Strings in AdS_3 and Black Hole microstates

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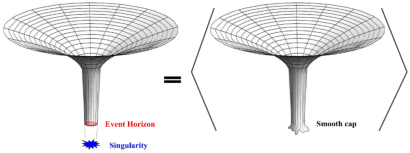
Based on ArXiv:
2208.00978 SciPost, 2105.02255, 2210.15313, 2212.05877, 2304.08361 JHEP, 2203.13828 PRL,
in collaboration with **Davide Bufalini, Sergio Iguri, Julián Toro** and **David Turton**.

Introduction and motivations

Fuzzballs and WS for BH microstates [Mathur et al,. . . 01-]

Fuzzball paradigm



$e^{S_{\rm BH}}$ **microstates**, some are smooth & horizonless geometries which have structure at the *horizon* scale.

Dynamics of *light* probes for
Microscopics? Evarporation?
Singularity? Observables?

Usual tools for this:

- Supergravity (wave eqs.)

- AdS/CFT (protected corrs)

But many microstates and observables are non-susy and/or highly curved!

We need an alternative description of string propagation!

- Some BH and BHμ admit solvable worldsheet theories:

We can study them exactly!

- The main tools come from string propagation in AdS$_3$.

JMaRT microstates: generalities [Jejjala et al 05]

We consider the JMaRT family of solutions. Pros / Cons:

- Microstates of asymptotically flat BHs in 5D.
  (in the grand canonical sense)
- Three-charge systems D1-D5-P / NS5-F1-P.
- IR AdS_3 region potentially understood from holography.
- Generically non-supersymmetric.
- Include BPS and 2-charge systems as limits: supertubes [GLMT 12]

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- These are **not** very *typical* microstates. (closer to BH for $k \gg 1$)
- AF solutions have ergoregion instabilities. [Cardoso et al 05]

Exact worldshet models for black hole microstates

[MMT 17-20] showed that they admit an **exact** (null) coset description

$$\frac{\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2) \times \mathbb{R}_t \times \mathrm{U}(1)_y}{\mathbb{R} \times \mathrm{U}(1)} \times T^4,$$

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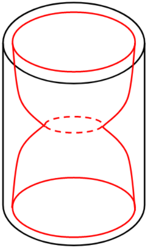
In this talk I will...

1. Describe the main building blocks of these WS models.
2. Present new results for the SL(2,$\mathbb{R}$) WZW model.
3. Discuss the applications to the BHμ coset models.
4. Obtain many exact HL...LH correlators in these microstates.

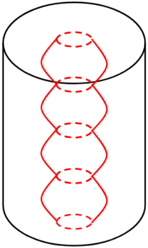
A proof for spectrally flowed 3pt-functions in AdS_3

String Spectrum for bosonic AdS_3 [SGK,Teschner,Maldacena-Ooguri 98-]

For pure NSNS fluxes, the worldsheet theory is the SL(2,$\mathbb{R}$)-WZW model:



- Continuum of **long string** *scattering* states with $j = \frac{1}{2} + i\mathbb{R}$, $m \in \mathbb{R}$ $(\sim H_3^+)$
- Discrete set of **short string** *bound* states $\frac{1}{2} < j < \frac{k-1}{2}$, $m = \pm(j+n)$ $(\sim SU(2))$
- Key aspect: **spectral flow** $\omega \sim$ *winding*



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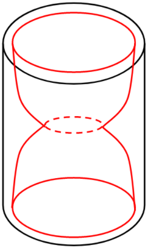

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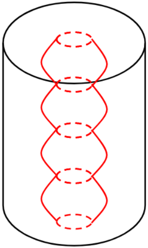


The unflowed sector in the x-basis $\to V_j(x;z)$

Zero-mode currents $\sim$ differential operators in the boundary coord,

$$J_0^+ \sim \partial_x \quad J_0^3 \sim x\partial_x + j \quad J_0^- \sim x^2\partial_x + 2jx$$

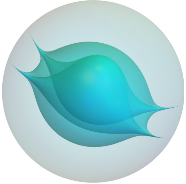
and structure constants $C(j_1, j_2, j_3)$ for $\omega = 0$ primaries are obtained by analytic continuation in $j(=h)$ from the H_3^+-model (Liouville).

