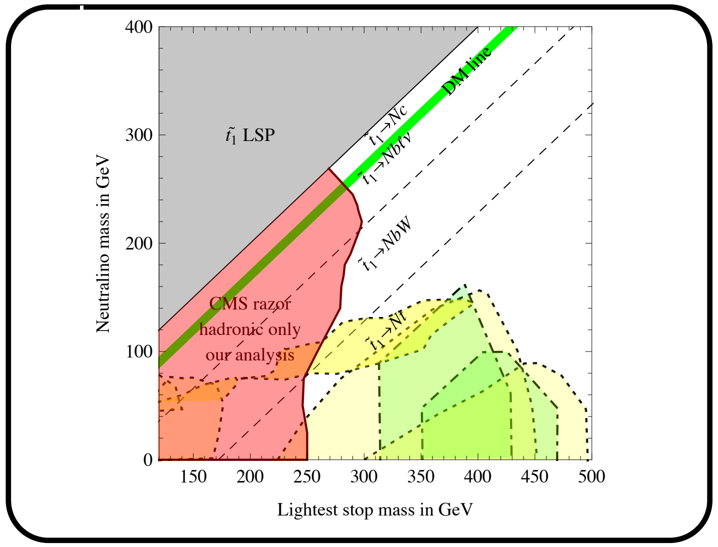
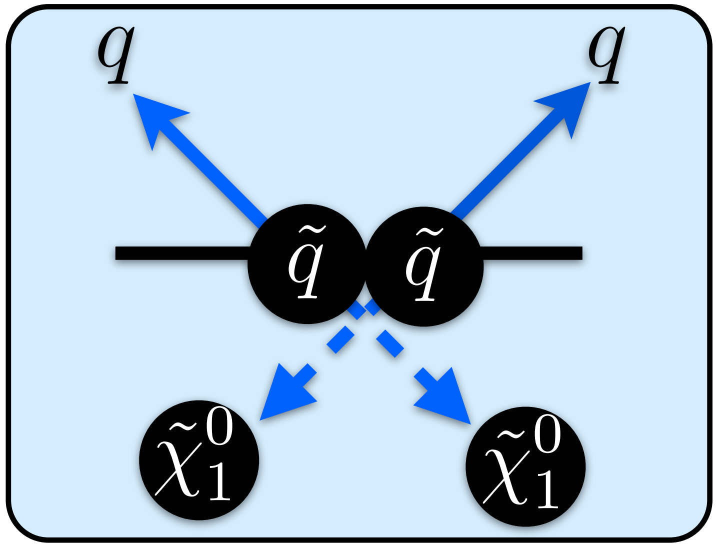
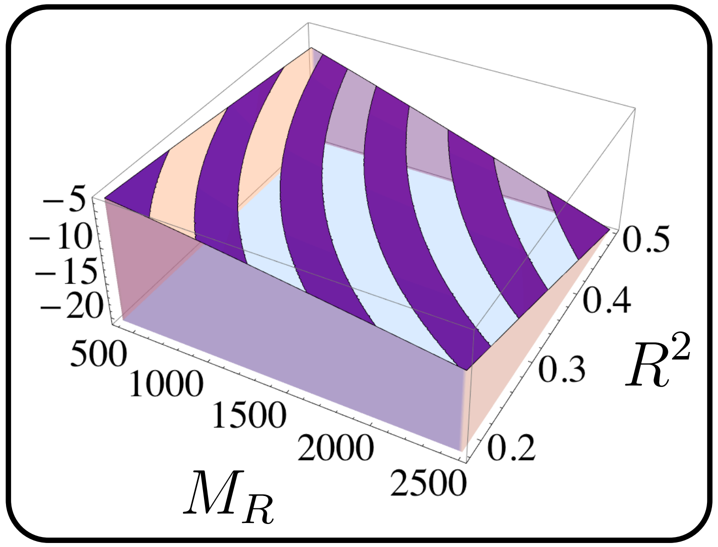
BSM Likelihoods in CMS







Likelihoods for the LHC Searches, CERN
January, 23, 2013

Javier Duarte
Caltech

Overview

- Likelihoods in BSM searches at CMS
  - Binned Cut and Count (Poisson)
  - Unbinned Shape Analysis (Analytic function)
  - Binned Cut and Count (Multinomial)
- Approximating the likelihood for reinterpretation
  - Simplify as binned cut and count (Poisson)
- Tools to help and future efforts

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Public Likelihoods in CMS

CMS SUSY Public Likelihoods

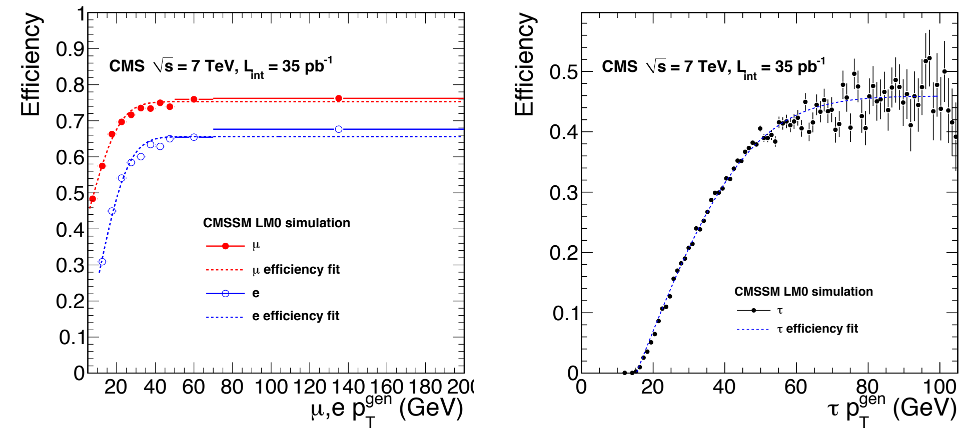
CMS SUSY Analysis	Reference	Likelihoods and Additional Information
Razor (7 TeV, 4.7 fb^{-1})	arXiv:1212.6961 twiki.cern.ch/twiki/bin/viewauth/CMSPublic/RazorLikelihoodHowTo	Binned Likelihood, Yields, **Forthcoming:** Detector Response, Efficiencies
SS Dilepton, 2 b-jets (8 TeV, 10.5 fb^{-1})	arXiv:1212.6194	Yields, Detector Response, Efficiencies
1 Lepton (7 TeV, 4.98 fb^{-1})	arXiv:1212.6428	Yields, Detector Response, Efficiencies
OS Dilepton	arXiv:1206.3949	Yields, Detector Response, Efficiencies
Z, Jets, MET	PLB 716, 260–284 (2012) arXiv:1204.3774	Yields, Detector Response, Efficiencies

Canonical Use of Likelihoods

Canonical Use

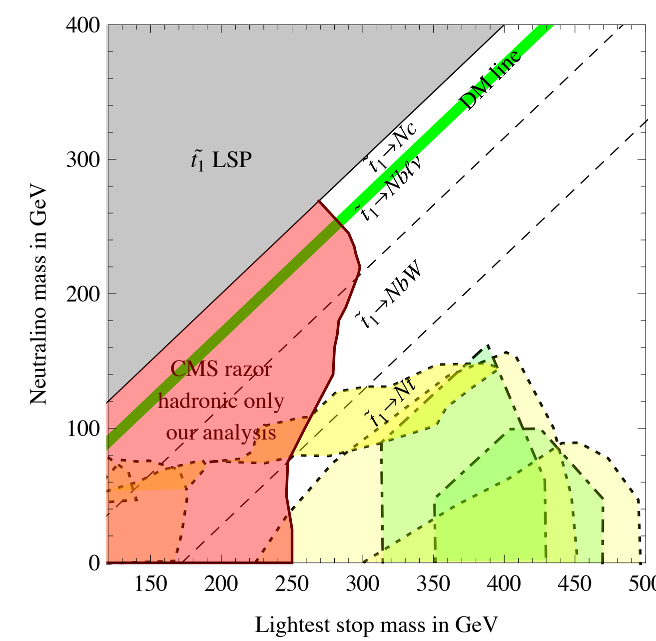
1. Use your favorite generator, Pythia8, MadGraph5, etc. for your BSM model

2. Apply cuts, efficiencies and smear with detector response



3. Reinterpret results for your BSM model with the likelihood

One possible simple application is a Bayesian Analysis...



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Bayesian Application of Likelihood

Recall from G. Cowan's talk

Bayes' theorem only needs $L(x|\theta)$ evaluated with a given data set (the 'likelihood principle').

Single bin counting experiment

Observe *N* events (data)

Interested in a signal, with Poisson likelihood and mean *s*+*b* (model)

$$\mathcal{L}(N|s,b) = \frac{(s+b)^N e^{-(s+b)}}{N!}$$

Update our priors in light of data with Bayes' rule

$$p(s,b|N) = \frac{\mathcal{L}(N|s,b)\pi(s,b)}{\int L(N|s,b)\pi(s,b)dbds}$$

normalization in model space

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Bayesian Application of Likelihood

Parameters play different roles

s - signal yield, parameter-of-interest

b - background yield, nuisance parameter

non-informative

$$\pi(s,b) = \pi_s(s)\pi_b(b)$$

informative

Usually only interested in signal yield, so we marginalize nuisance parameter

$$p(s|N) \propto \int \mathcal{L}(N|s,b)\pi_s(s)\pi_b(b)db$$

Compute 95% credibility intervals or whatever we want

upper limit on signal yield

$$\int_0^{s_{\rm up}} p(s|N)ds = 0.95$$

Example: cut and count

SS Dilepton

arXiv:1104.3168
JHEP 1106:077 (2011)
7 TeV/8 TeV Updates
arXiv:1205.3933
arXiv:1212.6194

Search in SS dilepton + jets + MET final states in several signal regions

- 2 isolated leptons, first lepton $p_T > 20$ GeV, second lepton $p_T > 10$ GeV
- RelIso< 0.1 for $p_T > 20$ GeV and IsoSum< 2GeV for $p_T < 20$ GeV
- 2 reconstructed jets, $p_T > 30$ GeV
- $E_T^{\text{miss}} > 30$ GeV (ee and $\mu\mu$) or $E_T^{\text{miss}} > 20$ GeV ($e\mu$)

After selection, estimate backgrounds

- Rare SM processes (from MC)
  e.g. $\mathrm{q\bar{q}} \to \mathrm{WZ}$ and ZZ
  $\mathrm{qq} \to \mathrm{q'q'W^{\pm}W^{\pm}}$
  $2 \times (\mathrm{q\bar{q}} \to \mathrm{W^{\pm}})$, $\mathrm{t\bar{t}W}$, and WWW
- 1 or 2 fake leptons (data-driven)
  e.g. semi-leptonic $\mathrm{t\bar{t}}$

- charge mis-reconstruction (data-driven)

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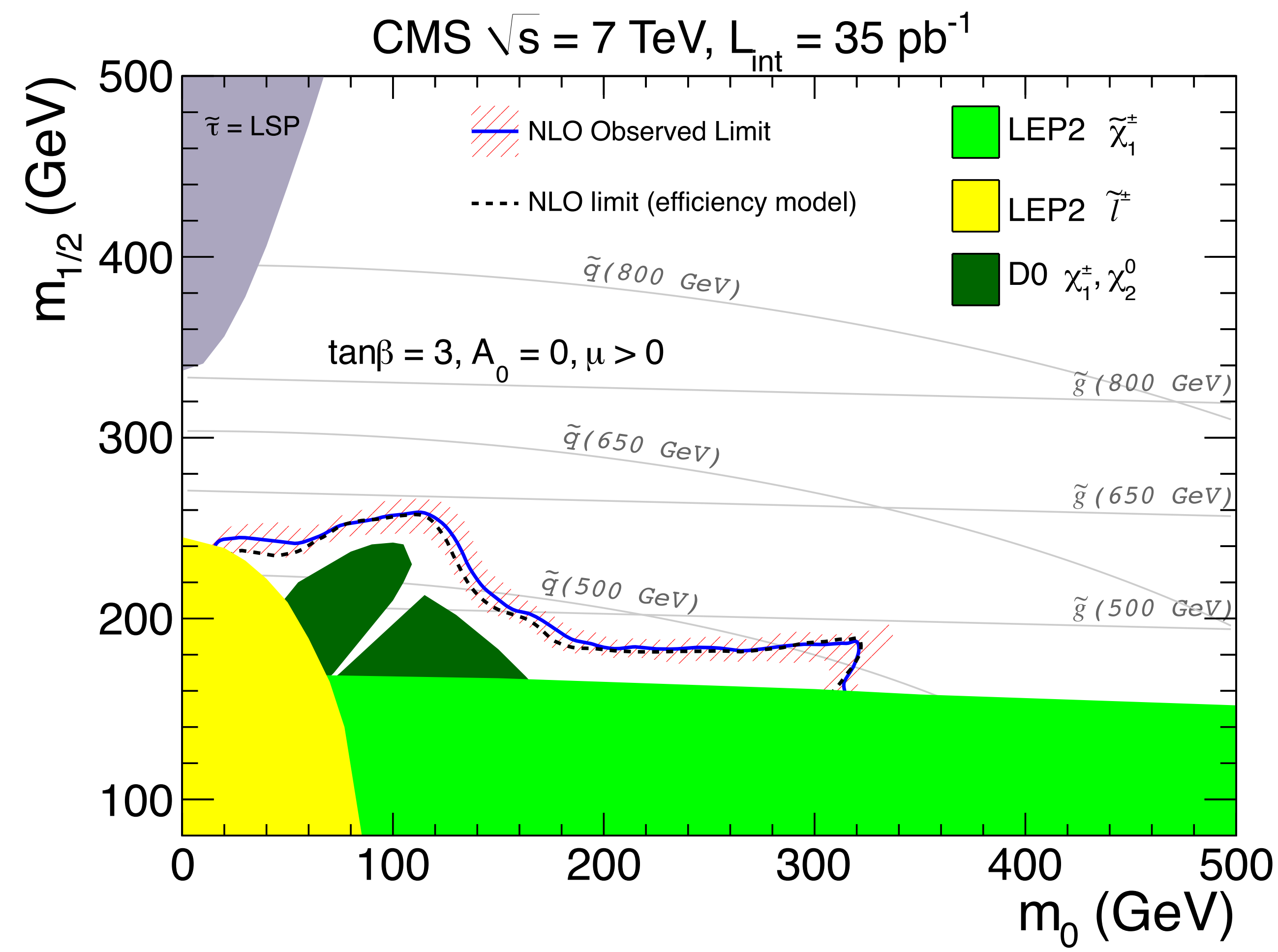
SS Dilepton Results

Search Region	ee	$\mu\mu$	$e\mu$	total	95% CL UL Yield
Lepton Trigger					
$E_T^{\rm miss} > 80$ GeV					
MC	0.05	0.07	0.23	0.35	
predicted BG	$\mathbf{0.23^{+0.35}_{-0.23}}$	$\mathbf{0.23^{+0.26}_{-0.23}}$	$\mathbf{0.74 \pm 0.55}$	$\mathbf{1.2 \pm 0.8}$	
observed	**0**	**0**	**0**	**0**	**3.1**
$H_T > 200$ GeV					
MC	0.04	0.10	0.17	0.32	
predicted BG	$\mathbf{0.71 \pm 0.58}$	$\mathbf{0.01^{+0.24}_{-0.01}}$	$\mathbf{0.25^{+0.27}_{-0.25}}$	$\mathbf{0.97 \pm 0.74}$	
observed	**0**	**0**	**1**	**1**	**4.3**
H_T Trigger					
Low-p_T					
MC	0.05	0.16	0.21	0.41	
predicted BG	$\mathbf{0.10 \pm 0.07}$	$\mathbf{0.30 \pm 0.13}$	$\mathbf{0.40 \pm 0.18}$	$\mathbf{0.80 \pm 0.31}$	
observed	**1**	**0**	**0**	**1**	**4.4**
	$e\tau_h$	$\mu\tau_h$	$\tau_h\tau_h$	total	95% CL UL Yield
τ_h enriched					
MC	0.36	0.47	0.08	0.91	
predicted BG	$\mathbf{0.10 \pm 0.10}$	$\mathbf{0.17 \pm 0.14}$	$\mathbf{0.02 \pm 0.01}$	$\mathbf{0.29 \pm 0.17}$	
observed	**0**	**0**	**0**	**0**	**3.4**

