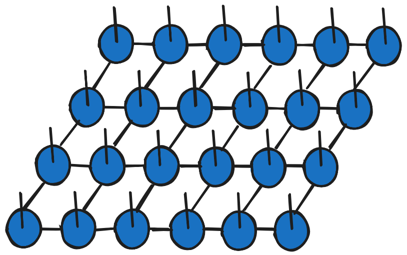
Introduction to Tensor Networks

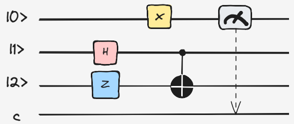
Ema Puljak

CERN | QUANTUM TECHNOLOGY INITIATIVE

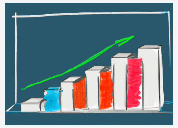
TENSOR NETWORKS



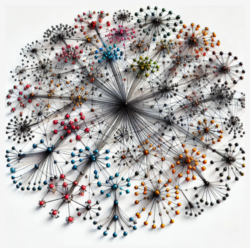
$|\psi\rangle$ efficiently represent quantum states



simulate quantum circuits

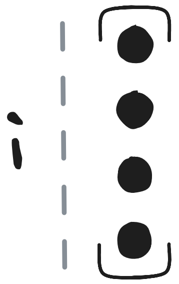


machine learning applications

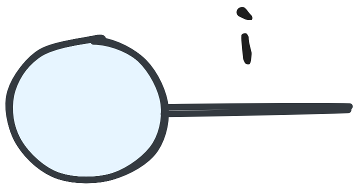


solve high-dimensional linear systems

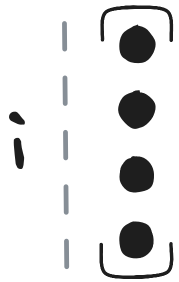
Vector



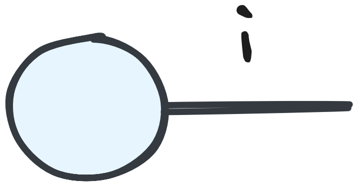
$$V_i$$



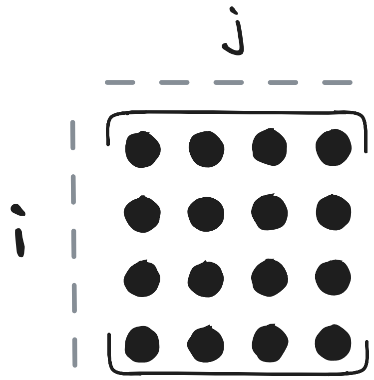
Vector



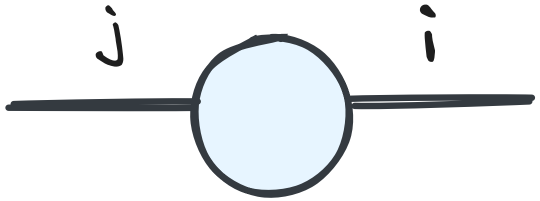
$$V_i$$



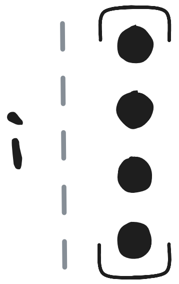
Matrix

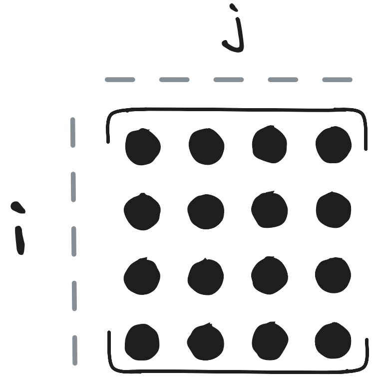


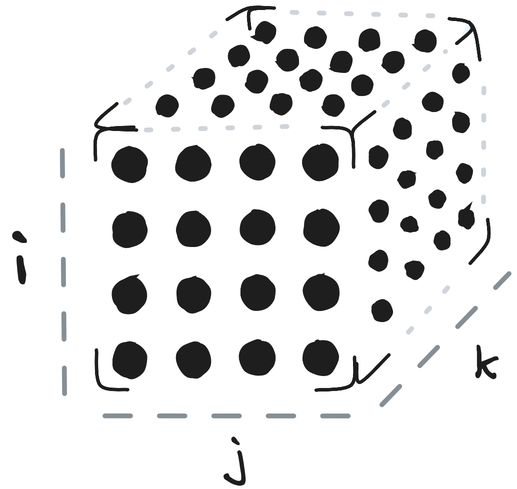
$$M_{ij}$$



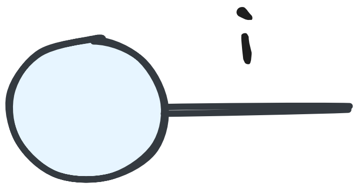
Vector | Matrix | 3-order tensor

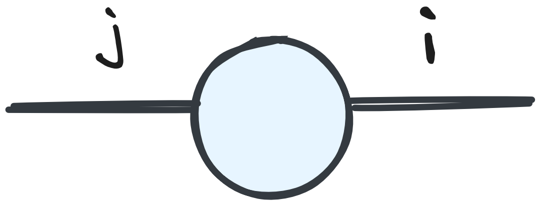


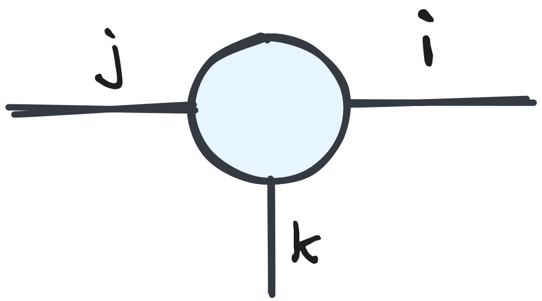




V_i | M_{ij} | T_{ijk}

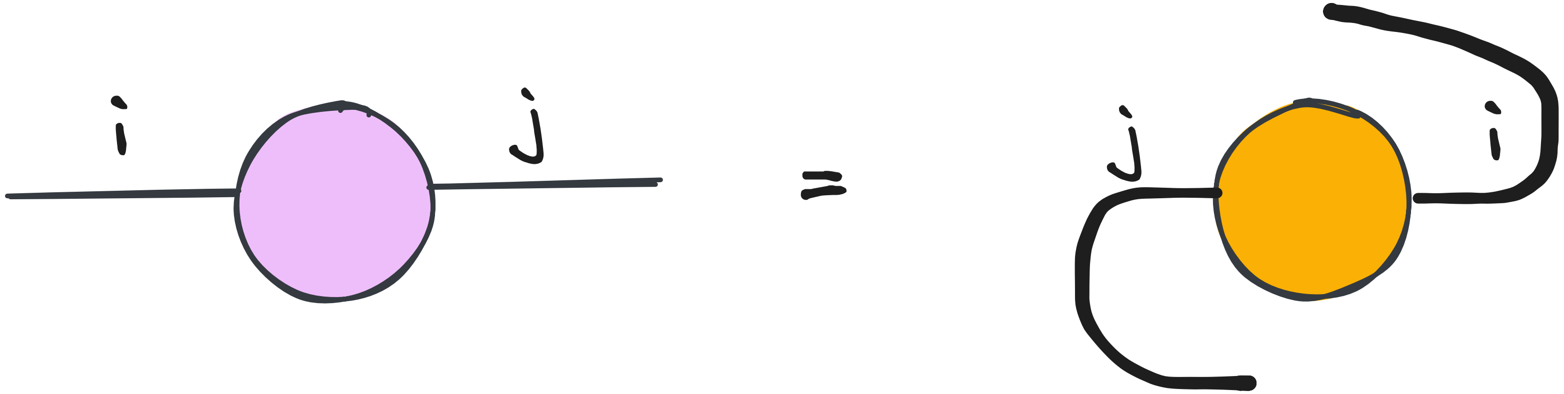






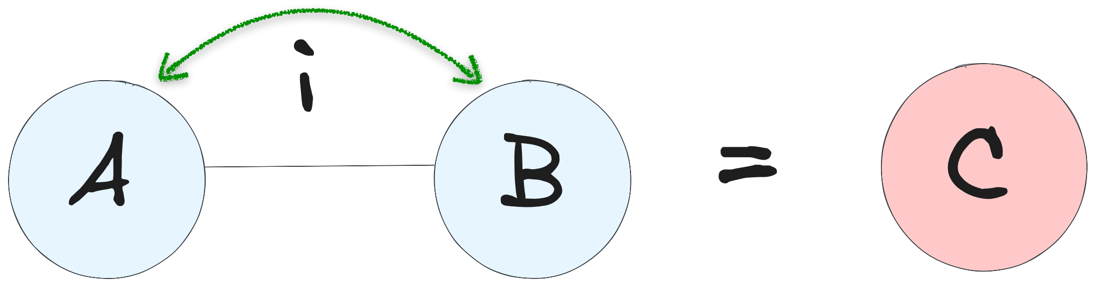
Transposition

$$A^T_{ij} = A_{ji}$$



Contraction

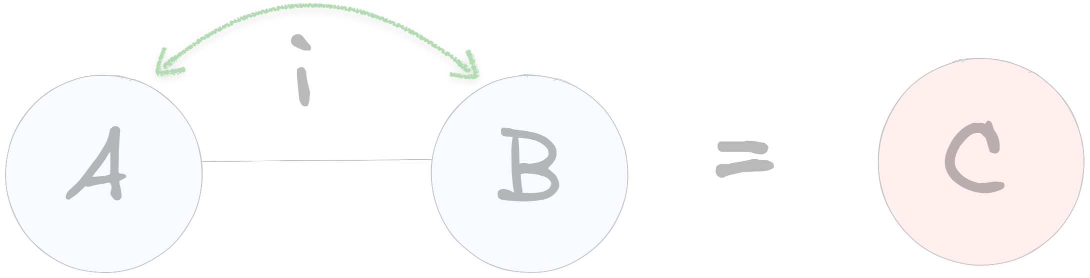
V - V



$$\sum_i A_i B_i$$

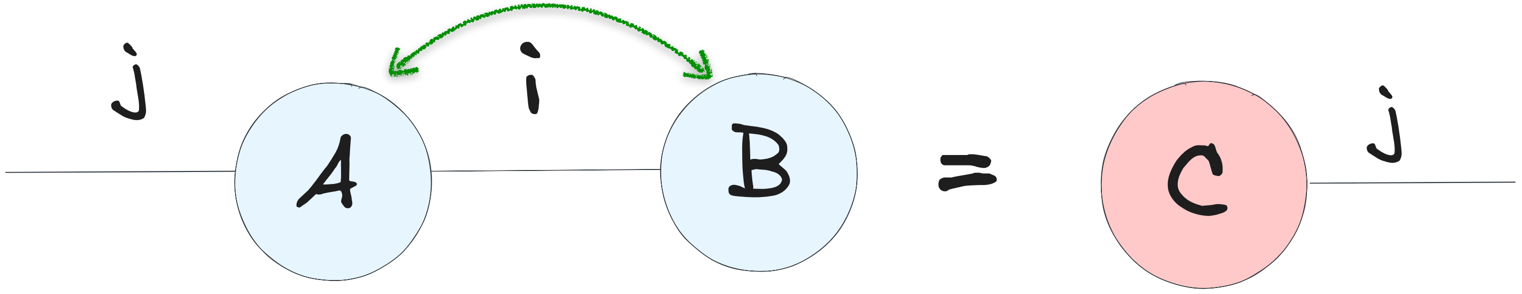
Contraction

V - V



$$\sum_i A_i B_i$$

M - V



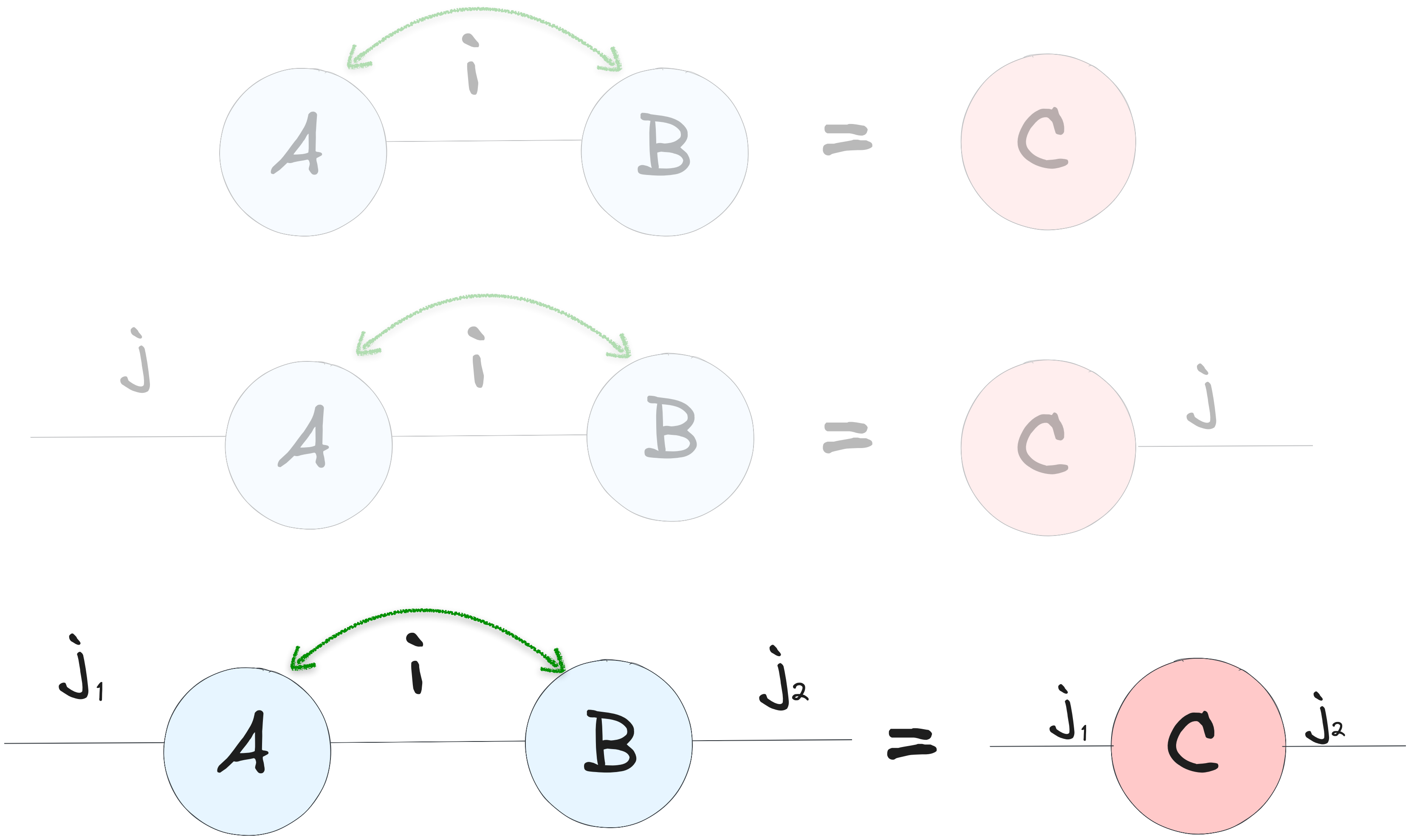
$$\sum_i A_{ji} B_i$$

Contraction

V - V

M - V

M - M



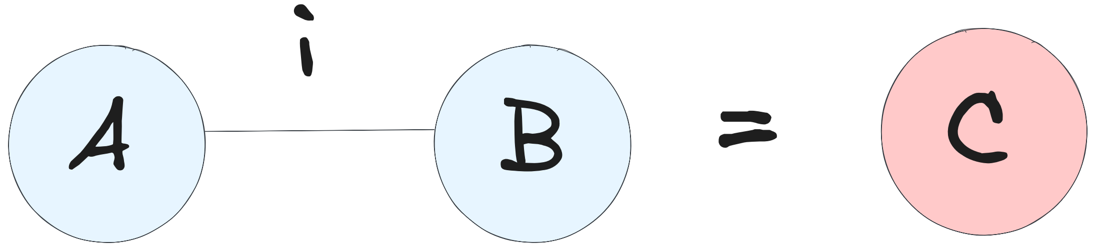
$$\sum_i A_i B_i$$

$$\sum_i A_{ji} B_i$$

$$\sum_i A_{j_1 i} B_{i j_2}$$

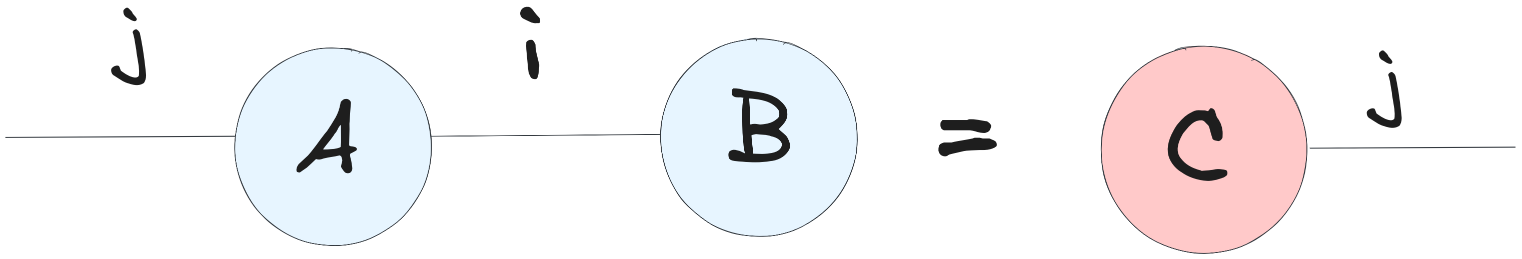
Contraction

V - V



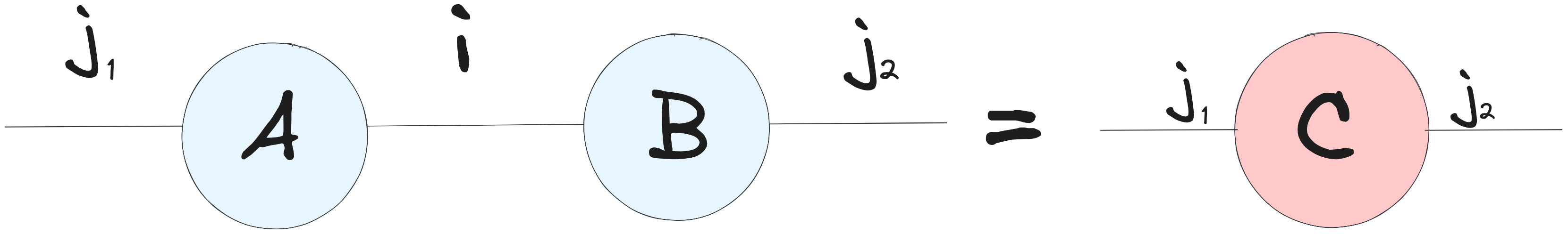
$$\sum_i A_i B_i$$

