Magnetization, hysteresis and dynamic effects

Ezio Todesco
MSC Group, TE Department
European Organization for Nuclear Research (CERN)

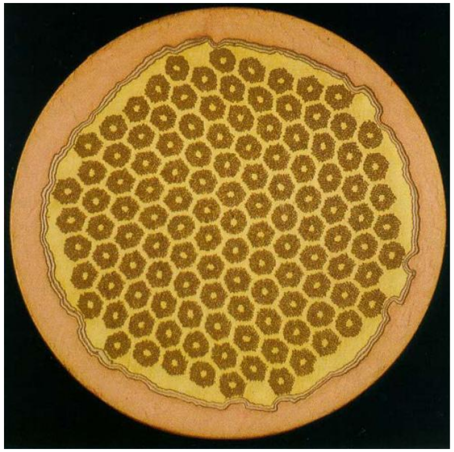
Mainly based on Mess-Schmüser-Wolff book
"Superconducting accelerator magnets", Chapter 2, 6 and 7
and based on the lectures by M. Sorbi at Milano University , Italy
and Units 6 and 7 of the Covid-19 lectures see https://indico.cern.ch/category/12408/

All the units will use International System (meter, kilo, second, ampere) unless specified

These slides made with a Mac, if you do not see the equations properly use the pdf

PROLOGUE

- A field variation induces a magnetization of the superconductor
  - This not due to Maxwell equations, but to Meissner effect
- The shielding currents flow without resistance (persistent currents) and produce a magnetization that has two main effects
  - It requires to have the superconductor in filaments of order of 0.01-0.1 mm, otherwise one has instabilities
  - It creates multipolar components at injection field (the lower the injection the larger these effects): this sets a limit in the energy increase in accelerators



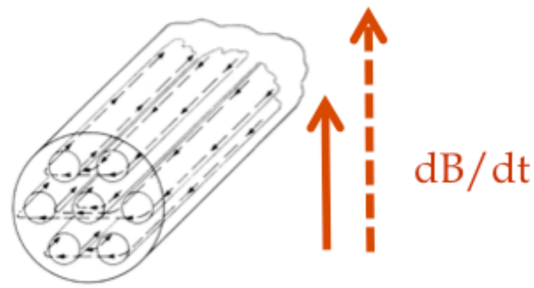
Filaments on a Nb_3Sn wire

- All these effects are not to the speed of the variation of B

PROLOGUE

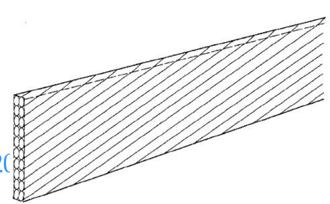
- Ramp rate effects are due to Maxwell equations

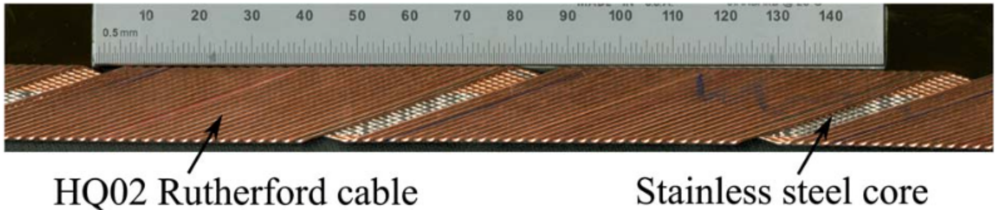
$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \propto \frac{dB}{dt}$$



Non twisted strand
M. N. Wilson, [4], p. 148.

- E.m. force induced in a loop
  - This effects depends on the derivative of the field with respect to the time (ramp rate effects)
  - The loop is partially superconductor and partially resistive
  - The resistive part is related to contact resistance between filaments (in a strand) and between strands (in a cable)
- This determines the architecture of strands and cables
  - They both need to be twisted to reduce the size of the loops
  - And may have to put a core



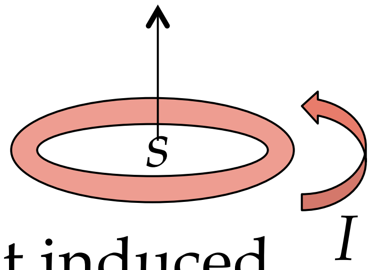


CONTENTS

MAGNETIZATION

- The magnetic moment is defined as the product of the surface of a current loop times the current
  - Unit is A m^2

$$m = Is$$



- Magnetization is the density of magnetic moment induced by an external magnetic field H
  - Unit is A/m

$$M(H) = \frac{dm(H)}{dV}$$

  - Magnetization can be opposed to the magnetic field *H* (diamagnetic, as in superconductors) or in the same direction of magnetic field *H* (paramagnetic/ferromagnetic as in iron)
- The magnetic field associated is $\mu_0 M$ and the total field is

Note: definition of H may vary in different textbooks

$$B = H + \mu_0 M$$

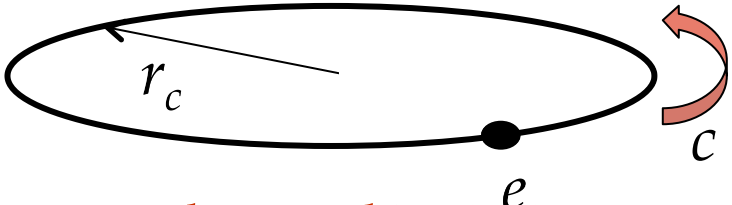
MAGNETIZATION OF IRON

- In ferromagnetic materials, the main source of magnetization is the electron intrinsic magnetic momentum

$$m_e \approx m_B \equiv \frac{eh}{4\pi m} = 9.3\times 10^{-24}\ \mathrm{A\ m^2}$$

  - In a first approximation, it can be seen as the momentum associated to a loop of current given by an electron at the speed of light on a radius = Compton radius (this is the Bohr magneton)

$$m_B = Is = e\frac{c}{2\pi r_c}\pi r_c^2 = \frac{ec}{2}r_c = \frac{ec}{2}\frac{h}{2\pi mc} = \frac{eh}{4\pi m} = 9.3\times 10^{-24}\ \mathrm{A\ m^2}$$



  - The difference between the actual intrinsic magnetic momentum and the Bohr magneton is the g-2 (for the electron, about 1 per mil)
  - Compton radius is about 1/137 of the Bohr radius (size of orbit of the electron around the proton in the H, i.e. "size of the atom")

MAGNETIZATION OF IRON

- We can estimate of the order of magnitude of the saturation (maximum contribution of iron magnetization)
  - The "volume" of iron an iron atom is

$$V_{Fe} \approx \frac{N_{Fe} m_p}{\rho} = \frac{55.87 \times 1.67 \times 10^{-27}}{7900} = 1.1 \times 10^{-29} \text{ m}^3$$

