Overview of Magnetic Measurements

Why, when, what and how to measure (and calibrate)

Marco Buzio, Test & Measurement Section, Magnet, Superconductors and Cryostats Group

CAS course on “Normal- and Superconducting Magnets”, 19.11–02.12.2023 Pölten, Austria

Introduction

Why and when to measure

Know your magnetic field

FEM/BEM simulations

– Highly idealized object
– Costs: mostly setup
(scripts → scalable parametric analysis)

Beam-based measurements

– Full set of actual objects
(+ all the rest around them …)
– Costs: beam time (expensive!)

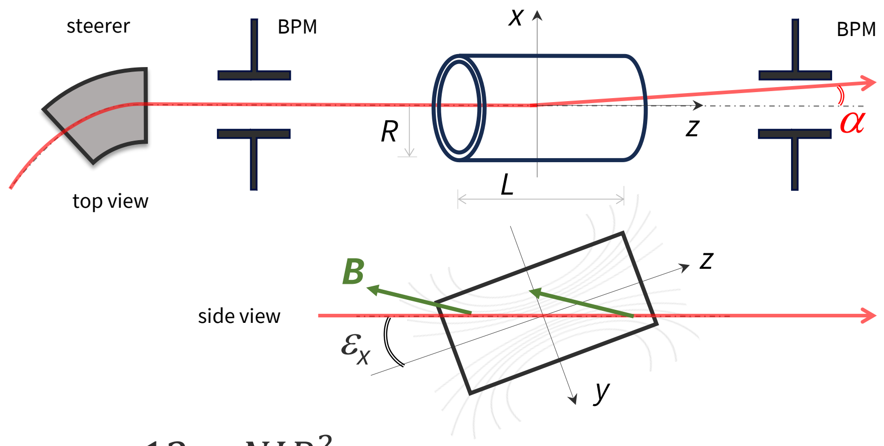
Magnetic measurements

– real object or close representative, under controlled laboratory conditions
– costs: development of instrumentation, power supplies, cooling, transport …. manpower $\propto$ n. of test cases

Beam-based measurement: examples

Solenoid axis tilt (ISOLDE REX)

- Tilt could not be measured in-situ due to mechanical constraints
- Beam Position Monitors measure beam deflection angle α by scanning the entry direction
- Radial field B_r approx. by 1st order Taylor expansion



$$B_r(r,z) \approx \frac{12}{\pi}\frac{\mu_0 N I R^2}{L^4} rz$$

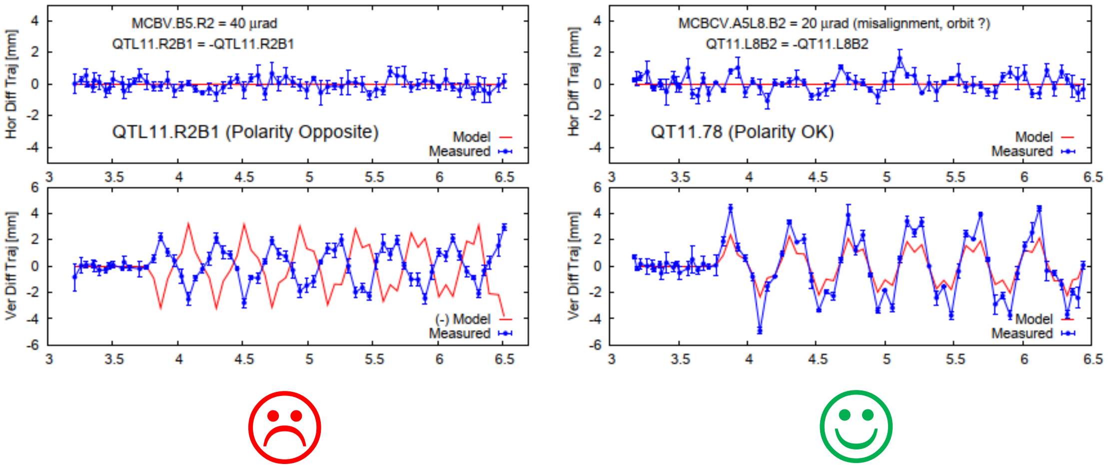
$$\int_{-L/2}^{L/2} B_y(\varepsilon_x z, z)\,dz \approx \frac{\mu_0 N I R^2}{\pi L}\varepsilon_x$$

$$p_{\mathrm{GeV/c}}\alpha = 0.3\int_{-L/2}^{L/2} B_y\,dz$$

$$\varepsilon_x = \frac{\pi}{0.3}\frac{L}{\mu_0 N I R^2} p_{\mathrm{GeV/c}}\alpha$$

LHC multipole polarity check

- Triggered by anomalies detected in early injection tests
- Beam prepared with low intensity and emittance; large momentum offset to enhance sensitivity
- Suspected multipoles set to excite betatron oscillations at opposite polarities
- Trajectory difference compared to MADX simulations
- Effective but €xp€n$iv€



R. Calaga , Polarity checks in Sectors 23 & 78, LHC Performance Note 010, 2009

Beam-based measurements: summary

Advantages

- by definition, **the beam sees all** that's relevant in the actual operating conditions
- Beam measurement can deal quickly with the **unexpected**
- They may be the **only option**: e.g. old magnets with no spare, no FE model

Limitations

- **difficult interpretation**: all effects integrated by the beam, unique inversion may not be possible
- require **very precise instrumentation** (e.g. ppm tune measurements, 10-50 μm BPM), stable machine, as many correctors as possible, dynamic aperture margin
- require well-known **optic model** (obtained from previous magnetic measurements !)
- only non-linearities small enough not to cause beam lifetime issues can be measured
- Good Field Region corners cannot be explored well due to the finite size of the beam
- **very time consuming** (e.g. LEP bump FFT took 2 months) hence extremely **expensive**

