

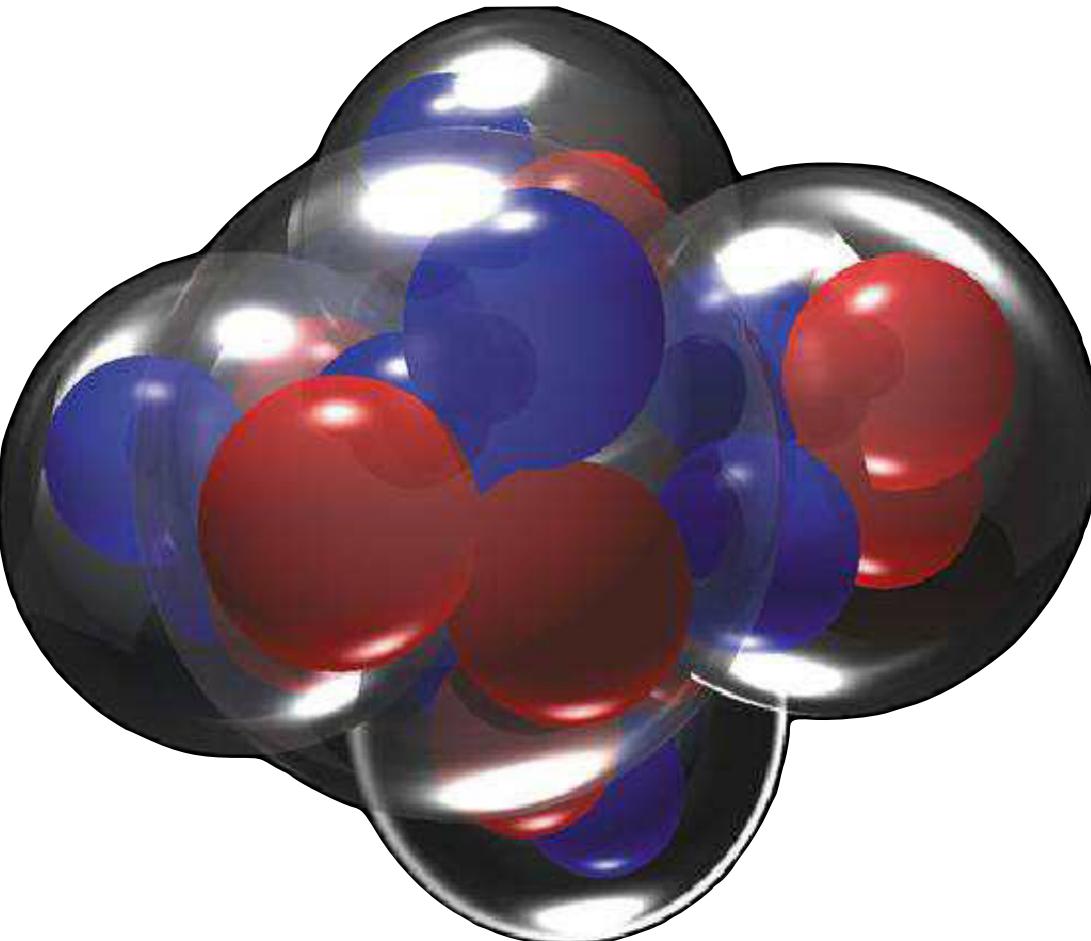


I | Illinois Center for Advanced Studies of the Universe

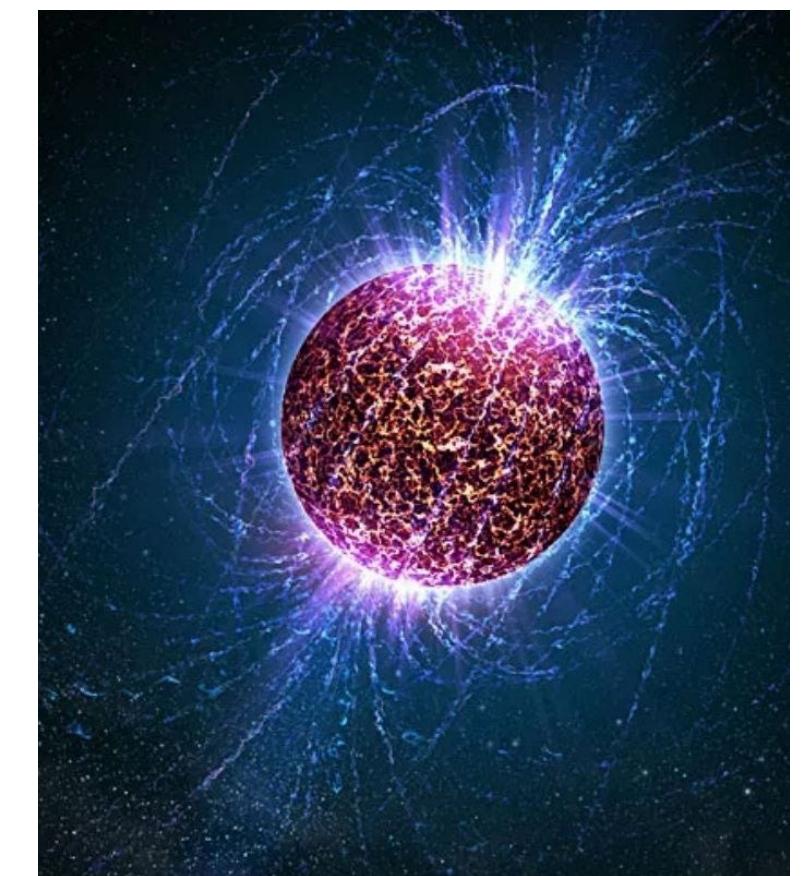
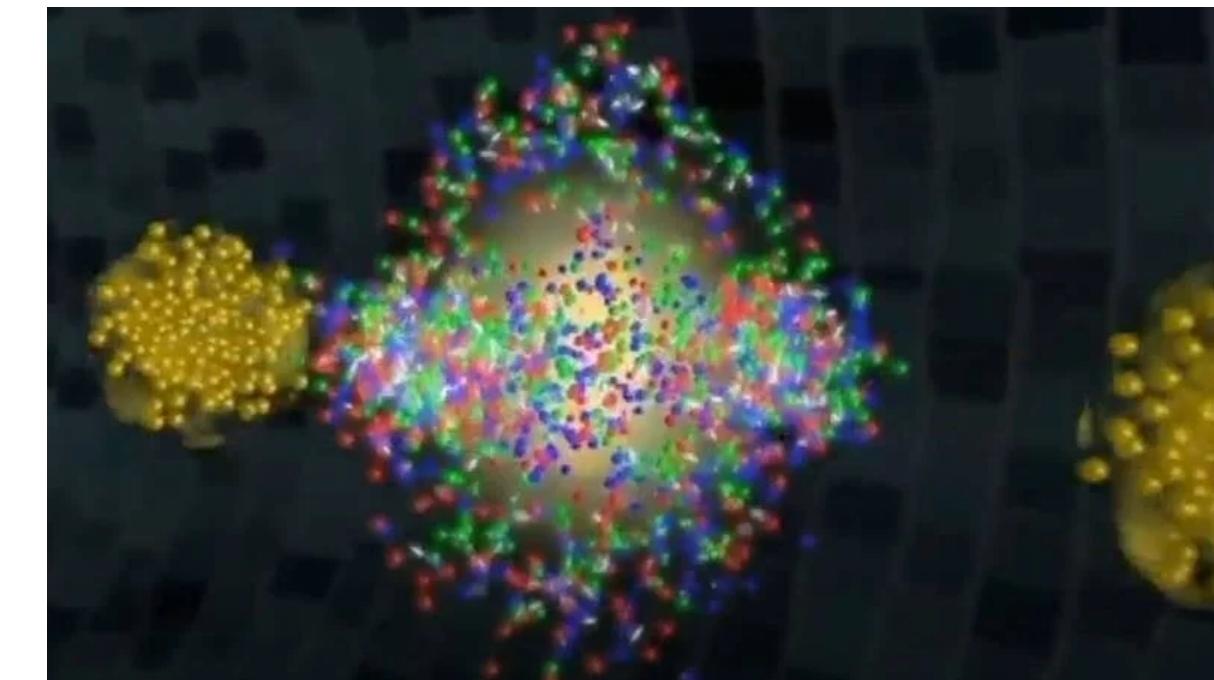


# The equation of state from Neutron Stars to Heavy Ion Collisions

Jacquelyn Noronha-Hostler



CERN - 13th Nov, 2024



GW170817

Animation



GW170817

Animation



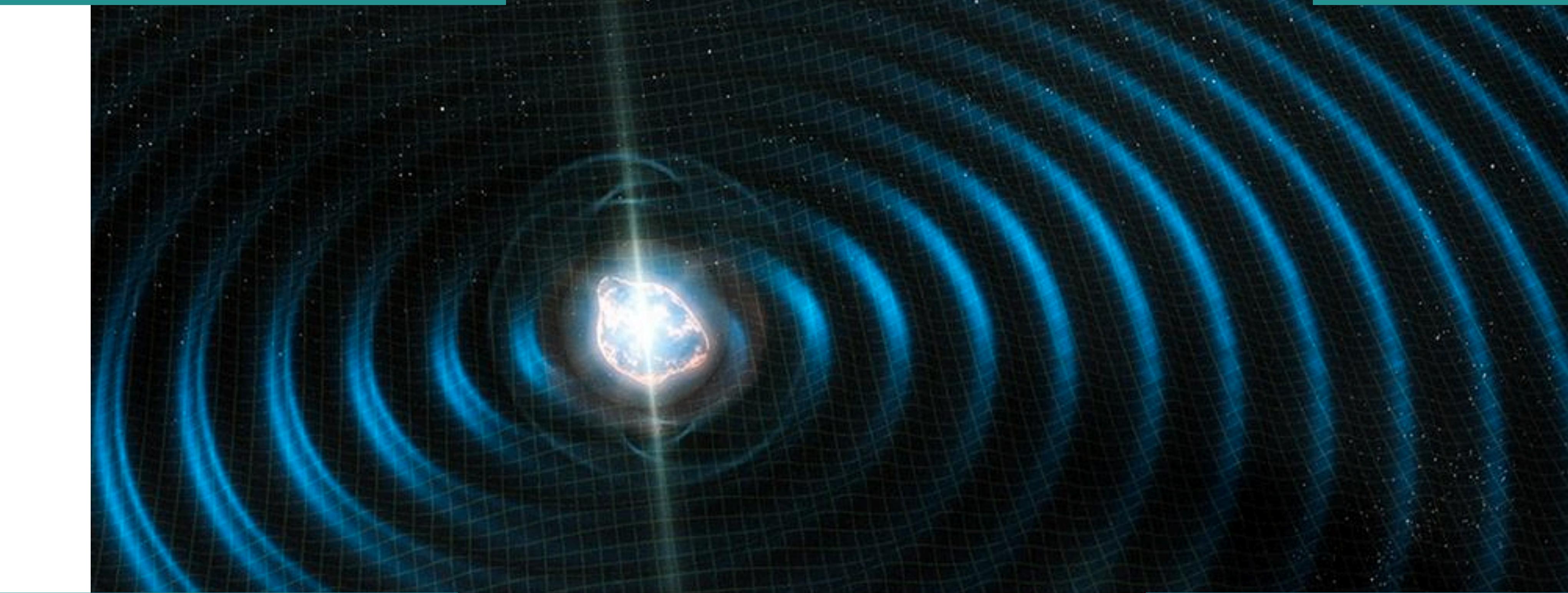
# Neutron star mergers combine the fundamental forces

Electromagnetism

Large B field, visual signals

Gravity

Gravitational Waves



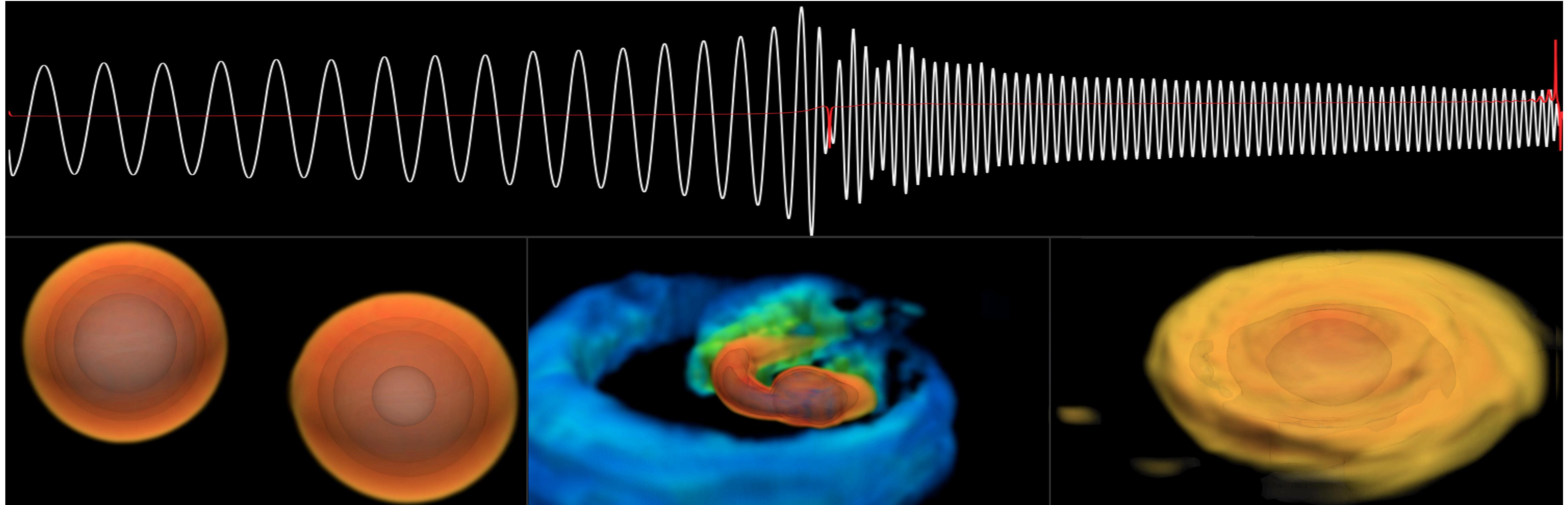
Strong Force

Nuclei, nucleons, quarks

Weak Force

Neutrinos, Strange decays

# How hot is a neutron star during a merger?



Cold (inspiral)

$$T \sim 10^6 K$$

$$T \sim 10^{-5} MeV$$

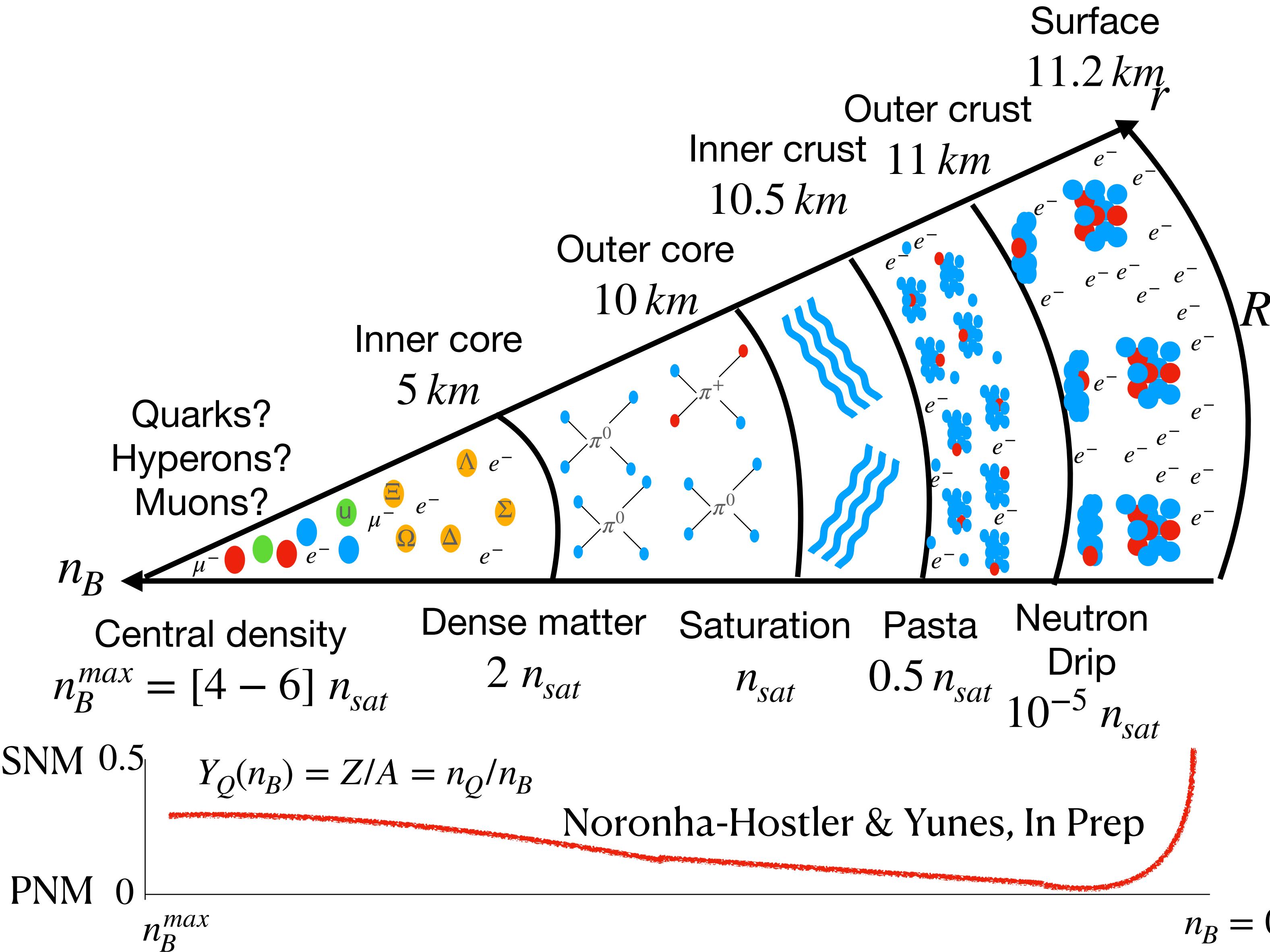
Hot (merger)

$$T \sim 10^{12} K$$

$$T \sim 100 MeV$$

Currently have inspiral data, but hopefully future detectors will probe mergers

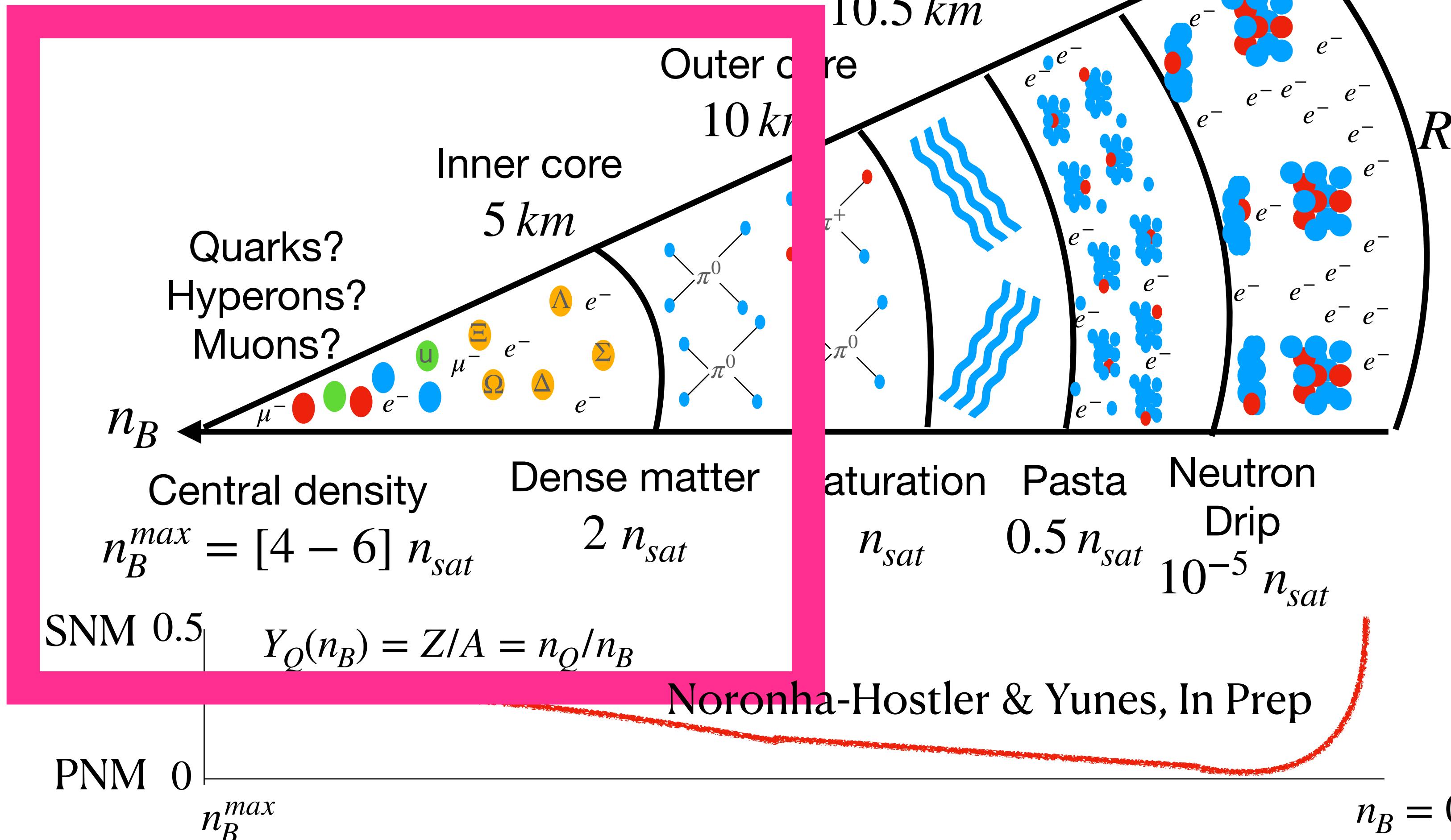
# What is inside neutron stars?



Equation of State (EOS)  
encodes degrees of freedom,  
phases of matter, interactions  
into the pressure vs energy  
density relationship  $p(\varepsilon)$

# What is inside neutron stars?

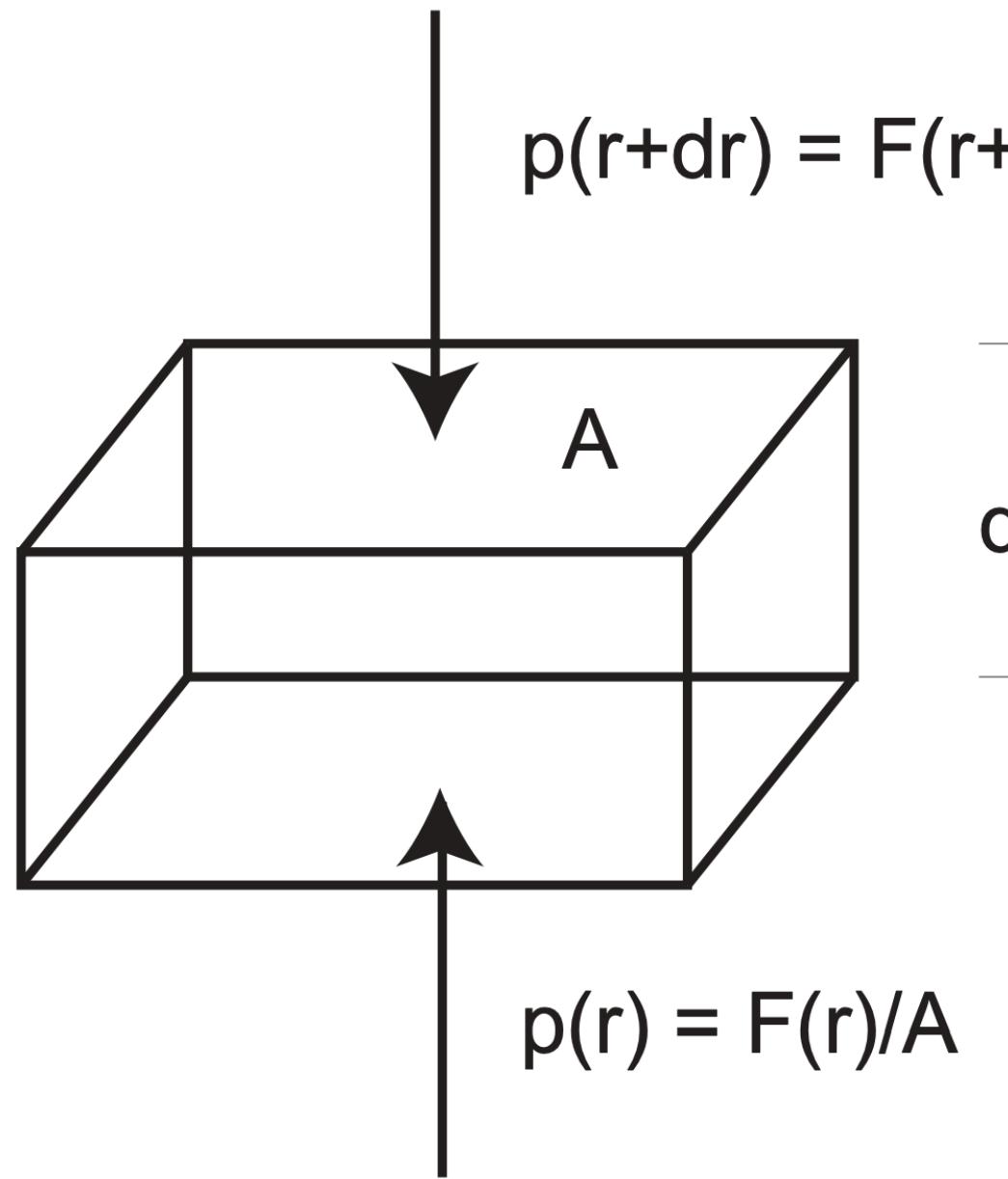
What happens here?



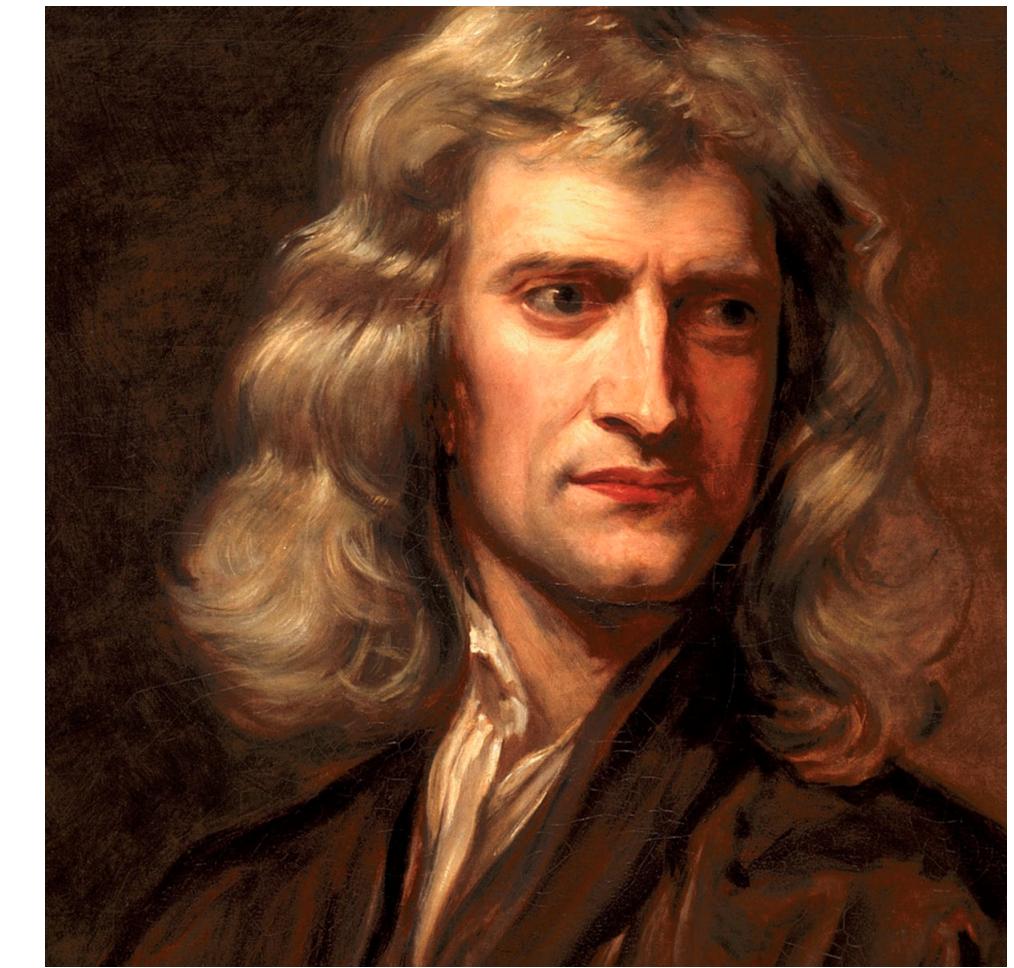
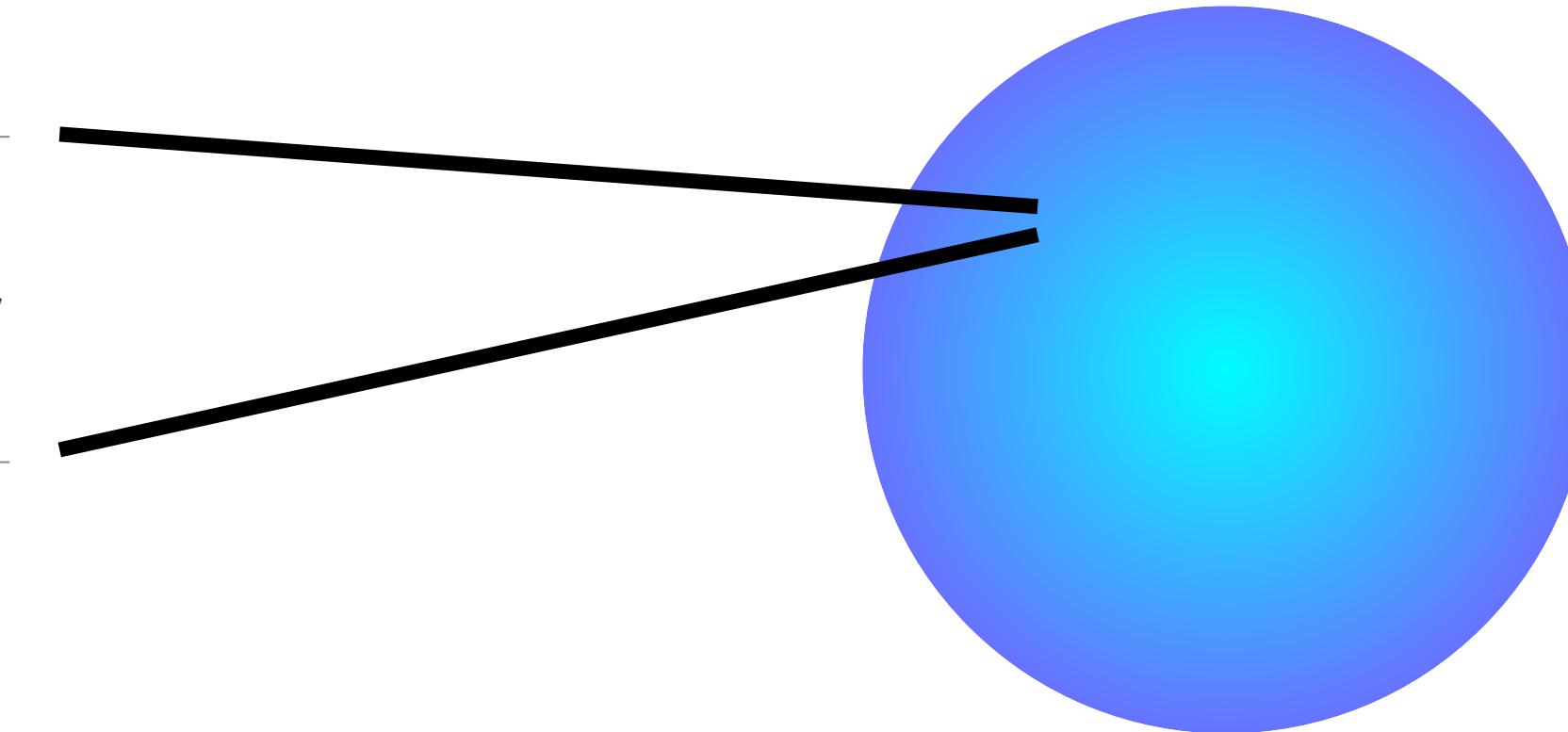
Equation of State (EOS)  
encodes degrees of freedom,  
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density relationship  $p(\varepsilon)$

# Mass and Radius: ideal fluid, non-rotating

## Newtonian Gravity



$dr$



$$\frac{dp}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2r^2}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \rho(r) = \frac{4\pi r^2 \epsilon(r)}{c^2}$$

$$\mathcal{M}(r) = 4\pi \int_0^r r'^2 dr' \rho(r') = 4\pi \int_0^r r'^2 dr' \epsilon(r')/c^2$$

Total mass inside the sphere of radius  $r$

# Mass and Radius: ideal fluid, non-rotating

**General Relativity: Tolman–Oppenheimer–Volkoff (TOV) equation**

Input:

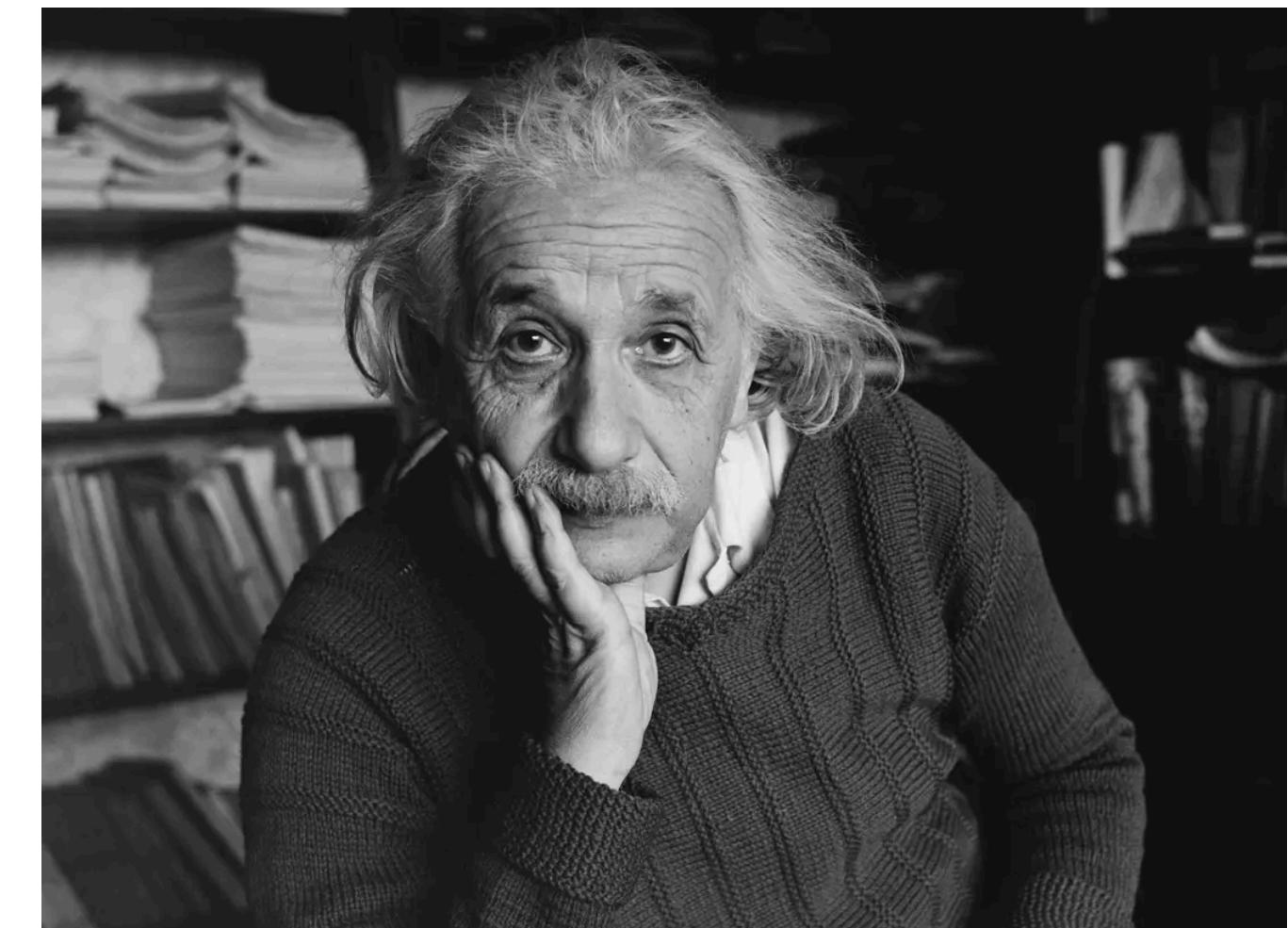
Equation of State  $p(\varepsilon)$

Output:

$M(R)$  when P=0

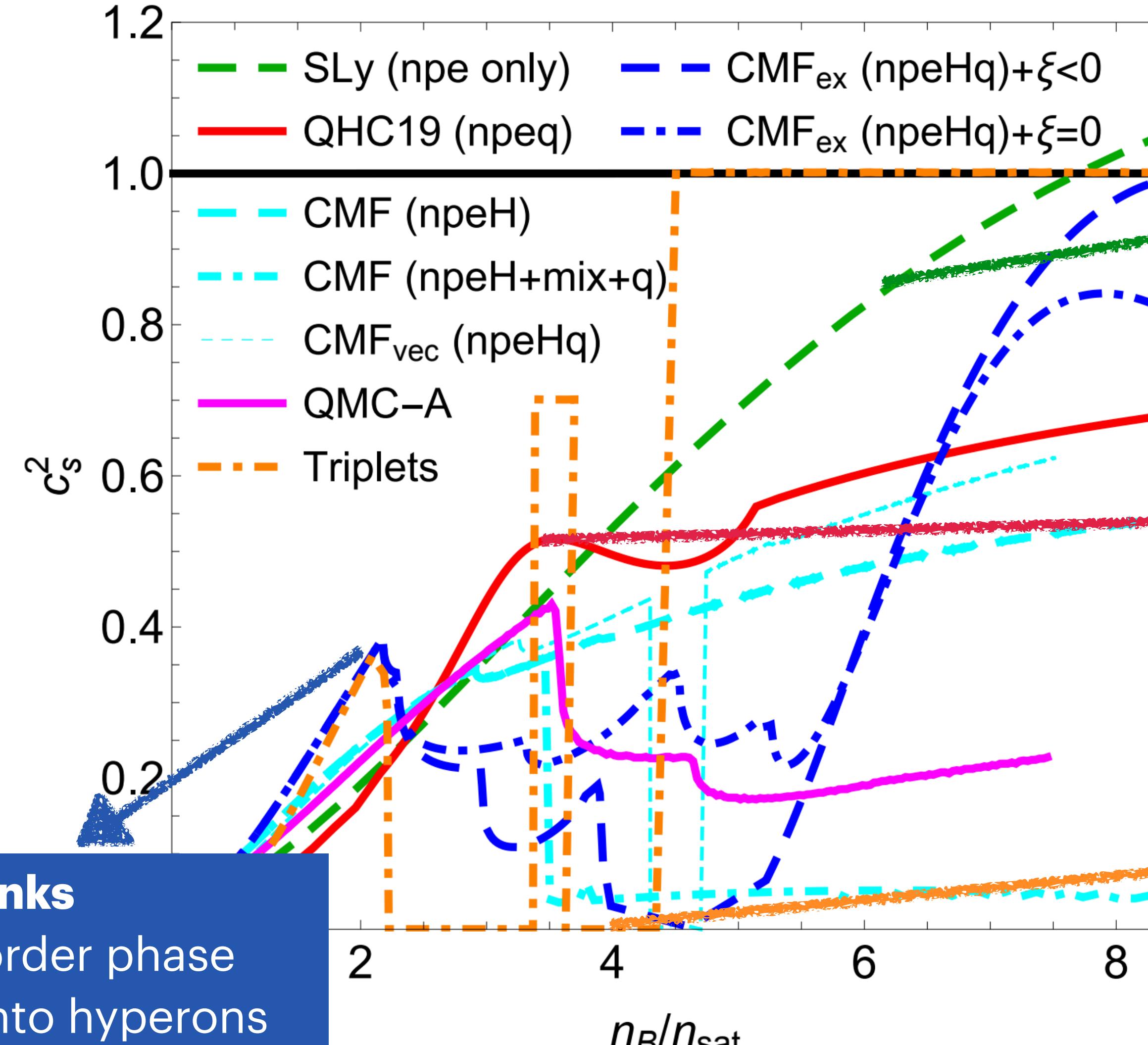
$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

$$M = m(R) = \int_0^R 4\pi r^2 \rho dr$$



# Possible EOS: Insights from effective models

Looking for “bumps” in the night



$$c_s^2 = \frac{dp}{d\varepsilon}$$

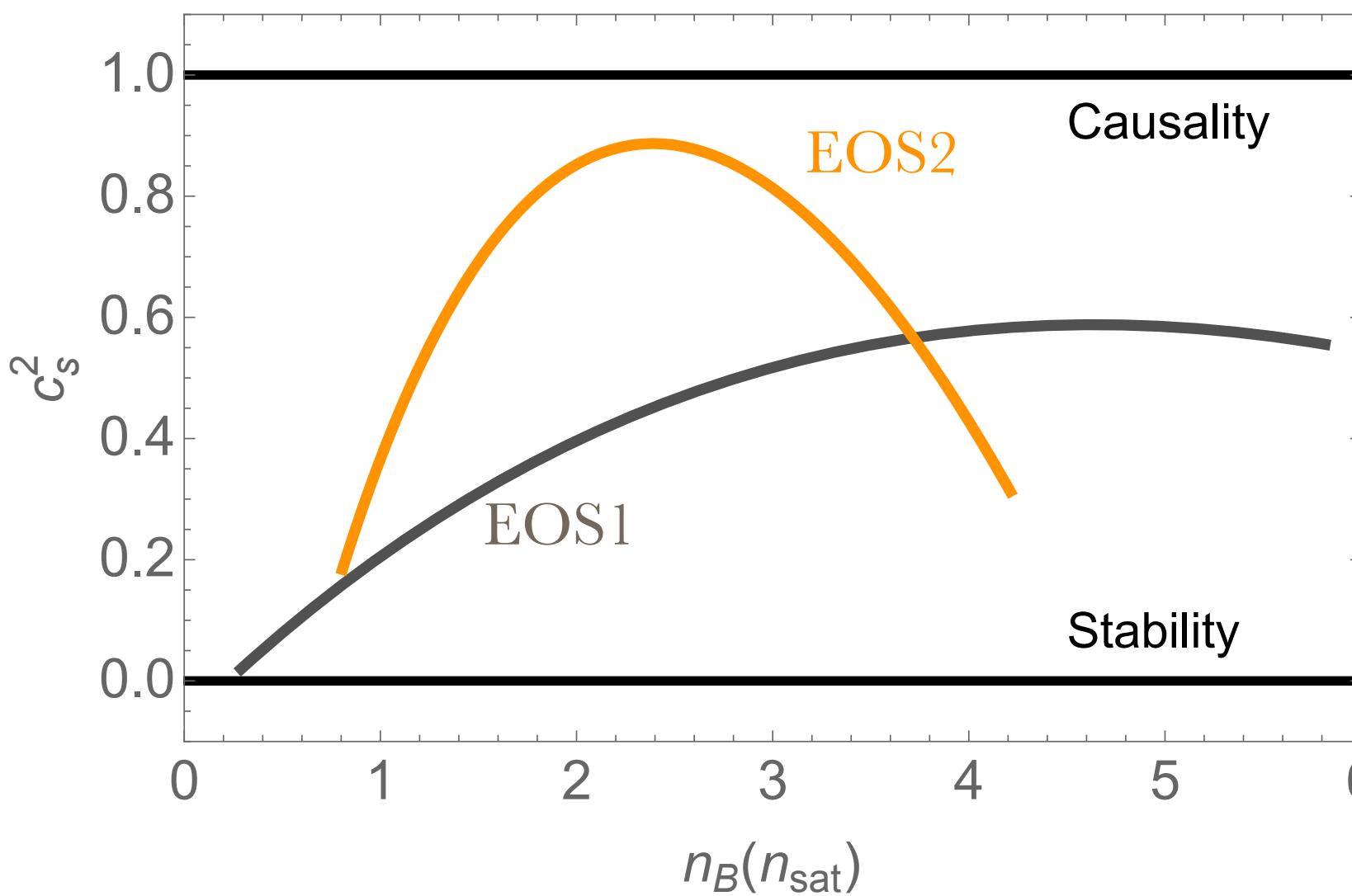
**Monotonic**  
Neutrons, protons,  $e^-$   
*Astron.Astrophys. 380 (2001) 151*

**Bump**  
cross-over phase transition into quarks  
*Phys.Rev.Lett. 122 (2019) 12, 122701;*  
*Astrophys.J. 885 (2019) 42*

$c_s^2 \rightarrow 0$   
1st-order phase transition (quarks or strange dominated)  
*Phys.Rev.D 88 (2013) 8, 083013*

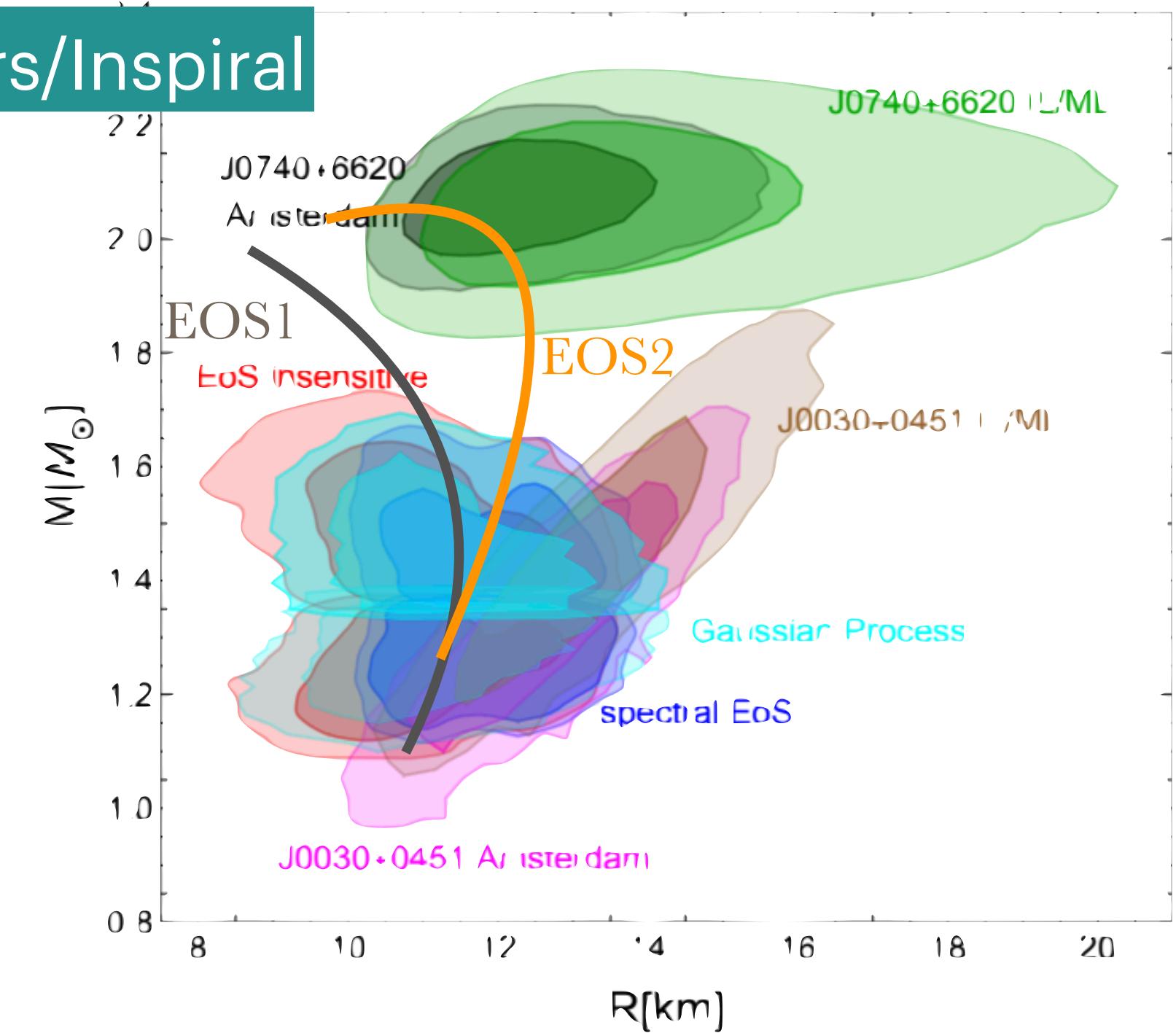
# Equation of State from astrophysics observation?

Build in desired features: particles, phase transitions, interactions etc



Isolated, cold neutrons stars/Inspiral

Astrophysical constraints



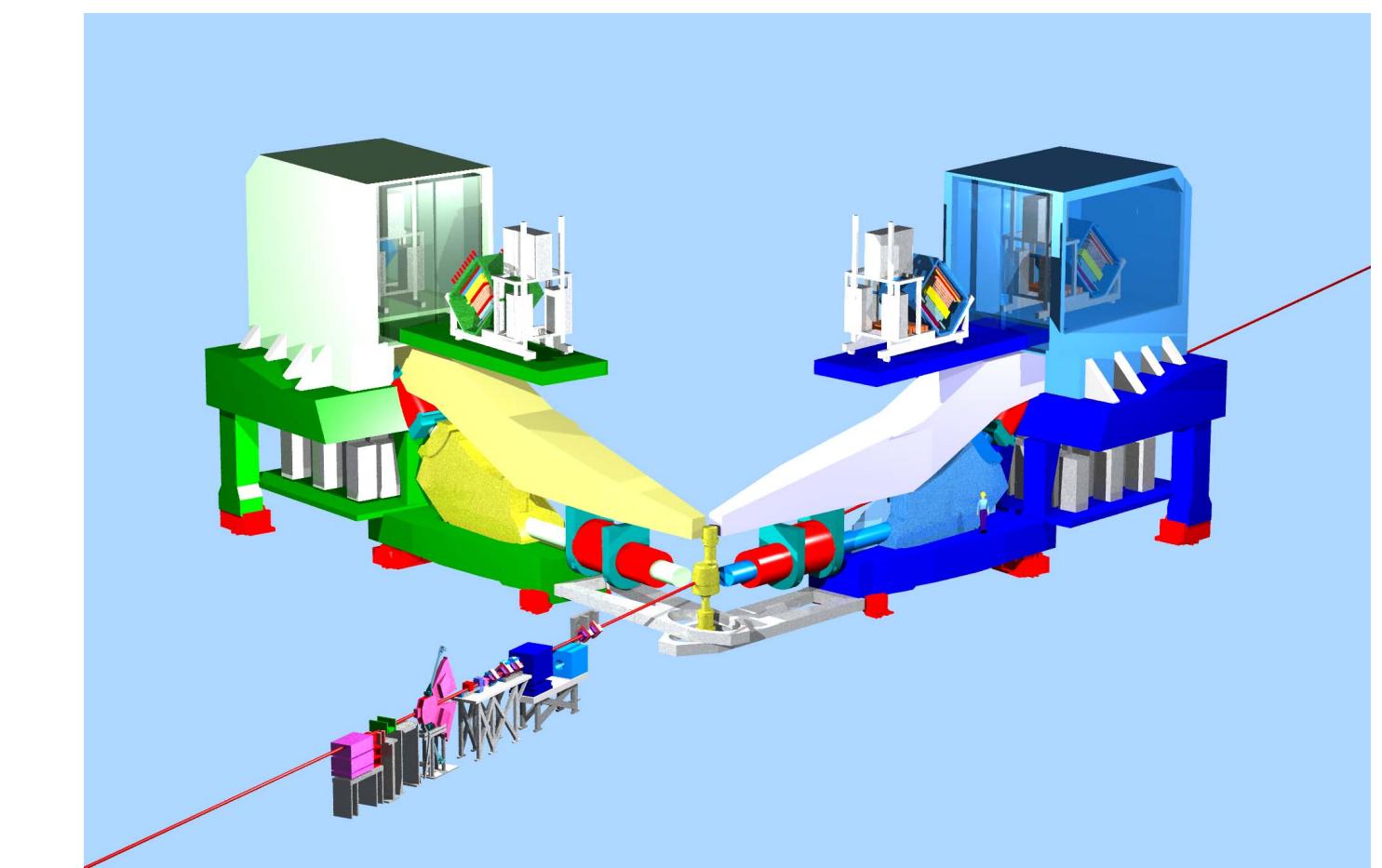
Assign a likelihood to each EOS

*Posterior  $\propto$  Likelihood  $\times$  Prior*

Independent measurements:

$$\mathcal{L} = \prod_{i=M,R,\dots} \left[ \prod_{j=1}^{j(i)} \mathcal{L}(i,j) \right]$$

Theory constraints



# Modified Gaussian Processes

**Building in structure in  $c_s^2$**

Original Gaussian Process approach  
R. Essick *Phys.Rev.D* 101 (2020) 6, 063007

- Model-agnostic approaches are common  
→ Gaussian processes (GPs):

EoS modeled via:  $\phi(x) = \log(1/c_s^2 - 1)$ , **stable and causal**

$$\phi \sim \mathcal{N}(\underline{\mu_i}, \Sigma_{ij})$$

Collection of functions, behavior specified by **a mean** and **covariance kernel**

**Squared-exponential is a common choice:**

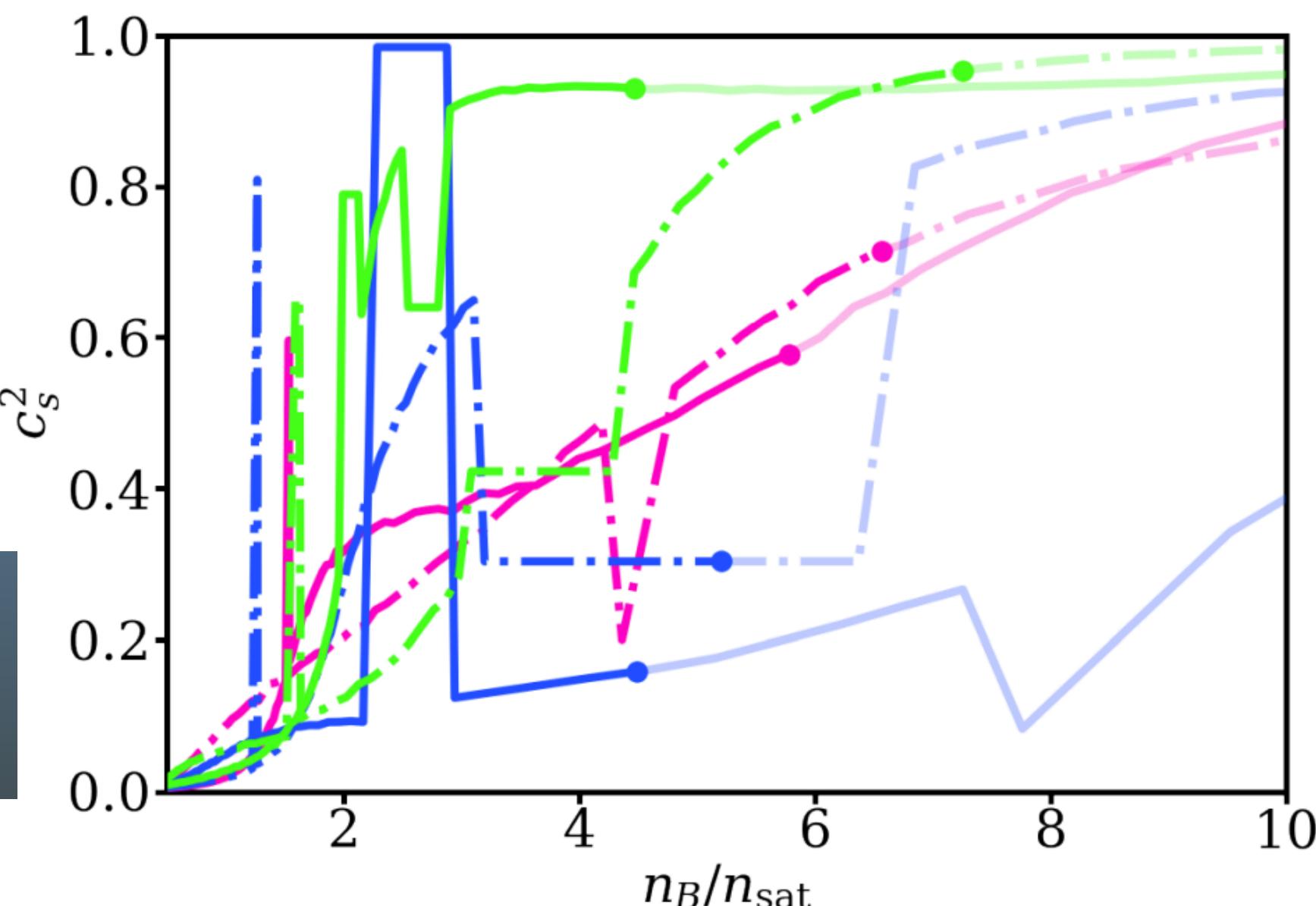
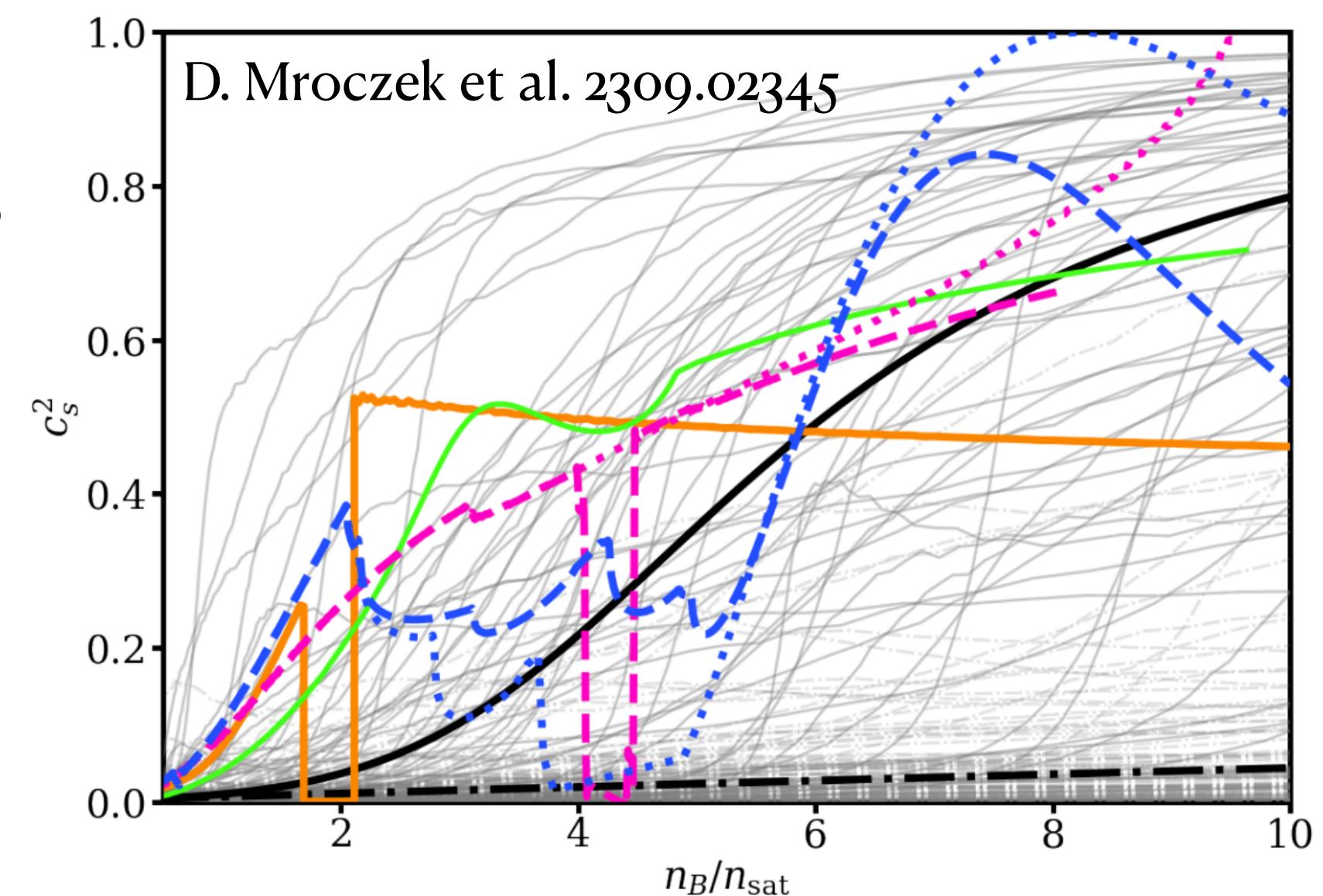
$$K_{\text{se}}(x_i, x_j) = \sigma^2 \exp \left[ -\left( x_i - x_j \right)^2 / 2\ell^2 \right]$$

$\ell$ : correlation length  
 $\sigma$ : correlation strength

Goal: Compare long range correlation (benchmark) versus multi-scale features

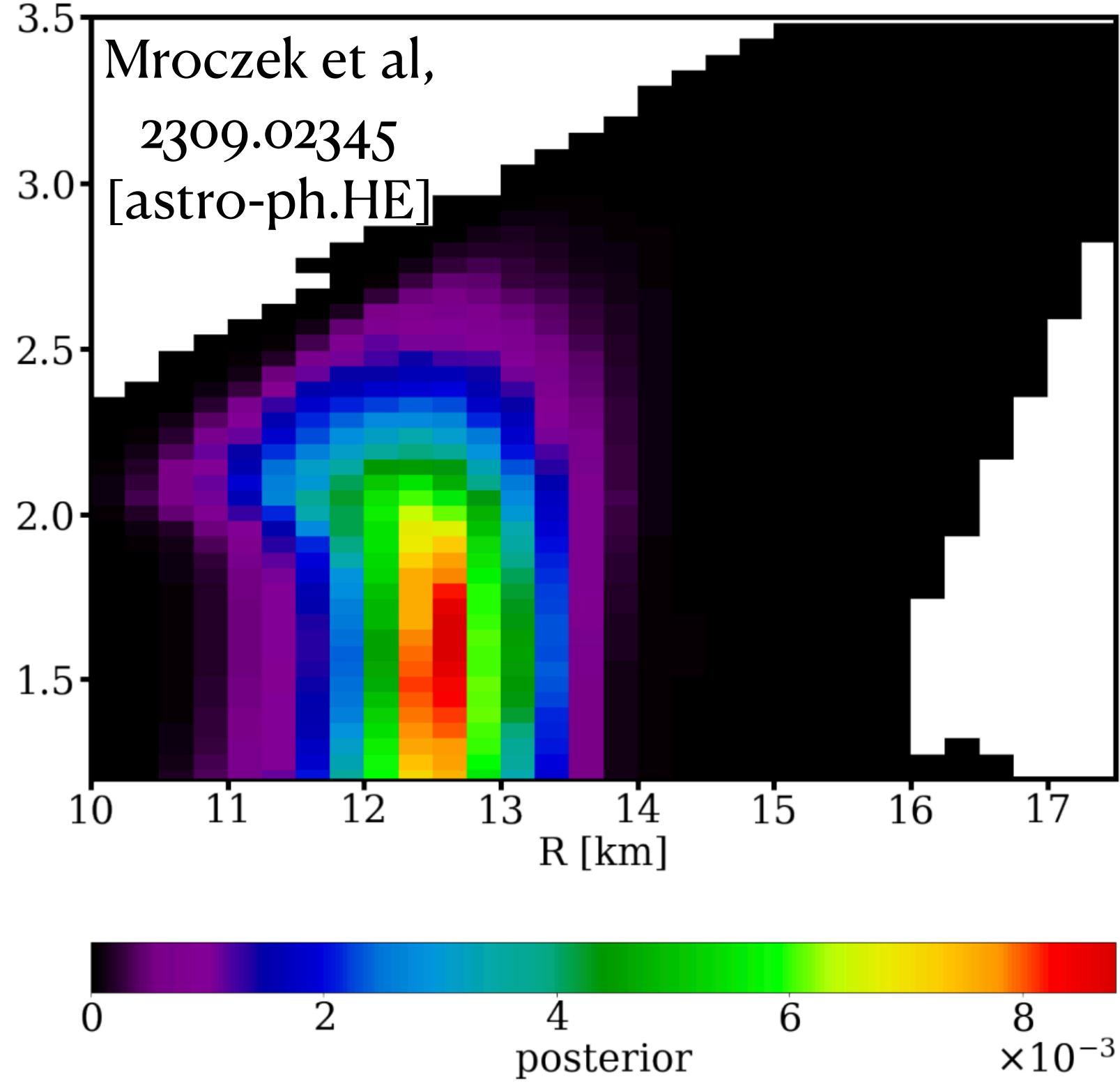
Gaussian Process  
(benchmark)

+ Structure

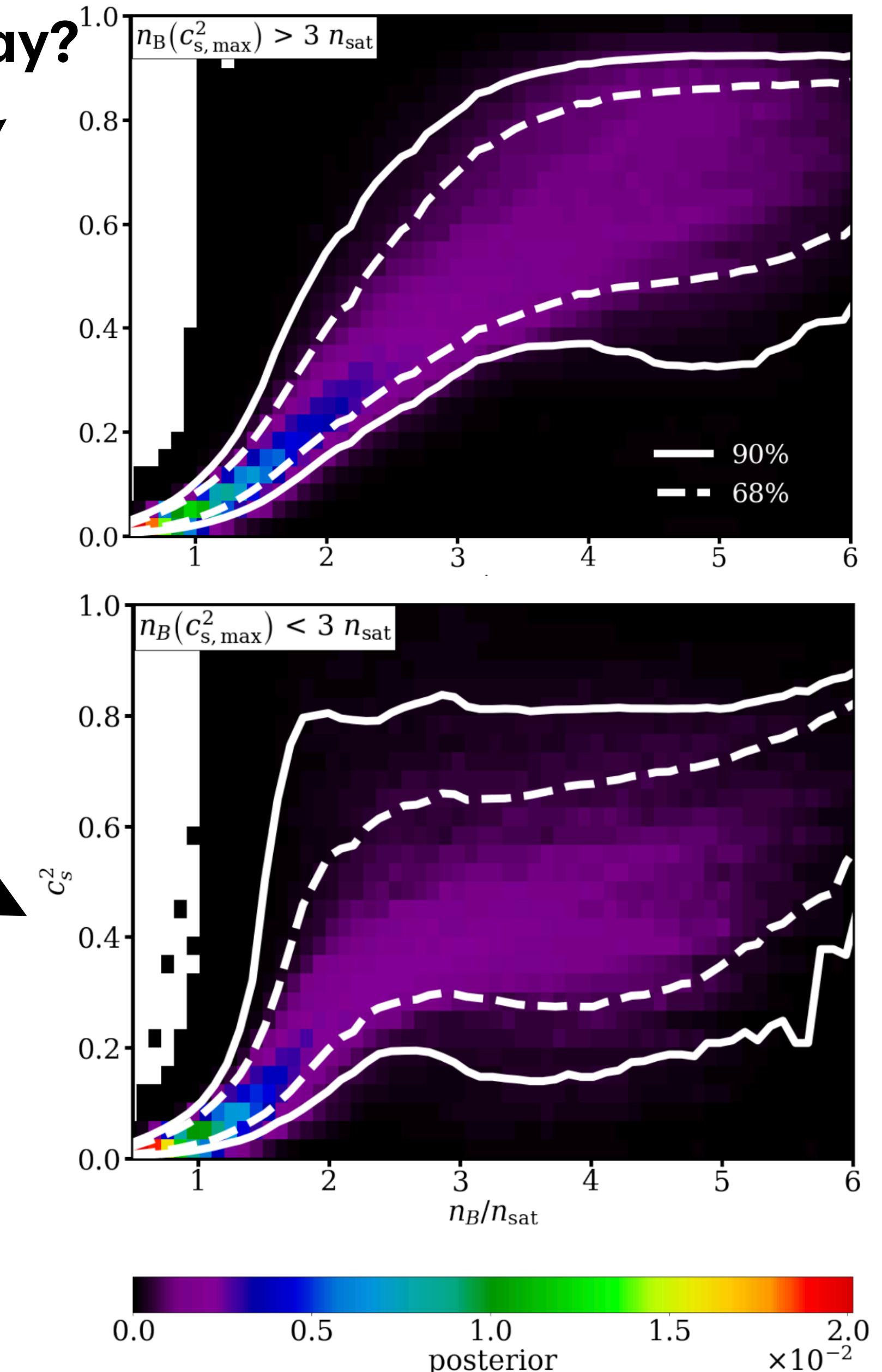


# Posteriors of Mass, Radius, $c_s^2(n_B)$

What does current data tell us today?



Both a monotonic  $c_s^2$  and one  
with a bump at  $n_B = [2,3] n_{sat}$



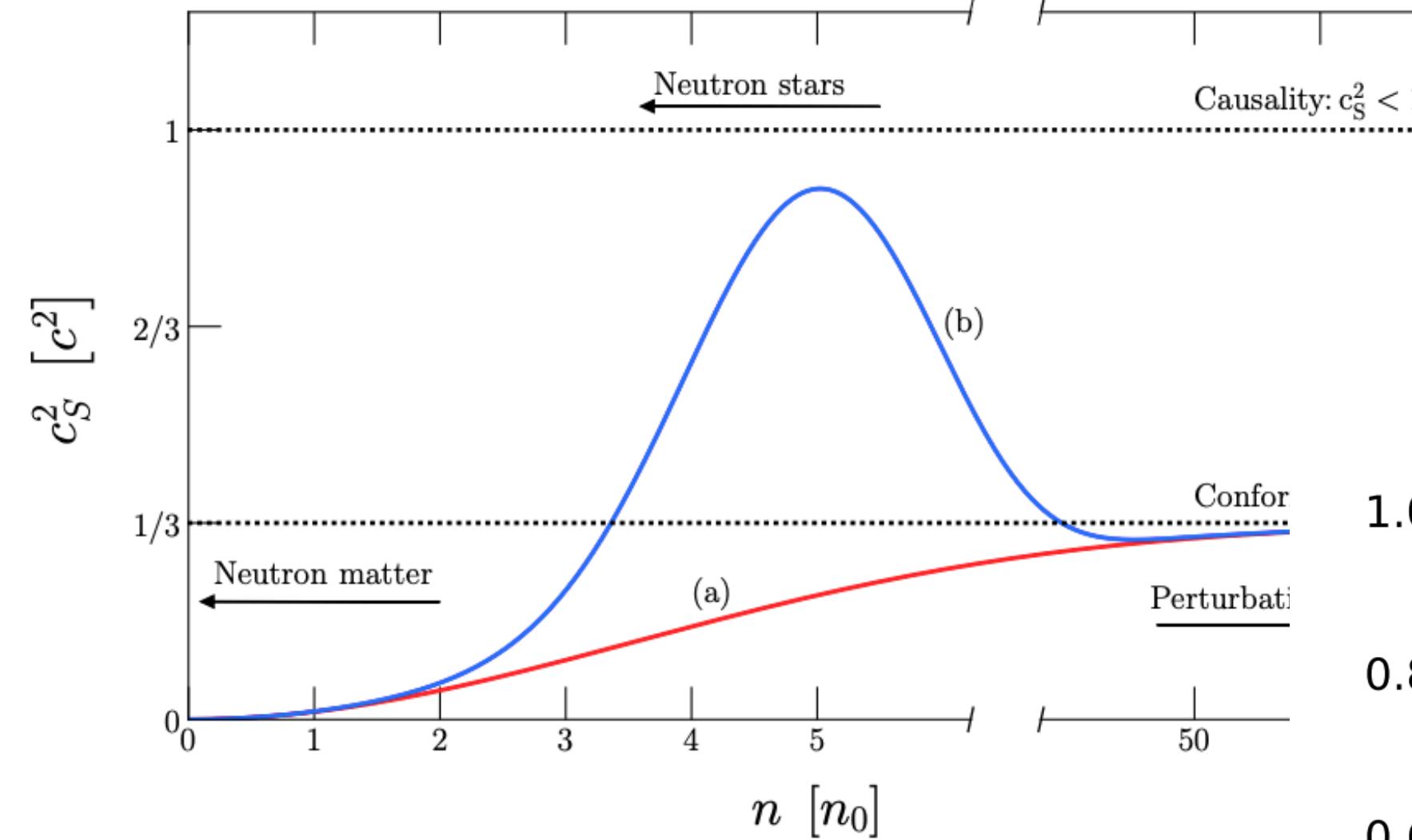
Current data is inconclusive, need more  
observations! Or new constraints!