Set Bayesian 95% credibility limits



Data: No excess

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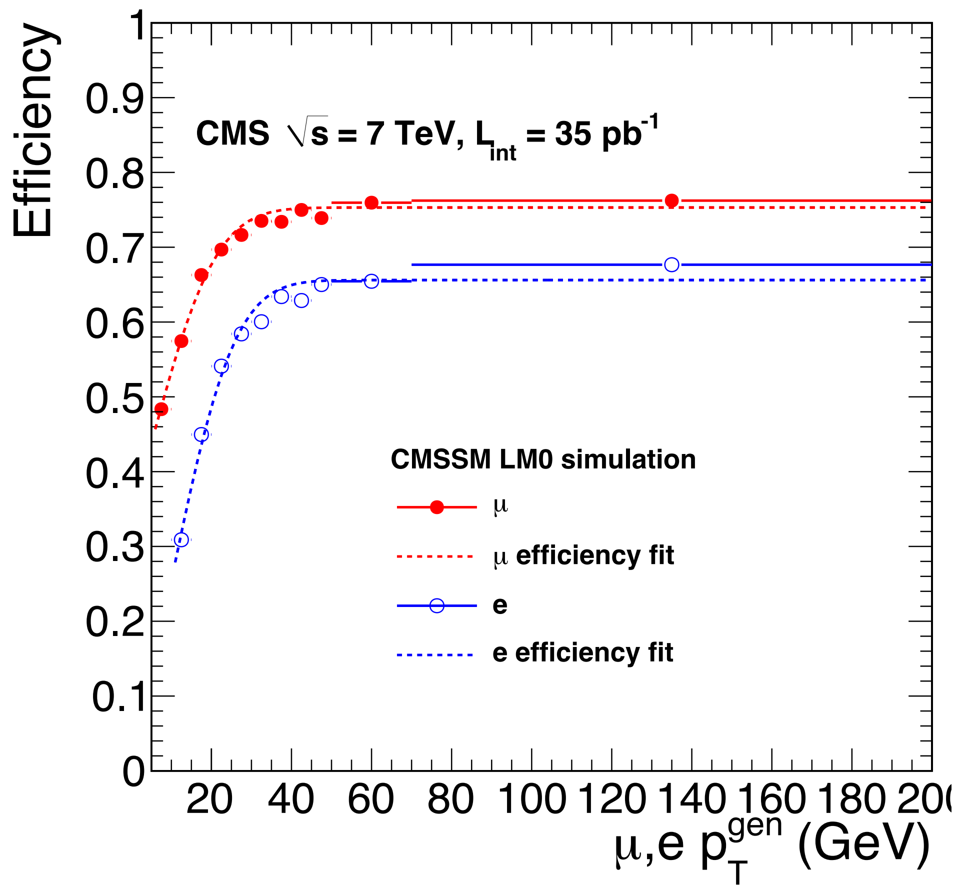
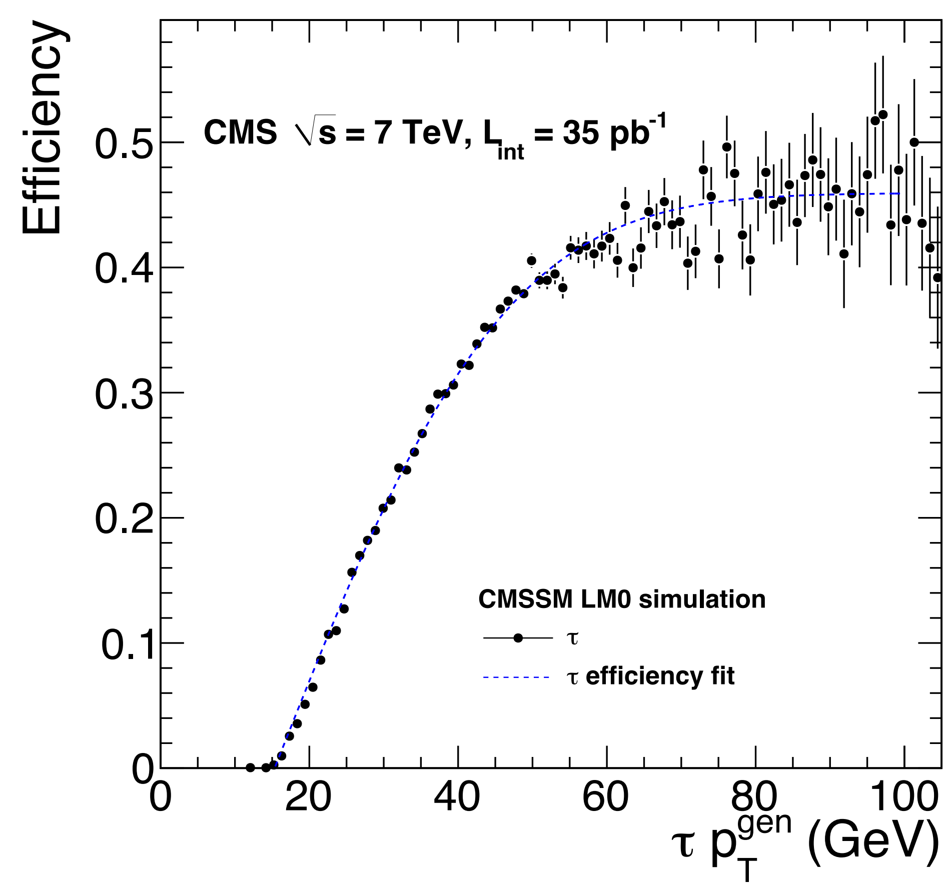
Additional Information for Gen-level Study





Parametrized curve describes analysis efficiency to canonical signal model **at gen-level**

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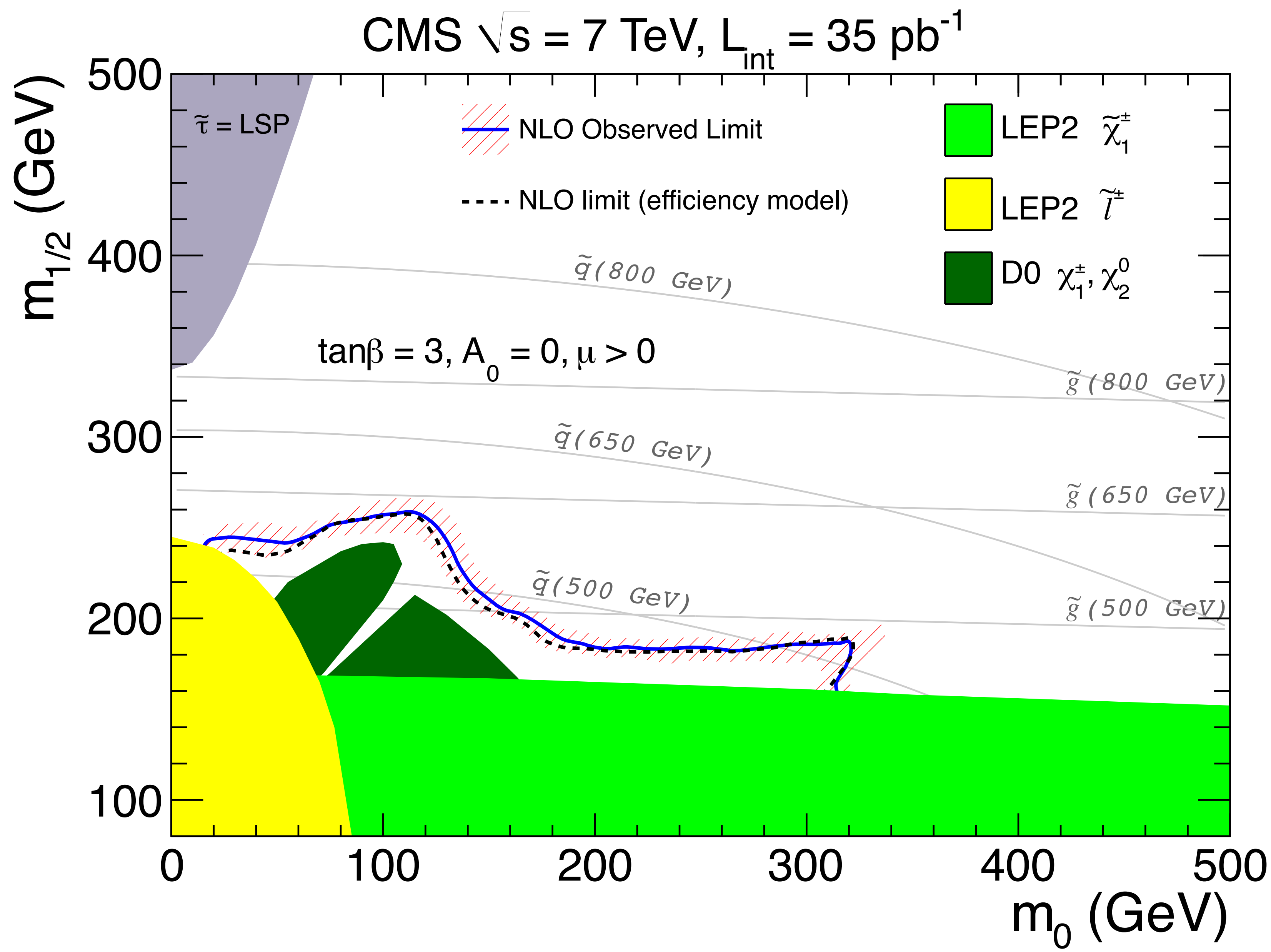
Validation - SS Dilepton

Using simple efficiency model of CMS detector, the 95% C.L. limits are reproduced



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Model Publication - SS Dilepton

arXiv:1104.3168
JHEP 1106:077 (2011)

clearly specified selection

- 2 isolated leptons, first lepton $p_T > 20$ GeV, second lepton $p_T > 10$ GeV
- RelIso< 0.1 for $p_T > 20$ GeV and IsoSum$<$ 2GeV for $p_T < 20$ GeV
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- $E_T^{\text{miss}} > 30$ GeV (ee and $\mu\mu$) or $E_T^{\text{miss}} > 20$ GeV ($e\mu$)

predicted background and observed yields

Search Region	ee	$\mu\mu$	$e\mu$	total	95% CL UL Yield
Lepton Trigger					
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MC	0.05	0.07	0.23	0.35	
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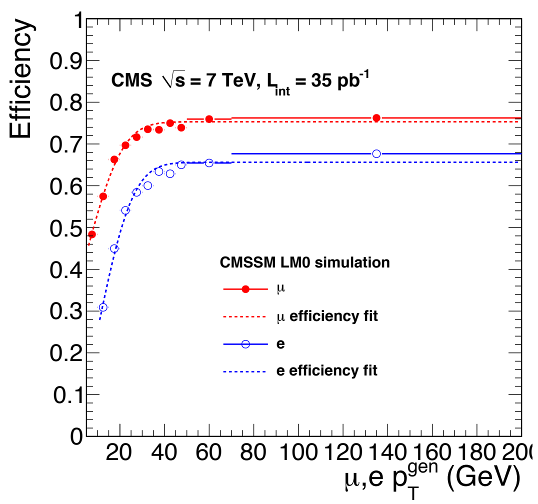
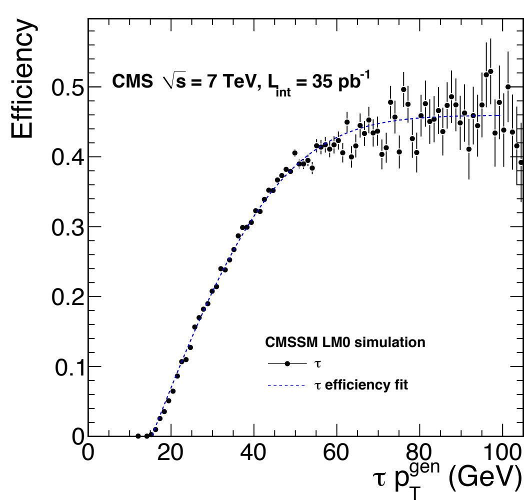
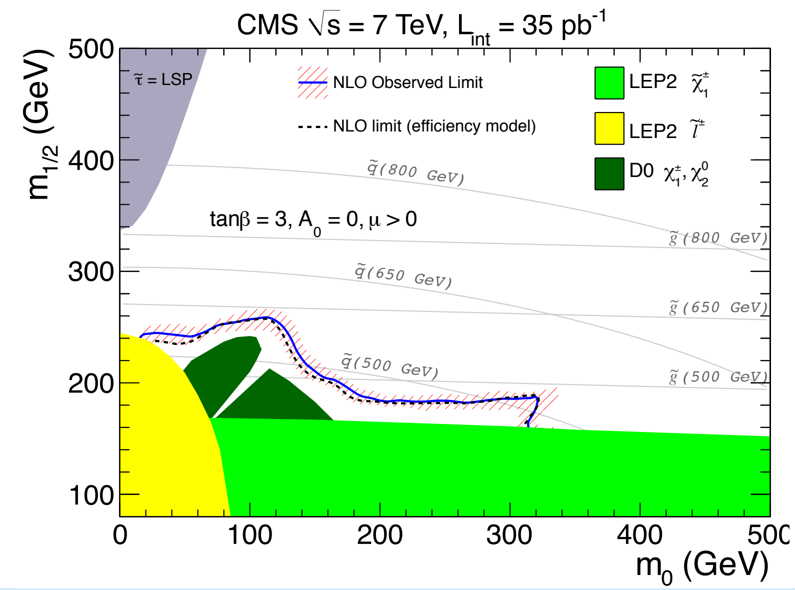
object efficiencies for a canonical signal model





reproducibility of limits



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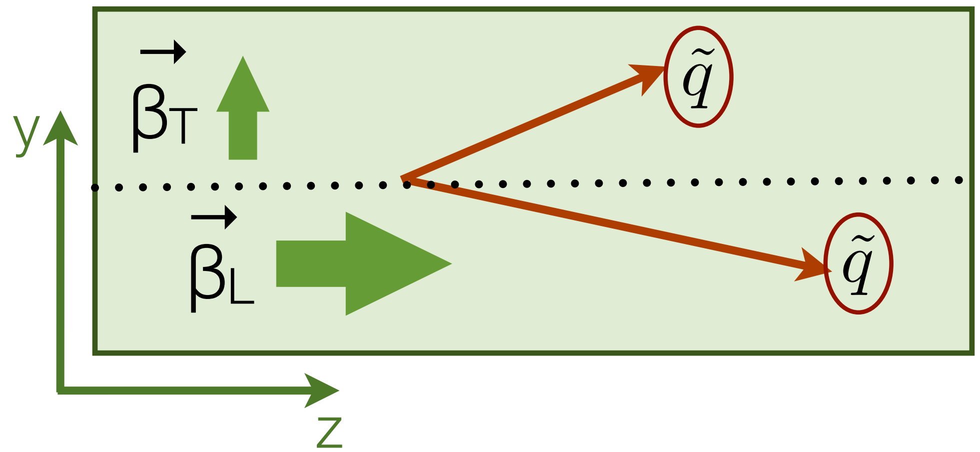
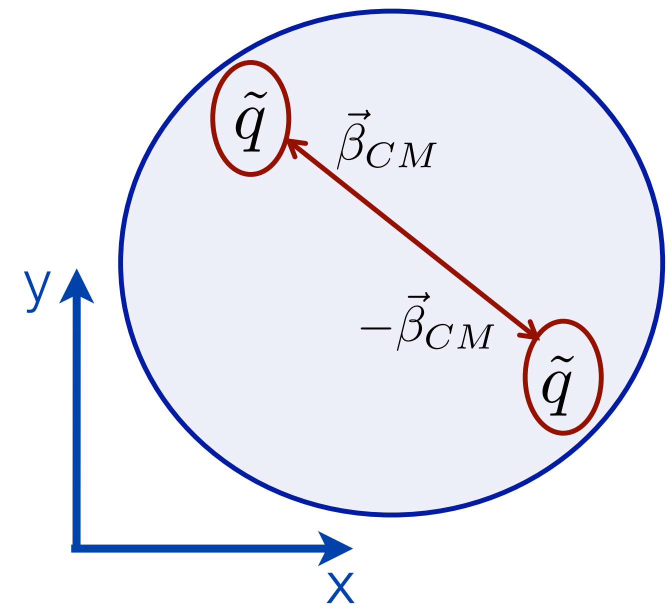
Reinterpreting a shape analysis in a new BSM model Example: 2011 Razor analysis

