Designing gravity via $Sym^N(C)$

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DAMTP

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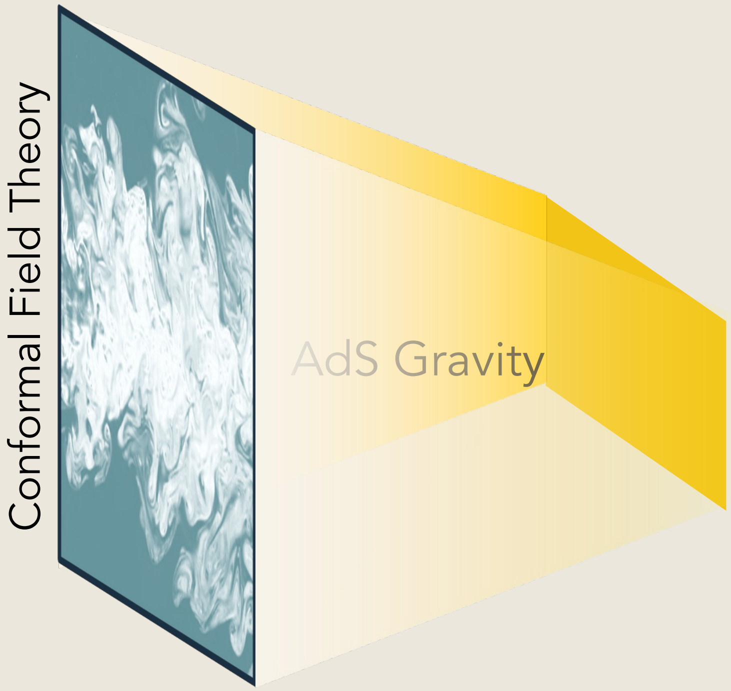
CFT_D → AdS_{D+1} Gravity



Holographic CFT

A CFT whose dual gravity theory that has a low-energy EFT description.

A few (but not all) properties associated to them are:

- Large central charge (large-N), which leads to a large number of d.o.f. (BHs)
- Sparse spectrum (degeneracy of light operators are not controlled by N).
- Factorization of correlation functions, i.e., Generalized Free Fields.(!!)
- …

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

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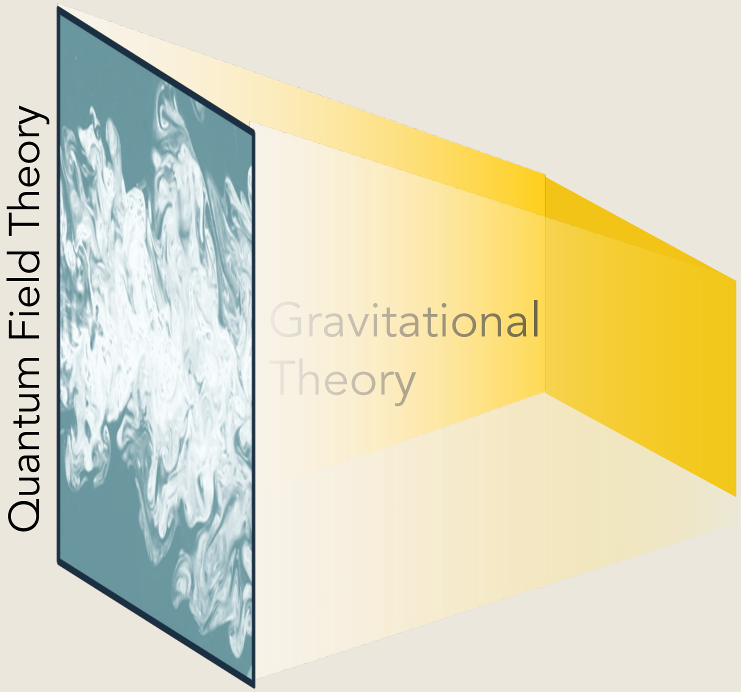
How many conditions do I need to impose?
How stringent are the conditions?

Designing AdS_3 Quantum Gravity

- Define gravity via the dual CFT_2
- Identify necessary conditions
- Determine possible designs we can achieve
- Focus on CFT_2 that we can quantify:
  Symmetric Product Orbifolds



Classification of Symmetric Product Orbifolds

Deformations of Symmetric Product Orbifolds

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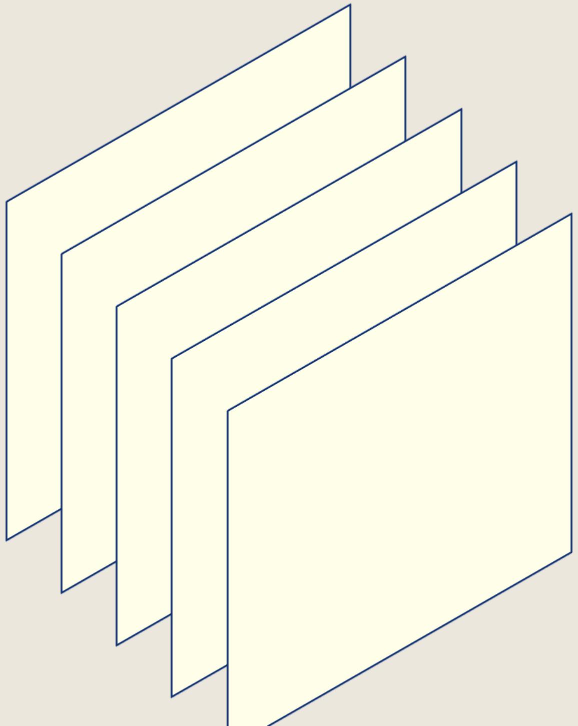
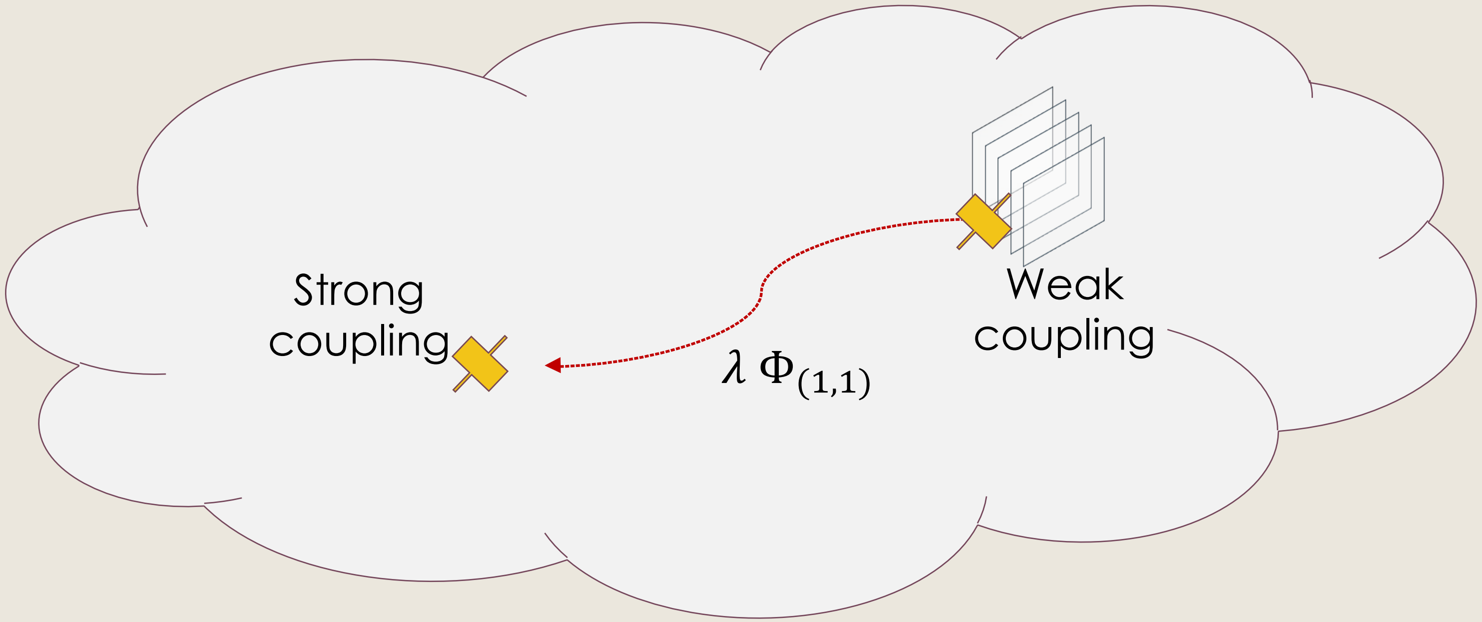