Numerical models ↔ Measurements

Numerical models

- **Object does not need to exist !**
  but beware: the final object will be always different from the model due to geometrical tolerances, material properties variations
- **Arbitrary operational parameter range**
- **Arbitrary range and resolution of results**
  (space/time)

- **Systematic (and "random") errors**
  - modelling errors: approximated or missing physical phenomena (e.g. hysteresis) and couplings (e.g. magnetothermal, -mechanical, -electrical);
  - approximation errors: truncation, simplified solutions
  - discretization errors: triangulation, outer domain boundaries
  - numerical errors: roundoff, instabilities (corner singularities)
- **Cost driver: setup and validation**
  - initial effort ranges from trivial to near-impossible
  - scripted iterations do not require manpower
  - computation cost normally scales with $h^{2\sim3}$, 1/dt

- **Best at: relative differences and changes**
  - parameter space exploration, optimization

Measurement

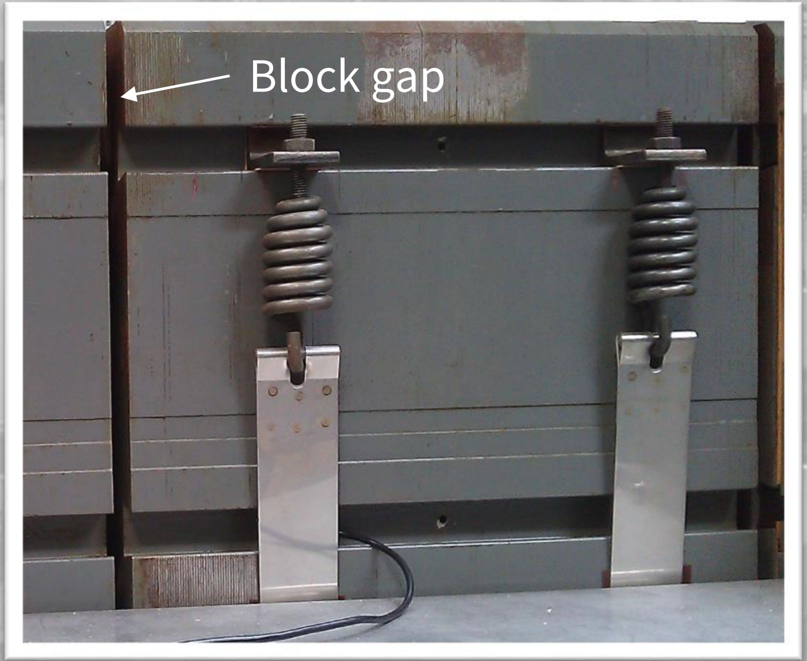
- **Object "as built"**
  but beware: a proto/spare may not represent well the mean of a series)
- **Practical limitations:**
  available test bench, power converter voltage/current, time …
- **Practical limitations:**
  - overall sensor size, sensing volume shape and size, mechanical positioning
  - sensor sensitivity, linear range, bandwidth

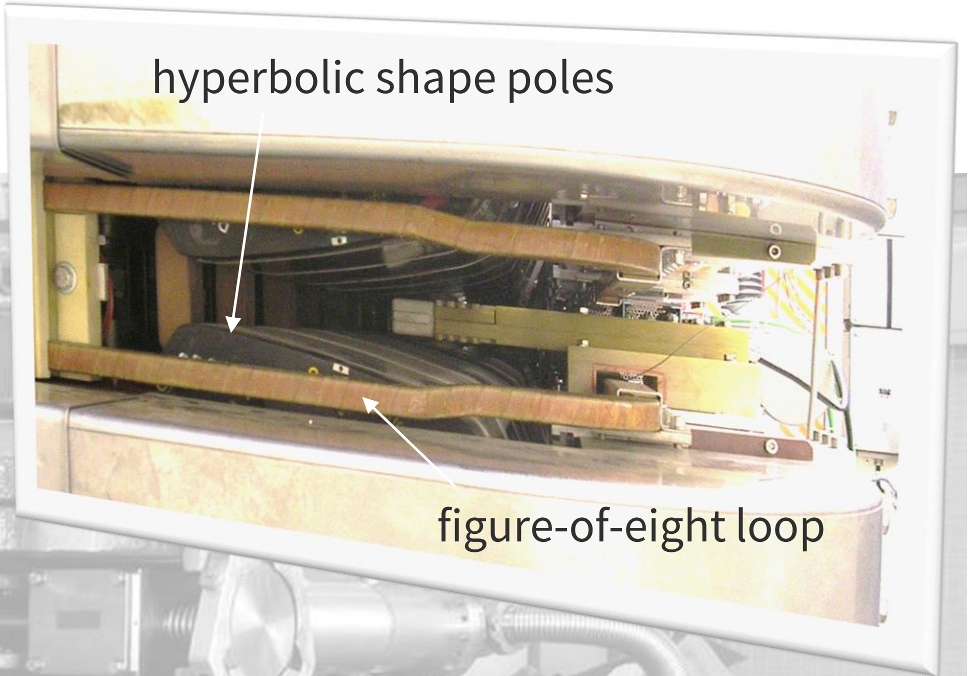
- **Systematic and random measurement errors:**
  - transduction (S/N) and acquisition noise (ADC linearity, quantization)
  - acquisition chain errors: preamplifier gain, frequency response
  - calibration and numerical post-processing errors