# More references on bump in $c_s^2$

From various microscopic theories and Bayesian analyses

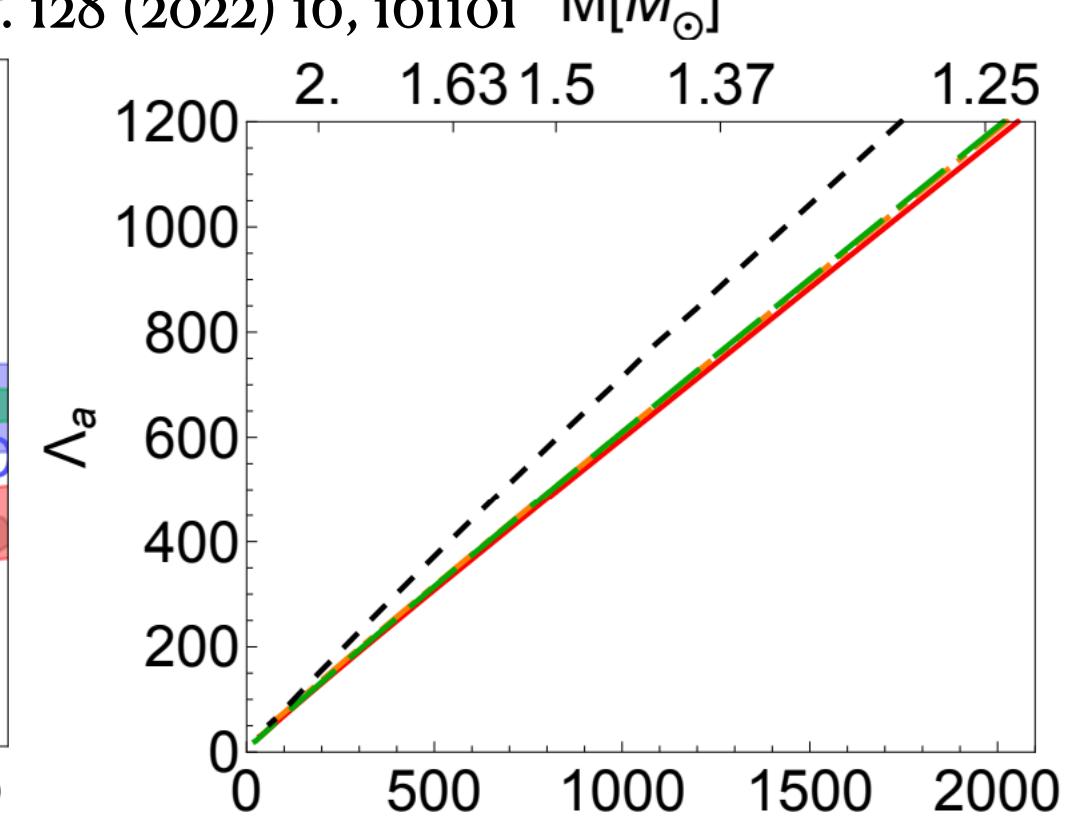
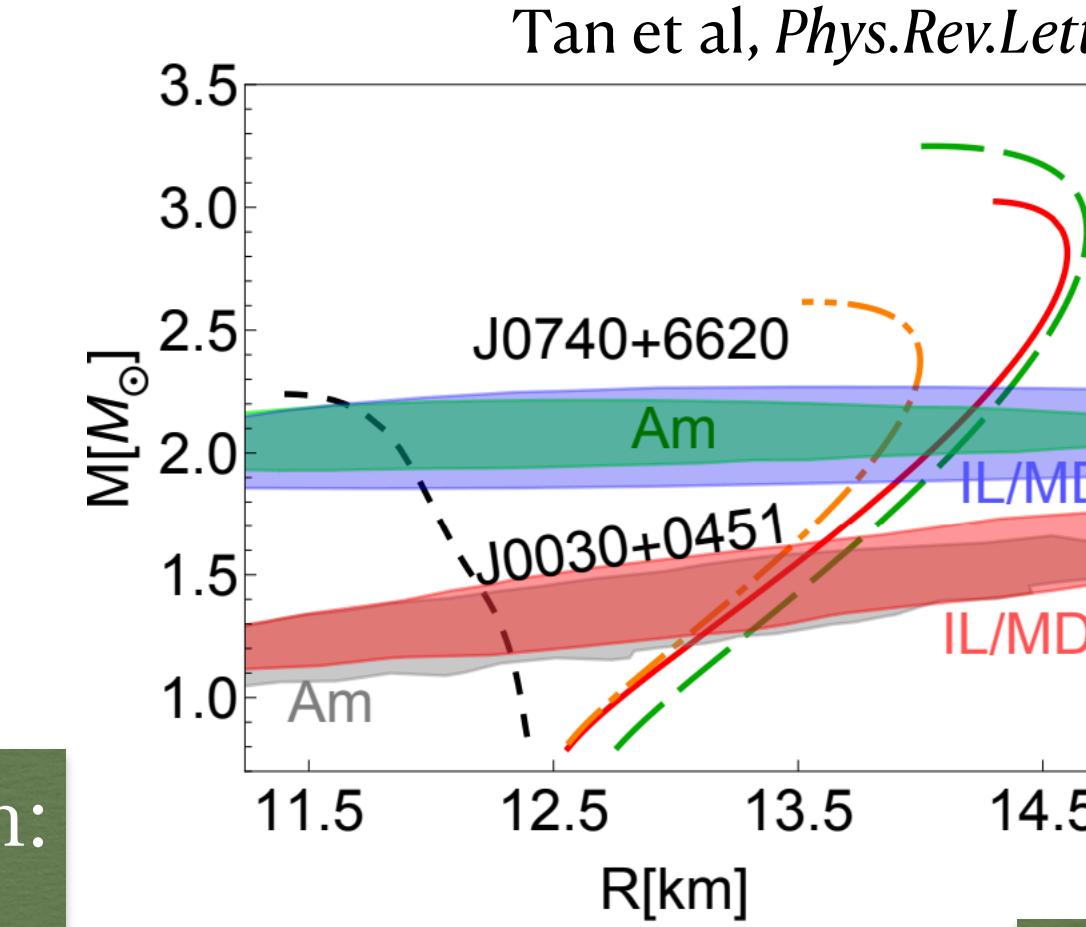
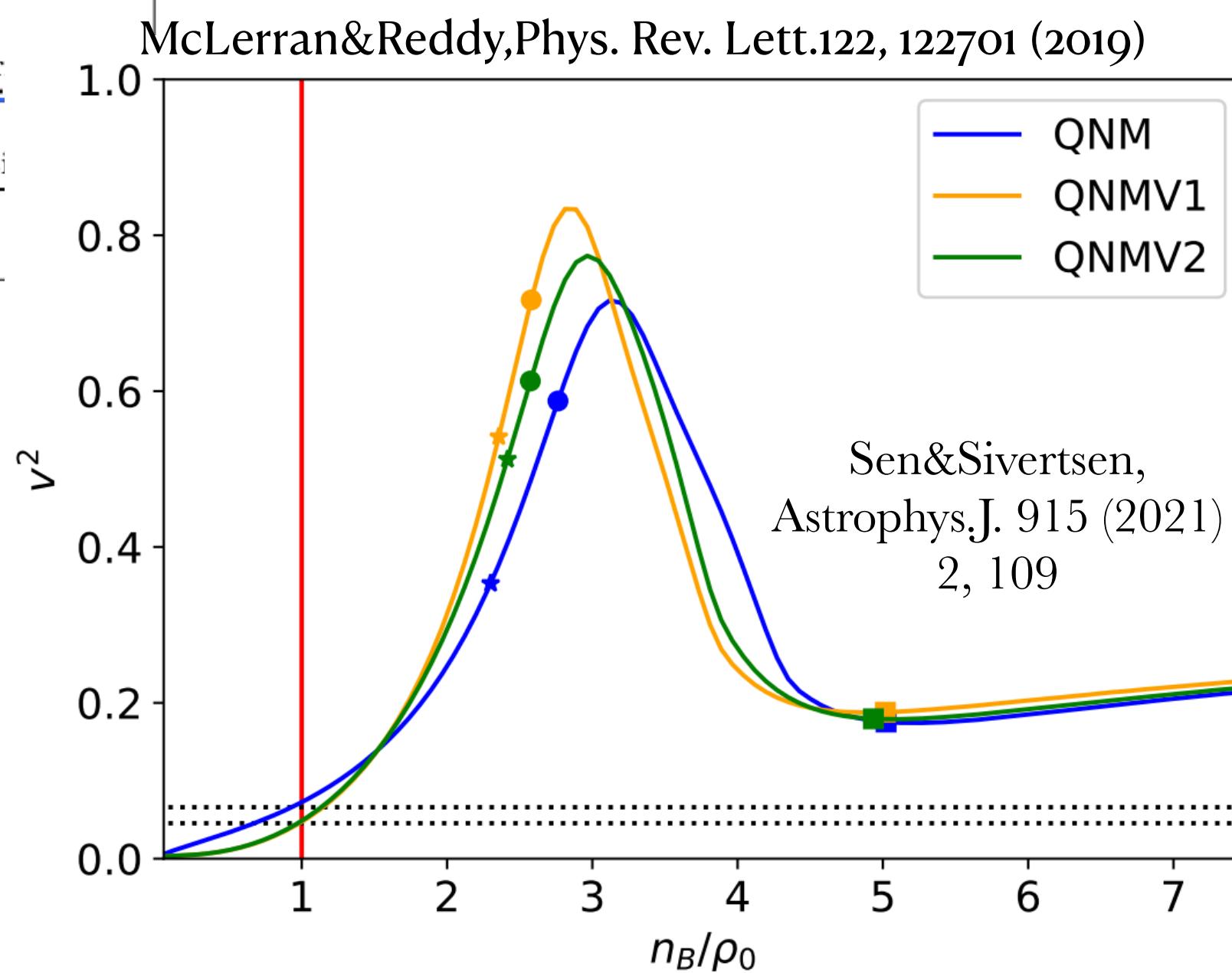
Measurements: breaking binary love relation

Steep rise in  $c_s^2$  to reach  $2M_\odot$

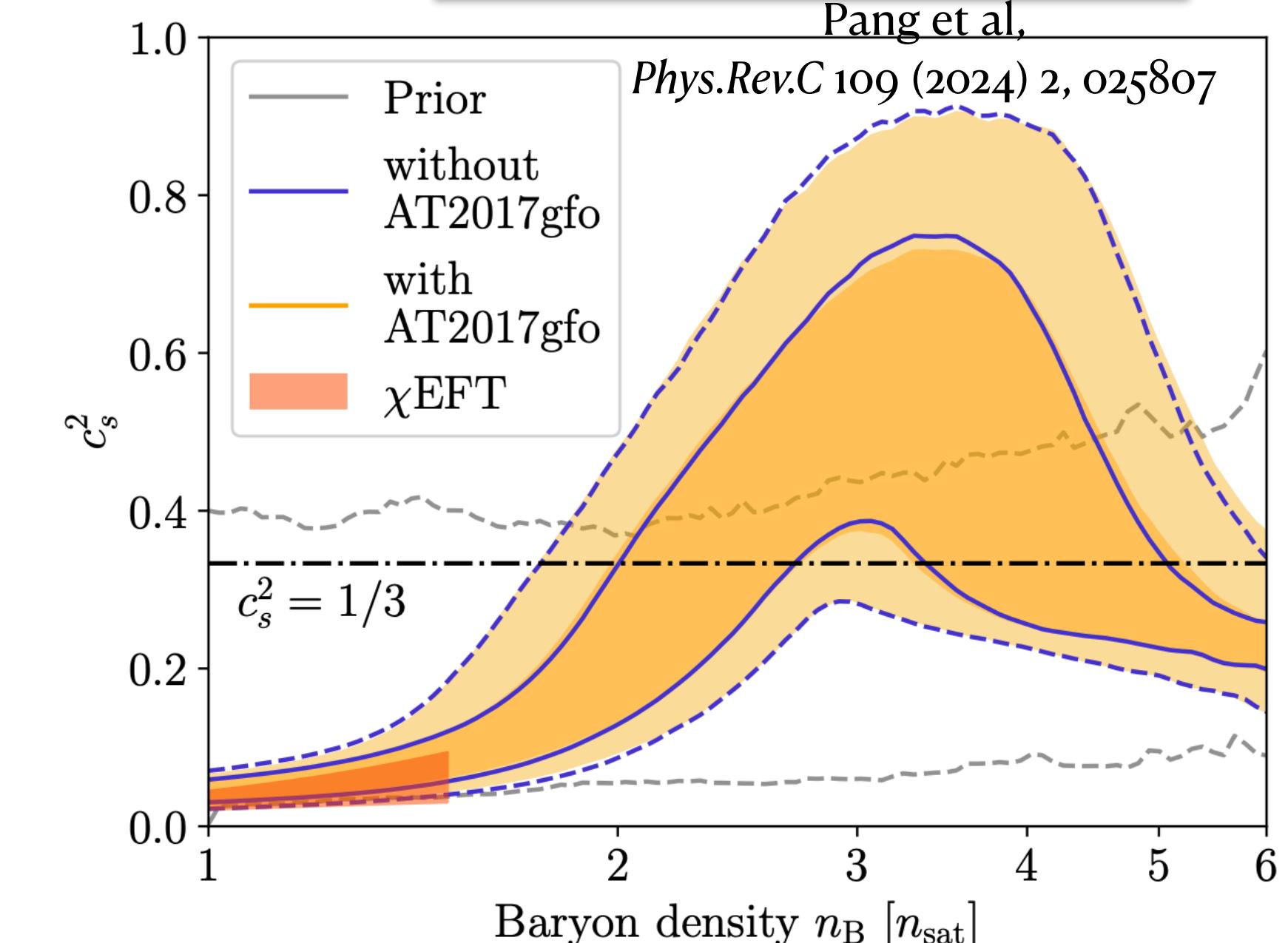


Bedaque & Steiner, Phys. Rev. Lett. 114 (2015) 3, 031103; Alford et al, Phys. Rev. D92, 083002 (2015), Ranea-Sandoval, et al, Phys. Rev. C93, 045812  
I. Tews, et al, Phys. Rev. C98, 045804 (2018)

One physical mechanism:  
Quarkyonic Matter

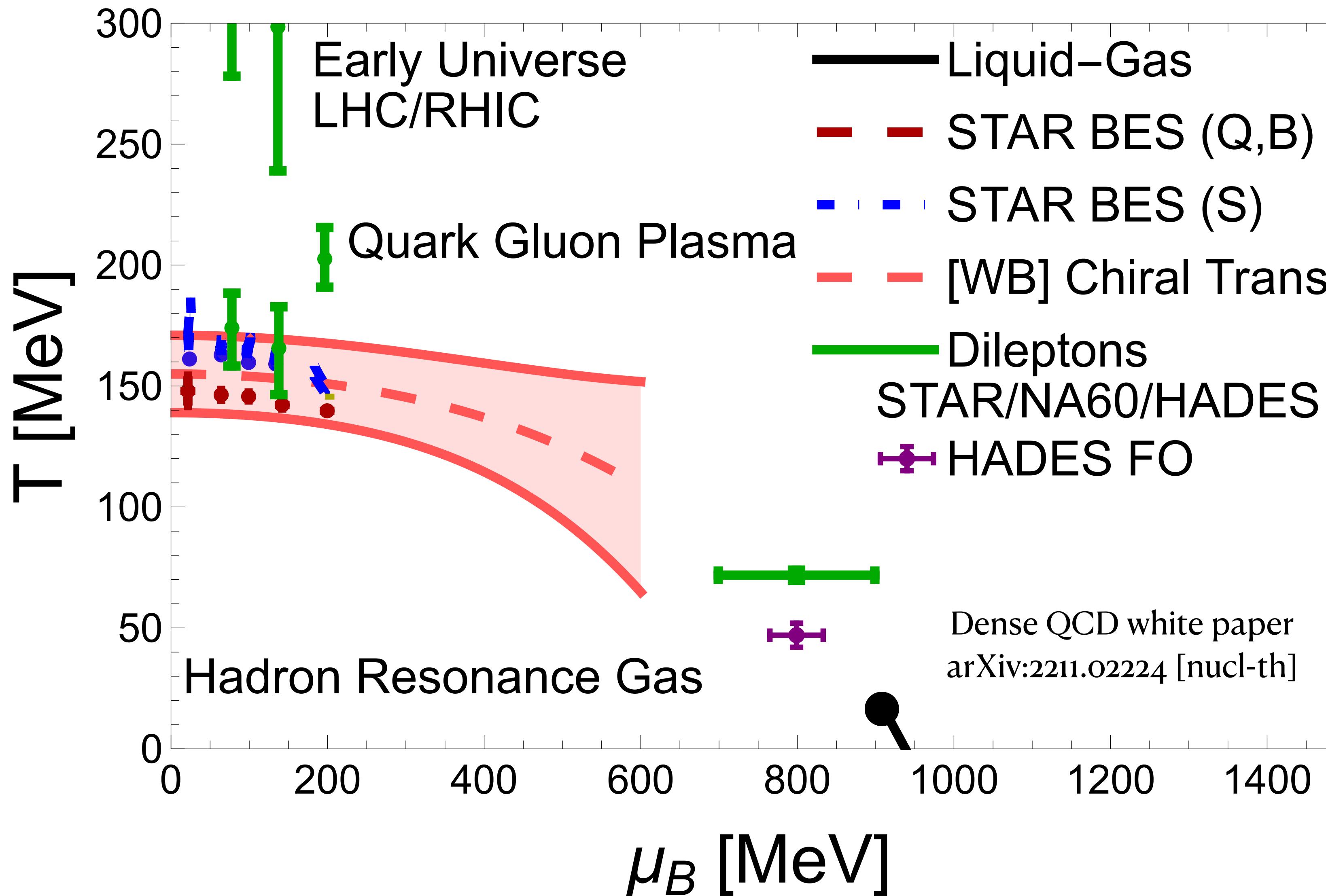


Bayesian analysis



# Connecting heavy-ion collisions and neutron stars

## QCD phase diagram

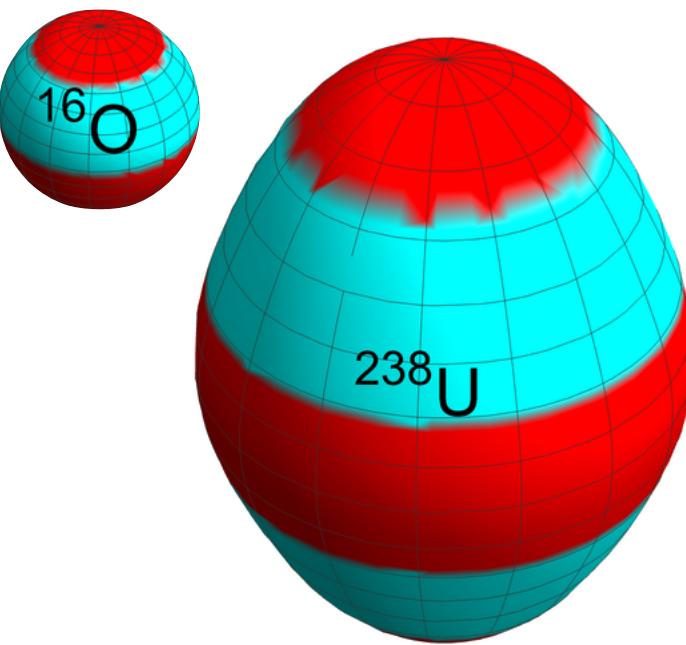


# Charge fraction of ions

## Isospin asymmetry

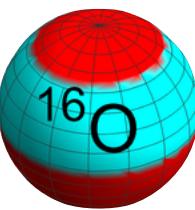
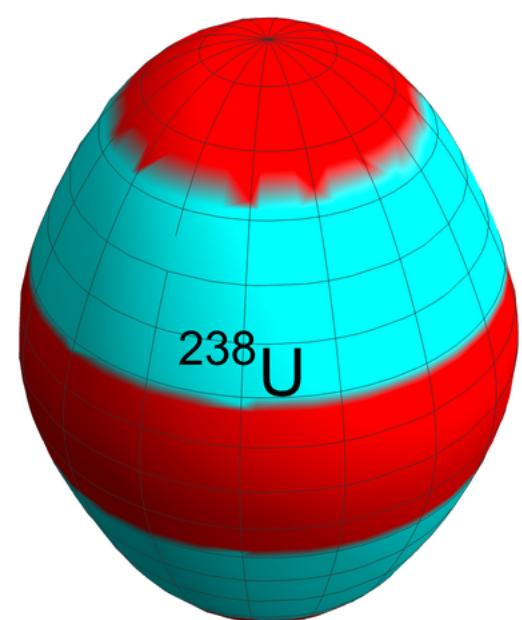
$$Y_Q = \frac{Z}{A} = \frac{n_Q}{n_B}$$

System	$Z$	$A$	$Y_Q$	Data?
O+O	8	16	0.5	some
Ne+Ne	10	20	0.5	no
Mg+Mg	12	24	0.5	no
Ca+Ca	20	40	0.5	no
Cu+Cu	29	63	0.46	yes
Ru+Ru	44	96	0.458	some
Ar+Ar	18	40	0.45	no
Xe+Xe	54	128	0.419	yes
Zr+Zr	40	96	0.417	some
Au+Au	79	198	0.399	yes
U+U	92	238	0.387	yes



$$Y_Q \lesssim 0.2$$

Heavy-Ions and Neutron Stars are all on the same phase diagram, but very different  $Y_Q$

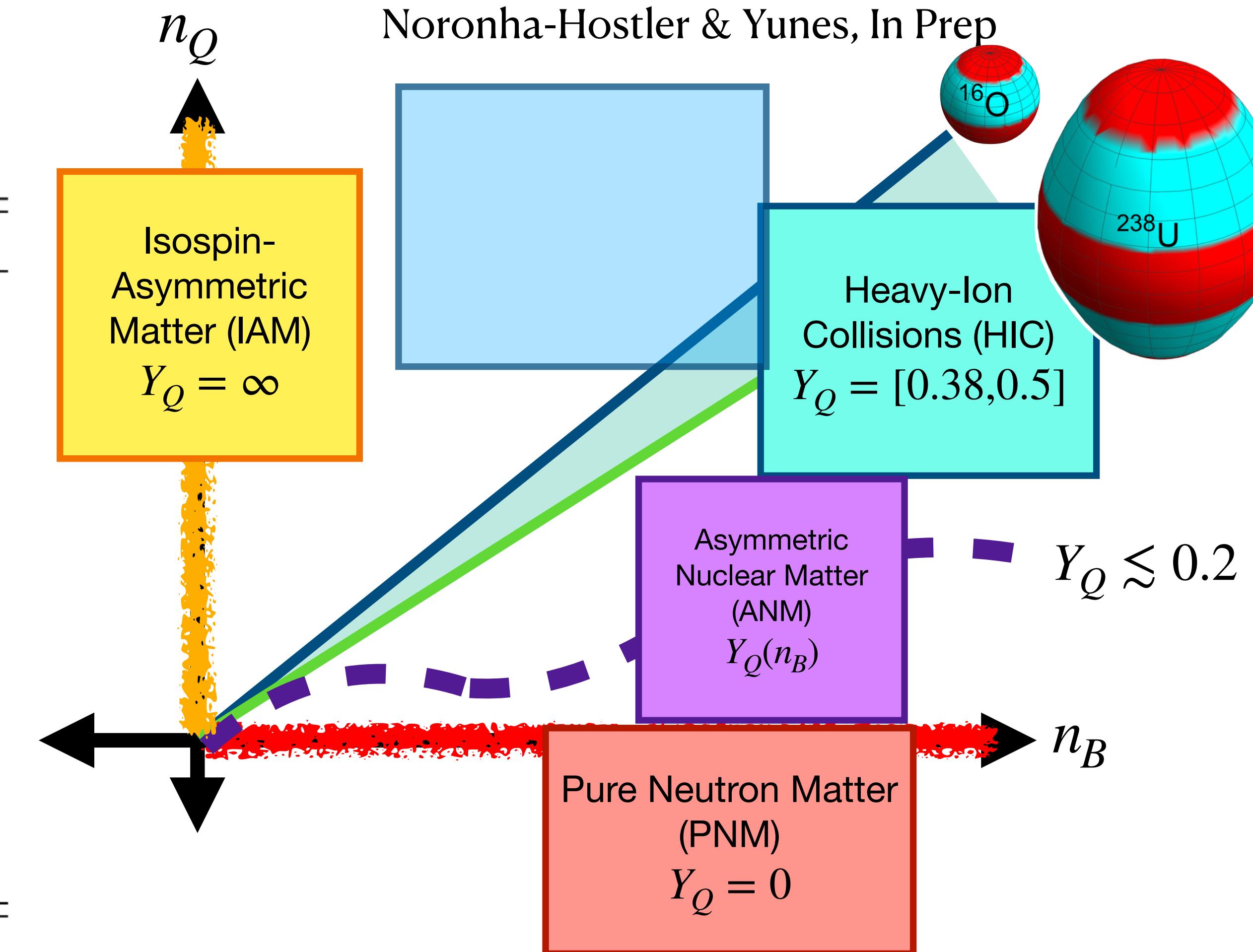


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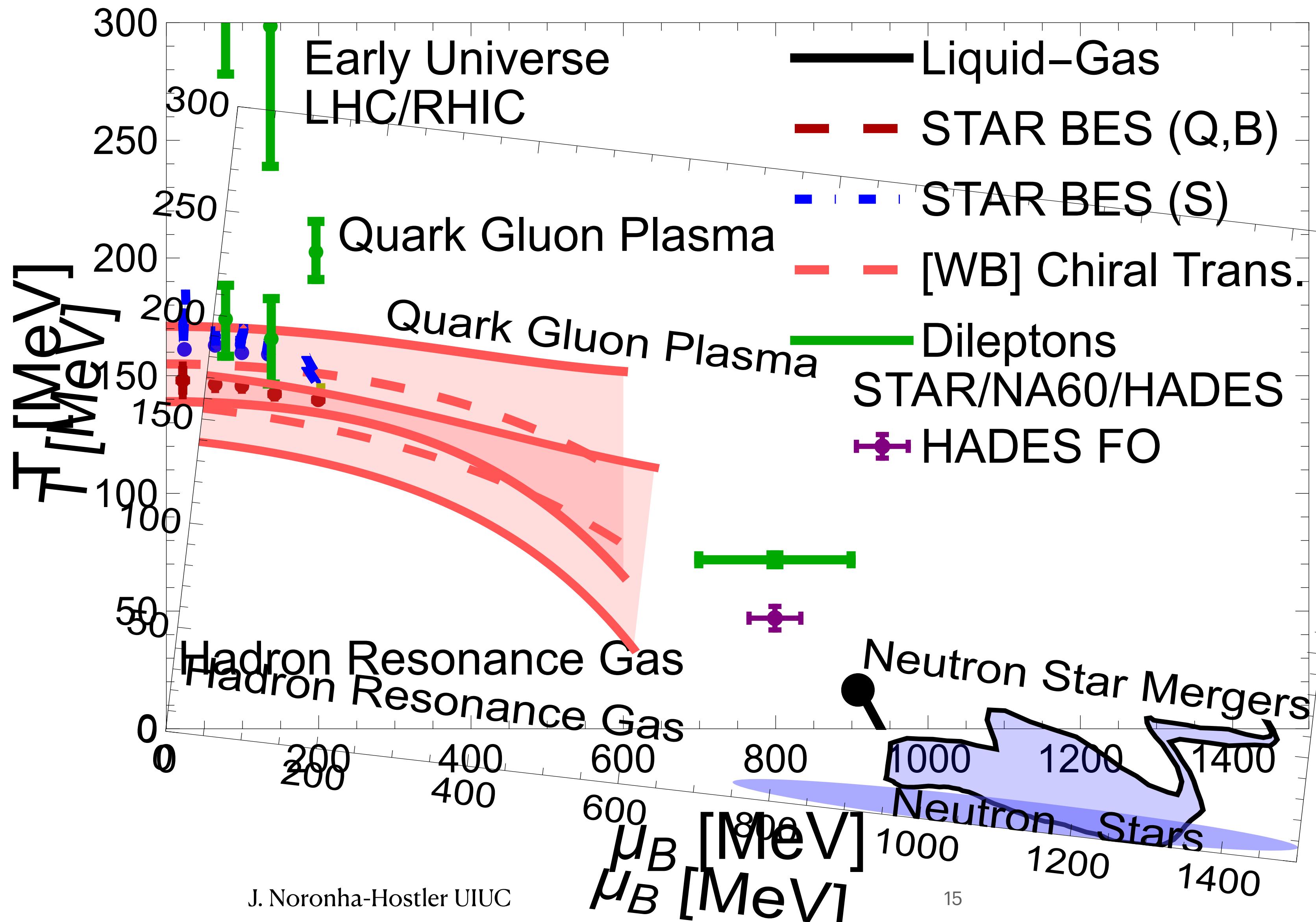
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Heavy-ions and Neutron Stars are all on the same phase diagram, but very different  $Y_Q$

# Connecting heavy-ion collisions and neutron stars

## QCD phase diagram



$$Y_Q \sim [0.01, 0.2]$$

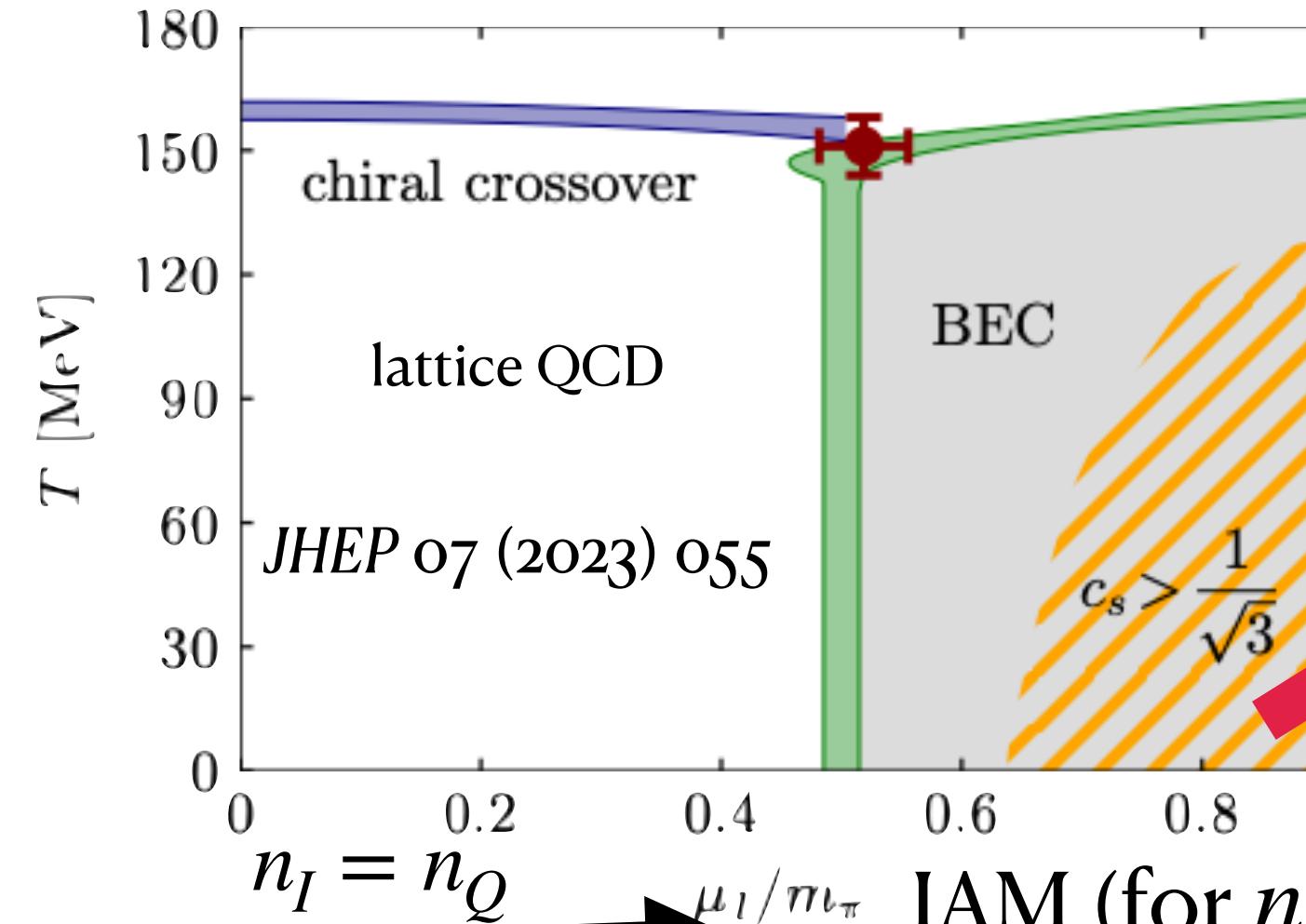
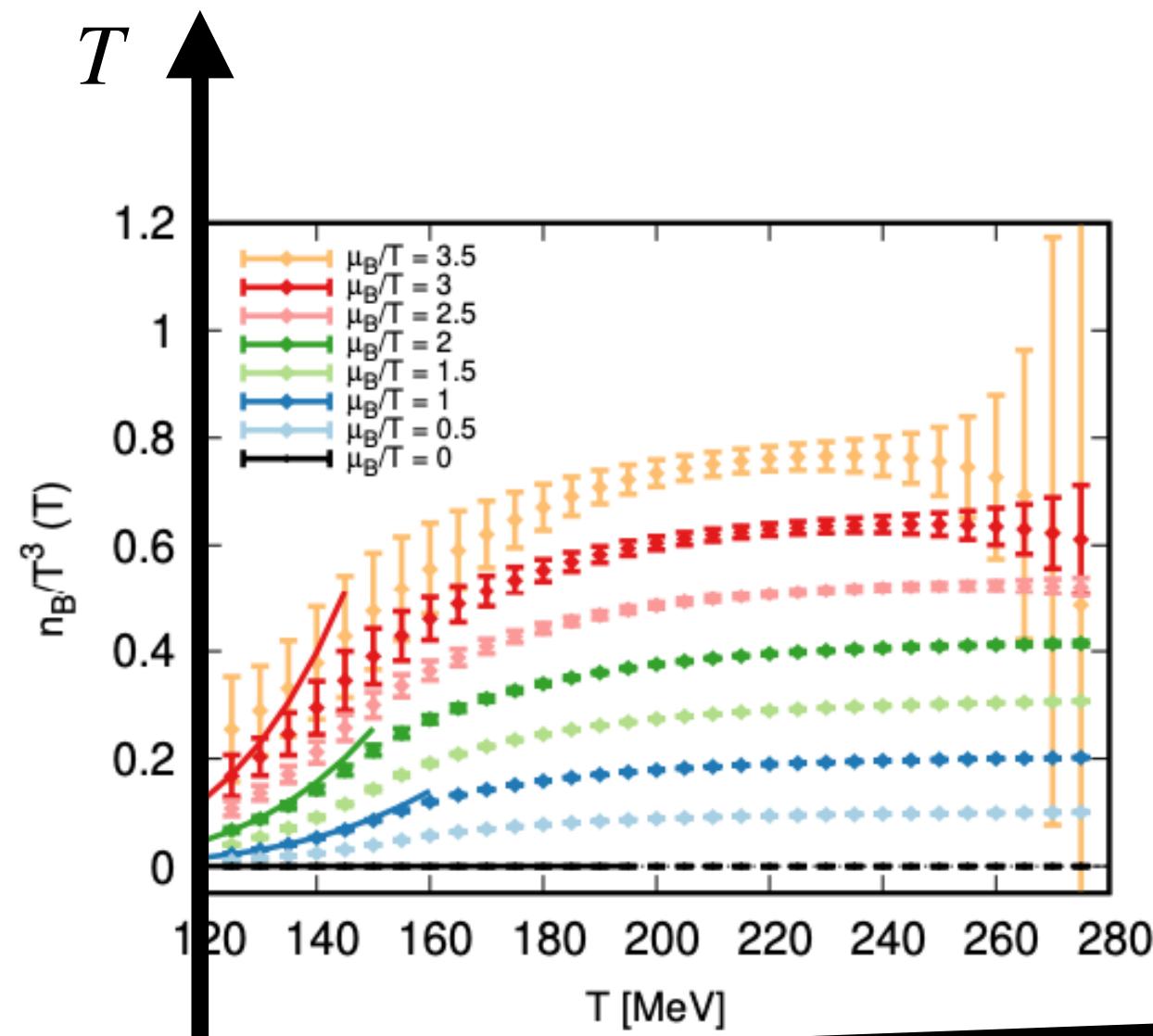
# First principle QCD constraints

