# Monte Carlo Generators and Soft QCD

## 1. Introduction and Parton Showers

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# Course Plan

**Improve understanding of physics at the LHC**

Complementary to the "textbook" picture of particle physics, since event generators are close to how things work "in real life". Notably "soft QCD", only realistically addressed by generators.

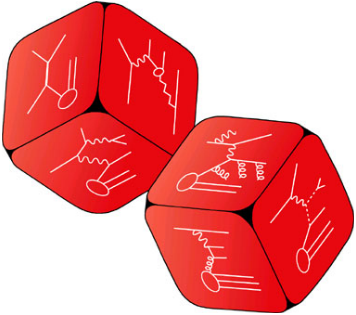
| | |
|---|---|
| Lecture 1 | Introduction and generator survey |
| | Parton showers: final and initial |
| Lecture 2 | Combining matrix elements and parton showers |
| Lecture 3 | Multiparton interactions and other soft physics |
| | Hadronization |
| | Conclusions |

Some prior contact with generators assumed. To learn more:

A. Buckley et al., "General-purpose event generators for LHC physics", Phys. Rep. 504 (2011) 145 [arXiv:1101.2599[hep-ph]]

or come to a MCnet summer school (see below).

# A tour to Monte Carlo

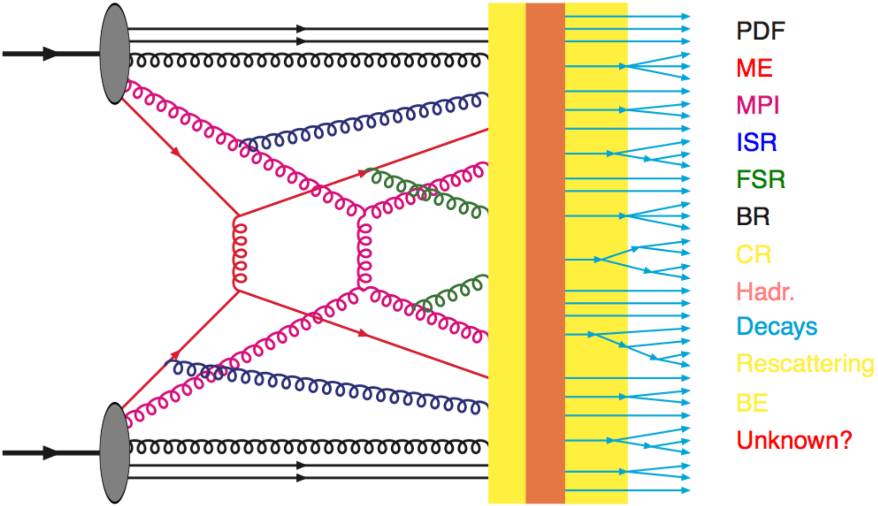


. . . because Einstein was wrong: God does throw dice!
Quantum mechanics: amplitudes $\Longrightarrow$ probabilities
Anything that possibly can happen, will! (but more or less often)

Event generators: trace evolution of event structure.
Random numbers $\approx$ quantum mechanical choices.

# The Structure of an Event

An event consists of many different physics steps, which have to be modelled by event generators:



# The Monte Carlo method

Want to generate events in as much detail as Mother Nature
$\Longrightarrow$ get average *and* fluctutations right
$\Longrightarrow$ make random choices, $\sim$ as in nature

$$\sigma_{\mathrm{final\ state}} = \sigma_{\mathrm{hard\ process}} \, \mathcal{P}_{\mathrm{tot,hard\ process} \to \mathrm{final\ state}}$$

(appropriately summed & integrated over non-distinguished final states)

where $\mathcal{P}_{\mathrm{tot}} = \mathcal{P}_{\mathrm{res}} \, \mathcal{P}_{\mathrm{ISR}} \, \mathcal{P}_{\mathrm{FSR}} \, \mathcal{P}_{\mathrm{MPI}} \mathcal{P}_{\mathrm{remnants}} \, \mathcal{P}_{\mathrm{hadronization}} \, \mathcal{P}_{\mathrm{decays}}$

with $\mathcal{P}_i = \prod_j \mathcal{P}_{ij} = \prod_j \prod_k \mathcal{P}_{ijk} = \ldots$ in its turn

$\Longrightarrow$ **divide and conquer**

an event with $n$ particles involves $\mathcal{O}(10n)$ random choices,
(flavour, mass, momentum, spin, production vertex, lifetime, ...)
LHC: $\sim 100$ charged and $\sim 200$ neutral (+ intermediate stages)
$\Longrightarrow$ several thousand choices
(of $\mathcal{O}(100)$ different kinds)

# The workhorses: what are the differences?

HERWIG, PYTHIA and SHERPA offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA (successor to JETSET, begun in 1978):
- originated in hadronization studies: the Lund string
- leading in development of MPI for MB/UE
- pragmatic attitude to showers & matching

HERWIG (successor to EARWIG, begun in 1984):
- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays

SHERPA (APACIC++/AMEGIC++, begun in 2000):
- own matrix-element calculator/generator
- extensive machinery for CKKW ME/PS matching
- hadronization & min-bias physics under development