- **Cost drivers: setup and operation**
  - adapted high-precision instrumentation often requires specific R&D
  - repeated tests (parametric studies, series runs) consume proportional resources

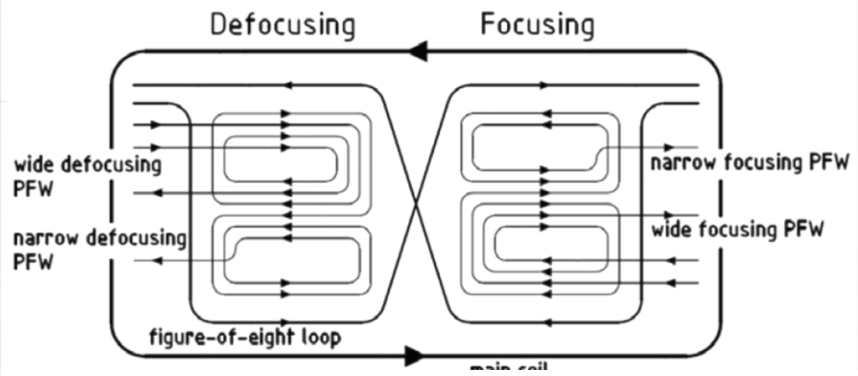
- **Best at: absolute results**
  with proper traceable calibration chain

Example: CERN PS main unit



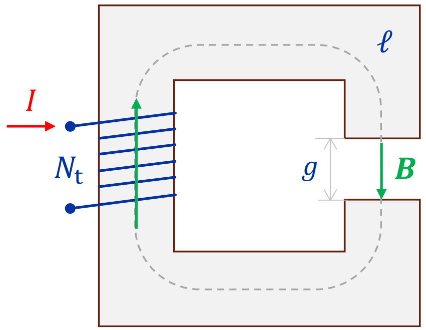


- Combined-function dipole + F/D quadrupole
- 10 × angled blocks – just one big end region
- 1+1+2+2 magnetically and inductively coupled excitation circuits
- Still no complete 3D dynamic model at age 60+ …



Pole face windings

Impact of model uncertainties

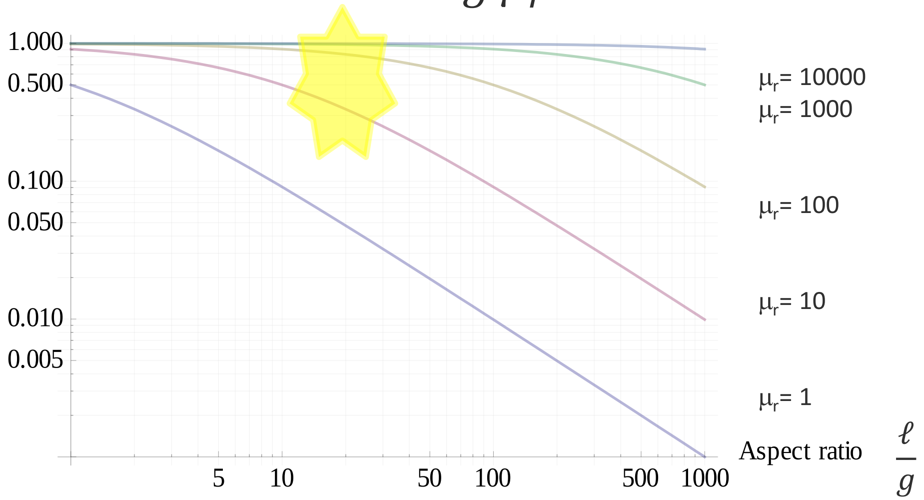


Analytical 1D model (neglecting leakage)

$$B = \frac{\mu_0 \mu_r N_t I}{\ell + \mu_r g}$$

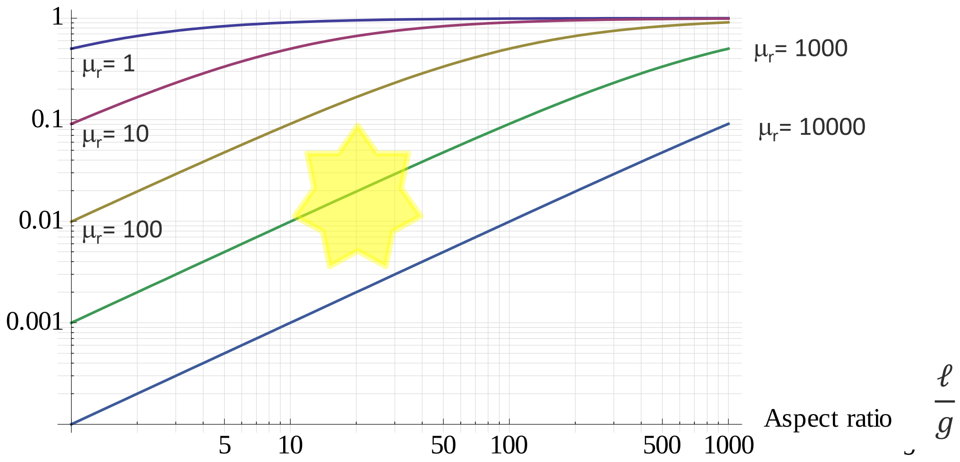
impact of geometrical uncertainty
(mechanical tolerances, assembly errors)

$$\frac{g}{B}\frac{\partial B}{\partial g} = -\frac{1}{\frac{\ell}{g}\frac{1}{\mu_r} + 1}$$