## Equation of State (EOS)

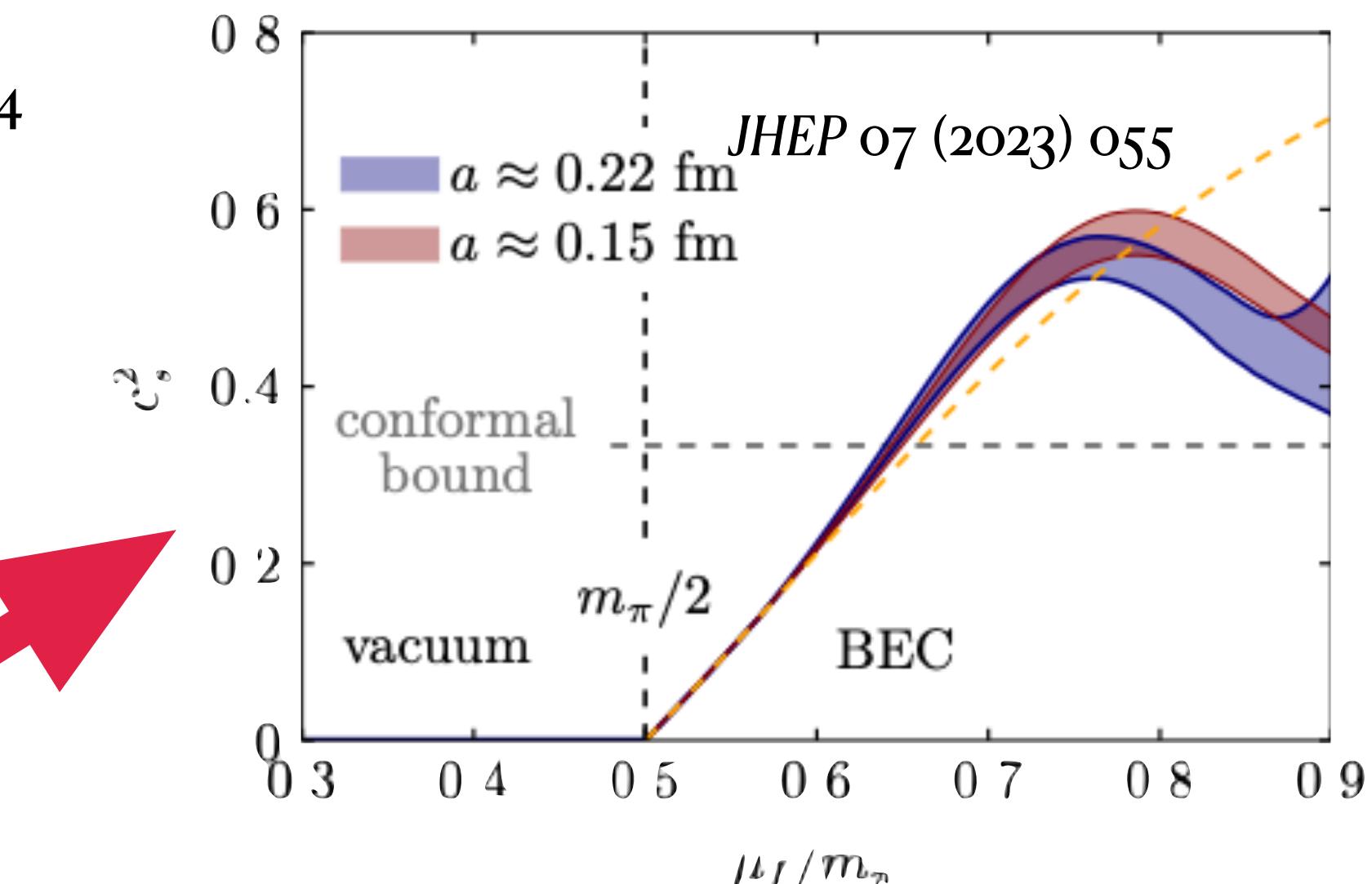
Resummed lattice QCD

Lattice *Phys.Rev.Lett.* 126 (2021) 23, 232001; *Phys.Rev.D* 105 (2022) 11, 114504

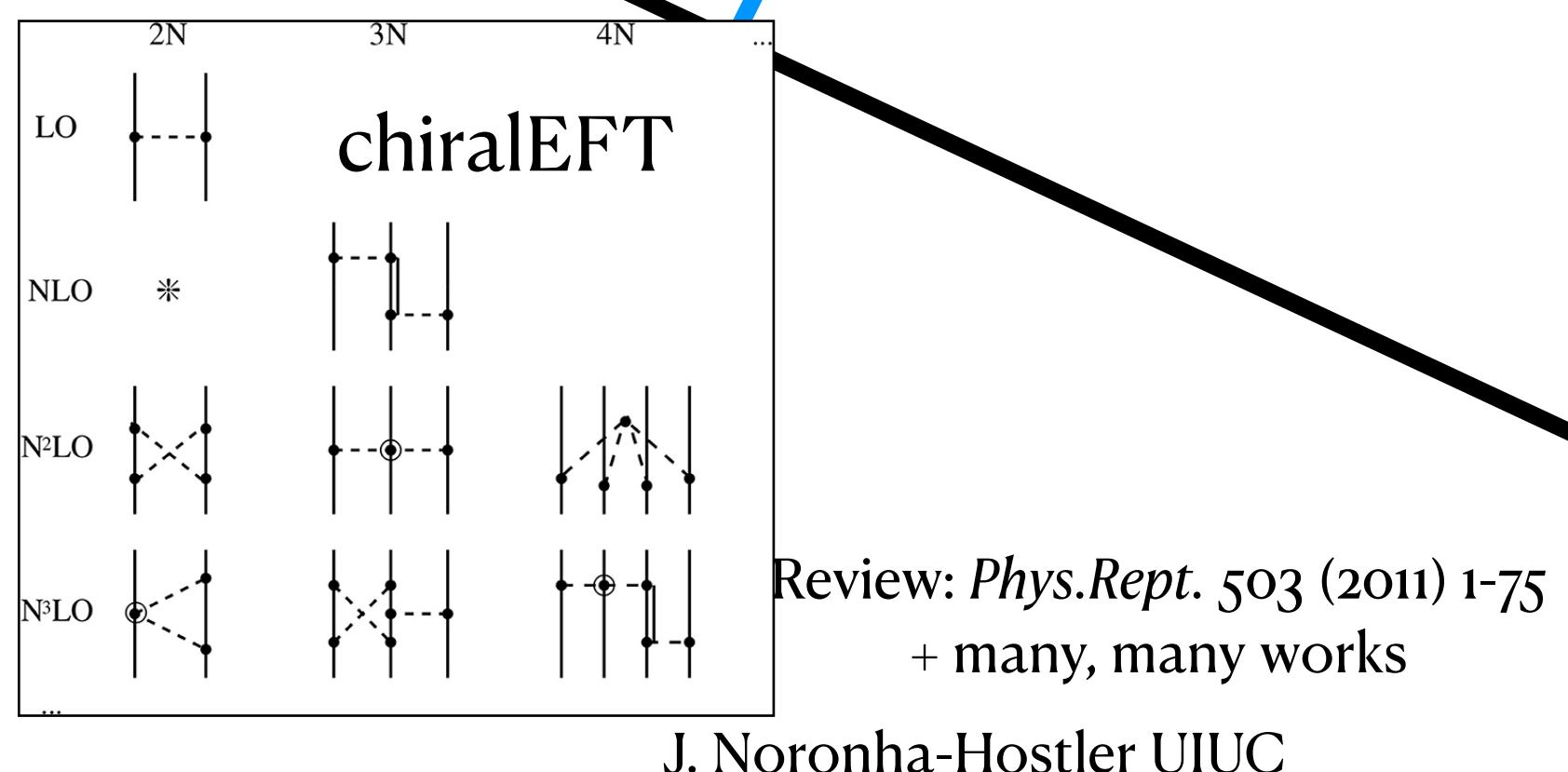
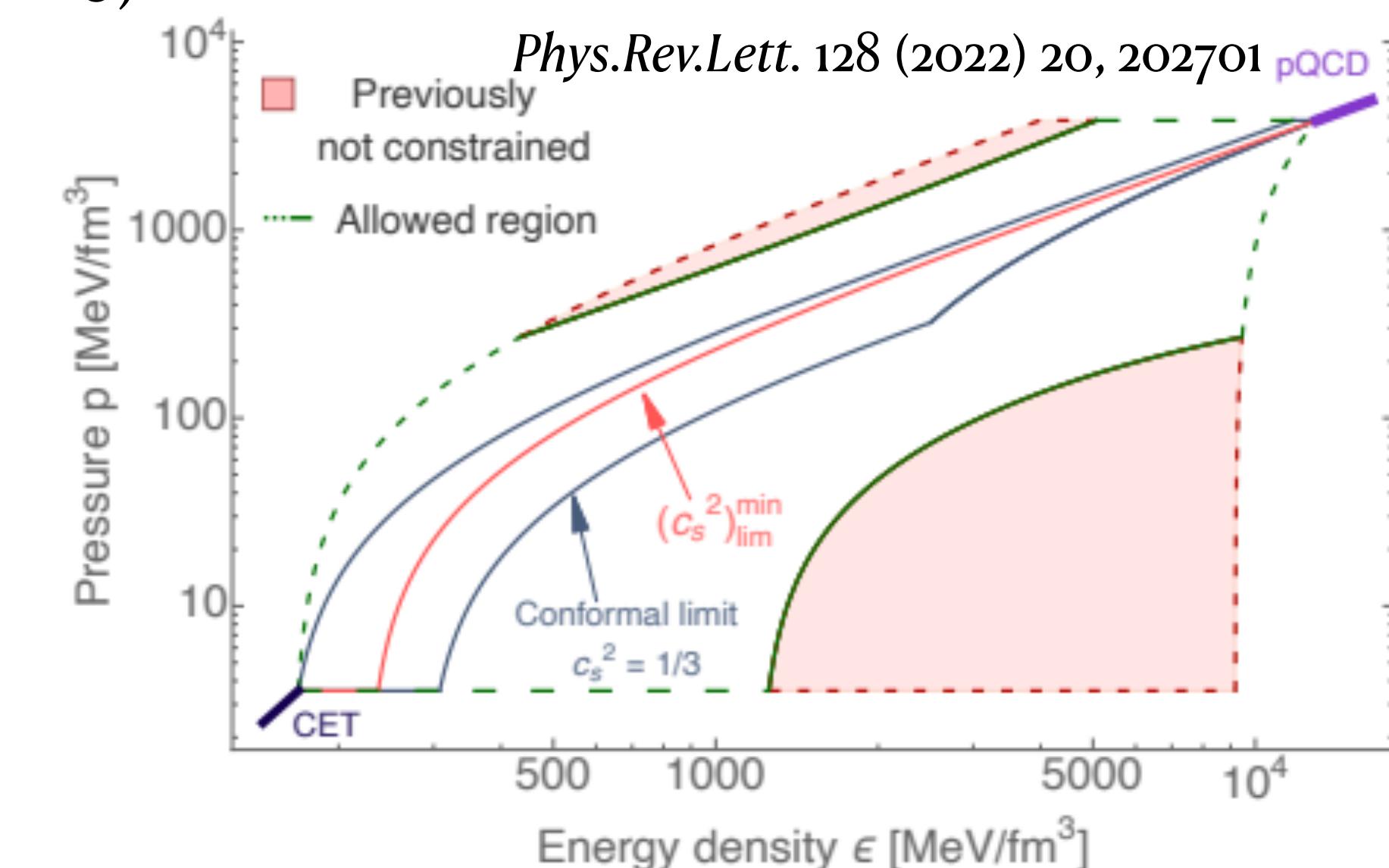
Open-source code *Phys.Rev.D* 109 (2024) 9, 094046



IAM (for  $n_S = n_B = 0$ )



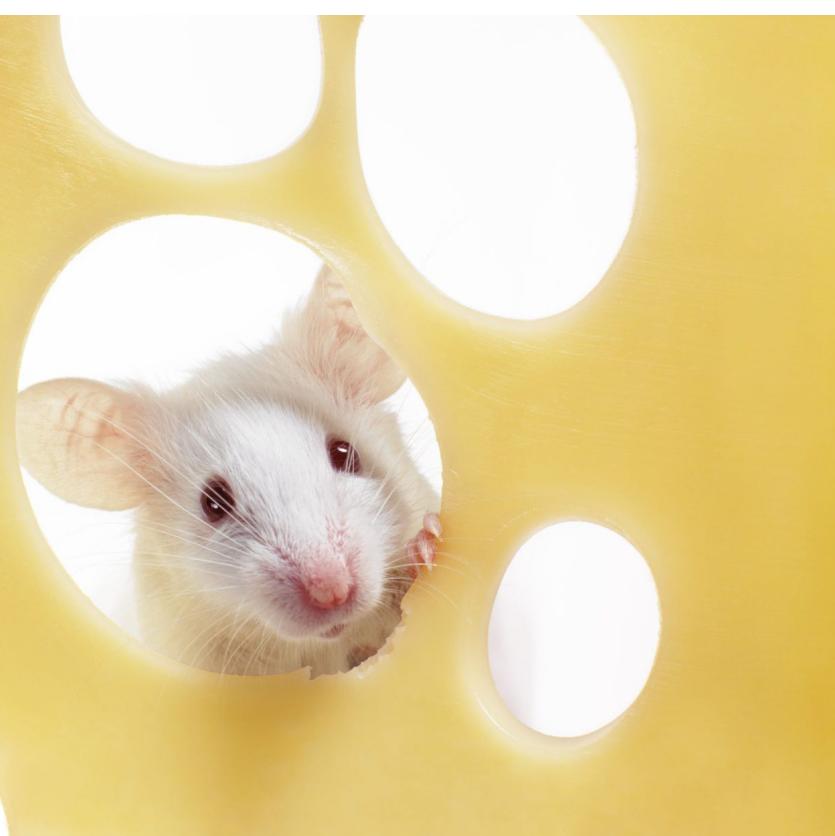
Perturbative QCD  $n_B \gtrsim 40 n_{sat}$



$n_B$   $\rightarrow$  SNM  $Y_Q = 0.5$

$n_B$   $\rightarrow$  PNM  $Y_Q = 0$

# Filling gaps in our knowledge



Expansions: symmetry energy, finite  $T$  or  $\mu$ , etc

$$\text{Lattice QCD } \mu_B = 0, \text{ then } p(T, \hat{\mu} = \vec{\mu}/T) = T^4 \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_S^j \hat{\mu}_Q^k$$

$$\text{Cold neutron stars } T = 0, \text{ then } p(T, \vec{\mu}) = p_{T=0} + \frac{1}{2} \frac{\partial s}{\partial T} \Big|_{T=0, \vec{\mu}} T^2 + \mathcal{O}(T^3)$$

[2404.01658 \[astro-ph.HE\]](#)

**Connect heavy-ions to neutron stars:**  $\frac{E_{ANM}}{N_B} = \frac{E_{SNM}}{N_B} + E_{sym} \delta^2 + \mathcal{O}(\delta^4)$

& expansion around  $n_{sat}$

Caveat: breaks down outside of regime of validity, struggles with phase transitions

Effective Models: relativistic mean field, NJL, quarkyonic, etc

Qualitative features: bump in  $c_s^2$ , phase transitions, ...

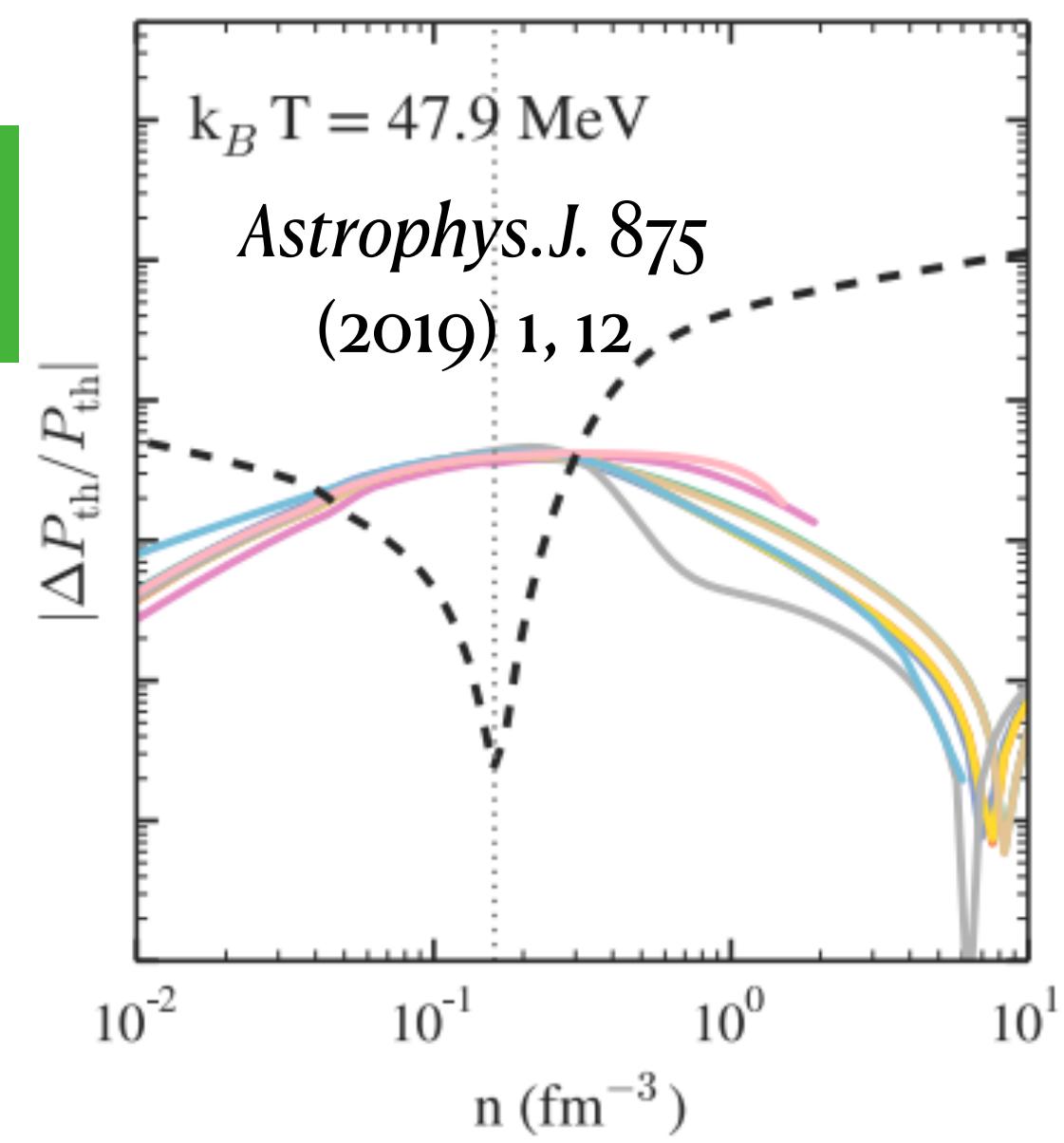
Constrain theory vs data; vary free parameters

Caveat: model assumptions and degrees of freedom

Minimalist models based on nuclear parameters

Effective Mass approach for neutron, proton, electron matter

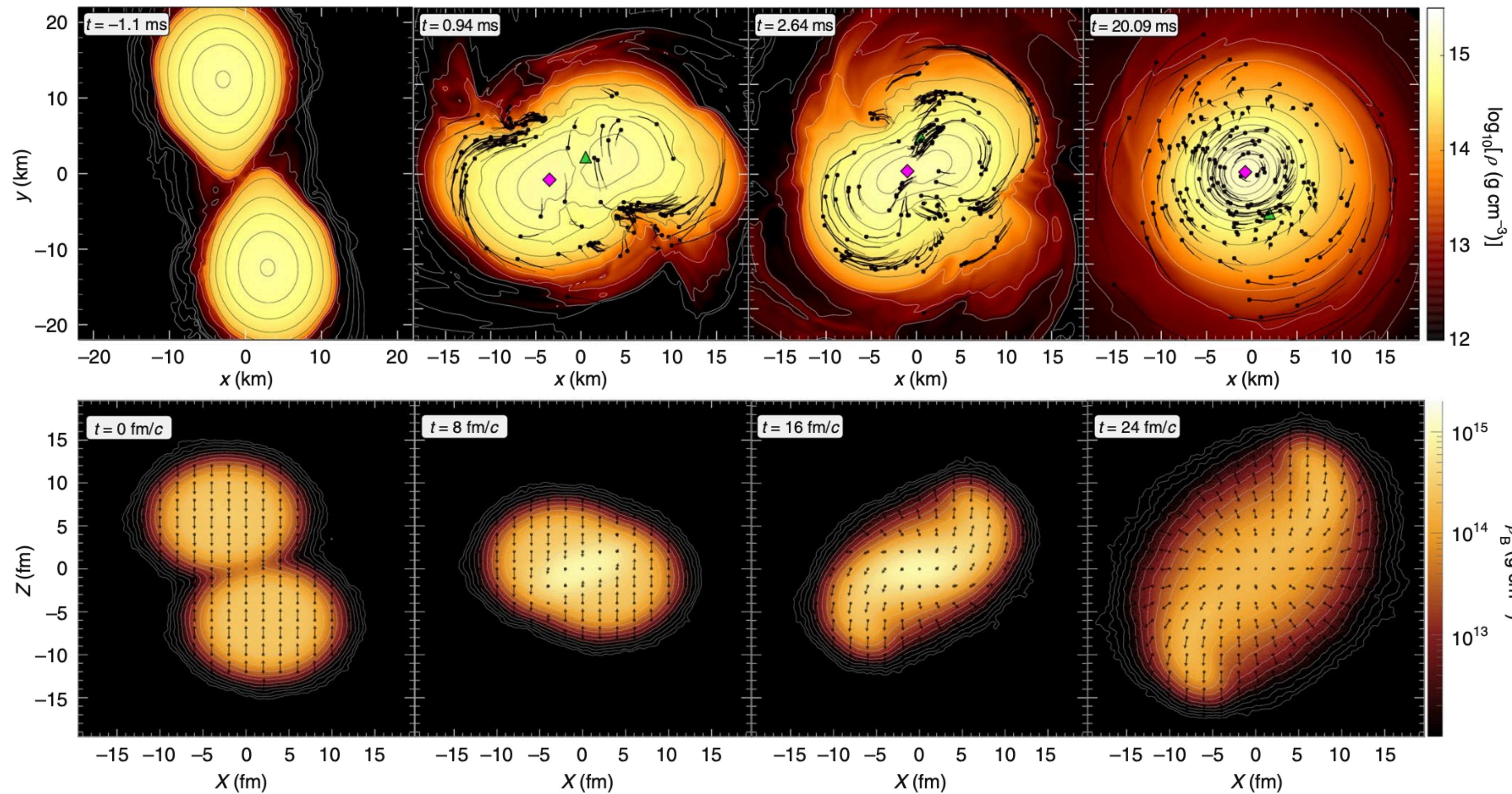
Caveat: breaks down outside of regime of validity, no phase transitions, hyperons etc



# What new connections can be made to understand the core of a neutron star?

[HADES] Nature Phys. 15 (2019) 10, 1040-1045

Neutron Star Mergers (Numerical Relativity)

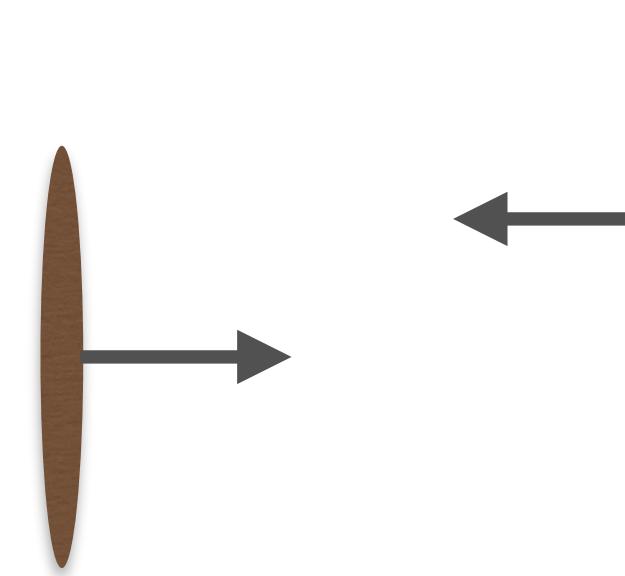


Low-energy Heavy-Ion Collisions (hadron transport)

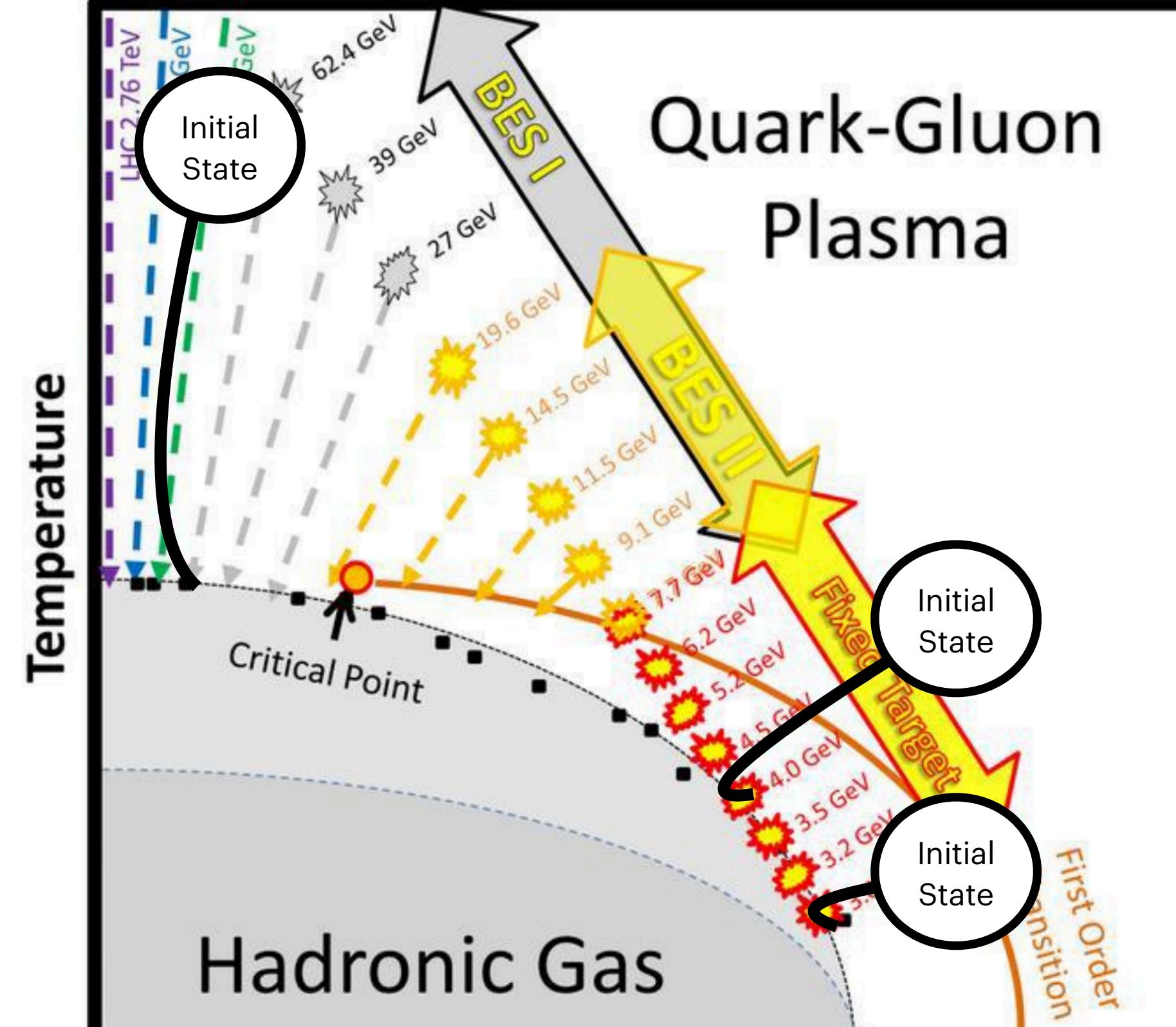
# Differences as you lower $\sqrt{s}$ in HIC

Quark-gluon degrees of freedom may matter significantly less

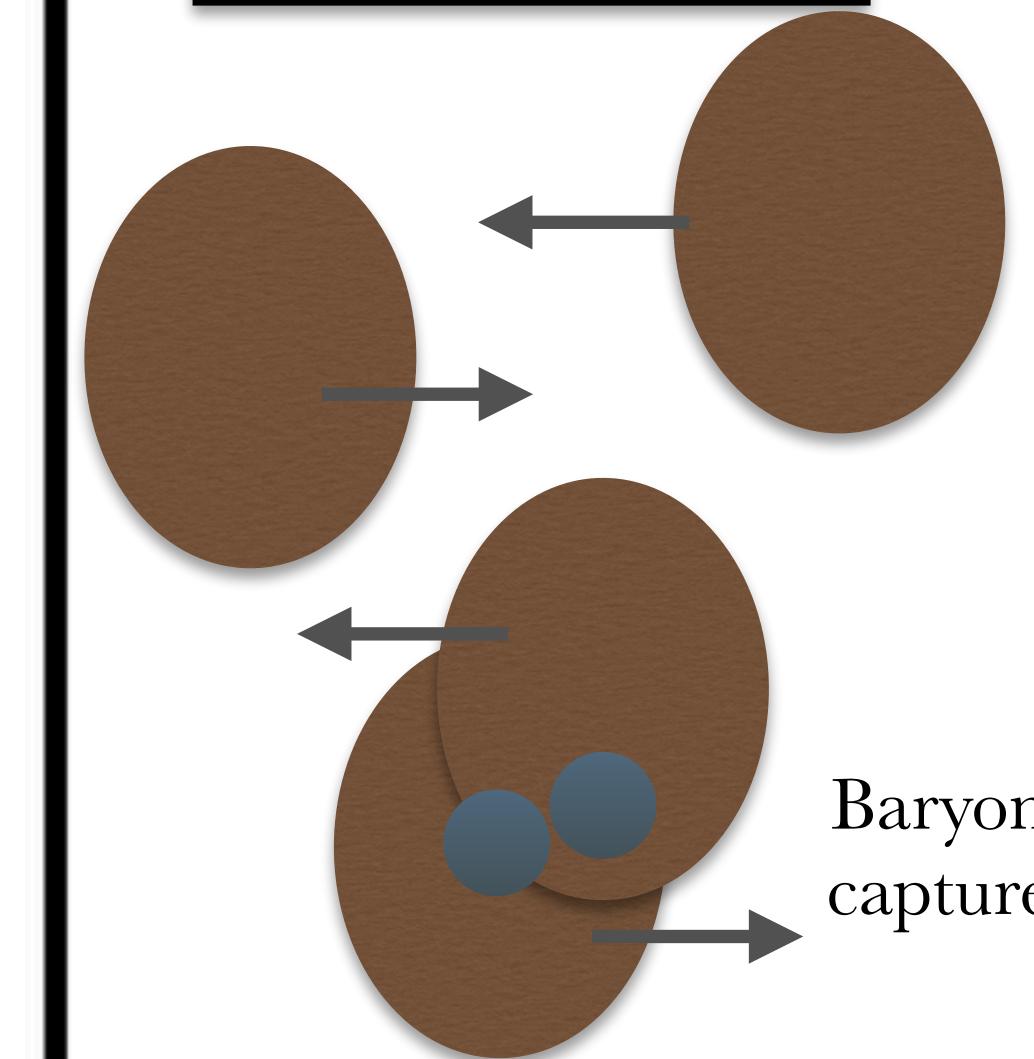
Large  $\sqrt{s_{NN}}$



- Lorentz contracted (2D)
- Nuclei pass through instantaneously
- Too quick to capture baryons



Small  $\sqrt{s_{NN}}$



- 3D nuclei pass slowly
- Time to capture baryons

Different  $\sqrt{s}$  have different time spent in quark/gluon phase vs hadron phase

# Low-energy heavy-ion collisions

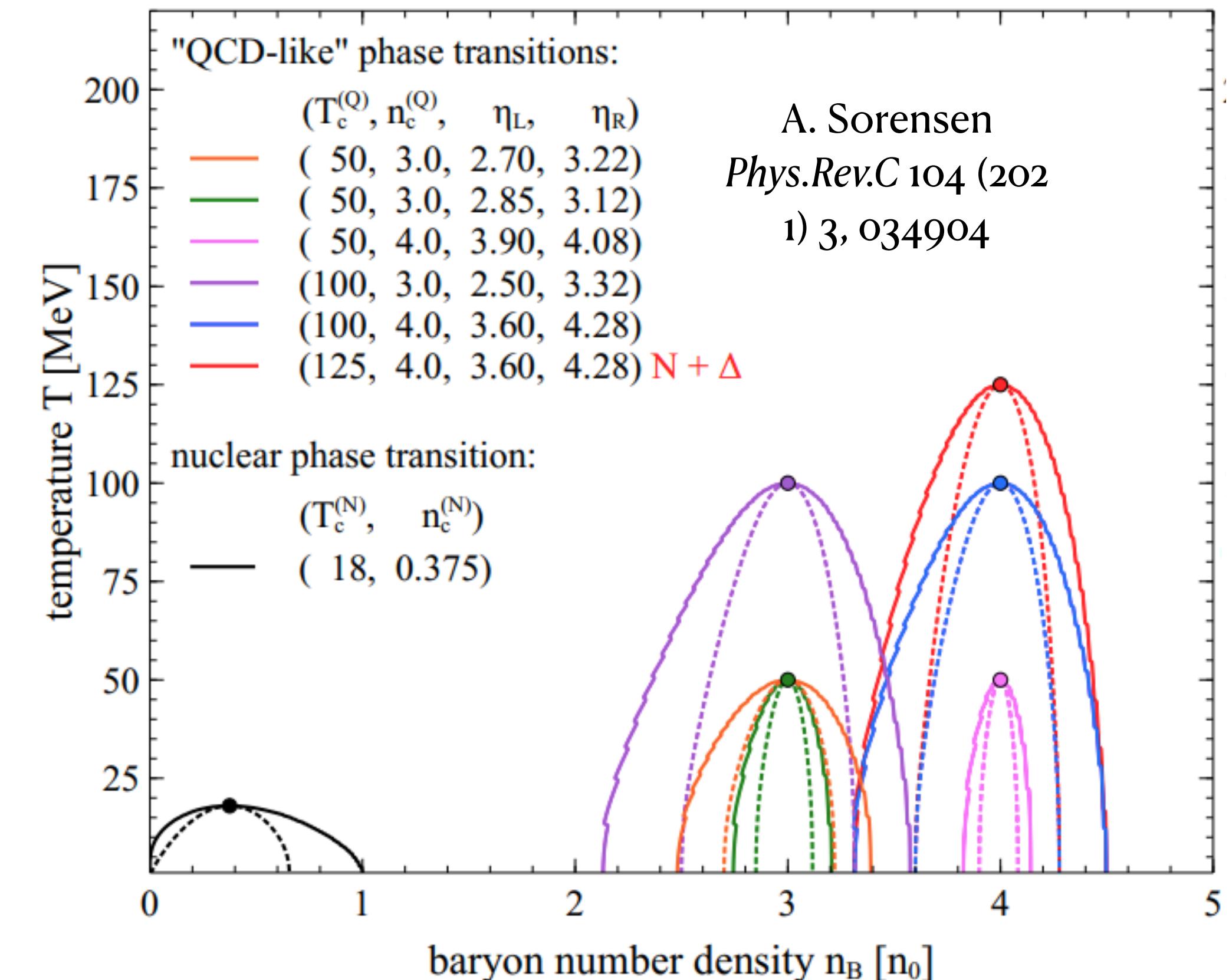
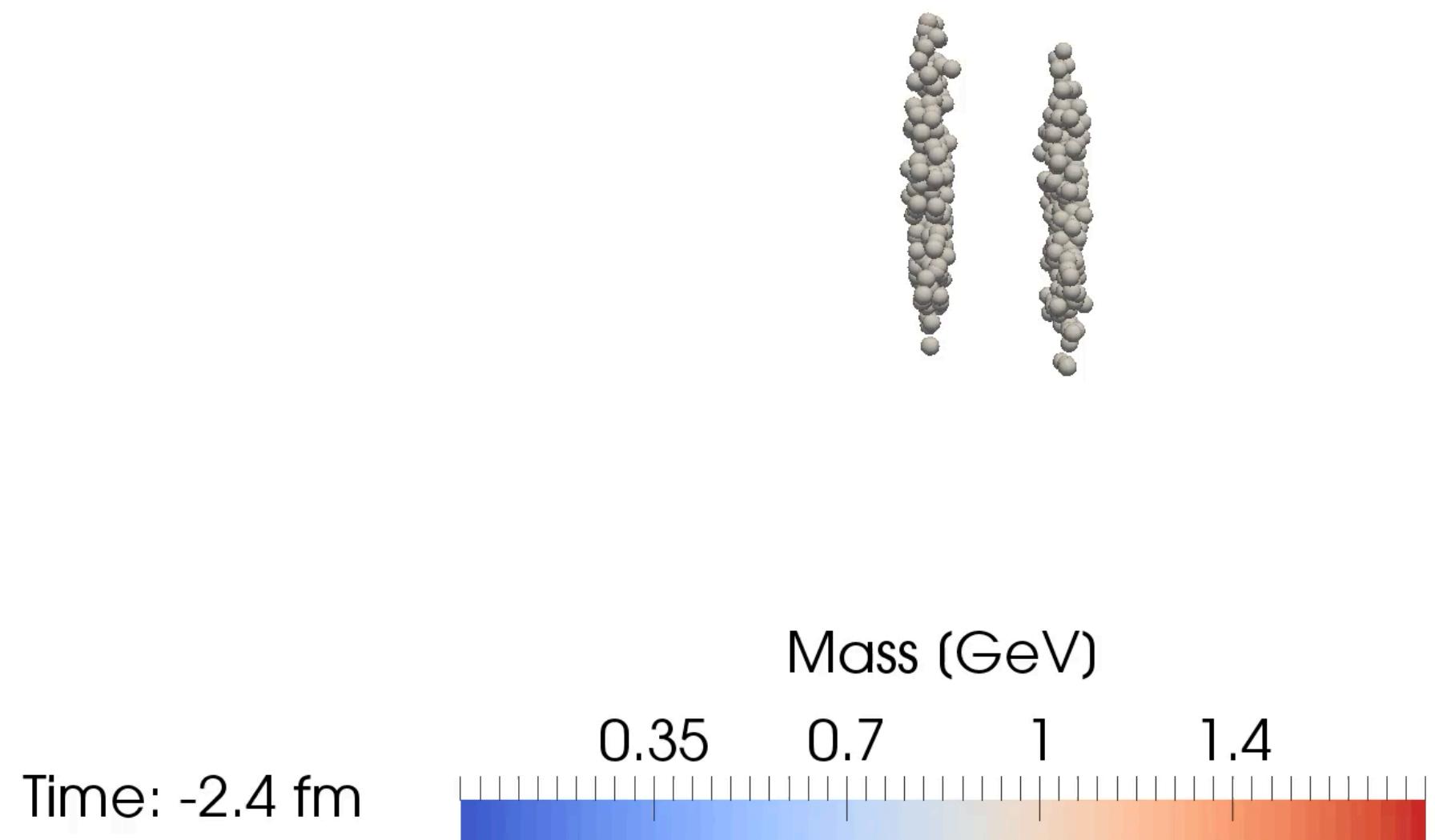
## How do we interpret Heavy-ion (HIC) data?