  - The magnetization when all spin are aligned is

$$M \approx \frac{m_B}{V_{Fe}} = \frac{9.3 \times 10^{-24}}{1.1 \times 10^{-29}} = 8.7 \times 10^5 \text{ A/m}$$

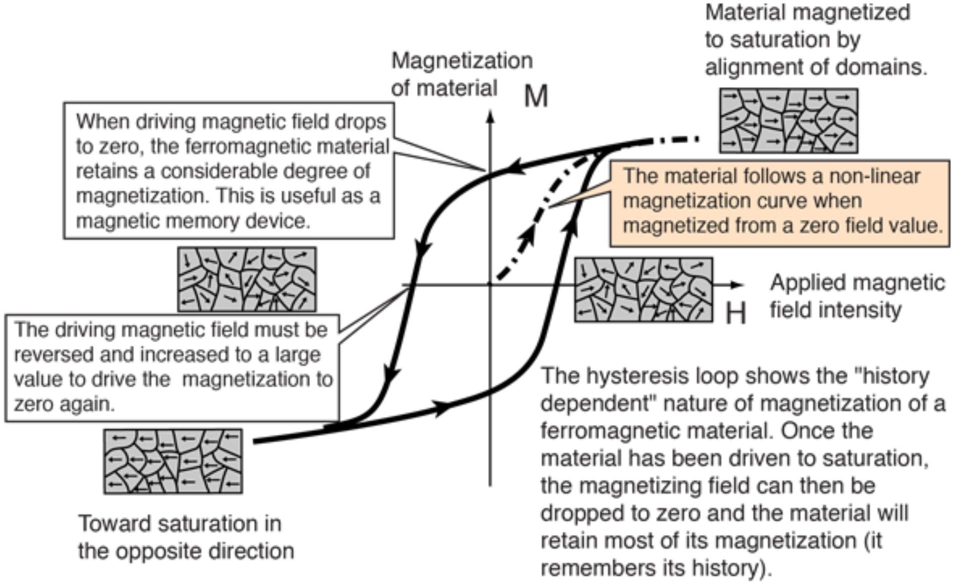
  - And the field generated by magnetization is

$$B_{sat} \approx \mu_0 M = 4\pi \times 10^{-7} \times 8.7 \times 10^5 = 1.1 \text{ T}$$

(see https://indico.cern.ch/category/12408/ Appendix B for further reading)

HYSTERESIS FOR IRON MAGNETIZATION

- We now move to hysteresis …
  - When making a cycle of external field, hysteresis phenomena appear: at zero external field one has a residual field



  - The energy dissipated is given by the integral $U = \oint M \, dB$

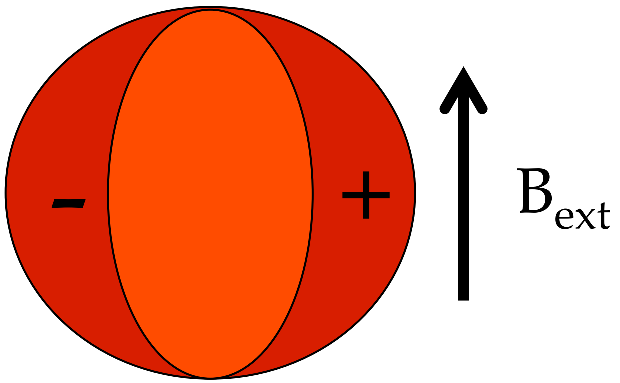
  - Infact, $MB=B^2/\mu_0$ has the dimension of an energy density

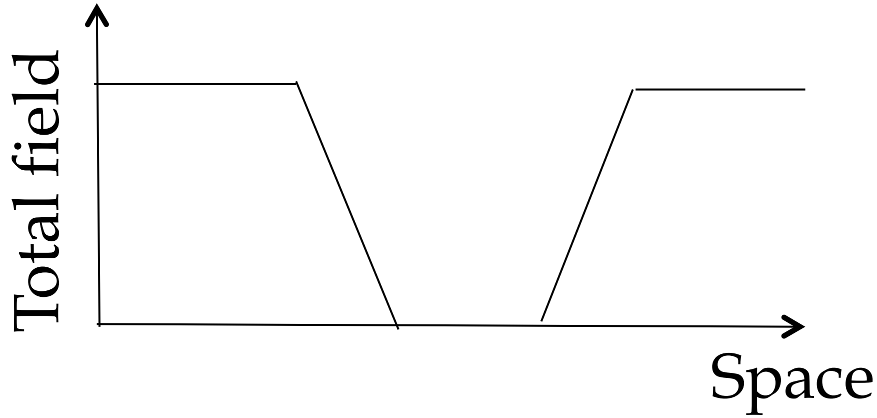
CONTENTS

- Magnetization in iron (ΔB)
- Magnetization in superconductors and stability (ΔB)
- Persistent currents (ΔB)
- Ramp rate effects (dB/dt)
- Appendix