M - V



$$\sum_i A_{ji} B_i$$

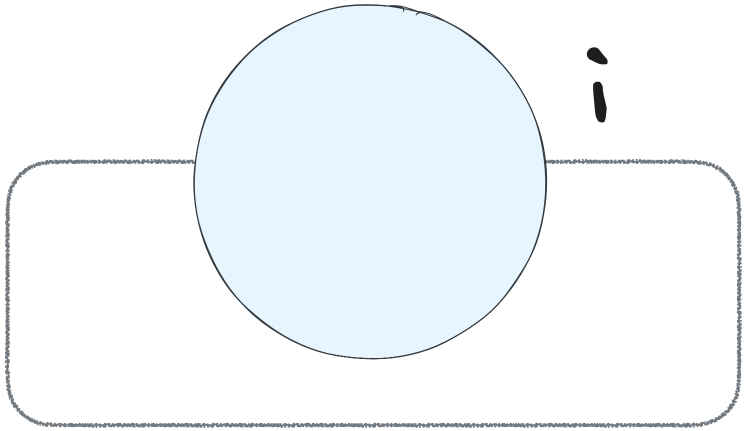
M - M



$$\sum_i A_{j_1 i} B_{i j_2}$$

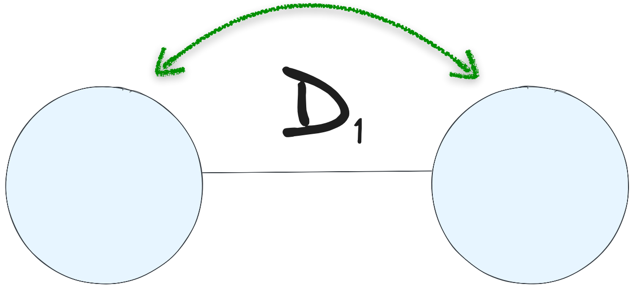
Trace

$$tr(A) = \sum_i A_{ii}$$



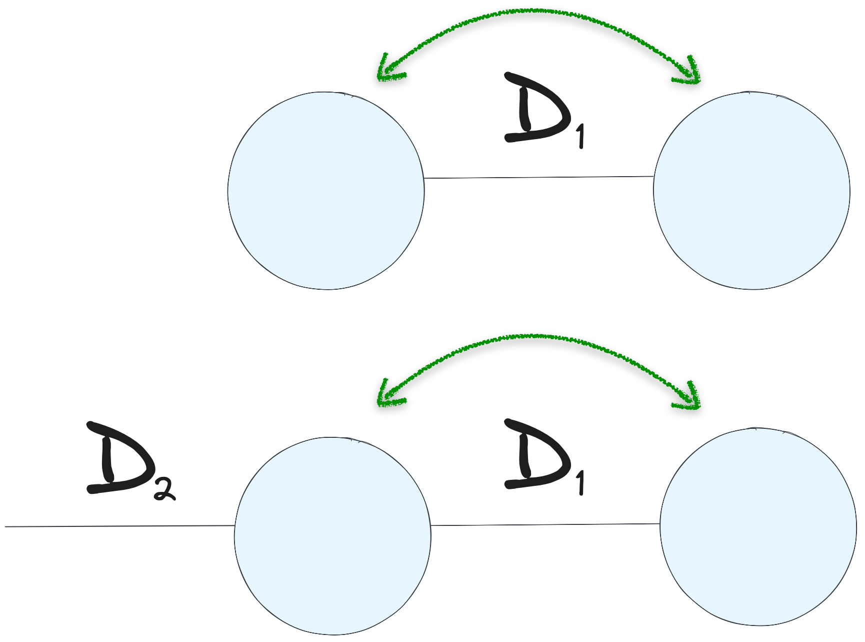
Contraction complexity

$\mathcal{O}(D_1)$



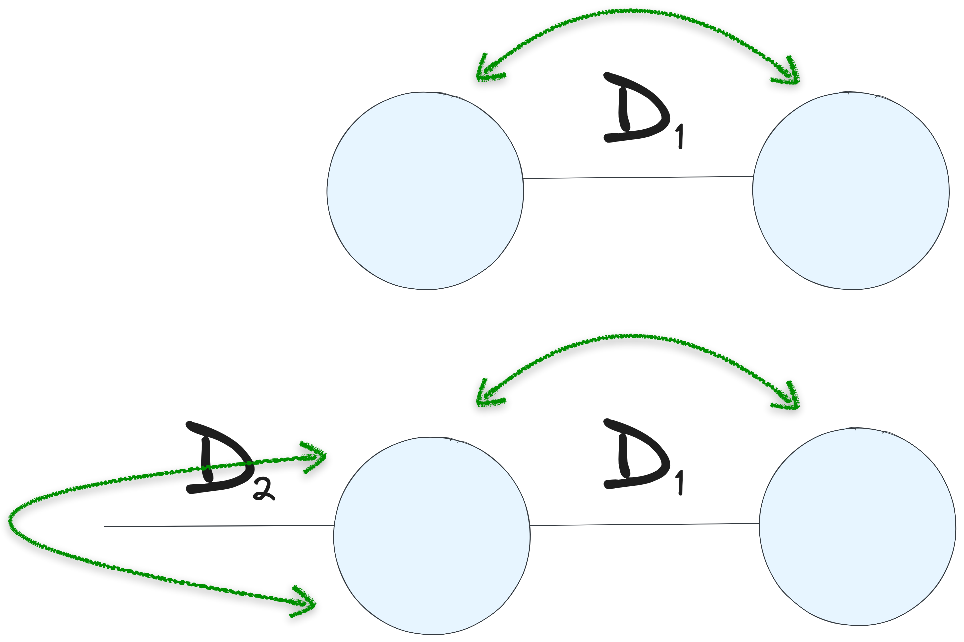
Contraction complexity

$$\mathcal{O}(D_1)$$



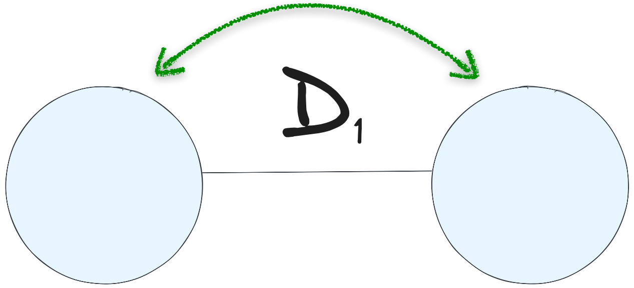
Contraction complexity

$\mathcal{O}(D_1)$

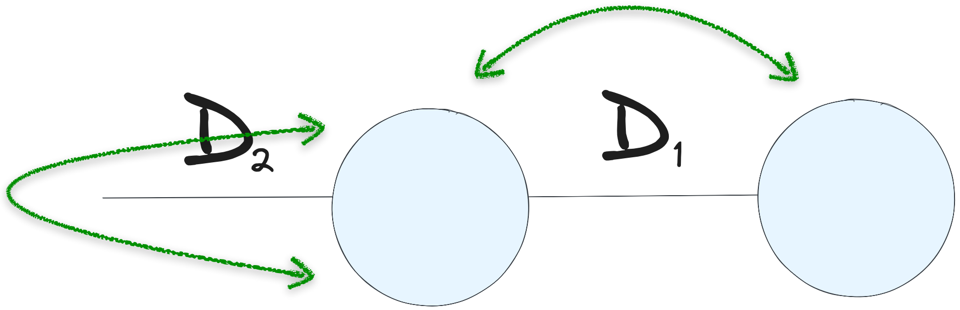


Contraction complexity

$$\mathcal{O}(D_1)$$

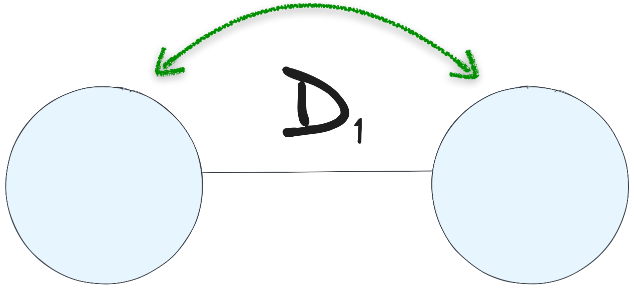


$$\mathcal{O}(D_1 D_2)$$

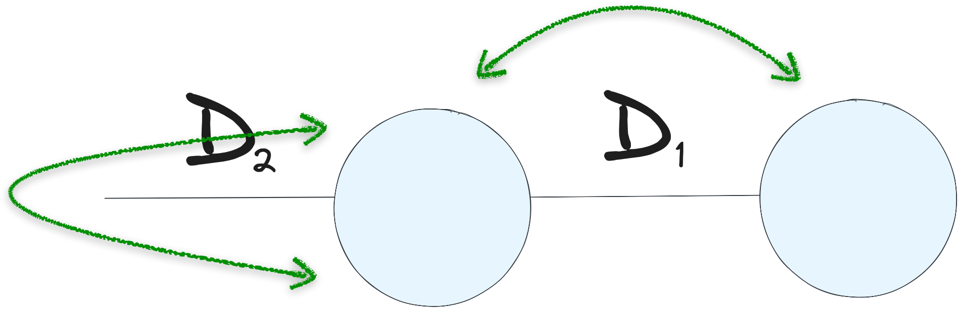


Contraction complexity

$$\mathcal{O}(D_1)$$



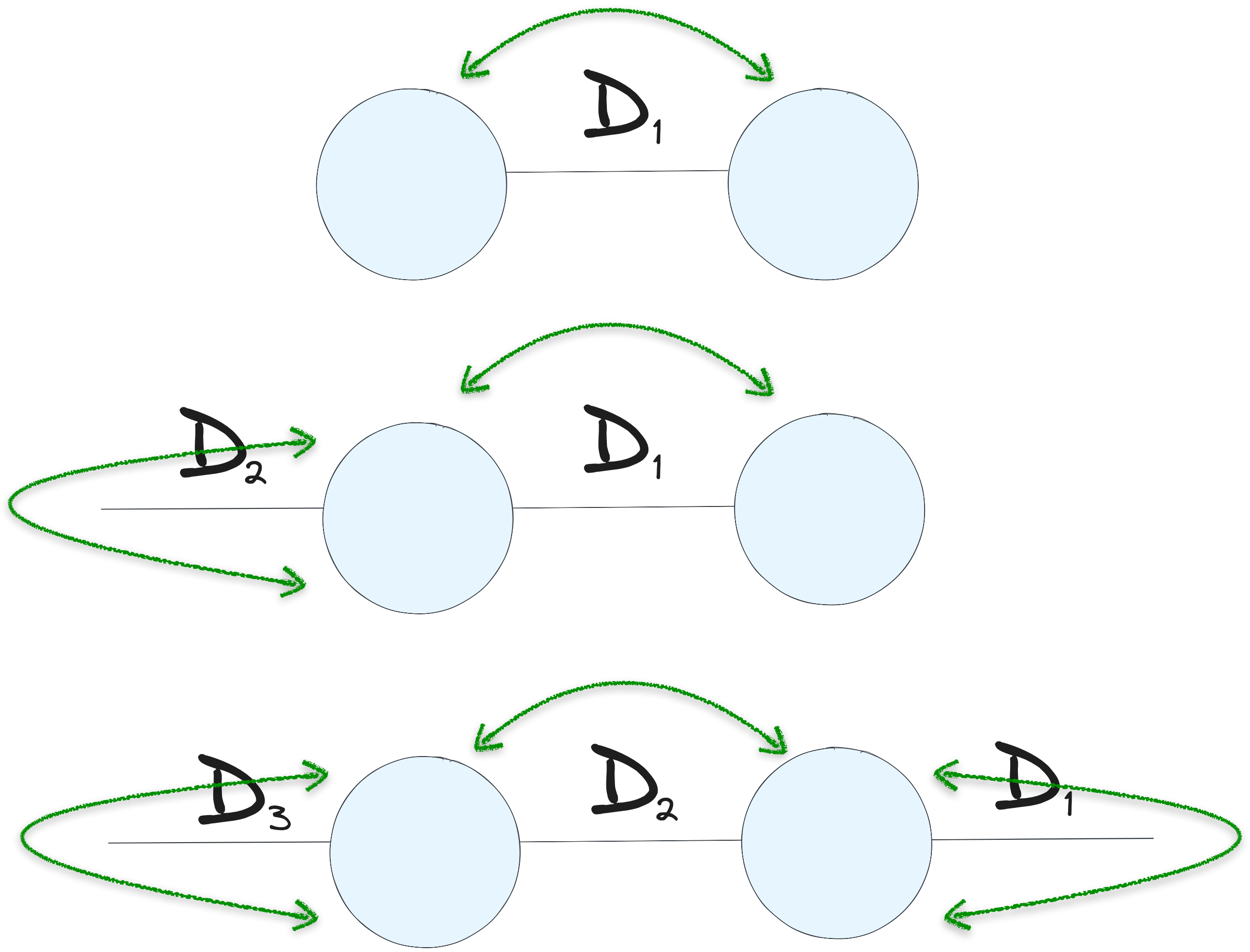
$$\mathcal{O}(D_1 D_2)$$



Contraction complexity

$$\mathcal{O}(D_1)$$

$$\mathcal{O}(D_1 D_2)$$

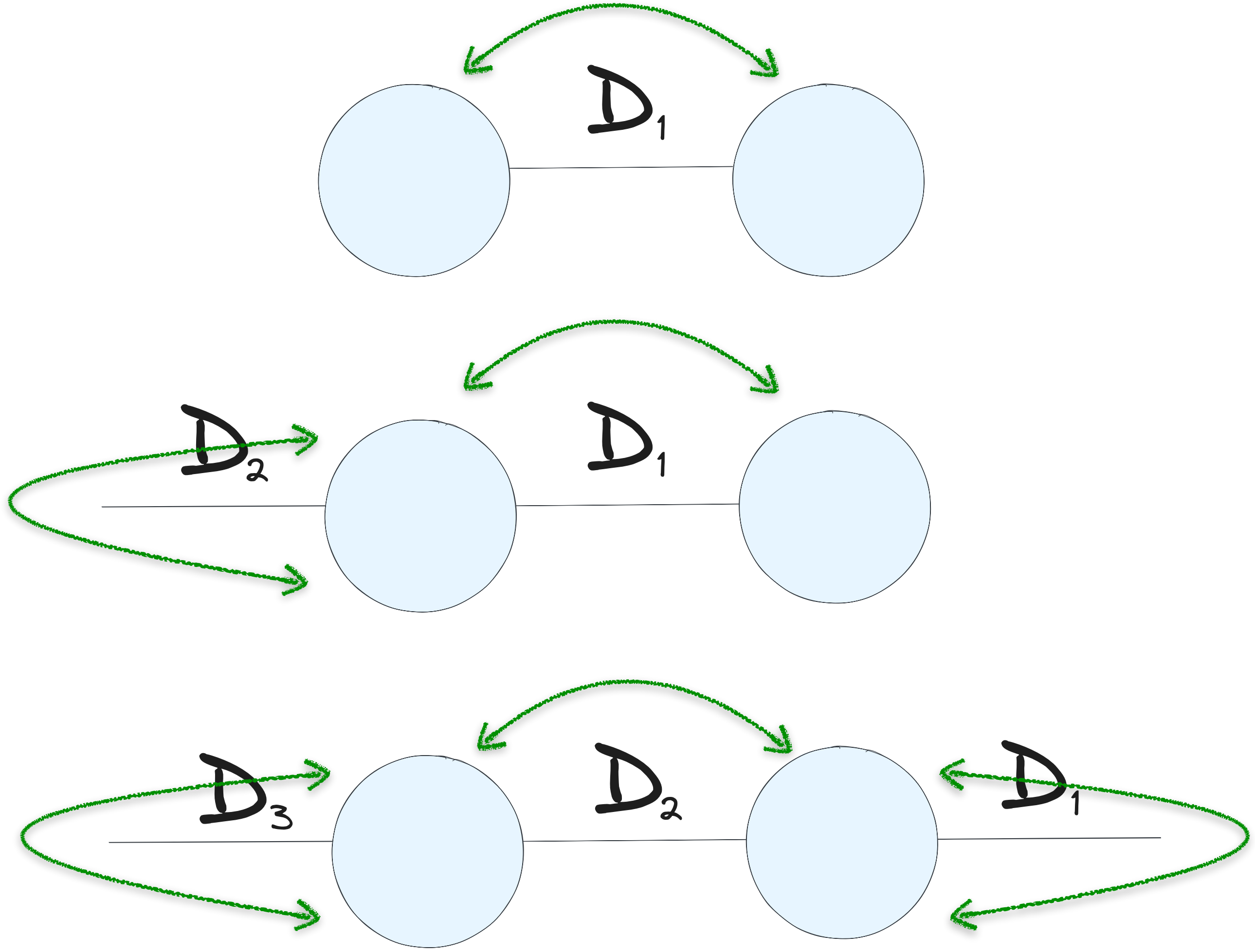


Contraction complexity

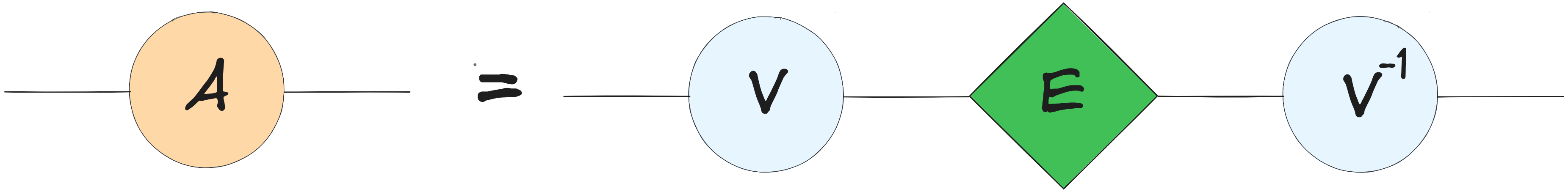
$$\mathcal{O}(D_1)$$

$$\mathcal{O}(D_1 D_2)$$

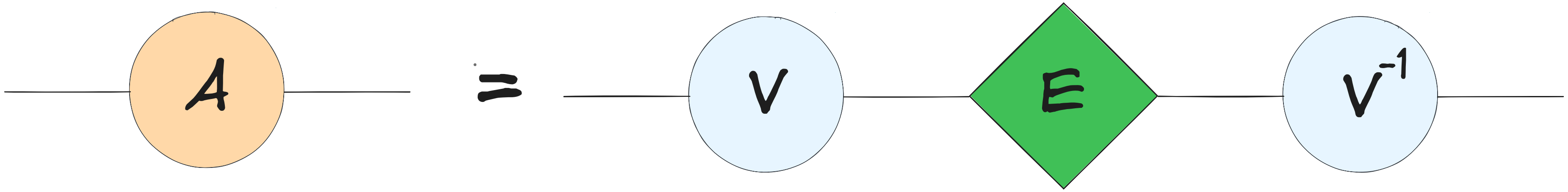
$$\mathcal{O}(D_1 D_2 D_3)$$



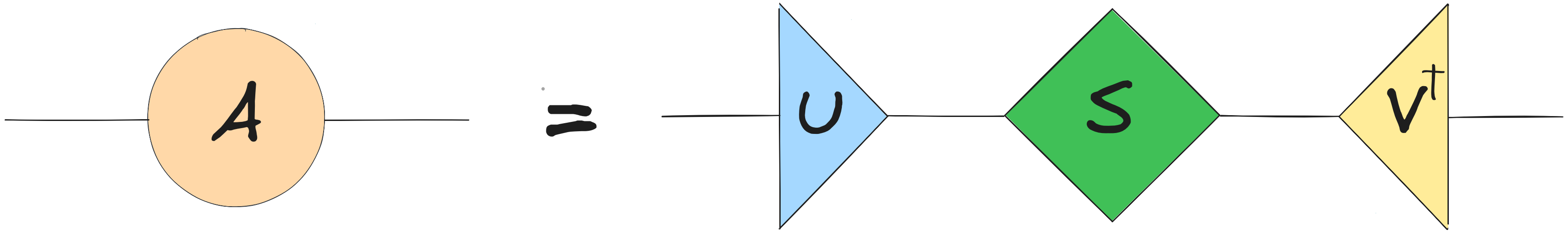
Eigendecomposition



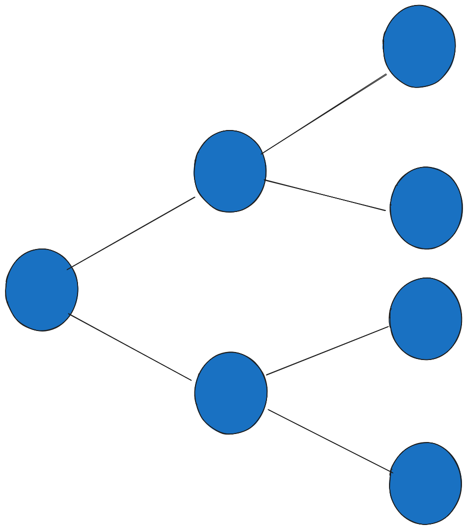
Eigendecomposition



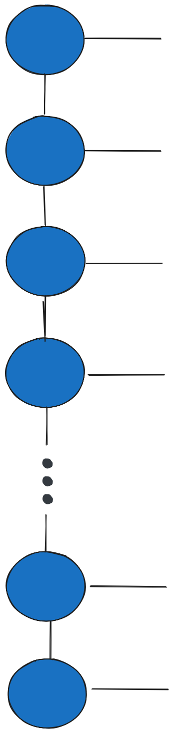
Singular Value Decomposition



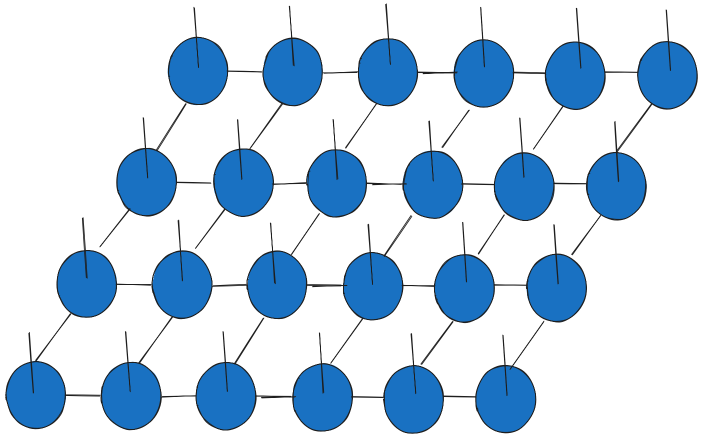
Tensor Networks



Tree

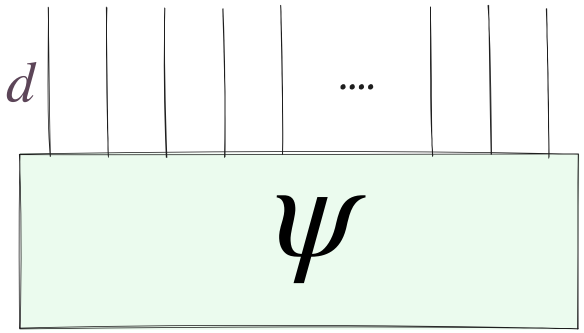


1D



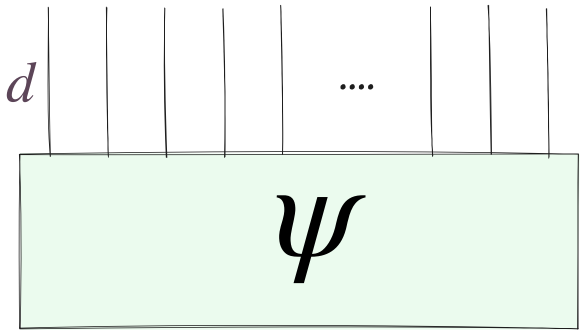
2D

Quantum state



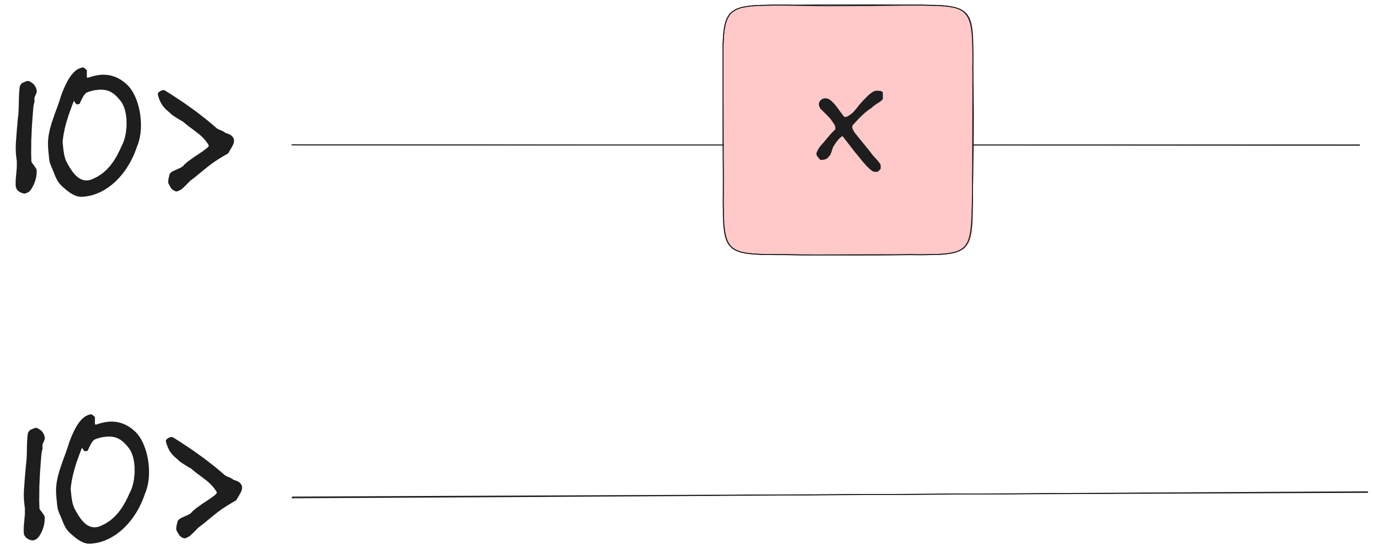
N-rank tensor of dimension d^N

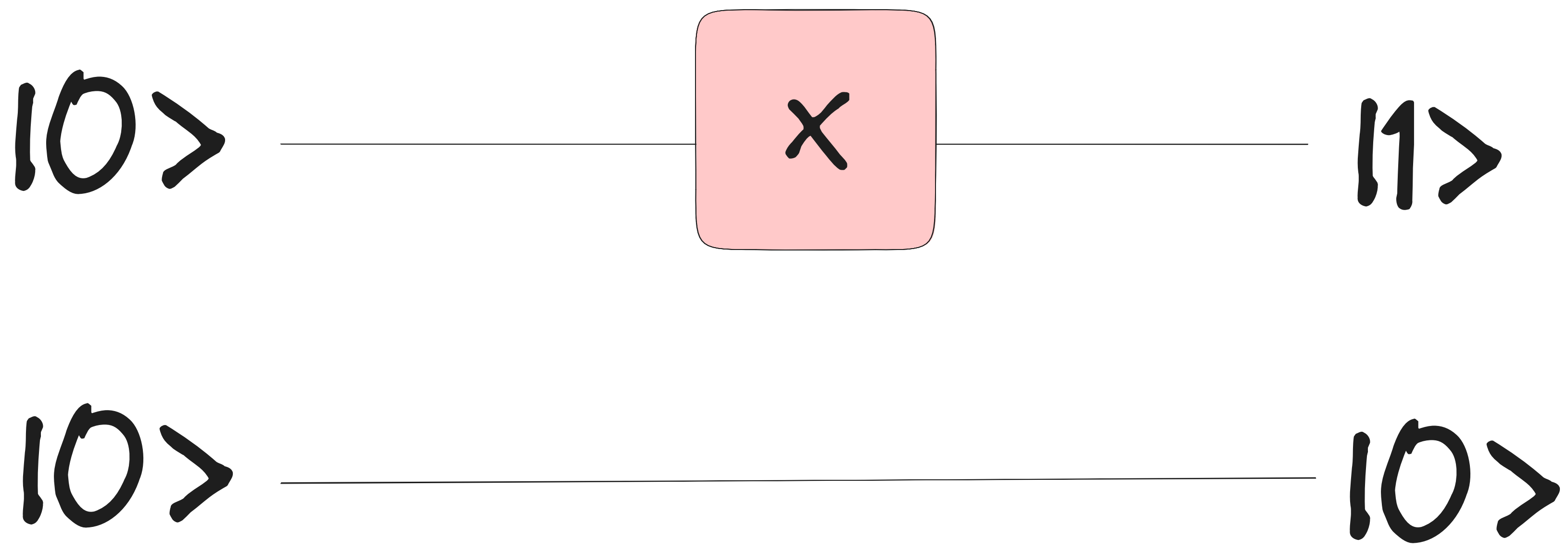
Quantum state



$$|\psi\rangle = \sum_{\{i_1,i_2...i_N\}=0}^{d-1} C_{i_1,i_2...i_N} |i_1, i_2 \ldots i_N\rangle$$