$\sqrt{s} \leq 7.7 \text{ GeV}$

### Fixed-target heavy-ion collisions

Open question: do quark-gluons d.o.f. matter?

One solution: build phase transitions into hadron transport



Bayesian analysis with heavy-ion flow data found a peak in  $c_s^2$  at  $n_B^{peak} = [2,3] n_{sat}$  for SNM  
Phys.Rev.C 108 (2023) 3, 034908

Systematic Hydro studies still needed

# Low-energy heavy-ion collisions

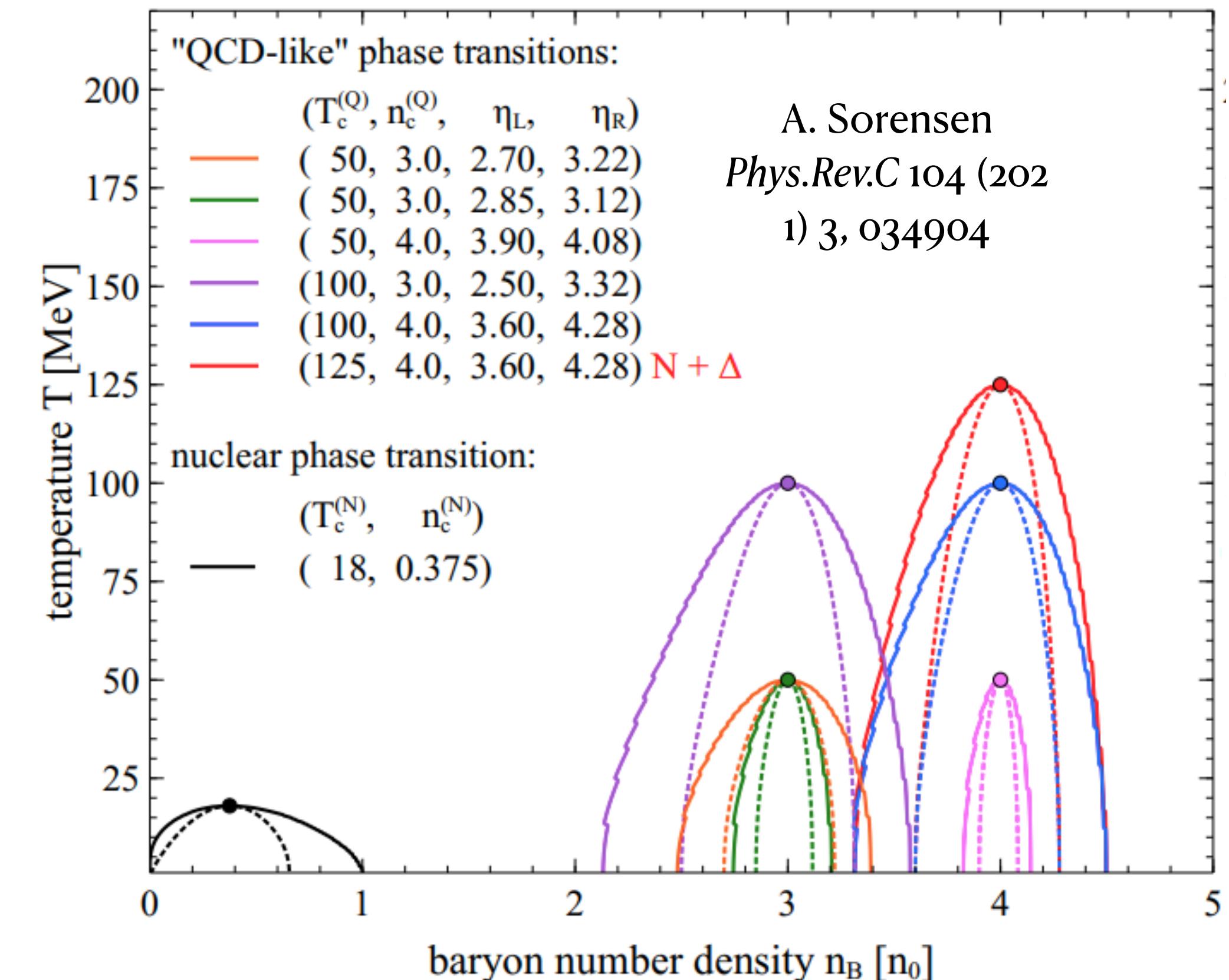
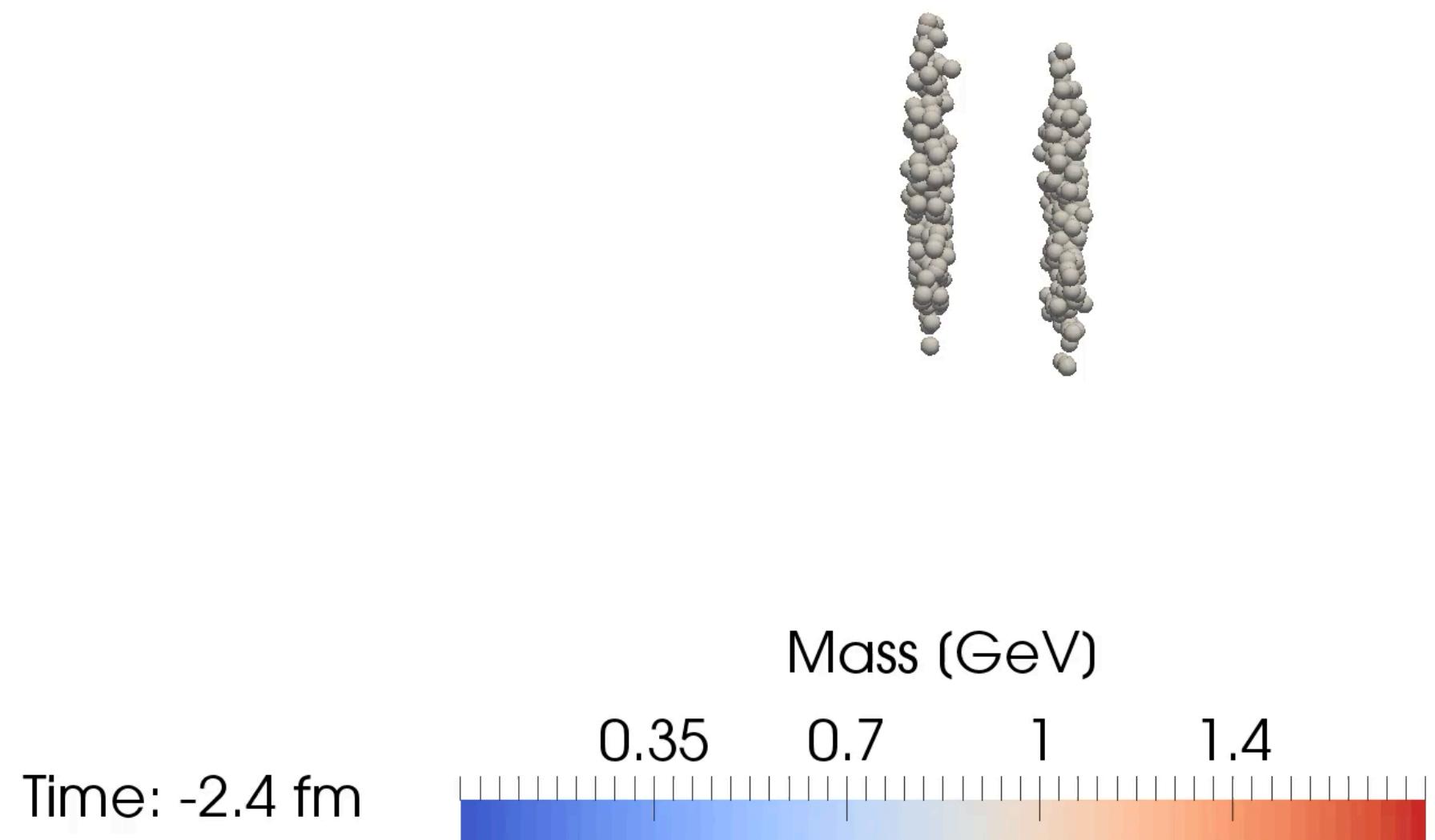
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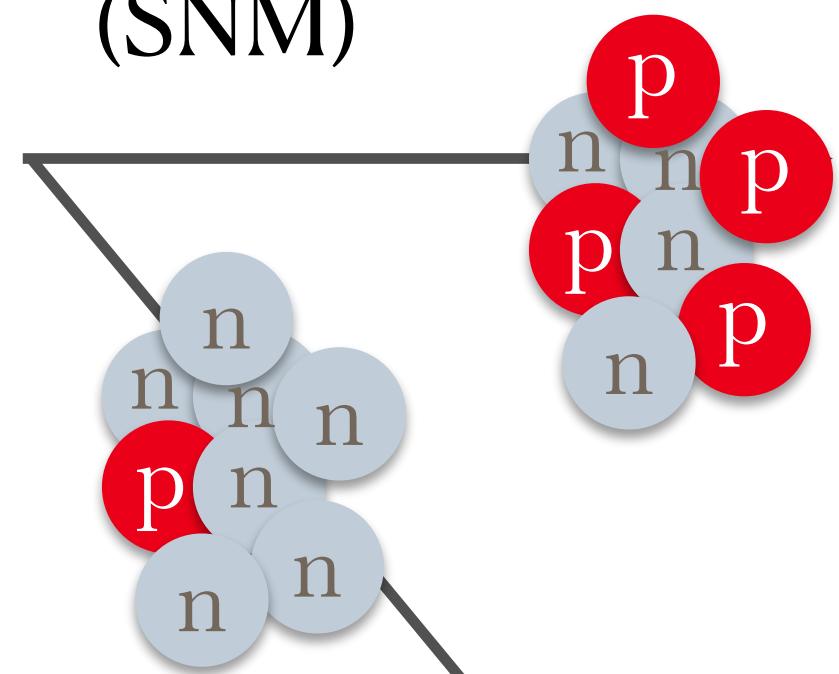
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Phys.Rev.C 108 (2023) 3, 034908

Systematic Hydro studies still needed

# Isospin asymmetry/Symmetry Energy Expansion

Symmetric matter

(SNM)



$$Y_Q = 0.5$$

$$\text{Isospin asymmetry } \delta = 1 - 2Y_Q$$

where  $\delta = 0$  for SNM and  $\delta = 1$  for PNM

Original symmetry energy expansion from binding energies  
Bombaci & Lombardo *Phys.Rev.C* 44 (1991) 1892-1900

Asymmetric matter (ANM)

$$Y_Q \lesssim 0.1$$

$$\frac{E_{ANM}}{N_B} = \frac{E_{SNM}}{N_B} + E_{sym,2} \delta^2 + \mathcal{O}(\delta^4)$$

Expand in  $\delta$  where odd terms drop due to isospin symmetry

Convert EOS from NS to HIC, expand around  $n_{sat}$

$$\epsilon_{HIC} = \epsilon_{NS} - 4n_B \left[ E_{sym,sat} + \frac{L}{3} \left( \frac{n_B}{n_{sat}} - 1 \right) + \frac{K}{18} \left( \frac{n_B}{n_{sat}} - 1 \right)^2 + \frac{J}{162} \left( \frac{n_B}{n_{sat}} - 1 \right)^3 \right] \left[ \left( Y_Q^{HIC} - Y_{Q,NS} \right) + \left( Y_{Q,NS}^2 - \left( Y_Q^{HIC} \right)^2 \right) \right]$$



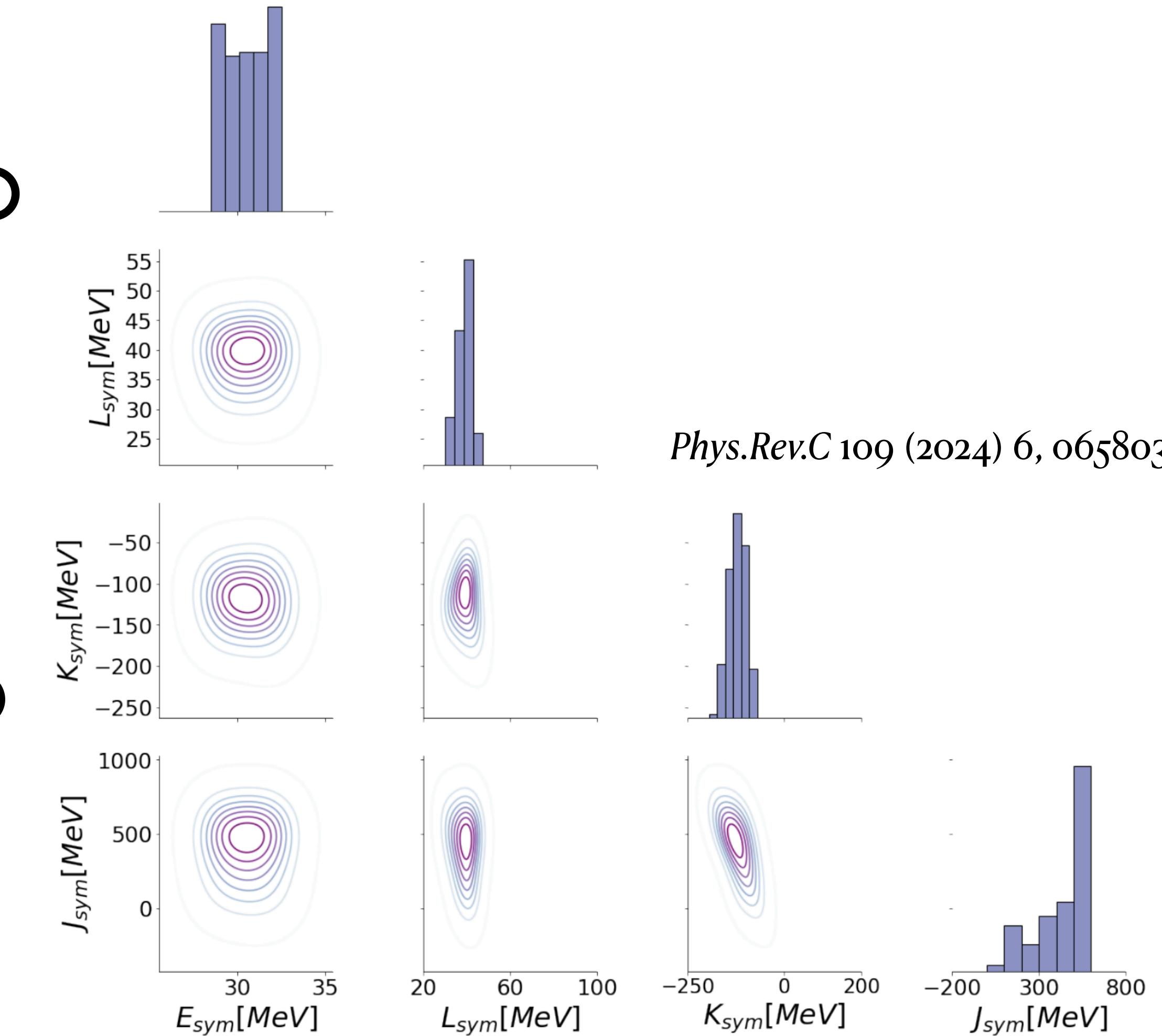
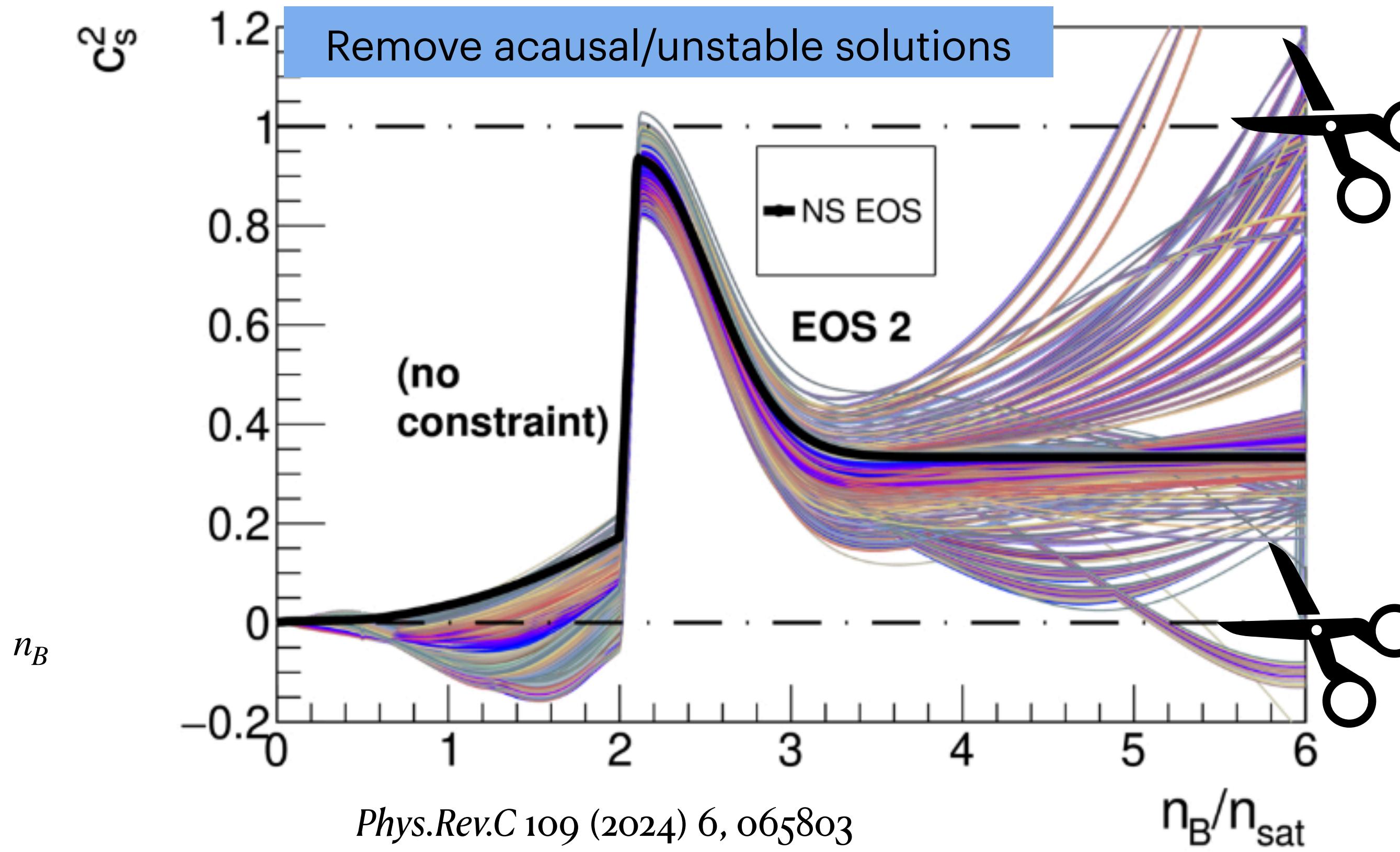
$E_{sym,2}(n_B)$     4 unknowns

To varying  $Y_Q^{HIC}$   
Yao et al, *Phys.Rev.C* 109 (2024) 6, 065803

# How do low-energy heavy-ion collisions at $T = 0$ connect to neutron stars?

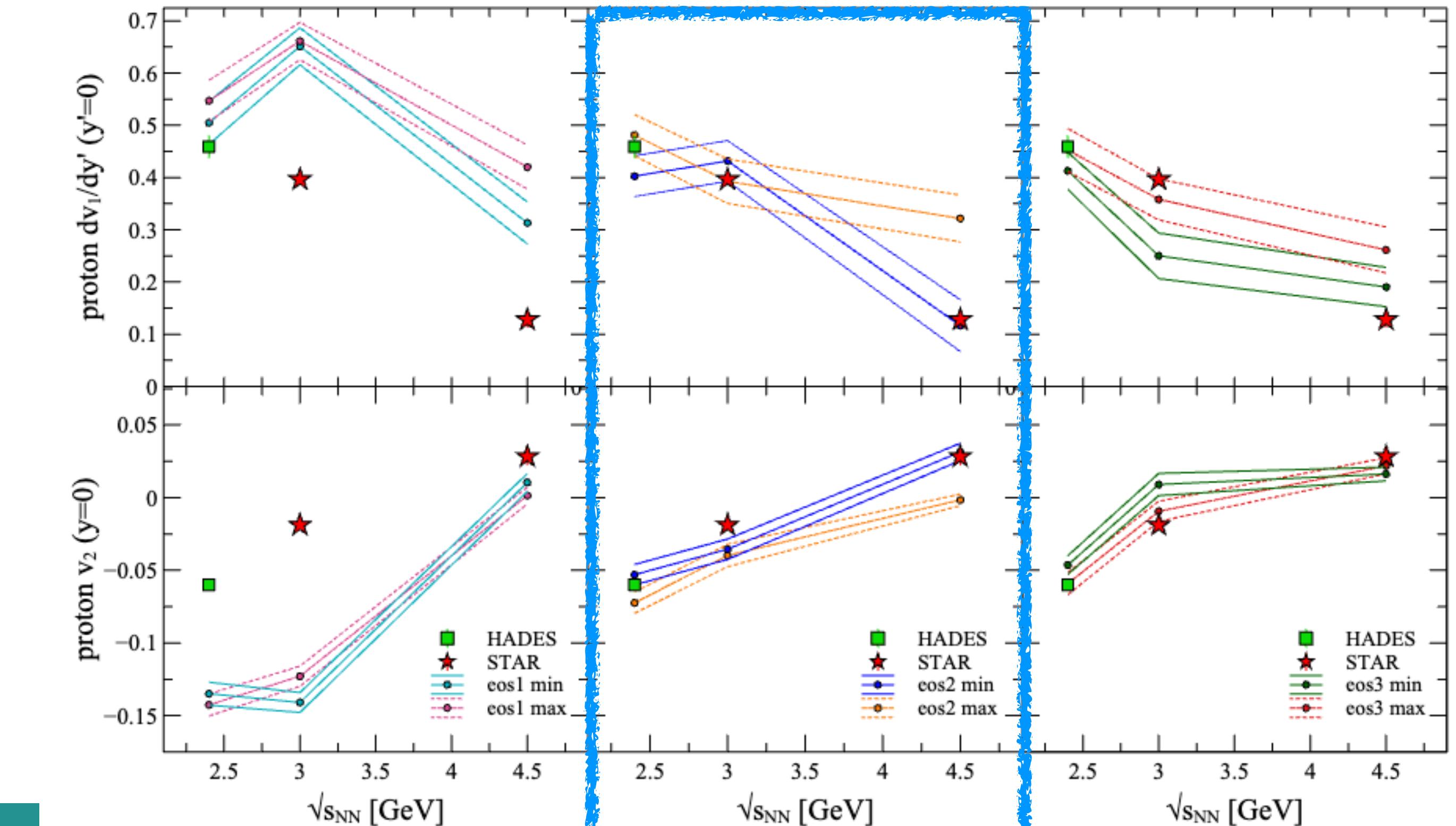
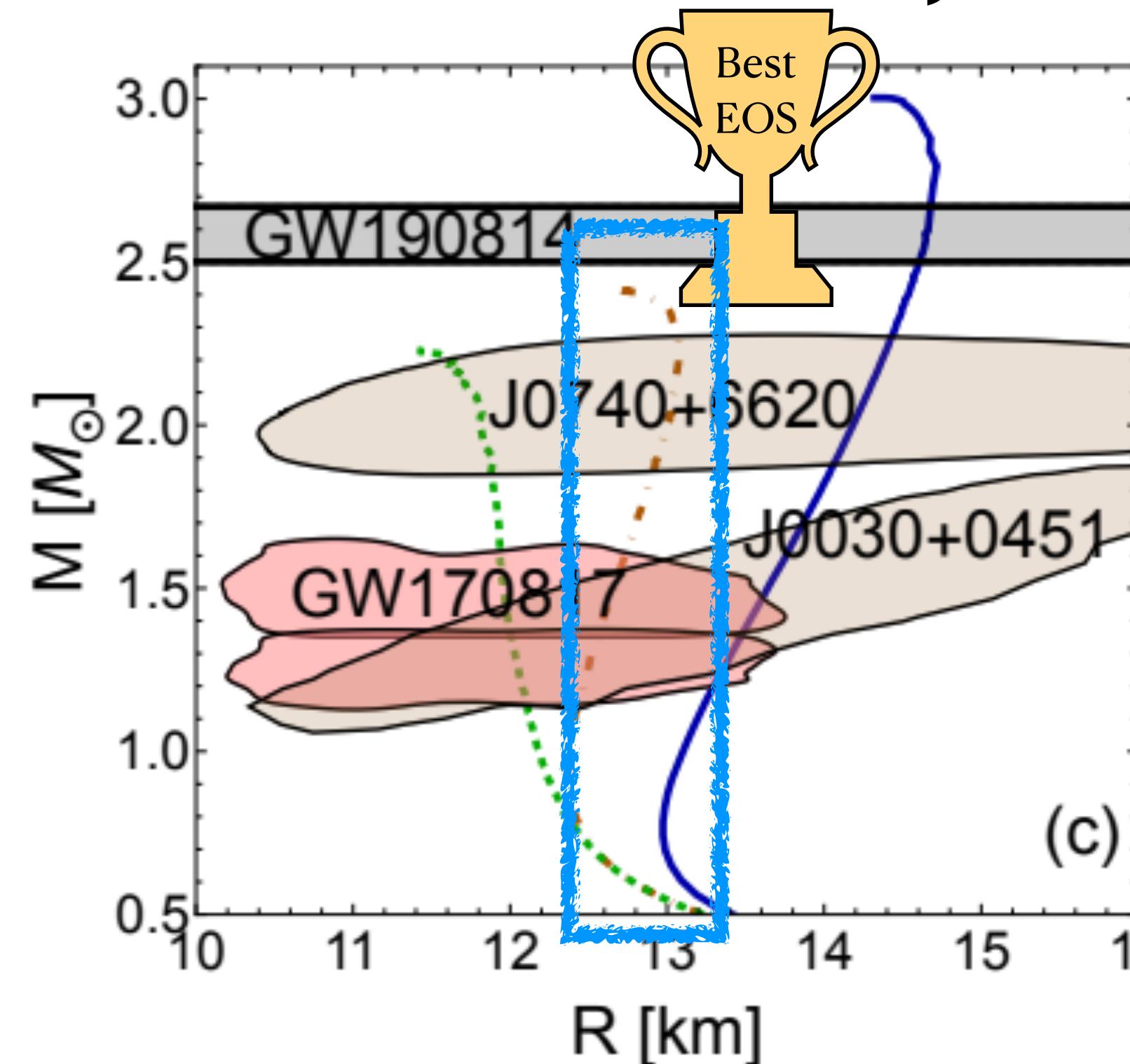
## Symmetry energy expansion

Given neutron star equation of state  $\rightarrow$  convert to HIC and can constrain by  $0 \leq c_s^2 \leq 1$  and saturation properties.

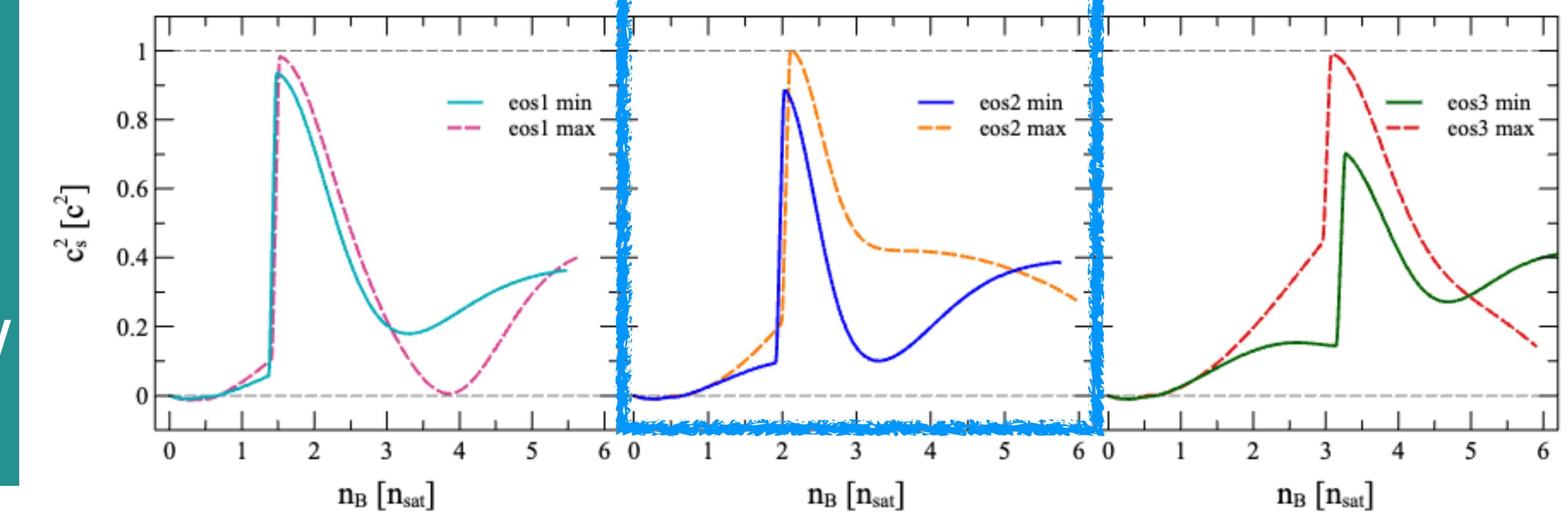


# Ultra heavy neutron star EOS works HIC

Phys.Rev.C 109 (2024) 6, 065803



Take extremes of the EOS band derived for heavy-ion collisions, run in hadron transport, and compare to heavy-ion collision flow data



# Many more possible connections with the QCD phase diagram

NS $\rightarrow$ HIC EOS constraints

HIC vs neutron star merger simulations

*Phys.Rev.D* 107 (2023) 4, 043034

Effective models + merger simulations

*Phys.Rev.Lett.* 122 (2019) 6, 061101

chiralEFT informed effective models

*Phys.Rev.C* 106 (2022) 5, 055804

Holographic predictions for the QCD critical point  
[2309.00579 \[nucl-th\]](#)

chiralEFT and pQCD constraints

*Phys.Rev.Lett.* 128 (2022) 20, 202701; *Phys.Rev.D* 109 (2024) 9, 094030;  
*Phys.Rev.C* 107 (2023) 5, L052801

Gaussian Process EOS in HIC

*Gong et al, 2410.22160 [nucl-th]*

Neutron skin and the neutron star EOS

*Phys.Rev.Lett.* 126 (2021) 17, 172503

Sign problem on quantum computers

*Phys.Rev.D* 97 (2018) 9, 094510; *JHEP* 08 (2022) 209

# Open-source tools for more cross-disciplinary connections!

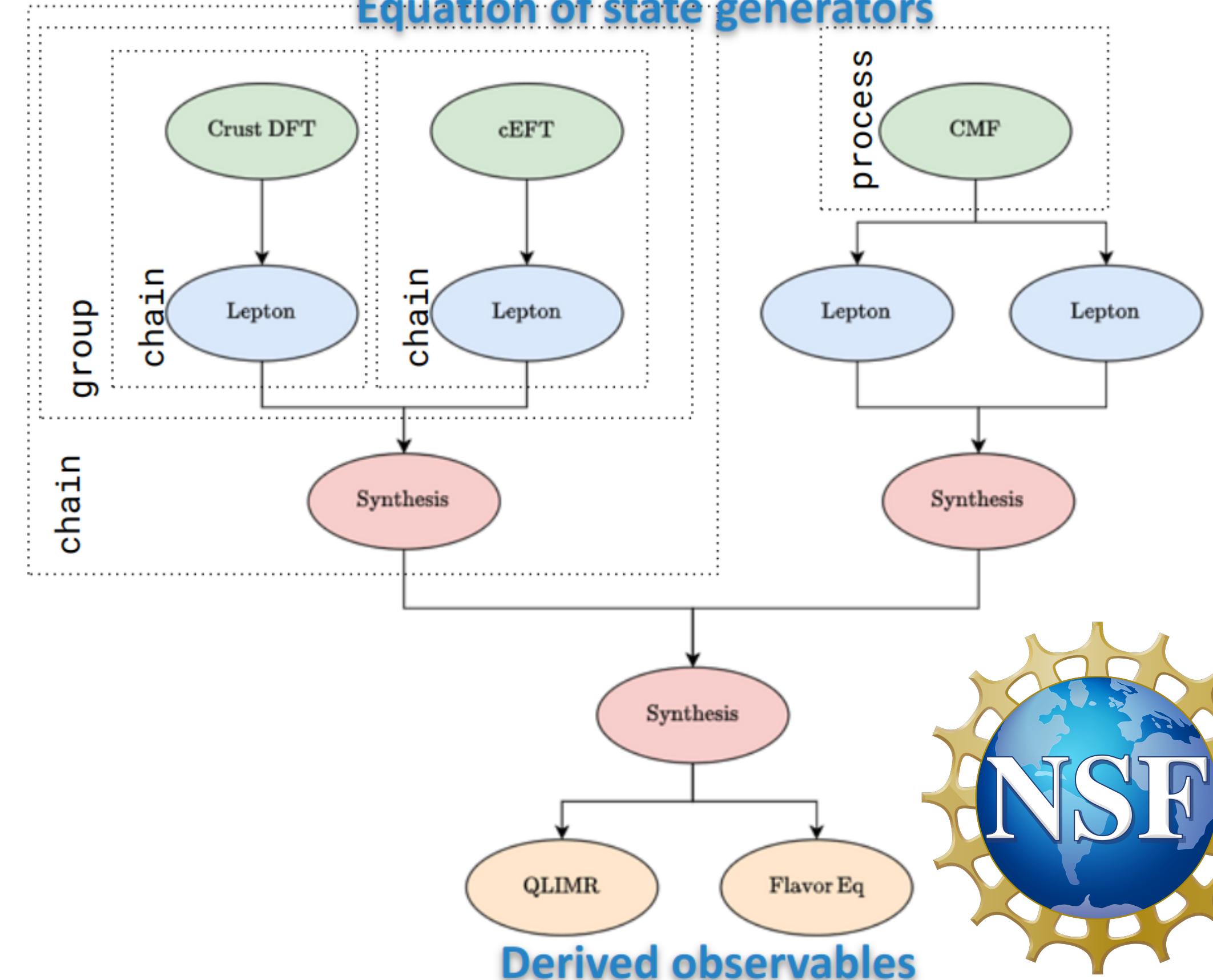


<https://musesframework.io/>

- 7 available equation of state (EOS) modules
- 3 available observable modules
- Both heavy-ion and neutron star EOS available
- $\alpha$ -release is out, being tested
- $\beta$ -release soon. Possible to run crust to core of a neutron star+calculate mass, radius, tidal deformability etc
- Looking for new collaborators!

Software across the QCD phase diagram

Equation of state generators



Later releases will connect heavy-ion and neutron star EOS across the entire phase diagram!

