Precision Holography for M2 branes

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Based on

[arXiv:2304.01734] & [arXiv:2210.09318]

with

N. Bobev and J. Hong

Motivation

- AdS/CFT provides a gauge theory description of string or M-theory on asymptotically locally AdS backgrounds. Immediate consequence:

$$Z_{\text{CFT}}[J] = Z_{\text{string/M}}[\phi]$$

- Tested thoroughly in the large N limit.

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- The correspondence is meant to be valid at finite N. Schematically,

$$\log Z_{\mathsf{CFT}} = F_0(\lambda) + \frac{1}{N^2}\, F_1(\lambda) + \ldots$$

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- At strong coupling, CFT observables encode information about supergravity. Corrections teach us about string/M-theory beyond the low-energy limit.
- Studying and understanding corrections on both sides of the duality:

Precision Holography

Precision holography

- Need concrete realizations where we can test and make predictions
  $\rightarrow$ use supersymmetry to gain computational mileage.

SCFT

Localization gives exact results

Solving the matrix models yields $1/N$ expansion to any order

Various techniques available

Analytic & numeric

Supergravity

Exact results out of reach*

Work order-by-order in the derivative expansion

Can also study one-loop effects

LO, NLO, NNLO tests

*except special low-d settings

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- New handle on AdS vacua of string/M-theory with non-trivial fluxes.

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SCFT	*Supergravity*
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- SCFT side under better control → use it to make predictions.
- New handle on AdS vacua of string/M-theory with non-trivial fluxes.
- This talk: $\text{AdS}_4/\text{CFT}_3$ holography from M2 branes.

AdS_4/CFT_3 from M2 branes

- Study 3d $\mathcal{N} \geq 2$ SCFTs arising from low-energy limit of N M2 branes.
- No λ in M-theory $\rightarrow$ compare numbers at each order in $1/N$.

$\text{AdS}_4/\text{CFT}_3$ from M2 branes

- Study 3d $\mathcal{N} \geq 2$ SCFTs arising from low-energy limit of N M2 branes.
- No λ in M-theory $\rightarrow$ compare numbers at each order in $1/N$.
- M-theory engineers many dual pairs:

3d Chern-Simons-matter theories

$\Updownarrow$

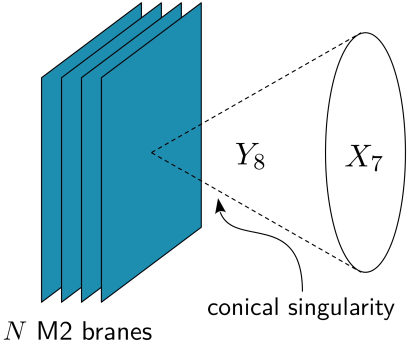
M-theory on $\text{AlAdS}_4 \times X_7$

Take X_7 to be Sasaki-Einstein $\rightarrow$ CSm

$X_7 = S^7/\mathbb{Z}_k$ (free) $\rightarrow$ ABJM

$X_7 = S^7/\mathbb{Z}_r$ (f.p.) $\rightarrow$ ADHM

$X_7 = N^{010},\ V^{52},\ Q^{111},\ \ldots$



- Put the gauge theory on compact M_3 and study susy partition functions. Use localization to compute them exactly for various (X_7, M_3).

Sphere partition functions

The squashed sphere

- Put the SCFT on the squashed 3-sphere: $M_3 = S^3_b$

$$\omega_1^2(x_1^2 + x_2^2) + \omega_2^2(x_3^2 + x_4^2) = 1 \quad \text{with} \quad b^2 = \omega_1/\omega_2$$

Preserves $\mathrm{U}(1) \times \mathrm{U}(1)$ isometry, symmetry $b \leftrightarrow 1/b$

- Compute the partition fct exactly using localization. [Hama,Hosomichi,Lee'11]

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- For ABJM corresponding to $X_7 = S^7/\mathbb{Z}_k$

$$Z_{\text{ABJM}}(b) = \frac{1}{(N!)^2} \int d^N\mu \, d^N\nu \, e^{\mathrm{i}\pi k \sum_i (\nu_i^2 - \mu_i^2)} \prod_{i>j} 4\sinh[\pi b(\mu_i - \mu_j)] \sinh[\pi b^{-1}(\mu_i - \mu_j)] \times (\mu \to \nu) \prod_{i,j} s_b\left[\tfrac{\mathrm{i}Q}{4} - \mu_j + \nu_i\right]^2 s_b\left[\tfrac{\mathrm{i}Q}{4} + \mu_j - \nu_i\right]^2$$

$Q = b + b^{-1}$ and s_b is the double sine function $s_b(x) = \prod_{m,n} \frac{mb + nb^{-1} + \frac{Q}{2} - \mathrm{i}x}{mb + nb^{-1} + \frac{Q}{2} + \mathrm{i}x}$