MAGNETIZATION IN SUPERCONDUCTORS

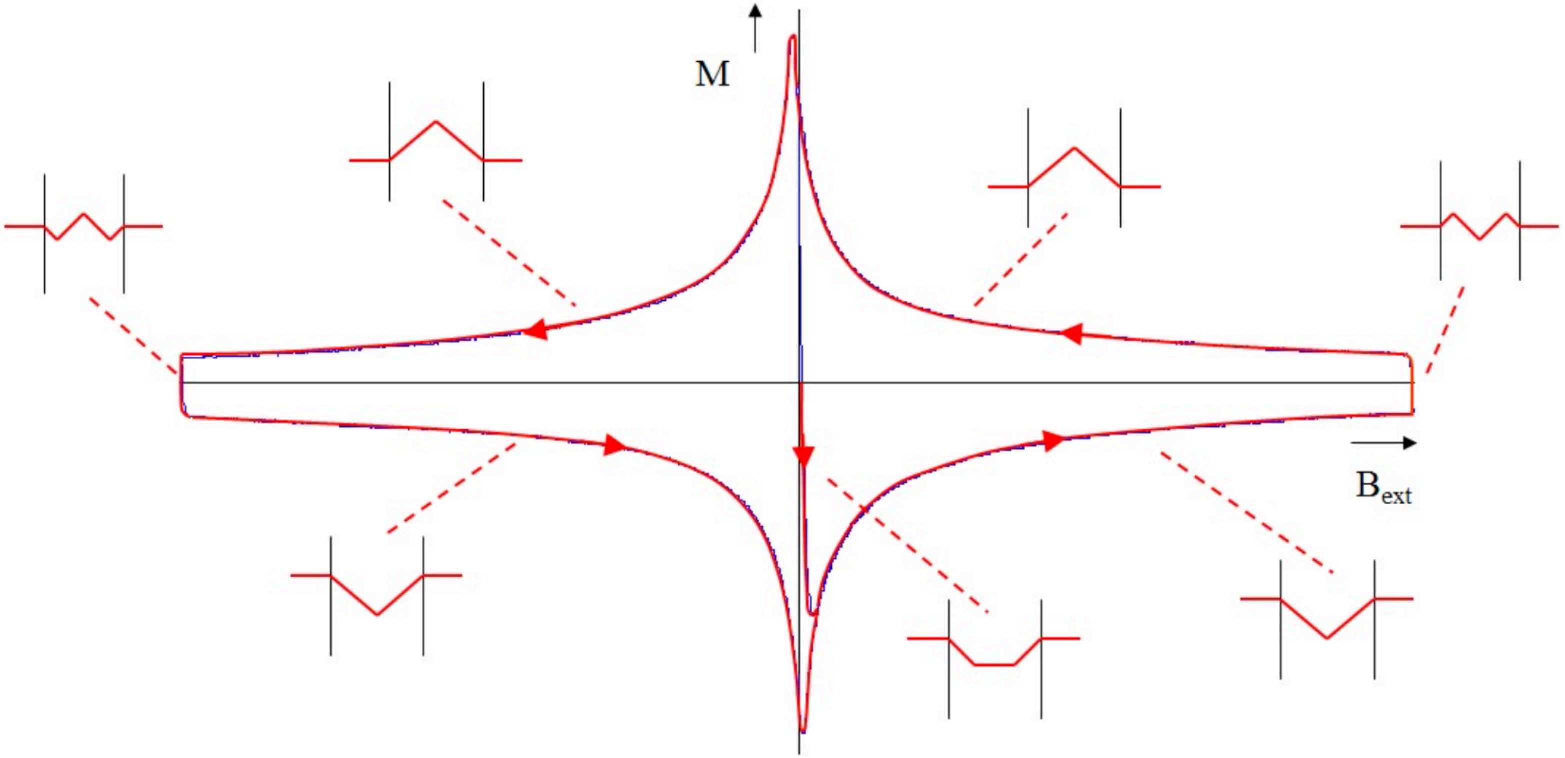
- In superconductors, magnetization is created by shielding currents opposing to the field to avoid penetration
  - Since resistance is zero, the current will circulate forever (persistent currents)
  - The currents circulate in the external shell, with a current density j_c equal to the critical current density (Bean model)
  - These currents produce a magnetization, that is larger at lower fields since the critical current is larger for lower fields





MAGNETIZATION IN SUPERCONDUCTORS

- This is a typical plot of magnetization in a LTS superconductor



Magnetization in a superconductor (courtesy of M. Sorbi)

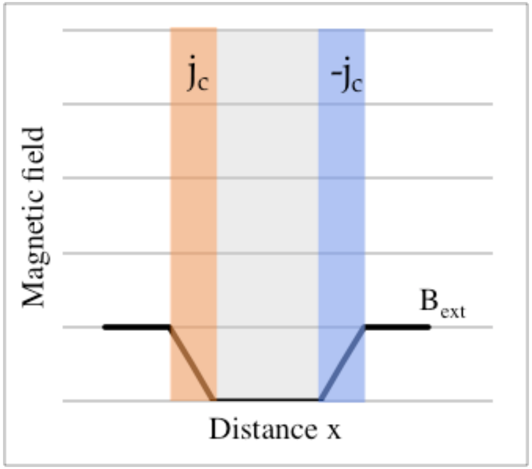
MAGNETIZATION IN SUPERCONDUCTORS

- Magnetization in superconductors immersed in magnetic fields has two adverse effects:
  - The dissipation induced by magnetization can induce a thermal runaway (also called instability): this imposes that superconductor wires are made by thin (order of 0.1 mm) filaments
    - This is a complex effect and this one of the reasons of the large delay between superconductivity discovery (1911) and the construction of first superconducting magnets (1960's) with fields of the order of 1 T
    - It will be described in this section, with the maths left to the appendix
  - The second adverse effect is to perturb the field quality (so-called persistent currents components in multipoles)
    - This effect is strong a injection field (lower field, larger critical current, larger magnetization)
    - Therefore this limits the energy increase that can be achieved in accelerators
    - It will be described in the next section

MAGNETIZATION IN SUPERCONDUCTORS

- Superconductor creates a magnetization to shield against external field
  - According to Bean theory, the current density of the shielding currents can be either zero or the critical current
- Let us consider a simple geometry of a infinite slab of superconductor, of thickness 2*d* in an external field *B*
  - Maximum field that can be created inside the slab is

$$B_p = \mu_0 j_c d$$

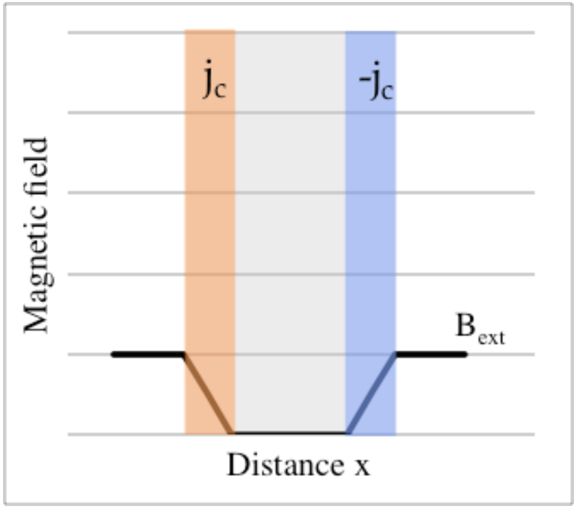


2*d*

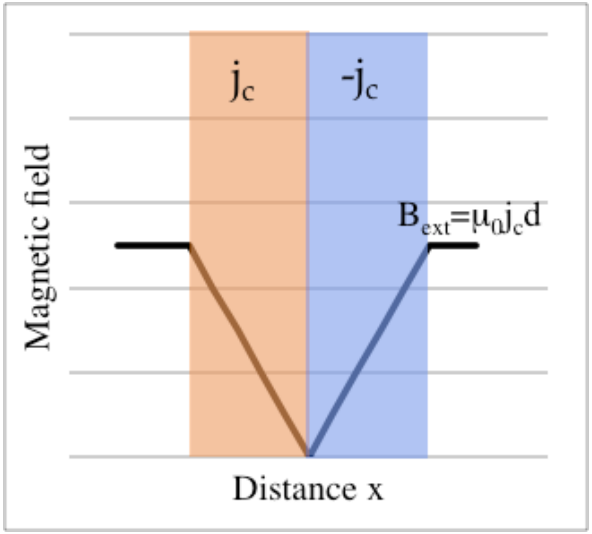
  - It is also called penetration field
  - So for a external field larger than the penetration field $B_{ext}>B_p$ one has a residual magnetic field