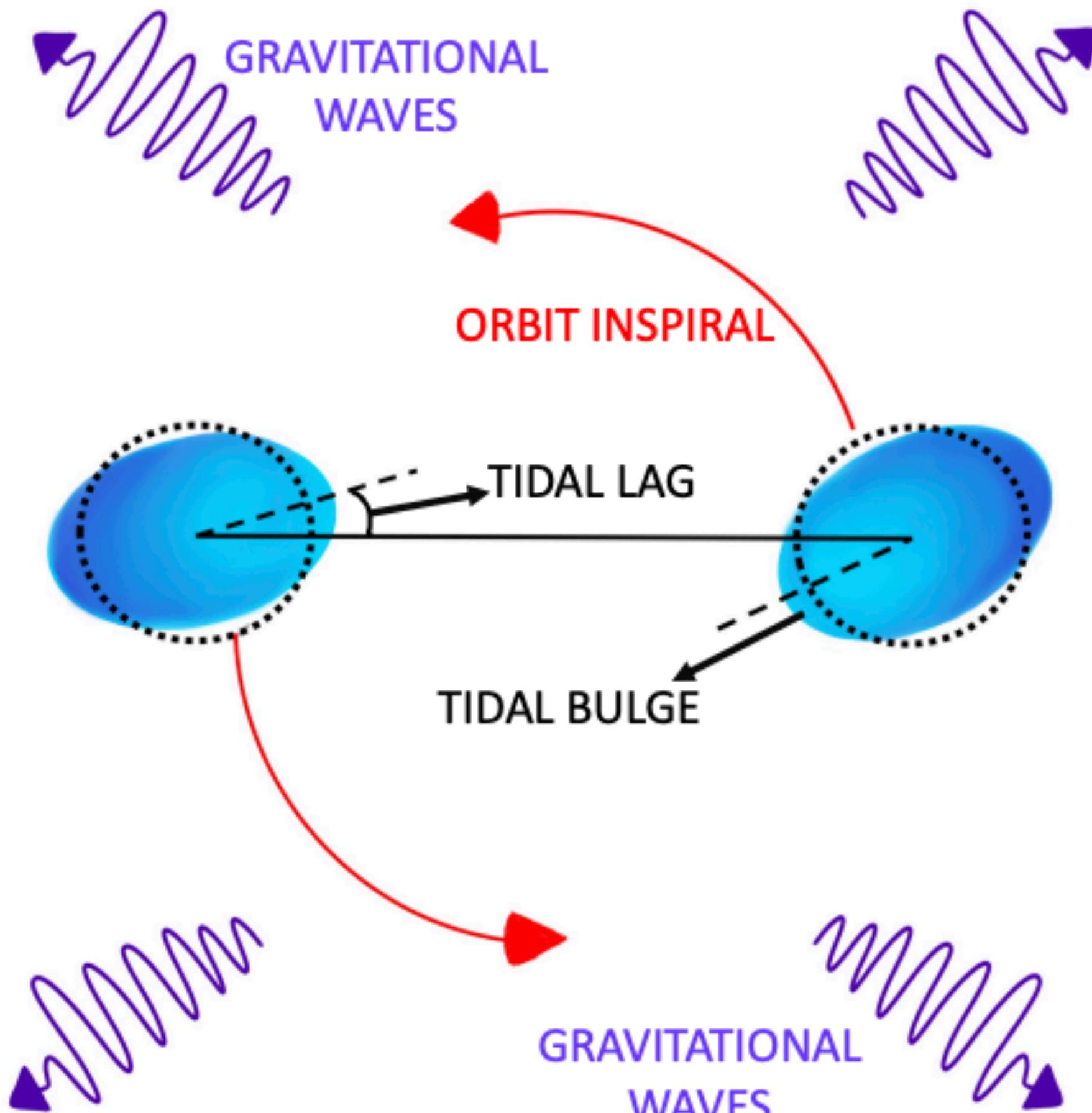
# Viscous effects from neutron star mergers

**Learning from heavy-ion collisions**



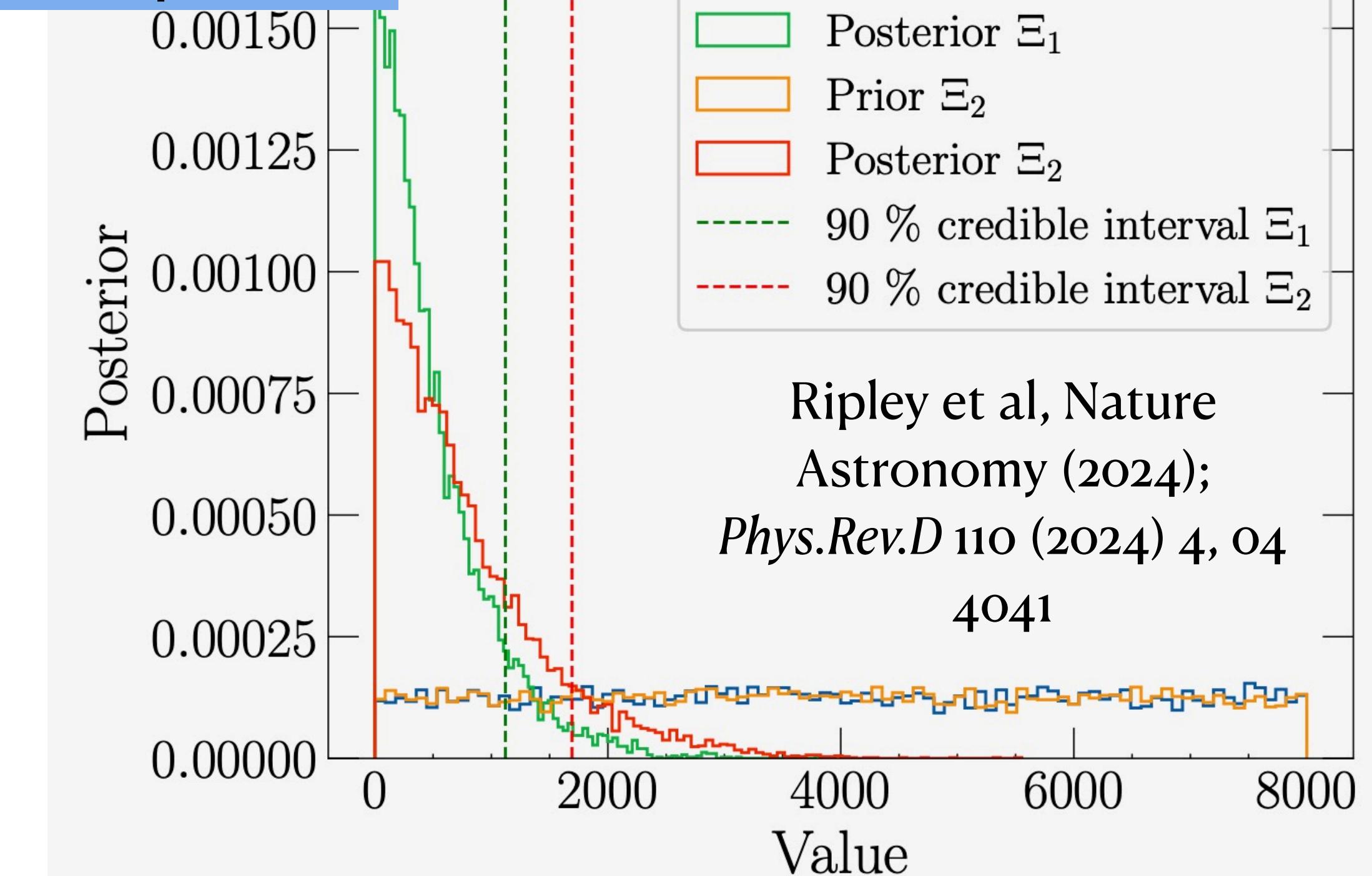
# Out-of-equilibrium: misalignment of tidal bulge

First constraints on viscosity from gravitational wave data!



Ripley, Hegade, Chandramouli,  
Nature Astronomy (2024)

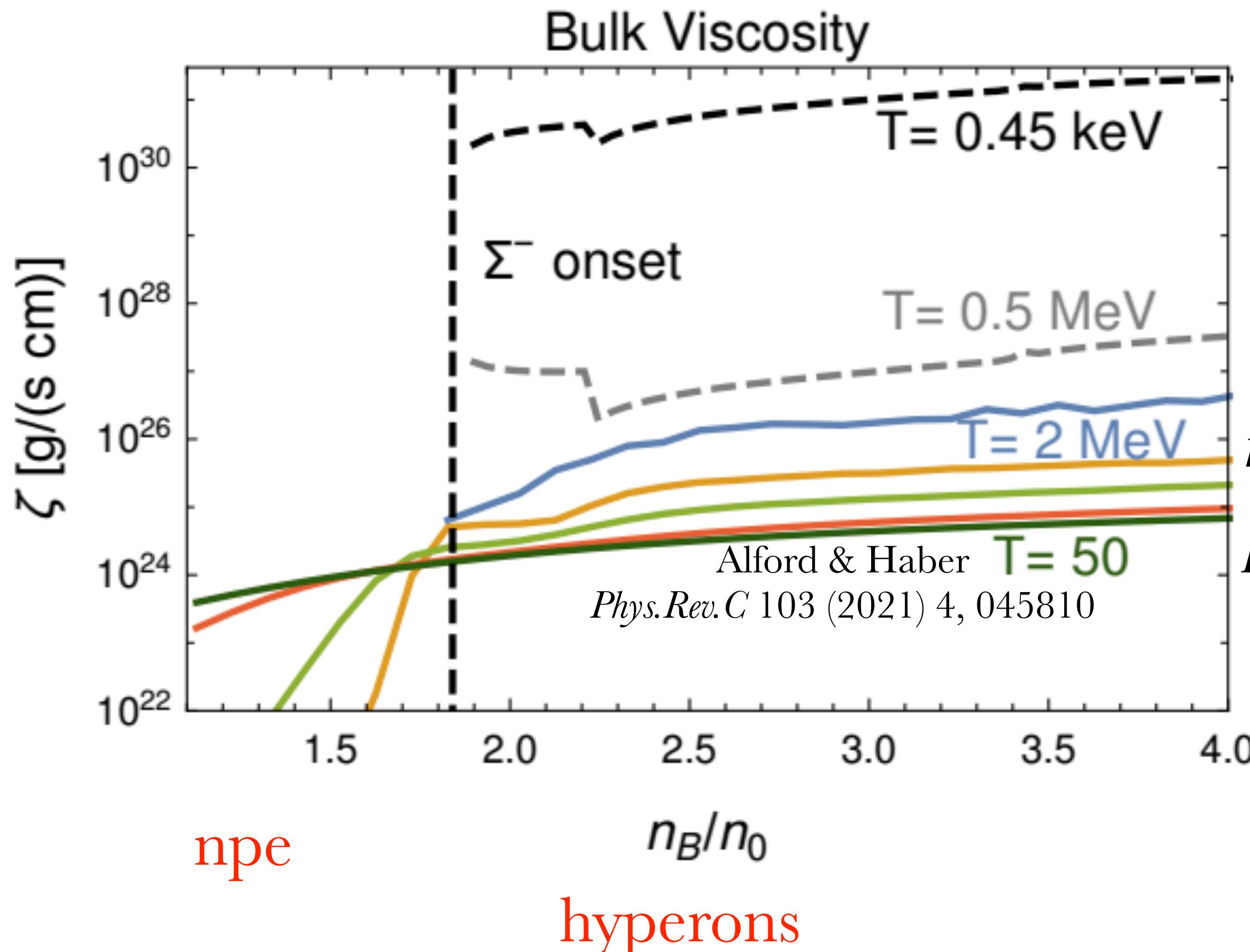
“Averaged” viscosity  
within the inspiral



Out-of-equilibrium measurements provide new  
way to probe neutron star degrees-of-freedom!

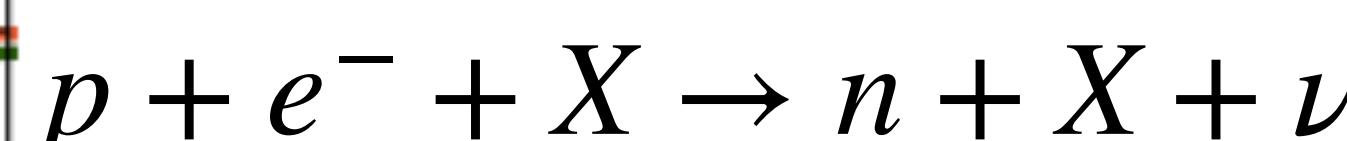
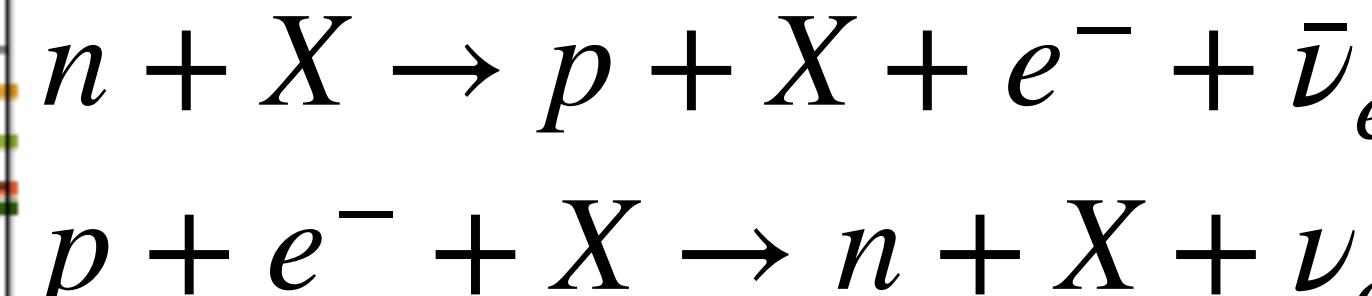
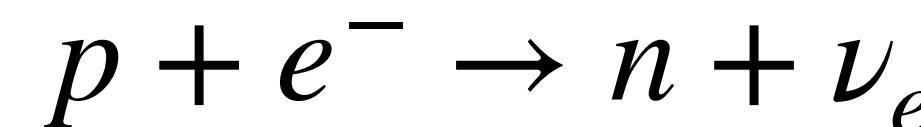
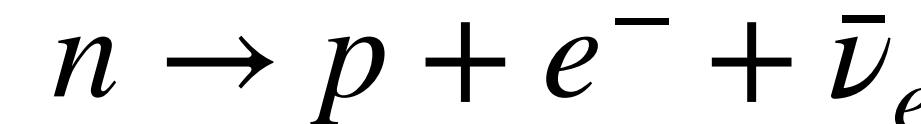
# Degrees-of-freedom and bulk viscosity

Depends on temperature, density, oscillations



New approach to understand the microscopic degrees of freedom in neutron stars

Delayed  $\beta$ -equilibrium in mergers



Bulk and Magnetic fields  
[2409.09423 \[nucl-th\]](#)

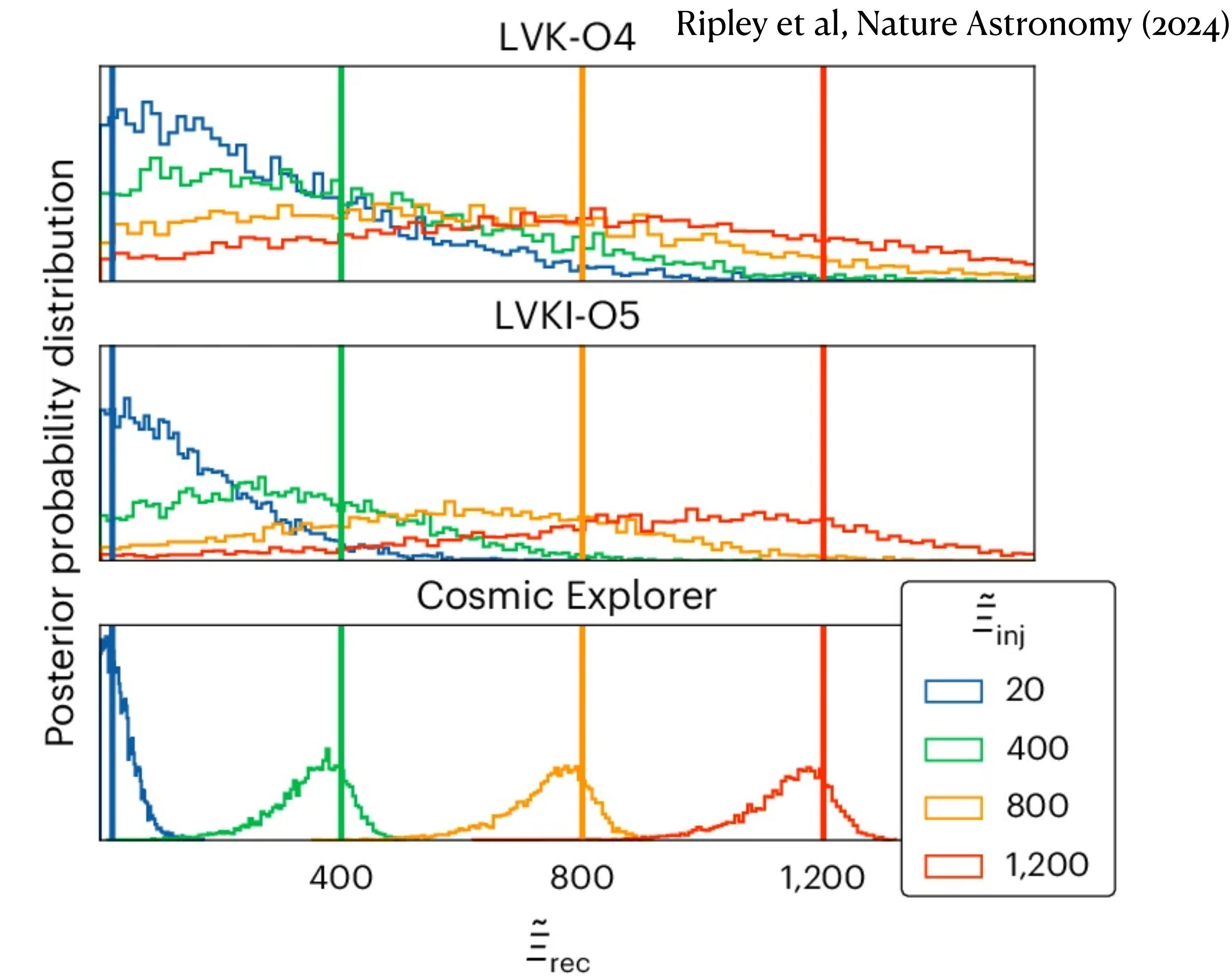
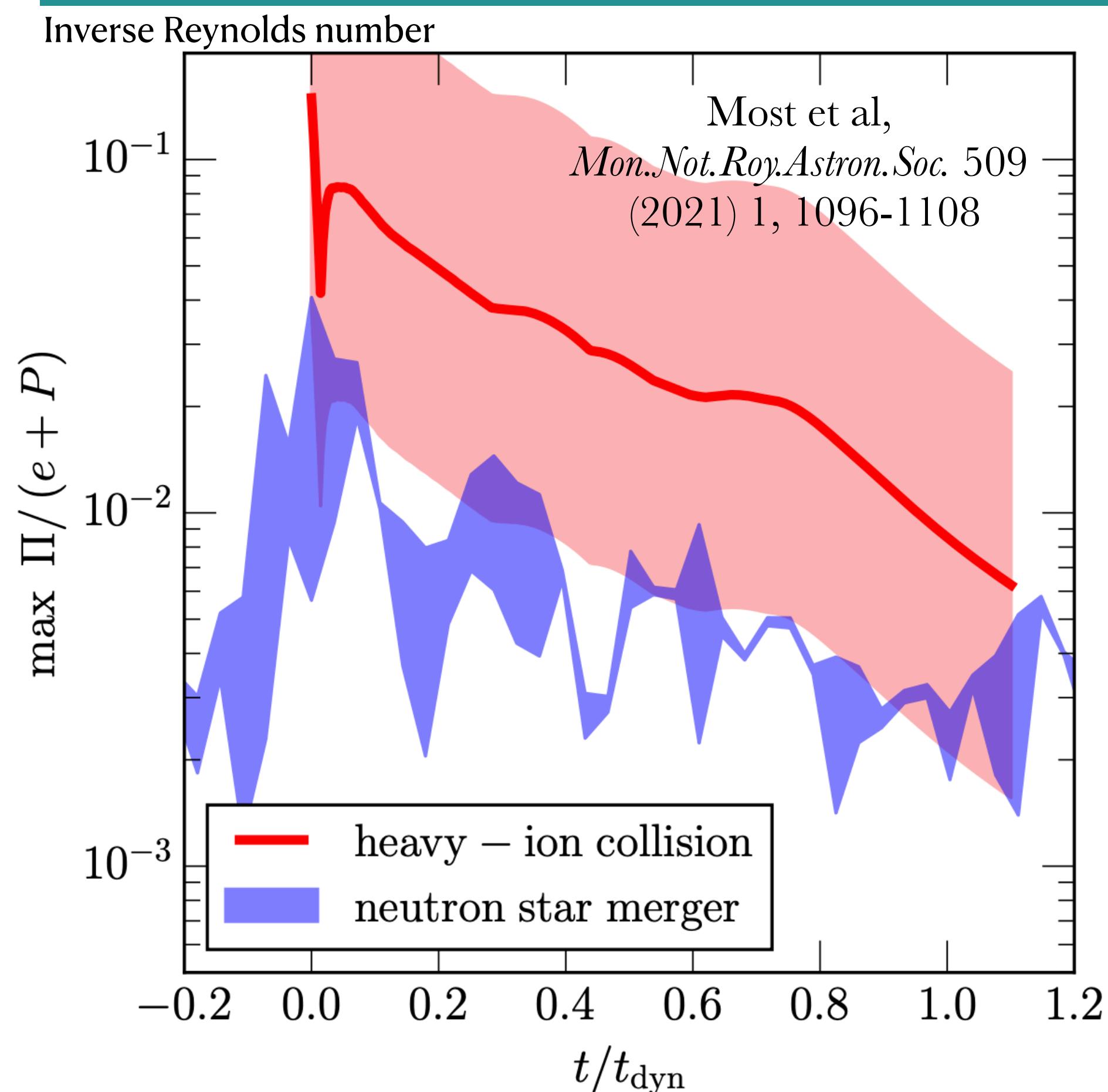
Bulk viscosity can cause phase shift in post-merger  
*Astrophys. J. Lett.* 967 (2024) 1, L14

Far-from-equilibrium and symmetry properties  
*Phys. Rev. C* 109 (2024) 1, 015805

# Bulk viscosity: heavy-ions vs neutron stars

## Comparisons of inverse Reynolds numbers

State-of-the-art heavy-ion collision simulations vs neutron star mergers find comparable effects from bulk viscosity



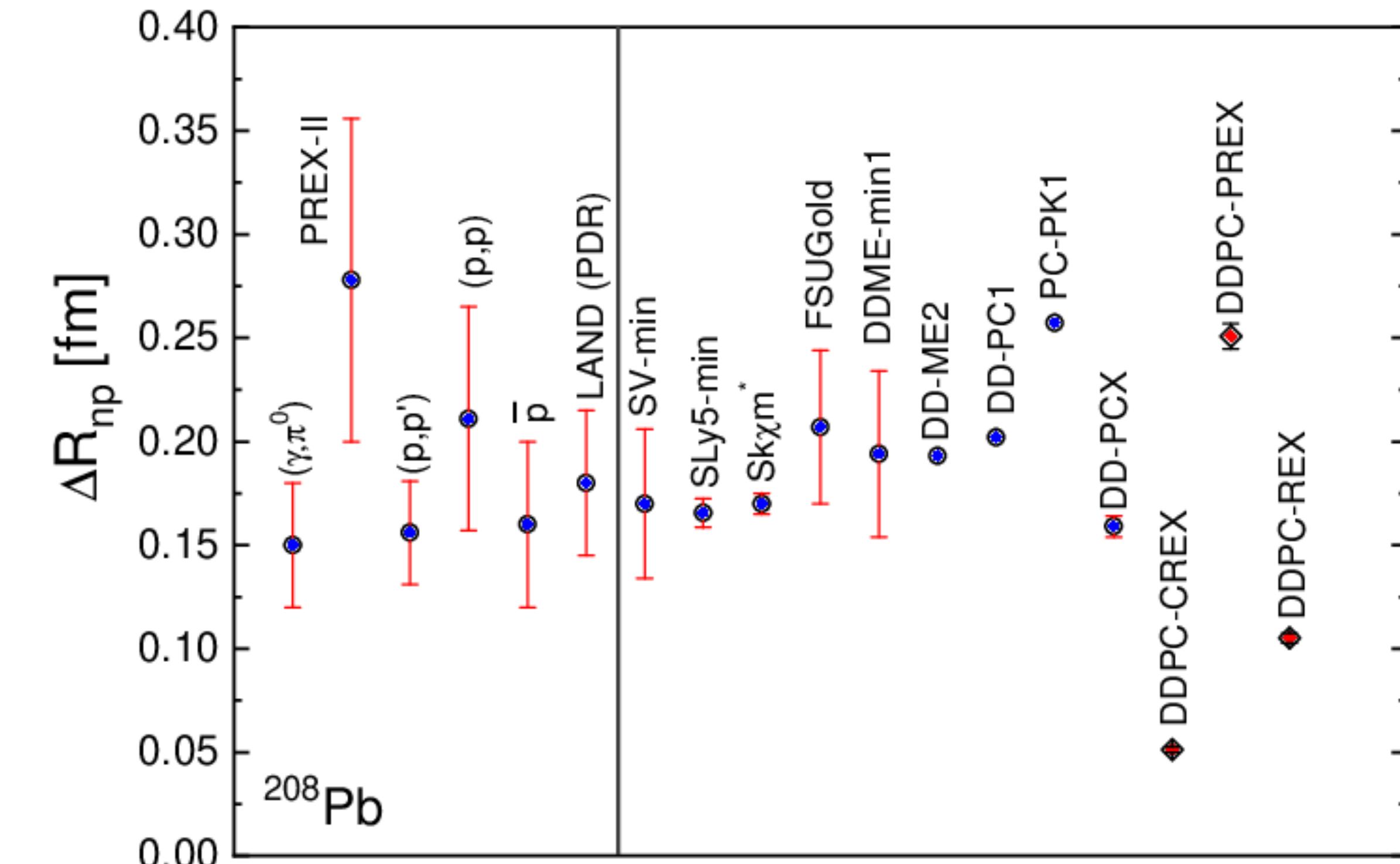
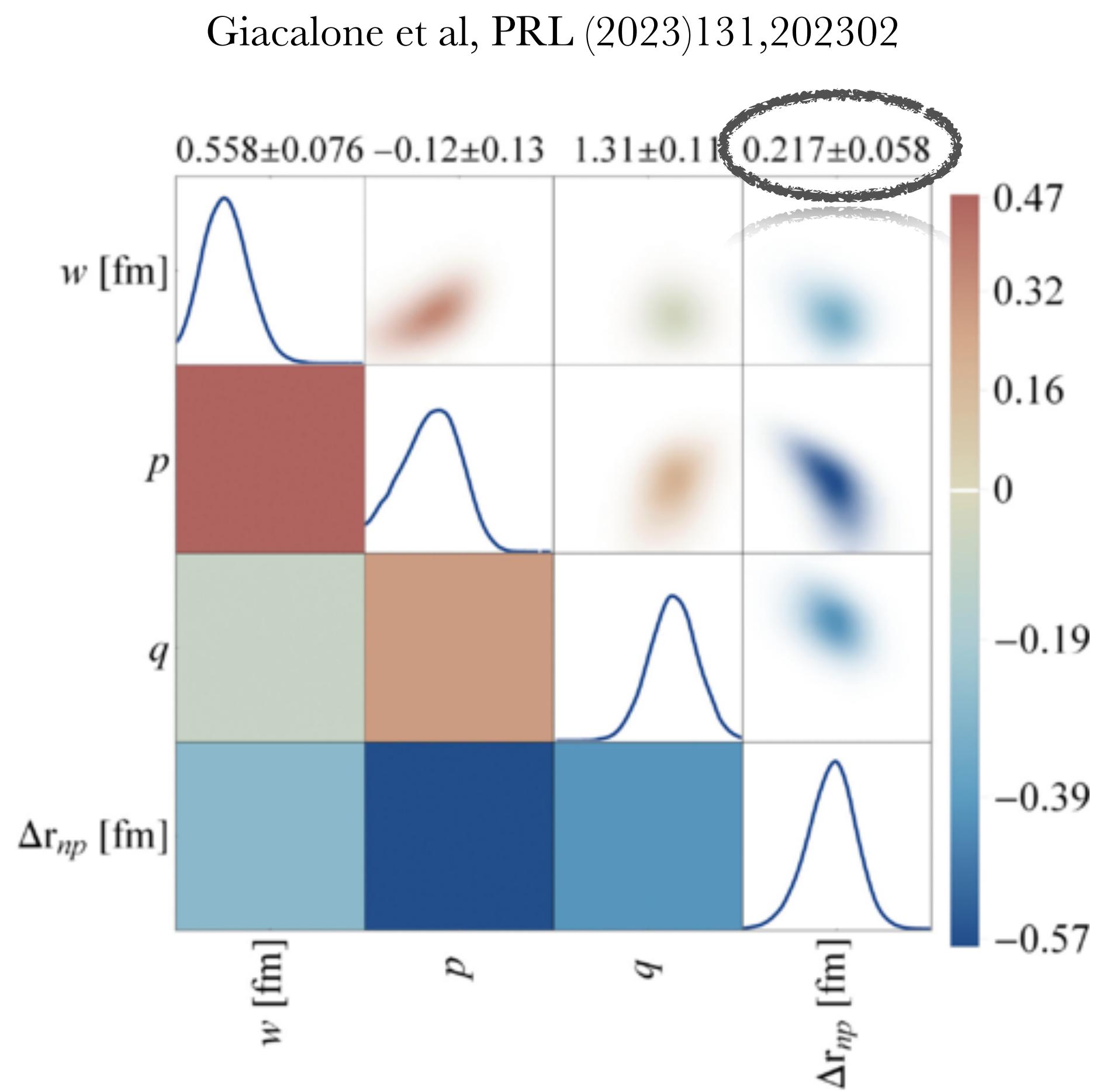
Future detectors/runs able to better constrain the averaged viscosity in the inspiral. Potential for future collaborations!

# **Other heavy-ion collision and neutron star connections**

# Extracting Neutron Skin from HIC data

Can be used to constrain symmetry energy coefficients

## Extraction from Bayesian analysis



## Extraction from Ultraperipheral heavy-ion collision data

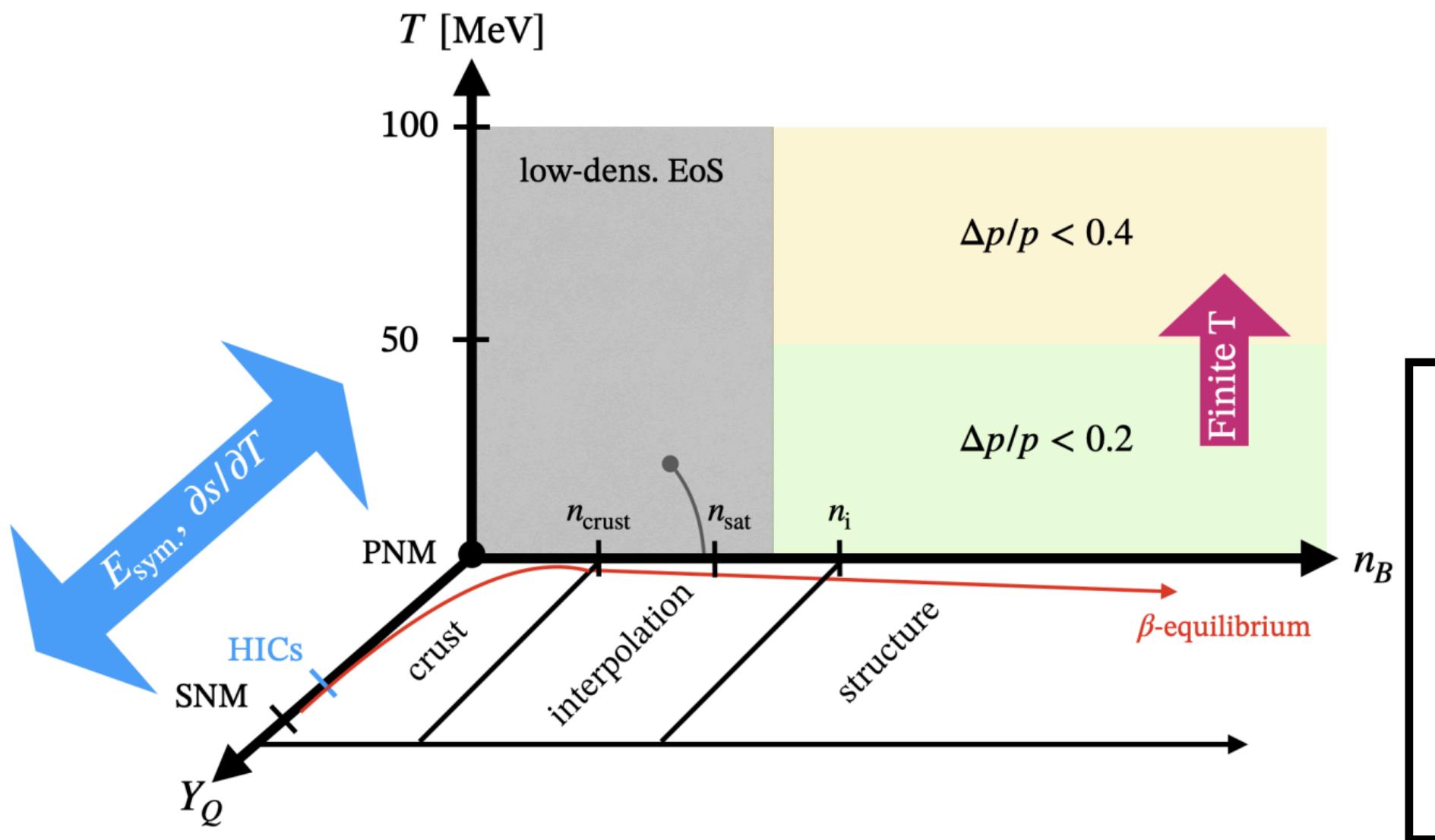
$$R_n - R_p = 0.17 \pm 0.03(\text{stat}) \pm 0.08(\text{syst}) \text{ for } {}^{197}\text{Au}$$

$$R_n - R_p = 0.44 \pm 0.05(\text{stat}) \pm 0.08(\text{syst}) \text{ for } {}^{238}\text{U}$$

[STAR] *Sci.Adv.* 9 (2023) 1, eabq3903

# Yields across $\frac{Z}{A}$ and the neutron star equation of state

Mroczek, Yao et al, 2404.01658 [astro-ph.HE]