N-rank tensor of dimension d^N


|0⟩
X
|0⟩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X
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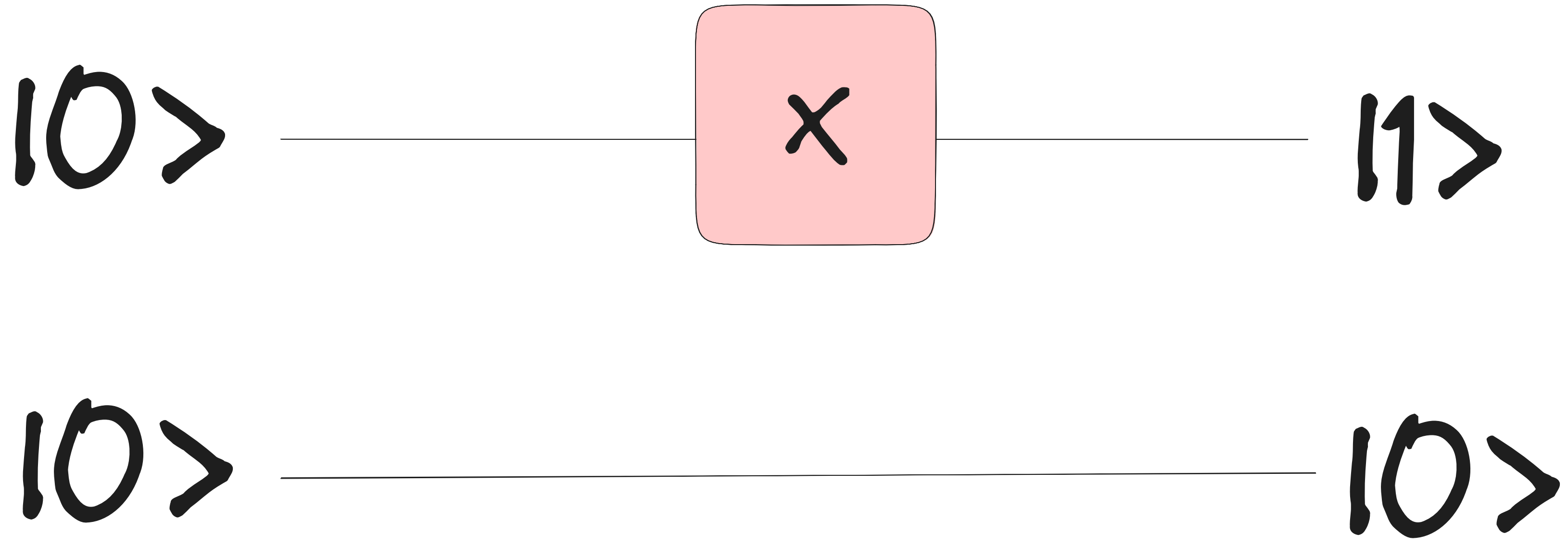
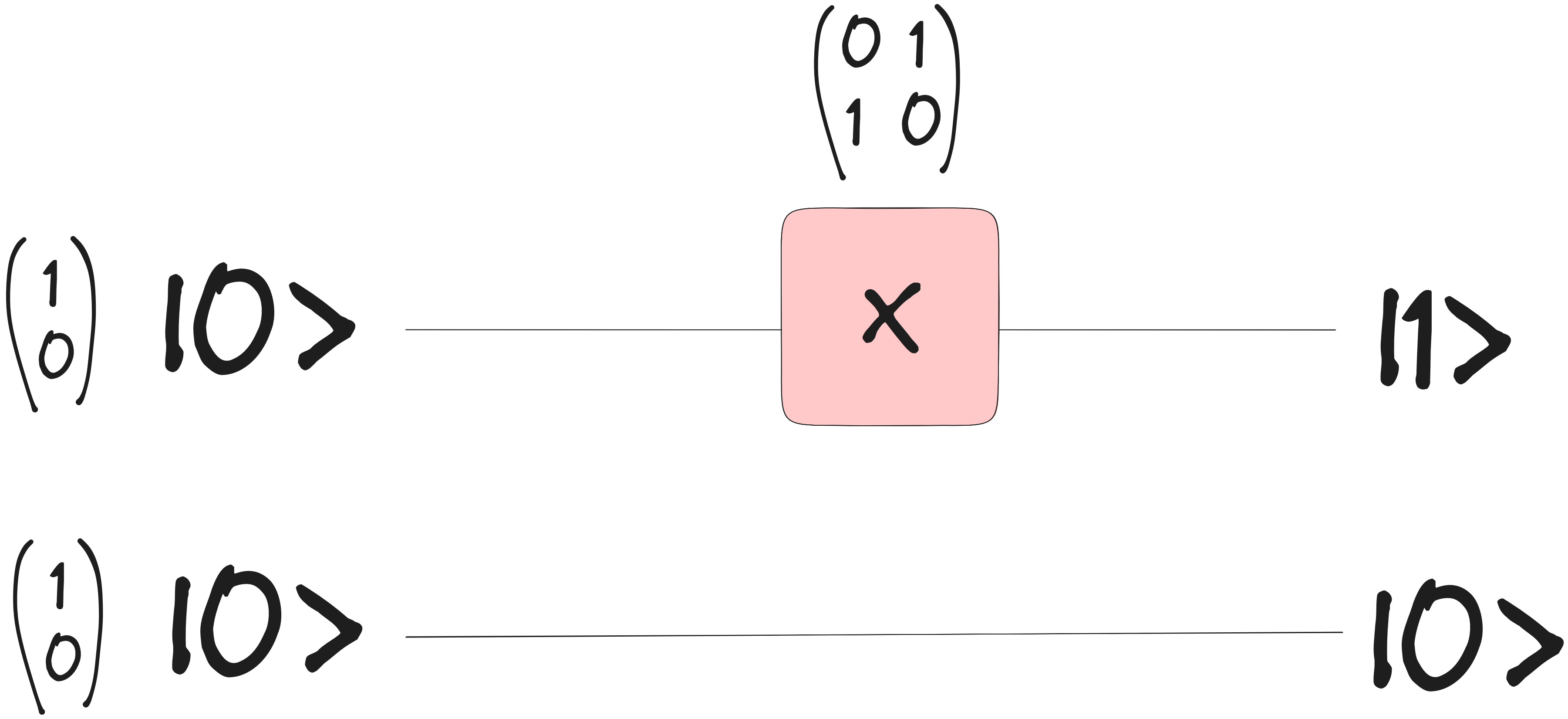
|0>



$$|\psi\rangle = |1\rangle \otimes |0\rangle$$



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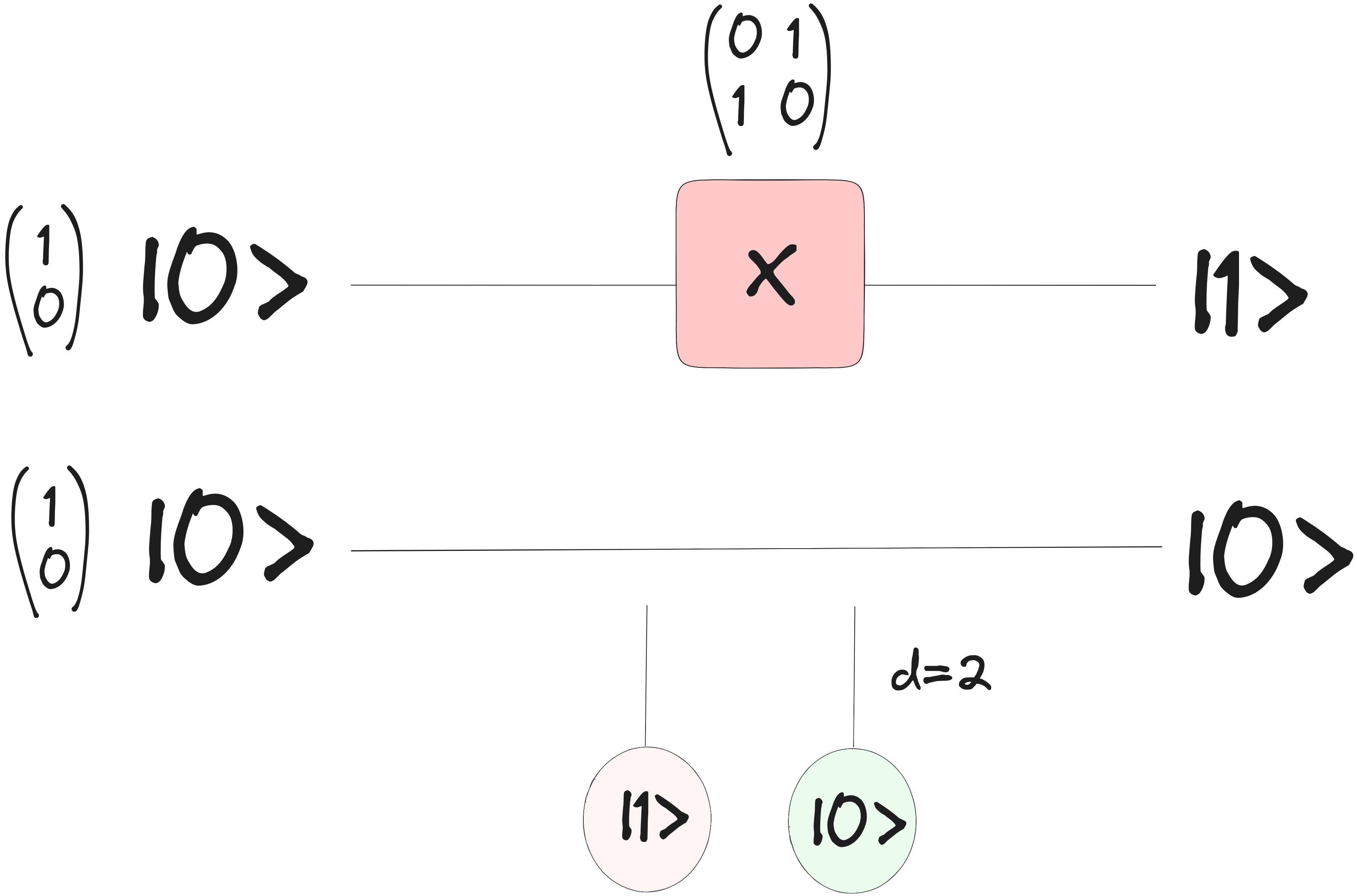
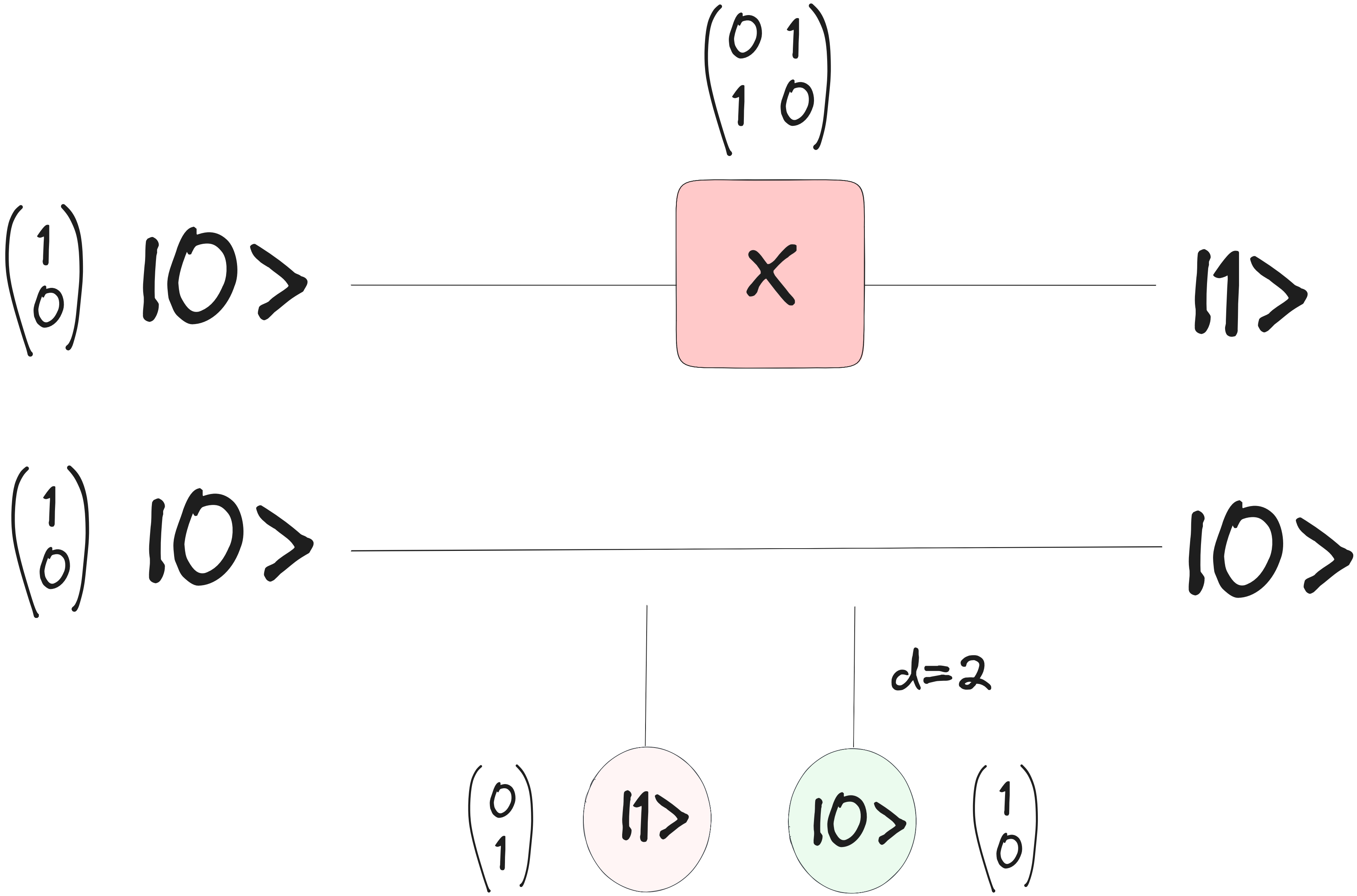
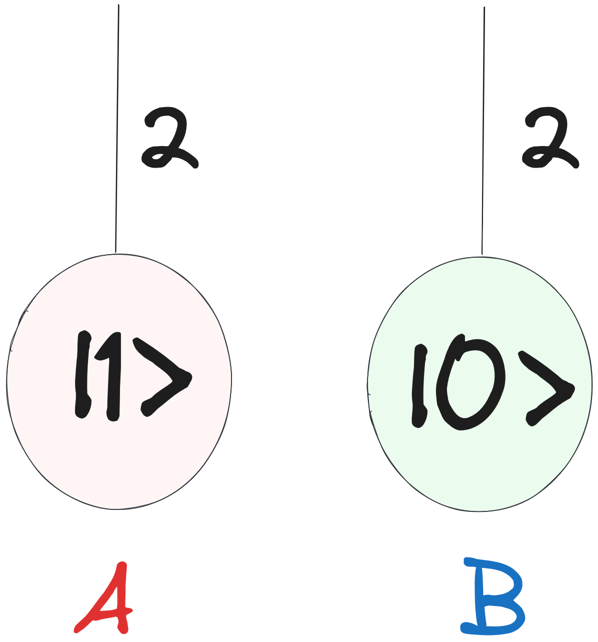


$\begin{pmatrix}0&1\\1&0\end{pmatrix}$
$\begin{pmatrix}1\\0\end{pmatrix}$ |0>
X
|1>
$\begin{pmatrix}1\\0\end{pmatrix}$ |0>
|0>
d=2
|1>
|0>



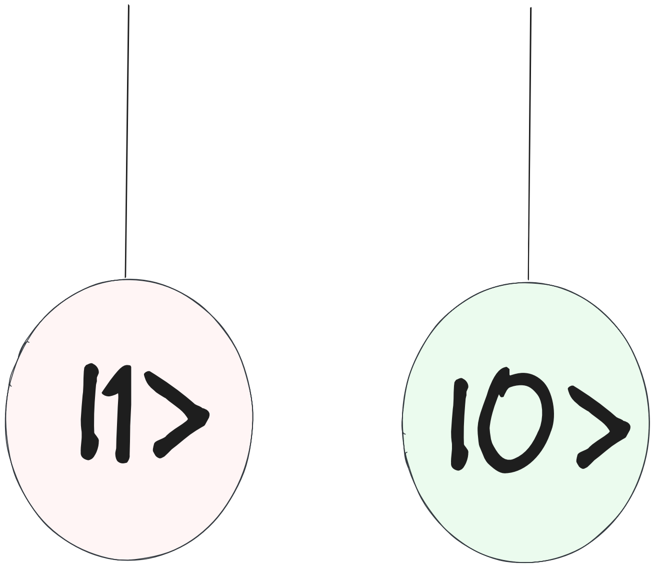
$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\begin{pmatrix}1\\0\end{pmatrix}$ |0>
X
|1>
$\begin{pmatrix}1\\0\end{pmatrix}$ |0>
|0>
d=2
$\begin{pmatrix}0\\1\end{pmatrix}$ |1>
|0> $\begin{pmatrix}1\\0\end{pmatrix}$



