ACTION and CHARGES in HIGHER-DERIVATIVE HOLOGRAPHY

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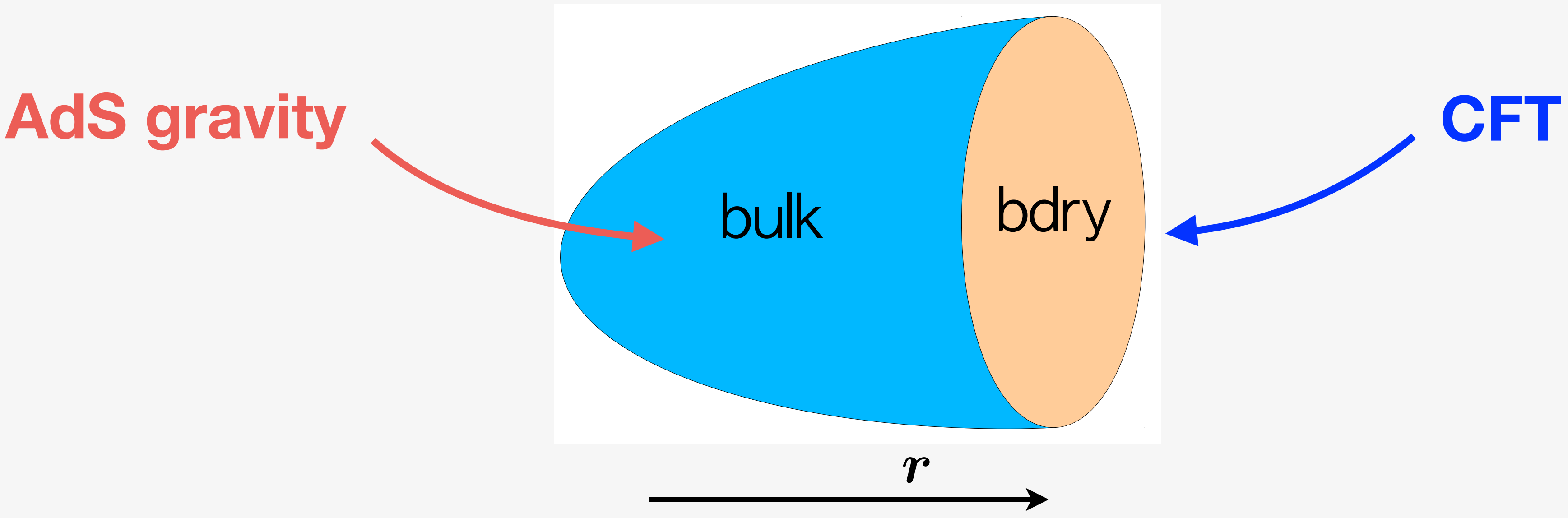
AdS/CFT correspondence

Non-perturbative definition of Quantum Gravity in asymptotically AdS space



$$Z_{\mathbf{gravity}} = Z_{\mathbf{CFT}}$$

AdS boundary conditions $\Longleftrightarrow$ CFT background fields

Use CFT to address black hole microstate counting

Outline

- Focus on $\mathrm{AdS}_5/\mathrm{CFT}_4$ d=4 SCFT's explicitly known and calculable

  $\mathcal{N}=4$ SYM , infinite $\mathcal{N}=1$ CFT's

- corrections to BPS black hole entropy from SCFT_4

- match these in higher-derivative $d=5$ supergravity

→ anomalies play a key role ←

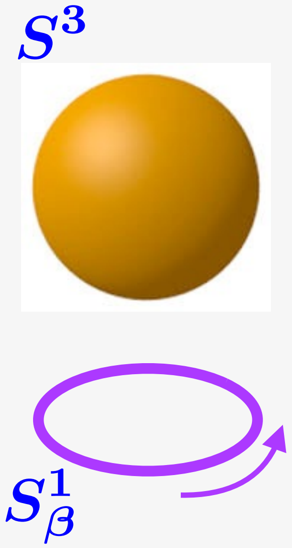
- along the way, will discuss issues that arise with higher-derivatives

Joint work with: A. Ruipérez, E. Turetta 2208.01007, 2304.06101

Z. Komargodski 2021;

A. Cabo-Bizet, S. Murthy, D. Martelli 2018-2020;

Black Hole from SCFT



$\mathcal{N}=1$ SCFT on $S^1 \times S^3$

Microcanonical ensemble

energy

angular momenta

R-charge

$$E\,,\ J_1\,,\ J_2\,,\ Q$$

Entropy S = log (degeneracy of states with assigned charges)