Spectral flow and string correlators for bosonic AdS_3

Long standing ?

$$\left\langle \prod_i V^{\omega_i}_{j_i h_i}(x_j, z_i) \right\rangle$$

- ω is the spectral flow charge
- j is the *unflowed* spin, fixing Δ
- h is the holographic spacetime weight ($\neq j$ for $\omega > 0$)



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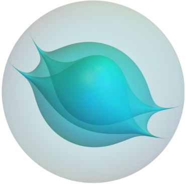

These operators are **highly non-canonical** from the worldsheet CFT point of view: they are Virasoro primaries but **not affine primaries**,

$$J^+(w)V^{\omega}_{jh}(x,z) \sim \frac{c^+ V^{\omega}_{j,h+1}(x,z)}{(w-z)} + \sum_{n=2}^{\omega} \frac{\left(J^+_{n-1} V^{\omega}_{jh}\right)(x,z)}{(w-z)^n} + \frac{\partial_x V^{\omega}_{jh}(x,z)}{(w-z)}$$

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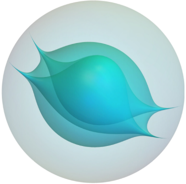

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where we have many unknowns, but also many constraints:

$$J^-(x,w)V^{\omega}_{jh}(x,z) = (w-z)^{\omega-1}\, c^- V^{\omega}_{j,h-1}(x,z) + \ldots$$

which leads to complicated recursion relations in h_i. [Eberhardt et al 19]

The y-basis for Flowed operators [Dei-Eberhardt 21]

By defining a (somewhat odd) new variable y such that

$$V_j(x,z) = \sum_m x^{m-j} V_{jm}(z) \;\Rightarrow\; V_j^{\omega}(x,y,z) \equiv \sum_h y^{h-\frac{k\omega}{2}-j} V_{jh}^{\omega}(x,z)$$

one recasts the recursions relations as differential equations

$$J_{\omega}^{+} \sim \partial_y \quad J_0^3 - \frac{k\omega}{2} \sim y\partial_y + j \quad J_{-\omega}^{-} \sim y^2\partial_y + 2jy$$

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Geometric interpretation of for y [Iguri-NK 22]

Diagonal J_0^3 + parafermions $\Rightarrow$ $V_{jm}^\omega(z) = \Psi_{jm}(z) e^{\left(m+\frac{k}{2}\omega\right)\sqrt{\frac{2}{k}}\,\phi(z)}$

which leads to the generalization of the Maldacena-Ooguri formula

$$V_j^\omega(x,y,z) \equiv \lim_{\varepsilon,\bar{\varepsilon}\to 0} |\varepsilon|^{2j\omega} V_j(x+y\varepsilon^\omega, z+\varepsilon) V^{\omega-1}_{\frac{k}{2},\frac{k}{2}\omega}(x,z)$$

- $V^{\omega-1}_{\frac{k}{2},\frac{k}{2}\omega} \sim \mathbb{1}^\omega_{\mathrm{st}}$ are the WS version of the HCFT twist operators σ^w
- the variable y implements spacetime point-splitting,
- it is related to holomorphic covering maps [Lunin-Mathur 00]

Covering maps and odd parity correlators [Dei-Eberhardt 21]

The fusion rules imply that for $\omega_i > 1$ we often have a **covering map**

$$\omega_1 + \omega_2 + \omega_3 \in 2\mathbb{Z} + 1 \;\Rightarrow\; \exists! \quad \Gamma(z \sim z_i) \approx x_i + a_i[\boldsymbol{\omega}](z - z_i)^{\omega_i} \quad \forall\, i$$

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$$\left\langle \prod_{i=1}^{3} V_{j_i}^{\omega_i}(y_i) \right\rangle \propto \prod_{i=1}^{3} (y_i - a_i)^{-2j_i} \left(\omega_1 \frac{y_1 + a_1}{y_1 - a_1} + \omega_2 \frac{y_2 + a_2}{y_2 - a_2} + \omega_3 \frac{y_3 + a_3}{y_3 - a_3} - 1 \right)^{\tilde{\jmath}}$$

with $\tilde{\jmath} = \frac{k}{2} - j_1 - j_2 - j_3$, and where a_i are simple numerical coefficients.

