Giant Gravitons and non-conformal vacua in Twisted Holography

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Based on arxiv:2106.14859, arxiv:2211.01419 with Davide Gaiotto

PERIMETER INSTITUTE

Twisting Supersymmetric QFTs

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- **Twisted holography:** holographic duals of these twists

Twisted Holography

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$\mathcal{N} = 4$ SYM with $U(N)$ $\approx$ type IIB on AdS$_5 \times S^5$

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4. Future directions:
   - "Bootstraping" to $\text{AdS}_5 \times S^5$
   - LLM type geometries
   - Holomorphic twist of $\mathcal{N} = 4$ SYM

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- D1-branes are **holomorphic curves** in $SL(2,\mathbb{C})$

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- We will be interested in correlation functions of determinant operators

$$\det(m + Z(u; z))$$

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- D-branes wrap **holomorphic submanifolds** eg. D1-branes are holomorphic complex lines

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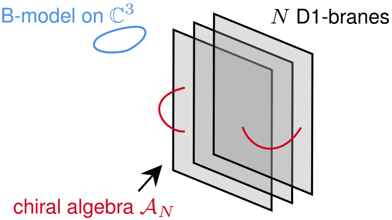

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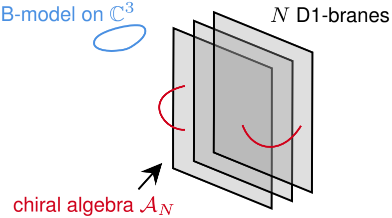

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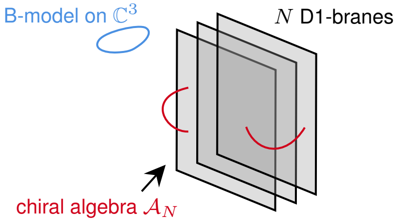

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[Costello, Gaiotto '18]

$$\text{B-model on } \mathbb{C}^3 + N \text{ D1-branes} \longrightarrow \text{B-model on } SL(2,\mathbb{C}) \approx \text{AdS}_3 \times S^3$$

$$\uparrow$$

$$\mathcal{A}_N$$

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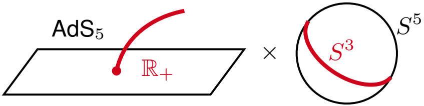

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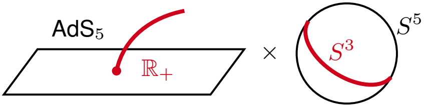

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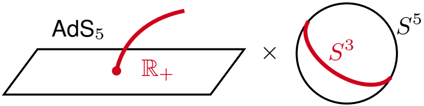

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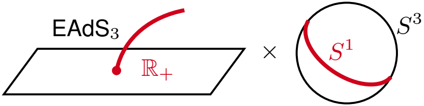

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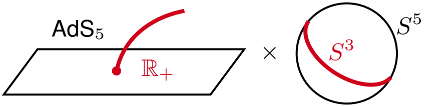

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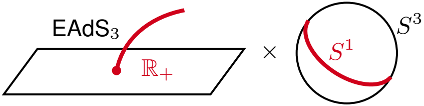


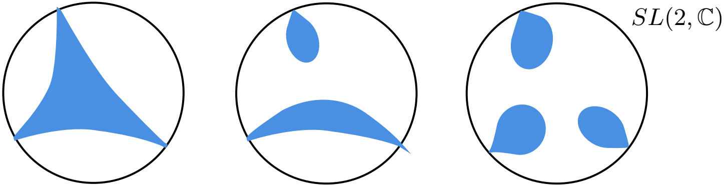
- Determinant operator:

$$\det(m + Z(u;z)), \qquad Z(u;z) = X(z) + uY(z)$$

  - z = position at the boundary of AdS$_3$
  - u controls orientation of $S^1 \subset S^3$
  - m controls size of $S^1 \subset S^3$