- Implement conditions
- Precise outcomes (with surprises)

A. Belin, J. Gomes, C. Keller, AC, 2016, 2018
A. Belin, C. Keller, B. Mühlmann, AC, 2019 (x2)
A. Belin, N. Benjamin, C. Keller and S. Harrison, AC, 2020
N. Benjamin, S. Bintanja, J. Hollander, AC 2022

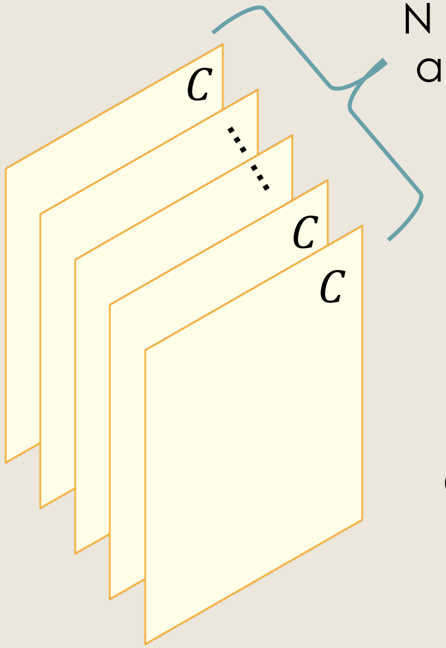


- New features in the design of AdS/CFT
- Breaking $Sym^N(\mathcal{C})$

L. Apolo, A. Belin, S. Bintanja, C. Keller, AC
2204.07590 and 2212.07436

Deformations and New Flavours of AdS/CFT

Symmetric Product Orbifolds

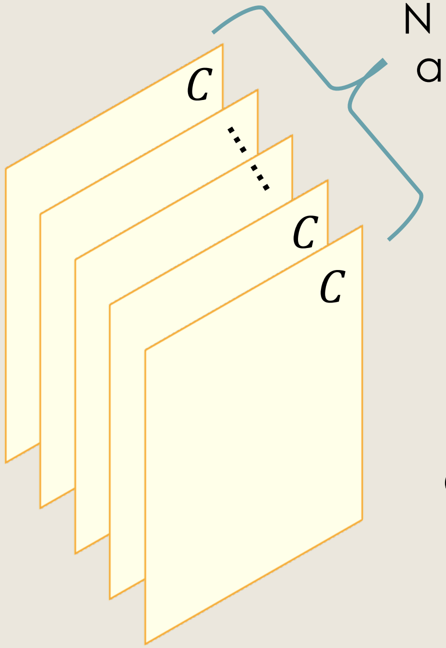


N copies of a unitary and compact CFT_2.
C = seed theory

$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

Orbifold by the permutation group S_N

Symmetric Product Orbifolds



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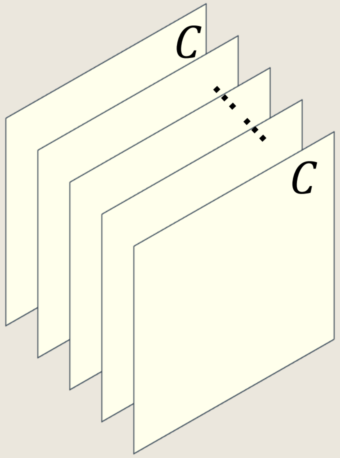
Orbifold by the permutation group S_N

The orbifold introduces two class of states:

- untwisted sector: it keeps states that are invariant under S_N.
- twisted sectors: new states labelled by conjugacy classes of S_N.

Symmetric Product Orbifolds

- Appeal: Mathematical and analytic control, e.g., DMVV formula.
- Familiarity: D1D5 CFT.
- Universality: large-N behavior is robust.
- Utility: compelling features for AdS/CFT.



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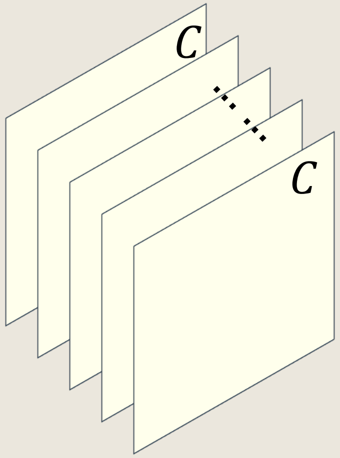
Today: non-universal properties. Demonstrate that there are different classes, and their features challenge the lore of AdS/CFT.