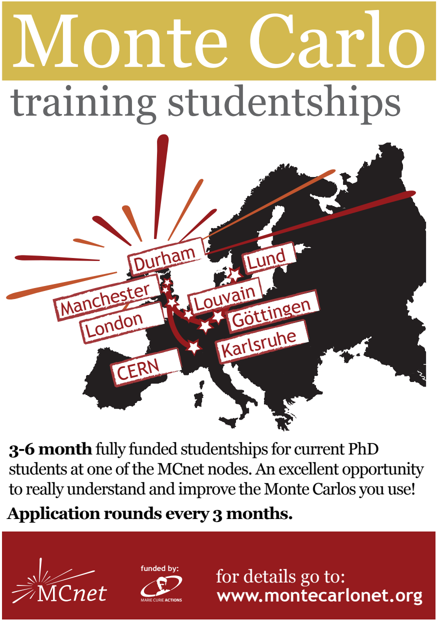
PYTHIA and HERWIG originally in Fortran, but now all in C++.

MCnet projects:

- PYTHIA (+ VINCIA)
- HERWIG
- SHERPA
- MadGraph
- Ariadne (+ DIPSY)
- Cedar (Rivet/Professor)

Activities include

- summer schools (2014: Manchester?)
- short-term studentships
- graduate students
- postdocs
- meetings (open/closed)



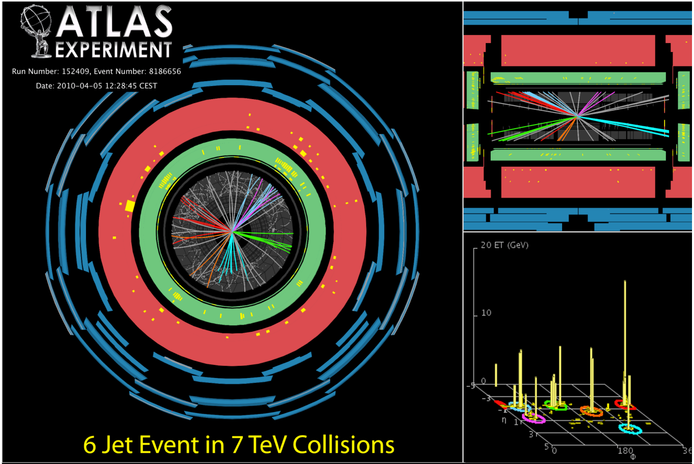
# Other Relevant Software

Some examples (with apologies for many omissions):

- Other event/shower generators: PhoJet, Ariadne, Dipsy, Cascade, Vincia
- Matrix-element generators: MadGraph/MadEvent, CompHep, CalcHep, Helac, Whizard, Sherpa, GoSam, aMC@NLO
- Matrix element libraries: AlpGen, POWHEG BOX, MCFM, NLOjet++, VBFNLO, BlackHat, Rocket
- Special BSM scenarios: Prospino, Charybdis, TrueNoir
- Mass spectra and decays: SOFTSUSY, SPHENO, HDecay, SDecay
- Feynman rule generators: FeynRules
- PDF libraries: LHAPDF
- Resummed ($p_\perp$) spectra: ResBos
- Approximate loops: LoopSim
- Jet finders: anti-$k_\perp$ and FastJet
- Analysis packages: Rivet, Professor, MCPLOTS
- Detector simulation: GEANT, Delphes
- Constraints (from cosmology etc): DarkSUSY, MicrOmegas
- Standards: PDF identity codes, LHA, LHEF, SLHA, Binoth LHA, HepMC

**Can be meaningfully combined and used for LHC physics!**

# Multijets – the need for Higher Orders

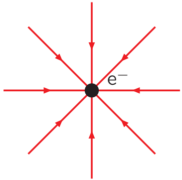



An event with 6 jets taken on April 4th, 2010. The jets have calibrated transverse momenta between 30 GeV and 70 GeV and are well separated in the detector. 12

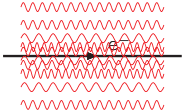
$2 \to 6$ process or $2 \to 2$ dressed up by bremsstrahlung!?

# In the beginning: Electrodynamics

An electrical charge, say an electron, is surrounded by a field:




For a rapidly moving charge this field can be expressed in terms of an equivalent flux of photons:



$$\mathrm{dn}_\gamma \approx \frac{2\alpha_{\mathrm{em}}}{\pi} \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}\omega}{\omega}$$

Equivalent Photon Approximation,
or method of virtual quanta (e.g. Jackson)
(Bohr; Fermi; Weiszäcker, Williams $\sim$1934)

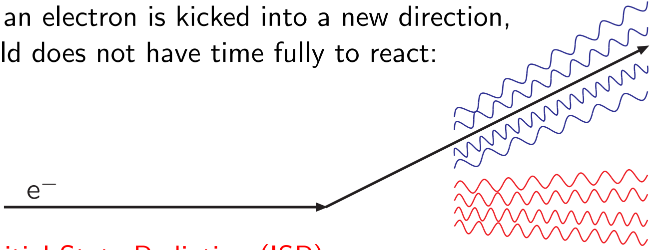
$\theta$: collinear divergence, saved by $m_{\mathrm{e}} > 0$ in full expression.