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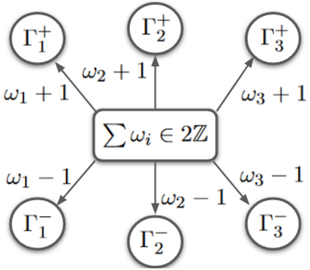
Some comments

- h-dependence is given in terms of Lauricella hypergeometrics.
- the y-basis diff eqs are null state conditions for twist ops
- singularities occur when $y_i \to a_i$ (geometric picture)
- this leaves the overall $C(j_i, \omega_i)$ factor unfixed
- and does not work for the even cases: there is no such map!

We can connect even and odd parity using SL(2,$\mathbb{R}$) series identifications:



The isomorphisms $\hat{\mathcal{D}}_j^{\pm,\omega} = \hat{\mathcal{D}}_{k/2-j}^{\mp,\omega\pm 1}$ read

$$y^{2j} V_j^{\omega}(x,y,z)|_{y\to\infty} = \mathcal{N}(j) V_{\frac{k}{2}-j}^{\omega-1}(x,0,z)$$

which allows us to fix all coefficients in the most general diff eqs from the near-by odd correlators obtained above.

Even correlators and the conjecture's proof [Bufalini-Iguri-NK 22]

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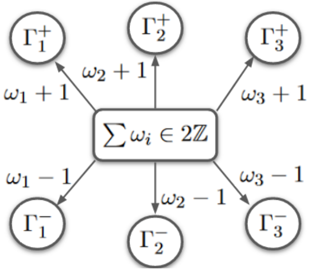

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$$\begin{aligned}
\langle V_{j_1}^{\omega_1}(y_1) V_{j_2}^{\omega_2}(y_2) V_{j_3}^{\omega_3}(y_3)\rangle \quad \propto \quad & \left(1 - \frac{y_2}{a_2[\Gamma_3^+]} - \frac{y_3}{a_3[\Gamma_2^+]} + \frac{y_2 y_3}{a_2[\Gamma_3^-]a_3[\Gamma_2^+]}\right)^{j_1-j_2-j_3} \\
\times \quad & \left(1 - \frac{y_1}{a_1[\Gamma_3^+]} - \frac{y_3}{a_3[\Gamma_1^+]} + \frac{y_1 y_3}{a_1[\Gamma_3^-]a_3[\Gamma_1^+]}\right)^{j_2-j_3-j_1} \\
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\end{aligned}$$

We also get the constants! Either $C(j_1,j_2,j_3)$ or $\mathcal{N}(j_1)C(k/2-j_1,j_2,j_3)$.

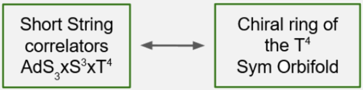
The exact AdS_3/CFT_2 chiral ring

Matching the full the chiral ring

[Iguri-NK-Toro 22]

A first non-trivial test

$\frac{1}{2}$-BPS sector protected $\Rightarrow$ can compare $\neq$ points in moduli space



$$\langle V^{\omega_1}_{j_1 h_1} V^{\omega_2}_{j_2 h_2} \left(j^{\omega_3} V^{\omega_3}_{j_3 h_3} \right) \rangle = ?$$

3 technical problems:

- Extend the y-basis to the SU(2) and fermionic sectors
- Compute descendant correlators appearing from picture changing
- Fix conjectured normalizations

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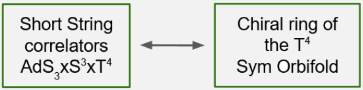

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Other important directions: [Dei-Eberhardt 21,22][Eberhardt 21]

- Similar (more involved) conjecture for 4pt-functions, to be proven.
- Recent proposal for a deformation of a symmetric orbifold of *Liouville th*.
- Matching obtained for residues of 3 and 4pt functions

The SUSY worldsheet theory sourced by n_5 five-branes and n_1 strings is

$$\mathrm{AdS}_3 \times S^3 \times T^4 \Rightarrow \mathrm{SL}(2,\mathbb{R})_{n_5+2} \times \mathrm{SU}(2)_{n_5-2} \times U(1)^4 + \text{free fermions}$$

