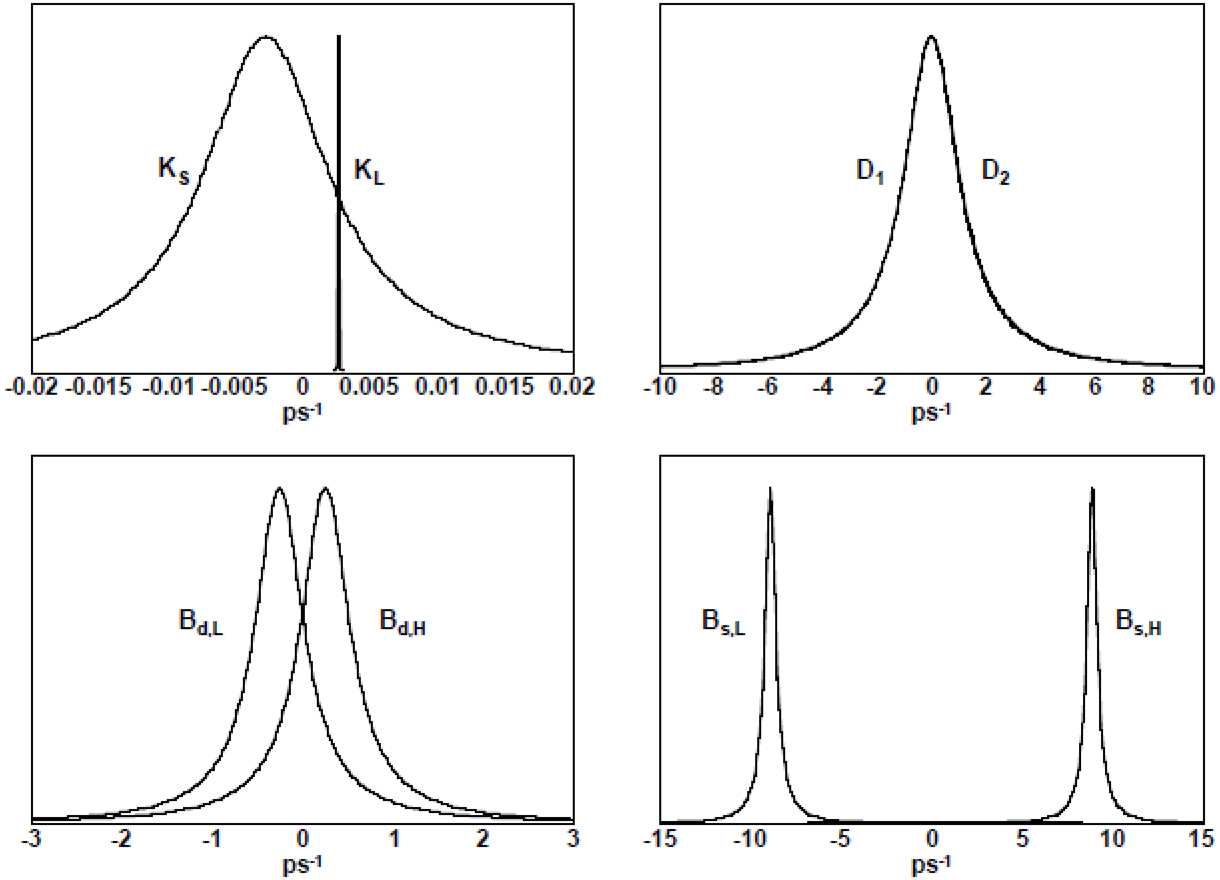
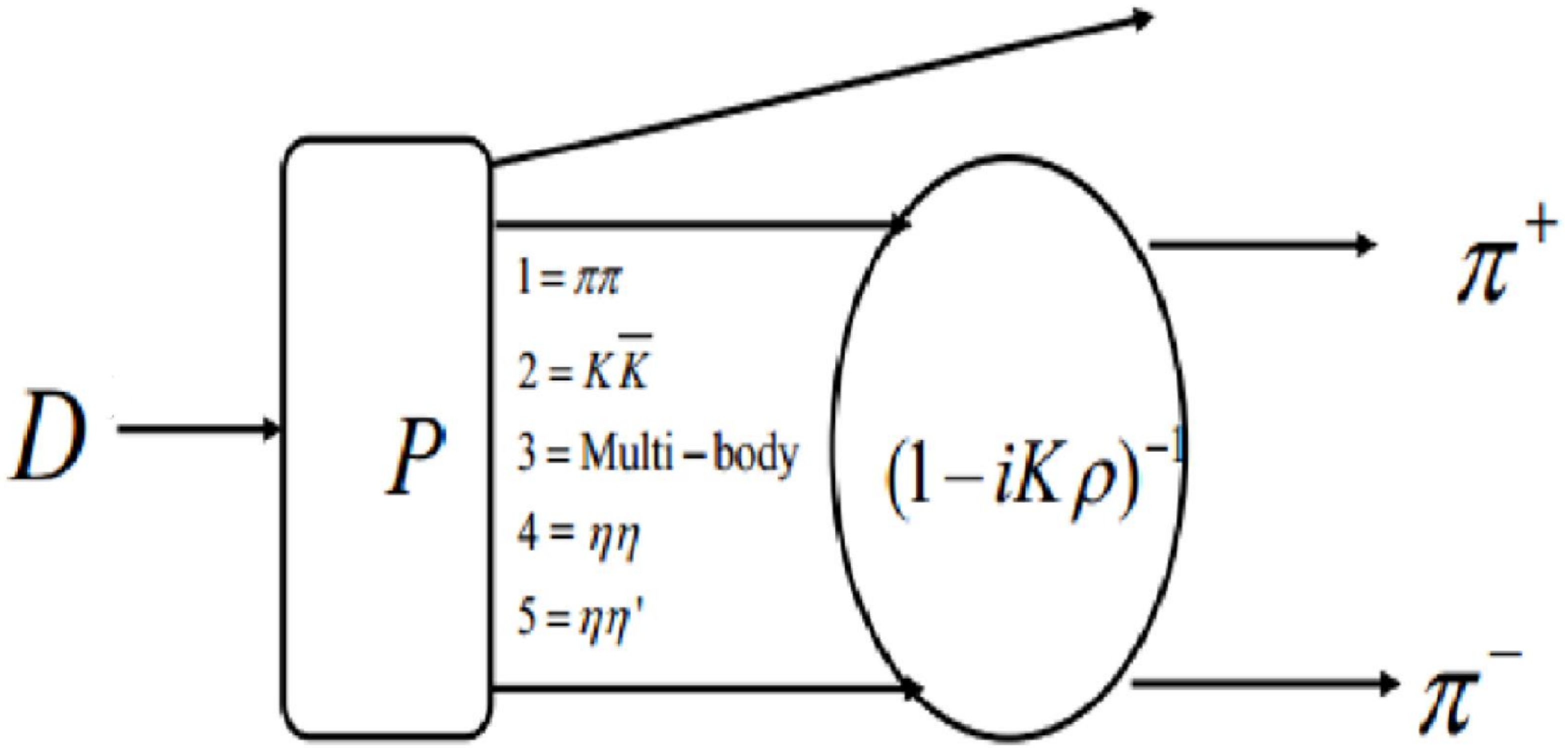


Fig. 1. The widths and mass differences of the physical states of the flavoured neutral mesons. The width corresponds to the inverse lifetime while the mass difference determines the oscillation frequency.


D
P
$1 = \pi\pi$
$2 = K\overline{K}$
$3 = \text{Multi-body}$
$4 = \eta\eta$
$5 = \eta\eta'$
$(1 - iK\rho)^{-1}$
π^+
π^-