MAGNETIZATION IN SUPERCONDUCTORS

- Shielding currents in a superconducting slab with external magnetic field



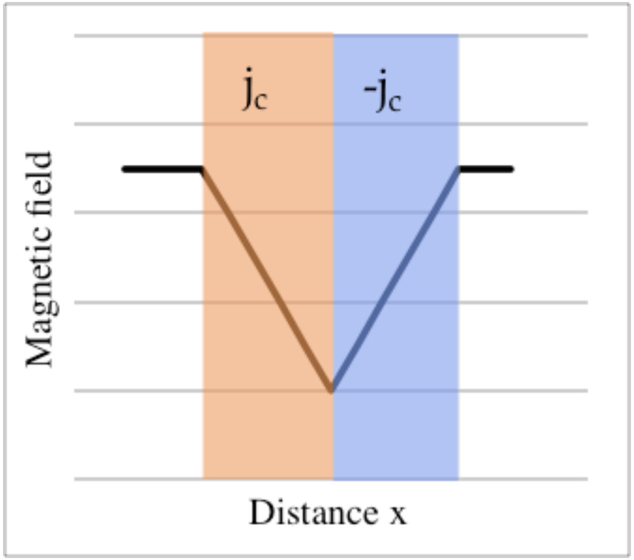
Shielding currents in presence of external field



Shielding currents at penetration field

$$B_p = \mu_0 j_c d$$

$$B(x) = B_{ext} - 2\mu_0 jx$$



Shielding currents above penetration field

- Stability is given by the following mechanism
  - We start from an increase of temperature ΔT_0 (local heat deposition by the beam, or mechanical movements and friction)
  - This induces a reduction of critical current density

$$B(x) = H - 2\mu_0 jx$$

  - Less critical current means less shielding, and therefore a change of magnetic field in the superconductor
  - The change of magnetic field flux induces a voltage according to Maxwell

$$\Phi = \int B\, ds \qquad V = \frac{d\Phi}{dt}$$

  - The voltage and current induce a dissipation VI
  - The dissipation induces heat and a change of temperature ΔT_1
- If the sum $\Sigma \Delta T_k$ converges we are stable, otherwise there is a mechanism providing a divergence of temperature

MAGNETIZATION AND STABILITY

- This gives a stability criterion on the slab size
  - C_p: specific heat γ: density
  - J_c: critical current density
  - T_c-T: difference between operational temperature and critical temperature (current sharing temperature

$$\frac{d}{2} < \frac{1}{j_c}\sqrt{\frac{3\gamma C_p(T_{cs}-T)}{\mu_0}}$$

- Finer superconductor more stability
  - If you are limited by magnetization instability, larger current density needs smaller filament size
- Larger critical current less stability
  - This is one of the adverse effect of large critical current …
  - … together with impact on stresses due to e.m. forces (see F. Toral on mechanics) and limits seen in protection (see the lecture on protection)
  - This equation is proved in the appendix

MAGNETIZATION AND STABILITY

- How much thin ?

$$\frac{d}{2} = \frac{1}{j_c}\sqrt{\frac{3\gamma C_p (T_{cs} - T)}{\mu_0}}$$

- Let us see the order of magnitudes:
  - Current density (in the sc) of 1000 A/mm^2, i.e. 10^9 A/m^2
  - We work at 2 K distance from critical surface (temperature margin)
  - Volumetric specific heat $\gamma\, C_p$ of Nb-Ti at 2 K is 2000 J/K/m^3

$$\frac{d}{2} = \frac{1}{10^9}\sqrt{\frac{3 \times 2000 \times 2}{4\pi \times 10^{-7}}} = 0.1 \text{ mm}$$

  - Note that at low current the condition is more stringent
    - At low field, temperature margin Nb-Ti of 10 K but current 10 times larger

$$\frac{d}{2} = \frac{1}{10^{10}}\sqrt{\frac{3 \times 2000 \times 10}{4\pi \times 10^{-7}}} = 0.02 \text{ mm}$$

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PERSISTENT CURRENTS

- The perturbation in the field caused by superconductor magnetization is order of per mil
  - It can be followed by appropriate powering, i.e. ramping the current in a slightly nonlinear way to compensate for it
- On the other hand, the influence on field quality (where the control is needed at 0.1 per mil) is significant
  - More affected are the allowed multipoles, i.e. b3 for the dipoles
  - This is effect is large at lower fields → sets a limit of the accelerating factor of a snychroton
  - A main superconducting magnet (dipole or quadrupole) cannot work around zero field

	Injection	Collision	Energy increase
Tevatron	150 GeV	980 GeV	6
HERA-p	40	920 GeV	23
LHC	450 GeV	7 TeV	15.6

PERSISTENT CURRENTS

- Since the magnetization is proportional to critical current and filament size

$$m \propto J_c D_{eff}$$

- For a Nb-Ti conductor one has

$$J_c(I) = \frac{1}{B}\left(\frac{B}{B_c(T)}\right)^p \left(1 - \frac{B}{B_c(T)}\right)^q \left[1 - \left(\frac{T}{T_{c0}}\right)^{1.7}\right]^m$$

(see also lecture by H. Gersem)