Grand-canonical ensemble

$$\beta\,,\ \Omega_1\,,\ \Omega_2\,,\ \Phi$$

inverse temperature

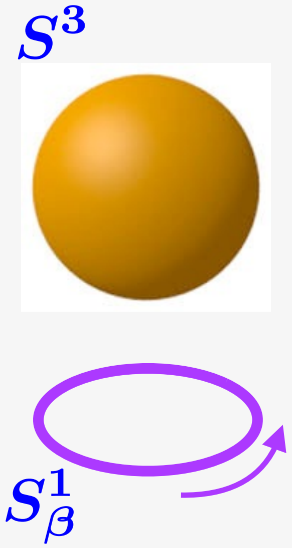
angular velocities

electrostatic potential

$$Z(\beta,\Omega_i,\Phi) \;=\; \mathrm{Tr}\, e^{-\beta(E-\Omega_1 J_1-\Omega_2 J_2-\Phi Q)}$$

Black Hole from SCFT

$\mathcal{N}=1$ SCFT on $S^1 \times S^3$



Microcanonical ensemble

supercharges

$$\{\mathcal{Q},\overline{\mathcal{Q}}\} = E - 1\,J_1 - 1\,J_2 - \tfrac{3}{2}\,Q$$

$$J_1\,,\; J_2\,,\; Q$$

Entropy S = log (degeneracy of states with assigned charges)

Grand-canonical ensemble

$$\beta\,,\quad \omega_1 = \beta(\Omega_1 - 1)\,,\quad \omega_2 = \beta(\Omega_2 - 1)\,,\quad \varphi = \beta(\Phi - \tfrac{3}{2})$$

$$Z(\beta,\omega_i,\varphi) = \mathrm{Tr}\,e^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+\omega_1 J_1+\omega_2 J_2+\varphi Q}$$

Black Hole from SCFT

A black hole is a large-N saddle, $Z(\beta, \omega_i, \varphi) \approx \mathrm{e}^{-I(\beta,\omega_i,\varphi)}$ ⟵ action

$$S = \mathrm{ext}_{\{\beta,\omega_1,\omega_2,\varphi\}} \left[-I + \beta\{\mathcal{Q}, \overline{\mathcal{Q}}\} - \omega_1 J_1 - \omega_2 J_2 - \varphi Q \right]$$

Supersymmetry

$\mathrm{e}^{\omega_1 J_1 + \omega_2 J_2 + \varphi Q}$ must commute with supercharge

Hosseini, Hristov, Zaffaroni

Cabo-Bizet, Murthy, DC, Martelli

Choi, Kim, Kim, Nahmgoong

$$\boxed{\varphi = \frac{\omega_1 + \omega_2}{2} - \pi i}$$

➜ $Z(\beta, \omega_i) = \mathrm{Tr}\ \mathrm{e}^{-\pi i Q}\ \mathrm{e}^{-\beta\{\mathcal{Q}, \overline{\mathcal{Q}}\} + \omega_i (J_i + \frac{1}{2} Q)}$

Supersymmetry

$e^{\omega_1 J_1 + \omega_2 J_2 + \varphi Q}$ must commute with supercharge

Hosseini, Hristov, Zaffaroni

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$$\boxed{\varphi = \frac{\omega_1 + \omega_2}{2} - \pi i}$$

$$\rightarrow \quad Z(\beta, \omega_i) = \mathrm{Tr}\, e^{-\pi i Q}\, e^{-\beta\{\mathcal{Q}, \overline{\mathcal{Q}}\} + \omega_i (J_i + \frac{1}{2} Q)}$$

$$= \underbrace{\mathrm{Tr}\, (-1)^F\, e^{-\beta\{\mathcal{Q}, \overline{\mathcal{Q}}\} + (\omega_1 - 2\pi i)(J_1 + \frac{1}{2} Q) + \omega_2 (J_2 + \frac{1}{2} Q)}}_{\text{Witten index}}$$

$e^{\pi i F} = e^{-2\pi i J_1}$

spin-statistics

Kinney, Maldacena, Minwalla, Raju

Romelsberger

only states with $\{\mathcal{Q}, \overline{\mathcal{Q}}\} = 0$ contribute

$Z = Z(\omega_i)$, no dependence on β

protected: does not depend on continuous parameters

TASK : look for saddles

Cardy-like limit

A device to isolate black hole saddle

lots of people, starting from

Choi, J. Kim, S. Kim, Nahmgoong '18

. . .