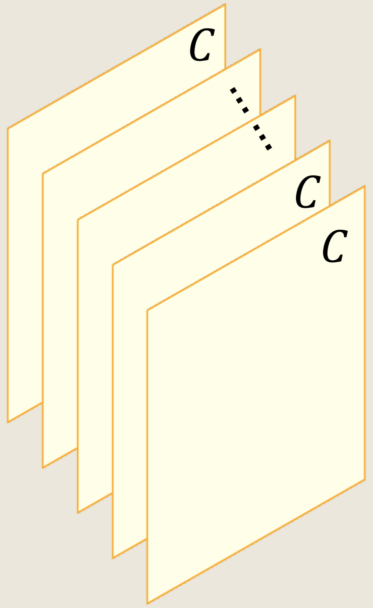
Universal Aspects

All symmetric product orbifolds satisfy:

- Correlation functions comply with large-N factorization.
  [Pakman et.al., Mathur et.al., Belin et.al., Hael et.al., ...]
- Hawking-Page transition at large-N.
  [Keller 2011; Hartman, Keller, Stoica 2014; Benjamin et.al. 2015]

- Higher spin currents due to orbifold structure.
- Universal Hagedorn growth of light states.
  [Keller 2011]

$d_{all}(\Delta) \sim e^{2\pi b\,\Delta}$ where $\Delta \gg 1,\ \Delta \sim O(N^0)$ and $b \sim O(N^0)$



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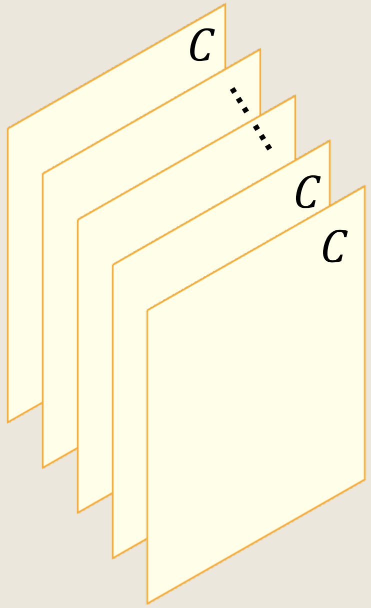

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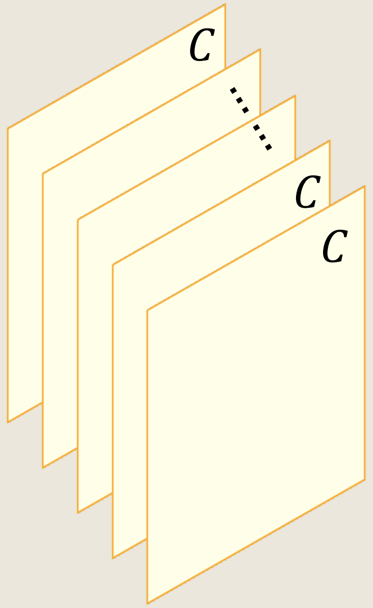
AdS/CFT interpretation: Dual of $Sym^N(C)$ looks like a tensionless string theory (or higher spin gravity).