Razor Variables Motivation

lab frame



disquark rest frame



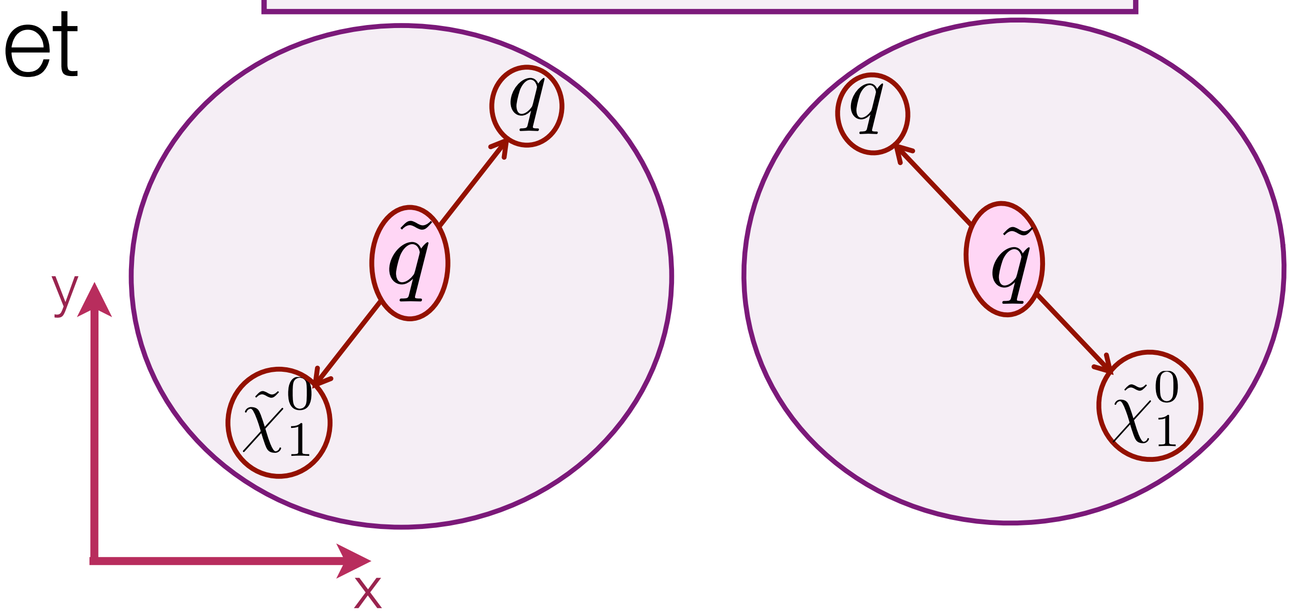
If we could see the LSPs, we could boost back by β_L, β_T, and β_{CM}

In this frame, we would then get

$$|p_{j1}| = |p_{j2}|$$

Too many missing degrees of freedom to do this

squark rest frame



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Razor Variables Motivation

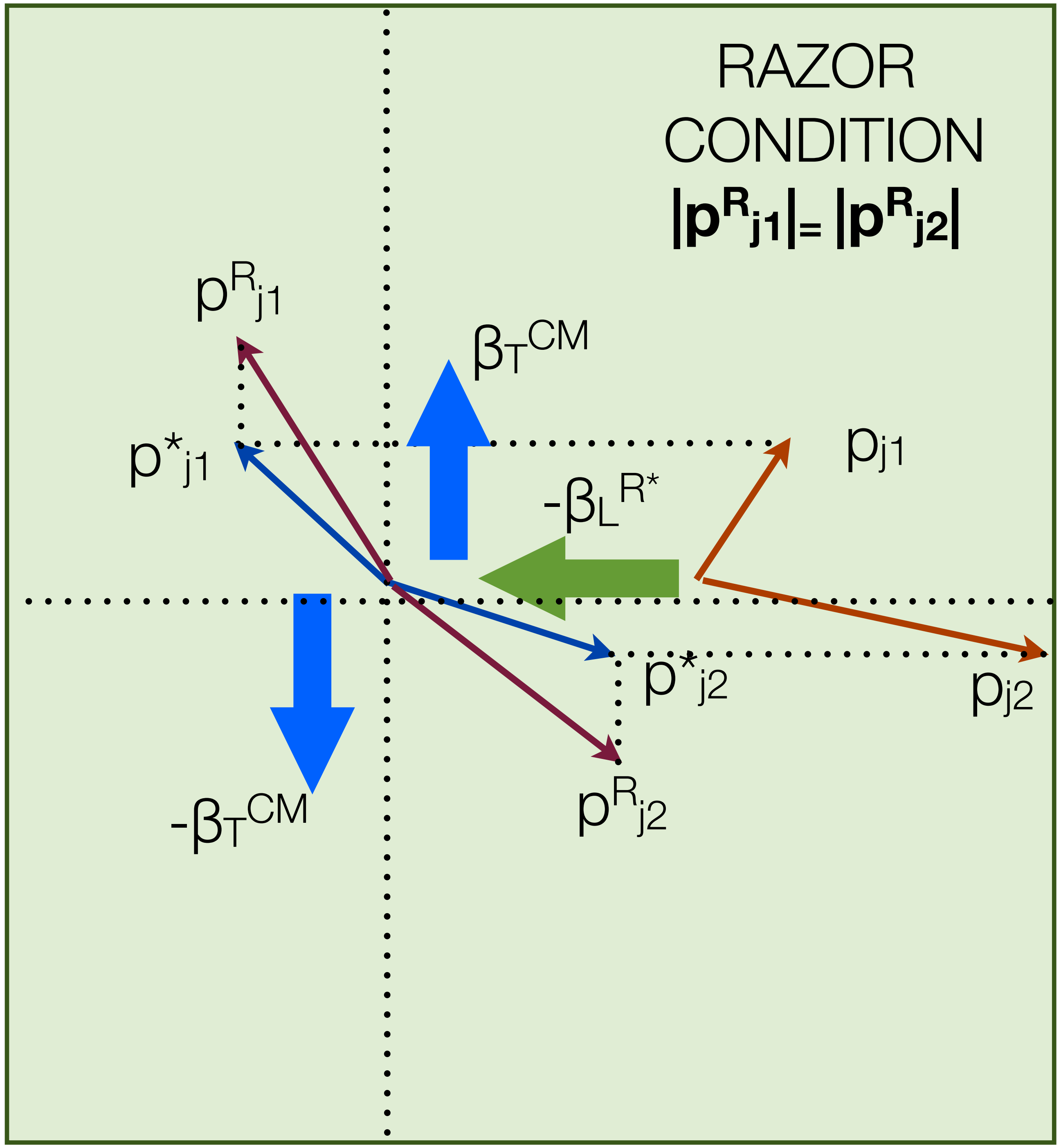
Approximate the squark rest frame by boosting to the frame where

$|p^R_{j1}| = |p^R_{j2}|$

Transformed momentum defines razor variable **M_R**

Estimates the momentum in the **true** squark rest frame

$$|\vec{p}_{j1}| = \frac{M^2_{\tilde{q}} - M^2_{\tilde{\chi}}}{2M_{\tilde{q}}}$$



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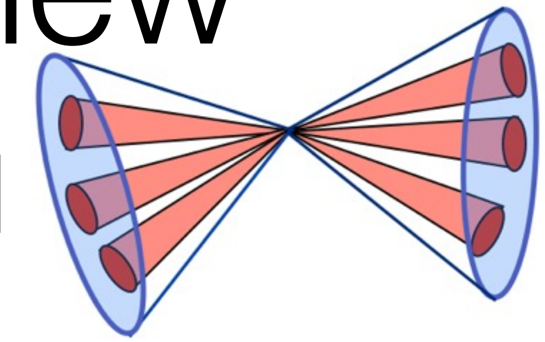
Razor Variables Overview

arXiv:1202.1503



Require 2 energetic jets

Cluster all jets into two megajets

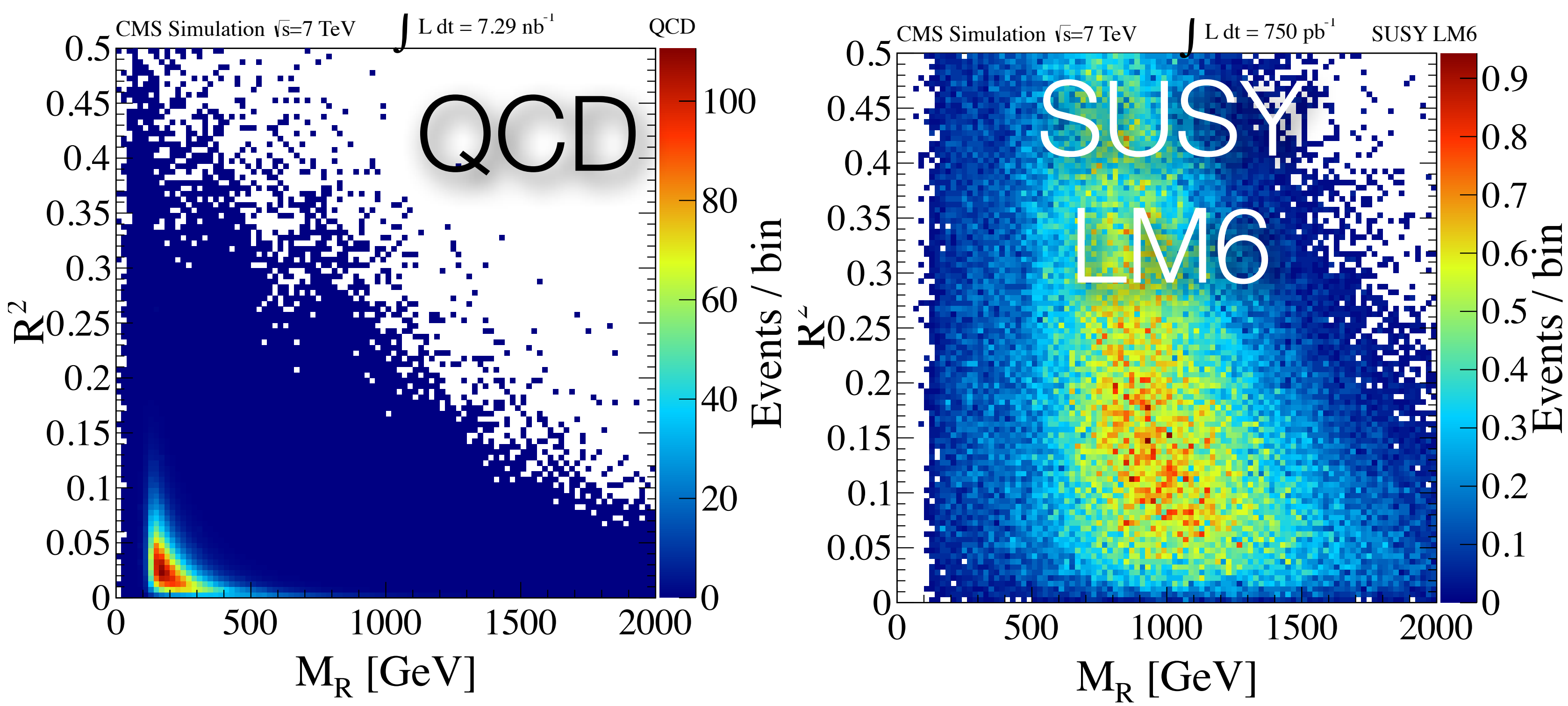
Compute R^2 and M_R

$$\boxed{M_R = \sqrt{(|\vec{p}^{j1}| + |\vec{p}^{j2}|)^2 - (p_z^{j1} + p_z^{j2})^2}}$$

$$M_T^R \equiv \sqrt{\frac{E_T^{\mathrm{miss}}(p_T^{j1} + p_T^{j2}) - \vec{E}_T^{\mathrm{miss}} \cdot (\vec{p}_T^{j1} + \vec{p}_T^{j2})}{2}}$$

$$\boxed{R \equiv \frac{M_T^R}{M_R}}$$

R^2 related to MET



M_R peaks at characteristic mass scale

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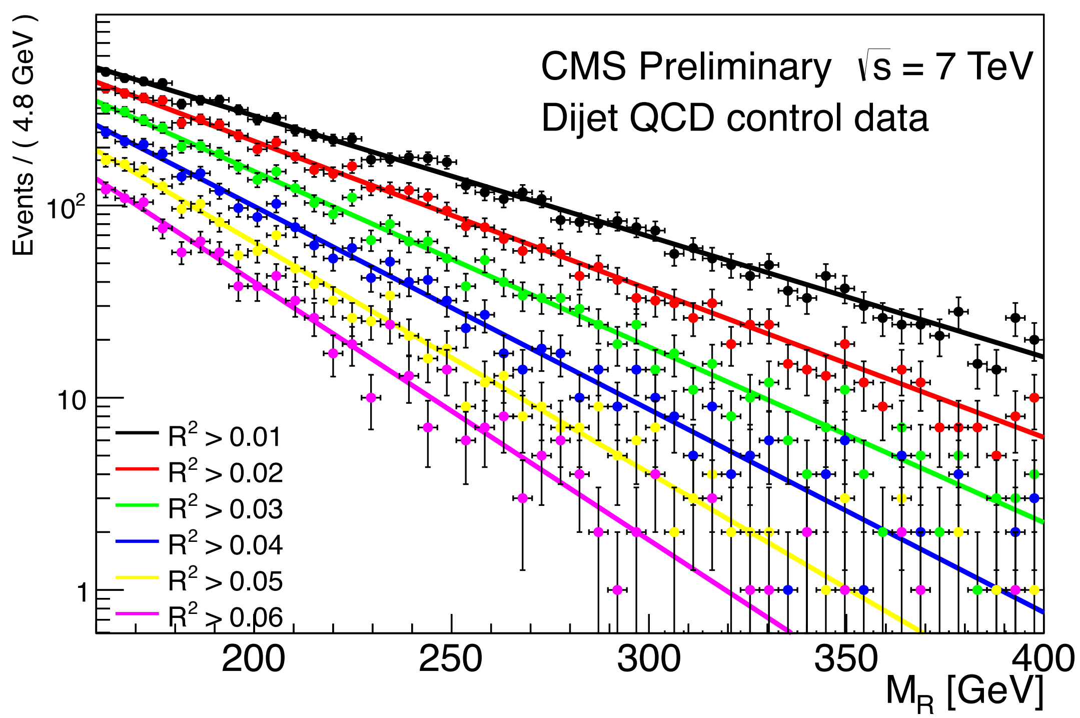
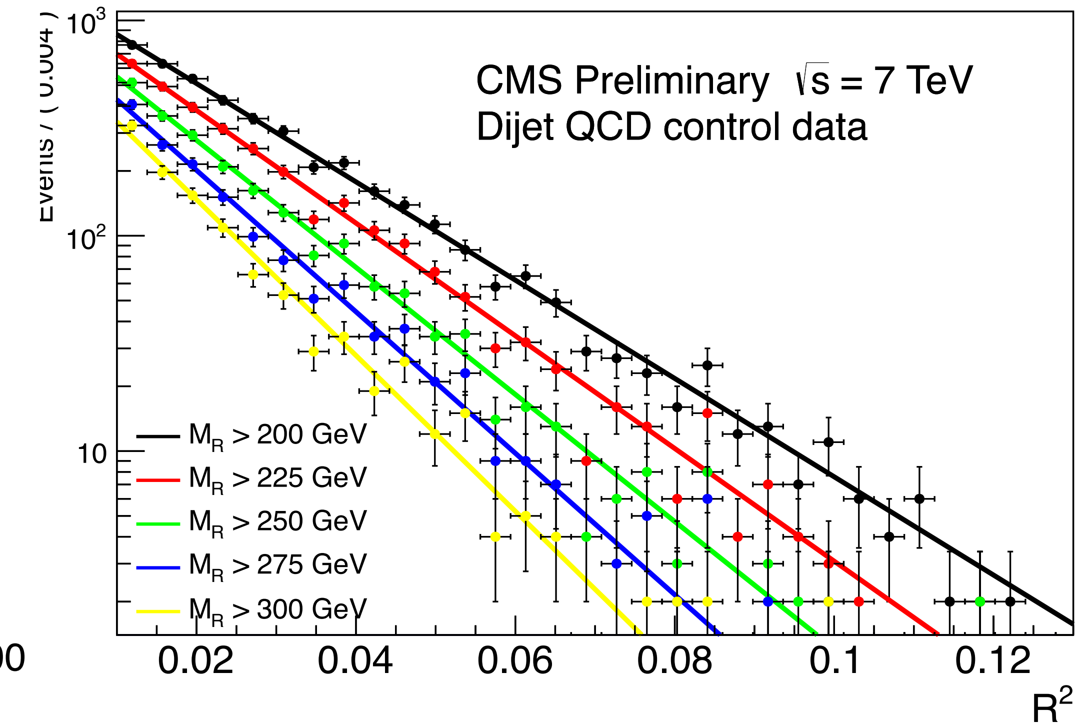
Modeling the Background in 2D