impact of material property uncertainties

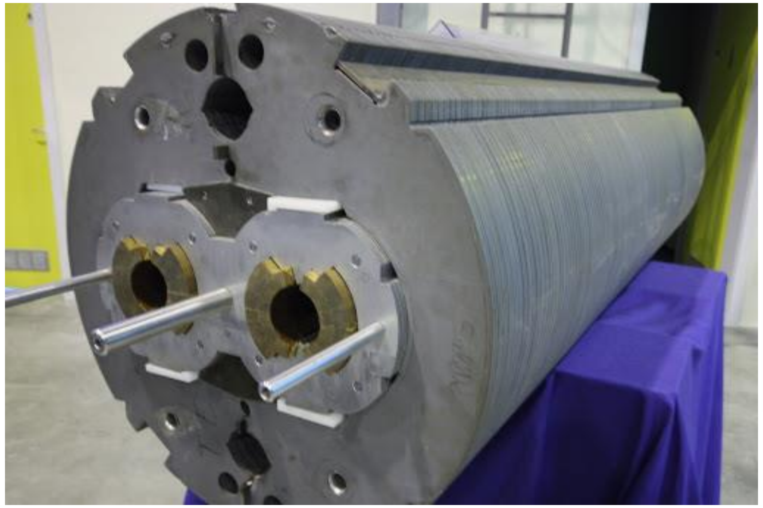
$$\frac{\mu_r}{B}\frac{\partial B}{\partial \mu_r} = \frac{1}{1 + \mu_r \frac{g}{\ell}}$$

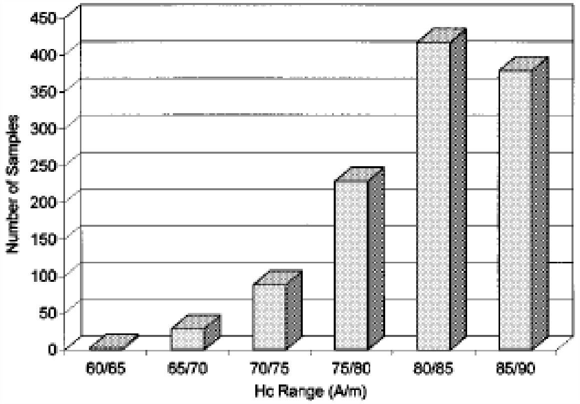


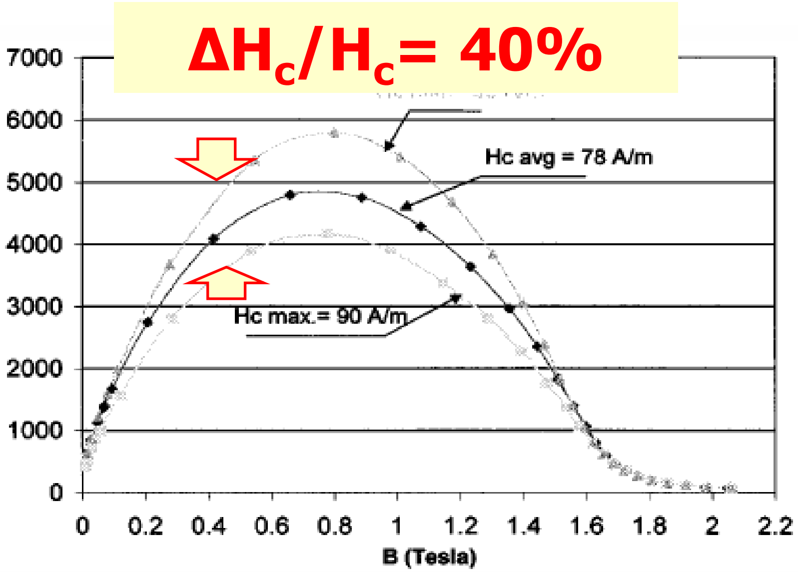
10 μm/100 mm gap error → **10^{-4} field error** at low field

5% μ_r error → **$5 \cdot 10^{-4}$ field error** at low field

Material properties uncertainty

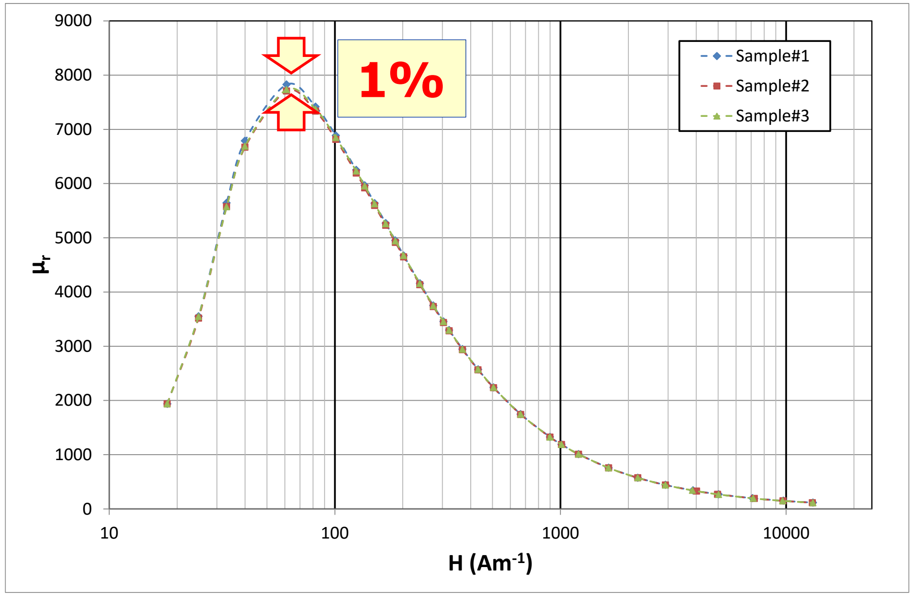






variation of the coercivity/peak permeability over 50 ktons of MAGNETIL steel sheets for LHC cryomagnets