$$A \otimes B$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

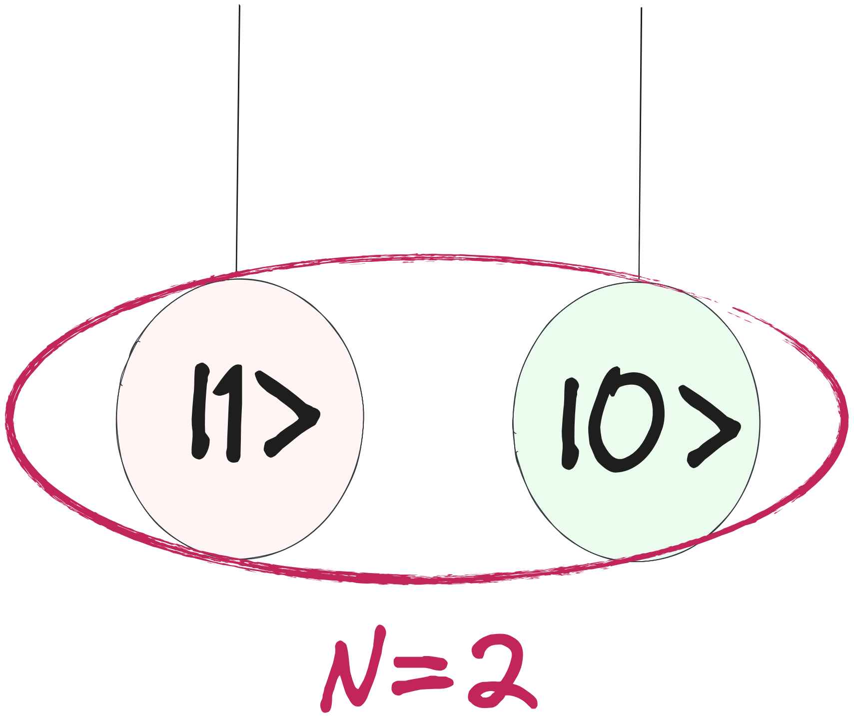
Product State



Number of parameters

$$\mathcal{O}(Nd)$$

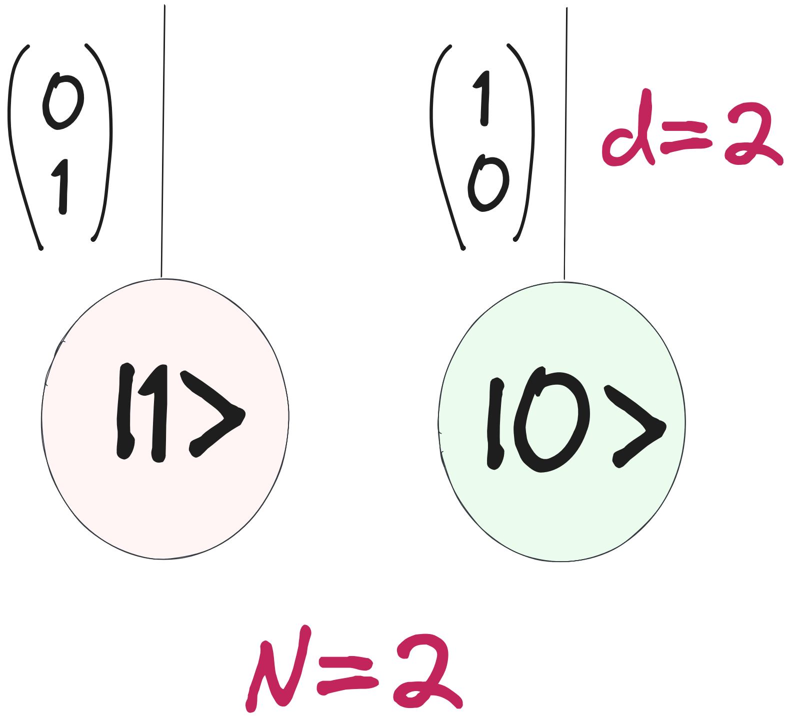
Product State



Number of parameters

$$\mathcal{O}(Nd)$$

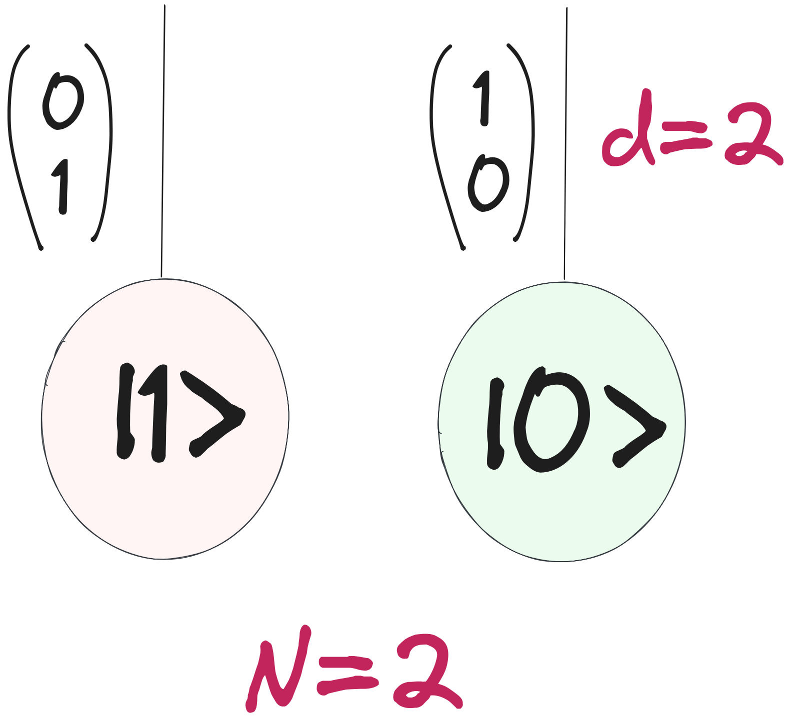
Product State



Number of parameters

$$\mathcal{O}(Nd)$$

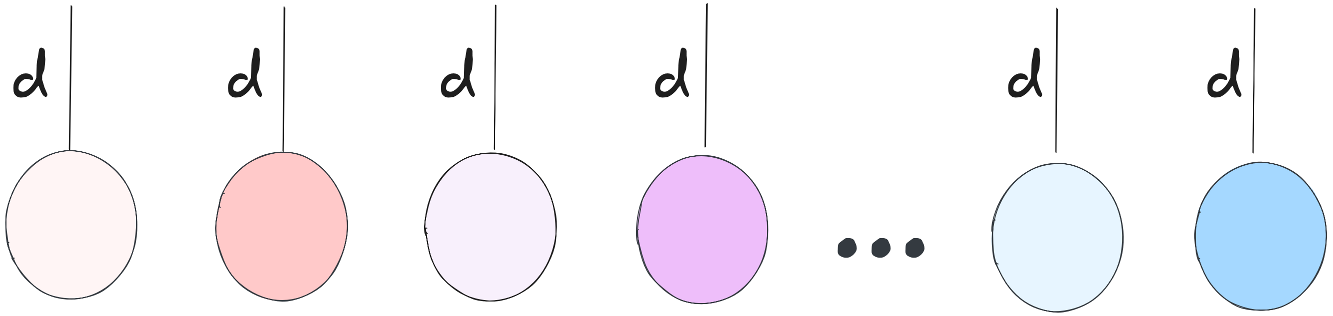
Product State



Number of parameters

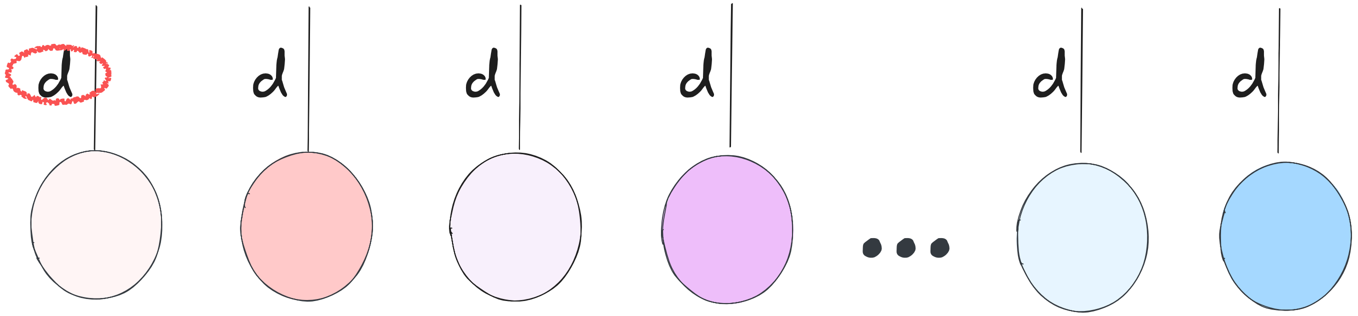
4

Product State

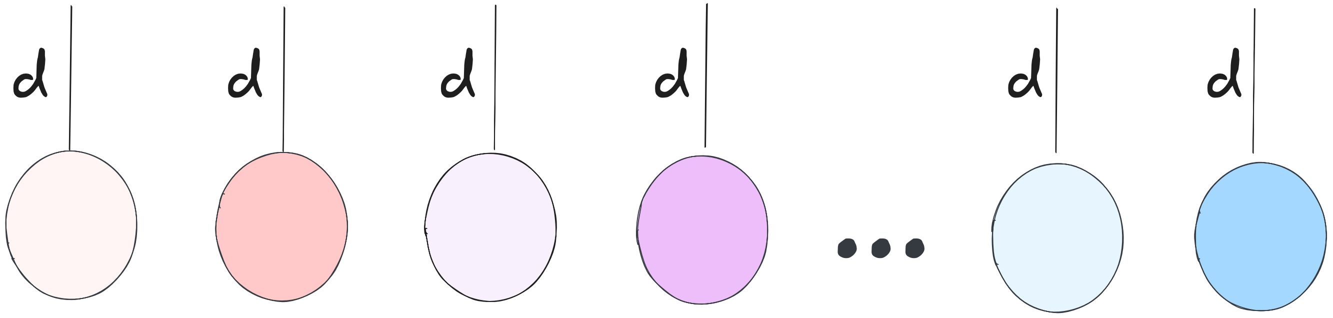


Product State

physical dimension

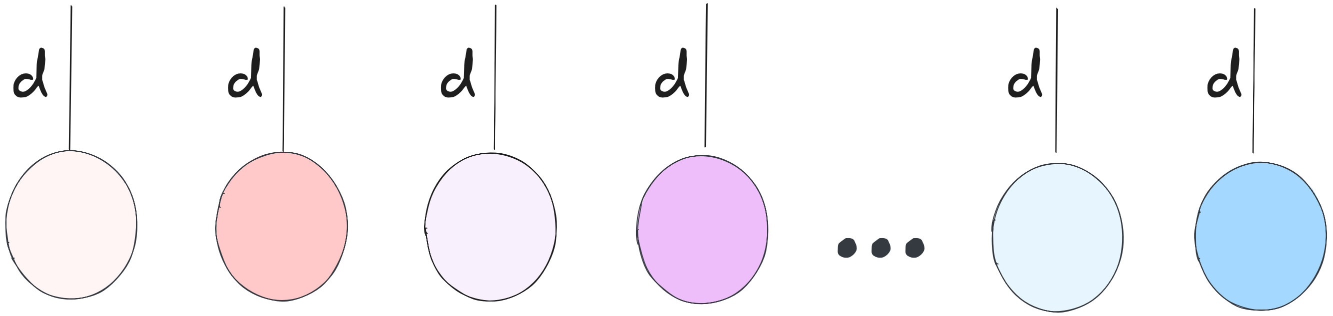


Product State



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

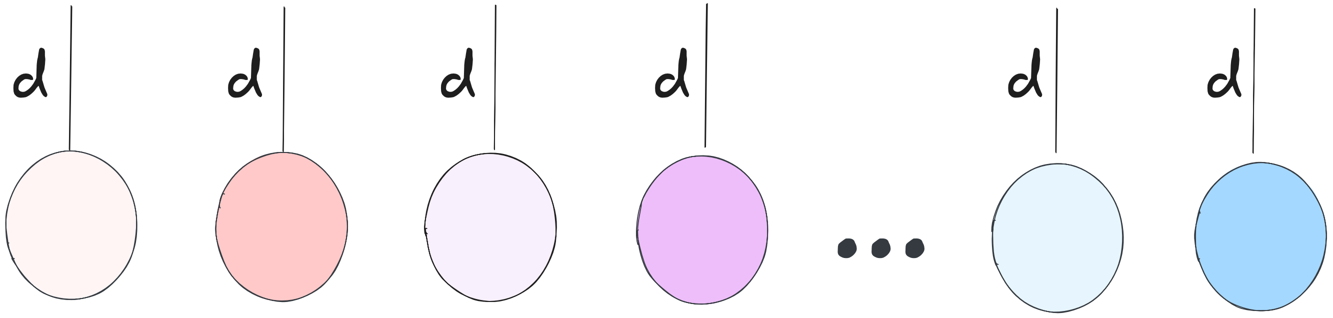
Product State



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

$$= \left(\sum_{i_1} c_1^{i_1} |i_1\rangle \right) \otimes \ldots \otimes \left(\sum_{i_N} c_N^{i_N} |i_N\rangle \right)$$

Product State



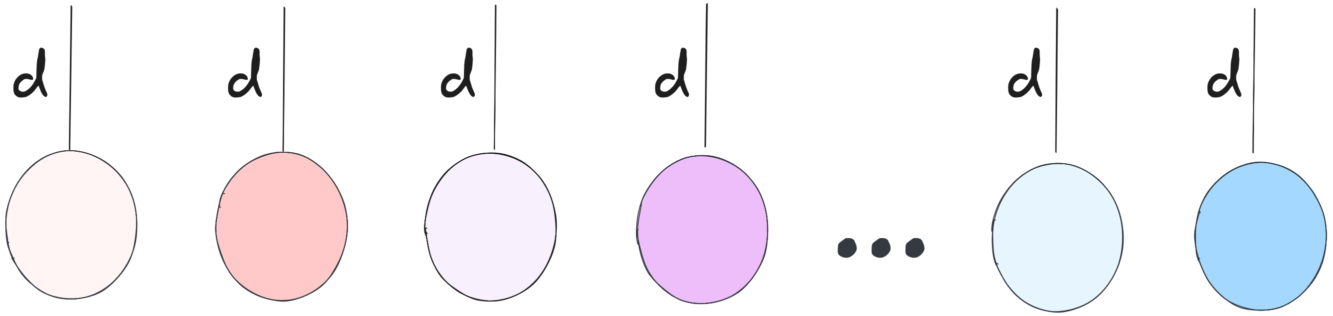
$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

$$= \left(\sum_{i_1} c_1^{i_1} |i_1\rangle \right) \otimes \ldots \otimes \left(\sum_{i_N} c_N^{i_N} |i_N\rangle \right)$$

$$|\Psi\rangle = |1\rangle \otimes |0\rangle$$

$$=$$

Product State



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

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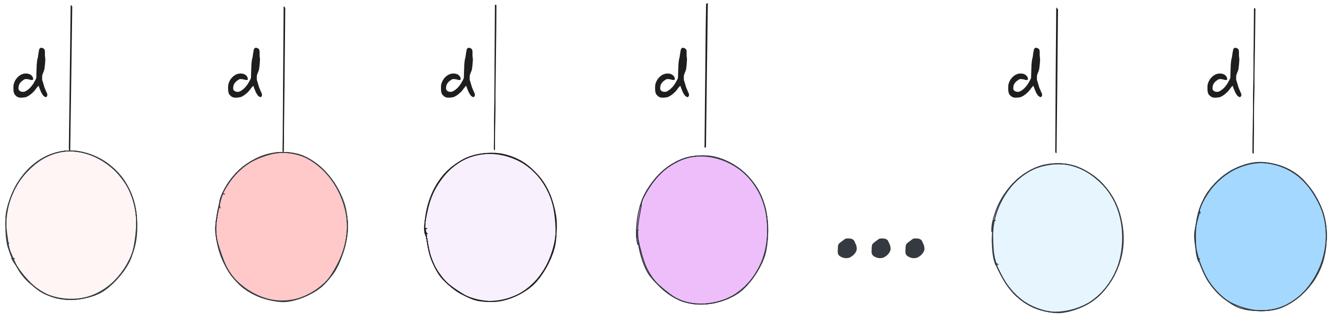
$$|\Psi\rangle = |1\rangle \otimes |0\rangle$$

$$= \left(0 \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \otimes$$

Product State



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

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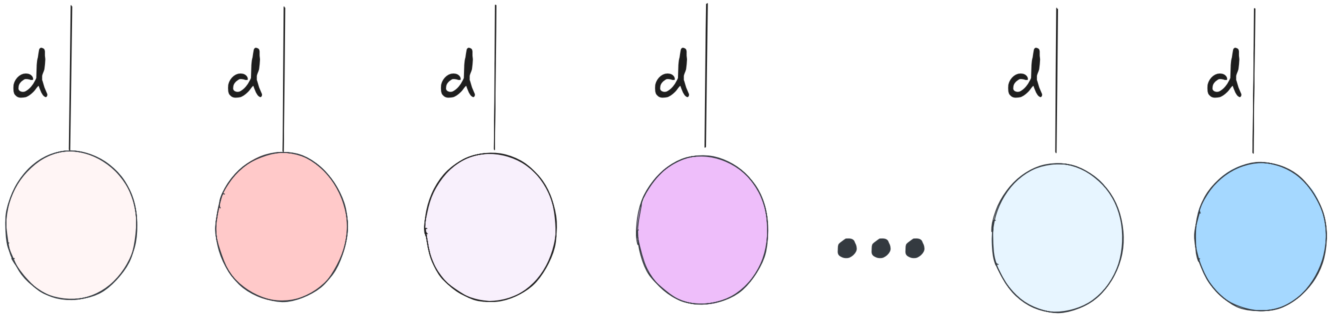
Product State



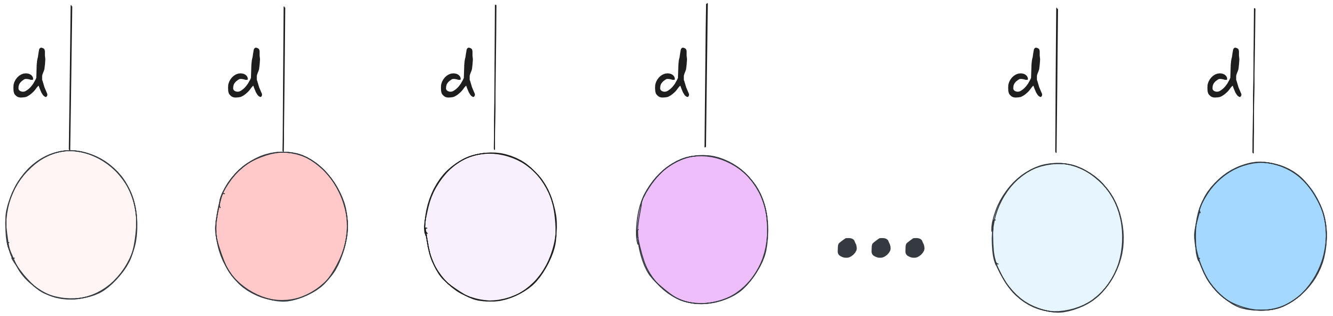
$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

$$= \left(\sum_{i_1} c_1^{i_1} |i_1\rangle \right) \otimes \ldots \otimes \left(\sum_{i_N} c_N^{i_N} |i_N\rangle \right)$$

$$|\Psi\rangle = |1\rangle \otimes |0\rangle$$

$$= \left(0 \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \otimes \left(1 \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

Product State



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

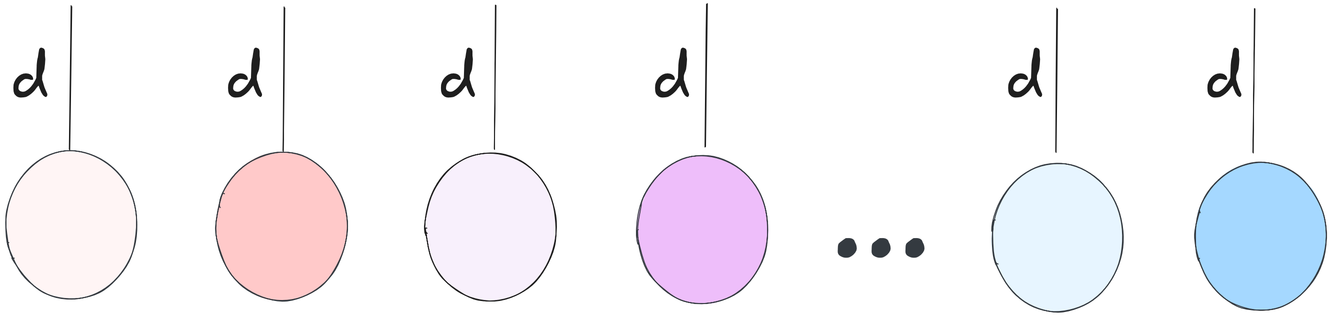
$$= \left(\sum_{i_1} c_1^{i_1} |i_1\rangle \right) \otimes \ldots \otimes \left(\sum_{i_N} c_N^{i_N} |i_N\rangle \right)$$

$$|\Psi\rangle = |1\rangle \otimes |0\rangle$$

$$= \left(0 \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \otimes$$

$$\left(1 \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

Product State

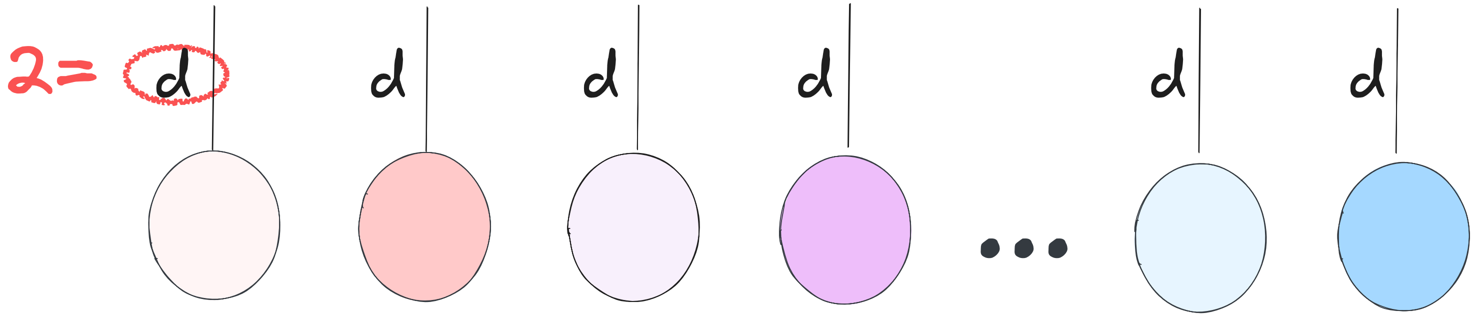


$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

$$= \left(\sum_{i_1} c_1^{i_1} |i_1\rangle\right) \otimes \ldots \otimes \left(\sum_{i_N} c_N^{i_N} |i_N\rangle\right) = \sum_{\{i_1, i_2 \ldots i_n\}=0}^{d-1} c_1^{i_1} c_2^{i_2} \ldots c_N^{i_N} |i_1, i_2 \ldots i_N\rangle$$

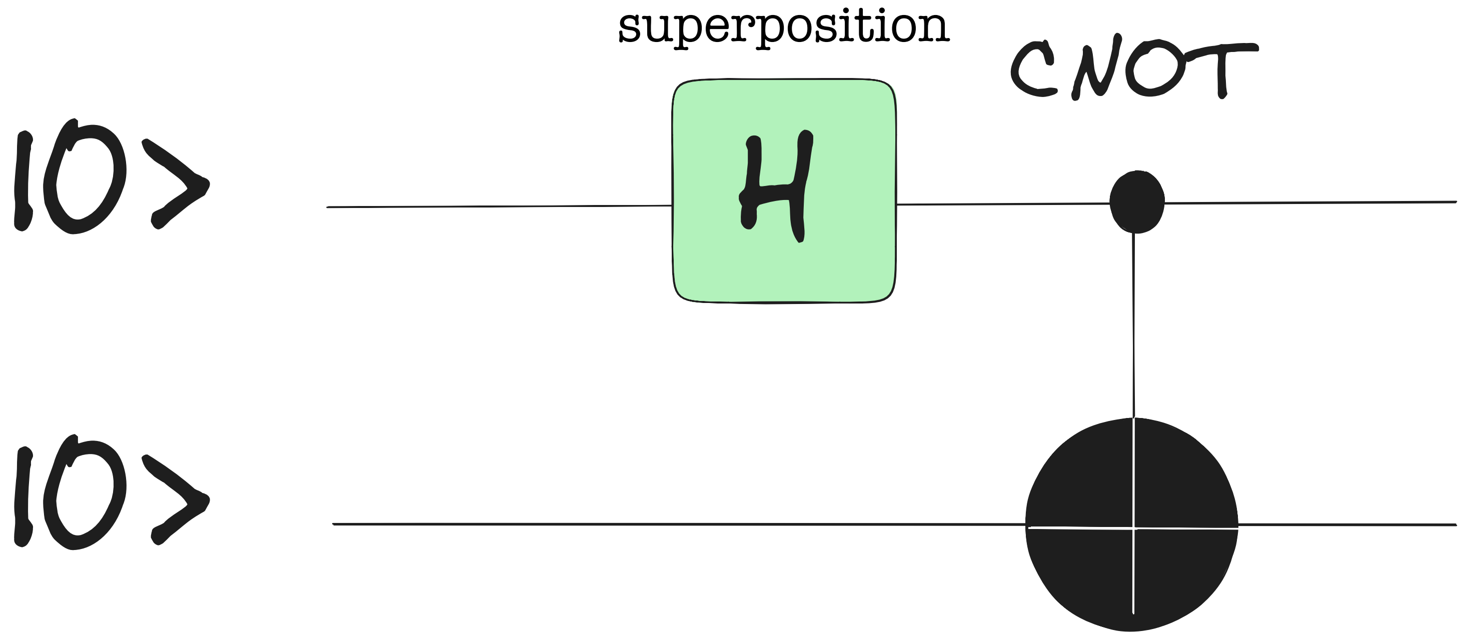
product of scalars

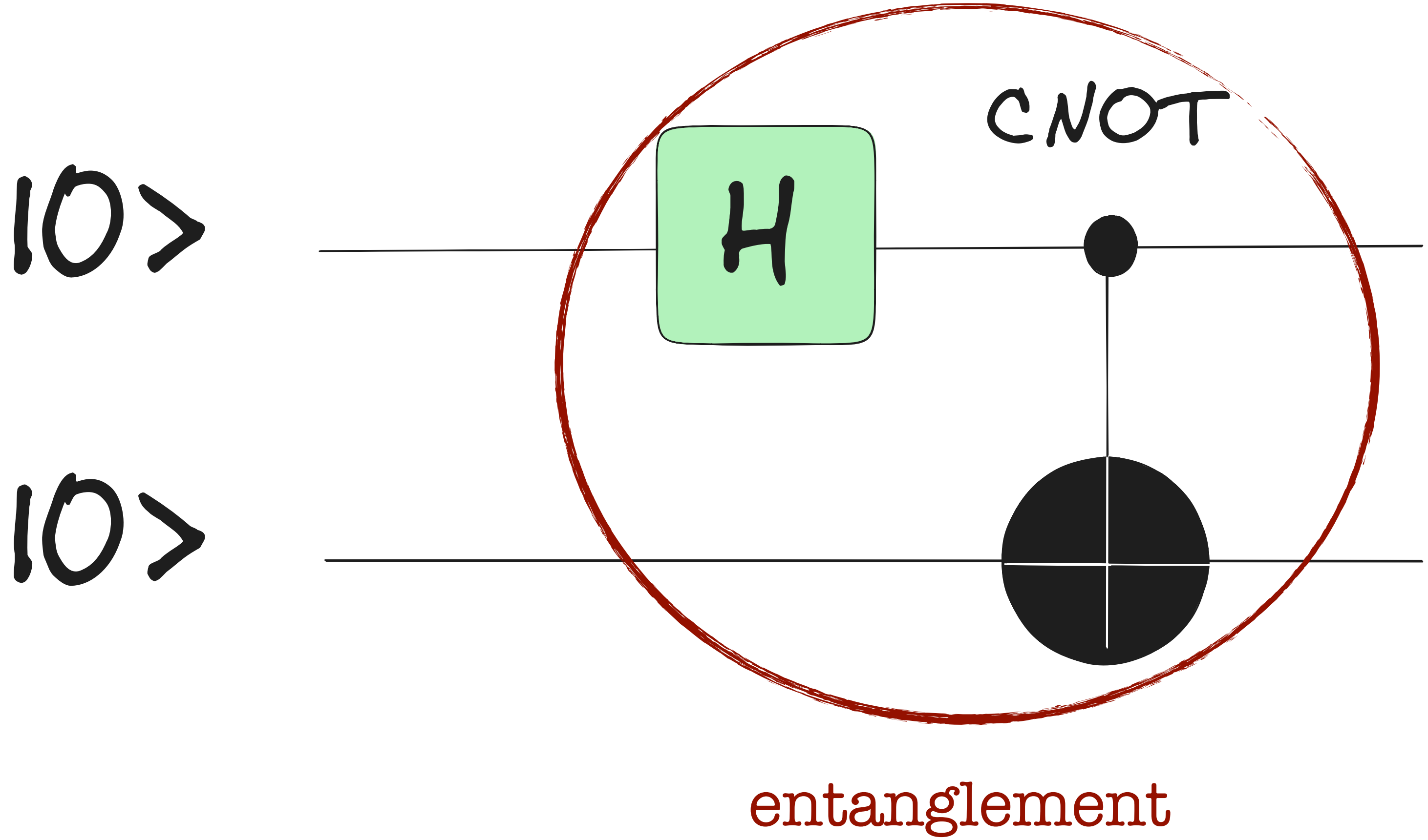
Product State

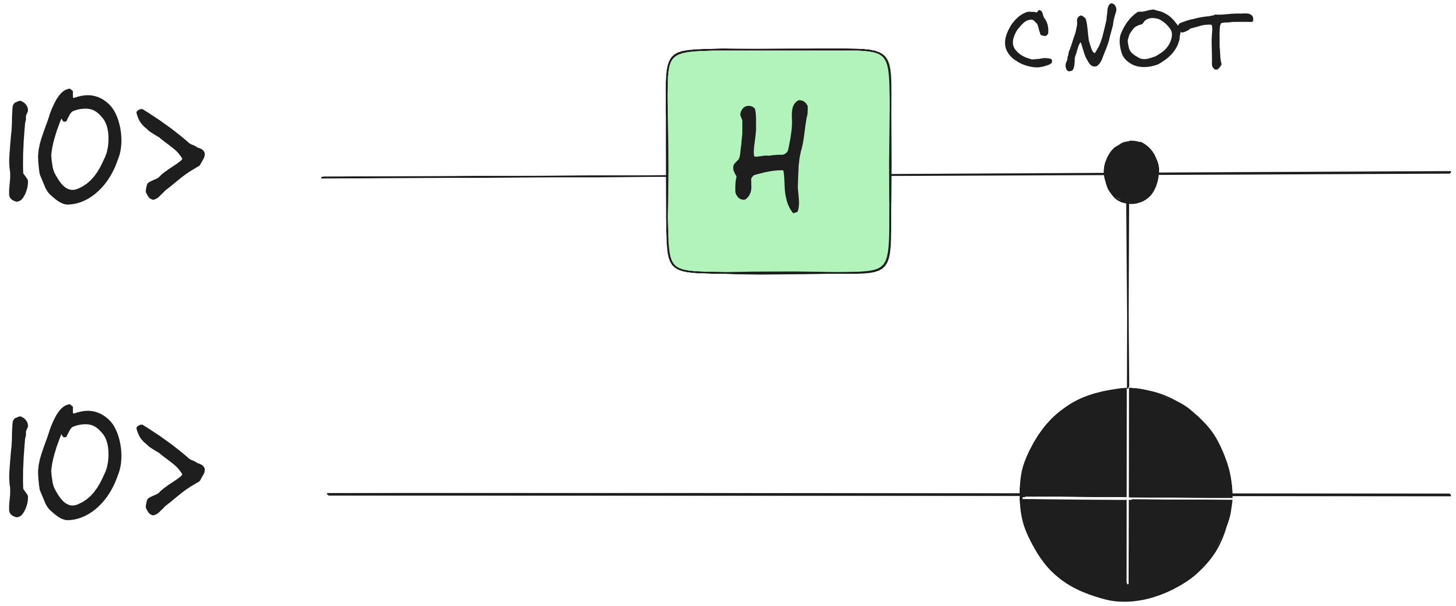


$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_N\rangle$$

$$= \left(\sum_{i_1} c_1^{i_1} |i_1\rangle\right) \otimes \ldots \otimes \left(\sum_{i_N} c_N^{i_N} |i_N\rangle\right) = \sum_{\{i_1, i_2 \ldots i_n\}=0}^{d-1} c_1^{i_1} c_2^{i_2} \ldots c_N^{i_N} |i_1, i_2 \ldots i_N\rangle$$