$f \propto \exp(-b\, M_R)$ $f \propto \exp(-c\, R^2)$

Properties of integration specify 2D form

$f \sim \exp(-k\, M_R\, R^2)$

$$f_j(M_R, R^2) = [k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j}) - 1]$$

$$\times \exp[-k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j})]$$

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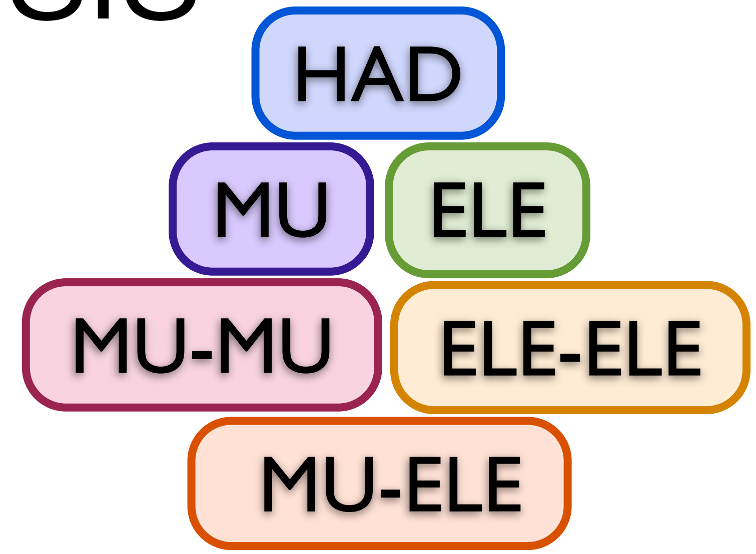
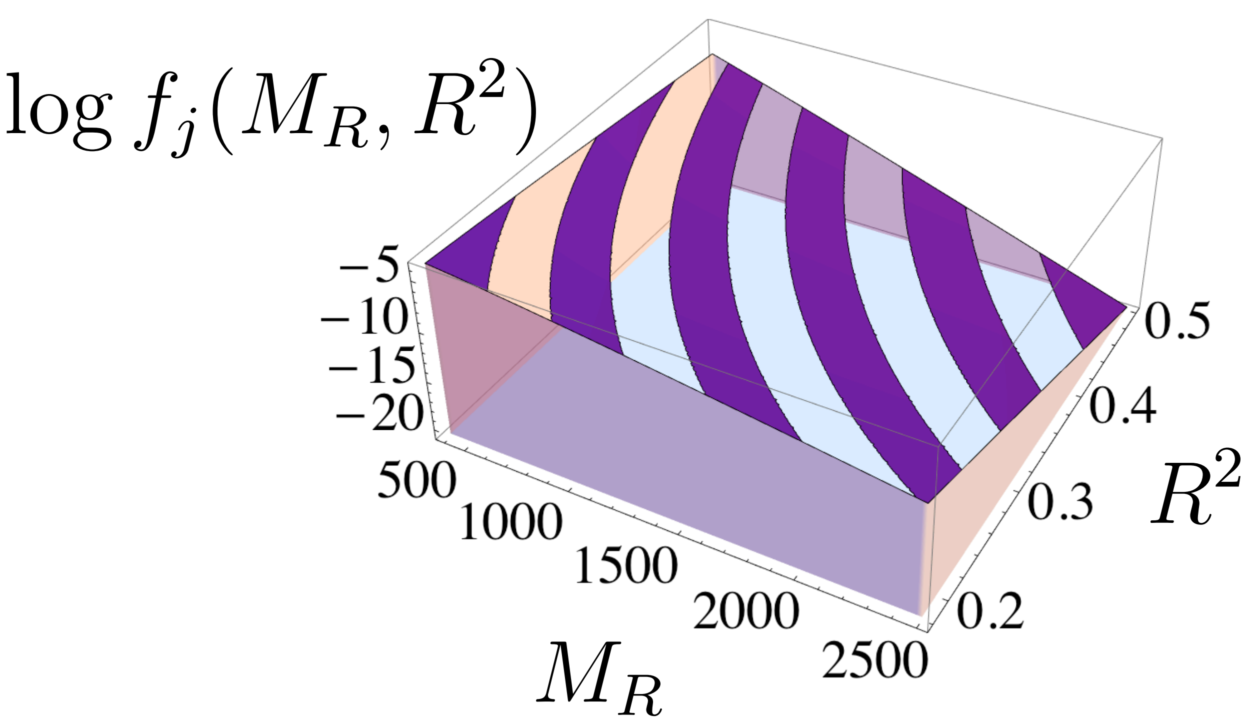
Razor Shape Analysis



Events are classified in 6 disjoint boxes based on lepton content

In each box, each SM background probability density is modeled by 1 or 2 instances of

$$f_j(M_R, R^2) = [k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j}) - 1]$$
$$\times \exp[-k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j})]$$



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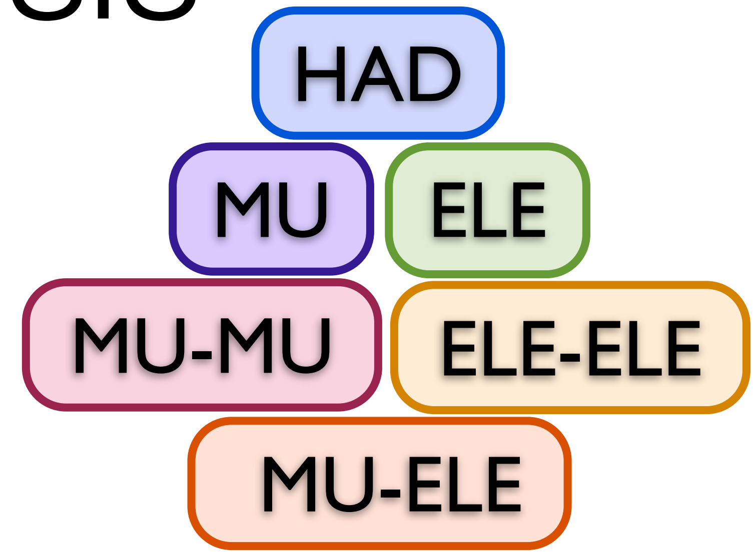
Razor Shape Analysis



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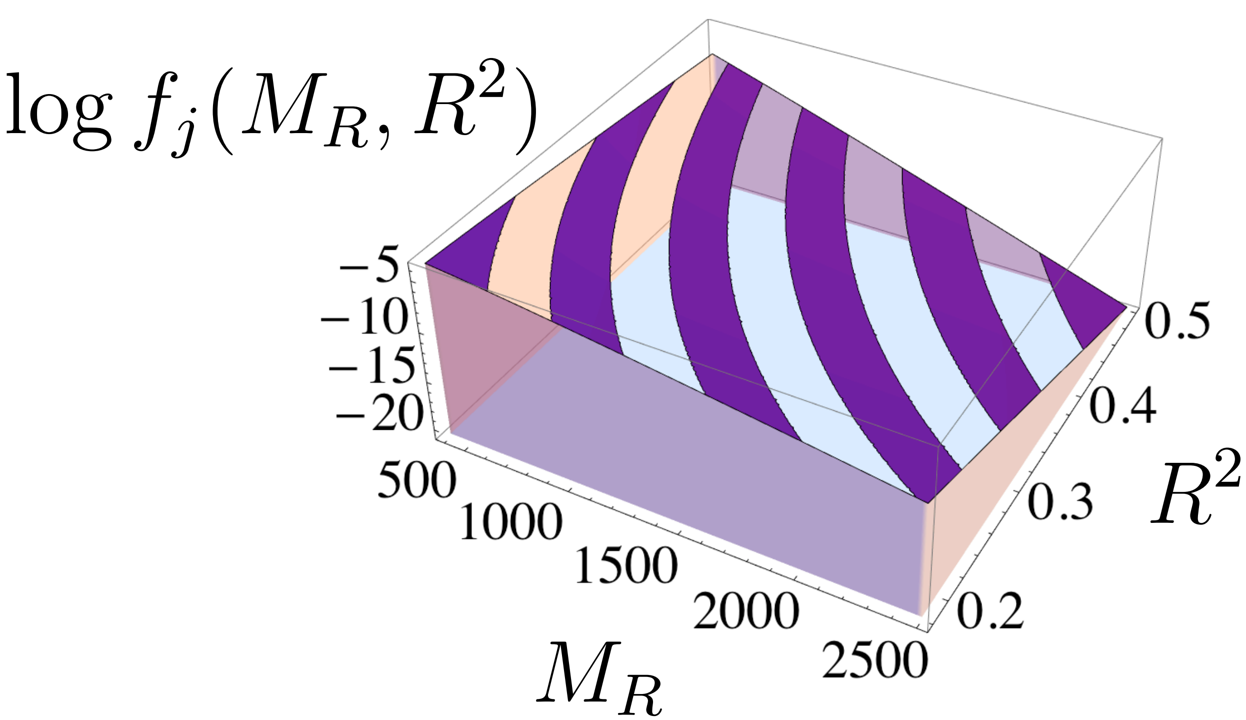
$$f_j(M_R, R^2) = [k_j(\boxed{M_R} - M^0_{R,j})(\boxed{R^2} - R^2_{0,j}) - 1] \times \exp[-k_j(\boxed{M_R} - M^0_{R,j})(\boxed{R^2} - R^2_{0,j})]$$



$\boxed{M_R}$ $\boxed{R^2}$ event-by-event observables

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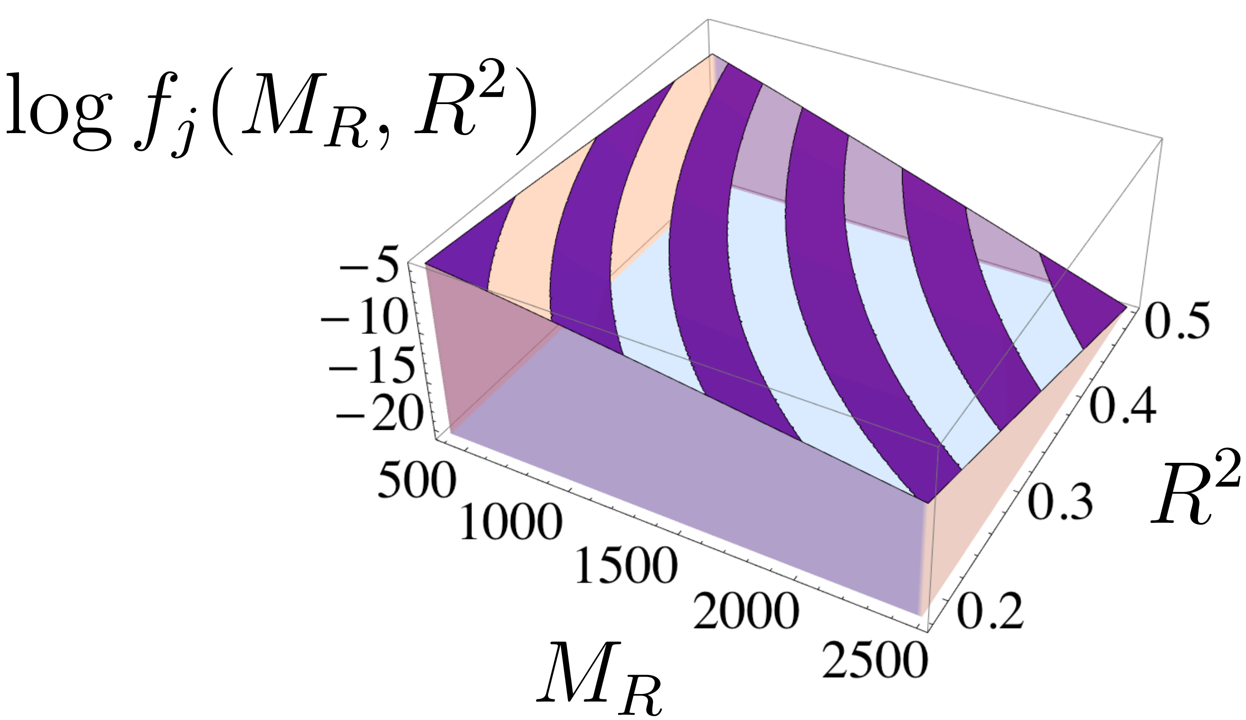
Razor Shape Analysis

HAD
MU ELE
MU-MU ELE-ELE
MU-ELE

Events are classified in 6 disjoint boxes based on lepton content

In each box, each SM background probability density is modeled by 1 or 2 instances of

$$f_j(M_R, R^2) = [k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j}) - 1] \times \exp[-k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j})]$$



j indexes the SM background, e.g. j = ttbar, W/Z+jets, etc.

$\boxed{k_j}$ $\boxed{M^0_{R,j}}$ $\boxed{R^2_{0,j}}$ fit parameters of bkgd model

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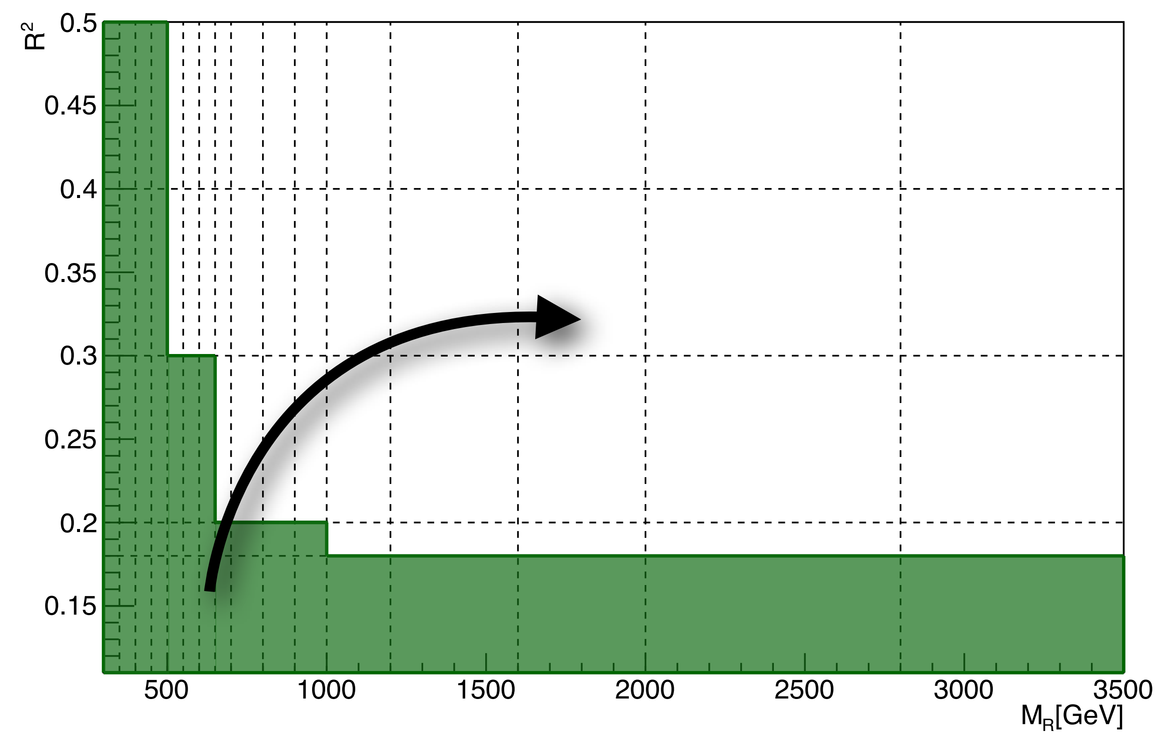
Razor Unbinned Likelihood

An unbinned, extended maximum likelihood fit is performed in a sideband fit region, and extrapolated

$$\mathcal{L}_b = \frac{\exp[-\sum_{j\in \mathrm{SM}} N_j]}{N!} \prod_{i=1}^{N} \left(\sum_{j\in \mathrm{SM}} N_j f_j(M_{R(i)}, R^2_{(i)}) \right)$$

i indexes an event in the dataset

N = total number of events
N_j = expected yield per SM bkgd
f_j = prob. density per SM bkgd.
$R^2_{(i)}$, $M_{R(i)}$ = per-event observables