Giuseppe Peiro *et al.*, Toward the Production of 50 000 Tonnes of Low-Carbon Steel Sheet for the LHC Superconducting Dipole and Quadrupole Magnets, IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY, VOL. 12, NO. 1, MARCH 2002



Add: difference between actual hysteresis loop and initial curve

scatter over a small set of three ARMCO steel samples taken from same batch

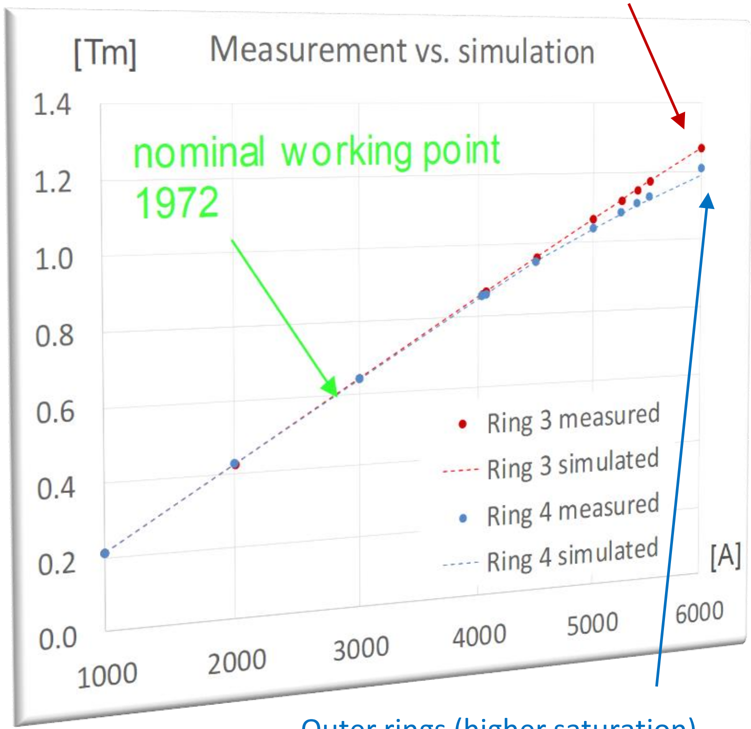
Courtesy Giuseppe Montenero, CERN

FE ↔ measurements: CERN PSB bending dipole

steel plates to be reinforced to
equalize the rings at high field
(+110% @ 2 GeV w.r.t. design value !)

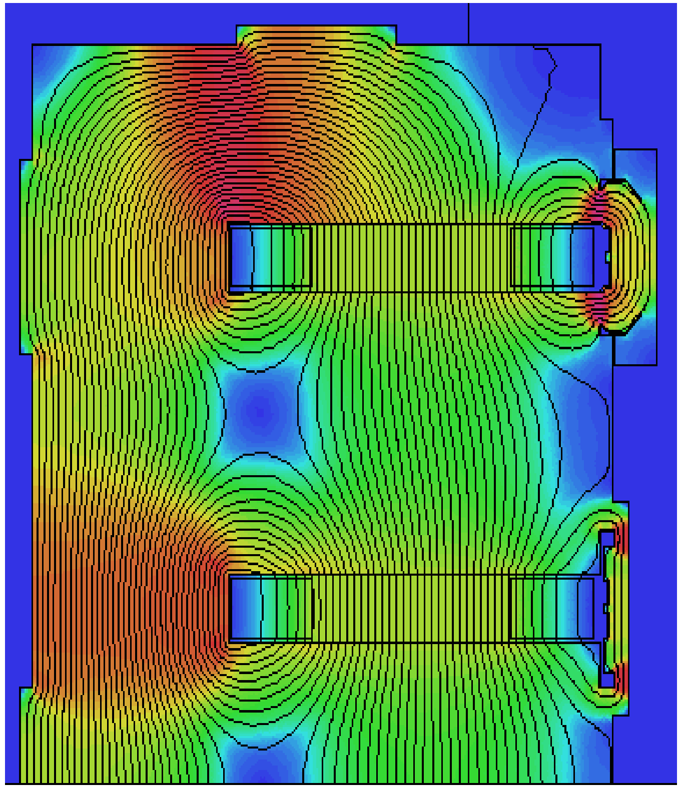
4-ring main bending dipole
of CERN PS Booster

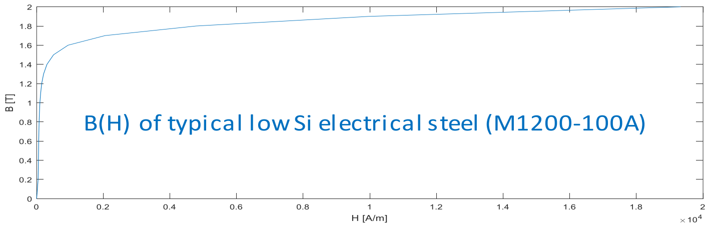
Inner rings
(high field, little saturation)
0.3% mismatch FE/MM



Outer rings (higher saturation)
2% mismatch FE/MM

2D FE with nominal *B(H)*
(tweaking the curve does not work !)