Spacetime CP operators are built from highest-weight short string states. Using y-basis techniques we get $\langle V_{j_1}^{\omega_1} V_{j_2}^{\omega_2} \left(j^{\omega_3} V_{j_3}^{\omega_3}\right)\rangle = \alpha_{\boldsymbol{\omega}} \langle V_{j_1}^{\omega_1} V_{j_2}^{\omega_2} V_{j_3}^{\omega_3}\rangle$

$$\alpha_{\boldsymbol{\omega}} \equiv \begin{cases} \dfrac{2a_3[\Gamma_{13}^{++}]\left[(\omega_1-\omega_2)(j_1-j_2)+(\omega_3+1)(\frac{k}{2}-j_3)\right]}{\omega_1+\omega_3-\omega_2+1} \\ \dfrac{2a_3[\Gamma_3^{+}]\left[(1+\omega_1+\omega_2)j_3-(1+\omega_3)(j_1+j_2)-\frac{k}{2}(\omega_1+\omega_2-\omega_3)\right]}{\omega_3-\omega_2-\omega_1} \end{cases}$$

Finally, we extend the method for flowed correlators to

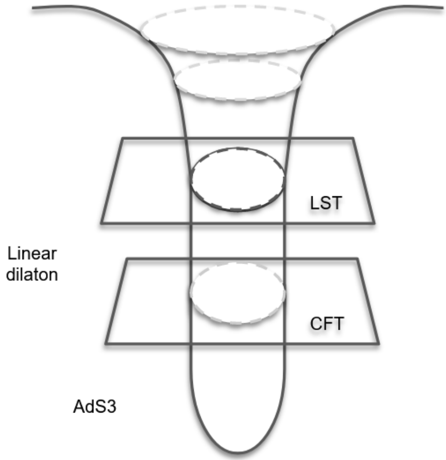
$$\mathrm{Fermions}:\ (k,j) \to (-2,-1), \quad \mathrm{SU}(2):\ (k,j) \to (-k',-l').$$

Most ω-dependent factors cancel in the relevant products of SL(2,$\mathbb{R}$), SU(2) and fermion correlators, giving the orbifold results

$$\langle \mathbb{V}_{j_1}^{\omega_1} \mathbb{V}_{j_3}^{\omega_2} \mathbb{V}_{j_3}^{\omega_3,(0)} \rangle = \frac{1}{\sqrt{N}} \left[\frac{(h_1+h_2+h_3-2)^4}{(2h_1-1)(2h_2-1)(2h_3-1)}\right]^{1/2}$$

Black hole microstates from the worldsheet

JMaRT: fields, regimes and dualities



- Quantized charges n_1, n_5, n_p,
- Radius R_y, and integers s, $\bar{s}$ (angular momenta) and k (orbifold structure).
- NS5-decoupling limit: $g_s \to 0$ with r/g_s fixed. Dual to LST. [Aharony et al 04+]
- AdS_3/CFT_2 limit: $R_y \to \infty$ with t/R_y and y/R_y fixed.
- Dual CFT: heavy states with **fractional** spectral flow.

An exact worldsheet description

The relevant worldsheet CFTs are gauged WZW models with target space

$$\frac{\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2) \times \mathbb{R}_t \times \mathrm{U}(1)_y}{\mathbb{R} \times \mathrm{U}(1)} \times T^4,$$

where we gauge the **null and chiral** currents 10+2 D → 9+1 D

$$J = J^3 + (2s+1)K^3 + i\mu\partial_t + ik_+\partial_y\,, \quad \bar{J} = \bar{J}^3 + (2\bar{s}+1)\bar{K}^3 + i\mu\partial_t + ik_-\bar{\partial}_y\,,$$

with

$$n_5(1 - s_\pm{}^2) + \mu^2 - k_\pm{}^2 = 0\,, \qquad k_\pm = \mp \mathsf{k} R_y + \mathsf{p}/R_y\,.$$

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Our results from [BIKT 21]:

1. All such consistent models give all JMaRT backgrounds.
2. CTC, Regularity and horizonless ↔ consistent WS spectrum.

An aside on $T\bar{T}$ deformations of the HCFT

Upon gauge fixing, this generates the WS marginal deformation

$$L_{\mathrm{WZW}} \to L_{\mathrm{gWZW}} = L_{\mathrm{WZW}} + \frac{1}{\Sigma(r,\theta)} J\bar{J}$$

- The **zeros** of Σ are the (possibly smeared) **locations of sources**.

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This is a slighly more complicated version of [Kutasov et al 17+], which

- *adds back* the "$1+$" in the harmonic $H_1(r)$ function,
- modifies the UV to include the linear dilaton region,
- is dual to a $T\bar{T}$-type deformation of the HCFT ($\lambda \sim 1/R_y$).