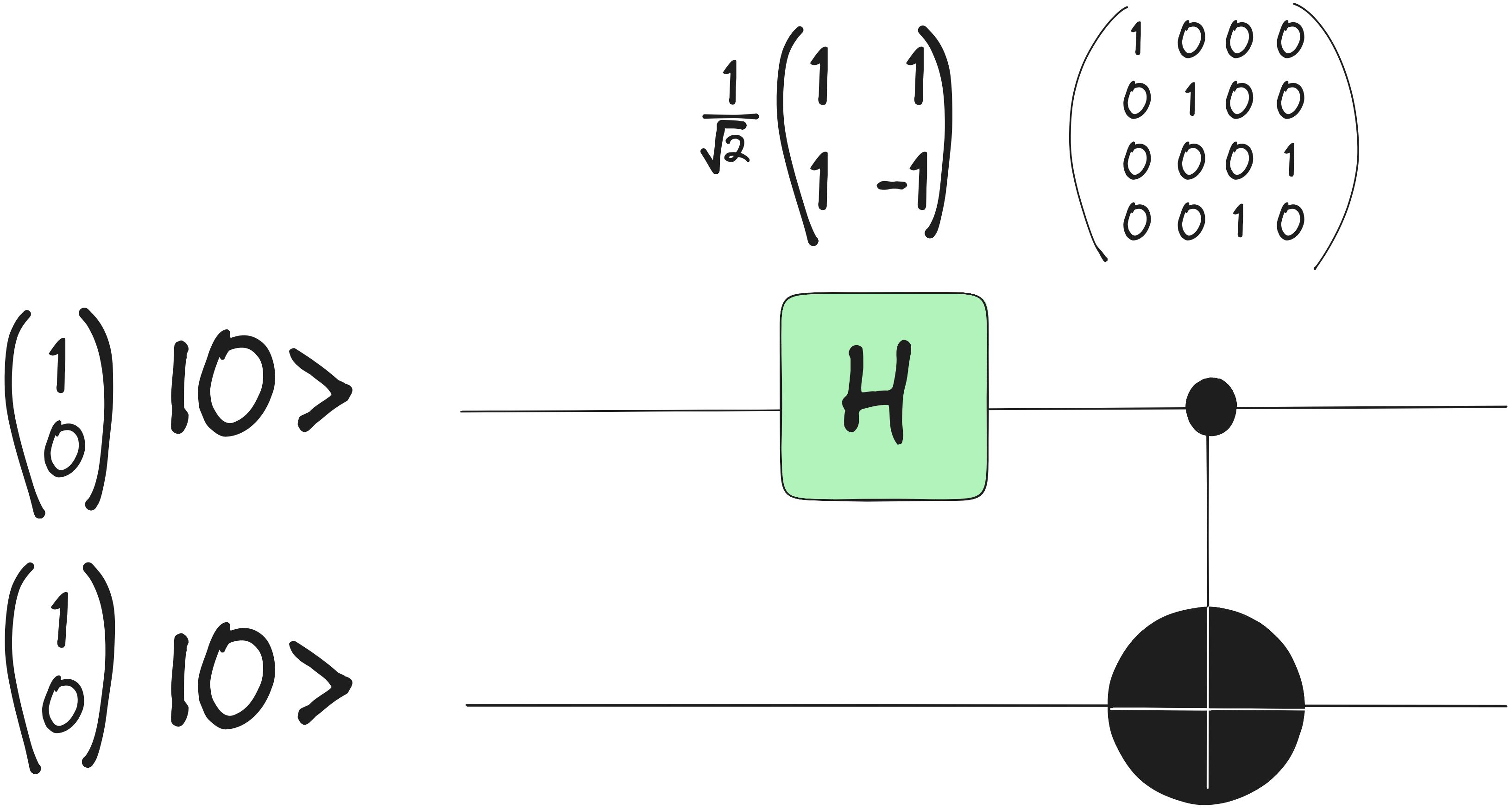

superposition
CNOT
|0>
|0>
H

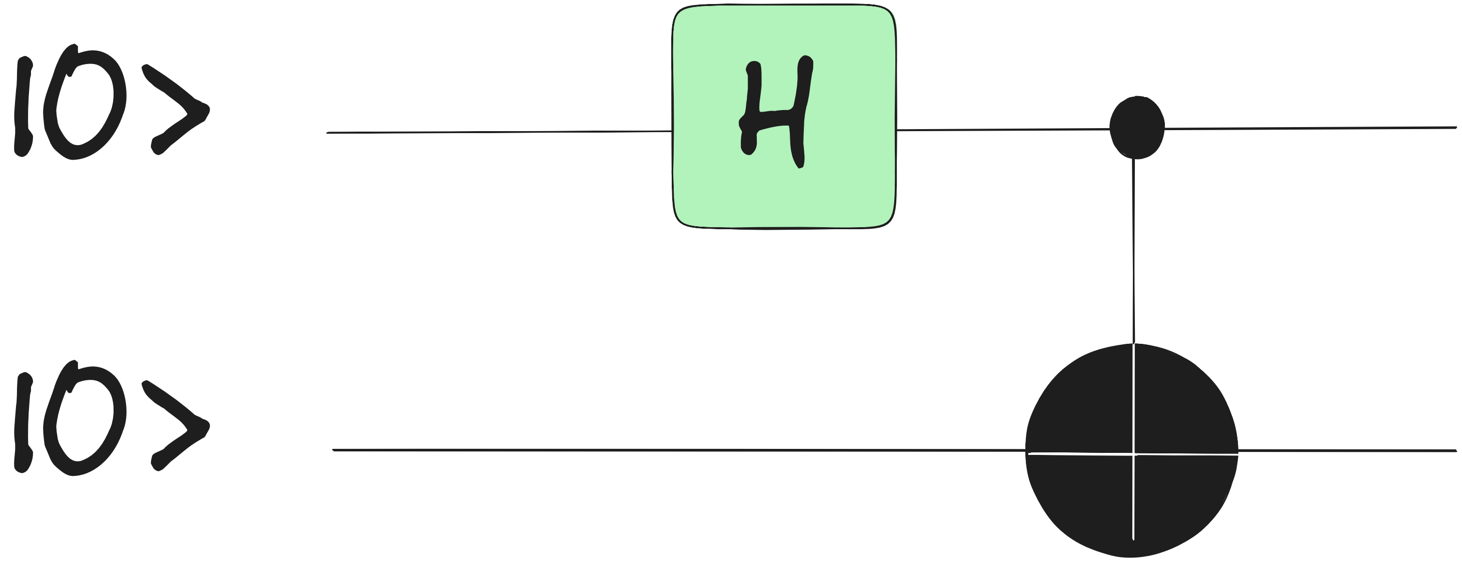

|0>
|0>
H
CNOT
entanglement



Bell state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$


$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$
$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$
$\begin{pmatrix}1\\0\end{pmatrix}$ |0>
$\begin{pmatrix}1\\0\end{pmatrix}$ |0>
H



$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

coefficient matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \xrightarrow{\text{reshape}} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

coefficient matrix

$1 \times |00\rangle$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \xrightarrow{\text{reshape}} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

coefficient matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \xrightarrow{\text{reshape}} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$1 \times |00\rangle$

$0 \times |01\rangle$

coefficient matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \xrightarrow{\text{reshape}} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$1 \times |00\rangle$

$0 \times |01\rangle$

$0 \times |10\rangle$

coefficient matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \xrightarrow{\text{reshape}} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$1 \times |00\rangle$

$0 \times |01\rangle$

$0 \times |10\rangle$

$1 \times |11\rangle$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \xrightarrow{\text{reshape}} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

coefficient matrix

$$\begin{array}{l} 1 \times |00\rangle \\ 0 \times |01\rangle \\ 0 \times |10\rangle \\ 1 \times |11\rangle \end{array}$$

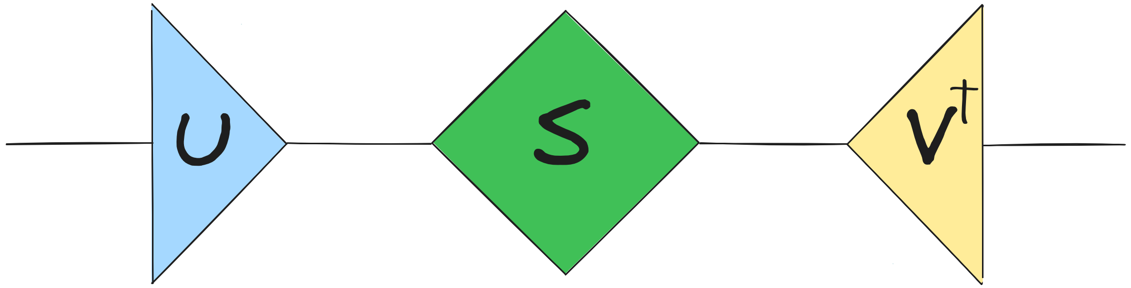
basis states

coefficient matrix

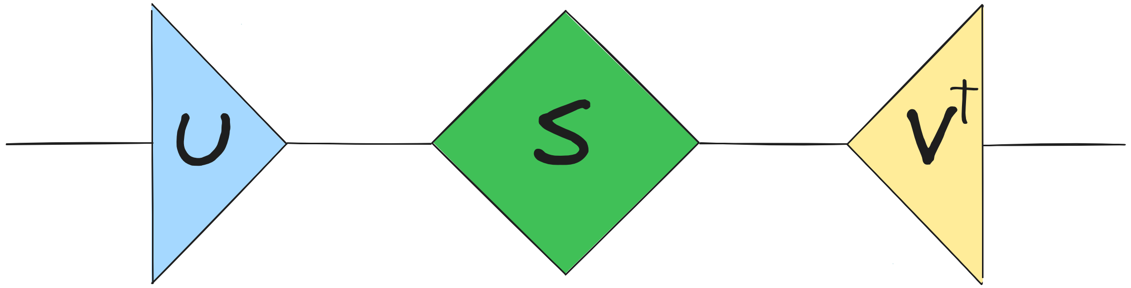
Bell state

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

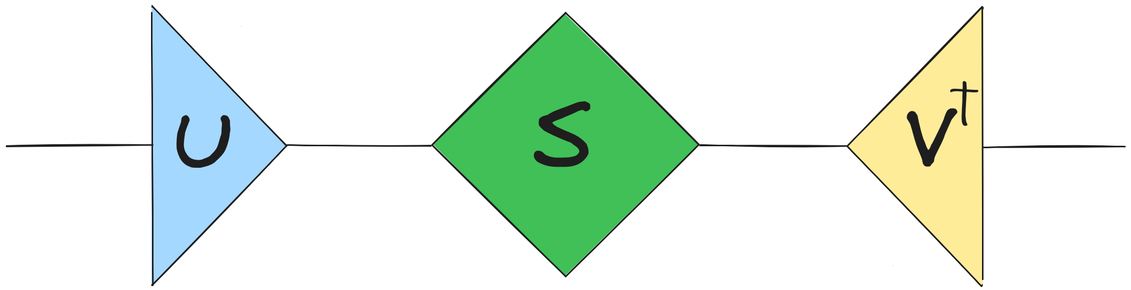


$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$



singular values

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$



singular values

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\lambda_i = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

singular values

$$\lambda_i = \frac{1}{\sqrt{2}} \longrightarrow D=2$$

max entanglement

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

singular values

$$\lambda_i = \frac{1}{\sqrt{2}} \longrightarrow D=2$$

max entanglement

Hilbert space dimension $\boxed{\log_2 d = D}$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \xrightarrow{\text{SVD}} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

singular values

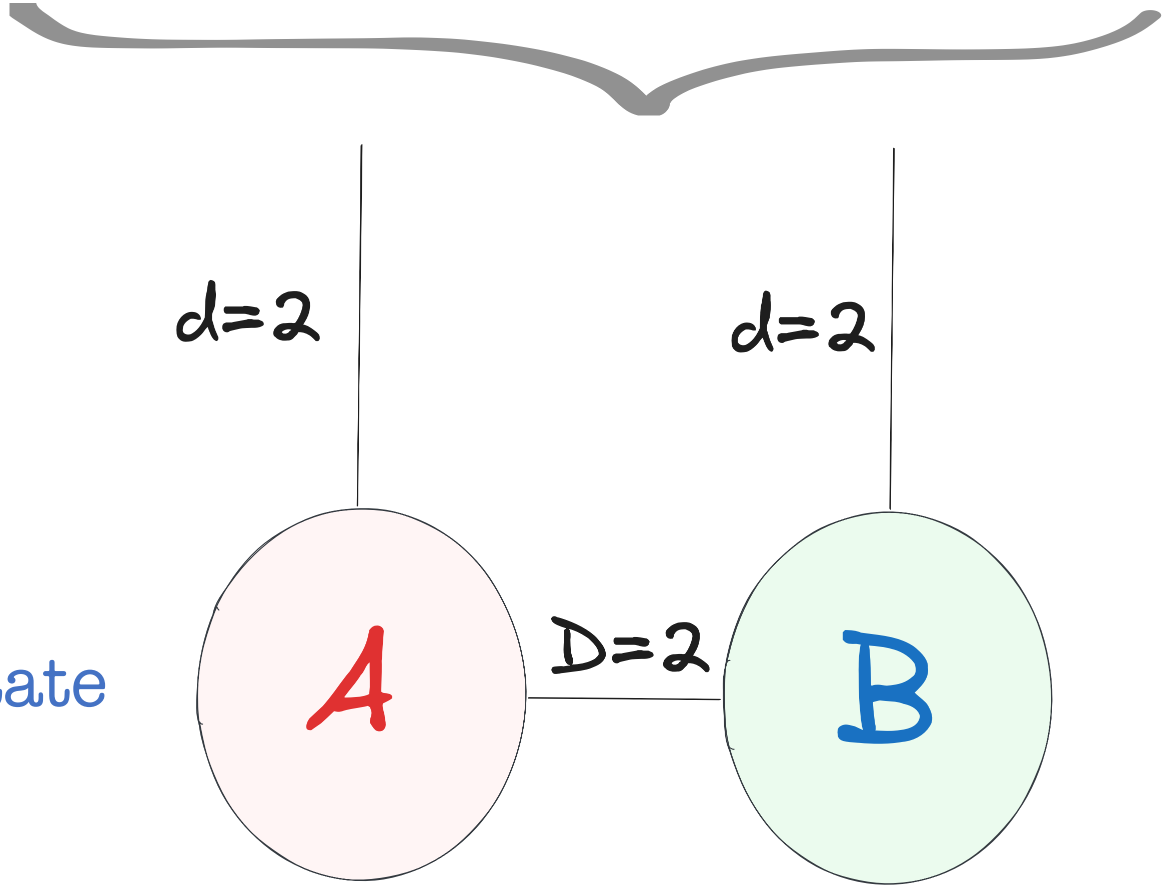
$$\lambda_i = \frac{1}{\sqrt{2}} \longrightarrow D=2$$

max entanglement

Hilbert space dimension $\log_2 d = D$

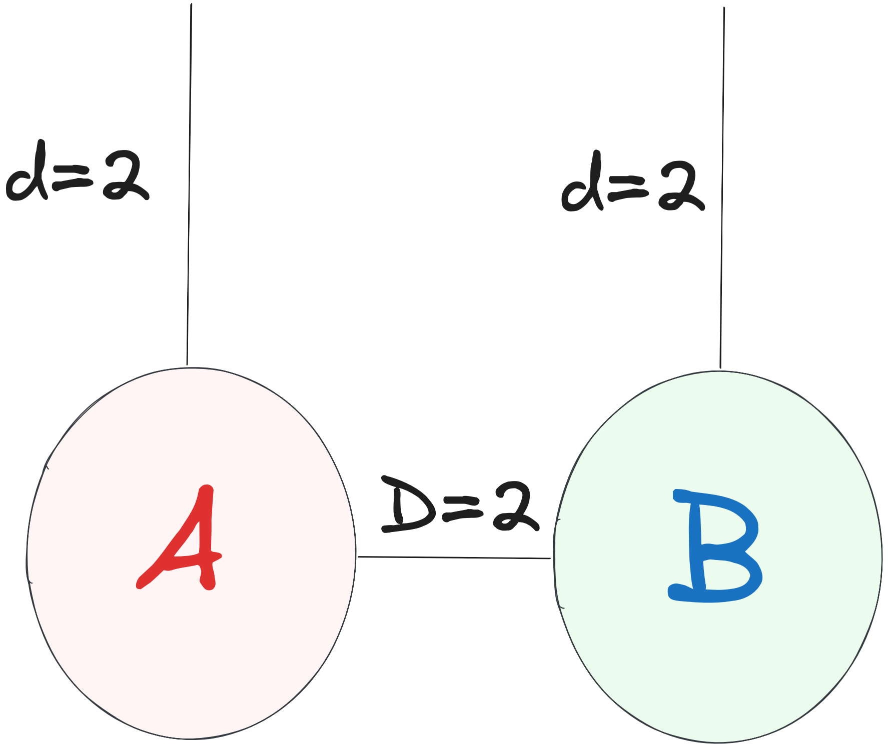
BOND DIMENSION

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \times \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