DC, Komargodski '21

Small ω_1, ω_2 , with $\varphi = \frac{\omega_1 + \omega_2}{2} - \pi i$

$$\boxed{\begin{aligned} I_{\text{CFT}} &= \frac{16}{27}(3c - 2a)\,\frac{\varphi^3}{\omega_1\omega_2} - \frac{4}{3}(a - c)\,\varphi\,\frac{2\pi i(\omega_1 + \omega_2) - \omega_1\omega_2}{\omega_1\omega_2} \\ &\quad + \log|\mathcal{G}| + \mathcal{O}(\mathrm{e}^{-1/\omega}) \end{aligned}}$$

rank of discrete 1-form symmetry

= *N* in many *SU(N)* examples

a , c *super*conformal anomaly coeff. General and finite

- Holographic SCFT at large-*N* ➜ $a = c$ ➜ $I_{\text{CFT}} = \frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}\,a$

Microcanonical entropy

$$S = \text{ext}_{\{\omega_1,\omega_2,\varphi,\Lambda\}}\left[-I - \omega_1 J_1 - \omega_2 J_2 - \varphi Q - \Lambda(\omega_1 + \omega_2 - 2\varphi - 2\pi i)\right]$$

linearizing in $a - c$

DC, Ruipérez, Turetta

$$S = \pi\sqrt{3Q^2 - 8a(J_1 + J_2) - 16a(a-c)\frac{(J_1 - J_2)^2}{Q^2 - 2a(J_1 + J_2)}} \quad + \mathcal{O}((a-c)^2)$$

also Bobev, Dimitrov, Reys, Vekemans

Reality of Legendre transform ↔ constraint between the charges

$$\left[3Q + 4\,(2a - c)\right]\left[3Q^2 - 8c\,(J_1 + J_2)\right]$$

$$= Q^3 + 16\,(3c - 2a)\,J_1 J_2 + 64a\,(a-c)\frac{(Q+a)(J_1 - J_2)^2}{Q^2 - 2a(J_1 + J_2)} \quad + \mathcal{O}((a-c)^2)$$

Microcanonical entropy

Setting $a = c$

$$S = \pi\sqrt{3Q^2 - 8a(J_1 + J_2)} + \mathcal{O}(a - c)$$

$$(3Q + 4a)\left[3Q^2 - 8a\,(J_1 + J_2)\right] = Q^3 + 16aJ_1J_2 + \mathcal{O}(a - c)$$

Microcanonical entropy

Simplification also for $J_1 = J_2 \equiv J$

$$S = \pi\sqrt{3Q^2 - 16a\,J} \quad + \mathcal{O}((a-c)^2)$$

$$[3Q + 4\,(2a - c)]\left[3Q^2 - 16c\,J\right] = Q^3 + 16\,(3c - 2a)\,J^2 + \mathcal{O}((a-c)^2)$$

Gravity side

Gravitational partition function

$$Z(\beta, \Omega_i, \Phi) \; = \; \int Dg_{\mu\nu} DA_\mu D\psi_\mu \, \mathrm{e}^{-\mathrm{Action}[g_{\mu\nu},\, A_\mu,\, \psi_\mu]}$$

Hard . . . resort to saddle point approximation

$$Z \; \approx \; \mathrm{e}^{-I}$$

EFT approach

work directly in $d = 5$ and do not commit to specific top-down models

(SCFT details are unimportant, only a , c matter)

Holographic anomaly matching

R-current anomalies are matched by Chern-Simons terms in the bulk

$$\mathcal{L}_{\mathbf{grav}} \supset (5a - 3c)\, \epsilon^{\mu\nu\rho\sigma\lambda} \underset{\substack{\uparrow \\ \boxed{\text{2 der.}}}}{F_{\mu\nu} F_{\rho\sigma}} A_\lambda + \frac{9}{8}(c-a)\, \epsilon^{\mu\nu\rho\sigma\lambda} \underset{\substack{\uparrow \\ \boxed{\text{4 der.}}}}{R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta}} A_\lambda$$

- 2∂ supergravity ↔ $a\,,\, c$ large & equal

$$a = c = \frac{\pi}{8g^3G} + \dots \quad \sim N^2 + \dots$$

in many *SU(N)* examples

- corrections to $a\,,\, c$

(in particular, to $a - c = 0 + \cdots$) ↔ 4∂ susy invariants !