$\omega$: true divergence, $n_\gamma \propto \int \mathrm{d}\omega/\omega = \infty$, but $E_\gamma \propto \int \omega \, \mathrm{d}\omega/\omega$ finite.

These are virtual photons: continuously emitted and reabsorbed.

# In the beginning: Bremsstrahlung

(Radio antenna: accelerated charges ⇒ emission of real photons.)
When an electron is kicked into a new direction,
the field does not have time fully to react:




- Initial State Radiation (ISR):
  part of it continues ∼ in original direction of e
- Final State Radiation (FSR):
  the field needs to be regenerated around outgoing e,
  and transients are emitted ∼ around outgoing e direction

Emission rate provided by equivalent photon flux in both cases.
Approximate cutoffs related to timescale of process:
the more violent the hard collision, the more radiation!

# In the beginning: Exponentiation

Assume $\sum E_\gamma \ll E_\mathrm{e}$ such that energy-momentum conservation is not an issue. Then

$$\mathrm{d}\mathcal{P}_\gamma = \mathrm{dn}_\gamma \approx \frac{2\alpha_\mathrm{em}}{\pi} \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}\omega}{\omega}$$

is the probability to find a photon at $\omega$ and $\theta$, *irrespectively* of which other photons are present.

Uncorrelated $\Rightarrow$ Poissonian number distribution:

$$\mathcal{P}_i = \frac{\langle n_\gamma \rangle^i}{i!} e^{-\langle n_\gamma \rangle}$$

with

$$\langle n_\gamma \rangle = \int_{\theta_\mathrm{min}}^{\theta_\mathrm{max}} \int_{\omega_\mathrm{min}}^{\omega_\mathrm{max}} \mathrm{dn}_\gamma \approx \frac{2\alpha_\mathrm{em}}{\pi} \ln\left(\frac{\theta_\mathrm{max}}{\theta_\mathrm{min}}\right) \ln\left(\frac{\omega_\mathrm{max}}{\omega_\mathrm{min}}\right)$$

Note that $\int \mathrm{d}\mathcal{P}_\gamma = \int \mathrm{dn}_\gamma > 1$ is not a problem: proper interpretation is that *many* photons are emitted.

Exponentiation: reinterpretation of $\mathrm{d}\mathcal{P}_\gamma$ into Poissonian.

# QED: Fixed Order Perturbation Theory

Order-by-order perturbative ME calculation contains fully differential distributions of multi-$\gamma$ emissions,

but integrating the main contributions (leading logs) gives

$$\begin{array}{rclllll} \frac{\sigma_{0\gamma}}{\sigma_0} & \approx & 1 & -\alpha_{\mathrm{em}} N & +\alpha_{\mathrm{em}}^2 \frac{N^2}{2} & -\alpha_{\mathrm{em}}^3 \frac{N^3}{6} \\ \frac{\sigma_{1\gamma}}{\sigma_0} & \approx & & +\alpha_{\mathrm{em}} N & -\alpha_{\mathrm{em}}^2 N^2 & +\alpha_{\mathrm{em}}^3 \frac{N^3}{2} \\ \frac{\sigma_{2\gamma}}{\sigma_0} & \approx & & & +\alpha_{\mathrm{em}}^2 \frac{N^2}{2} & -\alpha_{\mathrm{em}}^3 \frac{N^3}{2} \\ \frac{\sigma_{3\gamma}}{\sigma_0} & \approx & & & & +\alpha_{\mathrm{em}}^3 \frac{N^3}{6} \end{array}$$

which is the expanded form of the Poissonian $\mathcal{P}_i = \langle n_\gamma \rangle^i \, e^{-\langle n_\gamma \rangle} / i!$ with $\langle n_\gamma \rangle = \alpha_{\mathrm{em}} N$.

For practical applications two different regions

- large $\theta, \omega \Rightarrow$ rapidly convergent perturbation theory
- small $\theta, \omega \Rightarrow$ exponentiation needed, even if approximate

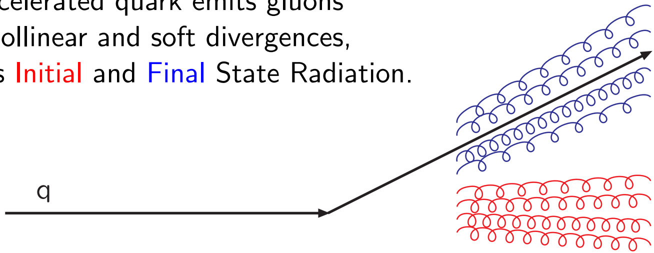
# So how is QCD the same?

- A quark is surrounded by a gluon field

$$\mathrm{d}\mathcal{P}_{\mathrm{g}} = \mathrm{dn}_{\mathrm{g}} \approx \frac{8\alpha_{\mathrm{s}}}{3\pi} \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}\omega}{\omega}$$

  i.e. only differ by substitution $\alpha_{\mathrm{em}} \to 4\alpha_{\mathrm{s}}/3$.