## Repurposing isobars

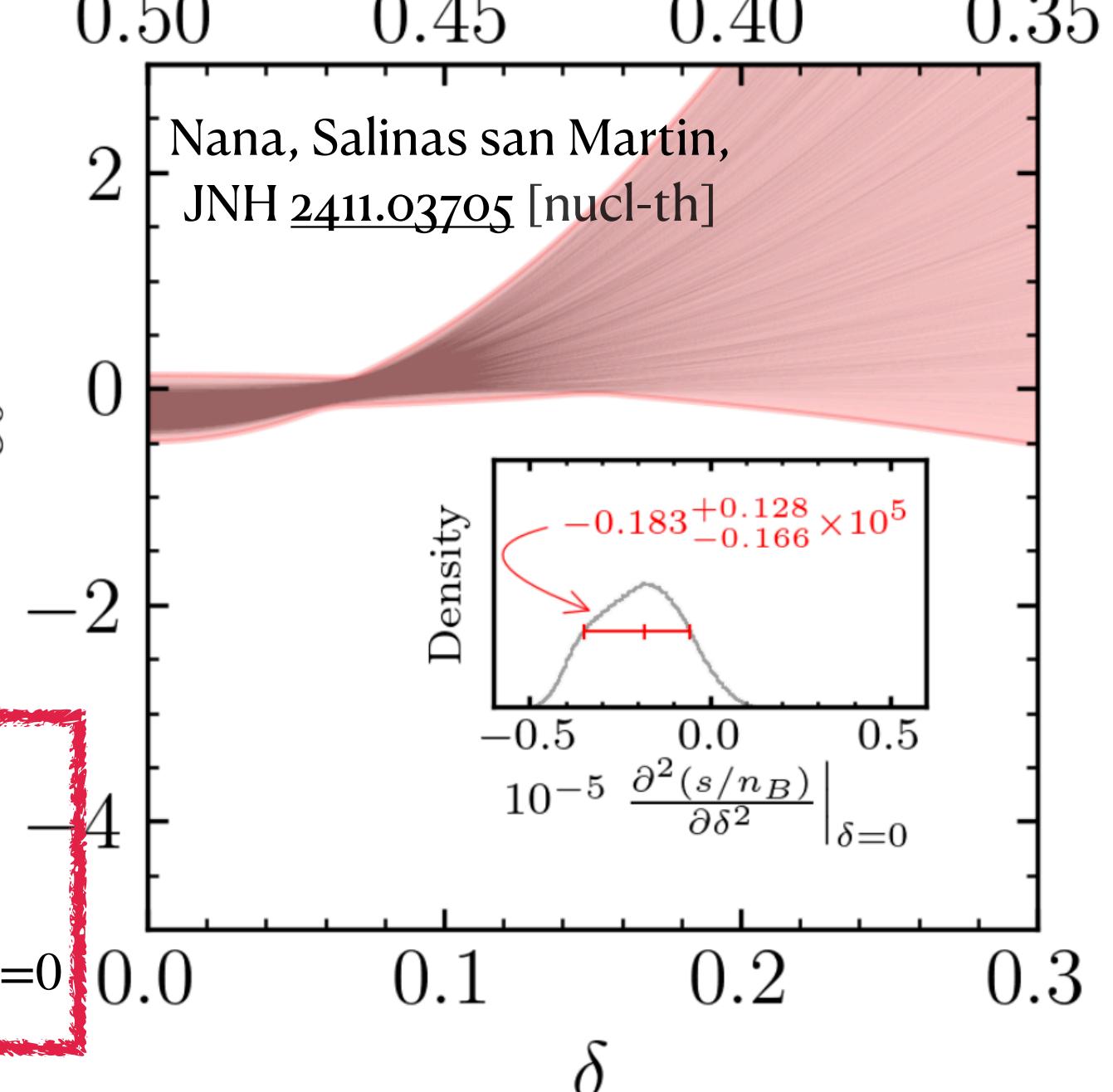
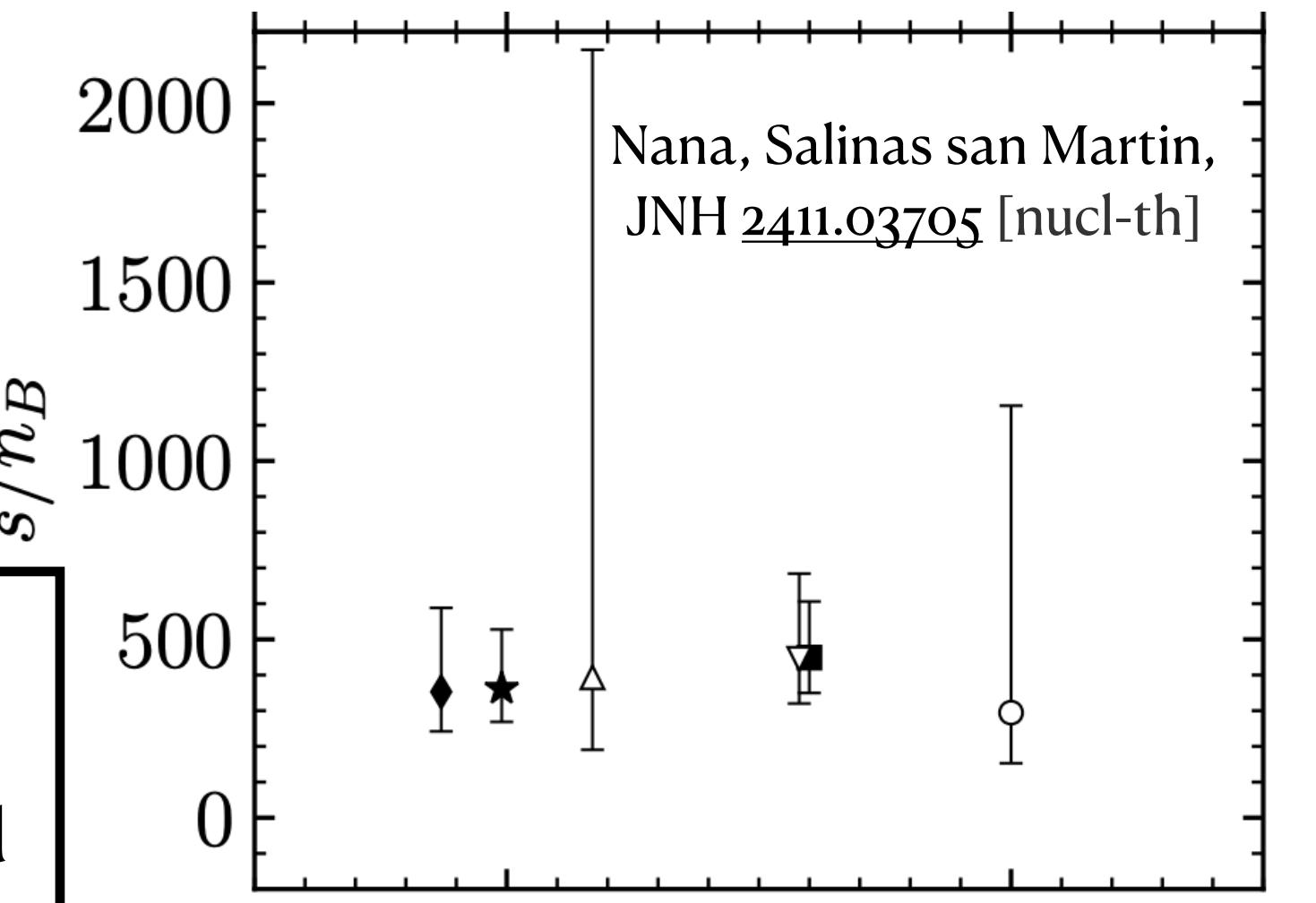
How do we get this information?  
Thermal models from heavy-ion collisions and  
varying  $Y_Q$ !  
Proof-of-principle from RHIC  $\sqrt{s} = 200$  GeV

Expand from a cold, neutron star EOS

$$p(T, \vec{\mu}) = p_{T=0} + \frac{1}{2} \frac{\partial s}{\partial T} \Big|_{T=0, \vec{\mu}} T^2 + \mathcal{O}(T^3)$$

Need information about how  $\partial s / \partial T$  varies with  $Y_Q$

$$\frac{\partial s / n_B(T, n_B, Y_Q)}{\partial T} \Bigg|_{T=0} = \frac{1}{n_B} \frac{\partial s_{\text{HIC}}(T, n_B, Y_Q)}{\partial T} \Bigg|_{T=\delta_{\text{HIC}}=0} + \frac{1}{2} \left( 1 - \frac{Y_Q}{Y_Q^{\text{HIC}}} \right)^2 \frac{\frac{\partial^3(s/n_B)_{\text{HIC},2}(T, n_B, \delta_{\text{HIC}})}{\partial T \partial \delta_{\text{HIC}}^2}}{\Bigg|_{T=\delta_{\text{HIC}}=0}}$$



# Summary and Outlook

- The QCD phase diagram is inherently interdisciplinary and requires collaborations amongst subfield to fill in gaps
  - More data anticipated from LIGO/Virgo/Karga run 4 (now) & run 5 (2028+)
  - Waiting on STAR Fixed Target
  - New fixed target heavy-ion detector: CMB at FAIR in Germany is being built to study the regime between heavy-ions and neutron stars → scans of Z/A
  - Possibilities at other fixed target experiments? Here at CERN? AGS? FRIB400?
  - (Bulk) Viscosity provides a new opportunity to learn about dense matter, opens up an interdisciplinary field of research
  - Strange baryon interaction constraints from ALICE *Nature* 588, 232–238 (2020); *Phys.Lett.B* 844 (2023) 137223

# Thank you!

## To my group + collaborators

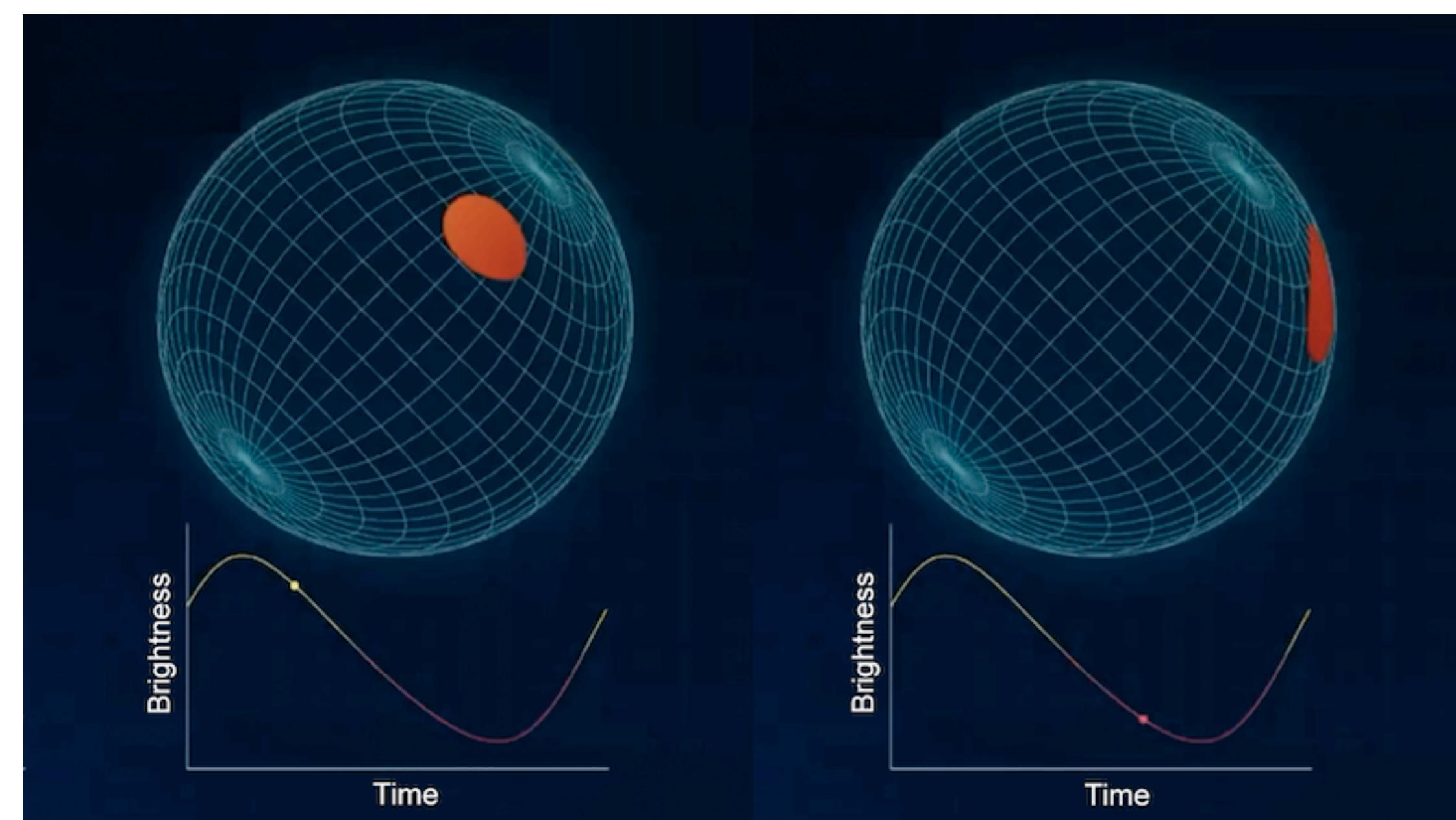


+Isaac Long,  
Leonardo Pena,  
David Olsen

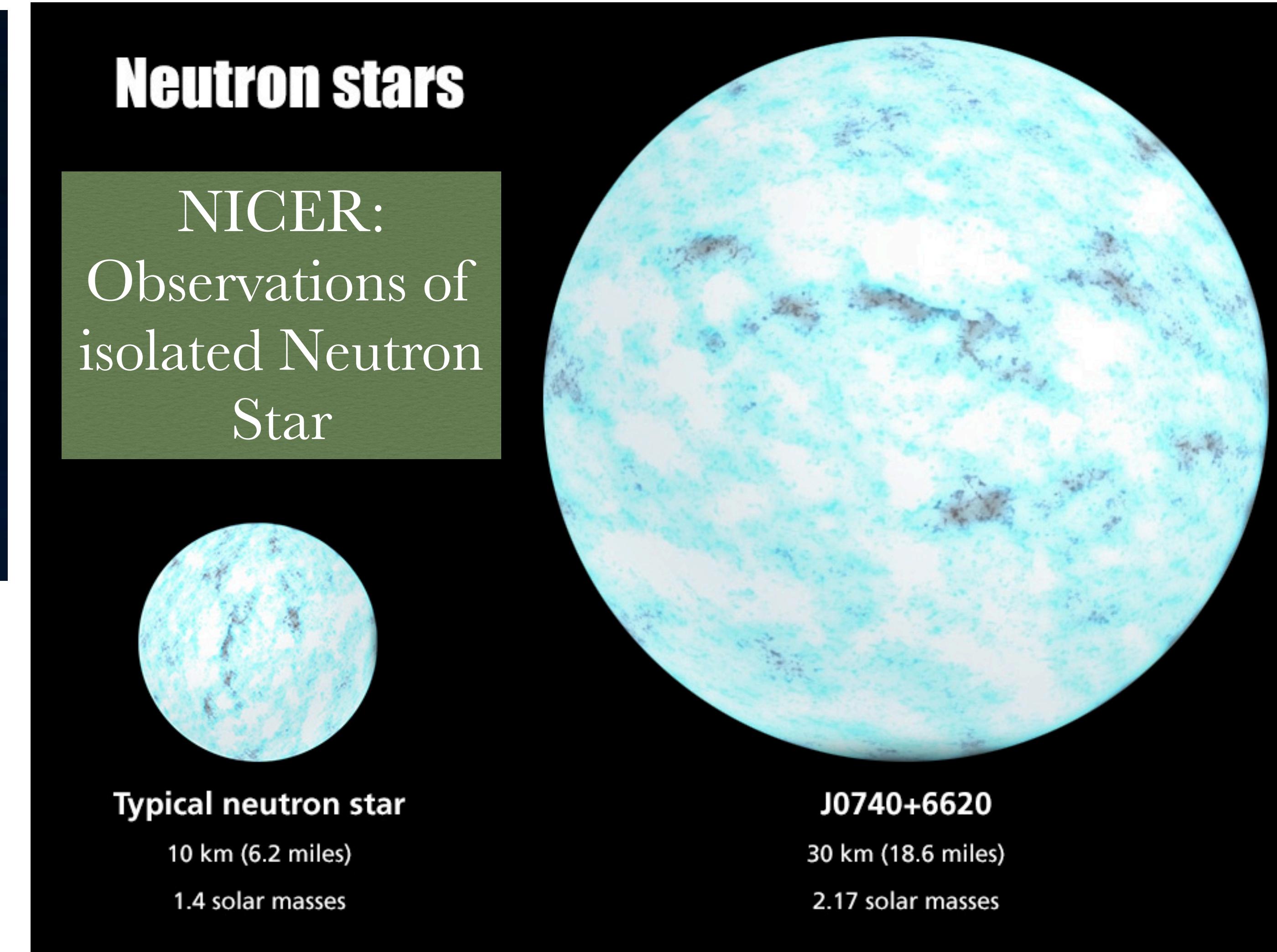


# Mass-Radius of Isolated Neutron Stars

NICER collaboration



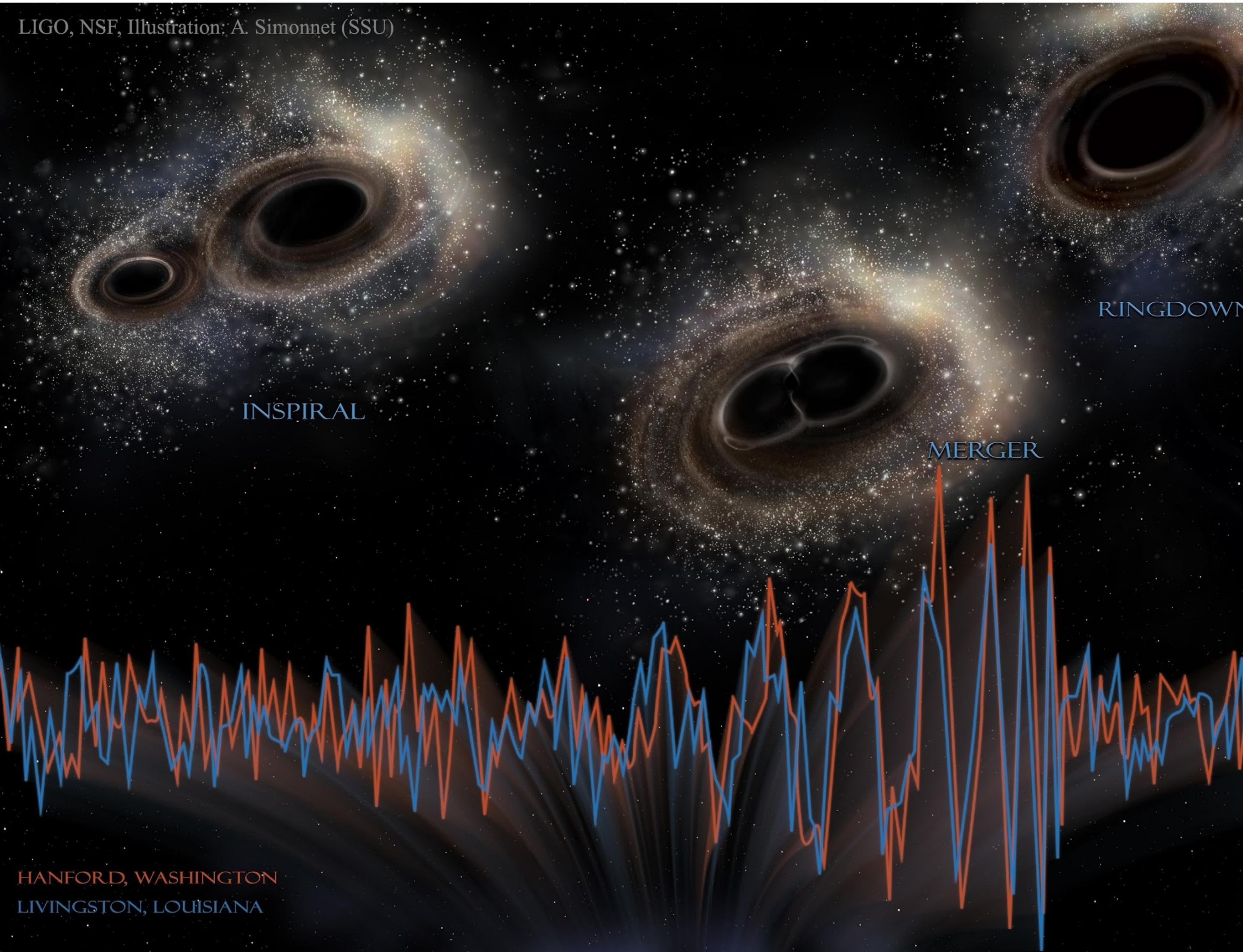
Heavier stars probe denser region of the EOS



Miller, arXiv:2105.06979 [astro-ph.HE]; Astrophys.J.Lett. 887 (2019) 1, L24;  
Raaijmakers, arXiv:2105.06981 [astro-ph.HE]; Astrophys.J.Lett. 887 (2019) 1, L22

# Tidal deformability

EOS information from gravitational waves



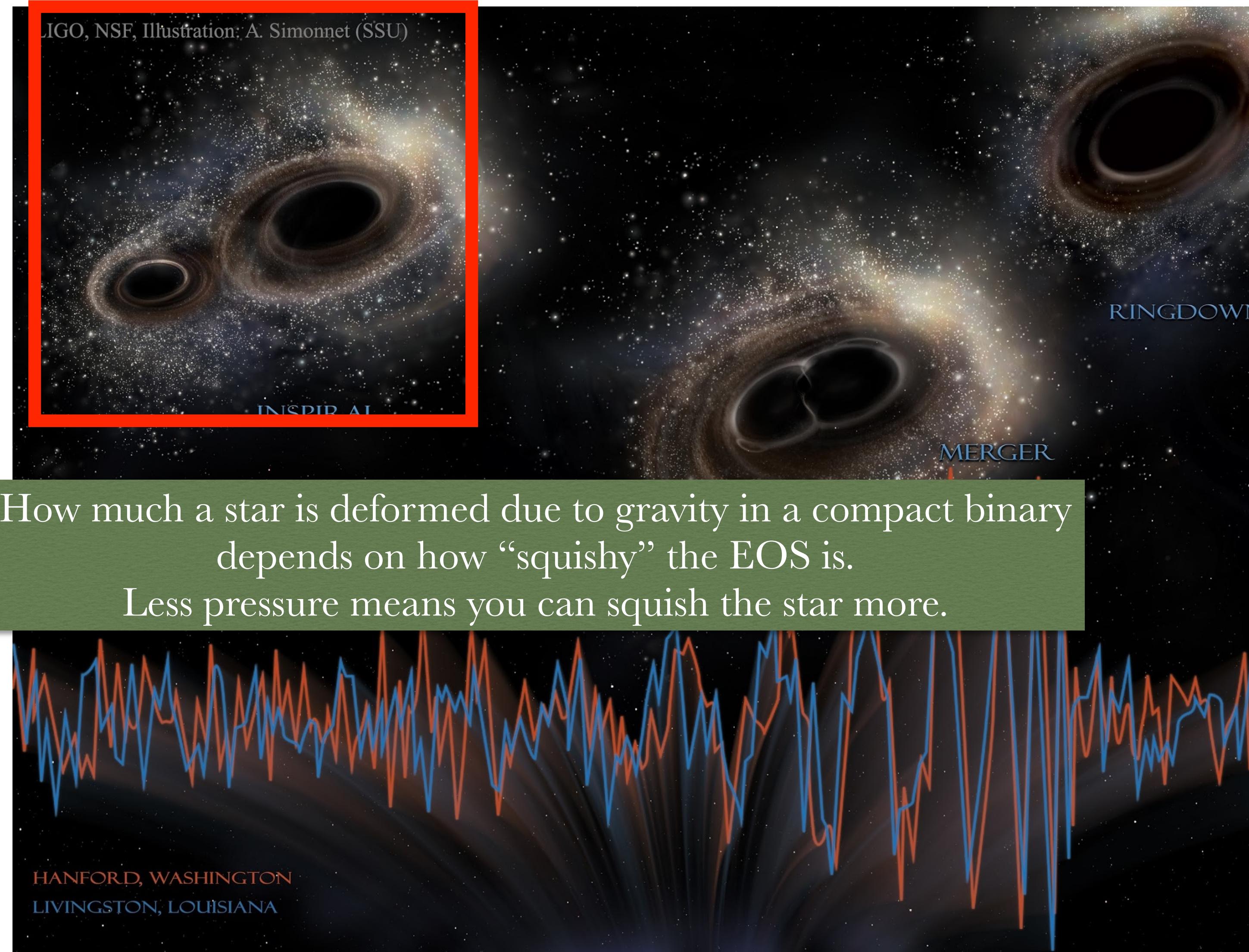
As the two objects approach,  
they are deformed (elongated)

The EOS dictates how much a neutron star can deform

- Black holes are not deformed  $\Lambda = 0$
- Light neutron stars more deformed (large  $\Lambda$ )
- Heavy neutron stars less deformed (small  $\Lambda$ )

# Tidal deformability

EOS information from gravitational waves



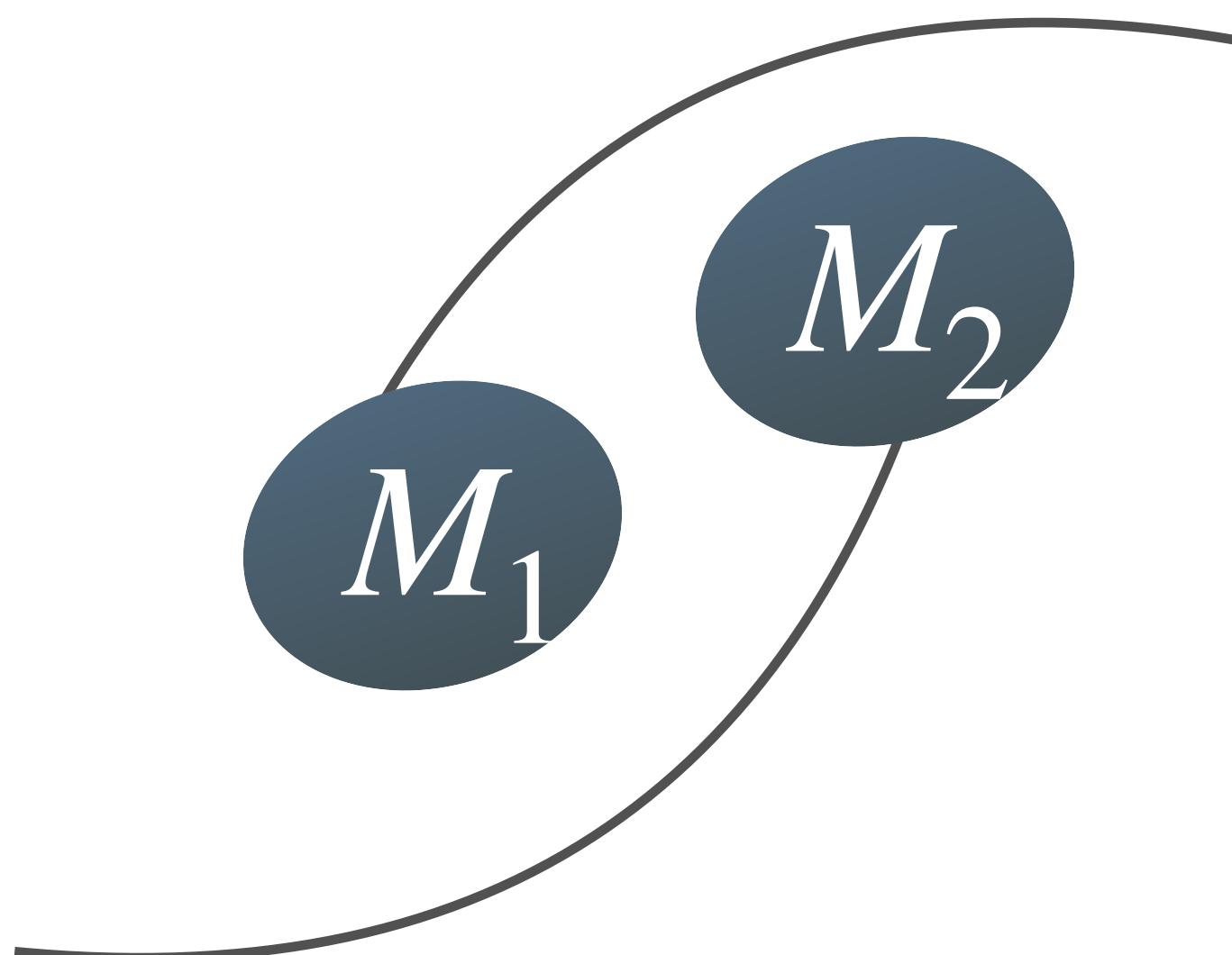
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- Black holes are not deformed  $\Lambda = 0$
- Light neutron stars more deformed (large  $\Lambda$ )
- Heavy neutron stars less deformed (small  $\Lambda$ )

# Breaking tidal deformability degeneracy

Binary love relation



We set  $m_1$  to be the lighter mass and  $m_2$  is the heavier star

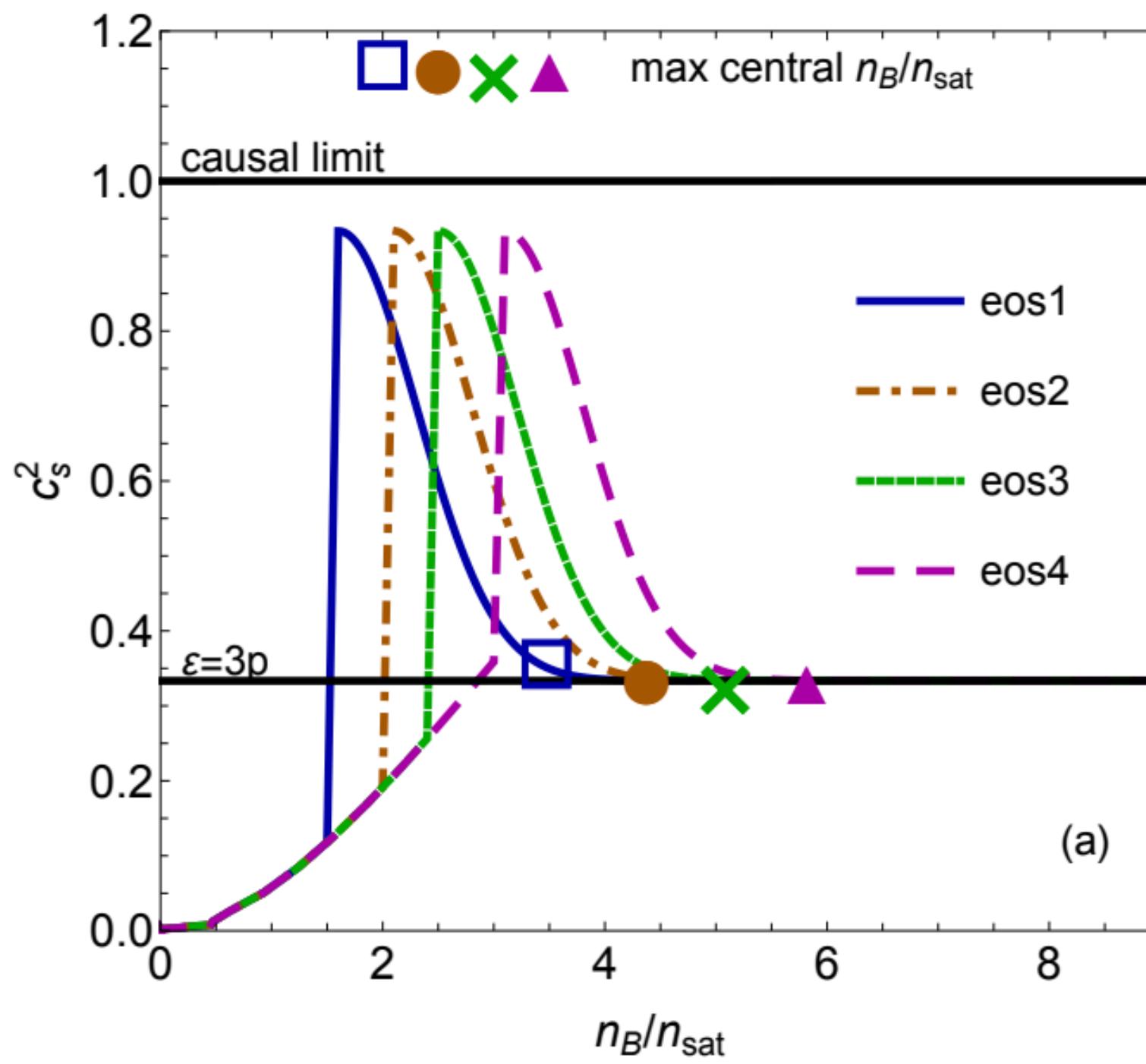
From gravitational waves, we obtain a combination of tidal deformabilities from each star

$$\tilde{\Lambda} \equiv \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_1 + 12M_2)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

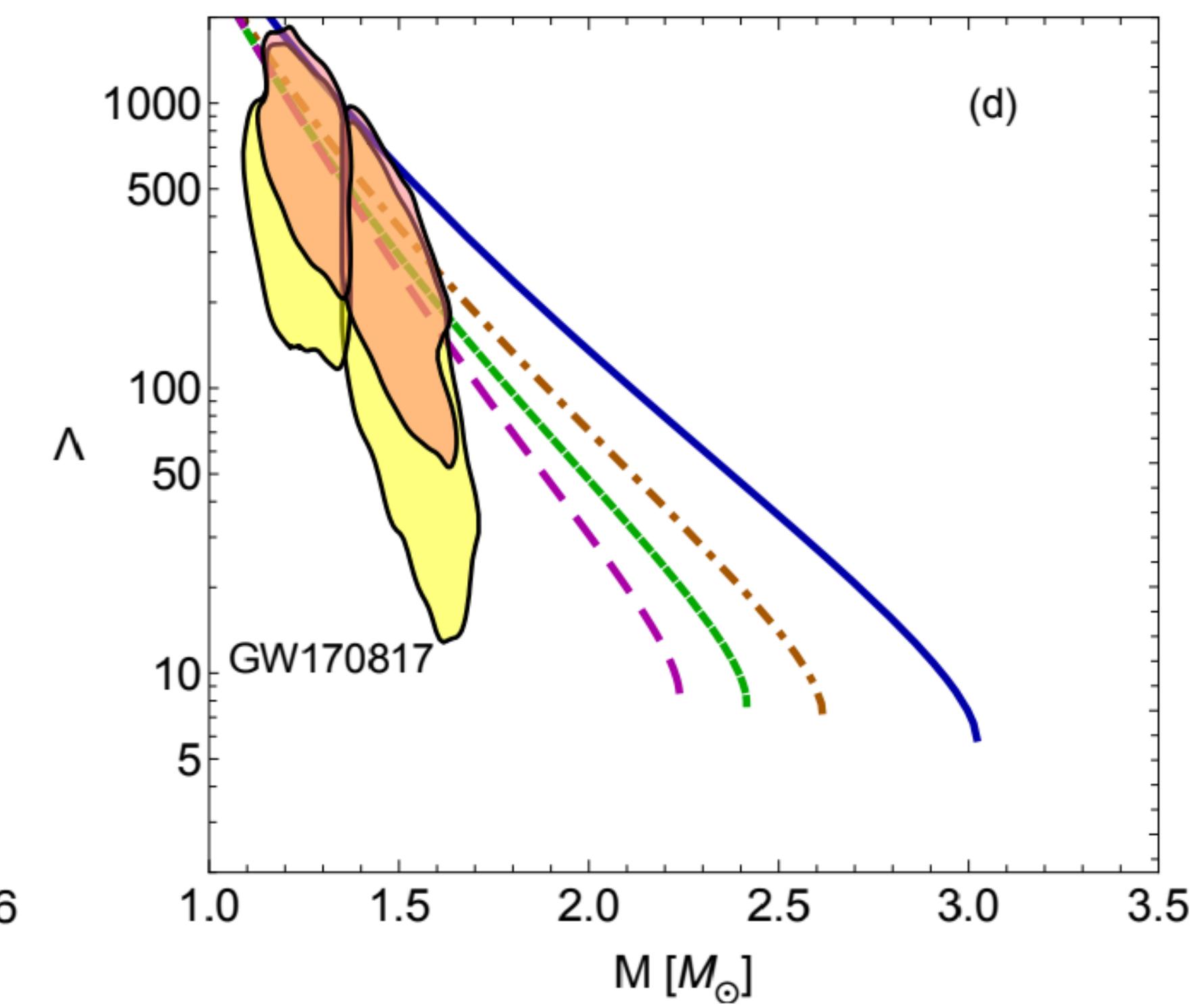
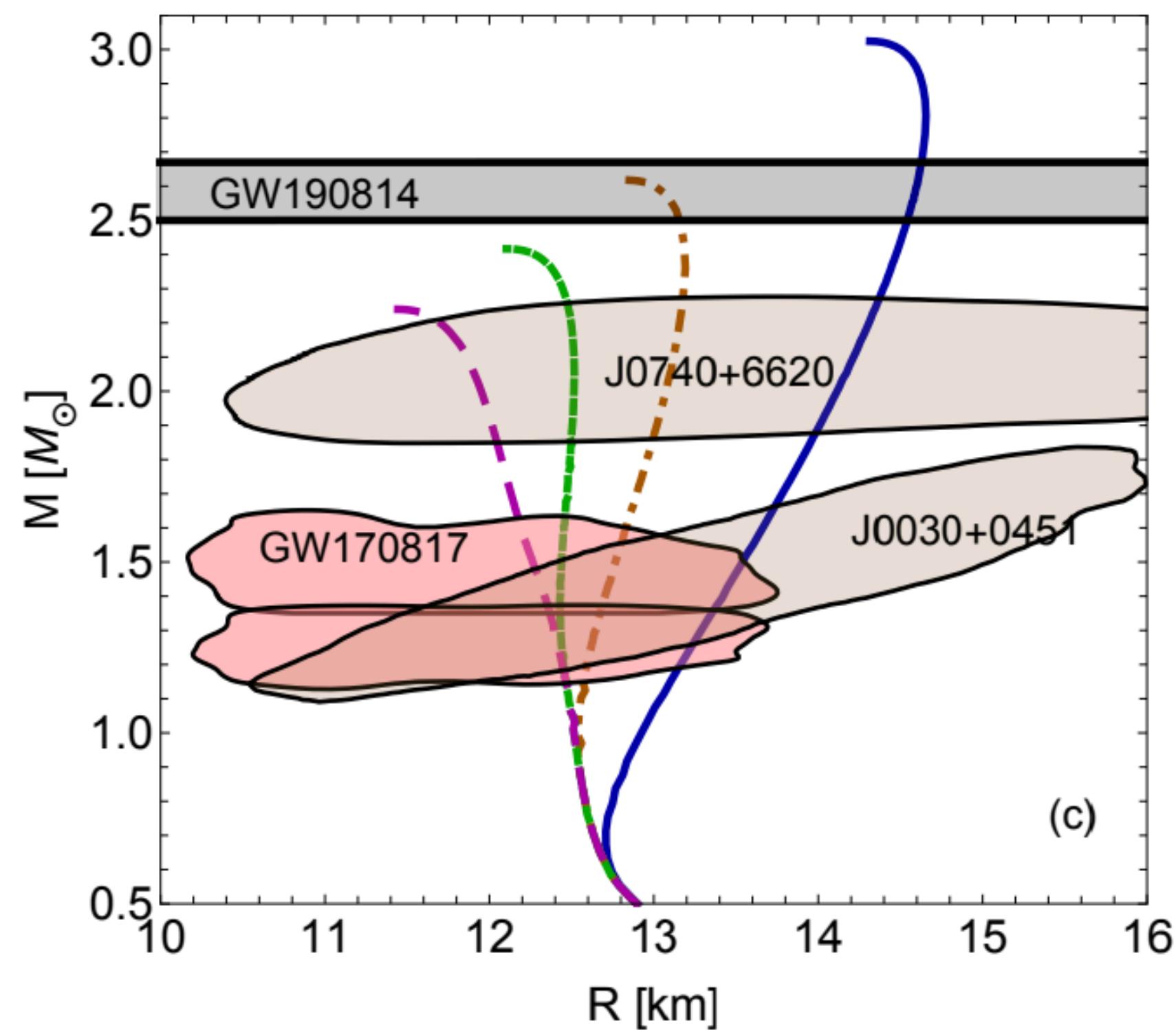
Need something to break the degeneracy between  $\Lambda_1$  and  $\Lambda_2$