Giant Gravitons

- Many possible brane configurations with the same boundary behaviour



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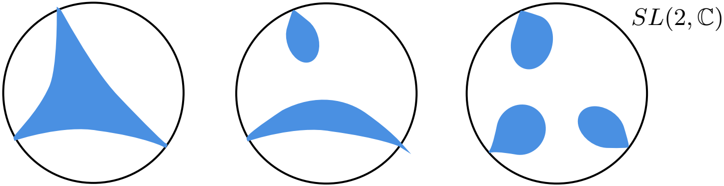

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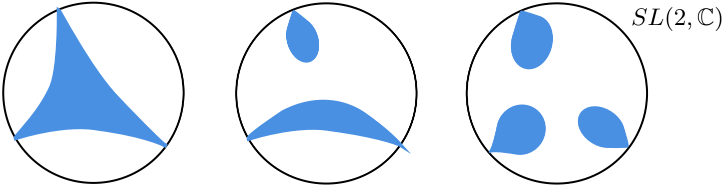

- We will match saddles ρ^* of correlation functions of determinants with brane configurations
  - m_i, u_i, z_i control boundary behaviour
  - Saddles ρ will control the shape in the bulk

Determinant correlation functions

[Jiang, Komatsu, Vescovi '19]

- Fermionize determinants

$$\det(m + Z(u;z)) = \int [\mathrm{d}\bar{\psi}\mathrm{d}\psi]\, e^{\bar{\psi}(m+Z(u,z))\psi}$$

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- Rewrite correlation function using **auxiliary bosonic variables** ρ^i_j for $i \neq j$, $\rho^i_i \equiv m_i$

$$\left\langle \prod_i^k \det(m_i + Z(u_i; z_i)) \right\rangle \sim \int [\mathrm{d}\rho]\; e^{N\, S[\rho]}$$

with action

$$S[\rho] = \frac{1}{2} \sum_{i \neq j} \frac{z_i - z_j}{u_i - u_j} \rho^i_j \rho^j_i + \log\det\rho$$

Saddles and branes

- **Saddle point equations** in the matrix form:

$$[\zeta, \rho] + [\mu, \rho^{-1}] = 0$$

where

$$\zeta = \begin{pmatrix} \mathsf{z}_1 & & \\ & \ddots & \\ & & \mathsf{z}_k \end{pmatrix}, \quad \mu = \begin{pmatrix} \mathsf{u}_1 & & \\ & \ddots & \\ & & \mathsf{u}_k \end{pmatrix}, \quad \rho = \begin{pmatrix} \mathsf{m}_1 & & ? \\ & \ddots & \\ ? & & \mathsf{m}_k \end{pmatrix}$$

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- $k > 2$ would be hard in $AdS_5 \times S^5$

Spectral curve

For each saddle ρ we define a spectral curve S_ρ:

- Define **commuting matrices**:

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- The matrices are defined so that:
  - They commute when ρ satisfies the saddle point equations
  - They satisfy

$$aD(a) - B(a)C(a) = 1$$

  - They have the expected boundary behavior

Holographic checks

- **Boundary behaviour** $a \to \infty$:

$$\frac{B(a)}{a} = \begin{pmatrix} u_1 - \frac{m_1}{a} & & \\ & \ddots & \\ & & u_k - \frac{m_k}{a} \end{pmatrix} + \dots, \qquad \frac{C(a)}{a} = \begin{pmatrix} z_1 & & \\ & \ddots & \\ & & z_k \end{pmatrix} + \dots$$

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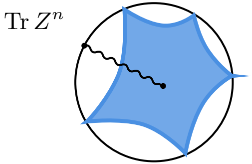
- **Various holographic checks:**



  - Correlation functions of determinants with a **single-trace** [Jiang, Komatsu, Vescovi '19]
  - Action $S[\rho]$ vs S[brane]
  - Modifications of determinants / excitations of the brane

Determinant modifications

- For example

$$\det X \to \frac{1}{N!}\varepsilon\varepsilon(X, X, X, \ldots, Y^2)$$

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- Create modifications by acting with the modes, eg.

$$\oint \mathrm{Tr}\, Y^4(z) \ \det X(0) \sim \varepsilon\varepsilon(X, X, X, \dots, Y^3) + \dots$$