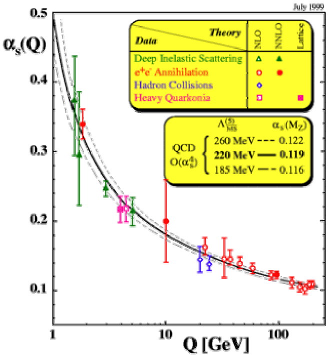
- An accelerated quark emits gluons with collinear and soft divergences, and as Initial and Final State Radiation.




- Typically $\langle n_{\mathrm{g}} \rangle = \int \mathrm{dn}_{\mathrm{g}} \gg 1$ since $\alpha_{\mathrm{s}} \gg \alpha_{\mathrm{em}}$
  $\Rightarrow$ even more pressing need for exponentiation.

# So how is QCD different?

- QCD is non-Abelian, so a gluon is charged and is surrounded by its own field:
  emission rate $4\alpha_\mathrm{s}/3 \to 3\alpha_\mathrm{s}$,
  field structure more complicated,
  interference effects more important.
- $\alpha_s(Q^2)$ diverges for $Q^2 \to \Lambda^2_\mathrm{QCD}$,
  with $\Lambda_\mathrm{QCD} \sim 0.2\,\mathrm{GeV} = 1\,\mathrm{fm}^{-1}$.
- Confinement: gluons below $\Lambda_\mathrm{QCD}$ not resolved $\Rightarrow$ de facto cutoffs.



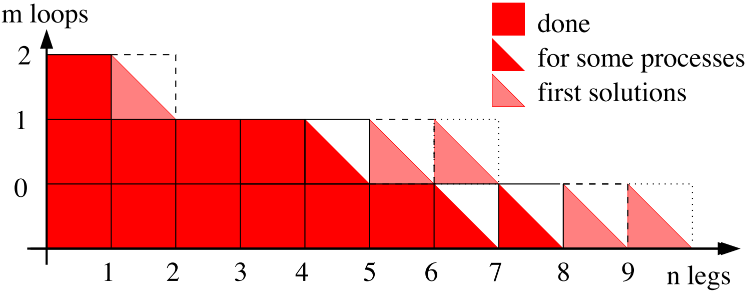

Unclear separation between
"accelerated charge" and "emitted radiation":
many possible Feynman graphs $\approx$ histories.

Next: matrix element (ME) and parton shower (PS) descriptions.

# Perturbative QCD – 1

Higher orders involve two frontiers
- more *legs* = final-state particles
- more *loops* = virtual corrections

Availability of “exact” calculations for hadron colliders:



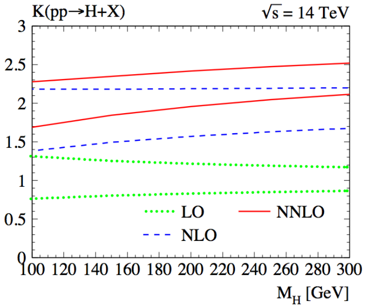

(courtesy Frank Krauss)

Note marked asymmetry between progress along the two axes!

# Perturbative QCD – 2

Order-by-order calculations: challenges more math than physics.

- LO: solved for all practical applications.
- NLO: in process of being automatized.
- NNLO: the current calculational frontier.
- Another bottleneck: efficient phase space sampling.




$gg \to H^0$ illustrates problems:

- Need high-precision calculations
- to search for BSM physics,
- but limited by poorly-understood slow convergence.

# Divergences

Perturbative calculations reliable for hard, well separated jets, but divergent behaviour for $\theta \to 0, \omega \to 0$.

With MEs need to calculate to high order *and* with many loops $\Rightarrow$ extremely demanding technically (not solved!), and involving big cancellations between positive and negative contributions.

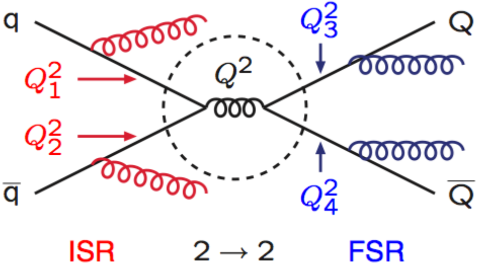
Two approaches address these issues:

- Resummation: analytical exponentiation;
- Parton showers: numerical exponentiation.

i.e. both reinterpret large probabilities as multiple emissions.

- Resummation: can be systematically improved order by order, but limited to a few observables;
- Parton showers: can address any (parton-level) observable, but typically with less accuracy.

$$2 \to n \quad = \quad (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$

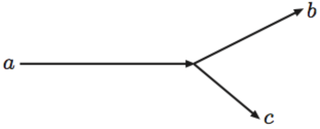



FSR = Final-State Radiation = timelike shower
$Q_i^2 \sim m^2 > 0$ decreasing
ISR = Initial-State Radiation = spacelike showers
$Q_i^2 \sim -m^2 > 0$ increasing