2-derivative AdS_5 Black Hole

- minimal AdS_5 supergravity

$$\mathcal{L} = R + 12g^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda \quad + \textit{fermions}$$

- *non*-supersymmetric, *non*-extremal black hole   Chong, Cvetic, Lu, Pope '05

- Supersymmetry: 2 supercharges, $\mathcal{Q}$, $\overline{\mathcal{Q}}$ Gutowski, Reall '04,

- Bekenstein-Hawking entropy

$$S = \frac{\textbf{Area}}{4G} = \pi\sqrt{3Q^2 - 8a(J_1 + J_2)}$$

$$a = \frac{\pi}{8g^3 G}$$

S. Kim, K.M. Lee

- Delicate part:
  on-shell action for $\beta = \infty$

CCMM

$$I_{\text{grav}} = \frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}a = I_{\text{CFT}}$$

✓

- constraints also agree

Dealing with higher derivatives

1. write down higher-derivative action
2. fix the boundary terms
3. evaluate it on solution
4. direct match of microcanonical entropy

1 4-derivative action

- start from *off-shell d* = 5 supergravity

- add two 4∂ invariants, controlled by λ_1, λ_2

$$\mathcal{L} = \mathcal{L}^{\text{off-sh}}_{2\partial} + \lambda_1 \tfrac{1}{g^2} \mathcal{L}^{\text{off-sh}}_{C^2} + \lambda_2 \tfrac{1}{g^2} \mathcal{L}^{\text{off-sh}}_{R^2}$$

Hanaki, Ohashi, Tachikawa '06

Ozkan, Pang '13

- integrate out auxiliary fields

- simplify result by field redefinitions

◆1 4-derivative action

Work at *linear* order in $\lambda_1, \ \lambda_2$

$$\Phi = \Phi^{(0)} + \lambda\, \Phi^{(1)}$$

$$I = I^{(0)}|_{\lambda=0} + \lambda \left(\partial_\lambda I^{(0)} + I^{(1)} \right)\Big|_{\lambda=0} + \mathcal{O}\left(\lambda^2\right)$$

1 4-derivative action

Work at *linear* order in λ_1, λ_2

$$\Phi = \Phi^{(0)} + \lambda\, \Phi^{(1)}$$

$$I = I^{(0)}|_{\lambda=0} + \lambda \left(\cancelto{0}{\partial_\lambda I^{(0)}} + I^{(1)} \right)\Big|_{\lambda=0} + \mathcal{O}\left(\lambda^2\right)$$

- EoM for auxiliary fields remain algebraic
- EoM dynamical fields remain second-order
- field redefinitions are perturbative ➜ simplifications

progress in:

Hanaki, Ohashi, Tachikawa '06

Cremonini, Hanaki, Liu, Szepietowski '08

Bobev, Hristov, Reys '21

Liu, Saskowski '22

◆1 4-derivative action

- arrive at 4∂ corrected supergravity action:

$$\mathcal{L} = c_0 R + 12 c_1 g^2 - \frac{1}{4} c_2 F^2 - \frac{1}{12\sqrt{3}} c_3\, \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda$$

$$+ \lambda_1 \frac{1}{g^2} \left[\mathcal{X}_{\mathrm{GB}} - \frac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right]$$

$$\mathcal{X}_{\mathrm{GB}} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$ Gauss-Bonnet

$$c_0 = 1 + 4\lambda_2\,, \quad c_1 = 1 - 10\lambda_1 + 4\lambda_2\,, \quad c_2 = 1 + 4\lambda_1 + 4\lambda_2\,, \quad c_3 = 1 - 12\lambda_1 + 4\lambda_2$$

1 4-derivative action

- arrive at 4∂ corrected supergravity action:

$$\boxed{\begin{aligned}\mathcal{L} \;=\; & c_0 R + 12 c_1 g^2 - \tfrac{1}{4} c_2 F^2 - \tfrac{1}{12\sqrt{3}} c_3\, \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda \\ & + \lambda_1 \tfrac{1}{g^2} \left[\mathcal{X}_{\mathrm{GB}} - \tfrac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \tfrac{1}{8} F^4 - \tfrac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right]\end{aligned}}$$

$$\mathcal{X}_{\mathrm{GB}} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \quad \text{Gauss-Bonnet}$$

$$c_0 = 1 + 4\lambda_2\,, \quad c_1 = 1 - 10\lambda_1 + 4\lambda_2\,, \quad c_2 = 1 + 4\lambda_1 + 4\lambda_2\,, \quad c_3 = 1 - 12\lambda_1 + 4\lambda_2$$