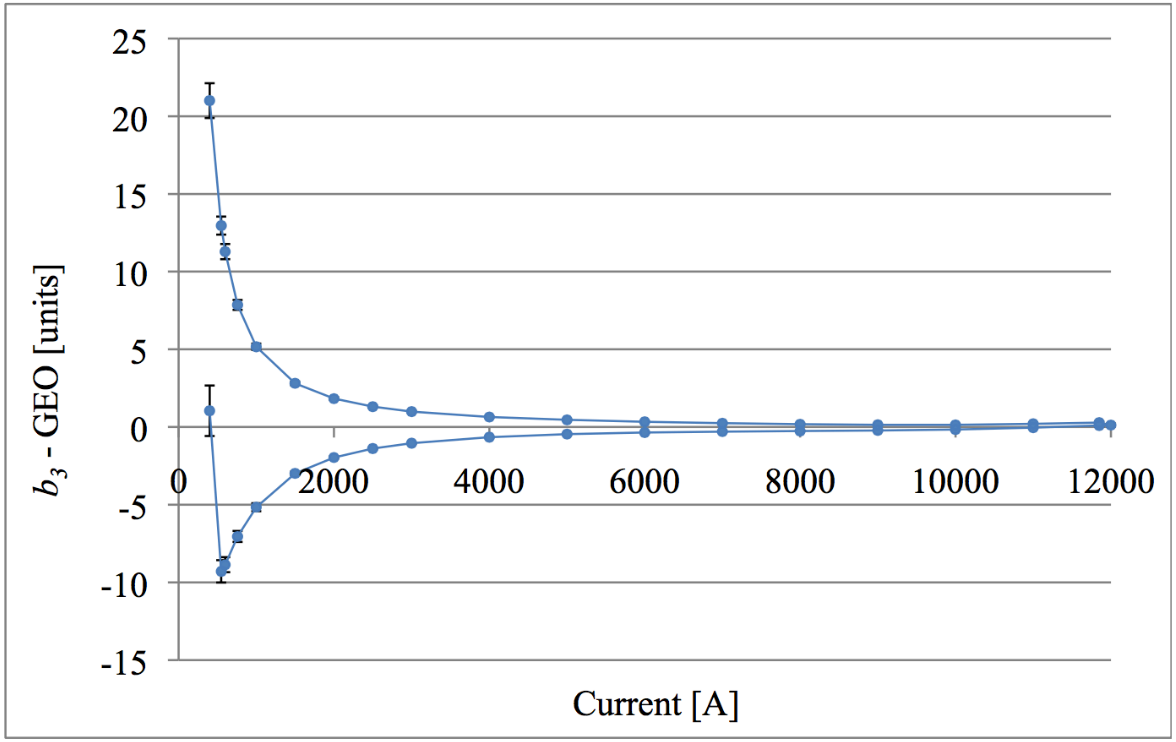
- And therefore the persistent current component can be fit via

$$b_n(I) = \mu_n \left(\frac{I}{I_{inj}}\right)^{p_n - 2} \left(\frac{I_c - I}{I_c - I_{inj}}\right)^{q_n}$$

N. Sammut, L. Bottura, J. Micalef *Phys. Rev. STAB* **9** 012402 (2006)

PERSISTENT CURRENTS

- Example of LHC dipoles



Variation of b3 along the LHC main dipole ramp (up and down) (L. Deniau for the FiDeL team)

- The 10 units b_3 variation along the ramp is corrected via dedicated corrector magnets (spool pieces)

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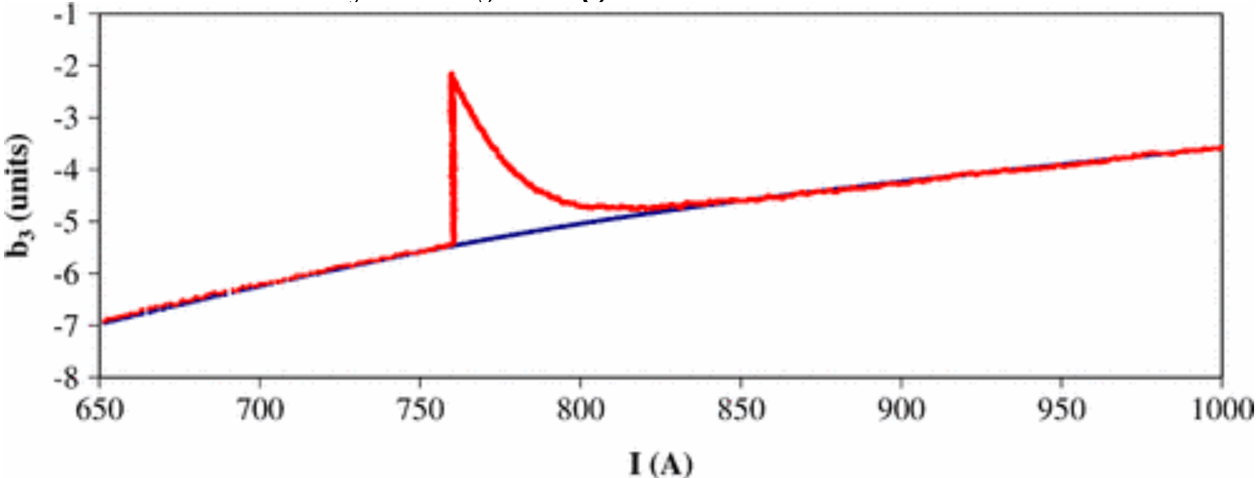
DECAY AND SNAPBACK

- Note that magnetization decays when the ramp is paused (for instance when we sit at injection current to inject the beam)
  - This is due to persistent current redistribution
  - The amount of decay for b_3 is significant and needs correction

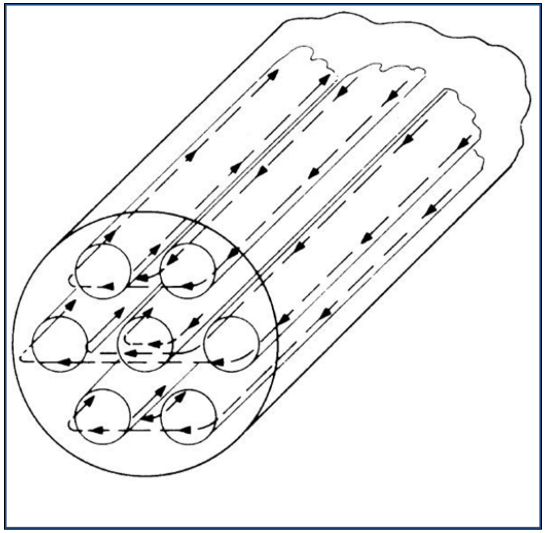


Variation of b3 along the LHC main dipole ramp with pause at 760 A to inject the beam
(N. Sammut, L. Bottura, et al., Phys. Rev. STAB 10 (2007) 082802

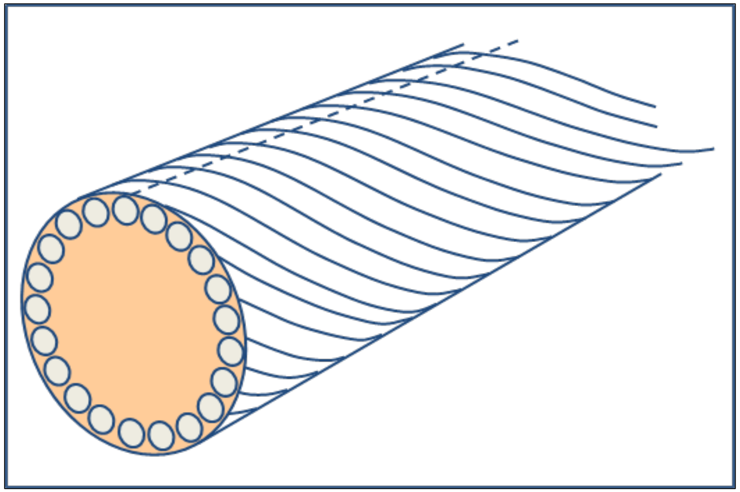
- When we restart the magnet ramp, this decay disappears (snaps-back) in a few s with exp dependence on current
  - Very rapid phenomenon that can induce beam losses at the end of the injection plateau – depends on ramp rates