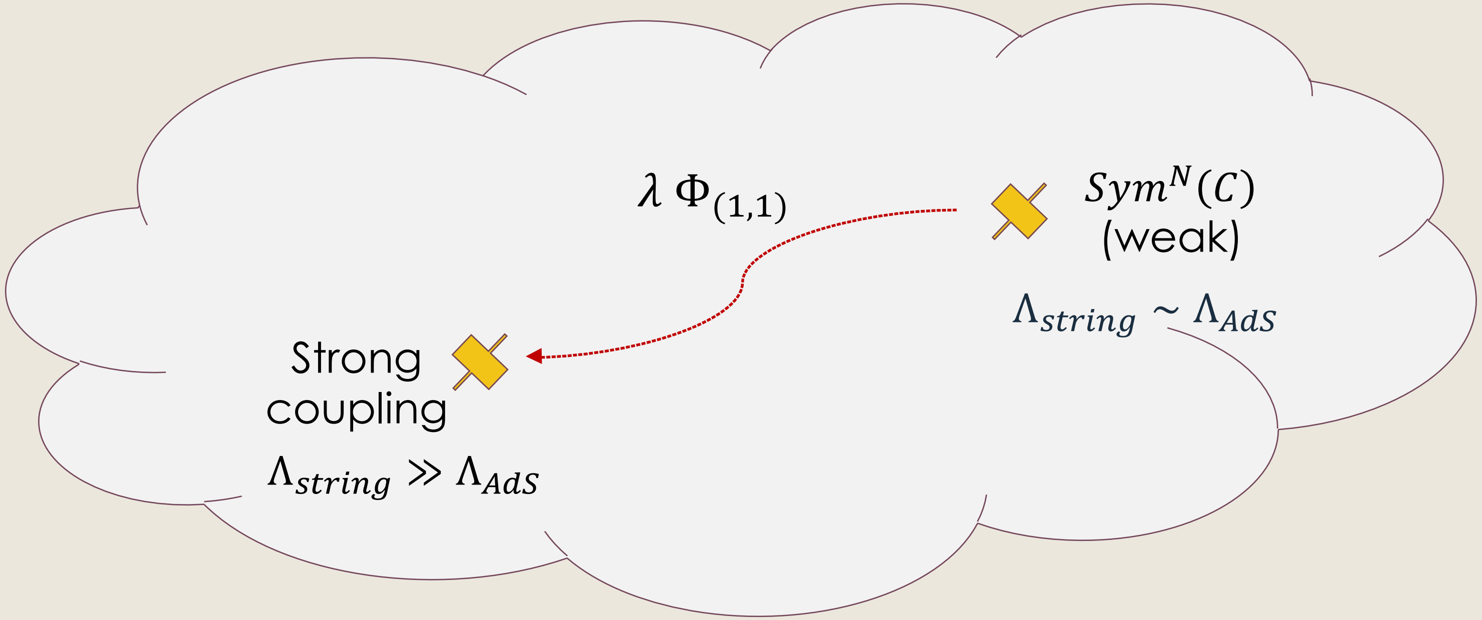


$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

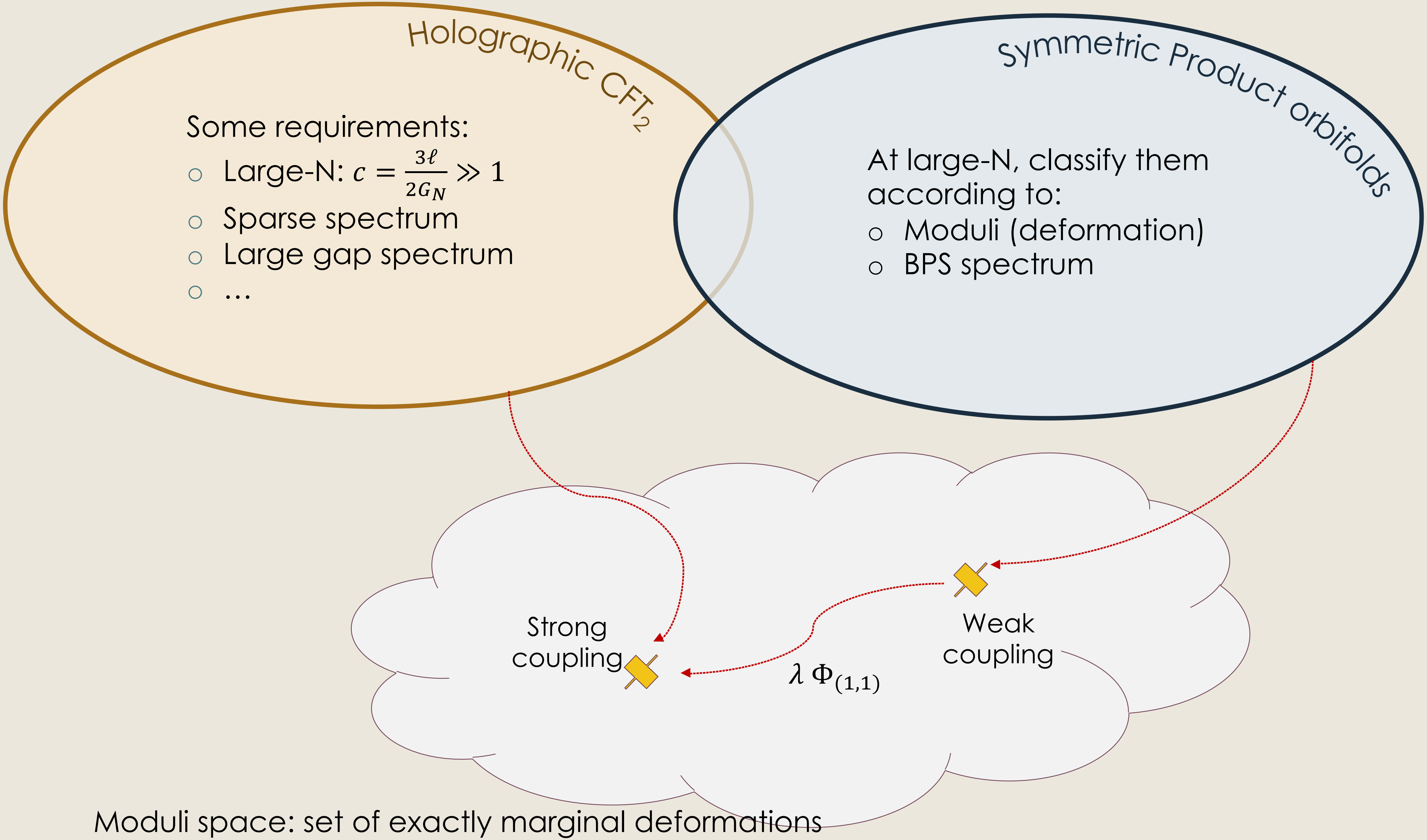
- Higher spin currents due to orbifold structure.
- Universal Hagedorn growth of light states.

Question: Which $Sym^N(C)$ could admit in their moduli space a dual supergravity point?

Strategy: Impose necessary conditions. Identify which $Sym^N(C)$ comply with them.



Moduli space: set of exactly marginal deformations


Holographic CFT$_2$
Some requirements:
Large-N: $c = \frac{3\ell}{2G_N} \gg 1$
Sparse spectrum
Large gap spectrum
…
Symmetric Product orbifolds
At large-N, classify them according to:
Moduli (deformation)
BPS spectrum
Strong coupling
Weak coupling
$\lambda\, \Phi_{(1,1)}$
Moduli space: set of exactly marginal deformations

Neccesary conditions

- Criterion 1: Existence of suitable moduli (single trace, twisted, BPS): $\lambda\, \Phi^{1t.tw.}_{(1,1)}$
- Criterion 2: Sparseness condition on the elliptic genera (index that captures ¼- BPS states).

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Three requirements on this operator $\Phi^{1t.tw.}_{(1,1)}$:

- ½-BPS: Supersymmetry protects the deformation everywhere in the conformal manifold.
- Twisted: break the orbifold structure of $Sym^N(C)$.
- Single-trace: have an effect at leading order at large-N.

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$$\chi(\tau, z; C) = \mathrm{Tr}_{RR}\left((-1)^F q^{L_0-\frac{c}{24}}\, y^{J_0}\, \bar{q}^{\bar{L}_0-\frac{c}{24}}\right) = \sum_{n,l} d(n,l) q^n y^l$$

$$Z(\rho, \tau, z) = \sum_N \chi\big(\tau, z; Sym^N(C)\big) e^{2\pi i \rho N} = \prod_{\substack{n,l,N\in\mathbb{Z}\\ N>0}} \frac{1}{(1-q^n y^l p^N)^{d(nN,l)}}$$

In the NS sector, for $Sym^N(C)$, we will distinguish them by the growth of light states:

- Slow growth: $d(\Delta) \sim e^{c_S \Delta^\gamma}$ with $\gamma < 1$
- Fast growth: $d(\Delta) \sim e^{c_H \Delta}$

For the regime
$\Delta \gg 1, \quad N \gg 1,$
$\Delta \sim O(N^0)$

Neccesary conditions

- Criterion 1: Existence of suitable moduli (single trace, twisted, BPS).
- Criterion 2: Sparseness condition on the elliptic genera (index that captures ¼- BPS states).

Based on these two criteria, we will classify $Sym^N(C)$ theories, and label them as

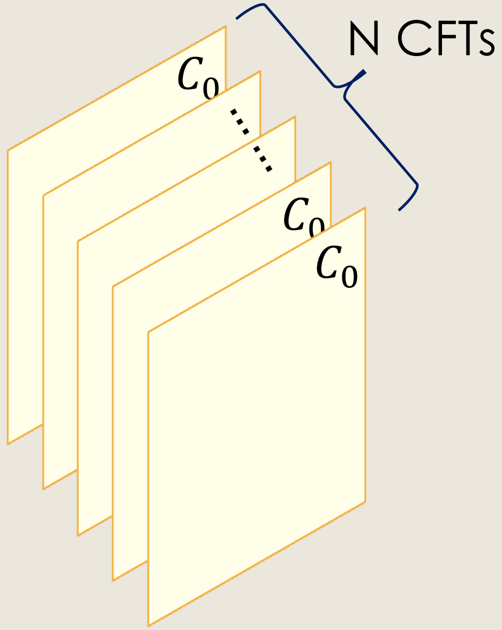
Type I: Both criteria	Type II: Only criterion 1	Type III: Neither criteria	Type IV: Only criterion 2

Neccesary conditions

- Criterion 1: Existence of suitable moduli (single trace, twisted, BPS).
- Criterion 2: Sparseness condition on the elliptic genera (index that captures ¼-BPS states).