Null-gauged WZW models

For chiral null gaugings all anomalies cancel [Chung-Tye 92]

$$S_{\mathrm{gWZW}}[g, \mathcal{A}, \bar{\mathcal{A}}] = S_{\mathrm{WZW}}[\tilde{g}] + \mathrm{ghosts}$$

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Physical states of the coset model correspond to vertex operators of the upstairs theory that are BRST-closed under

$$\mathcal{Q}_{\mathrm{BRST}} = \oint dz : \Big[c \left(T + T_{\beta\gamma\tilde{\beta}\tilde{\gamma}} \right) + \gamma G + \tilde{c} J + \tilde{\gamma} \boldsymbol{\lambda} + \text{ ghosts } \Big] : ,$$

where the last terms implement bosonic and fermionic gauge invariance.

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where the last terms implement bosonic and fermionic gauge invariance.

We can construct vertex operators in the WS theory from SL(2,$\mathbb{R}$) and SU(2) WZW models. They will be dual to AdS_3 light states in the IR, and LST operators in the full coset.

Vertex operators in the coset models (NS sector)

They are excitations of the center-of-mass wave-function

$$\Phi_0 = V_{jm} V'_{j'm'} e^{i(-E\,t + P_y y)}$$

Virasoro and **gauge** constraints

$$\frac{-j(j-1) + j'(j'+1)}{n_5} - \frac{1}{4}\left(E^2 - P_y^2\right) = 0 = m + s_+\, m' + \frac{1}{2}\left(\mu E + k_+ P_y\right)$$

However, $\mathrm{AdS}_3 \times S^3$ isometries are broken since $[J^{\pm}, \mathcal{Q}_{\mathrm{BRST}}] \neq 0$
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We find $\Phi_{\rm AdS} = e^{-\varphi}\left(\psi_{\perp} V_j\right)_{jm} V'_{j'm'} e^{i(-E\,t + P_y y)}$ with

$$\psi_{\perp}^3 = \psi^3 + c^t\lambda^t + c^y\lambda^y \ , \ c^t = \frac{n_5 P_y}{k_+ E + \mu P_y} \ , \ c^y = -\frac{n_5 E}{k_+ E + \mu P_y} \ .$$

- Modified $j \to j(E, P_y)$ obtained from the quadratic Virasoro.
- New terms $c^t\lambda^t$, $c^y\lambda^y$ needed for transversality in t and y dirs.
- Gauge invariance relates the different quantum numbers.

Heavy-Light correlators at all orders in α'

Worldsheet correlation functions

Each **heavy background** *defines* a coset model:

$$\langle \mu\mathrm{BH}| O_1 \dots O_n |\mu\mathrm{BH}\rangle \leftrightarrow \langle \Phi_1 \dots \Phi_n \rangle_{\text{WS vacuum}}$$

- encode the dynamics of light probes in the BH microstates,
- define Heavy-Light LST correlators (*in progress*)
- Characterize the flow of these objects under the marginal deformation of the WS theory (st $T\bar{T}$-HCFT)

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In the IR, these become HL correlators in the dual CFT_2.

From now on I focus on their explicit computation.

Coset states in the IR

A **modified** version of the $\mathrm{AdS}_3 \times S^3$ symmetries emerges in the IR:

$$R_y \to \infty \quad \text{with} \quad \mathcal{E} = ER_y \;\text{ and }\; n_y = P_y R_y \in \mathbb{Z} \;\text{ held fixed}$$

we have

1. Virasoro has $\frac{1}{4}\left(E^2 - P_y^2\right) \sim R_y^{-2} \to 0 \Rightarrow j = j' + 1$ as usual.
2. The coefficients of the extra terms $c_{t,y}$ are $\sim R_y^{-1} \to 0$.

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But we know that even in the simplest case, the HLLH correlator with O_L the $h = 1/2$ untwisted CP and $|H\rangle$ a SUSY background ($\bar{s} = 0$), it **should be quite non-trivial**! [Galliani et al 16]

$$\langle O_{\frac{1}{2}}(1)\bar{O}_{\frac{1}{2}}(x)\rangle_{s,k} = \frac{x^{(\hat{s}-s)/k}}{|x||1-x|^2}\,\frac{1-|x|^{2(1-\hat{s}/k)} + \bar{x}\left(|x|^{-2\hat{s}/k} - 1\right)}{1-|x|^{2/k}}$$

where $\hat{s} = s \bmod k$, computed from SUGRA. How can this be?