RAMP RATE EFFECTS

- Ramp rate effects are given by "local" current loops created in the superconductor bridged with a small conducting part
  - Either between the filaments through the copper matrix



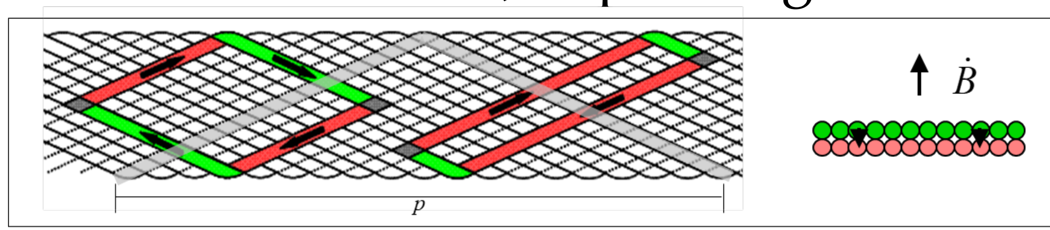
Current loops between filaments in an untwisted strand (from M. Wilson book "Superconducting magnets")

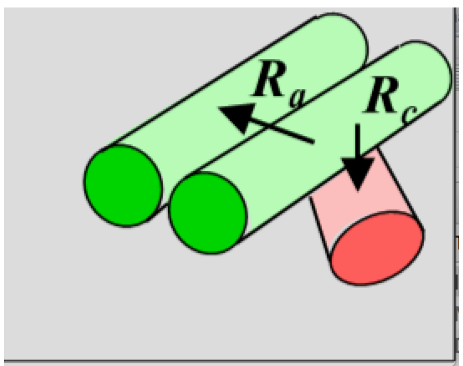


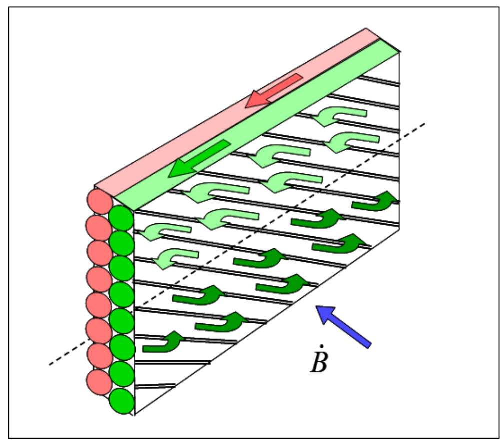
Strand twisting in a cable around core (easier to see) (courtesy of L. Bottura)

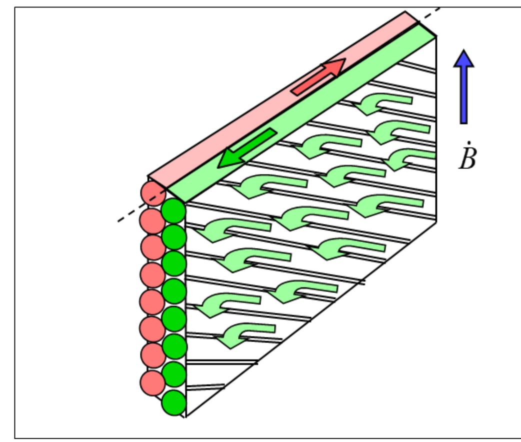
- That's why the filaments in the strand are twisted

- … or between between strands in the cable, depending on the interstrand resistance









Current induced in the Rutherford cable (courtesy of P. Ferracin)

- That's why the Rutherford cable is twisted
- We will see later how interstrand resistance can be controlled
- Note that for some accelerator magnets (for instance correctors) these effects are not significant and therefore a ribbon is used

RAMP RATE EFFECTS

- Ramp rate effects are given by "local" current loops created in the superconductor bridged with a small conducting part
  - Either between filaments in the strand or between strand in the cable
  - Associated magnetization is

$$M = \frac{2\dot{B}\tau}{\mu_0}$$

  - And dissipated power density scales with the square of ramp rate

$$Pdt = MdB = \frac{2\dot{B}\tau}{\mu_0}\frac{dB}{dt}dt = \frac{2\dot{B}^2\tau}{\mu_0}dt \qquad P = \frac{2\dot{B}^2\tau}{\mu_0}$$

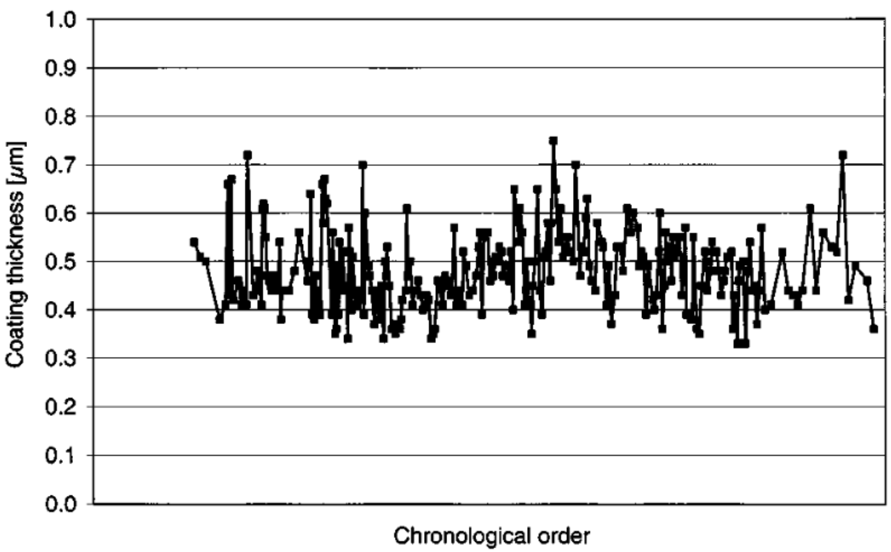
- where τ is the time constant of the decay of these currents, that scales according to