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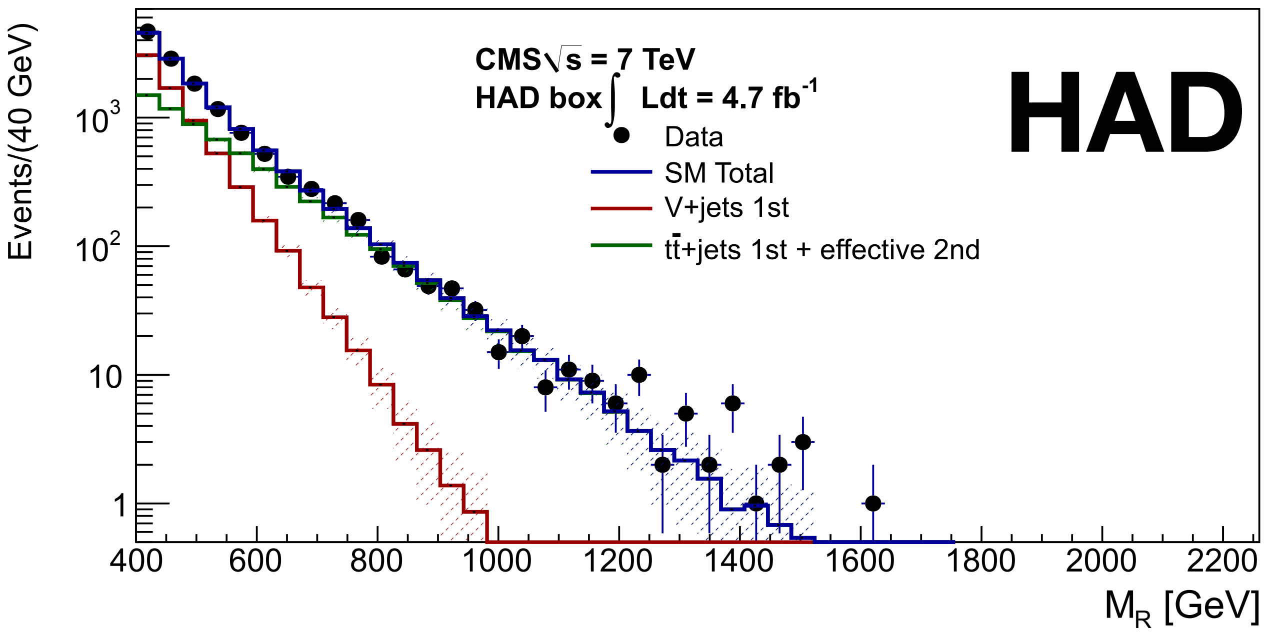
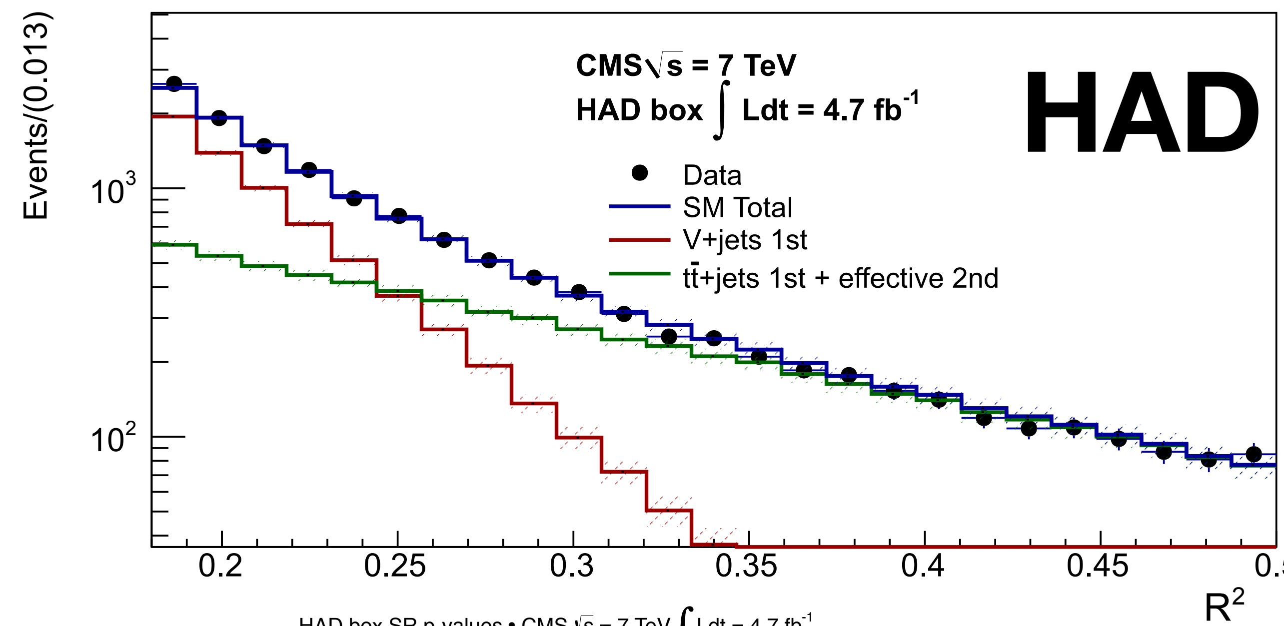
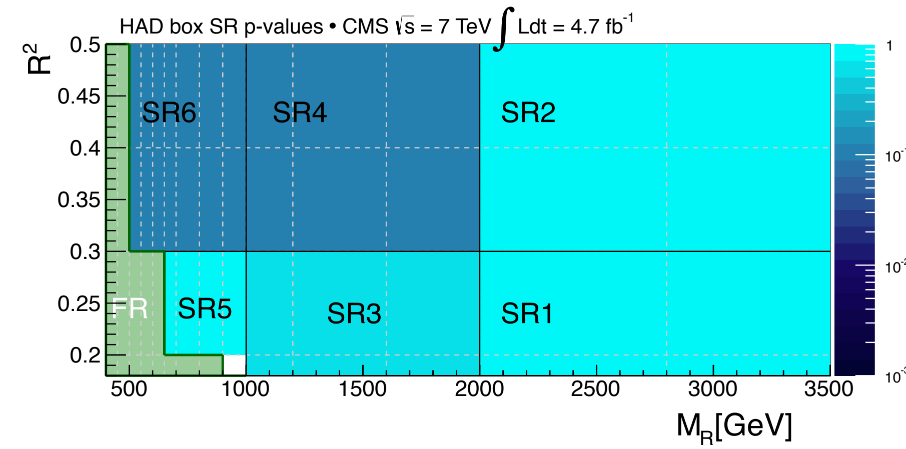
Razor Fit Results







Sum of W/Z+jets and tt+jets backgrounds described by sum of 3 PDFs of the form

$$f_j(M_R, R^2) = [k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j}) - 1] \times \exp[-k_j(M_R - M^0_{R,j})(R^2 - R^2_{0,j})]$$

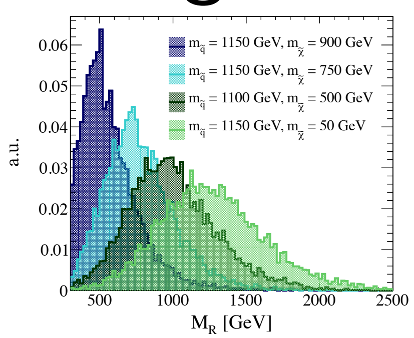
HAD	68% range	mode	median	observed	p-value
SR1	(0, 0.7)	0.5	0.5	0	0.99
SR2	(0, 0.7)	0.5	0.5	0	0.99
SR3	(45, 86)	73	69	74	0.68
SR4	(4, 15)	9.5	10.5	20	0.12
SR5	(530, 649)	566	593	581	0.82
SR6	(886, 1142)	987	1020	897	0.10

Signal + Background Likelihood

An unbinned, extended likelihood for the signal +background hypothesis is formed from the sum

$$\mathcal{L}_{s+b} = \frac{\exp[-N_s - \sum_{j\in \mathrm{SM}} N_j]}{N!} \prod_{i=1}^{N} \left(N_s f_s(M_{R(i)}, R^2_{(i)}) + \sum_{j \in \mathrm{SM}} N_j f_j(M_{R\,i}, R^2_i) \right)$$

signal + background



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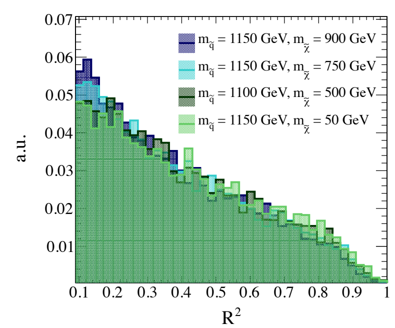

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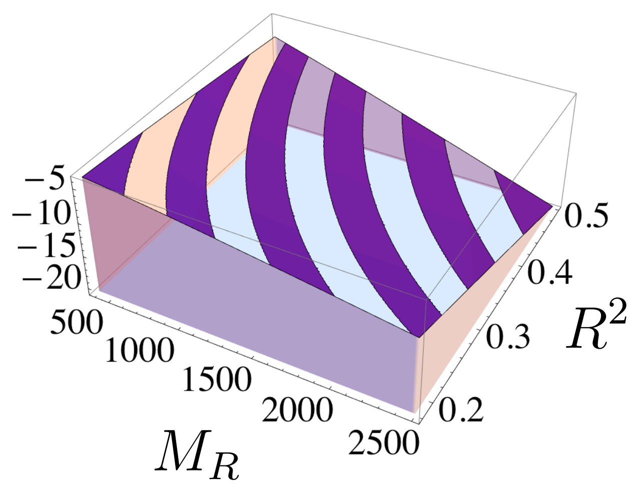

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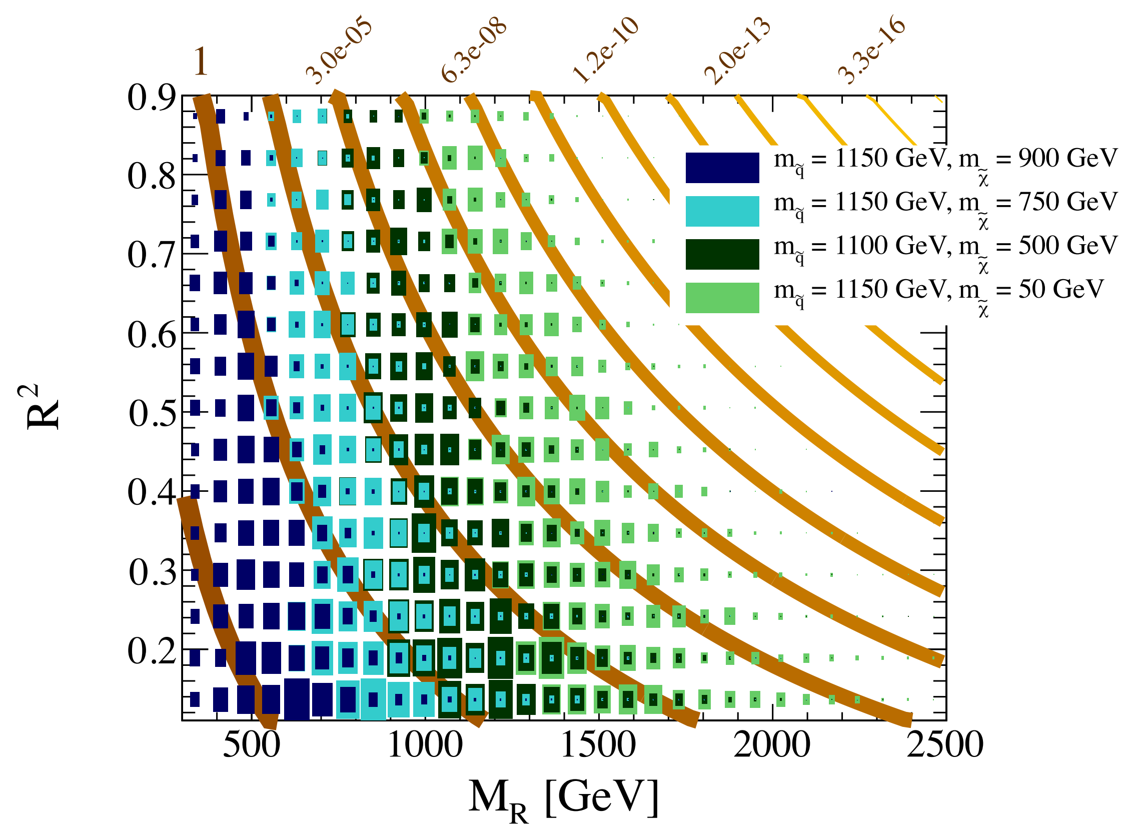

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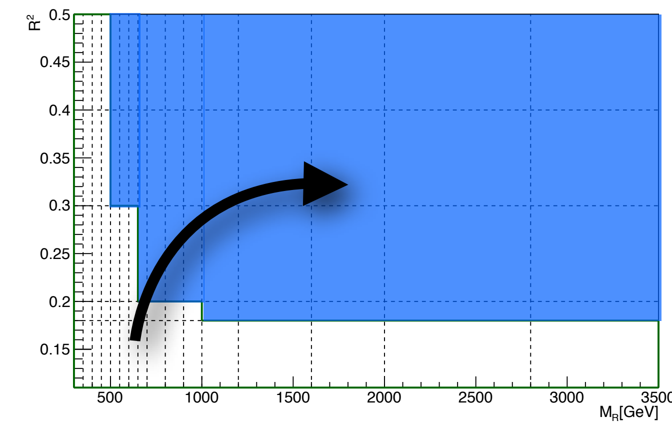
Limit Setting

We use the ratio of *marginal* likelihoods as the test statistic, evaluated on data in the signal region

$$\boxed{\log\left(\frac{\mathcal{L}^{(m)}_{s+b}}{\mathcal{L}^{(m)}_{b}}\right)}$$

$$\mathcal{L}_{s+b} = \frac{\exp[-N_s - \sum_{j\in\mathrm{SM}} N_j]}{N!}\prod_{i=1}^{N}\left(N_s f_s(M_{R(i)}, R^2_{(i)}) + \sum_{j\in\mathrm{SM}} N_j f_j(M_{R\,i}, R^2_i)\right)$$



$$\mathcal{L}^{(m)}_{s+b} = \int \mathcal{L}_{s+b}\, d\boldsymbol{\nu_b}\, d\boldsymbol{\nu_s}$$

Done *numerically*, by varying the distributions of the signal and background in pseudo-experiments according to the nuisance parameter priors*

$\boldsymbol{\nu_b}$ bkgd fit parameters k_j $M^0_{R,j}$ $R^2_{0,j}$ etc.

$\boldsymbol{\nu_s}$ jet energy scale, PDFs, etc.

*More on this procedure later

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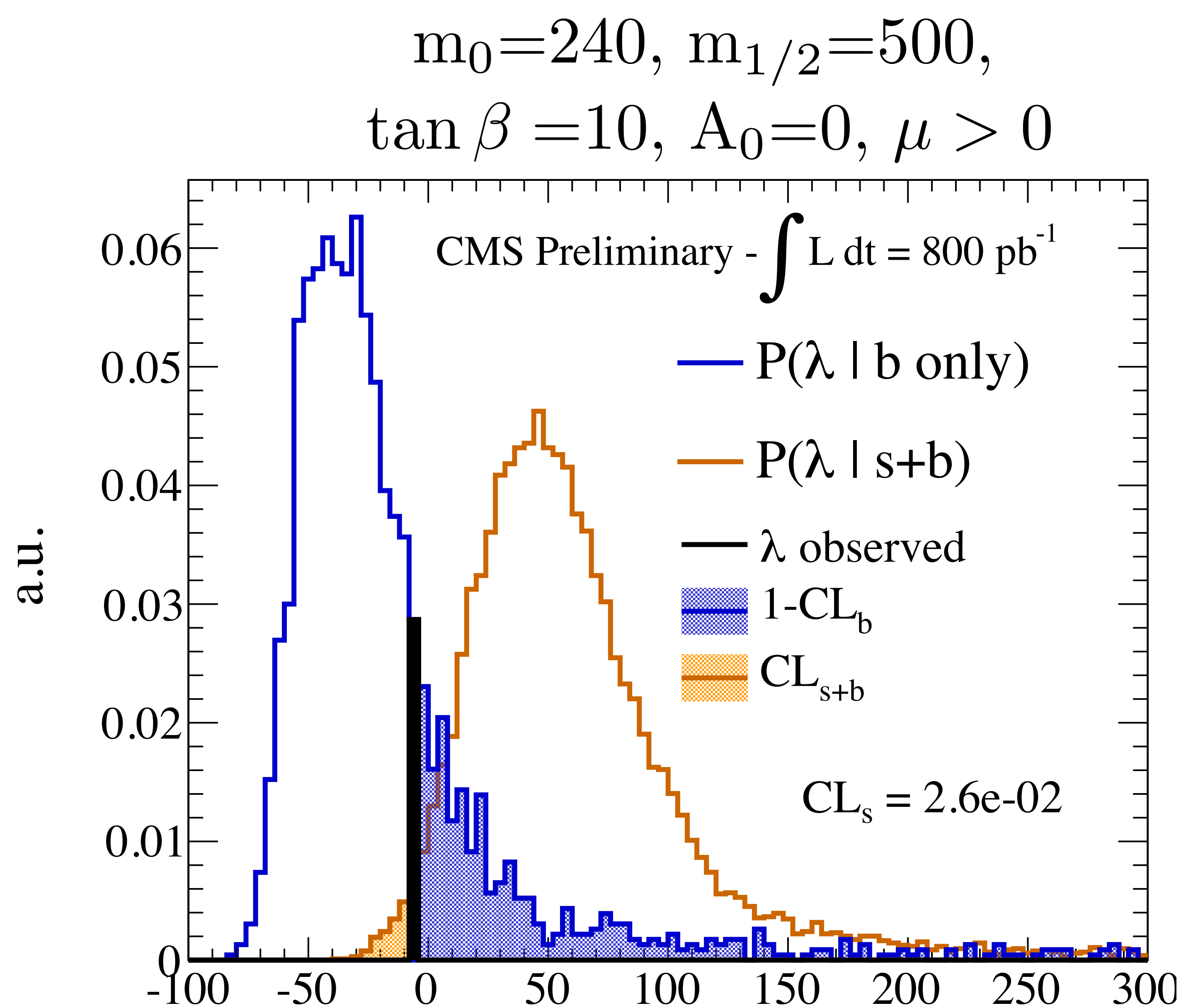
Example Hypothesis Test



$$\lambda = \log\left(\frac{\mathcal{L}^{(m)}_{s+b}}{\mathcal{L}^{(m)}_{b}}\right)$$