# Why "time" like and "space" like?

Consider four-momentum conservation in a branching $a \to b\, c$

$$\mathbf{p}_{\perp a} = 0 \quad \Rightarrow \quad \mathbf{p}_{\perp c} = -\mathbf{p}_{\perp b}$$
$$p_{+} = E + p_{\mathrm{L}} \quad \Rightarrow \quad p_{+a} = p_{+b} + p_{+c}$$
$$p_{-} = E - p_{\mathrm{L}} \quad \Rightarrow \quad p_{-a} = p_{-b} + p_{-c}$$




Define $p_{+b} = z\, p_{+a}, \quad p_{+c} = (1 - z)\, p_{+a}$

Use $p_{+}p_{-} = E^2 - p_{\mathrm{L}}^2 = m^2 + p_{\perp}^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z\, p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1 - z)\, p_{+a}}$$

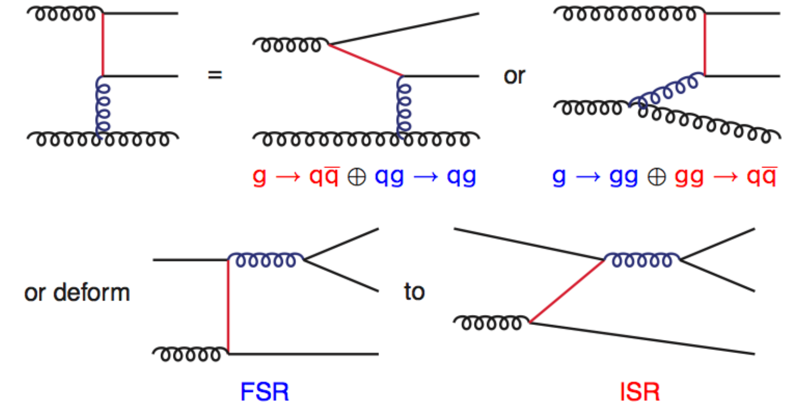
$$\Rightarrow \quad m_a^2 = \frac{m_b^2 + p_{\perp}^2}{z} + \frac{m_c^2 + p_{\perp}^2}{1 - z} = \frac{m_b^2}{z} + \frac{m_c^2}{1 - z} + \frac{p_{\perp}^2}{z(1 - z)}$$

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_{\perp}^2}{z(1-z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_{\perp}^2}{1-z} < 0 \Rightarrow$ spacelike

# Doublecounting

A $2 \to n$ graph can be "simplified" to $2 \to 2$ in different ways:




Do not doublecount: $2 \to 2$ = most virtual = shortest distance

Conflict: theory derivations assume virtualities strongly ordered; interesting physics often in regions where this is not true!

# The DGLAP equations

Probability of branchings $a \to bc$ described by

## DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z$$

$$P_{\mathrm{q}\to\mathrm{qg}} = \frac{4}{3}\frac{1+z^2}{1-z} \qquad \text{(neglecting quark masses)}$$

$$P_{\mathrm{g}\to\mathrm{gg}} = 3\,\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{\mathrm{g}\to\mathrm{q}\overline{\mathrm{q}}} = \frac{n_f}{2}\,(z^2+(1-z)^2) \qquad (n_f = \text{no. of quark flavours})$$

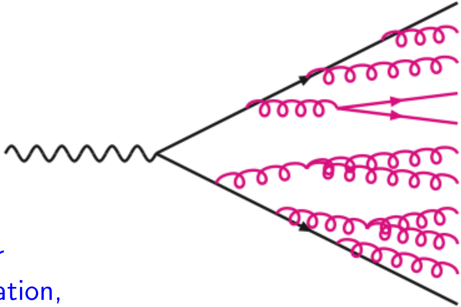
Universality: any matrix element reduces to DGLAP in collinear limit.

$$\text{e.g.} \quad \frac{\mathrm{d}\sigma(\mathrm{H}^0\to\mathrm{q}\overline{\mathrm{q}}\mathrm{g})}{\mathrm{d}\sigma(\mathrm{H}^0\to\mathrm{q}\overline{\mathrm{q}})} = \frac{\mathrm{d}\sigma(\mathrm{Z}^0\to\mathrm{q}\overline{\mathrm{q}}\mathrm{g})}{\mathrm{d}\sigma(\mathrm{Z}^0\to\mathrm{q}\overline{\mathrm{q}})} \quad \text{in collinear limit}$$

# The iterative structure

One-emission expression generalizes to many consecutive emissions if strongly ordered, $Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots$ ($\approx$ time-ordered).
To cover "all" of phase space use DGLAP in whole region $Q_1^2 > Q_2^2 > Q_3^2 \ldots$.

Iteration gives (final-state) parton showers:



Iterative structure allows for energy–momentum conservation, unlike simple exponentiation.

Need soft/collinear cuts to stay away from nonperturbative physics. Details model-dependent, but around 1 GeV scale.

# The ordering variable

In the evolution with

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z$$

$Q^2$ orders the emissions (memory).