The Fermi gas

- For precision holography, need access to $1/N$ corrections.
- Simplifications for some values of the parameters:

theory	parameters $\mathcal{F}$	susy	cf.
$X_7 = S^7/\mathbb{Z}_k$	$b^2 = 1$ and $k \geq 1$	$\mathcal{N} = 6$	[Mariño,Putrov'11]
$X_7 = S^7/\mathbb{Z}_r$	$b^2 = 1$ and $r \geq 1$ $b^2 = 3$ and $N_f \geq 1$	$\mathcal{N} = 4$	[Mezei,Pufu'13] [Hatsuda'16]
$X_7 = N^{010}/\mathbb{Z}_k$	$b^2 = 1$ and $k \geq 1$	$\mathcal{N} = 3$	[Mariño,Putrov'11]

- For these cases, reformulation in terms of free Fermi gas

$$Z_{S_b^3}(\mathcal{F}) = \frac{1}{N!} \sum_{\sigma \in S_N} (-1)^\sigma \int d^N x \prod_{i=1}^{N} \rho_b(x_i, x_{\sigma(i)})$$

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- Leads to an Airy function encoding all perturbative terms

$$Z_{S_b^3}(\mathcal{F}) = e^A C^{-\frac{1}{3}}\,\mathrm{Ai}\big[C^{-\frac{1}{3}}(N-B)\big] + \mathcal{O}(e^{-\sqrt{N}})$$

Dual supergravity

- Asymptotic expansion of the free energy at large N:

$$-\log Z_{S^3_b}(\mathcal{F}) = \frac{2}{3\sqrt{C}} N^{\frac{3}{2}} - \frac{B}{\sqrt{C}} N^{\frac{1}{2}} + \frac{1}{4} \log N + \mathcal{O}(N^0)$$

- Dual to 4d minimal supergravity solutions (g_4, A) with $\Lambda < 0$ and $\partial\mathcal{M} = S^3_b$

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- LO $N^{\frac{3}{2}}$ term matches the two-derivative regularized on-shell actions.
  [Emparan,Johnson,Myers'99; Martelli,Passias,Sparks'11]

- NLO $N^{\frac{1}{2}}$ term reproduced by including bulk four-derivative terms.
  [Bobev,Charles,Hristov,VR'21]

- NNLO $\log N$ term is a one-loop effect from summing over the KK modes around the 11d backgrounds. [Bhattacharyya,Grassi,Mariño,Sen'12]

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- NNLO $\log N$ term is a one-loop effect from summing over the KK modes around the 11d backgrounds. [Bhattacharyya,Grassi,Mariño,Sen'12]
- Precision holography $\rightarrow$ prediction for all higher-derivative and higher-loop effects in the bulk!

Beyond the Fermi gas?

- For general b there is no known Fermi gas.
- Conjecture: there is again an Airy function [Bobev,Hong,VR'22 & '23]

$$Z_{S^3_b} = e^{A} C^{-\frac{1}{3}} \,\mathrm{Ai}\left[C^{-\frac{1}{3}}(N-B)\right] + \mathcal{O}(e^{-\sqrt{N}})$$

theory	C	B	susy
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$X_7 = S^7/\mathbb{Z}_r$	$\frac{32}{\pi^2 r} Q^{-4}$	$-\frac{7r}{24}\left(1-\frac{16}{7}Q^{-2}\right) - \frac{1}{3r}\left(1-10Q^{-2}\right)$	$\mathcal{N}=4$
$X_7 = N^{010}/\mathbb{Z}_k$	$\frac{12}{\pi^2 k} Q^{-4}$	$-\frac{5k}{48}\left(1-\frac{16}{5}Q^{-2}\right) - \frac{1}{3k}\left(1-5Q^{-2}\right)$	$\mathcal{N}=3$
$X_7 = V^{52}/\mathbb{Z}_r$	$\frac{81}{4\pi^2 r} Q^{-4}$	$-\frac{r}{6}\left(1-\frac{9}{2}Q^{-2}\right) - \frac{1}{4r}\left(1-9Q^{-2}\right)$	$\mathcal{N}=2$

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- Consistent with ongoing detailed numerical studies of the matrix model.
- Question: what structure produces this Airy function in M-theory?
  Is there a Fermi gas? A topological string? Something new?

A word on correlators

- The conjecture has implications for SCFT dynamics since the squashing parameter b couples to the stress tensor.