Reject signal model when

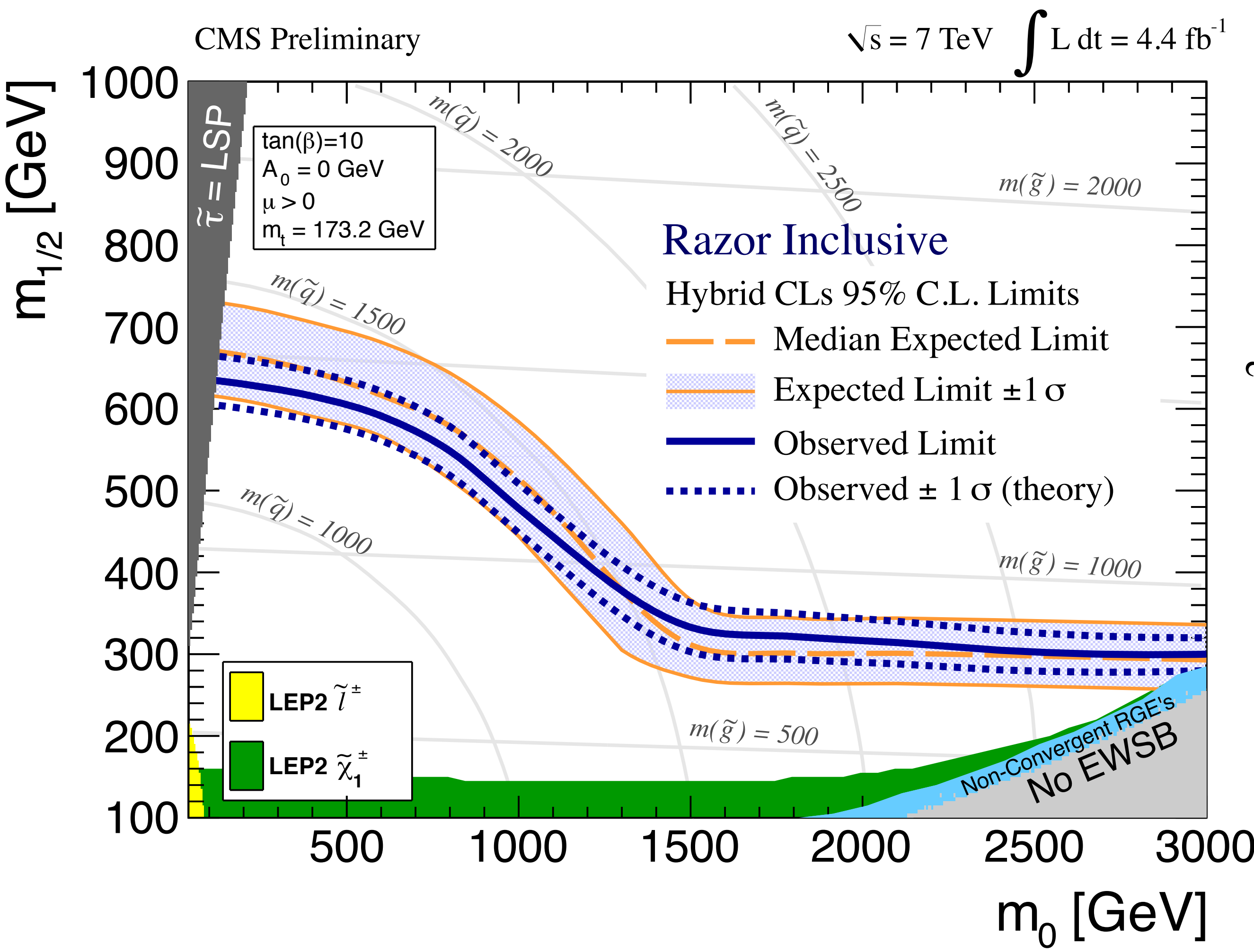
$$CL_s = \frac{CL_{s+b}}{CL_b} < 0.05$$

CL_{s+b}
measure of incompatibility of data with s+b hypothesis

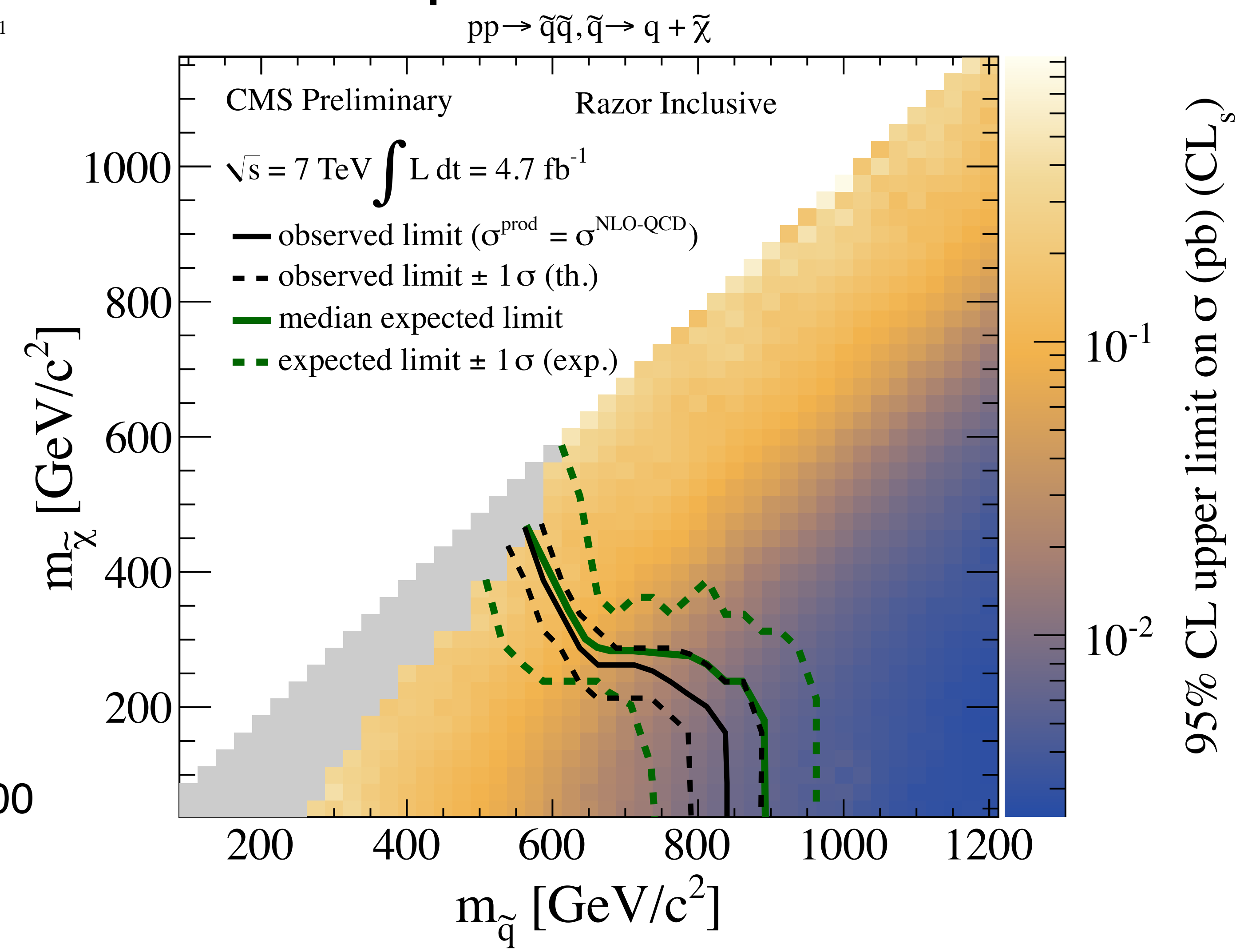
$1 - CL_b$
measure of incompatibility of data with b-only hypothesis

2011 Razor Limits

Constrained MSSM



Disquark Production Simplified Model



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Razor Binned Likelihood

Since CMS data is not public, unbinned likelihood is of limited use

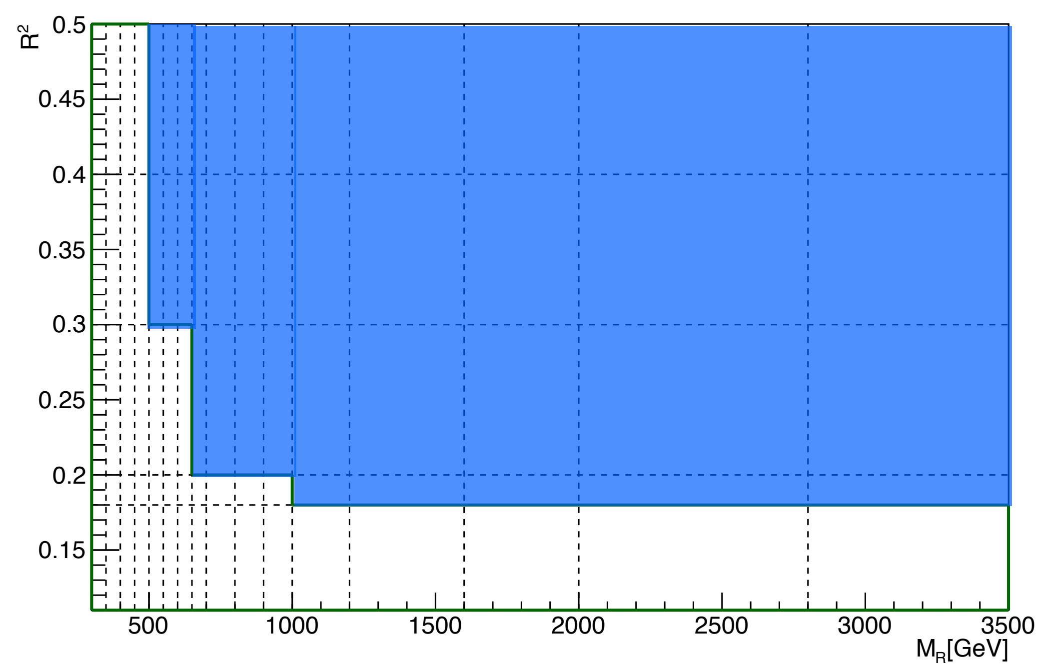
Instead, one can construct an binned likelihood as the product of many independent poisson likelihoods

$$\mathcal{L}^{(m)}_{s+b} = \prod_{\text{bin } i} \int \text{Poisson}(n_i|s_i, b_i)\ \pi(b_i|\bar{b}_i, \delta b_i) db_i$$

Mean expected background count is marginalized with your choice of prior

Gamma, Gaussian, LogNormal, etc.

Had Box	Observed	Predicted Mode	Predicted Median	$b \pm \delta b$
bHad_4_3	56	64.5	64.5	64.3 ± 1.4
bHad_4_4	27	23.5	23.5	22.7 ± 1.1
bHad_5_3	30	39.5	39.5	38.6 ± 1.3
bHad_5_4	18	12.5	12.5	12.2 ± 0.8
bHad_6_3	21	23.5	23.5	23.4 ± 1.0
bHad_6_4	4	7.5	7.5	6.6 ± 0.8
bHad_7_2	44	57.5	58.5	57.6 ± 1.5
bHad_7_3	11	14.5	14.5	14.1 ± 0.8



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Additional Information - Razor Analysis

arXiv:1202.1503 twiki.cern.ch/twiki/bin/viewauth/CMSPublic/RazorLikelihoodHowTo

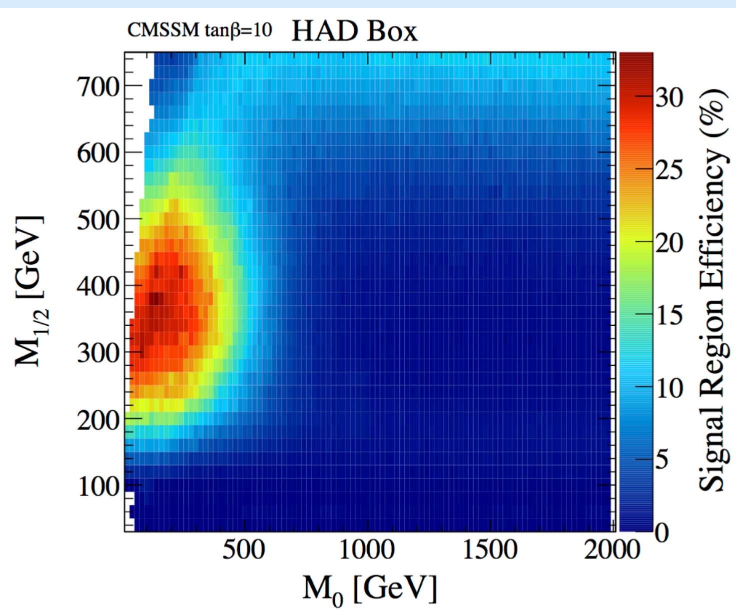
clearly specified selection

- 2, jets with $p_T > 60$ GeV, cluster all jets ($p_T > 40, |\eta| < 3.0$) into megajets
- $R^2 > 0.18$, $M_R > 400$ GeV (Had)
- Tight electron ($p_T > 20, |\eta| < 2.5$), Loose electron ($p_T > 10, |\eta| < 2.5$)
- Tight muon ($p_T > 15, |\eta| < 2.1$), Loose muon ($p_T > 10, |\eta| < 2.1$)
- MuEle: one Tight electron, one Tight muon
- MuMu: one Tight muon, one Loose muon
- EleEle: one Tight electron, one Loose electron
- Mu: one Tight muon
- Ele: one Tight electron
- Had: all other events

predicted background and observed yields

Had Box	Observed	Predicted Mode	Predicted Median	$b \pm \delta b$
bHad_4_3	56	64.5	64.5	64.3 ± 1.4
bHad_4_4	27	23.5	23.5	22.7 ± 1.1
bHad_5_3	30	39.5	39.5	38.6 ± 1.3
bHad_5_4	18	12.5	12.5	12.2 ± 0.8
bHad_6_3	21	23.5	23.5	23.4 ± 1.0
bHad_6_4	4	7.5	7.5	6.6 ± 0.8
bHad_7_2	44	57.5	58.5	57.6 ± 1.5
bHad_7_3	11	14.5	14.5	14.1 ± 0.8
bHad_7_4	1	3.5	3.5	3.3 ± 0.8
bHad_8_2	50	64.5	64.5	63.5 ± 1.5
bHad_8_3	18	14.5	14.5	13.9 ± 0.9
bHad_8_4	4	3.5	3.5	3.0 ± 0.7
bHad_9_2	18	29.5	29.5	28.7 ± 1.1
bHad_9_3	4	5.5	5.5	5.0 ± 0.7
bHad_9_4	2	1.5	1.5	0.7 ± 0.7
bHad_10_2	8	13.5	13.5	13.1 ± 0.9
bHad_10_3	2	2.5	2.5	1.7 ± 0.8
bHad_10_4	0	0.5	0.5	0.3 ± 0.3

sel. efficiencies for CMSSM



marginal binned likelihood function

$$\mathcal{L}^{(m)}_{s+b} = \prod_{\text{bin } i} \int \text{Poisson}(n_i|s_i, b_i)\ \pi(b_i|\bar{b}_i, \delta b_i) db_i$$