# Connecting an EOS to data

Need TOV and solve Einstein equations up to 2nd order in slow rotations



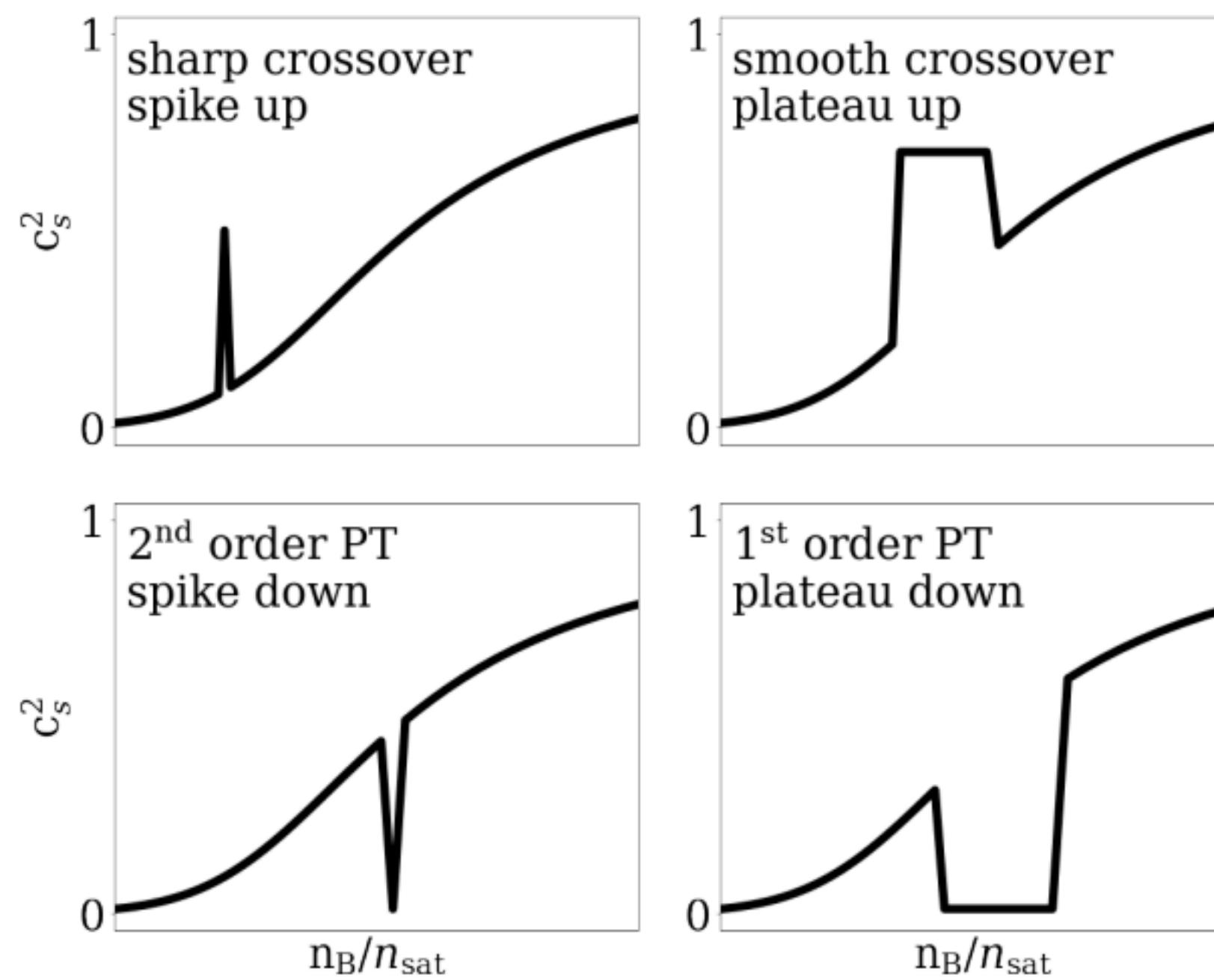
Given an EOS we can obtain a specific  $M(R)$  and  $\Lambda(M)$



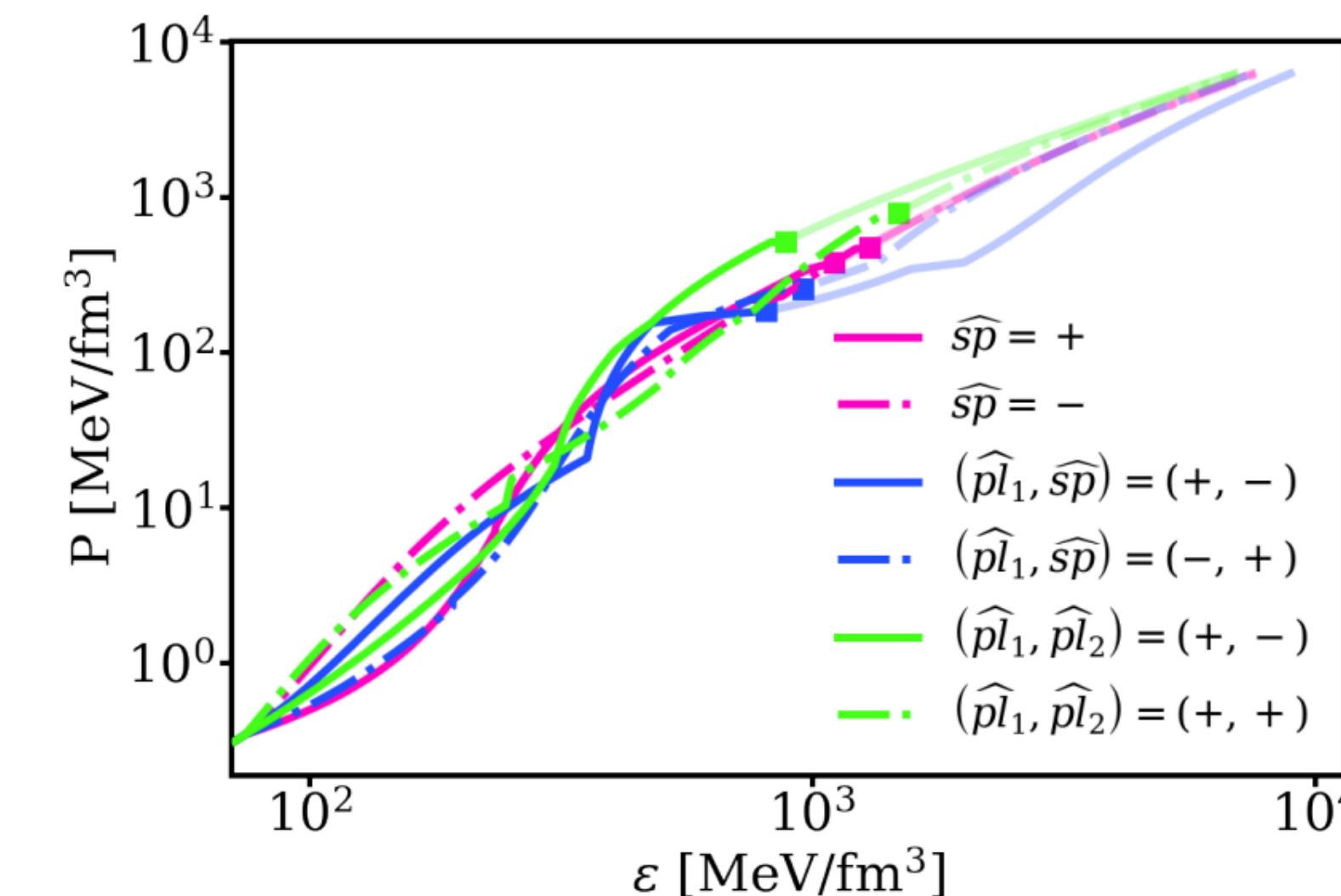
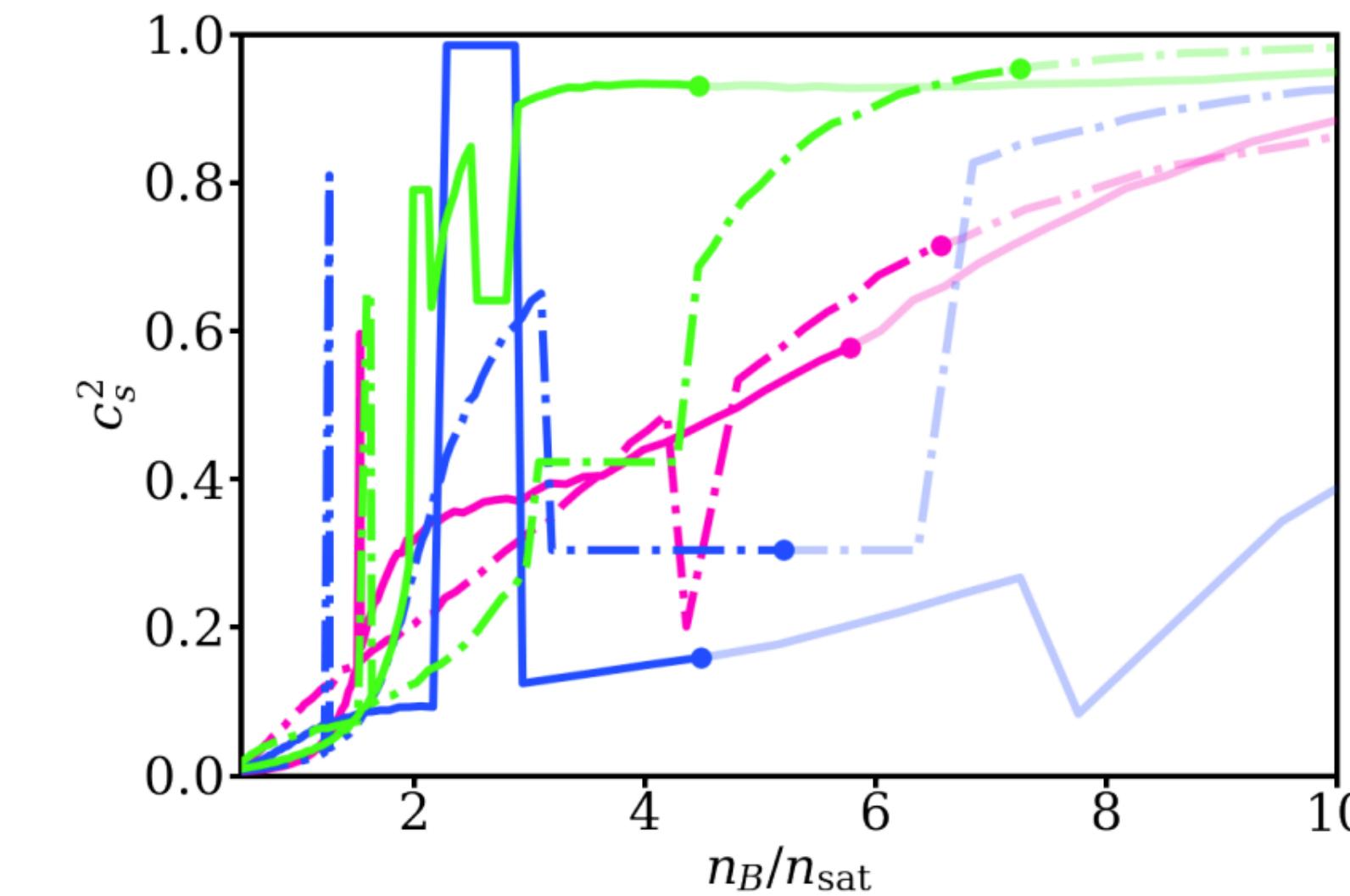
# Neutron stars with structure in EOS

## Functional forms

Mroczek et al, 2309.02345  
[astro-ph.HE]



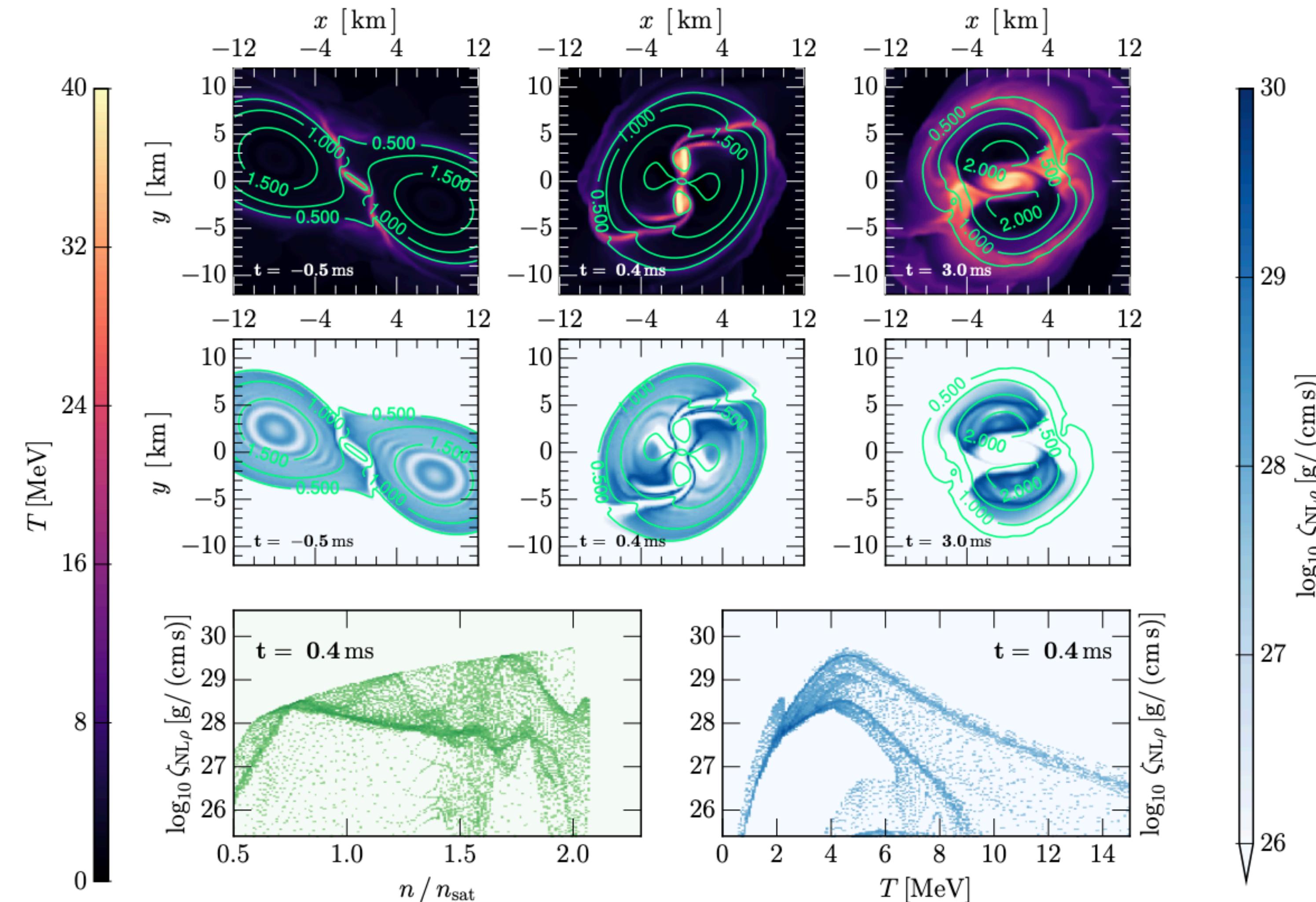
Features built into the EOS and example EOS using modified Gaussian Processes



# Bulk viscosity: heavy-ions vs neutron stars

## Post-merger influence of bulk viscosity

Most et al, *Mon.Not.Roy.Astron.Soc.* 509 (2021) 1, 1096-1108



# Gaps in our knowledge

## How do we interpret data?

Collider mode Heavy-ion collisions

$$\sqrt{s} \geq 7.7 \text{ GeV}$$

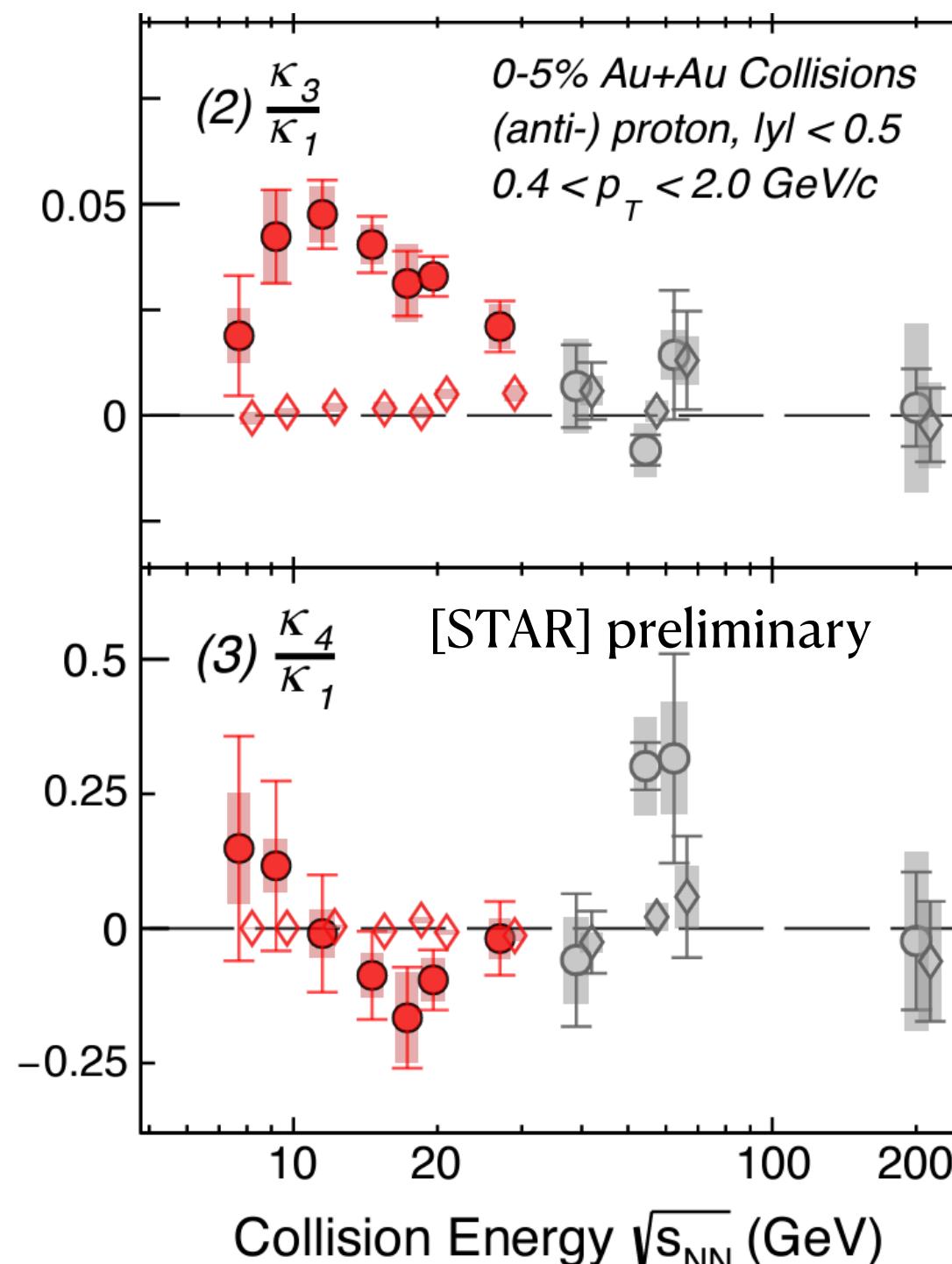
Exp. Observables: flow, multiplicity, fluctuations, HBT, ... = Hundreds of data points!

Theory constraints: *thermodynamic* stability in 4D (Appendix E in [2409.06837](#) [nucl-th]),  
*out-of-equilibrium* causality/stability (*Phys.Rev.Lett.* **126** (2021) 22, [222301](#), [2209.11210](#) [hep-th])

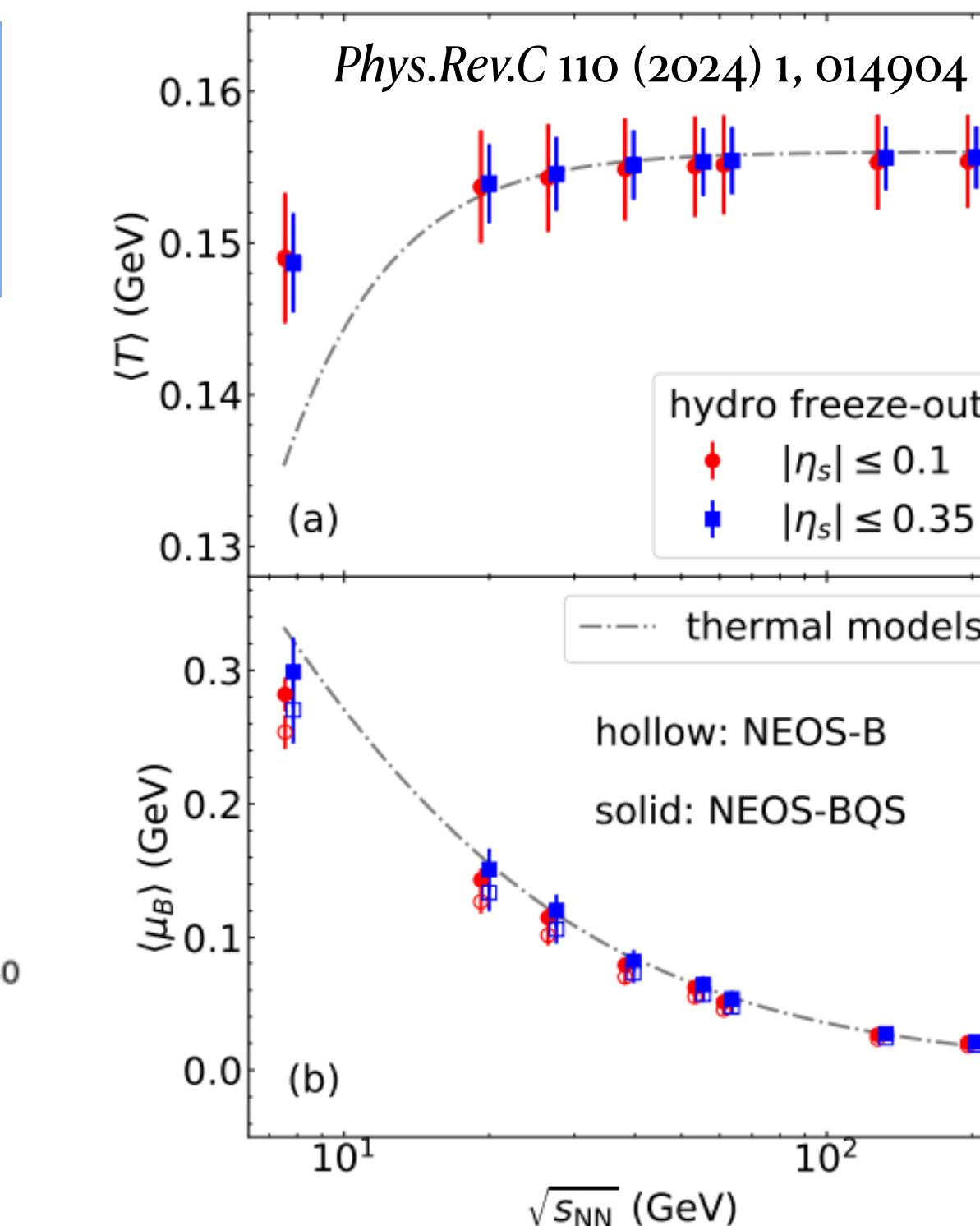
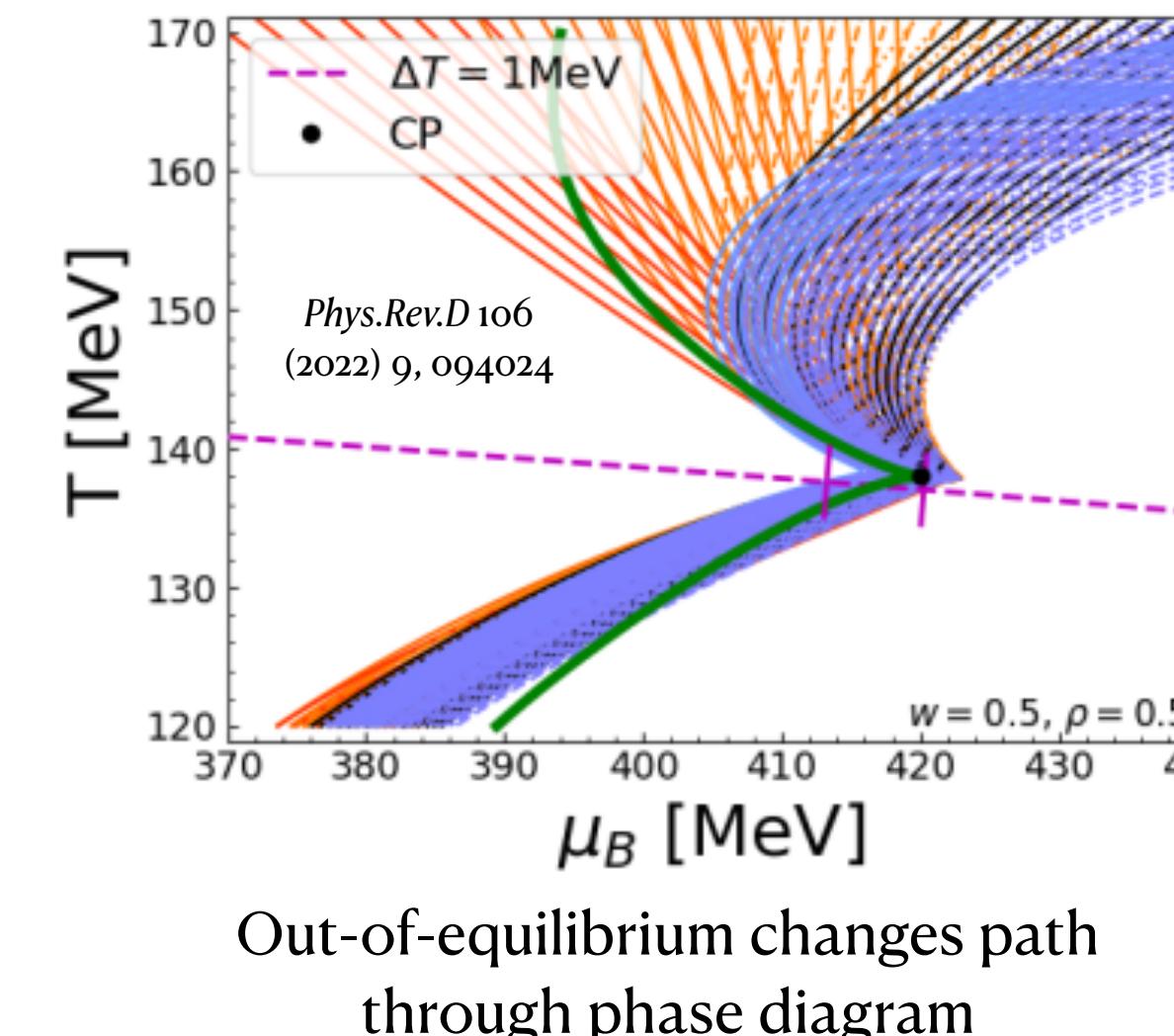
Model: Rel. viscous hydrodynamics+conserved charges, many unknowns in the initial state, EOS, transport coefficients etc

**Desperately needed:** collaboration to put together these tools and conduct a Bayesian analysis with the data!

Further details: [2211.02224](#) [nucl-th]; *Nucl.Phys.A* **1017** (2022) [122343](#)



Hints of a critical point, but  
 need full scale analysis of  
 out-of-equilibrium effects



# Head on collisions and deformations

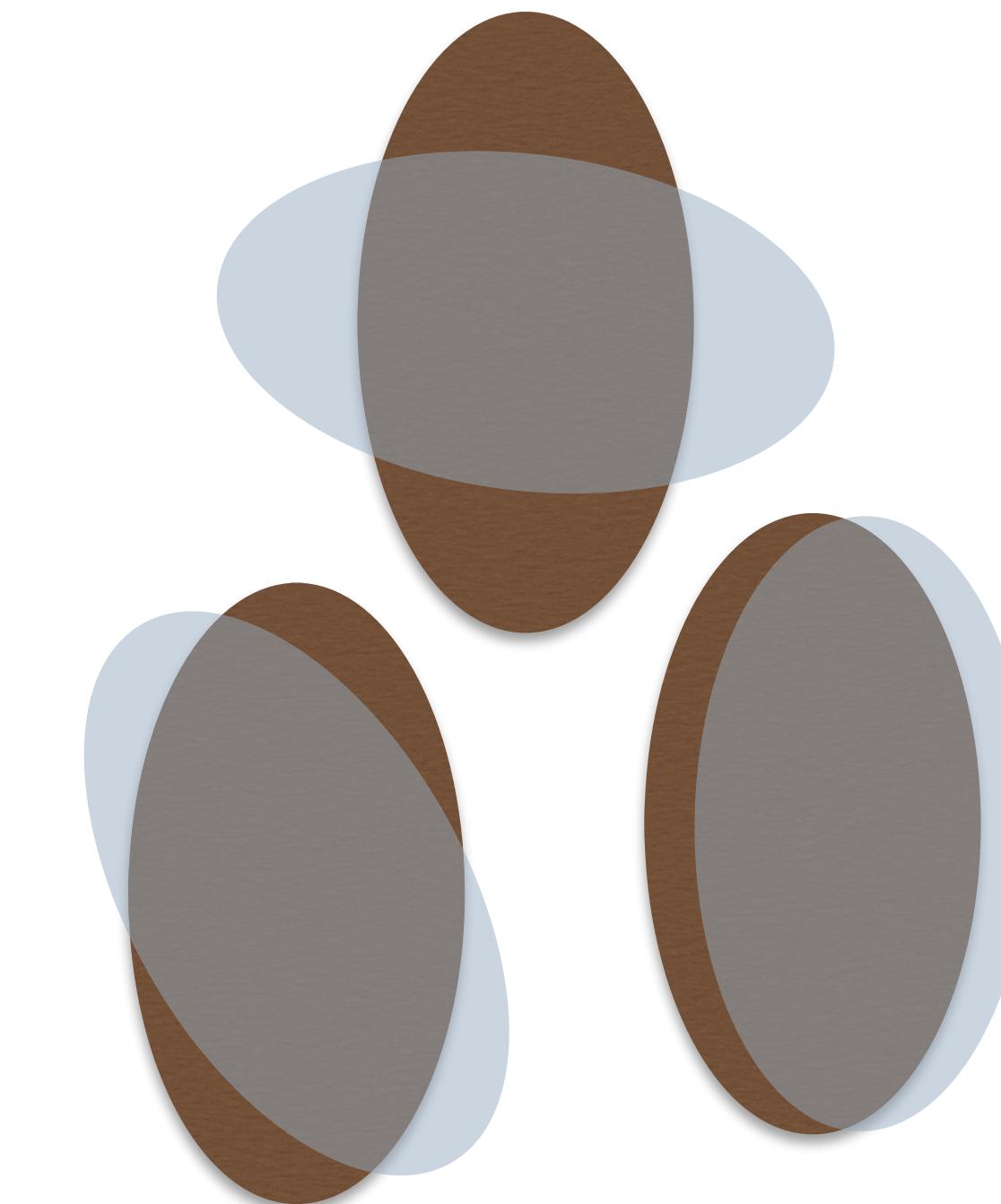
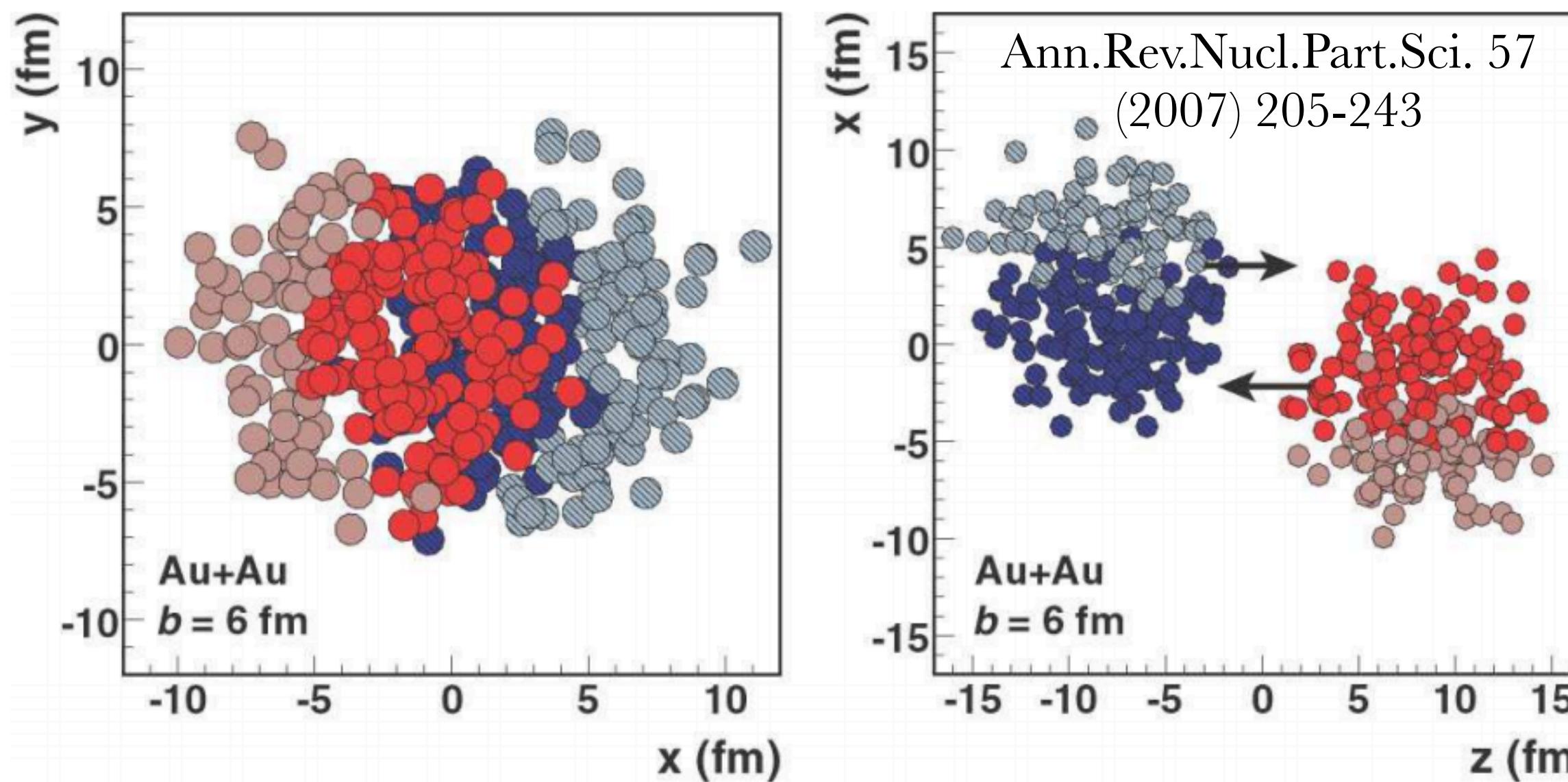
Centrality % = 0 for head on collisions

Terminology:

-participants (colliding nucleons)

-spectators (fly off to the detector)

All  $b = 0$  impact parameters,  
very different shapes



Head-on collisions most sensitive to structure, but  $b = 0$  does not necessarily mean ultra central!