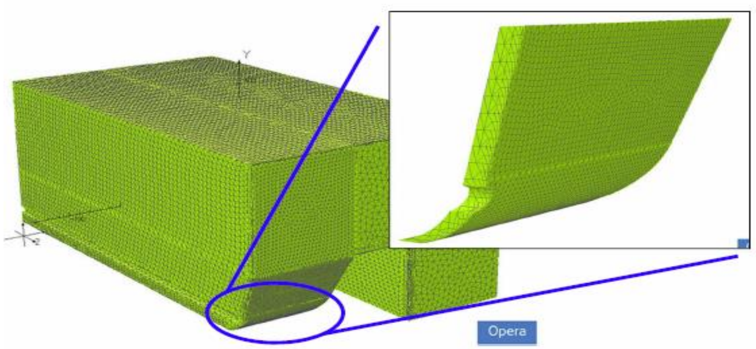




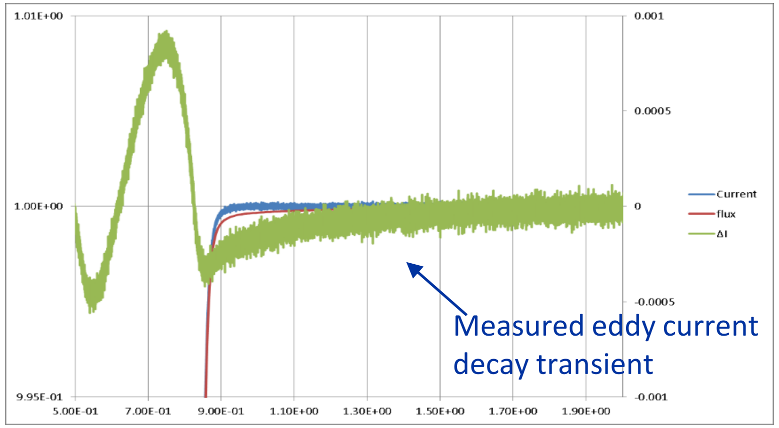
Courtesy A. Newborough, R Chritin

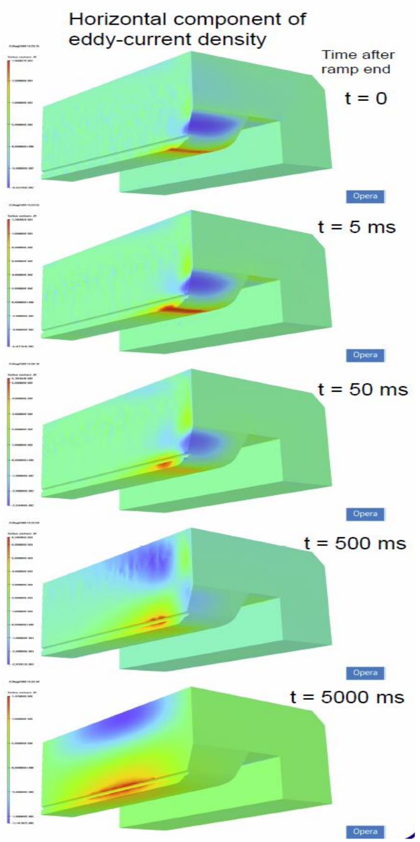
FE $\leftrightarrow$ measurements: MedAustron Bending Dipole

- modelling issues more complex for dynamic phenomena (eddy currents)
- medical hadrontherapy machine requirements: fast energy changes, high accuracy and stability
- settling time: measured **200 ± 20 ms**, computed **150 ms**



~ 2M elements, 80 h running time



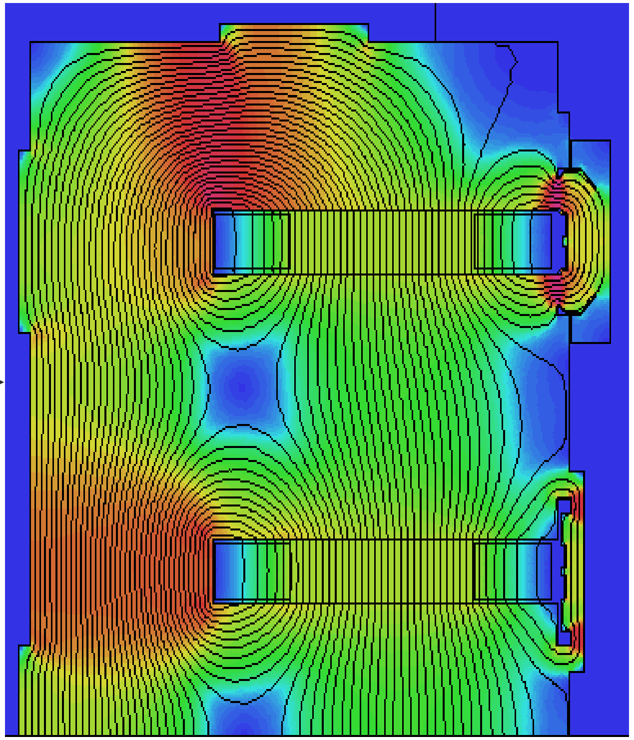


G. Golluccio, A. Beaumont *et al.*, Overview of the magnetic measurements status for the MedAustron project, IMMW18
T. Zickler *et al.*, Design and Optimization of the MedAustron Synchrotron Main Dipoles, IPAC11

Traditional (complementary) approach

Requirements

design



manufacturing

(mech. tolerances, material properties, excitation history …)

4-ring bending dipole
CERN PS Booster

testing

Performance characterization

feedback

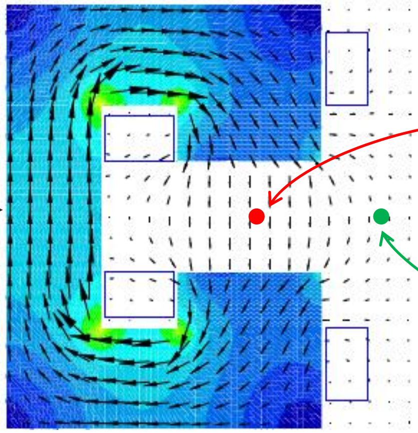
Hybrid approach: data-driven Digital Twins