- $\mathcal{N} = 2$ Ward identities imply [Closset,Dumitrescu,Festuccia,Komargodski'12]

$$\langle T_{\mu\nu}(x)\, T_{\rho\sigma}(0)\rangle = c_T\, \mathcal{I}_{\mu\nu\rho\sigma}(x) \quad \text{and} \quad c_T = \frac{\partial^2 \log Z(b)}{\partial b^2}\Big|_{b=1}$$

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- Taking more derivatives gives access to integrated correlators of stress tensors over the round S^3.
- An appropriate flat space limit gives a way to study scattering amplitudes in M-theory $\rightarrow$ access HD structures in the M-theory EFT.

[Chester,Pufu,Yin'18; Binder,Chester,Pufu'18]

Twisted indices

The topologically twisted index (TTI)

- Put the SCFT on $M_3 = S^1 \times S^2$
- Turn on flux for exact R-sym $\int_{S^2} dA_R = 2\pi \to$ topological twist.
- Localization gives an exact result: [Benini,Zaffaroni'15 & '17; Closset,Kim'16]

$$Z_{S^1\times S^2} = \sum_{\mathfrak{m}_1,\ldots,\mathfrak{m}_p \in \mathbb{Z}^N} \oint_{\mathcal{C}} Z_{\text{int}}(u,\mathfrak{m})$$

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- After some manipulations, the TTI can be written in the form

$$Z_{S^1\times S^2} = \sum_{\{x_i,\tilde{x}_j\}\in\text{BAE}} \mathcal{B}(x_i,\tilde{x}_j)$$

- Sum over sols to transcendental eqs: Bethe Ansatz Equations. For ABJM,

$$x_i^k \prod_{j=1}^{N} \frac{(1-\mathrm{i}\frac{\tilde{x}_j}{x_i})^2}{(1+\mathrm{i}\frac{\tilde{x}_j}{x_i})^2} = (-1)^N \quad \text{and} \quad \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1-\mathrm{i}\frac{\tilde{x}_j}{x_i})^2}{(1+\mathrm{i}\frac{\tilde{x}_j}{x_i})^2} = (-1)^N$$

Numerical evaluation

- Known solution to BAE at large N:

  [Benini,Hristov,Zaffaroni‘15; Liu,Pando Zayas,Rathee,Zhao‘17]

$$\log x_i = N^{\frac{1}{2}}\, t_i - \mathrm{i} v_i\,, \quad \log \tilde{x}_j = N^{\frac{1}{2}}\, t_j - \mathrm{i}\tilde{v}_j\,.$$

  where t_i, v_i and $\tilde{v}_j$ do not scale with N.

- Use as `init` $\to$ numerically solve BAE at finite (but large) N and fixed k.
- Evaluate TTI numerically and fit

$$\log Z_{S^1\times S^2} = f_{3/2}(k)\, N^{\frac{3}{2}} + f_{1/2}(k)\, N^{\frac{1}{2}} + f_{\log} \log N + \sum_{s=0}^{L} f_{-s/2}\, N^{-\frac{s}{2}}$$

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- Observe that the integer powers can be resummed in terms of a "shifted" N

$$f_{\log} \log N + \sum_{s=1}^{L/2} f_{-s}\, N^{-s} \simeq -\frac{1}{2} \log \widehat{N}$$

- Fit with respect to $\widehat{N}$ instead $\rightarrow$ the fit terminates!

The ABJM twisted index

- Detailed numerics → we can propose an analytic formula: [Bobev,Hong,VR'22]

$$-\log Z_{S^1\times S^2} = \frac{\pi\sqrt{2k}}{3}\left(\widehat{N}^{\frac{3}{2}} - \frac{3}{k}\widehat{N}^{\frac{1}{2}}\right) + \frac{1}{2}\log\widehat{N} + f(k) + \mathcal{O}(e^{-\sqrt{N}})$$

with $\widehat{N} = N - \frac{k}{24} + \frac{2}{3k}$

k	$R_{3/2}$	$R_{1/2}$	$f(k)_{\text{num}}$	σ_0
1	2.436×10^{-39}	5.319×10^{-37}	3.045951	7.834×10^{-36}
2	9.935×10^{-28}	4.336×10^{-25}	1.786596	4.310×10^{-24}
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- Expand at large $N \to N^{\frac{3}{2}}$, $N^{\frac{1}{2}}$, $\log N$ terms... and all perturbative corrections.

Dual supergravity

- The index captures the path integral of M-theory on the 11d background

$$ds_{11}^2 = \frac{L^2}{4} ds_4^2 + L^2 ds_{\mathbb{CP}^3}^2 + L^2 \Big(d\psi + \sigma + \frac{1}{4} A \Big)^2$$
$$G_4 = \frac{3L^3}{8} \mathrm{vol}_4 - \frac{1}{4} \star_4 F \wedge J$$

with $(g_4, F = dA)$ the Euclidean Romans solution of 4d $\mathcal{N} = 2$ minimal gauged supergravity. [Romans'92; Genolini,Ipiña,Sparks'19; Bobev,Charles,Min'20]

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- LO, NLO and NNLO for BH entropy match the corresponding terms in TTI. [Benini,Hristov,Zaffaroni'16; Bobev,Hong,VR'22; Liu,Pando Zayas,Rathee,Zhao'17]

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- LO, NLO and NNLO for BH entropy match the corresponding terms in TTI.
[Benini,Hristov,Zaffaroni'16; Bobev,Hong,VR'22; Liu,Pando Zayas,Rathee,Zhao'17]

- Precision holography → prediction for the corrected Bekenstein-Hawking entropy of this BPS black hole to all orders in $1/N$.