λ_2 renormalizes G_{Newton}

◆1 4-derivative action

- arrive at 4∂ corrected supergravity action:

$$\boxed{\begin{aligned}\mathcal{L} =\ & c_0 R + 12 c_1 g^2 - \tfrac{1}{4} c_2 F^2 - \frac{1}{12\sqrt{3}} c_3\, \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda \\ & + \lambda_1 \frac{1}{g^2} \Big[\mathcal{X}_{\rm GB} - \tfrac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \tfrac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \Big]\end{aligned}}$$

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$$c_0 = 1 + 4\lambda_2\,, \quad c_1 = 1 - 10\lambda_1 + 4\lambda_2\,, \quad c_2 = 1 + 4\lambda_1 + 4\lambda_2\,, \quad c_3 = 1 - 12\lambda_1 + 4\lambda_2$$

1 4-derivative action

- arrive at 4∂ corrected supergravity action:

$$\mathcal{L} = c_0 R + 12 c_1 g^2 - \frac{1}{4} c_2 F^2 - \frac{1}{12\sqrt{3}} c_3\, \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda$$

$$\boxed{+\lambda_1} \frac{1}{g^2} \left[\mathcal{X}_{\rm GB} - \frac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right]$$

$$\mathcal{X}_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \quad \text{Gauss-Bonnet}$$

$$c_0 = 1 + 4\lambda_2\,, \quad c_1 = 1 - 10\lambda_1 + 4\lambda_2\,, \quad c_2 = 1 + 4\lambda_1 + 4\lambda_2\,, \quad c_3 = 1 - 12\lambda_1 + 4\lambda_2$$

λ_1 from auxiliary fields & field redefinitions

1 4-derivative action

- arrive at 4∂ corrected supergravity action:

$$\mathcal{L} = c_0 R + 12 c_1 g^2 - \frac{1}{4} c_2 F^2 - \frac{1}{12\sqrt{3}} c_3\, \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda$$
$$\boxed{+\lambda_1} \frac{1}{g^2} \left[\mathcal{X}_{\rm GB} - \frac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right]$$

$$\mathcal{X}_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \quad \text{Gauss-Bonnet}$$

$$c_0 = 1 + 4\lambda_2\,, \quad c_1 = 1 - 10\lambda_1 + 4\lambda_2\,, \quad c_2 = 1 + 4\lambda_1 + 4\lambda_2\,, \quad c_3 = 1 - 12\lambda_1 + 4\lambda_2$$

λ_1 from auxiliary fields & field redefinitions

- Holographic anomaly matching ➜ dictionary :

$$a = \frac{\pi}{8 G g^3} (1 + 4\lambda_2)\,, \qquad c = \frac{\pi}{8 G g^3} (1 + 8\lambda_1 + 4\lambda_2)$$

2 Boundary terms

$$I_{\mathrm{grav}} = \underbrace{\int_{M_5} \mathcal{L} + \int_{\partial M_4} \mathcal{L}_{\mathrm{GHY}}}_{\substack{\text{divergences only in} \\ 2\partial \text{ sector and Gauss-Bonnet term}}} + \int_{\partial M_4} \mathcal{L}_{\mathrm{ct}}$$

- 2∂ sector ➜ standard holographic counterterms
- Gauss-Bonnet term ➜ Myers '87
  Liu, Sabra '08; Cremonini, Liu, Szepietowski '09
- These are all the relevant terms DC, Ruipérez, Turetta '23
- Variational principle is also under control

◆3 On-shell action

corrected EoM : hard in general.

Luckily not necessary when working at linear order in corrections:

enough to evaluate $$I_{\text{grav}} = \int_{M_5} \mathcal{L} + \int_{\partial M_4} \mathcal{L}_{\text{GHY}} + \int_{\partial M_4} \mathcal{L}_{\text{ct}}$$

on *uncorrected* solution

Reall, Santos

using "*susy first, extremal later*" prescription CCMM

$$I_{\text{grav}} = \frac{16}{27}(3c-2a)\frac{\varphi^3}{\omega_1\omega_2} - \frac{4}{3}(a-c)\,\varphi\,\frac{2\pi i(\omega_1+\omega_2)-\omega_1\omega_2}{\omega_1\omega_2}$$

$$+\mathcal{O}((a-c)^2) \quad = I_{\text{CFT}}$$

➜ **matches CFT !**

also: Bobev, Dimitrov, Reys, Vekemans

4 Direct match of the entropy

- corrected EoM hard in general, except when there is enough symmetry.