Physical operators in the new x-basis

It's all hidden in the *coset* x-basis. Gauge constraints do *not* trivialize!

define the spacetime modes

$$m_y = \frac{\mathcal{E} + n_y}{2}\ ,\ \bar{m}_y = \frac{\mathcal{E} - n_y}{2}$$

$$0 = m + s_+ m' - \mathsf{k}\, m_y$$

$$m = u\partial_u + h - \beta \qquad m_y = x\partial_x + h.$$

where "x" is the physical boundary coordinate, "u" is the auxiliary *upstairs* one, and $\beta = h(1 - \mathsf{k}) + s_+ m'$ is an extra shift.

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For the above β, this becomes $u\partial_u = \mathsf{k} x\partial_x$, solved by $u^{\mathsf{k}} = x$.

$$V_j(x) = \sum_{m=j+n} x^{m-j}\, V_{jm} \to O_h(x) \equiv \sum_{u^{\mathsf{k}}=x} u^{\beta}\, \bar{u}^{\bar{\beta}} \mathcal{V}_h(u) \mathcal{V}'_{h'\, m'\, \bar{m}'}$$

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The role of β is **two-fold**:

1. it gives the Jacobian factor for the coordinate change from x to u,
2. and also the appropriate rescaling under spectral flow (in u-space).
3. The modes m_y are fractional, as expected from spacetime k-twist.

HL correlators with arbitrary light insertions

Based on all this, we obtain a formula for all higher-point-function:

$$\langle O_1(x_1)\dots O_n(x_n)\rangle_H = \sum_{u_i^k = x_i} \left(\prod_{i=1}^{n} u_i^{\beta_i}\, \bar{u}_i^{\bar{\beta}_i} \right) \langle O_1(u_1)\dots O_n(u_n)\rangle$$

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1. For $n = 2, 3$ light fields, these are closed formulas, exact in α'
2. Valid for **any** JMaRT background (BPS or not),
3. and **any** CP of weight h_i and of any twist / spectral flow!
4. Reproduces [Galliani et al 16] for $n = 2$ and $\bar{s} = 0$,
5. They match all known orbifold results, **including non-susy cases**.
6. We used it to study the analogue of Hawking radiation.
7. As a further test, we have computed the first HLLLH correlator in a microstate background $\vec{h}_L = (\frac{1}{2}, \frac{1}{2}, 1)$ from both sides.

Matching with the D1D5CFT: untwisted vs twisted sectors

For untwisted ops this parallels the **symmetric orbifold** [Lunin-Mathur 01]

$$X_{(1)} \to X_{(2)} \to \cdots \to X_{(\mathrm{k})} \to X_{(1)}$$

with **fractional modes** since JMaRT states $\in$ k-twisted sector:

$$O_{\frac{m}{\mathrm{k}}} = \oint dx \sum_{r=1}^{\mathrm{k}} O_{(r)}(x) e^{\frac{2\pi i m}{\mathrm{k}}(r-1)} x^{h+\frac{m}{\mathrm{k}}-1} \Rightarrow O(x) = \sum_{r=1}^{\mathrm{k}} O_{(r)}(x) \to \sum_{u^k=x} O(u)$$

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A twist-2 example [Lima et al 20+]

- Here using the map $u^{\mathrm{k}} = x$ is surprising.
- Various structures contribute, at large N $\langle R_{\mathrm{k}} R_{\mathrm{k}} O_2 O_2^\dagger R_{\mathrm{k}}^\dagger R_{\mathrm{k}}^\dagger \rangle$.
- The more complicated covering map is $x(u) = \left(\frac{u+1}{u-1}\right)^{2\mathrm{k}}$.
- One has to sum over pre-images.

Remarkably, our result matches their formula exactly!

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Still plenty to learn from the AdS_3 WZW model and related cosets!

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- Is it possible to match correlation functions beyond their residues? [Dei-Eberhardt 22]
- Can we embed the old $H_3^+ \leftrightarrow$ Liouville duality into the new proposal for the holographic CFT at $n_5 > 1$? [Ribault-Teschner 05]

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- Study more complicated processes such as the Penrose process? [Bianchi 19] What about the partition functions?
- How do these correlators flow to the UV? Locality in x breaks down, as it should in LST. [Giveon et al 17-23]
- Can we obtain WS models for the new NSNS superstrata? [Čeplak 22]

Thank you! Any questions?