1. We proved that both criteria (independently) imply that seed theory must have

$$1 \leq c_0 \leq 6$$



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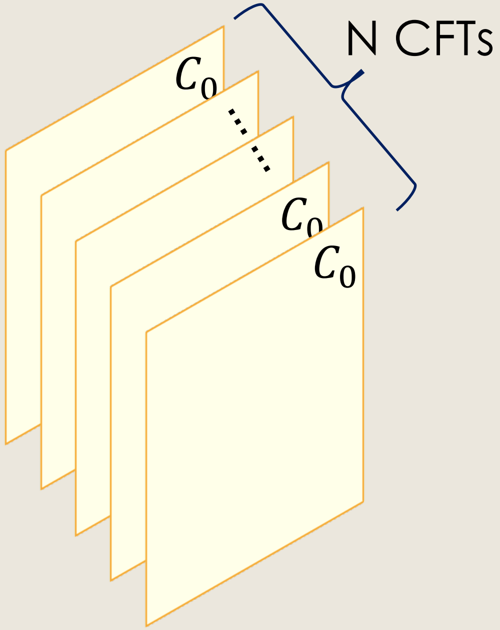

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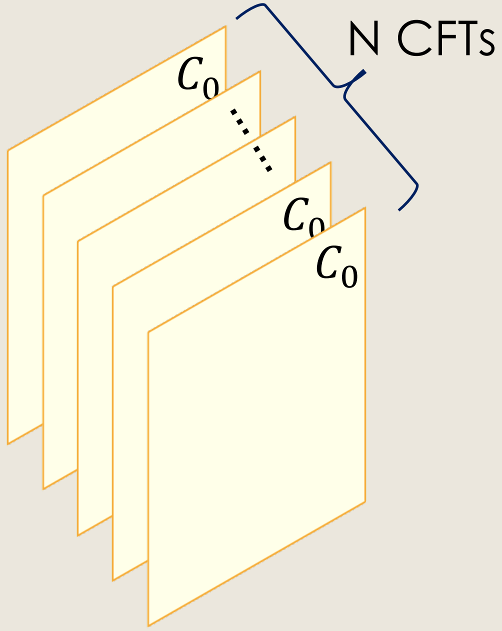

2. Criterion 2 can be done systematically and is exhaustive.

3. If Criterion 2 is satisfied, we proved that one always gets

$$d_{\frac{1}{4}BPS}(\Delta) \sim e^{\sqrt{\Delta}} \quad \text{where} \quad \Delta \gg 1,\ \Delta \sim O(N^0)$$

Classification

Type I: Both criteria →	Needles in a haystack. Comply with necessary conditions compatible with a holographic CFT.
Type II: Only criterion 1 →	Strange and counter-intuitive. Moduli exists, but Hagedorn behavior persists.
Type III: Neither criteria →	Generic, most abundant. They will never lead to a supergravity point in moduli space.
Type IV: Only criterion 2 →	Unicorns. No unitary example yet. Modular invariance does not rule it out.

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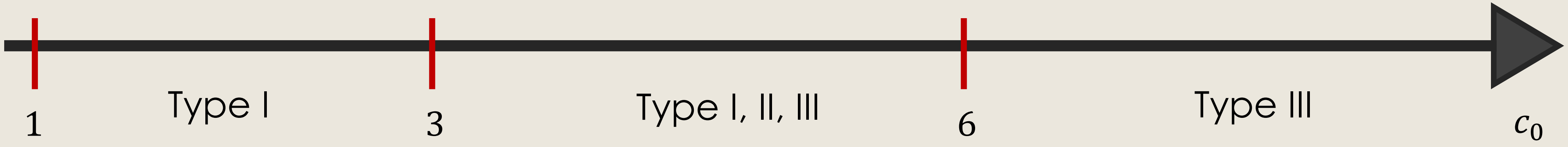
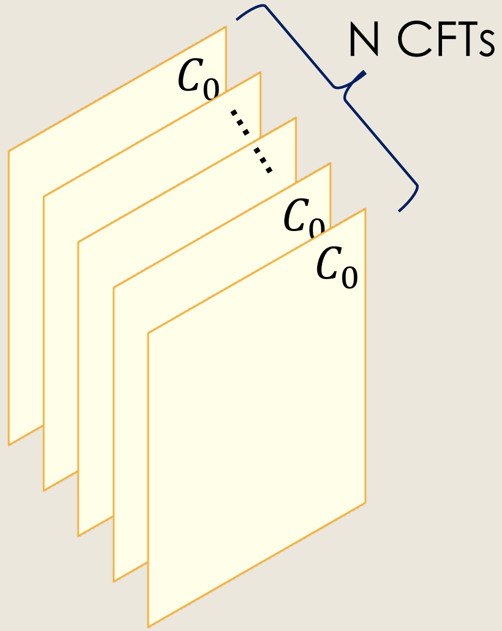
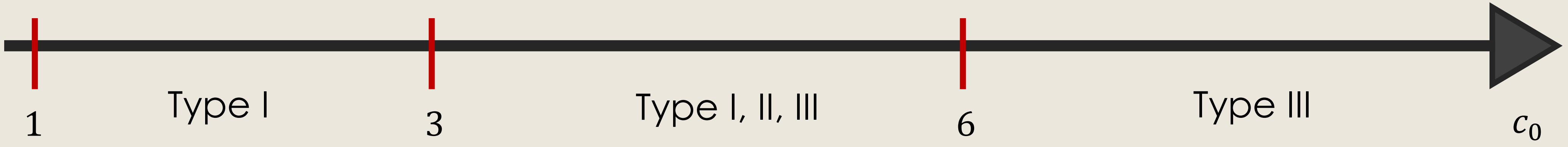
Summary



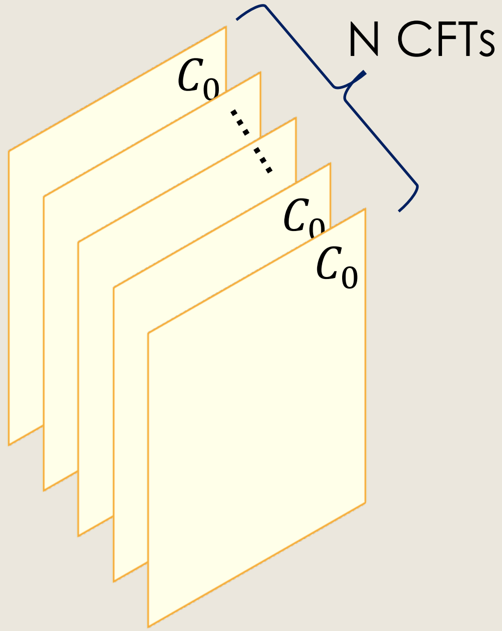


Summary



Comments:

- Only consider CFTs that are unitary and compact.
- Assume that the elliptic genus does not vanish.
- D1D5 on K3 sits at $c_0 = 6$.
- Search between $1 \leq c_0 < 3$ is exhaustive: N=2 Minimal Models.
- Search between $3 \leq c_0 \leq 6$ is not exhaustive (but systematic).