- Combine optimally the strong points of computer modeling and measurement
- Formalize and automate the comparison of computed and measured quantities
- Example: CERN ISOLDE 90° High Resolution Separator

Melvin Liebsch
this CAS

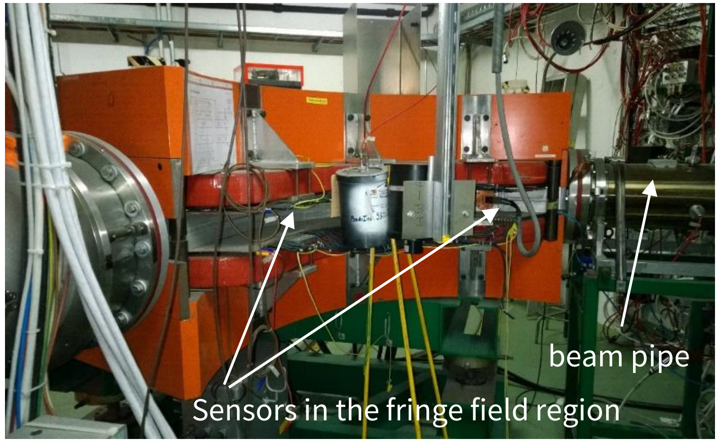
Parameters *p*
(g, μ_r)

Bayesian priors
(g_0, σ_g), $(\mu_{r0}, \sigma_{\mu r})$



… and we want the field here …

… but we can only measure B_j there !

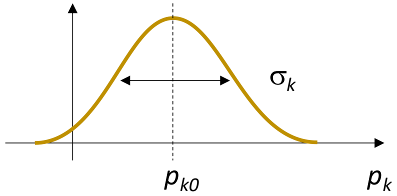


Measurement uncertainty
σ_B

computation

measurement

$$\chi^2(\boldsymbol{p}) = \sum_k \frac{(p_k - p_{k0})^2}{\sigma_k^2} + \sum_j \frac{\left(B_j^{\mathrm{comp}}(\boldsymbol{p}) - B_j^{\mathrm{meas}}\right)^2}{\sigma_B^2}$$



We don't know well these …

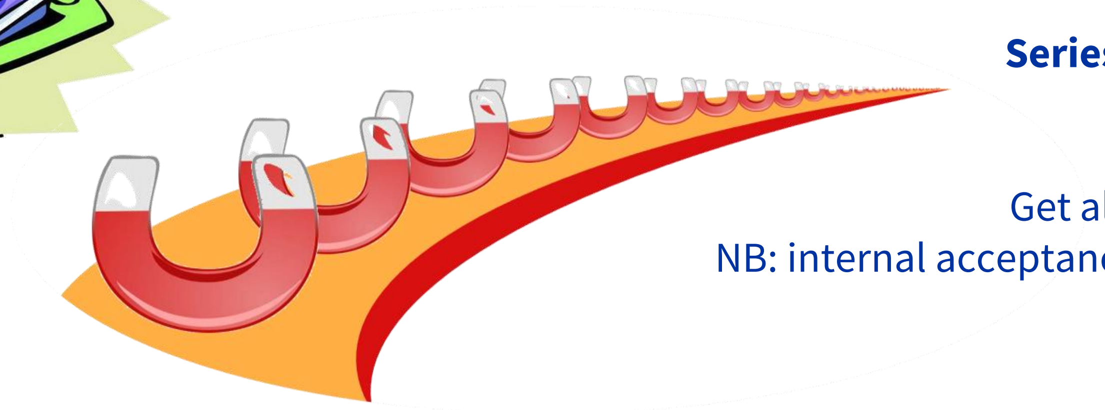
Minimization w.r.t. ***p***

Indications for experiment design

$$J = \frac{\partial B_j^{\mathrm{comp}}}{\partial \boldsymbol{p}}, \qquad H = \frac{\partial^2 B_j^{\mathrm{comp}}}{\partial \boldsymbol{p}^2}$$

When to measure

Design phase: test material samples for permeability, coercivity etc…; test prototypes or models (scaled down versions) to validate computer simulations and specific design choices (e.g. chamfers, shims, details …)



Series acceptance tests: monitor production quality, tooling wear …
trap errors as early as possible to steer manufacturing.
Build up statistics to reduce tests and minimize total cost.
Get all data required for fiducialization (installation) and beam optics.
NB: internal acceptance criteria might be flexible, but contractual acceptance is binary