Matrix Product State

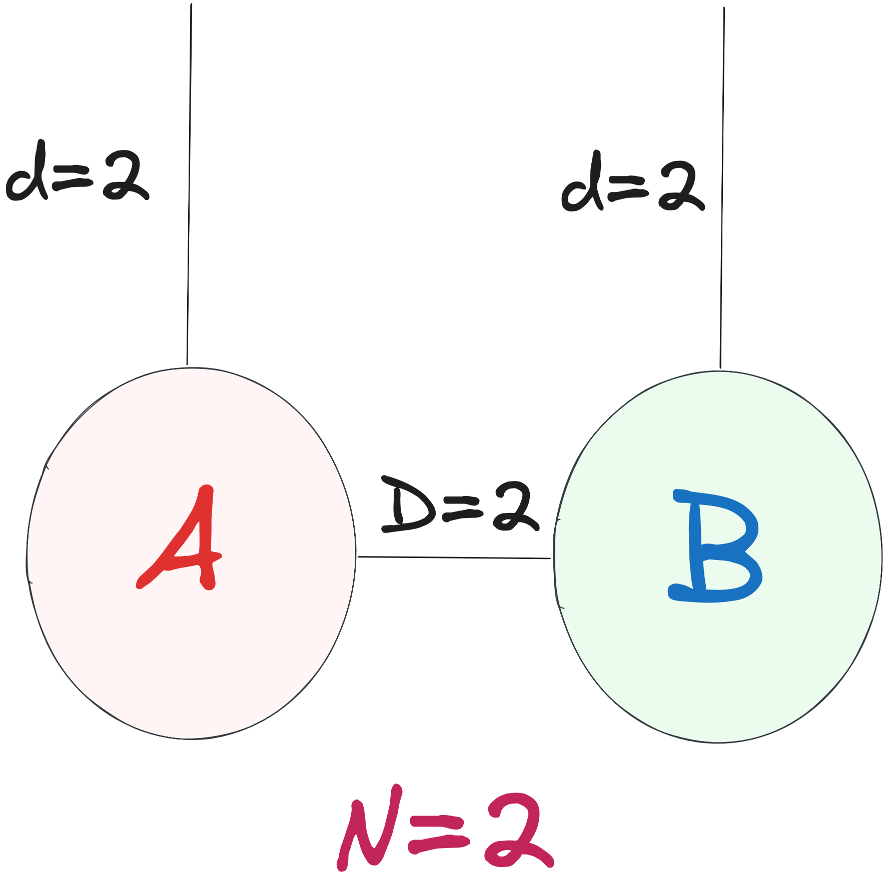
Matrix Product State



Number of parameters

$$\mathcal{O}(NdD^2)$$

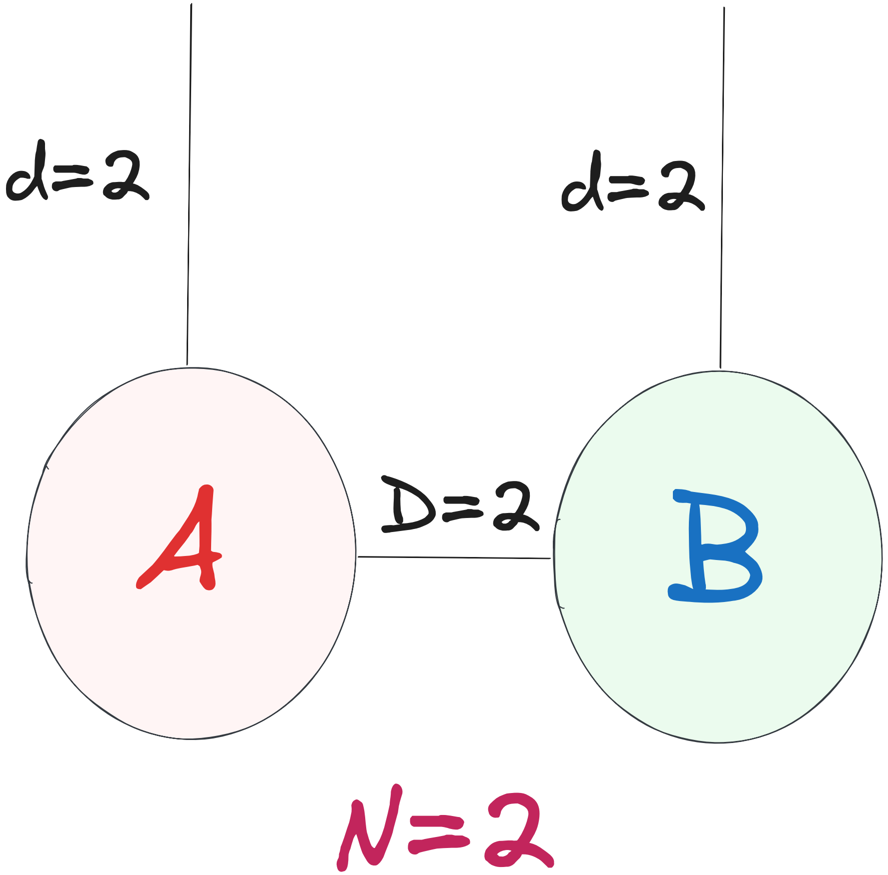
Matrix Product State



Number of parameters

$$\mathcal{O}(NdD^2)$$

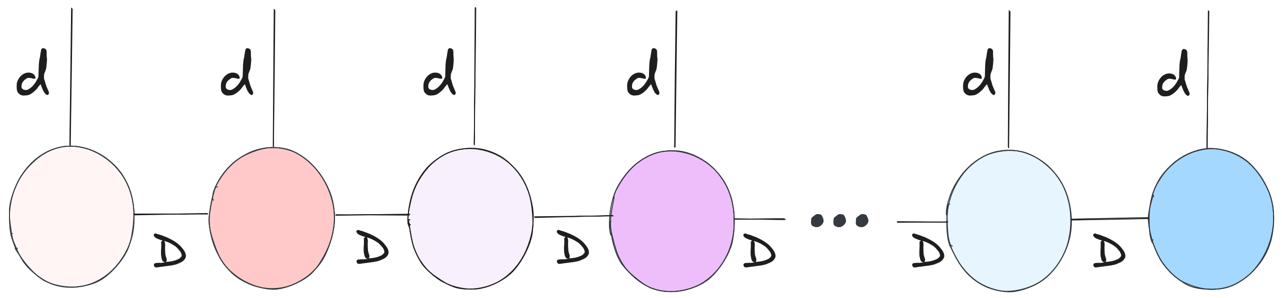
Matrix Product State



Number of parameters

16

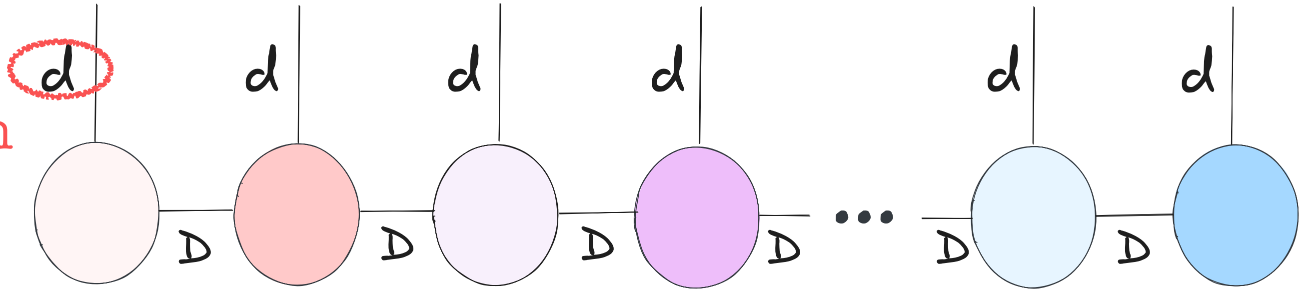
Matrix Product State



$$|\psi\rangle = \sum_{\{i_1,i_2\ldots i_N\}=0}^{d-1} C_{i_1,i_2\ldots i_N}|i_1, i_2 \ldots i_N\rangle$$

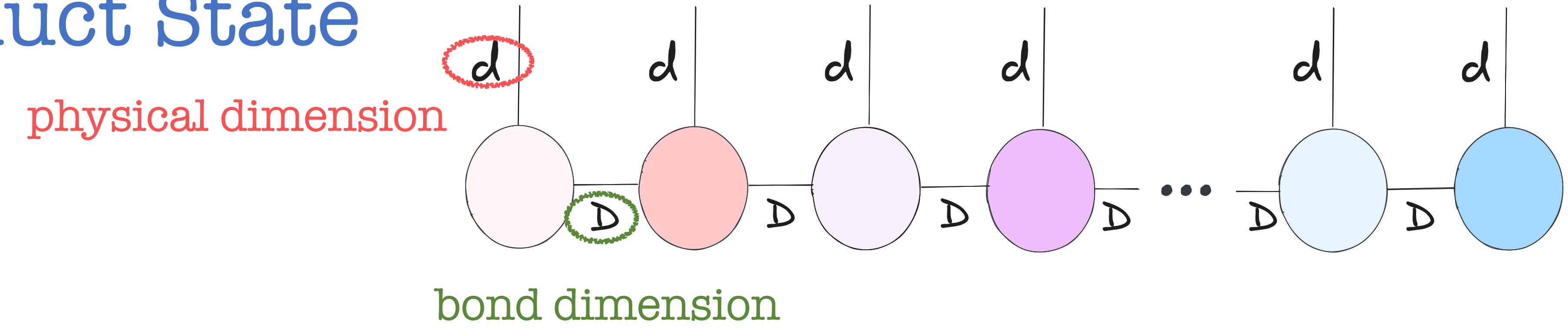
Matrix Product State

physical dimension



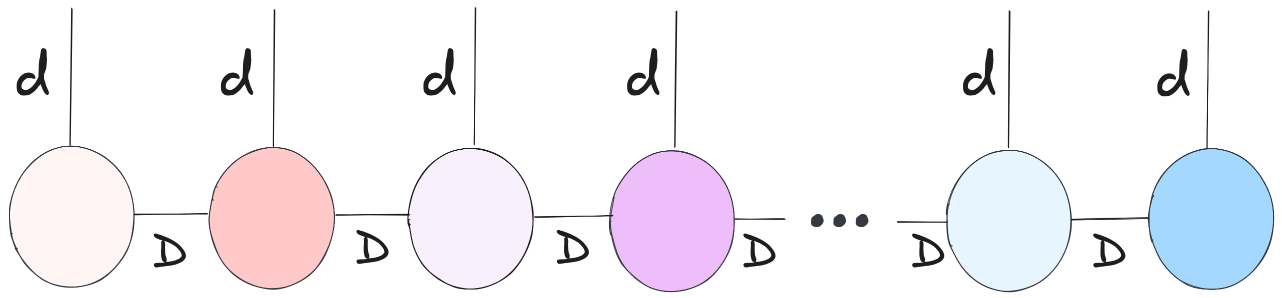
$$|\psi\rangle = \sum_{\{i_1,i_2\ldots i_N\}=0}^{d-1} C_{i_1,i_2\ldots i_N} | i_1, i_2 \ldots i_N\rangle$$

Matrix Product State



$$|\psi\rangle = \sum_{\{i_1,i_2\ldots i_N\}=0}^{d-1} C_{i_1,i_2\ldots i_N} |\, i_1, i_2 \ldots i_N\rangle$$

Matrix Product State

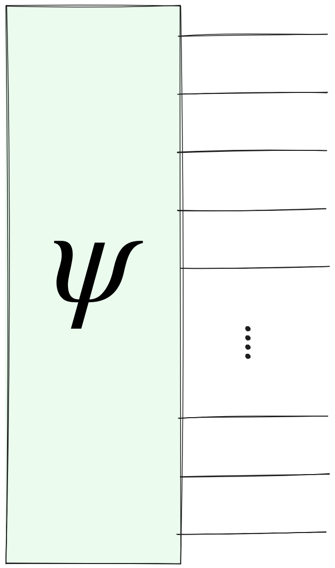


$$|\psi\rangle = \sum_{\{i_1, i_2 \ldots i_N\}=0}^{d-1} C_{i_1, i_2 \ldots i_N} |i_1, i_2 \ldots i_N\rangle$$

product of matrices

Matrix Product State

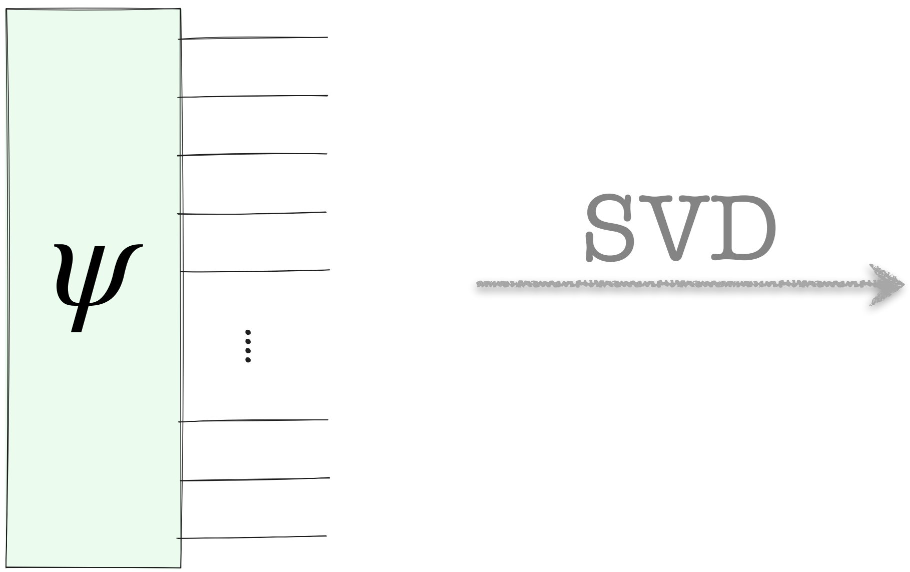
N-rank tensor



params = d^N

Matrix Product State

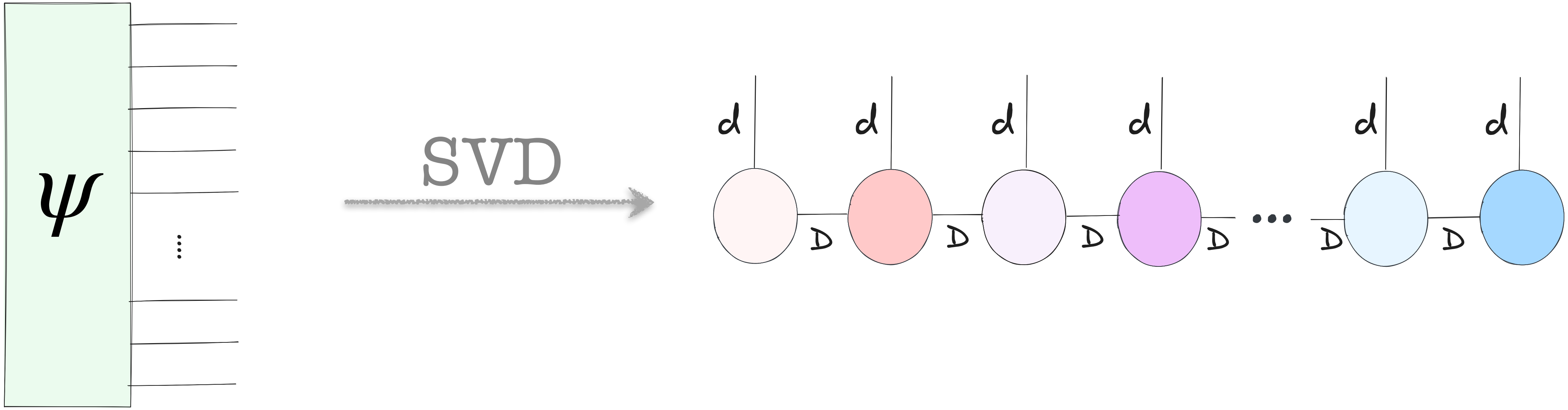
N-rank tensor



params = d^N

Matrix Product State

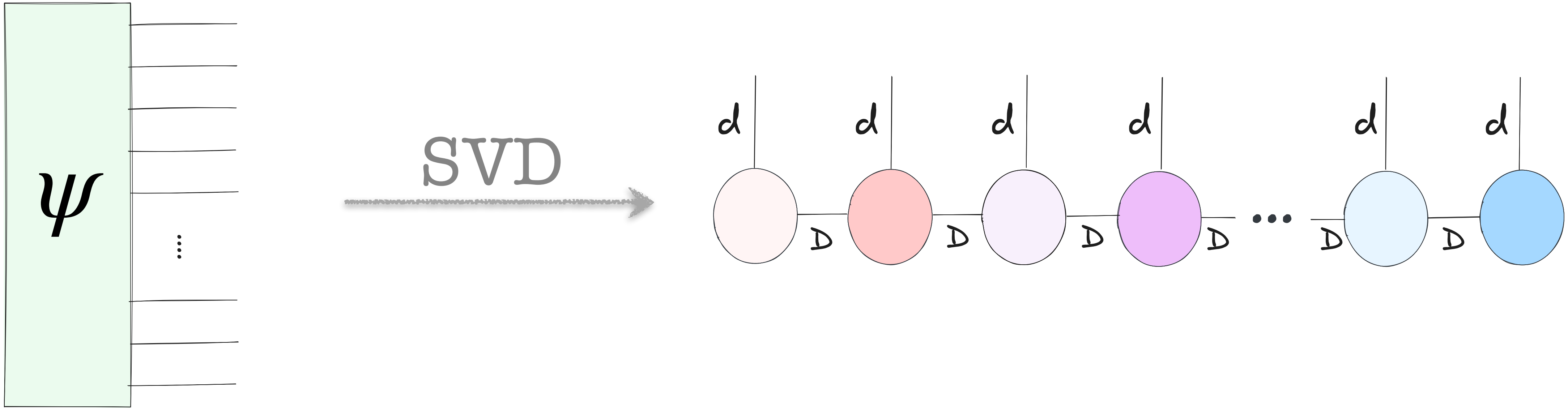
N-rank tensor



params = d^N

Matrix Product State

N-rank tensor

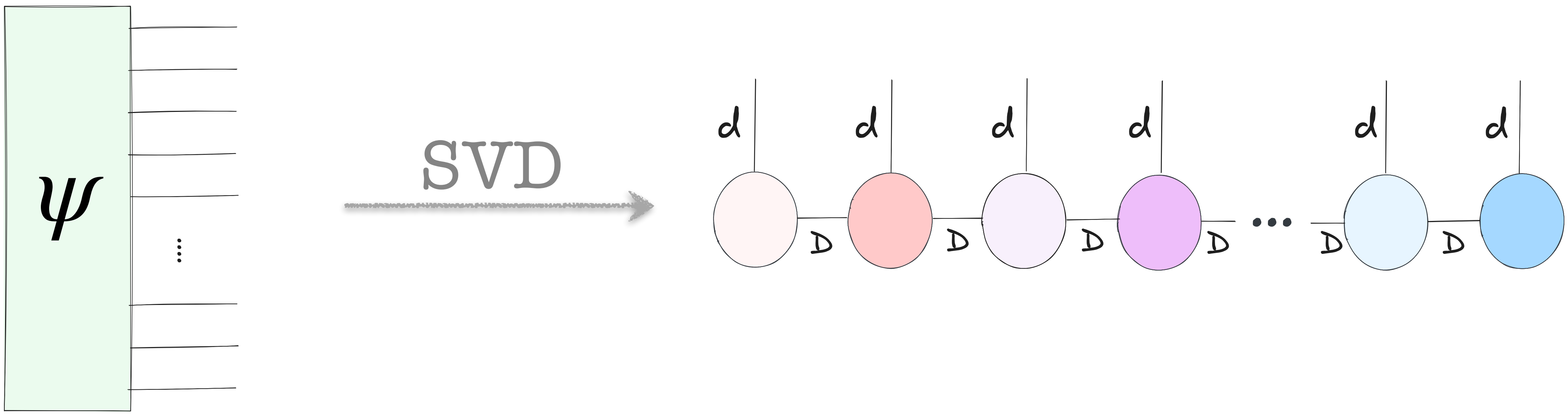


params = d^N

params = NdD^2

Matrix Product State

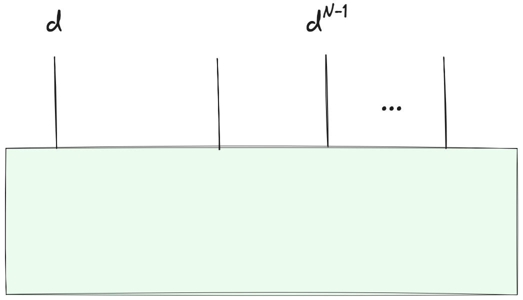
N-rank tensor



params = d^N >> # params = NdD^2

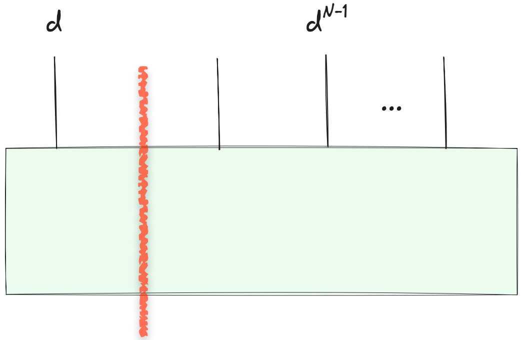
Singular Value Decomposition

$$d \times d^{N-1}$$



Singular Value Decomposition

$$d \times d^{N-1}$$

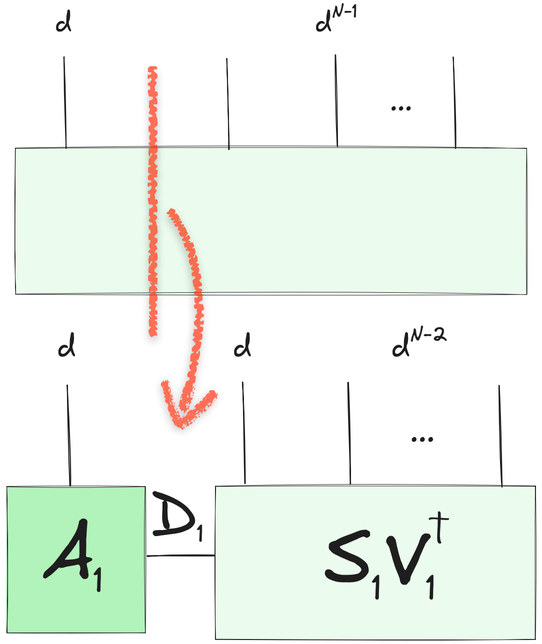


Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$A_1 S_1 V_1^{\dagger}$$



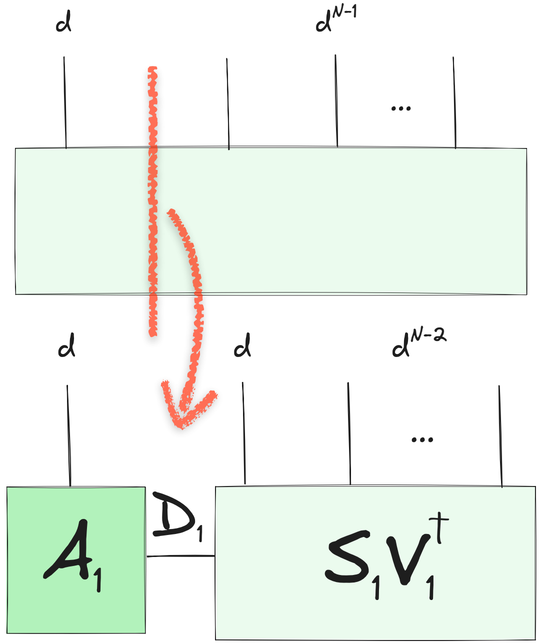
Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \quad A_1 S_1 V_1^\dagger$$

$$D_1 \leq d$$



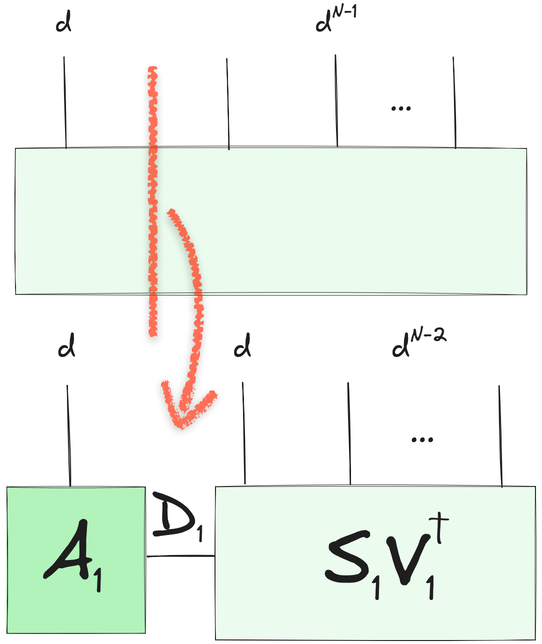
Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \quad A_1 S_1 V_1^{\dagger} \quad D_1 \times d^{N-1}$$

$$D_1 \leq d$$

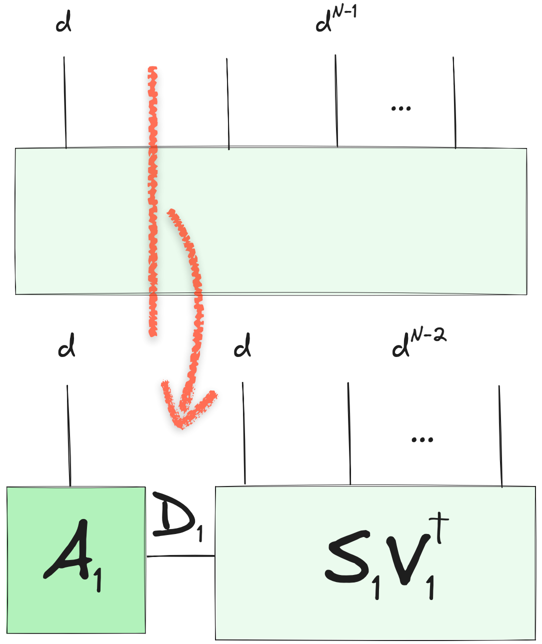


Singular Value Decomposition

$$d \times d^{N-1}$$

$$d \times D_1 \quad A_1 S_1 V_1^\dagger \quad D_1 \times d^{N-1}$$

$$D_1 \leq d$$



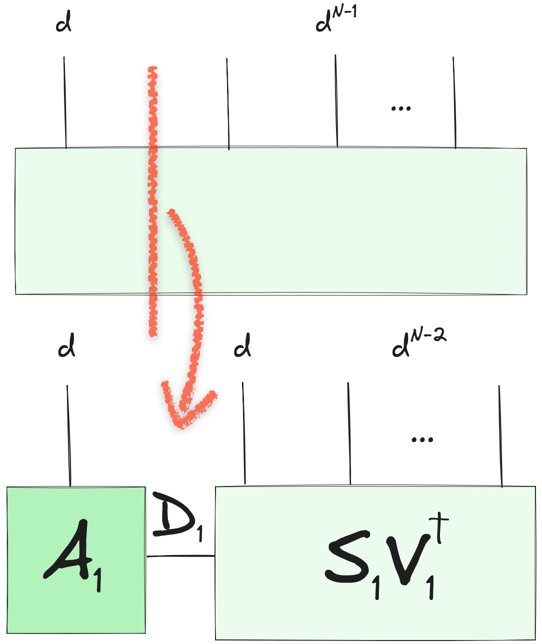
Singular Value Decomposition

$$d \times d^{N-1}$$

$$d \times D_1 \quad A_1 S_1 V_1^{\dagger} \quad D_1 \times d^{N-1}$$

$$D_1 d \times d^{N-2}$$

$$D_1 \leq d$$



Singular Value Decomposition

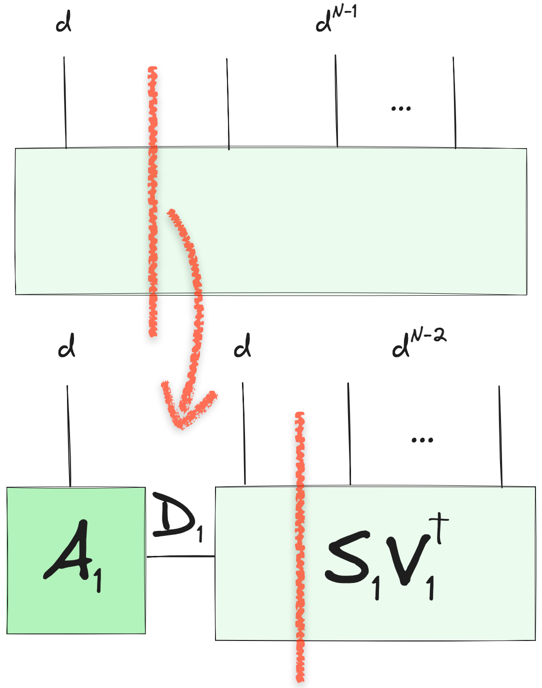
$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \qquad A_1 S_1 V_1^\dagger \qquad D_1 \times d^{N-1}$$