Other SCFTs

- $\mathcal{N} = 4$ with $X_7 = S^7/\mathbb{Z}_r \to \widehat{N} = N + \frac{7r}{24} + \frac{1}{3r}$ and

$$\frac{\log Z_{S^1\times\Sigma_{\mathfrak{g}}}}{\mathfrak{g}-1} = \frac{\pi\sqrt{2r}}{3}\left(\widehat{N}^{\frac{3}{2}} - \left(\frac{r}{2} + \frac{5}{2r}\right)\widehat{N}^{\frac{1}{2}}\right) + \frac{1}{2}\log\widehat{N} + g(r)$$

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- In all cases, black hole solution exists $\to$ prediction for entropy at finite N.

Type IIA string theory

ABJM on S^3 in the IIA limit

- Reorganize the M-theory expansion in a IIA expansion

$$F_{S^3} = -\sum_{g\geq 0}(2\pi i)^{2g-2}\, F_g(\lambda)\, k^{2-2g}$$

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- Obtain genus-g free energies of IIA strings on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ (up to $\mathcal{O}(e^{-\sqrt{\lambda}})$)

$$F_0(\lambda) = \frac{4\pi^3\sqrt{2}}{3}\,\hat{\lambda}^{\frac{3}{2}} + \frac{\zeta(3)}{2}$$

$$F_1(\lambda) = \frac{\pi}{3\sqrt{2}}\,\hat{\lambda}^{\frac{1}{2}} - \frac{1}{4}\log\hat{\lambda} + \frac{1}{6}\log\lambda + 2\zeta'(-1) - \frac{3}{4}\log 2 + \frac{1}{6}\log\frac{\pi}{2}$$

$$F_2(\lambda) = \frac{5}{96\pi^3\sqrt{2}}\,\hat{\lambda}^{-\frac{3}{2}} - \frac{1}{48\pi^2}\,\hat{\lambda}^{-1} + \frac{1}{144\pi\sqrt{2}}\,\hat{\lambda}^{-\frac{1}{2}} - \frac{1}{360}$$

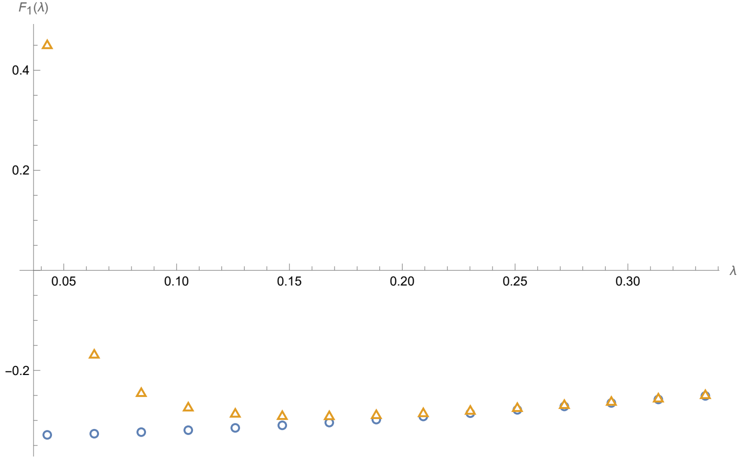
$$\ldots$$

with $\hat{\lambda} = \lambda - \frac{1}{24}$ the shifted 't Hooft coupling.

Comparing to the topological string

- Topo string on local $\mathbb{P}^1 \times \mathbb{P}^1$ gives non-perturbative genus-1 free energy

[Huang,Klemm'06; Mariño,Pasquetti,Putrov'09; Drukker,Mariño,Putrov'11]



Triangles: perturbative result; circles: topological string.

ABJM on S_b^3 and $S^1 \times S^2$ in the IIA limit

- Our proposals give access to free energies of IIA string theory on backgrounds of the form $\mathcal{M}_4 \times \mathbb{CP}^3$

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- Suggests that the free energies can be obtained to all orders in α'.
- Question: is there a worldsheet string theory computation?

Conclusions

- Localized partition functions in SCFTs can be studied very precisely.
- Precision holography uncovers structures in M-theory: Airy, “shifted” N...
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Thank you for your attention!