If $Q^2 = m^2$ (for FSR) is one possible evolution variable then $Q'^2 = f(z)Q^2$ is also allowed, since

$$\left|\frac{\mathrm{d}(Q'^2,z)}{\mathrm{d}(Q^2,z)}\right| = \begin{vmatrix} \frac{\partial Q'^2}{\partial Q^2} & \frac{\partial Q'^2}{\partial z} \\ \frac{\partial z}{\partial Q^2} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} f(z) & f'(z)Q^2 \\ 0 & 1 \end{vmatrix} = f(z)$$

$$\Rightarrow \mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{f(z)\mathrm{d}Q^2}{f(z)Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q'^2}{Q'^2}\,P_{a\to bc}(z)\,\mathrm{d}z$$

- $Q'^2 = E_a^2\theta_{a\to bc}^2 \approx m^2/(z(1-z))$; angular-ordered shower
- $Q'^2 = p_\perp^2 \approx m^2 z(1-z)$; transverse-momentum-ordered

# The Sudakov form factor – 1

Time evolution, conservation of total probability:
$\mathcal{P}(\text{no emission}) = 1 - \mathcal{P}(\text{emission})$.

Multiplicativeness, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$\begin{aligned}
\mathcal{P}_{\text{no}}(0 \leq t < T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{no}}(T_i \leq t < T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{em}}(T_i \leq t < T_{i+1})) \\
&= \exp\left( - \lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{em}}(T_i \leq t < T_{i+1}) \right) \\
&= \exp\left( - \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{em}}(t)}{\mathrm{d}t} \mathrm{d}t \right) \\
\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) &= \mathrm{d}\mathcal{P}_{\text{em}}(T) \exp\left( - \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{em}}(t)}{\mathrm{d}t} \mathrm{d}t \right)
\end{aligned}$$

# The Sudakov form factor – 2

Expanded, with $Q \sim 1/t$ (Heisenberg)

$$
\begin{aligned}
\mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\mathrm{d}Q^2}{Q^2}\,\frac{\alpha_\mathrm{s}}{2\pi}\,P_{a\to bc}(z)\,\mathrm{d}z \\
&\times \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_\mathrm{max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int\frac{\alpha_\mathrm{s}}{2\pi}\,P_{a\to bc}(z')\,\mathrm{d}z'\right)
\end{aligned}
$$

where the exponent is (one definition of) the Sudakov form factor

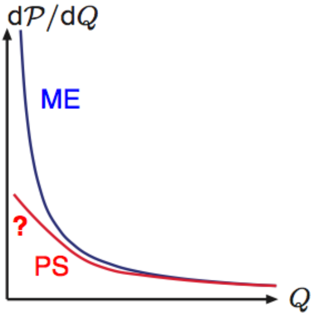
A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c}\int\int\,\mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo*
($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).

Intimately related to $e^{-\langle n\rangle}$ factor of Poissonian (exponentiation).

# The Sudakov form factor – 3

Sudakov regulates singularity for *first* emission ...




... but in limit of *repeated soft* emissions $\mathrm{q} \to \mathrm{qg}$ (but no $\mathrm{g} \to \mathrm{gg}$) one obtains the same inclusive $Q$ emission spectrum as for ME,

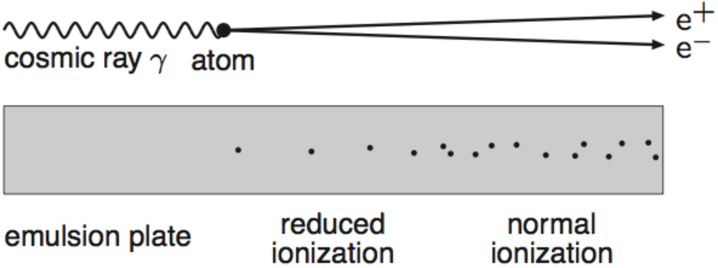
i.e. divergent ME spectrum
$\Longleftrightarrow$ infinite number of PS emissions

Naively exponentiation like in QED, but more complicated in reality:

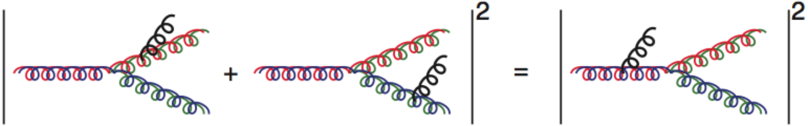
- energy-momentum conservation effects big since $\alpha_{\mathrm{s}}$ big, so hard emissions frequent
- $\mathrm{g} \to \mathrm{gg}$ branchings leads to accelerated multiplication of partons
- coherence effects important

# Coherence

QED: Chudakov effect (mid-fifties)




QCD: colour coherence for **soft** gluon emission

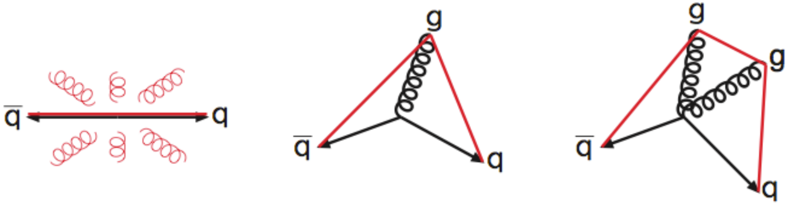



solved by • requiring emission angles to be decreasing
or • requiring transverse momenta to be decreasing

Standard shower language with $a \to bc$ successive branchings:


HERWIG: $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$
old PYTHIA: $Q^2 = m^2$ (+ brute-force coherence)

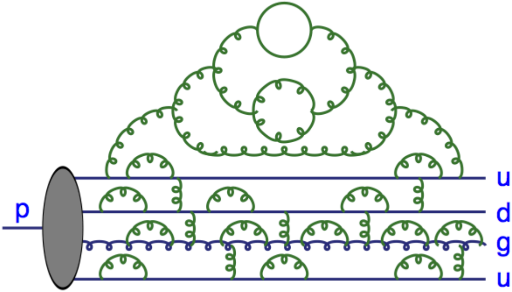
Newer ARIADNE picture of dipole emission $ab \to cde$:




is the basis for most current-day algorithms (HERWIG excepted)

# Parton Distribution Functions

Hadrons are composite, with time-dependent structure:




$f_i(x, Q^2)$ = number density of partons $i$
at momentum fraction $x$ and probing scale $Q^2$.

Linguistics (example):

$$F_2(x, Q^2) = \sum_i e_i^2 \, x f_i(x, Q^2)$$

structure function — parton distributions

# PDF evolution

Initial conditions at small $Q_0^2$ unknown: nonperturbative.

Resolution dependence perturbative, by DGLAP:

**DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)**

$$\frac{\mathrm{d}f_b(x, Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(y, Q^2) \frac{\alpha_\mathrm{s}}{2\pi} P_{a\to bc}\left(z = \frac{x}{y}\right)$$

DGLAP already introduced for (final-state) showers:

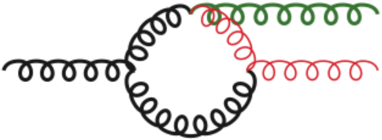
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z$$

Same equation, but different context:

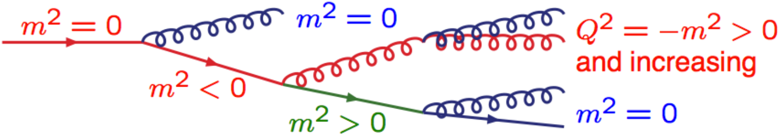
- $\mathrm{d}\mathcal{P}_{a\to bc}$ is probability for the individual parton to branch; while
- $\mathrm{d}f_b(x, Q^2)$ describes how the ensemble of partons evolve by the branchings of individual partons as above.

# Initial-State Shower Basics

- Parton cascades in $\mathrm{p}$ are continuously born and recombined.
- Structure at $Q$ is resolved at a time $t \sim 1/Q$ *before* collision.
- A hard scattering at $Q^2$ probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



- Convenient reinterpretation:



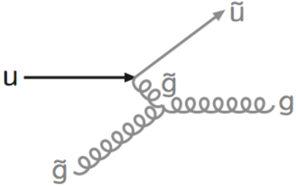
# Forwards vs. backwards evolution

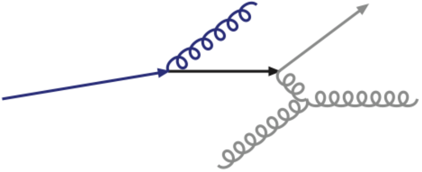
Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low $Q_0$ and evolve,
consider collisions at different $Q^2$ and pick by $\sigma$ of those.

**Inefficient:**

1. have to evolve and check for *all* potential collisions, but 99.9...% inert
2. impossible (or at least very complicated) to steer the production, e.g. of a narrow resonance (Higgs)

**Backwards evolution** is viable and ∼equivalent alternative:
start at hard interaction and trace what happened "before"



# Backwards evolution master formula

Monte Carlo approach, based on *conditional probability*: recast

$$\frac{\mathrm{d}f_b(x, Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} \, f_a(x', Q^2) \, \frac{\alpha_s}{2\pi} \, P_{a \to bc}(z)$$

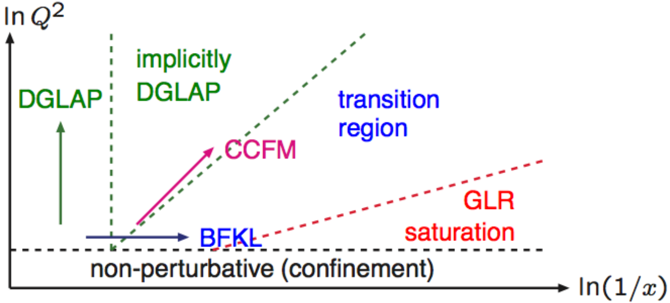
with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \, \frac{x' f_a(x', t)}{x f_b(x, t)} \, \frac{\alpha_s}{2\pi} \, P_{a \to bc}(z)$$

then solve for *de*creasing $t$, i.e. backwards in time,
starting at high $Q^2$ and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Webber: can be recast by noting that total change of PDF at $x$ is difference between gain by branchings from higher $x$ and loss by branchings to lower $x$.