$$\tau = \frac{\mu_0}{2\rho}\left(\frac{l_{twist}}{2\pi}\right)^2$$

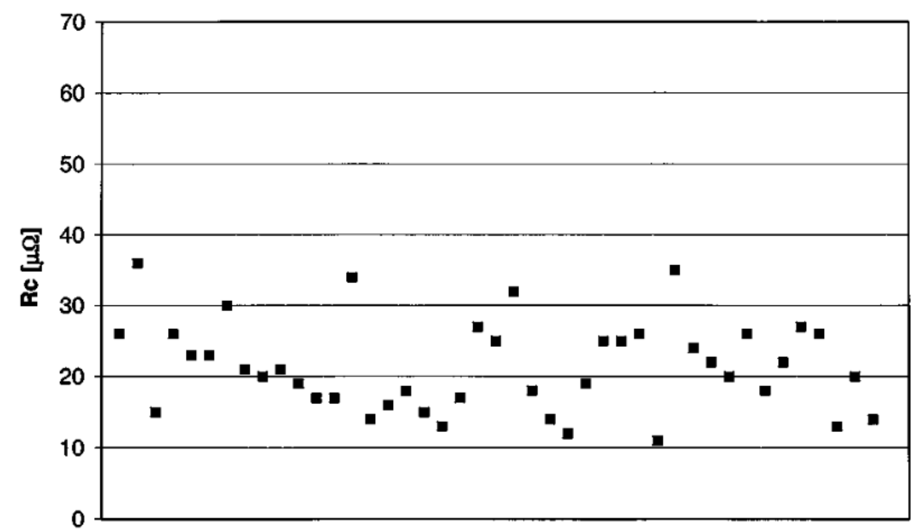
- Where ρ is transverse effective resistivity, and l_{twist} is the twist pitch

CONTROL OF INTERSTRAND RESISTANCE

- In Nb-Ti main magnets for accelerators, the strand is coated with SnAg to increase the interstrand resistance
  - Thickness of the order of 0.5 μm
  - Interstrand resistance should be not too large to allow current redistribution
  - (J. Adam, D. Leroy, et al. IEEE TAS 12 (2002) 1056-1063)



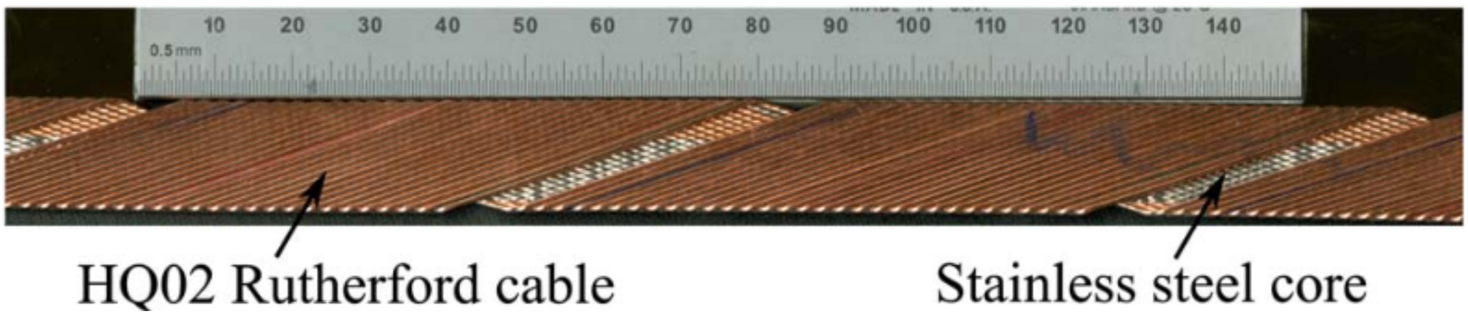
Coating thickness of LHC strands along the production



Interstrand resistance of LHC cable along the production

RUTHERFORD CABLES: CORE

- For Nb_3Sn, coating is not an option due to heat treatment: a core is required in Rutherford cables to avoid ramp rate effects

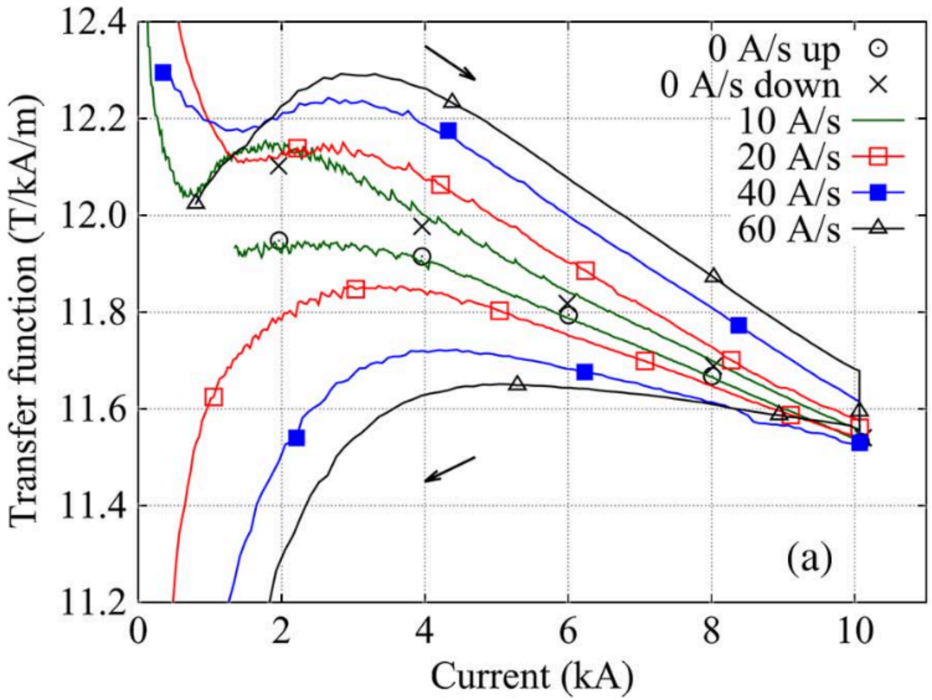


Stainless steel in the HQ02 cable (X. Wang, et al, IEEE TAS 24 (2014) 4002607)

  - A typical core is made of 25-μm-thick stainless steel sheet, covering about 2/3 of the cable width
  - Its function is to cut the interstrand currents between the two layers of the Rutherford
  - It is part of the design of the Nb_3Sn cables in MQXF and 11 T HL-LHC projects

RUTHERFORD CABLES: CORE

- Here you see the strong dependence of transfer function (ratio between main component and current) on ramp rate for HQ01 (R&D for the HL-LHC triplets) if you do not use a cored cable



Effect of ramp rate on the transfer function for HQ01, a Nb_3Sn magnet without cored cable
(S. Wang, et al, IEEE TAS 24 (2014) 4002607)

CONCLUSIONS

- Magnetization and ramp rate effects determine the cable and strand design
  - Magnetization due to shielding currents is given by Meissner effect and Bean theory – it is large at low fields
  - Limits the injection energy (or better, the energy increase) of accelerators based on superconducting magnets
  - Ramp rate effects are due to Maxwell equations (current induced by the variation of field with time in a loop, that is partially superconductive and partially resistive)
- Magnetization requires to have the superconductor in fine filaments (0.1-0.01 mm)
  - Otherwise it creates instabilities for temperature small variations
  - This blocked the development of sc magnets for 50 years
- Ramp rate effects requires twisting the filaments, the cables, and controlling the interstrand resistance
  - For Nb-Ti main magnets a coating is used on the strand
  - For Nb_3Sn a core is included in the cable