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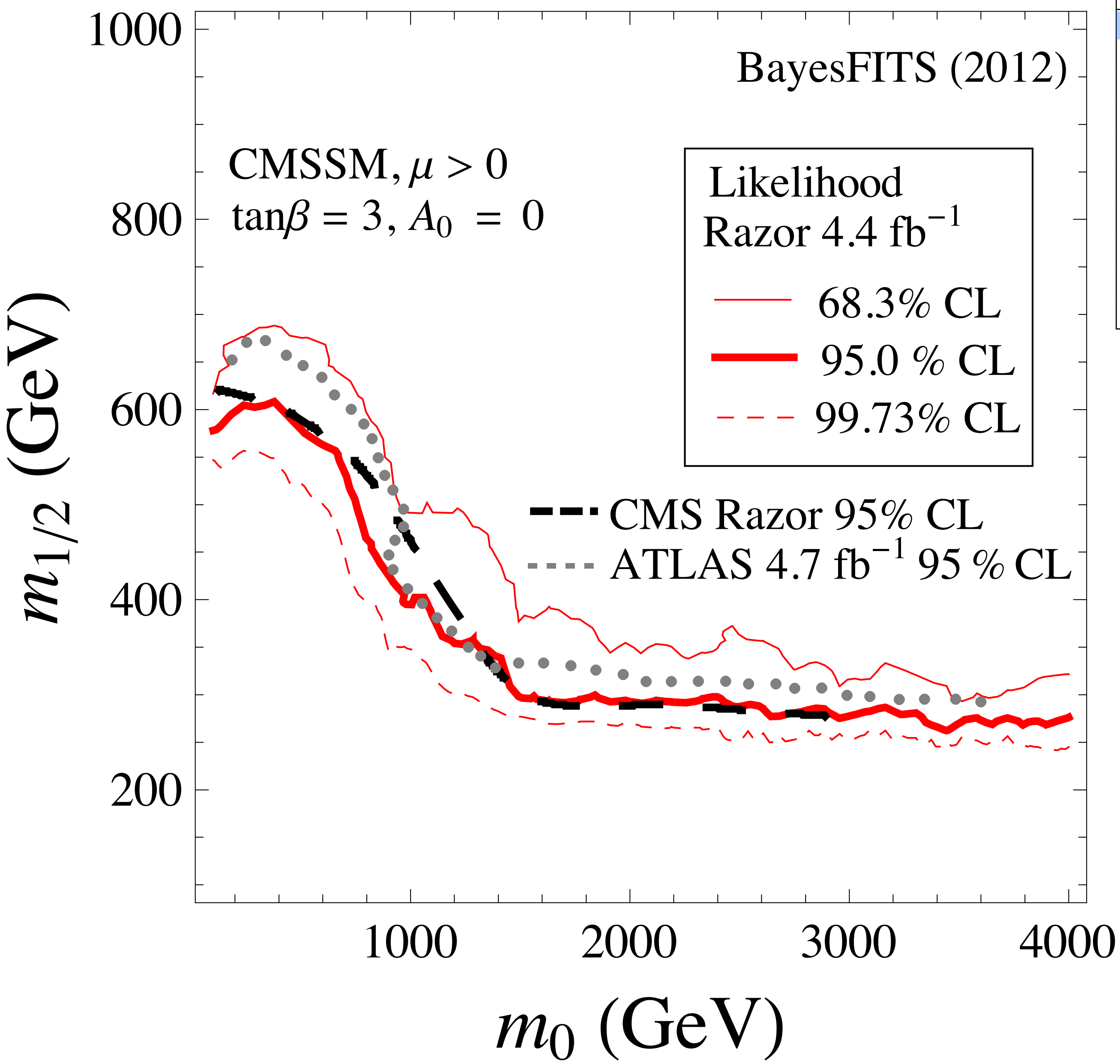
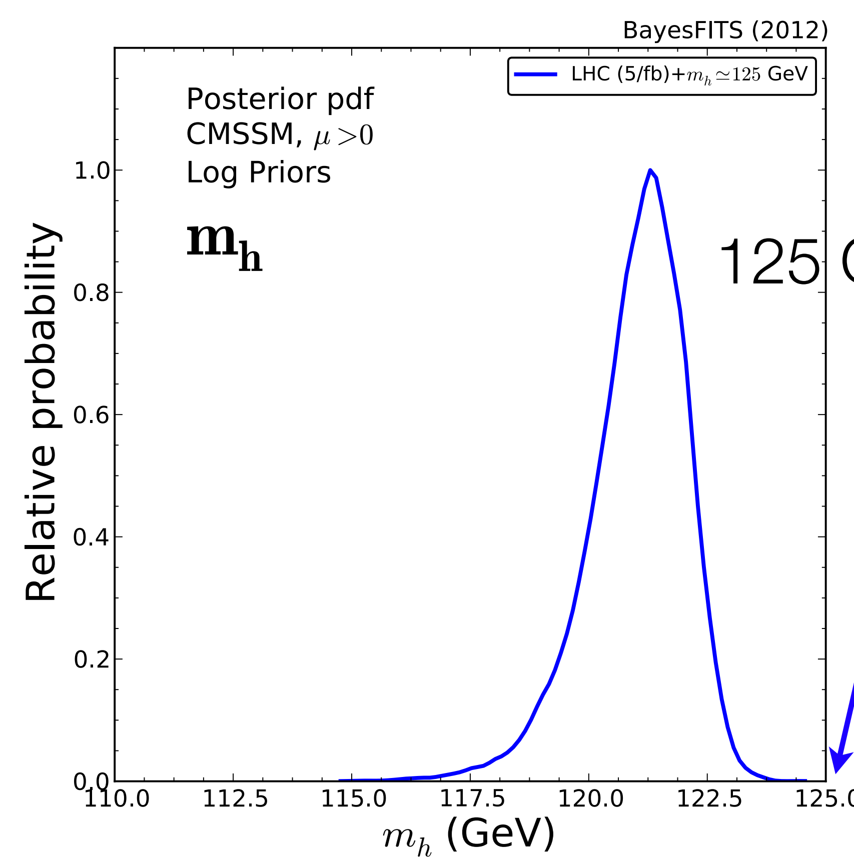
Validation and Combination of Razor Binned Likelihood

arXiv:1206.0264



Measurement	Mean or Range	Exp. Error	Th. Error	Likelihood Distribution	Ref.
CMS razor 4.4/fb analysis	See text	See text	0	Poisson	[2]
SM-like Higgs mass m_h	125	2	2	Gaussian	[8, 9, 44]
$\Omega_\chi h^2$	0.1120	0.0056	10%	Gaussian	[46]
$\sin^2\theta_{\rm eff}$	0.23116	0.00013	0.00015	Gaussian	[47]
m_W	80.399	0.023	0.015	Gaussian	[47]
$\delta(g-2)_\mu^{\rm SUSY}\times 10^{10}$	28.7	8.0	1.0	Gaussian	[47, 48]
BR $(\overline{\rm B}\to {\rm X_s}\gamma)\times 10^4$	3.60	0.23	0.21	Gaussian	[47]
BR $({\rm B_u}\to\tau\nu)\times 10^4$	1.66	0.66	0.38	Gaussian	[49]
ΔM_{B_s}	17.77	0.12	2.40	Gaussian	[47]
BR $({\rm B_s}\to\mu^+\mu^-)$	$< 4.5\times 10^{-9}$	0	14%	Upper limit – Error Fn	[23]



125 GeV very unlikely in CMSSM

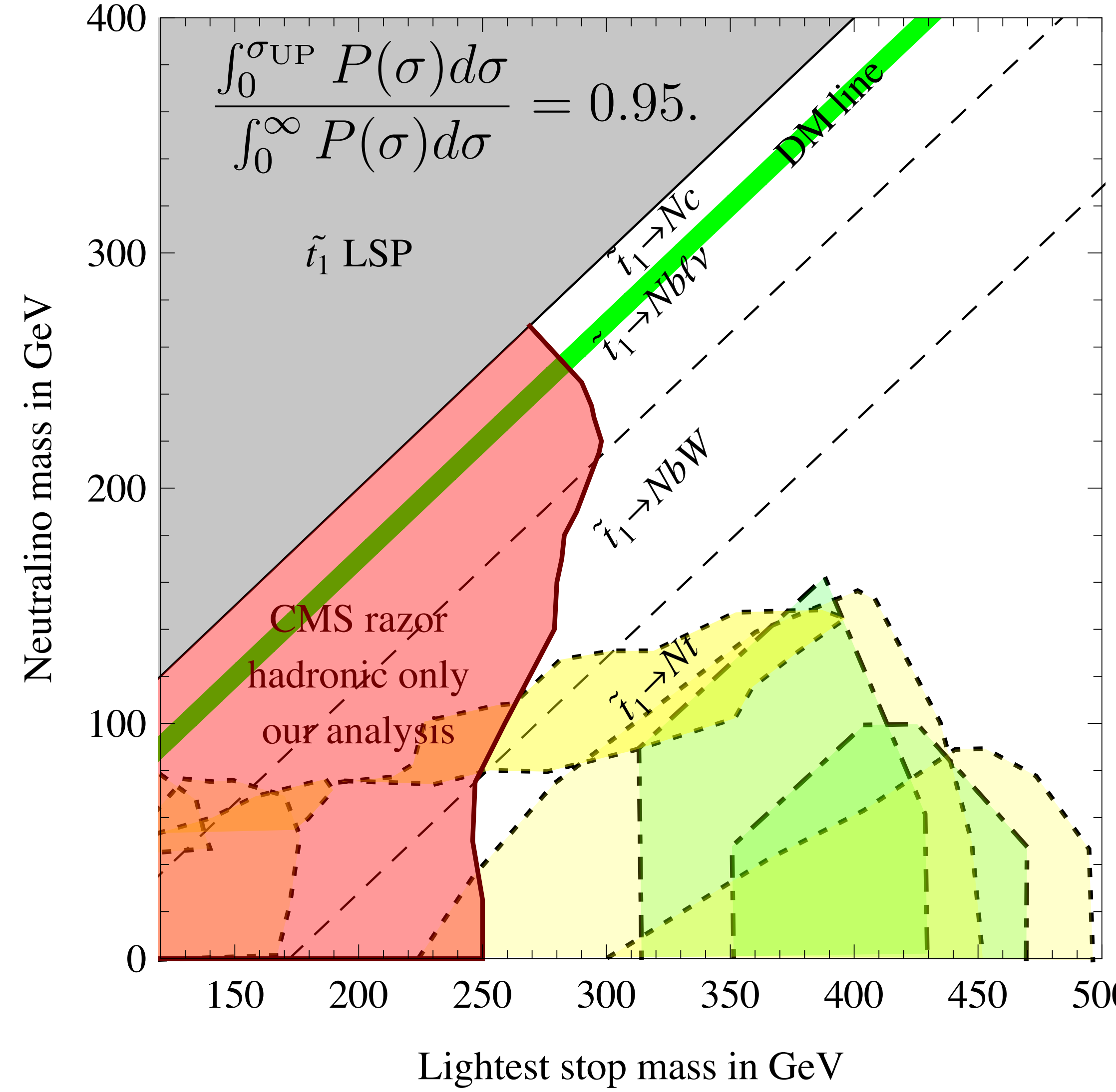
Reinterpretation for light stops

arXiv:1212.6847

Search for right-handed stop with

$$m_{\tilde{t}_1} = 200\text{–}400\,\mathrm{GeV}$$

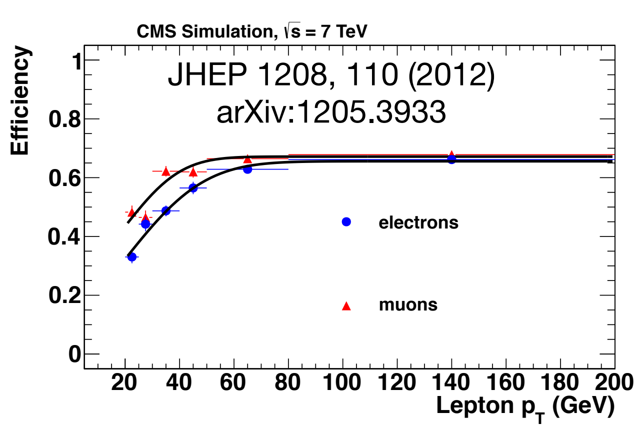
Motivated by LHC data and flavor constraints, RG equations, and thermal abundance for DM



1. Generated pair-produced stops with Pythia 8, clustered into jets with FastJet

$$\tilde{t} \to \ell \nu_\ell b N.$$

2. Applied lepton efficiencies from SS Dilepton



3. Derived posterior probability on cross section from likelihood

$$P(\sigma) = \int_0^\infty db \int_0^1 d\epsilon \frac{(b + L\sigma\epsilon)^n e^{-b-L\sigma\epsilon}}{n!} \mathrm{Ln}(\epsilon|\bar{\epsilon}, \delta_\epsilon)\,\mathrm{Ln}(b|\bar{b}, \delta_b)$$

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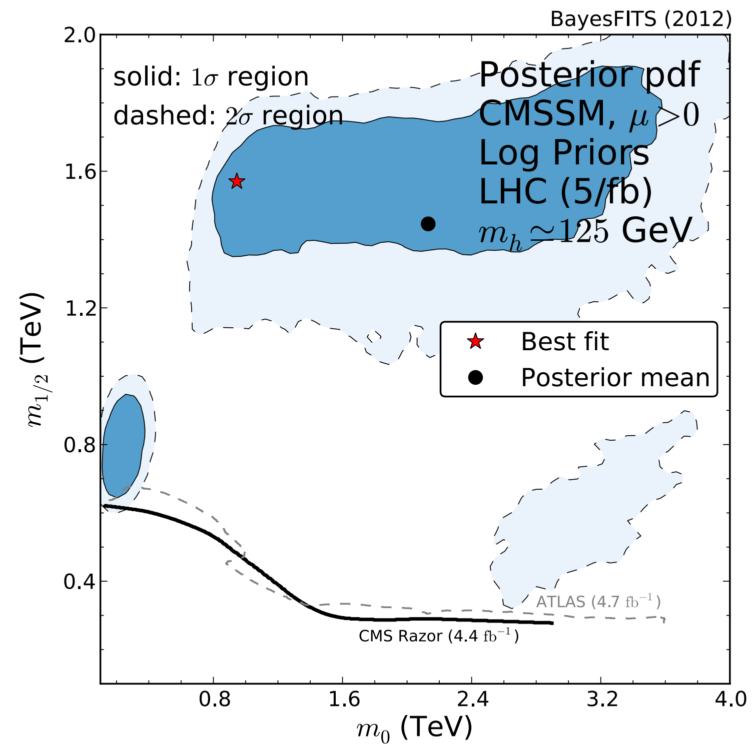
Recap of Uses

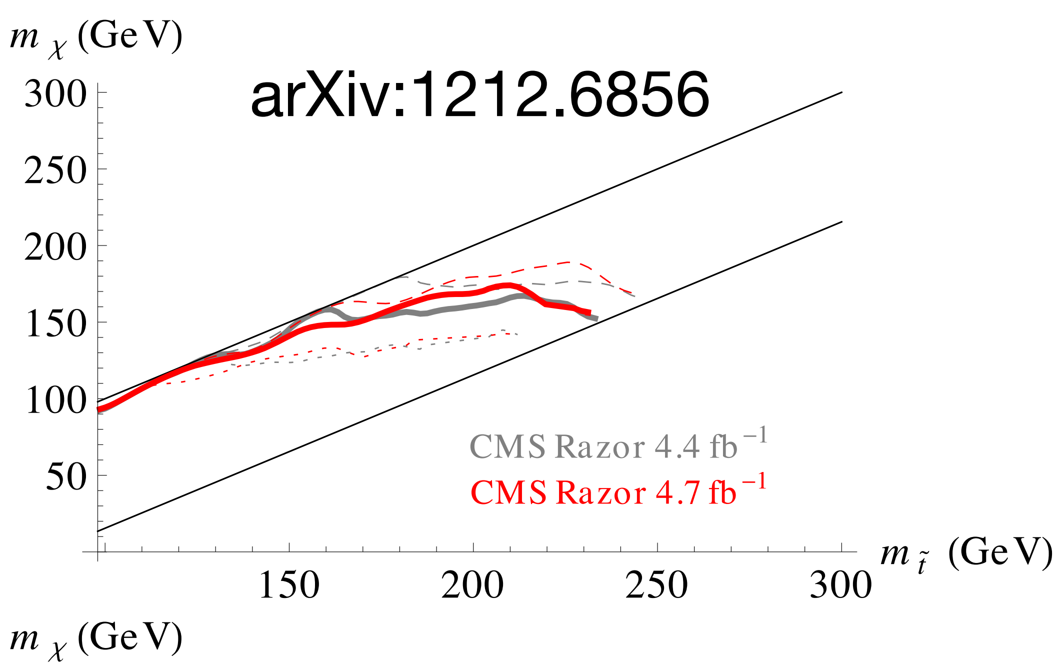
arXiv:1206.0264

BayesFITS combination of Razor with Higgs, $B_s \to \mu\mu$, etc. to derive posterior probabilities in CMSSM