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Determinant modifications

- We computed 2-pt functions of determinant modifications

$$\left\langle [J^{(n')}_{p',q'}, \det Y(\infty)][J^{(n)}_{p,q}, \det X(0)] \right\rangle \Big|_{N\to\infty}$$

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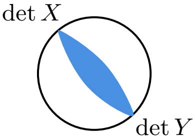
$$\begin{aligned} J^{n}_{p,p-1} \;:\quad & \det X \longrightarrow n\, \varepsilon\varepsilon(X, X, \ldots, Y^{1-2p}) \\ J^{n}_{p,p+1} \;:\quad & \det X \longrightarrow n\, \varepsilon\varepsilon(X, X, \ldots, Y^{-2p-1}\partial X) \\ & \qquad\qquad + n\, \varepsilon\varepsilon(X, X, \ldots, \partial^2 Y^{-2p-3}) \end{aligned}$$

Brane excitations

- $\langle \det Y(\infty) \det X(0) \rangle$ has a single nontrivial saddle corresponding to the brane:

$$\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} \subset SL(2,\mathbb{C})$$



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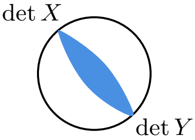

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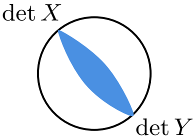

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For standard $SL(2, \mathbb{C})$ geometry:

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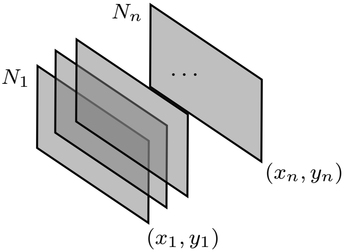

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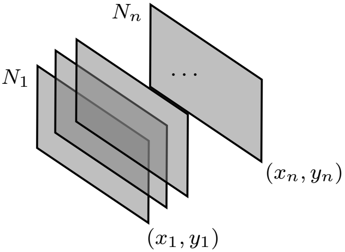

- Holographic check:
  - Determinant correlation functions
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- Mathematical question:

Solutions of matrix equations $[\zeta, \rho] + [\mu, \rho^{-1}] = 0$	$\Longleftrightarrow$	Holomorphic curves in $SL(2, \mathbb{C})$

$\Rightarrow$ Spectral curve construction

$\Leftarrow$ For genus $g = 0$, we can go back

$\Leftarrow$ For genus $g > 0$, we don't know

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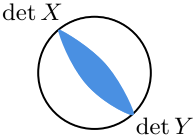

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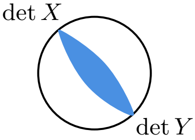

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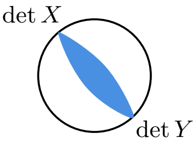

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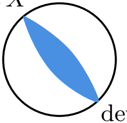

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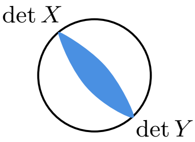

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Thank you!

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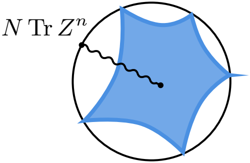
Correlation function of determinants with a **single-trace**
[Jiang, Komatsu, Vescovi '19]:

$$\left\langle \prod_i \mathcal{D}(m_i; u_i; z_i)\, N\,\mathrm{Tr}\, Z^n \right\rangle\Big|_{N\to\infty} \approx -e^{NS[\rho^*]}\; N\,\mathrm{Tr}_{k\times k}(-\rho\frac{\mu-u}{\zeta-z})^n\Big|_{\rho=\rho^*}$$



We can rewrite it to a form

$$\int_{S_{\rho^*}} \partial^{-1}\alpha,$$

where α is a Kodaira-Spencer field sourced by $N\,\mathrm{Tr}\,Z^n$:

$$\alpha = \partial\Big((b-ua)^n \delta_{\frac{c}{a}=z} + (za-c)^{-n}\delta_{\frac{b}{a}=u}\Big)$$