Type I: Examples

Series	k	untwisted moduli	twisted moduli	single trace twisted
A_2	1	1	28	1 twist 5, 1 twist 7
A_3	2	3	26	1 twist 3, 1 twist 4, 1 twist 5
A_5	4	9	24	1 twist 2, 1 twist 3, 1 twist 4
A_{k+1}	odd, ≥ 3	$P(k+2)-2$	9	1 twist 3
A_{k+1}	even, ≥ 6	$P(k+2)-2$	$10+\sum_{r=1}^{\frac{k}{2}+2} P(r)$	1 twist 2, 1 twist 3
D_4	4	6	20	1 twist 2, 2 twist 3, 1 twist 4
$D_{\frac{k}{2}+2}$	0 mod 4, ≥ 8	$P(\frac{k}{2}+1)+P(\frac{k}{4}+1)$	$8+\sum_{r=1}^{\frac{k}{4}+1} P(r)$	1 twist 2, 1 twist 3
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E_6	10	4	5	1 twist 2
E_7	16	6	5	1 twist 2
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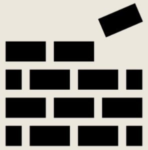
N=2 Virasoro Minimal Models

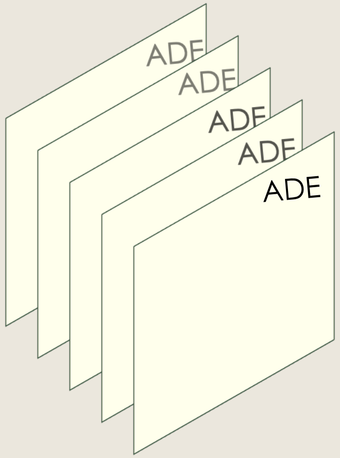
$$c_0 = \frac{3k}{k+2} < 3$$

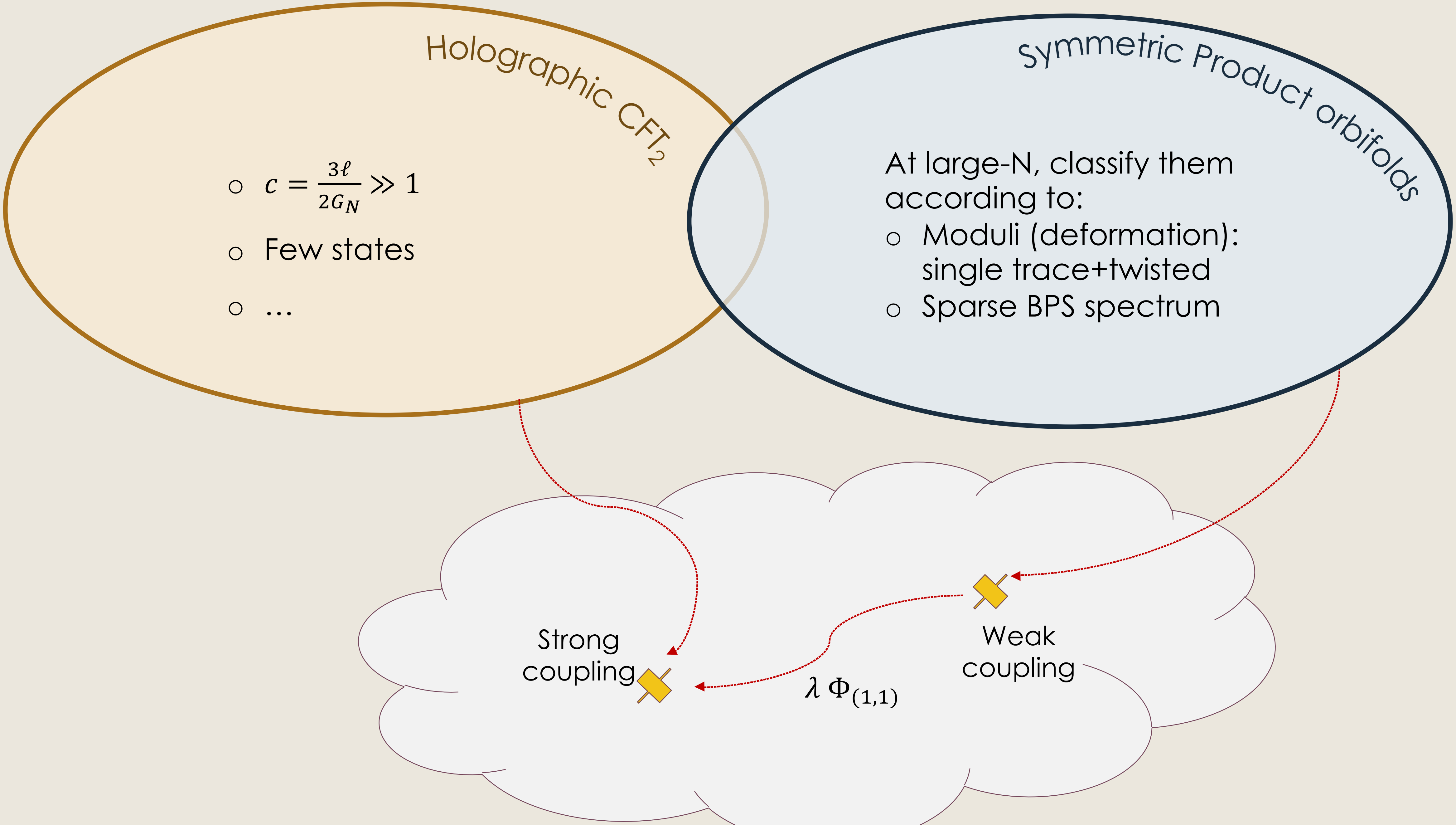
where $k = 1, 2, \ldots$



Necessary conditions:

- Criterion 1: Exactly marginal operator
- Criterion 2: Sparse spectrum for elliptic genera





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Responsible of lifting most states.
Breaks higher spin symmetry

Effects of single trace deformation

Turn on deformation

$$S \to S + \lambda\sqrt{N} \int d^2z\, \Phi_{1,1}(z,\bar{z})$$

+

Effects on 2pt function

$$\langle \mathbb{O}_a(z)\mathbb{O}_a(z')\rangle_\lambda = \frac{1}{(z-z')^{2(h+\mu_a(\lambda))}(\bar{z}-\bar{z}')^{2(\bar{h}+\bar{\mu}_a(\lambda))}}$$

Deformation preserves supersymmetry and conformal symmetry.

Further expectations of this operator:

- to induce anomalous dimensions on most operators,
- reduce the Hagedorn growth.