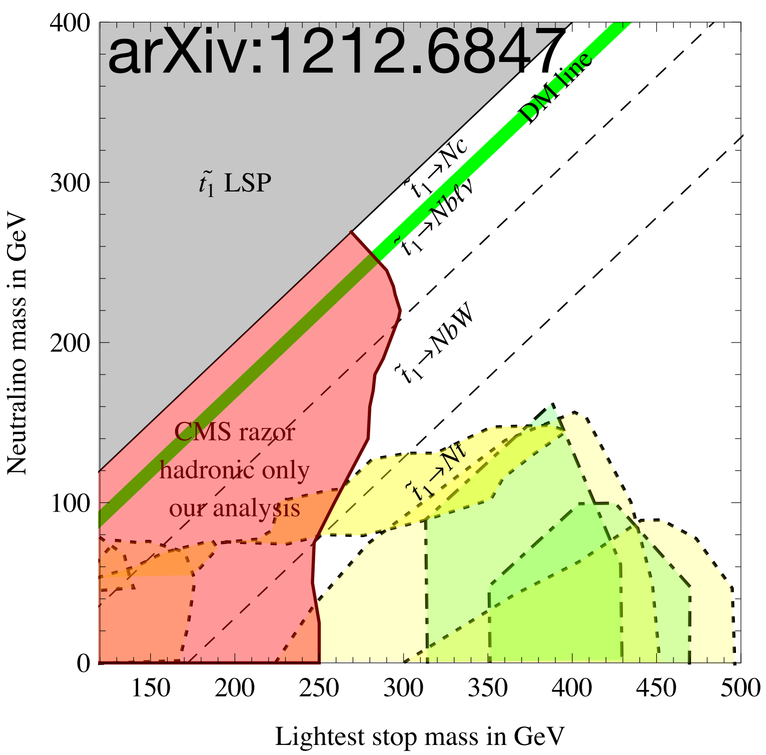
Reinterp. for very light stops*

$$m_{\tilde{t}} < m_{\tilde{\chi}} + m_t$$



*Used a coarser binned likelihood

Reinterp. for light stops, ~ degen. with neutralino



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New

Example: Search for contact interactions in jet p_T spectrum

Contact Interactions

New quark-gluon physics at a high mass scale Λ

⇒ effective 4-fermion interaction

$$L = \zeta \frac{2\pi}{\Lambda^2} (\bar{q}_L \gamma^\mu q_L)(\bar{q}_L \gamma_\mu q_L)$$

+1 destructive

-1 constructive

Reparametrize

$$\lambda = 1/\Lambda^2$$

$$a = a_{\rm SM} + \lambda\, a_{\rm CI}$$

cross section proportional to squared amplitude

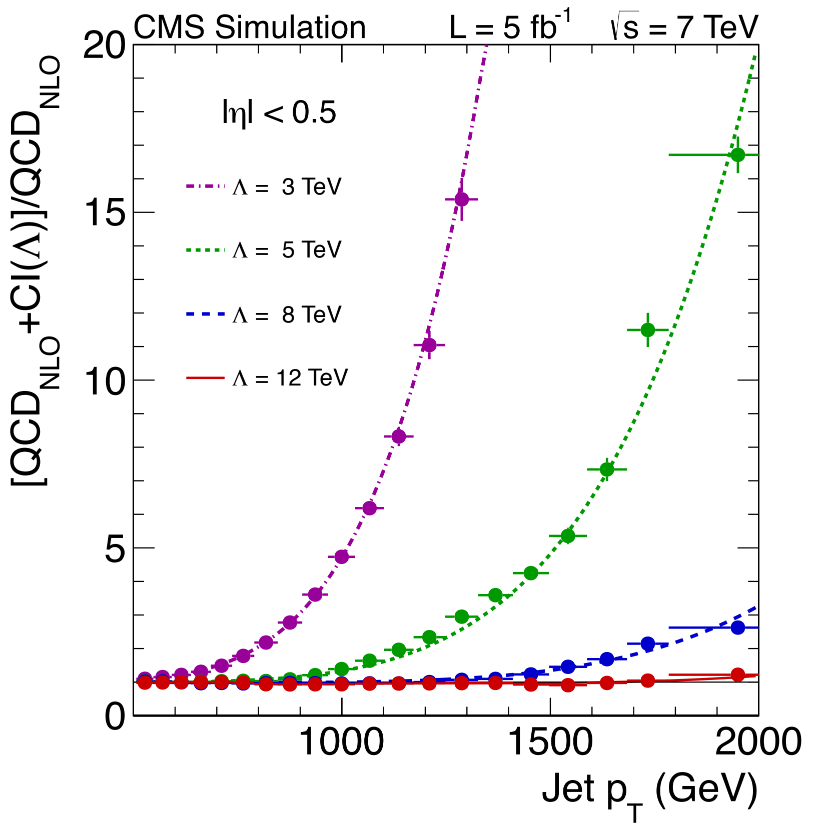
$$\Rightarrow \sigma_k = \underset{\mathrm{QCD_{NLO}}}{c_k} + \underbrace{b_k \lambda + a_k \lambda^2}_{\mathrm{CI}(\Lambda)}$$

Statistical Model

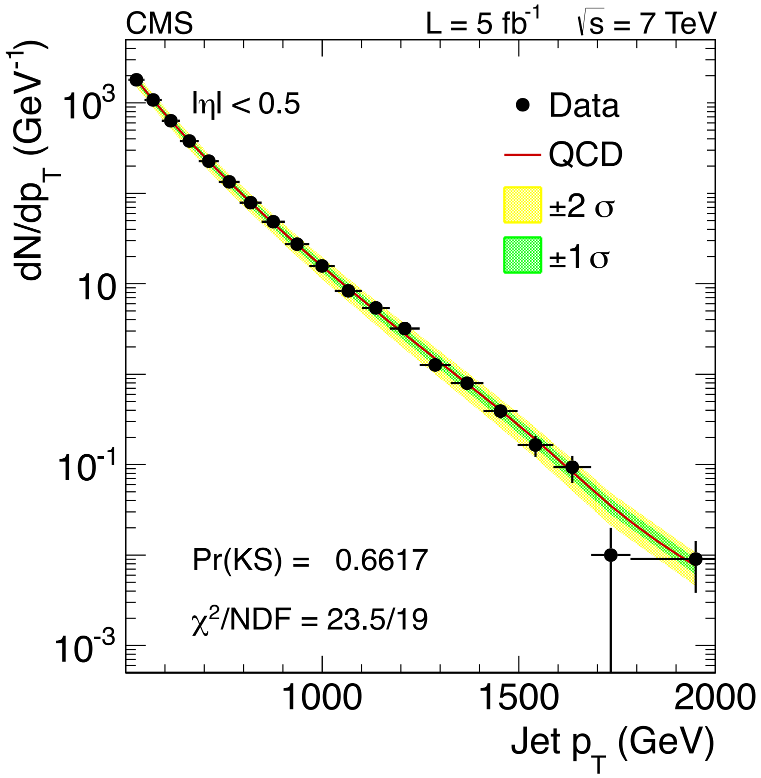
Analytic model of the signal for any λ

$$f = 1 + p_1 \left(\frac{p_T}{100\,\mathrm{GeV}}\right)^{p_2} \left(\frac{\lambda}{1\,\mathrm{TeV}^{-2}}\right) + p_3 \left(\frac{p_T}{100\,\mathrm{GeV}}\right)^{p_4} \left(\frac{\lambda}{1\,\mathrm{TeV}^{-2}}\right)^2$$

$\omega = p_1, p_2, p_3, p_4$ parameters are estimated in a simultaneous fit



Data: no significant deviations



Multinomial Dist. Model

$$p(D|\lambda,\omega) = \frac{N!}{N_1! \cdots N_K!} \prod_{j=1}^{K} \left(\frac{\sigma_j}{\sigma}\right)^{N_j}$$

Data are the counts in each p$_T$ bin $D \equiv N_1, \cdots, N_K$

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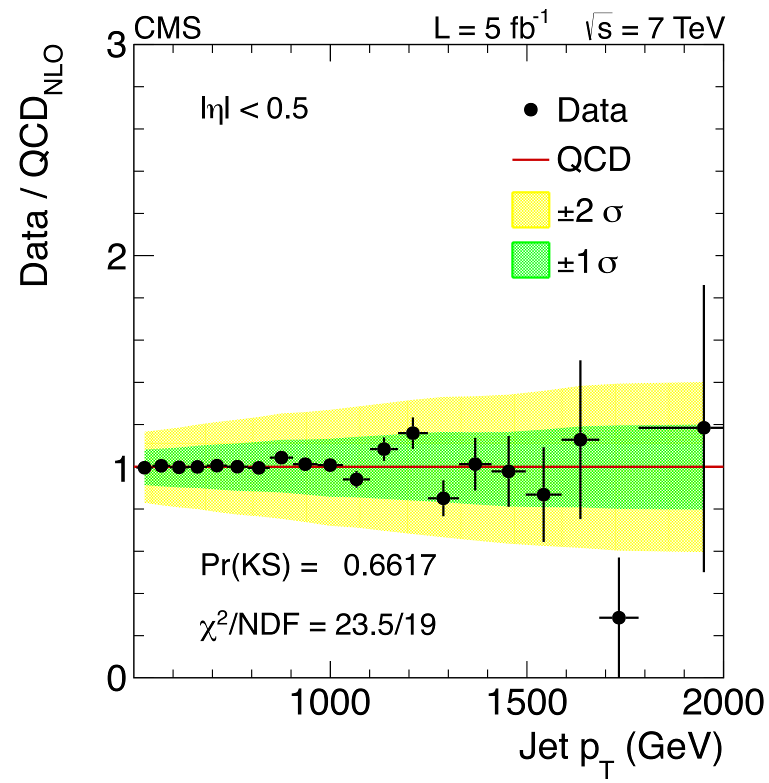
Handling Nuisances

Likelihood is marginalized discretely by creating ensemble of **correlated** background and signal spectra, in which parameters vary randomly

jet energy scale
jet energy resolution
PDFs
renormalization scale
factorization scale



Refit resulting distributions 500 times, arrive at a discrete approximation of marginal model

$$p(D|\lambda) = \int p(D|\lambda,\omega)\,\pi(\omega)\,d\omega,$$
$$\approx \frac{1}{M}\sum_{m=1}^{M} p(D|\lambda,\omega_m),$$

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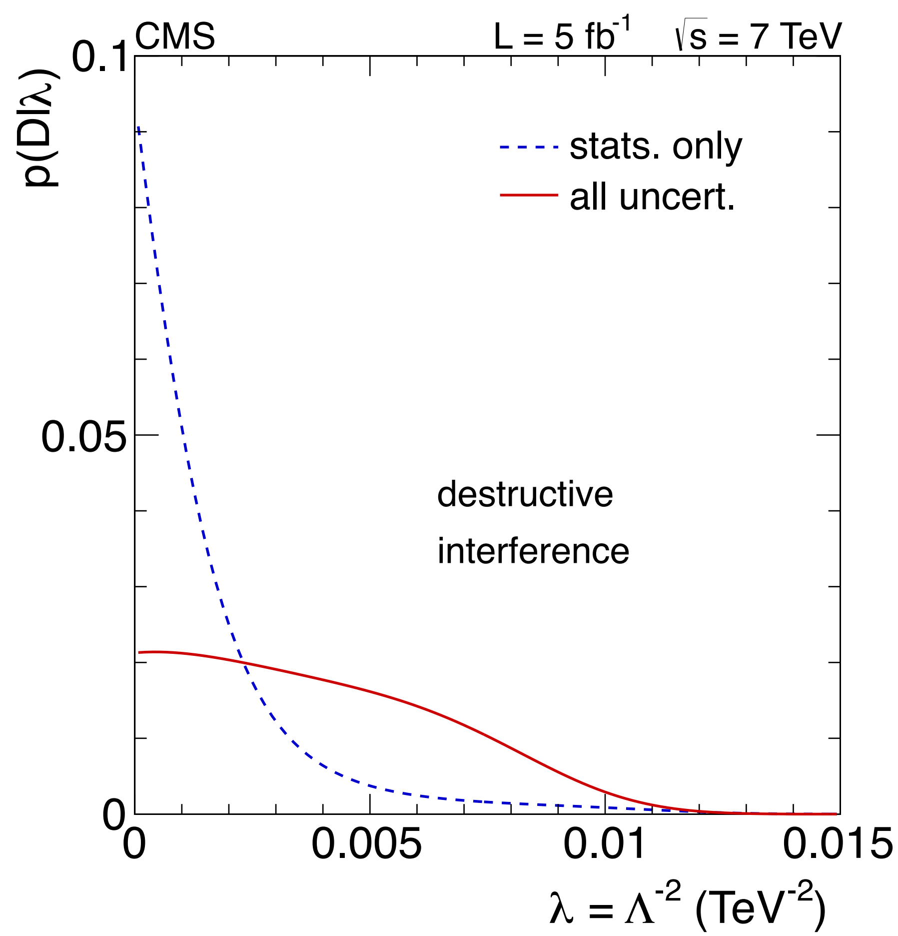
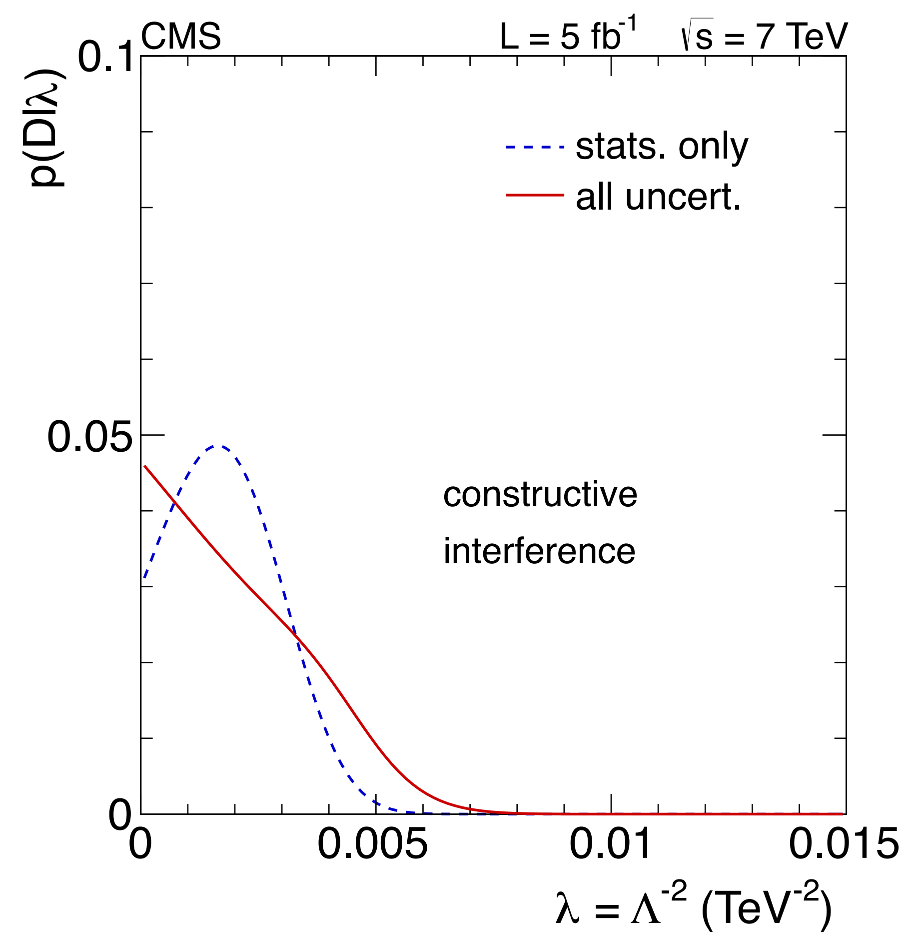
Results





Set 95% lower limits of 9.9 TeV (destructive) and 14.3 TeV (constructive) using ratio of marginal model and CL$_s$

$$-2\log\left(\frac{p(x|\lambda)}{p(x|0)}\right)$$

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Summary and Outlook

Summary and Outlook

- CMS BSM searches use a variety of likelihoods and handle nuisances / compute limits in different ways
  - Bayesian (Marginalize), Frequentist (Profile), Hybrid
- Several analyses provide public likelihoods and details for generator-level study of your own BSM model
- Even in cases where full model are not provided, one can derive an approximate likelihood
- Working to provide public likelihoods in future CMS BSM analyses
  - Razor Analysis will provide full details and code to implement binned likelihood in python

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