$$D_1 d \times d^{N-2}$$

$$D_1 \leq d$$



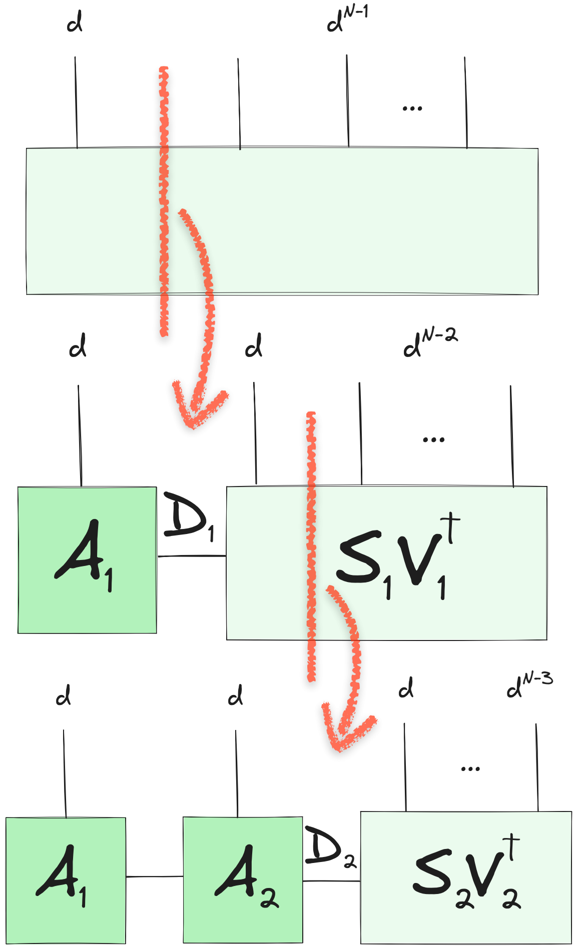
Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \quad A_1 S_1 V_1^{\dagger} \quad D_1 \times d^{N-1} \qquad D_1 \leq d$$

$$\searrow \; D_1 d \times d^{N-2}$$



Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

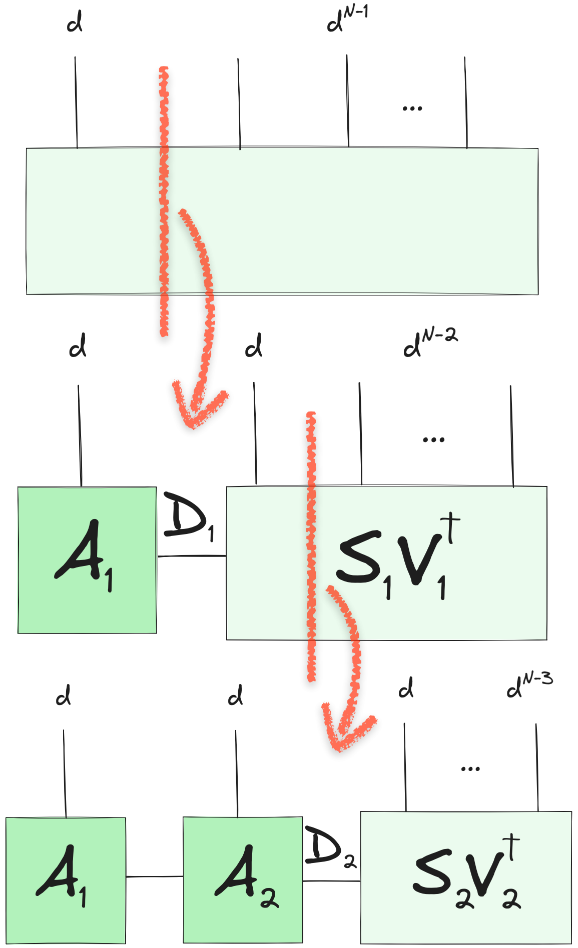
$$d \times D_1 \quad A_1 S_1 V_1^{\dagger} \quad D_1 \times d^{N-1}$$

$$D_1 \leq d$$

$$D_1 d \times d^{N-2}$$

$$\downarrow$$

$$A_1 A_2 S_2 V_2^{\dagger}$$



Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \quad A_1 S_1 V_1^\dagger \quad D_1 \times d^{N-1}$$

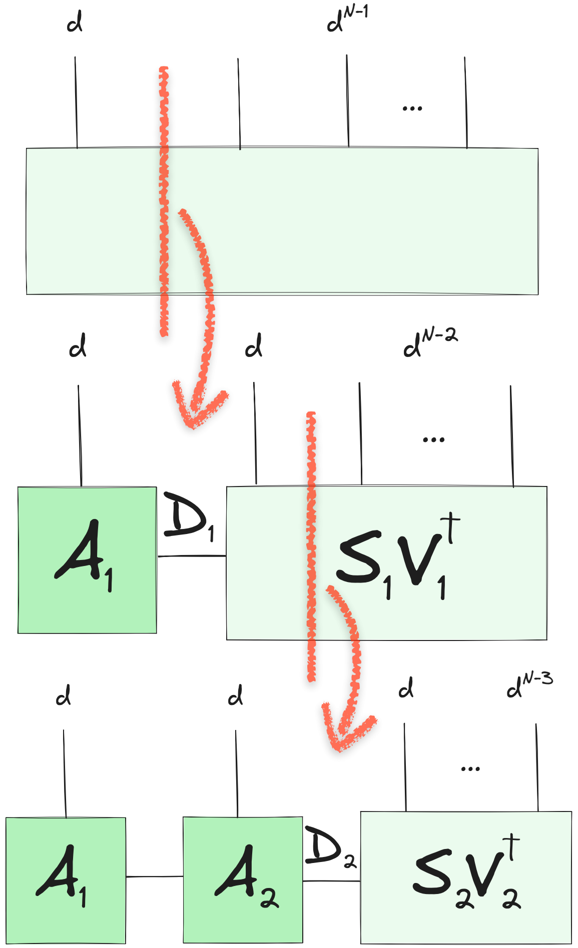
$$D_1 d \times d^{N-2}$$

$$\downarrow$$

$$A_1 A_2 S_2 V_2^\dagger$$

$D_1 \leq d$

$D_2 \leq d^2$



Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \quad A_1 S_1 V_1^{\dagger} \quad D_1 \times d^{N-1}$$

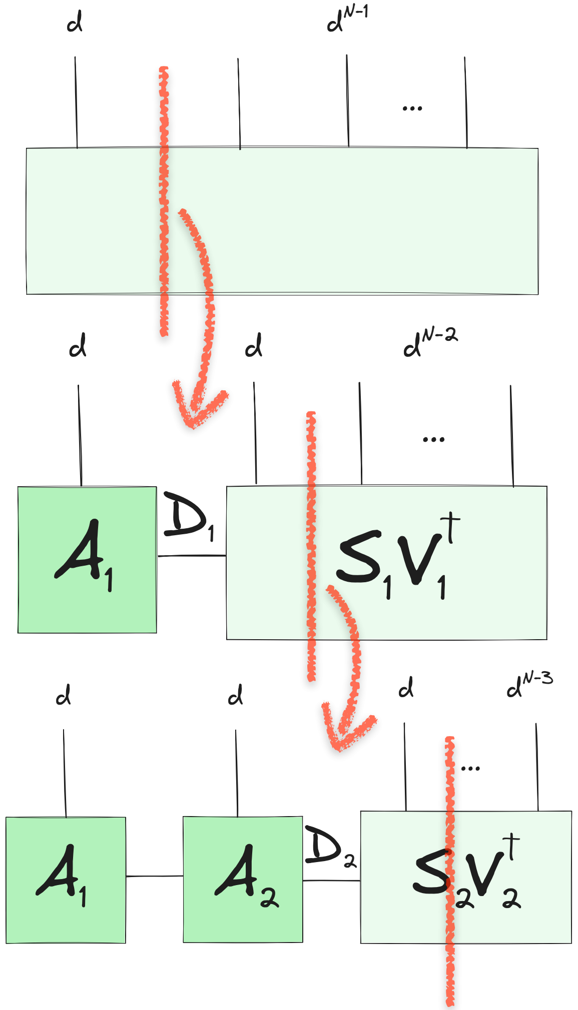
$$D_1 d \times d^{N-2}$$

$$\downarrow$$

$$A_1 A_2 S_2 V_2^{\dagger}$$

$$D_1 \leq d$$

$$D_2 \leq d^2$$



Singular Value Decomposition

$$d \times d^{N-1}$$

$$\downarrow$$

$$d \times D_1 \quad A_1 S_1 V_1^\dagger \quad D_1 \times d^{N-1}$$

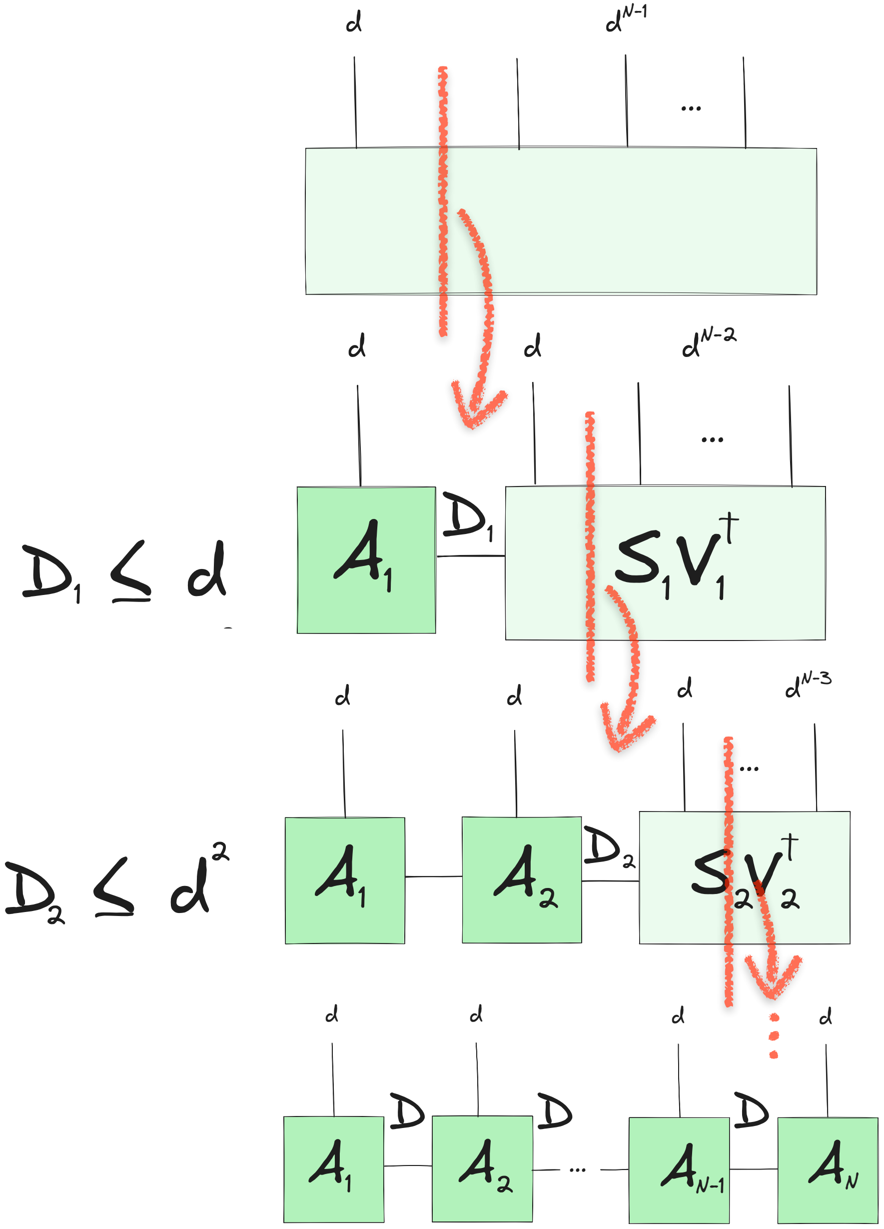
$$D_1 d \times d^{N-2}$$

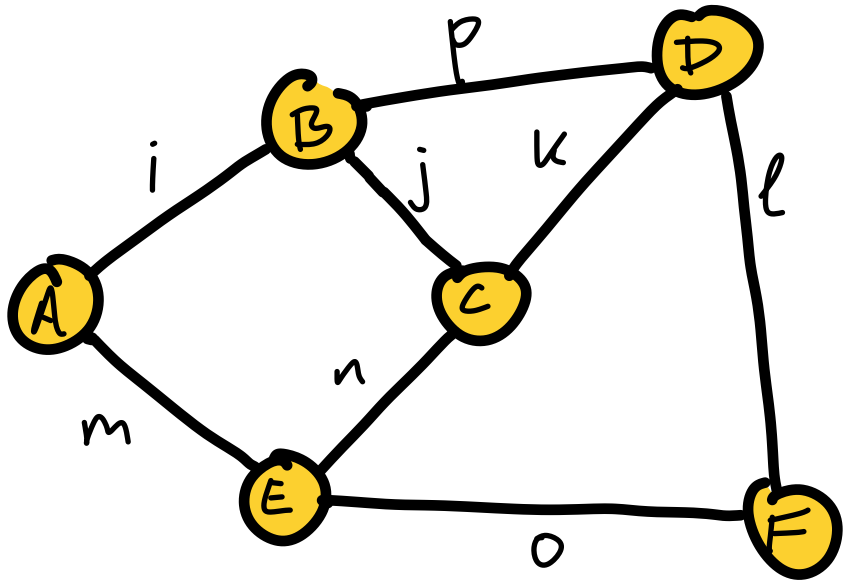
$$\downarrow$$

$$A_1 A_2 S_2 V_2^\dagger$$

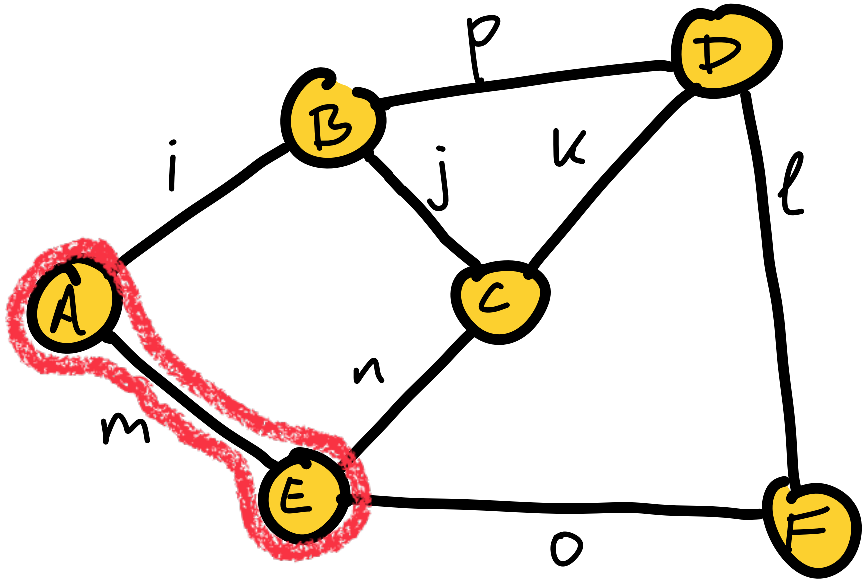
$$\vdots$$

$$A_1 A_2 \cdots A_{N-1} A_N$$

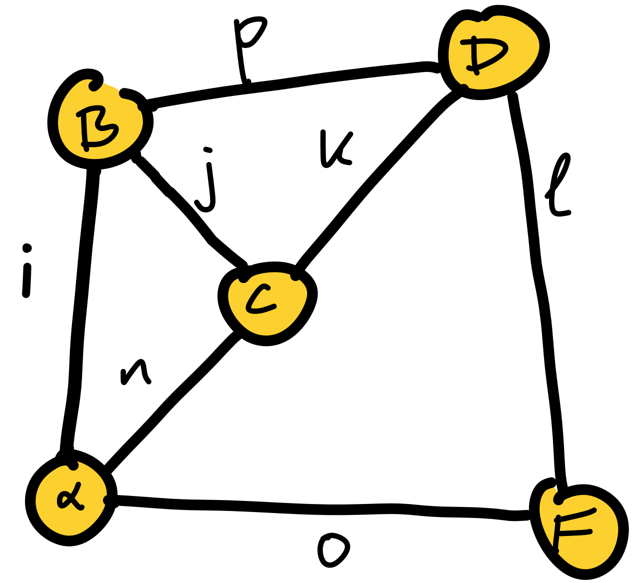




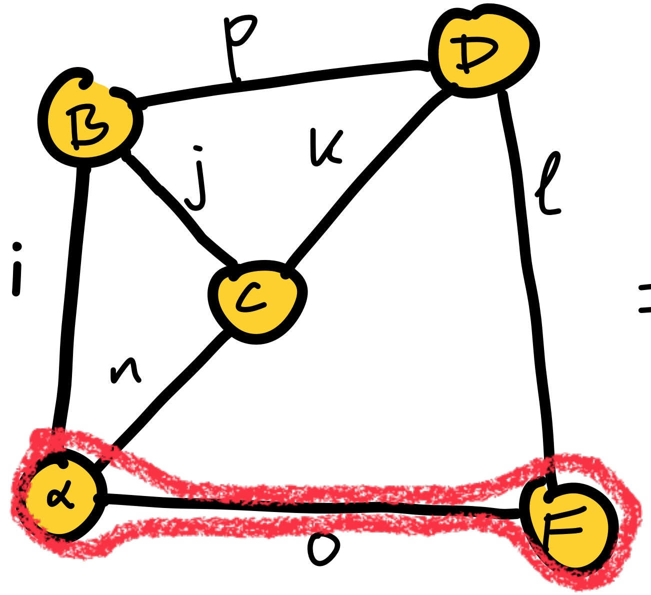
$$= \sum_{ijklmnop} A_{mi} B_{ijp} C_{jkn} D_{pkl} E_{mno} F_{ol}$$



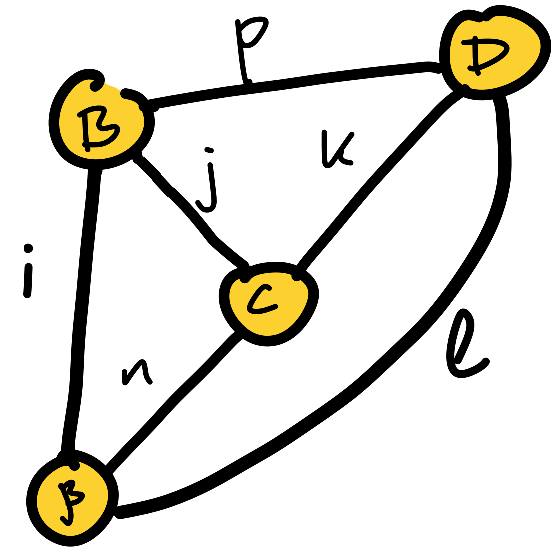
$$= \sum_{ijklmnop} A_{mi} B_{ijp} C_{jkn} D_{pkl} E_{mno} F_{ol}$$



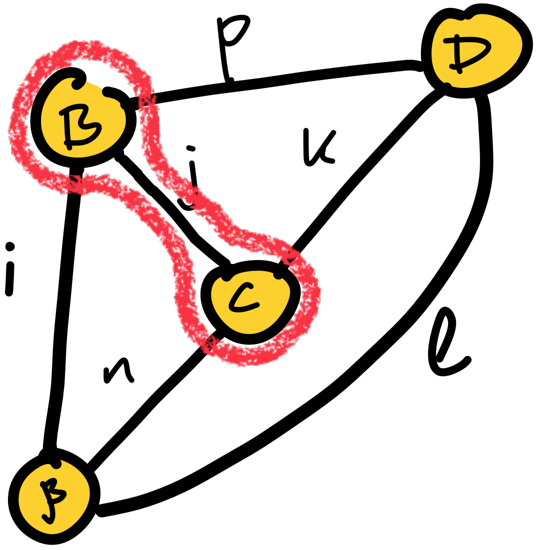
$$= \sum_{ijklnop} \alpha_{ino} B_{ijp} C_{jkn} D_{pkl} F_{ol}$$



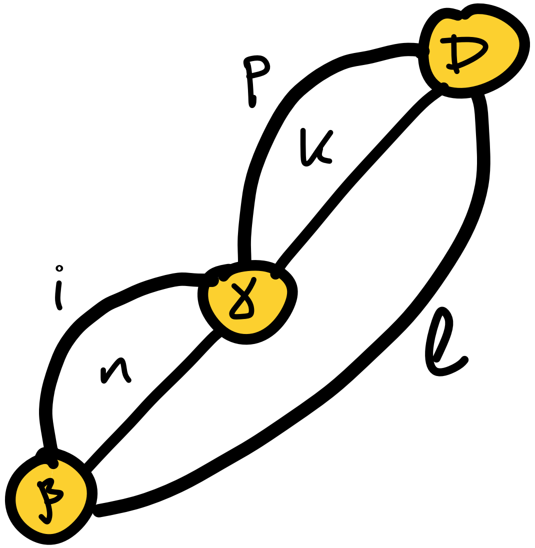
$$= \sum_{ijkln\textcolor{red}{o}p} \textcolor{red}{\alpha}_{in\textcolor{red}{o}} B_{ijp} C_{jkn} D_{pkl} \textcolor{red}{F}_{\textcolor{red}{o}l}$$



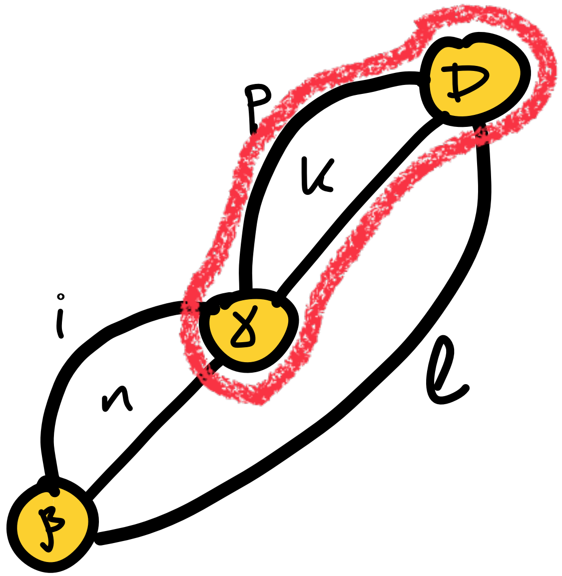
$$= \sum_{ijklnp} \beta_{inl} B_{ijp} C_{jkn} D_{pkl}$$



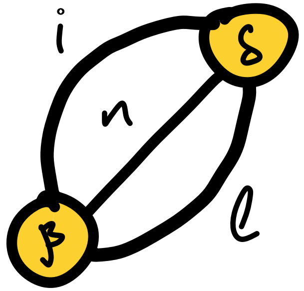
$$= \sum_{ijklnp} \beta_{inl} B_{ijp} C_{jkn} D_{pkl}$$