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APPENDIX: STABILITY

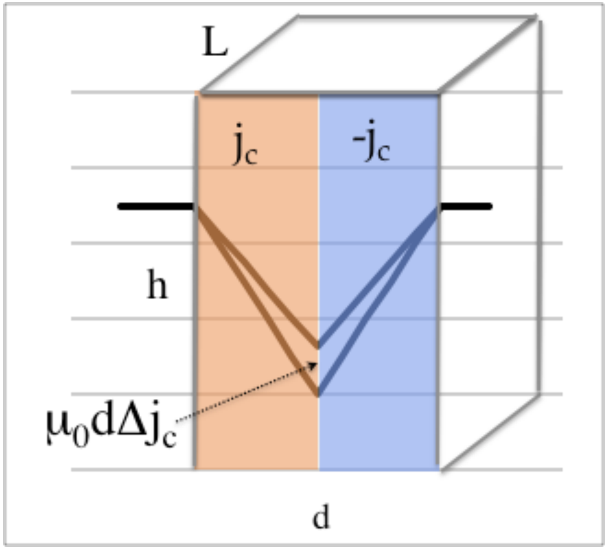
- We now give a proof of the equation for stability
- Ingredients:
  - Reduction of critical current induced by increase of temperature (approximation)

$$\frac{\Delta j_c}{j_c} = \frac{\Delta T}{T - T_c}$$

  - Relation between current density and current (trivial)

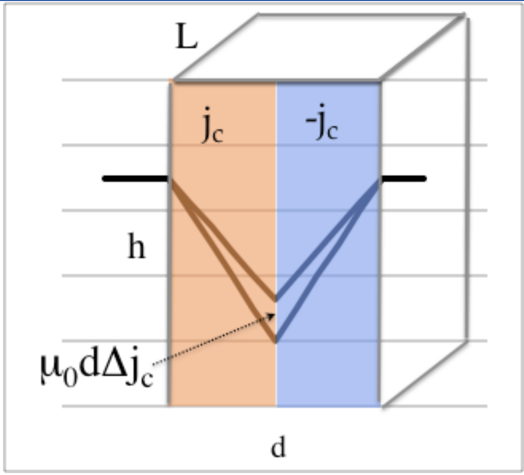
$$I = dhj_c$$

  - Change of flux through the area *L d*



$$\Delta\Phi = 2L\mu_0 \frac{d}{2}\Delta j_c \frac{d}{6} = L\mu_0 \frac{d^2}{12}\Delta j_c$$

APPENDIX: STABILITY



- Heat increase

$$\Delta Q = \int VI\,dt = dhj_c \int \frac{d\Phi}{dt} dt = dhj_c \Delta\Phi$$

- Since

$$\Delta\Phi = L\mu_0 \frac{d^2}{12} \Delta j_c$$

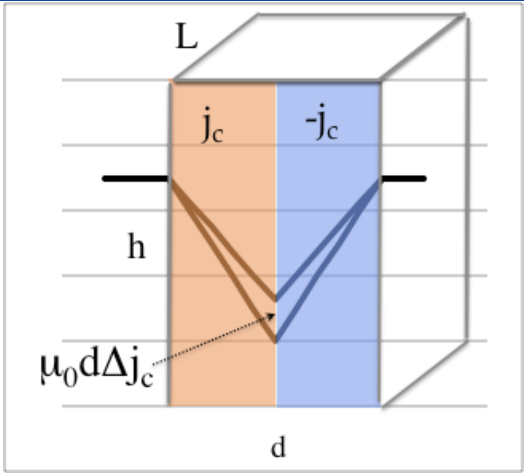
- So heat increase is

$$\Delta Q = dhj_c L\mu_0 \frac{d^2}{12} \Delta j_c = \left[dhL\right]\mu_0 \frac{d^2}{12} j_c \Delta j_c$$

- And the heat density increase is

$$\Delta q = \mu_0 \frac{d^2}{12} j_c \Delta j_c = \frac{\mu_0}{3}\left(\frac{d}{2}\right)^2 j_c \Delta j_c$$

APPENDIX: STABILITY



- Summarizing: initial dT_0 gives

$$\Delta q_1 = \frac{\mu_0}{3}\left(\frac{d}{2}\right)^2 j_c \Delta j_c$$

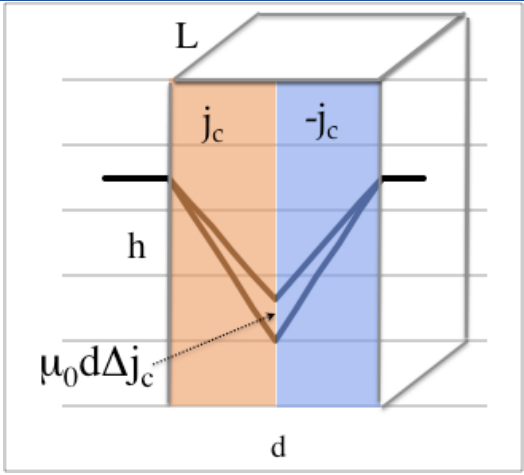
- Since we assumed

$$\frac{\Delta j_c}{j_c} = \frac{\Delta T_0}{T - T_c}$$

- The dT_0 gives an additional temperature increase dT_1

$$\Delta T_1 = \frac{\Delta q_1}{\gamma C_p} = \frac{\mu_0 j_c^2}{3\gamma C_p (T - T_c)}\left(\frac{d}{2}\right)^2 \Delta T_0$$

APPENDIX: STABILITY



- Summarizing: initial ΔT_0 gives

$$\Delta T_1 = \frac{\Delta q_1}{\gamma C_p} = \frac{\mu_0 j_c^2}{3\gamma C_p (T - T_c)} \left(\frac{d}{2}\right)^2 \Delta T_0$$

- And ΔT_1 will give

$$\Delta T_2 = \left[\frac{\mu_0 j_c^2}{3\gamma C_p (T - T_c)} \left(\frac{d}{2}\right)^2\right] \Delta T_1$$

- After n steps one has

$$\Delta T_n = \left[\frac{\mu_0 j_c^2}{3\gamma C_p (T - T_c)} \left(\frac{d}{2}\right)^2\right]^n \Delta T_0$$

- The series converges if

$$\frac{\mu_0 j_c^2}{3\gamma C_p (T - T_c)} \left(\frac{d}{2}\right)^2 < 1$$