Anomalous dimension for spin-2

k	$c = \frac{3k}{k+2}$	n	$\mu_{(2)}$
1	1	5	—
		7	
2	$\frac{3}{2}$	3	$\frac{20\pi^2\lambda^2(3N-2)}{27(N-1)}$
		4	$\frac{187\pi^2\lambda^2(3N-2)}{256(N-1)}$
		5	$\frac{4\pi^2\lambda^2(3N-2)}{5(N-1)}$
3	$\frac{9}{5}$	3	$\frac{44\pi^2\lambda^2(9N-5)}{243(N-1)}$
4	2	2	$\frac{39\pi^2\lambda^2(2N-1)}{64(N-1)}$
		3	$\frac{19\pi^2\lambda^2(2N-1)}{27(N-1)}$
		4	$\frac{207\pi^2\lambda^2(2N-1)}{256(N-1)}$
$5, 6, \ldots$	$2 < c < 3$	3	$\frac{4\pi^2\lambda^2(c^2+12c-9)(cN-1)}{27c^2(c-1)(N-1)}$
$6, 8, \ldots$	$2 < c < 3$	2	$\frac{3\pi^2\lambda^2(24+c)(cN-1)}{64c(c-1)(N-1)}$

$$\langle \mathbb{O}_a(z)\mathbb{O}_a(z')\rangle_\lambda = \frac{1}{(z-z')^{2(h+\mu_a(\lambda))}(\bar{z}-\bar{z}')^{2(\bar{h}+\bar{\mu}_a(\lambda))}}$$

- First correction in perturbation theory
- Sensitivity on the twist and central charge.
- Still, currents are lifting. Good sign!

$$W_2(z) = T(z) - \frac{3}{2}(JJ)(z) + \frac{3(cN-1)}{2c(N-1)}\sum_{i\neq j}^{N} J^{(i)}(z)J^{(j)}(z)$$

$$= T(z) + \frac{3(c-1)}{2c(N-1)}(JJ)(z) - \frac{3(cN-1)}{2c(N-1)}\sum_{i=1}^{N}(J^{(i)}J^{(i)})(z)\,.$$

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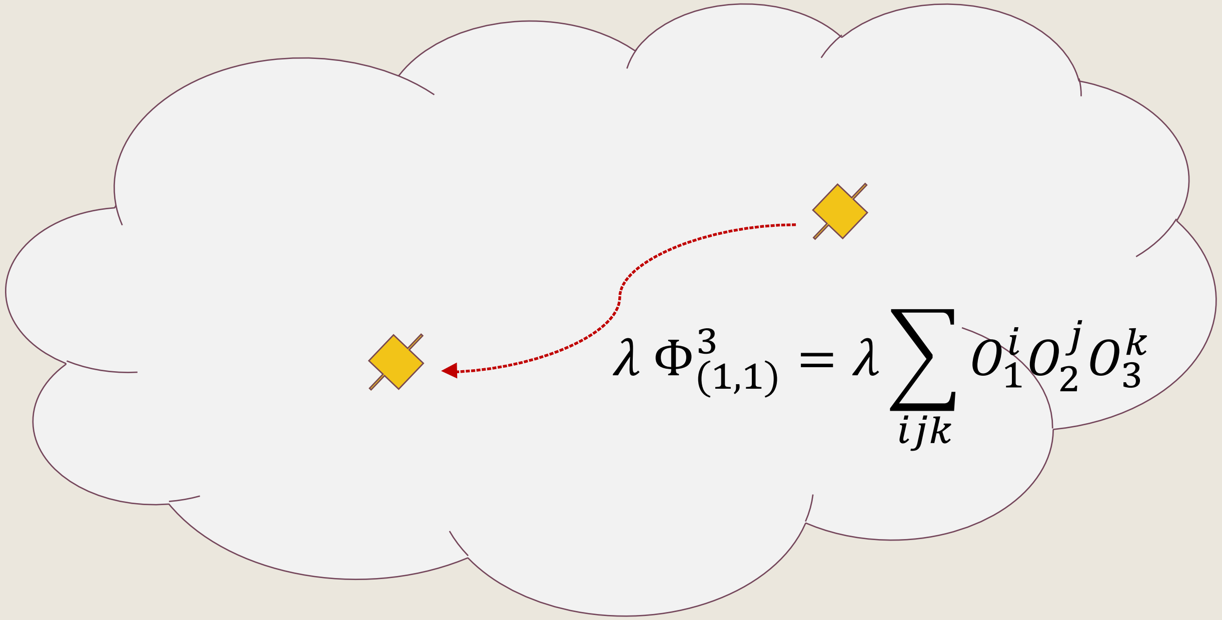
Multi-trace deformations.
Explicit example of CFT with these BPS deformations.

Destroy Factorization

Consider any CFT that complies with Large-N and Factorization



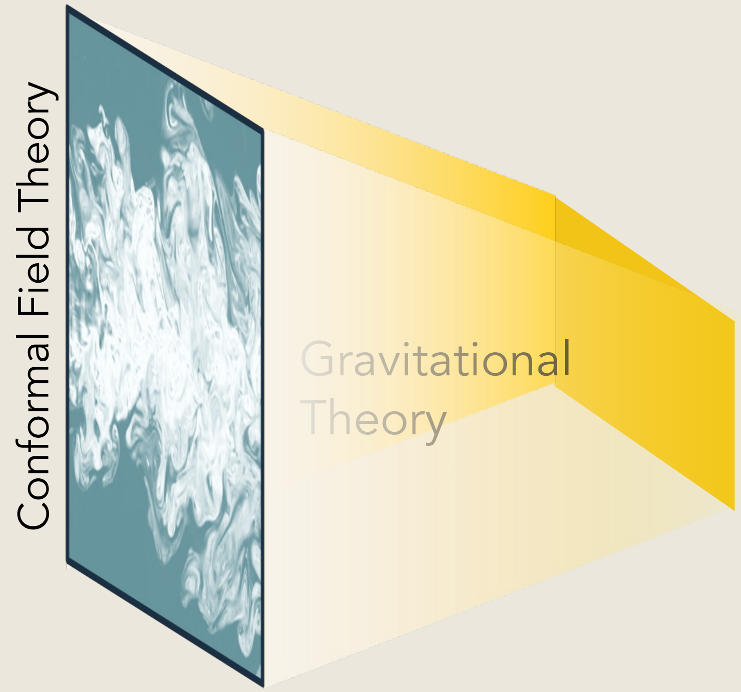
$$\langle O_1 O_2 O_3 \rangle_\lambda \sim \lambda$$

- Breaks large-N factorization
- Interactions that are not controlled by G_N
- Type I theories have these deformations
- Argument is general: applies to $\mathrm{CFT_D}$

The coupling λ is independent of N.
This deformation does not affect the large-N limit (observables converge).