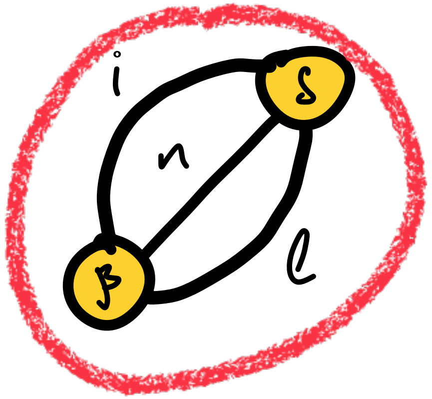
$$= \sum_{iklnp} \gamma_{inpk} \beta_{inl} D_{pkl}$$



$$= \sum_{iklnp} \gamma_{inpk} \beta_{inl} D_{pkl}$$



$$= \sum_{iln} \delta_{inl}\beta_{inl}$$



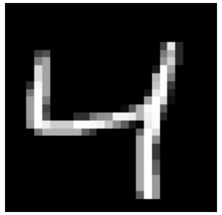
$$= \sum_{iln} \delta_{inl}\beta_{inl}$$

$$= \mathcal{E}$$

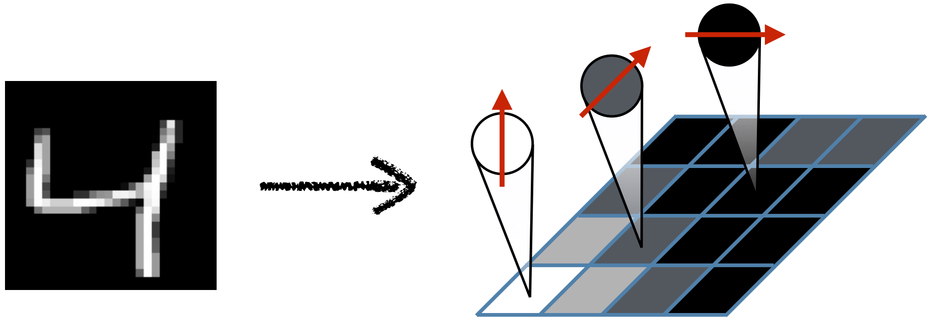
Contraction path

m → o → j → k → p → i → l → n

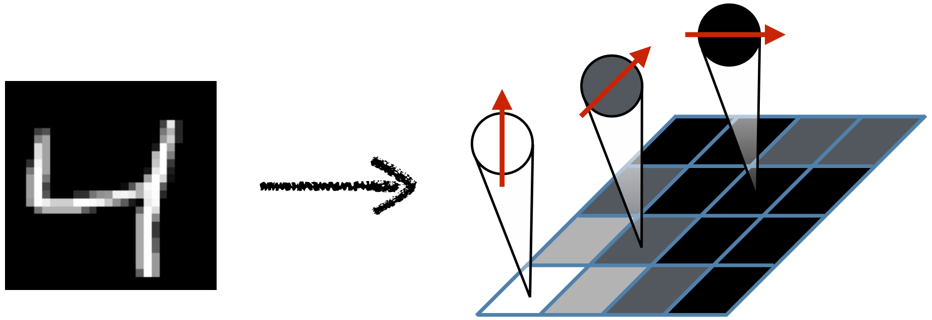
MNIST clasiffication



MNIST clasiffication

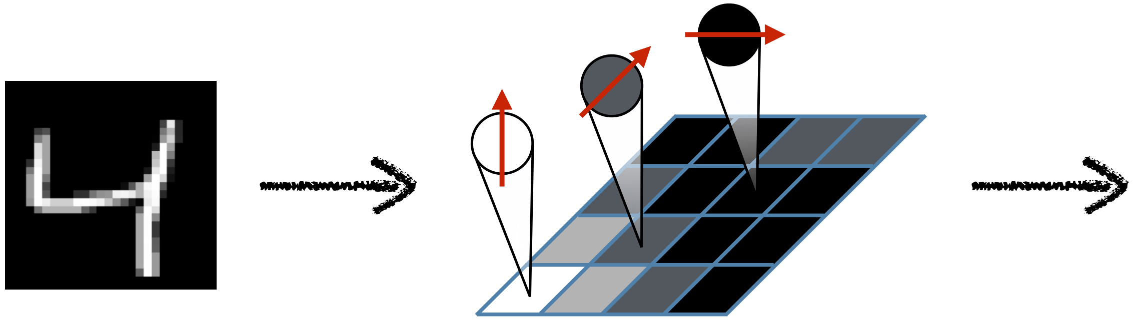


MNIST clasiffication



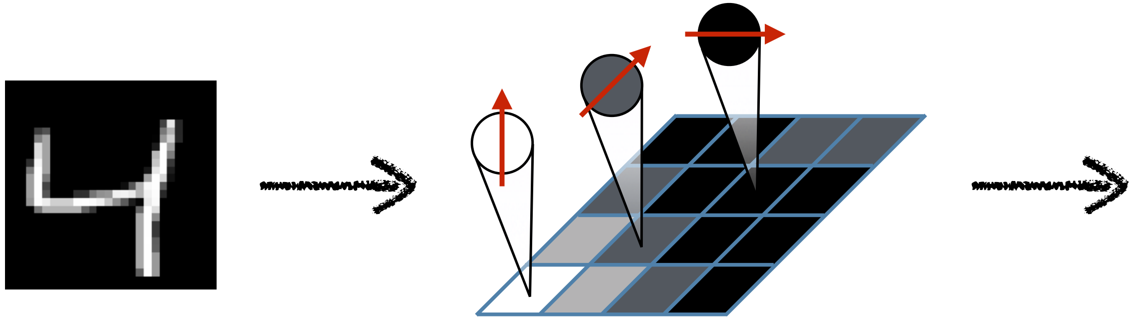
$$\phi(x_i) = \left[cos\left(\frac{\pi}{2}x_i\right), sin\left(\frac{\pi}{2}x_i\right) \right]$$

MNIST clasiffication

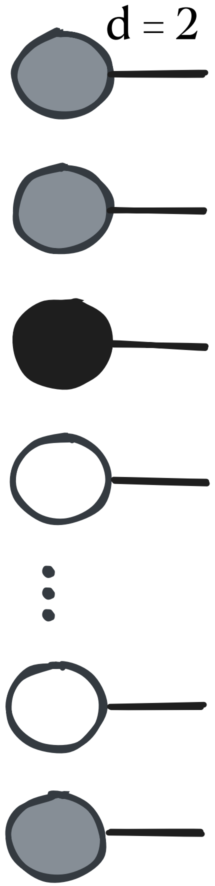


$$\phi(x_i) = \left[cos\left(\frac{\pi}{2}x_i\right), sin\left(\frac{\pi}{2}x_i\right) \right]$$

MNIST clasiffication

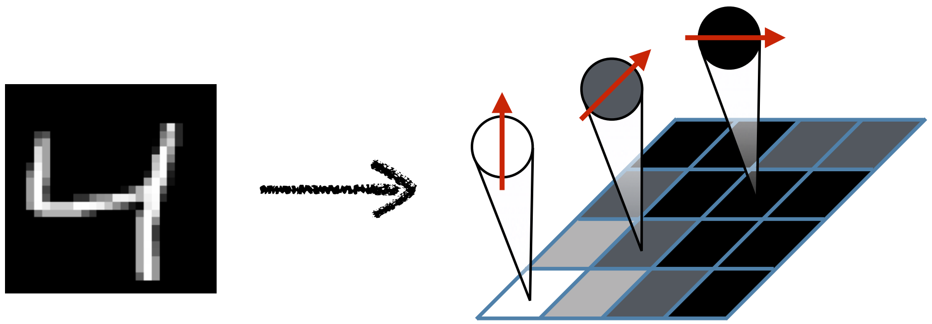


$$\phi(x_i) = \left[cos\left(\frac{\pi}{2}x_i\right), sin\left(\frac{\pi}{2}x_i\right) \right]$$

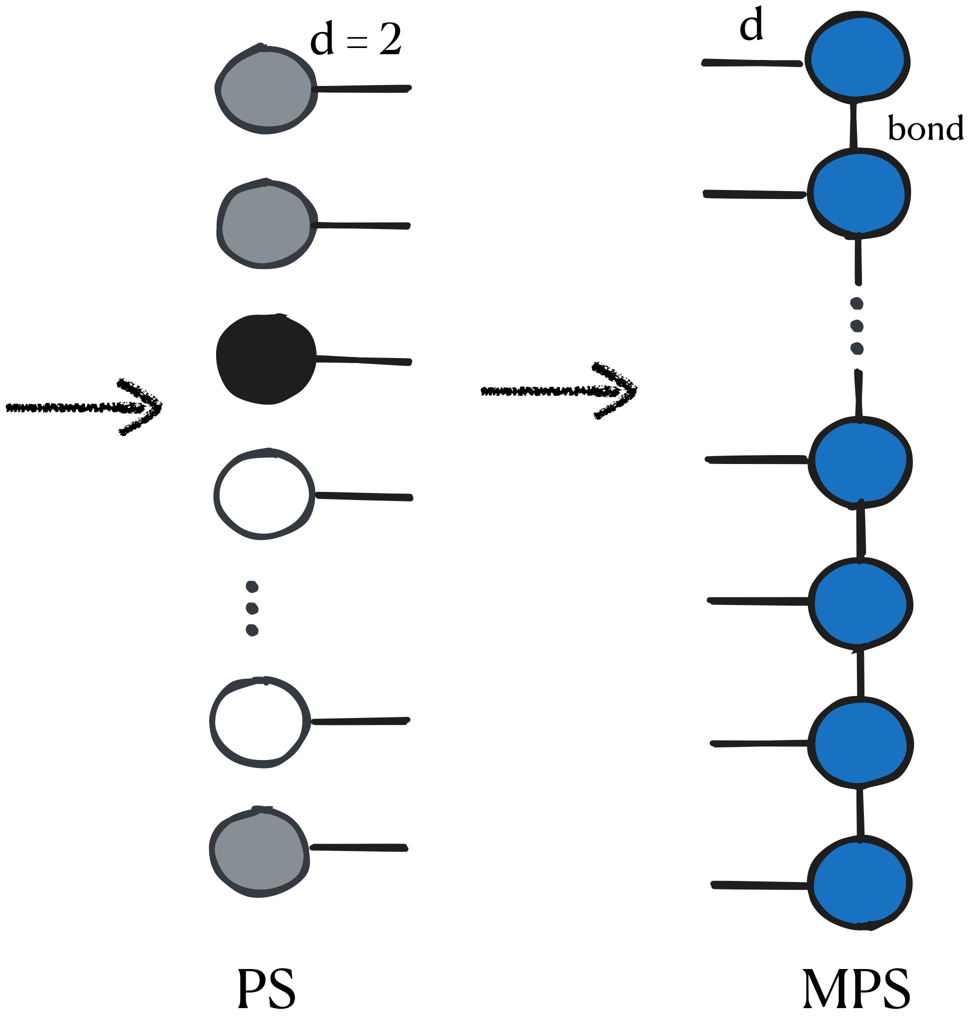


product state

MNIST clasiffication



$$\phi(x_i) = \left[\cos\left(\frac{\pi}{2}x_i\right), \sin\left(\frac{\pi}{2}x_i\right)\right]$$



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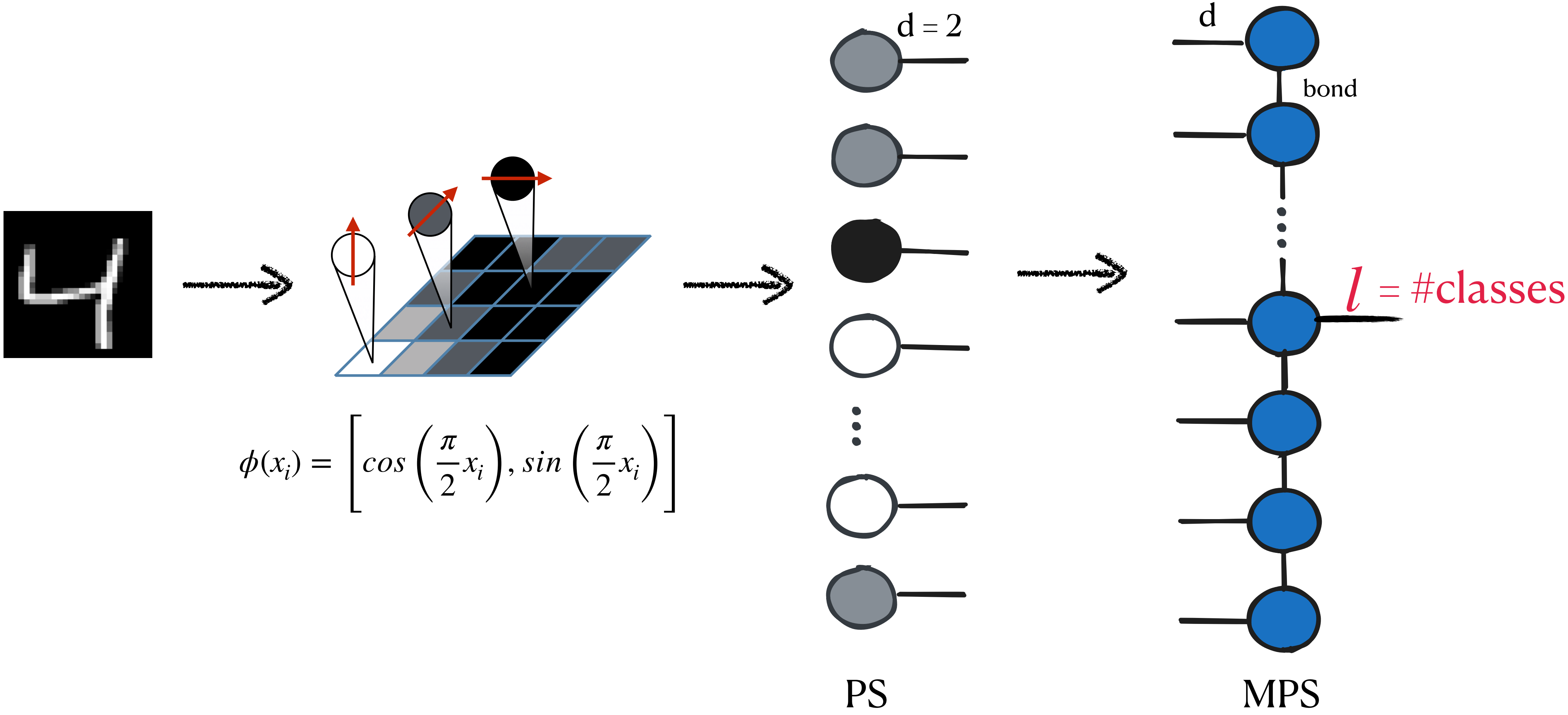
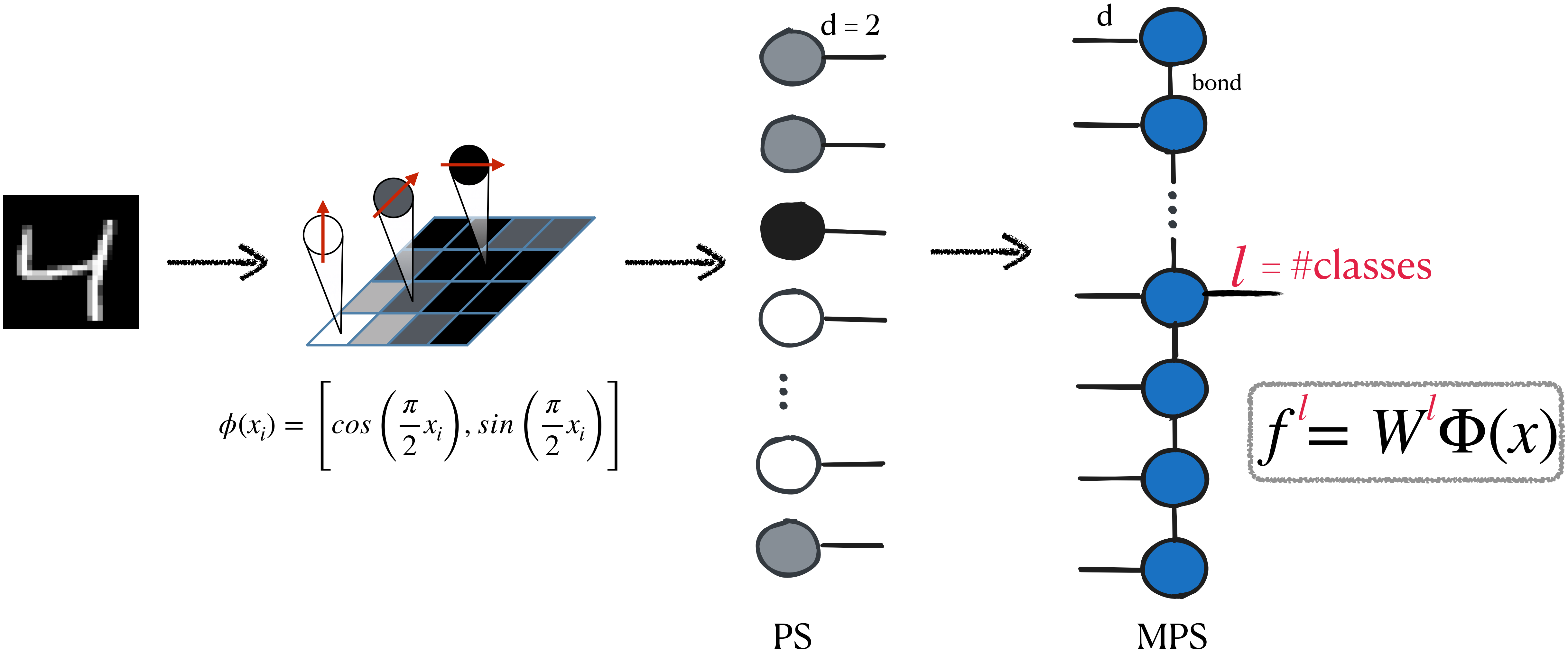
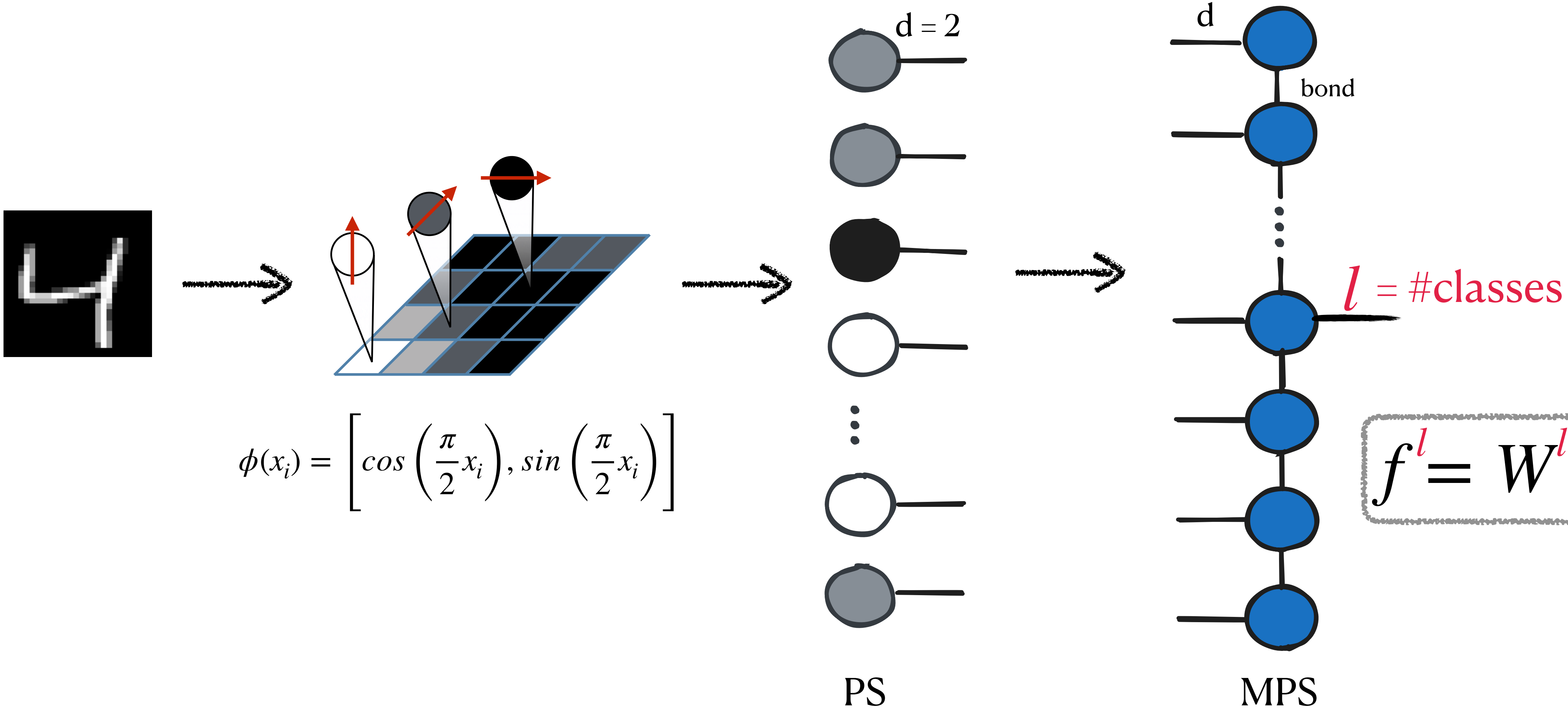


$$\phi(x_i) = \left[cos\left(\frac{\pi}{2}x_i\right), sin\left(\frac{\pi}{2}x_i\right) \right]$$

MNIST clasiffication



MNIST clasiffication



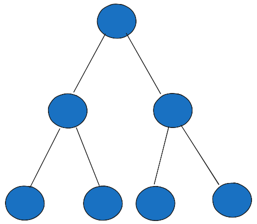
$$\phi(x_i) = \left[cos\left(\frac{\pi}{2}x_i\right), sin\left(\frac{\pi}{2}x_i\right) \right]$$

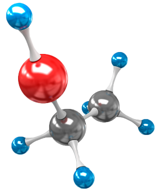
$$f^l = W^l \Phi(x)$$

Supervised Learning with Quantum-Inspired Tensor Networks [1605.05775]

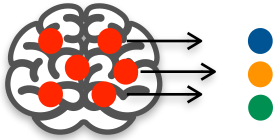
Different types of TNs

Tree TN





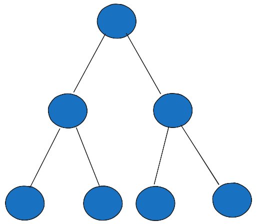
quantum chemistry



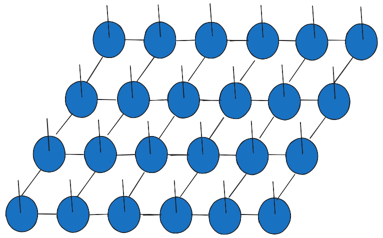
ML feature extraction

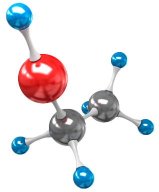
Different types of TNs

Tree TN

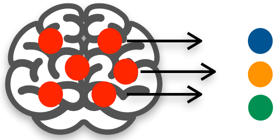


PEPS (2D) TN

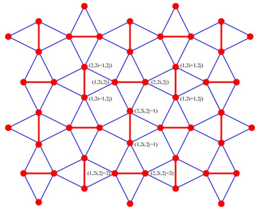




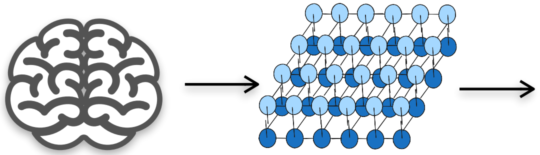
quantum chemistry



ML feature extraction



quantum lattice models



classification

QUIMB

TensorKrowch

Tenet.jl

tn4ml

QUESTIONS?

ema.puljak@cern.ch