We study *near-horizon* geometry with $J_1 = J_2 \equiv J$

Ansatz for metric and gauge field:

$$ds^2 = v_1 \left(-r^2 \mathrm{d}t^2 + \frac{dr^2}{r^2} \right) + \frac{v_2}{4} \left[\sigma_1^2 + \sigma_2^2 + v_3 \left(\sigma_3 + w\, r\, dt \right)^2 \right]$$

$$A = e\, r\, dt + p\, \left(\sigma_3 + w\, r\, dt \right)$$

EoM are then algebraic ➜ we find corrected *near-horizon* solution

Keep boundary conditions unchanged.

4 Direct match of the entropy

We checked that BH thermodynamics works in higher-der theory with CS terms

- entropy given by Wald's formula

$$S_{\text{Wald}} = -2\pi \int_{\mathcal{H}} \mathrm{d}^3x \sqrt{\gamma}\, \mathcal{P}^{\mu\nu\rho\sigma} \underset{\nearrow}{n_{\mu\nu}}\, n_{\rho\sigma} \,, \qquad \mathcal{P}^{\mu\nu\rho\sigma} = \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}}$$

binormal

$$S_{\text{Wald}} = \frac{\pi^2}{g^3 G} \frac{\tilde{a}\sqrt{\tilde{a}(\tilde{a}+2)}}{(1-\tilde{a})^2} \left[1 + 4\lambda_2 + 48\lambda_1 \frac{2\tilde{a}^2 + 5\tilde{a} + 2}{11\tilde{a}^2 + 8\tilde{a} - 1} \right]$$

To obtain microcanonical form we need the corrected charges

Strategy: use formulae for charges that obey Gauss law

$$Q = \int_{\infty} *F \underset{\substack{\uparrow \\ F(\infty)=0}}{=} \int_{\infty} (*F + A \wedge F) \underset{\substack{\uparrow \\ \mathrm{d}(*F + A\wedge F) = 0}}{=} \int_{\mathcal{H}} (*F + A \wedge F)$$

4 Direct match of the entropy

$$Q = \frac{1+4\lambda_2}{16\pi G}\int_{\mathcal{H}}\left(*\mathcal{F} + \frac{1-12\lambda_1}{\sqrt{3}} A\wedge F + \frac{2\lambda_1}{\sqrt{3}g^2}\Omega_{\mathrm{CS}}\right) \qquad \mathcal{F}^{\mu\nu} = -2\,\frac{\partial(\mathcal{L}-\mathcal{L}_{\mathrm{CS}})}{\partial F_{\mu\nu}}$$

Lorentz-Chern-Simons

$$J = \frac{1+4\lambda_2}{32\pi G}\int_{\mathcal{H}}\sqrt{\gamma}\,n_{\mu\nu}\left(-4\nabla_\sigma\mathcal{P}^{\mu\nu\sigma\rho}\eta_\rho + 2\mathcal{P}^{\mu\nu\sigma\rho}\nabla_\sigma\eta_\rho + \iota_\eta A\left(\mathcal{F}^{\mu\nu} + \frac{1-12\lambda_1}{3\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}A_\rho F_{\sigma\lambda}\right)\right)$$

rotational Killing vector

Evaluate these on the corrected near-horizon solution

$$S_{\mathrm{Wald}} = \pi\sqrt{3Q^2 - 16aJ}$$ ✔

Outlook

- SCFT$_4$ prediction beyond Bekenstein-Hawking works

  ➜ a step towards understanding **full Quantum Entropy**

- Multi-charge black holes: action I still controlled by anomalies

  only multi-charge d=5 BPS black hole known with no hypers is for IIB on S^5

  ➜ very simple anomalies

- Go beyond linearization in $a-c$

  conjecture: only susy Chern-Simons matter,

  globally well-defined invariants don't contribute

  Reminiscent of 3d rigid susy Closset, Dumitrescu, Festuccia, Komargodski

  ↓

  localization of the full gravitational partition function (in Cardy limit)?

Outlook

? How to determine $I(\omega_i, \varphi_K)$ using a minimal set of data?

- gravitational blocks Hosseini, Hristov, Zaffaroni
- integration of anomaly polynomial Ohmori, Tizzano
- localization of the gravitational action Benetti Genolini, Perez Ipiña, Sparks
- Alberto' s talk

thank you !