Outlook

Quantify the space of type I theories:
- Different from known examples
- Systematic and tractable
- Infinite family
- New possibilities in AdS/CFT

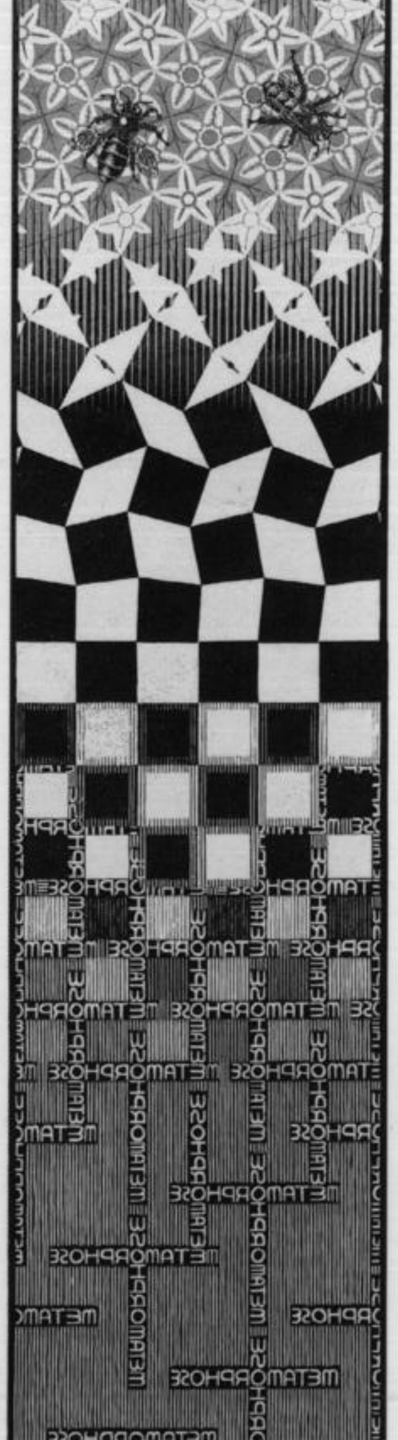


Type I $Sym^N(C)$

Conditions:
- Large-N
- Sparse elliptic genera
- Moduli

Holographic CFT_2

Some requirements:
- Large-N
- Sparse spectrum
- Large gap spectrum



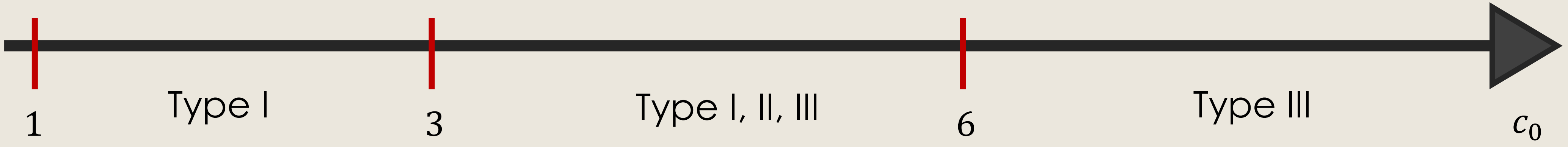
- Which CFTs capture classical (geometric) properties of gravity?
- What are possible theories of quantum gravity that can be designed?
- What are the materials needed to assemble them?

Next steps:

- String theory and supergravity description.
- Heavy states: contrast black holes among type I, II and III.
- Effects of multi-trace deformation.
- Type I vs II: lifting of generic operators.
- Non-compact CFTs.

EXTRA

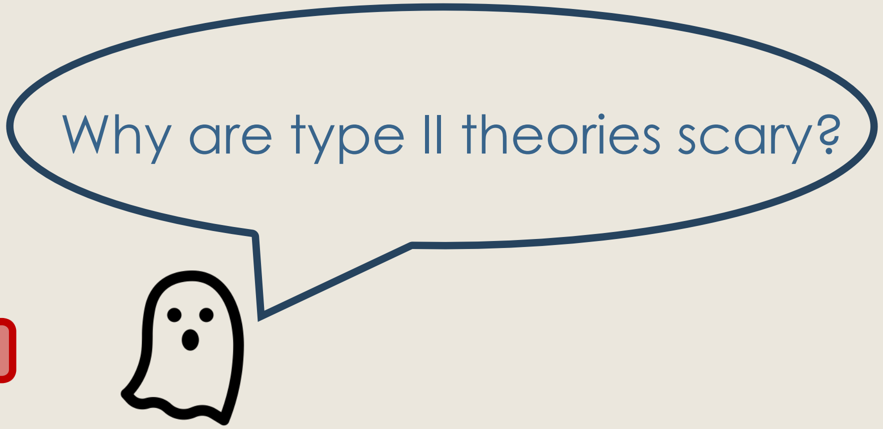
Re-cap



Type II:
Only criterion 1

→ Strange and counter-intuitive.
Moduli exists, but Hagedorn behavior persists.

	Theory	Sparse?	Moduli?	Composition
	$A_6 \otimes A_{41}$	✓	✓	(11,88), (22,22)
	$A_7 \otimes A_{23}$	✓	✓	(11,55),(22,22)
	$A_8 \otimes A_{17}$	✓	✓	(11,44),(22,22)
	$A_9 \otimes A_{14}$	✓	✓	(22,22)
	$A_{11} \otimes A_{11}$	✓	✓	(11,33),(33,11),(22,22)
	$A_6 \otimes D_{22}$	✗	✗	
	$A_7 \otimes D_{13}$	✗	✓	(11,55)
	$A_{23} \otimes D_5$	✗	✓	(55,11)
	$A_8 \otimes D_{10}$	✗	✗	
	$A_{14} \otimes D_6$	✗	✗	
	$A_{11} \otimes D_7$	✓	✓	(11,33),(33,11)
	$A_8 \otimes E_7$	✗	✗	
Type II	$A_{11} \otimes E_6$	✗	✓	(33,11)
	$D_5 \otimes D_{13}$	✗	✓	(11,55)
	$D_7 \otimes D_7$	✓	✓	(11,33),(33,11)
Type I	$D_7 \otimes E_6$	✓	✓	(33,11)
	$E_6 \otimes E_6$	✗	✗	
	$A_2 \otimes A_5 \otimes A_5$	✓	✓	(11,11,22),(11,22,11)
	$A_2 \otimes A_5 \otimes D_4$	✓	✓	(11,22,11)
	$A_2 \otimes D_4 \otimes D_4$	✗	✗	
	$A_3 \otimes A_3 \otimes A_5$	✓	✓	(11,11,22)
	$A_3 \otimes A_3 \otimes D_4$	✗	✗	



Examples of theories where the seed has $c_0 = 5$

Comparisson

Type I: Both criteria → Needles in a haystack.
Comply with necessary conditions to lead to a holographic CFT.

Type II: Only criterion 1 → Strange and counter-intuitive.
Moduli exists, but Hagedorn behavior persists.

- We evaluated anomalous dimension of several holomorphic operators (currents).
- Type I and II theories exhibit no difference at leading order in perturbation theory. 😖
- What is the key feature that guarantees a supergravity point in moduli space?