# Evolution procedures

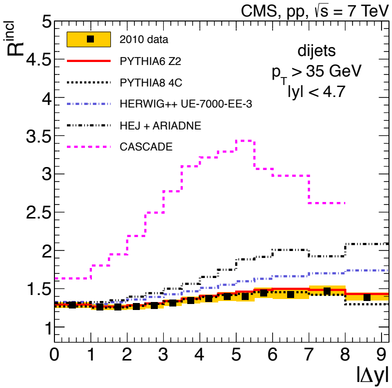



DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger $Q^2$ and (implicitly) towards smaller $x$
BFKL: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller $x$ (with small, unordered $Q^2$)
CCFM: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL
GLR: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region,
where partons recombine, not only branch

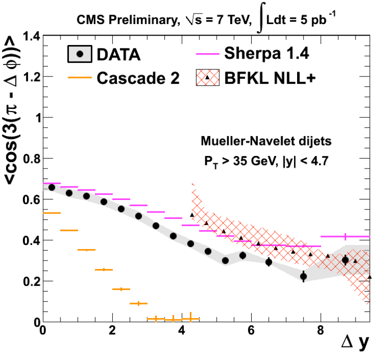
# Did we reach BFKL regime?

Study events with $\geq 2$ jets as a function of their $y$ separation.

Ratio of the inclusive to exclusive dijet cross sections:




Azimuthal decorrelation:




No strong indications for BFKL/CCFM behaviour onset so far!

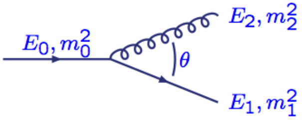
# Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \cdot (\mathrm{Sudakov})$$

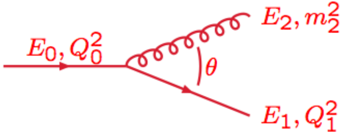
**but**

Final-state showers:
$Q^2$ timelike ($\sim m^2$)



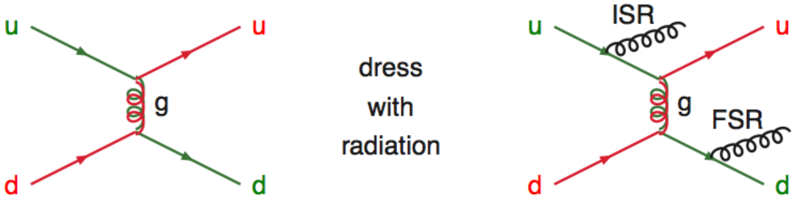

decreasing $E, m^2, \theta$
both daughters $m^2 \geq 0$
physics relatively simple
$\Rightarrow$ "minor" variations:
$Q^2$, shower vs. dipole, ...

Initial-state showers:
$Q^2$ spacelike ($\approx -m^2$)

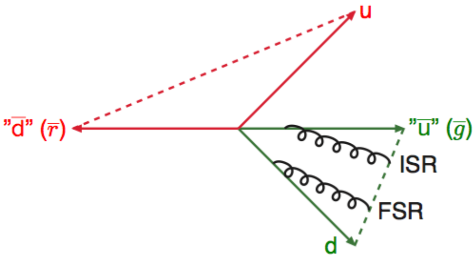



decreasing $E$, increasing $Q^2, \theta$
one daughter $m^2 \geq 0$, one $m^2 < 0$
physics more complicated
$\Rightarrow$ more formalisms:
DGLAP, BFKL, CCFM, GLR, ...

# Combining FSR with ISR




Separate processing of ISR and FSR misses interference
($\sim$ colour dipoles)

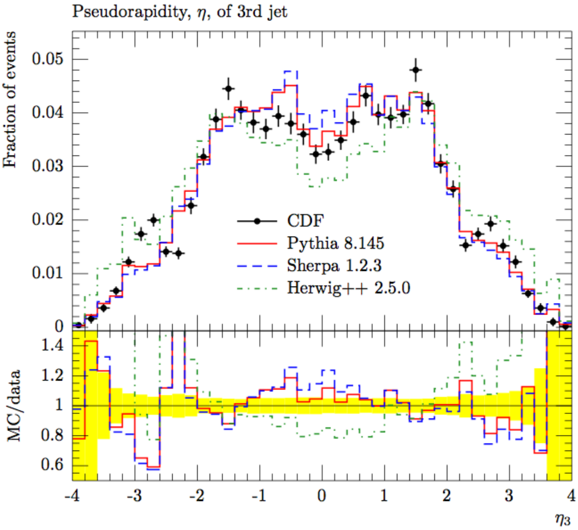



ISR+FSR add coherently
in regions of colour flow
and destructively else

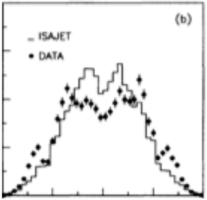
in "normal" shower by
azimuthal anisotropies

automatic in dipole
(by proper boosts)

Current-day generators for pseudorapidity of third jet:




and past incoherent:



# Summary and Outlook

- A multitude of physics mechanisms at play in pp collisions.
- Event generators separate problem into manageable chunks.
- Random numbers $\approx$ quantum mechanical choices.
- Often need to combine several software packages.
- Matrix element calculations at core of process selection.
- Parton shower offers convenient alternative to HO ME's.
- Unitarity by Sudakov form factor.

Next (this afternoon):

- Combining matrix elements and parton showers.