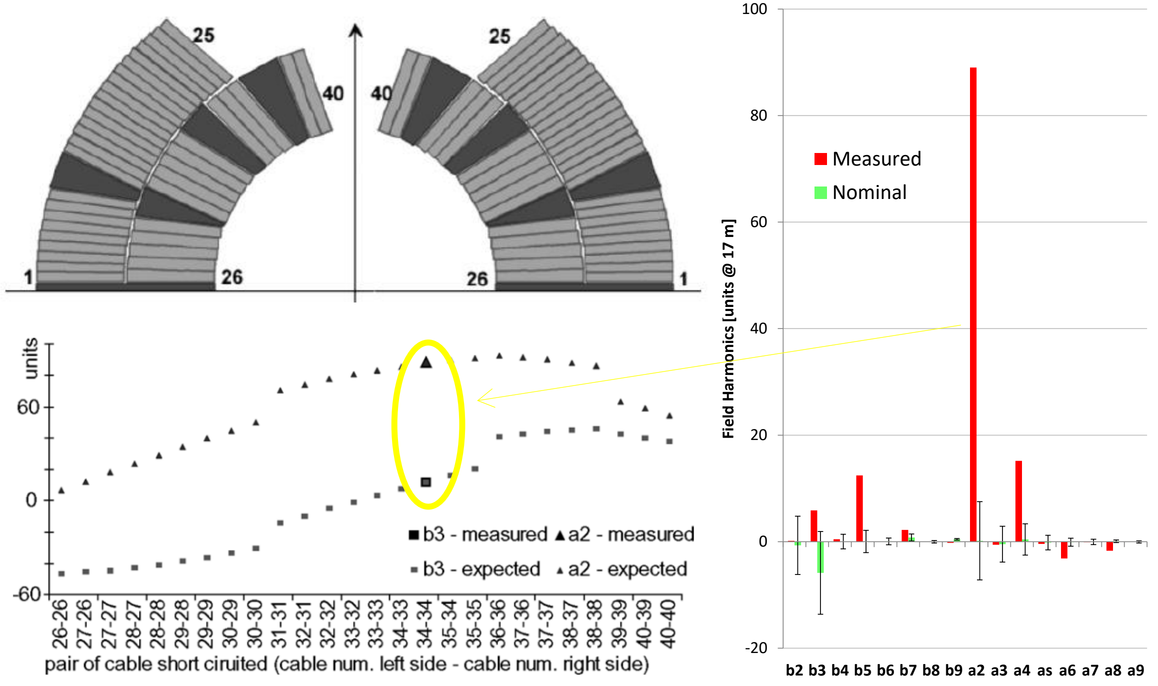
Prototypes/pre-series: test field quality to verify the respect of mechanical tolerances (inverse problem), give feedback to designer and manufacturing firms. Carry out a fully detailed magnetic characterization (often no time to do so during series tests)

throughout lifetime: characterize magnets after repairs, or to allow use in different ways than originally intended

different trade-offs between accuracy and resources at different times

From field quality to mechanical issues: LHC dipoles

- **Electrical QA**: fault detection of LHC dipoles via harmonic field measurements: 18 shorts, 4 assembly faults, 14 non-conform components detected
- **Inter-turn short circuits**: harmonics due to all possible cases pre-computed and matched to measurements



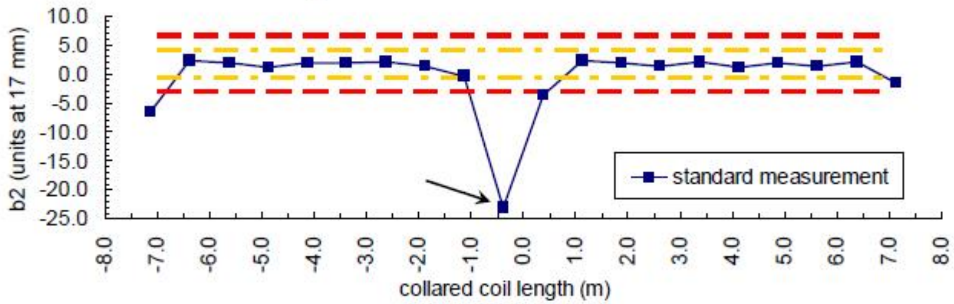


Fig. 6. Measurement of multipole b2 along the magnet axis with alarm limits at 4σ and 8σ (dashed lines) in the straight part.

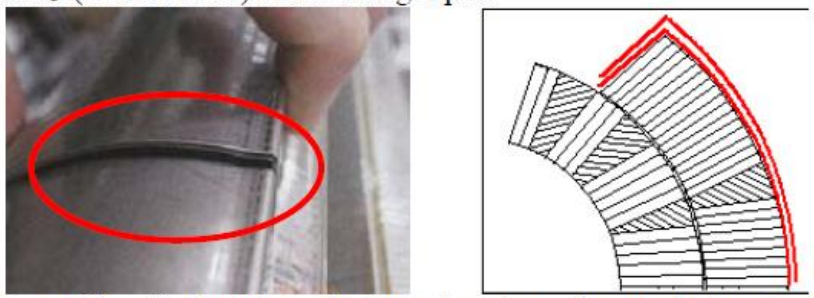


Fig. 7. Assembly of a double coil protection sheet observed after de-collaring (left); illustration of the defect (right).

Insulation thickness QA

anomaly identification threshold set at $\frac{|b_n - \langle b_n \rangle|}{\sigma(b_n)} \geq 4$

→ one false alarm expected in a population of 1232

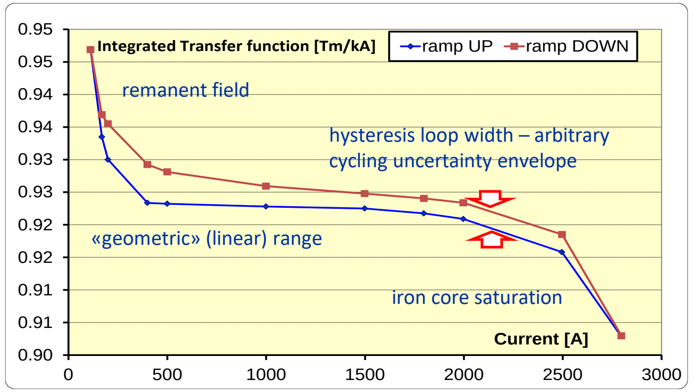
Measurement goals

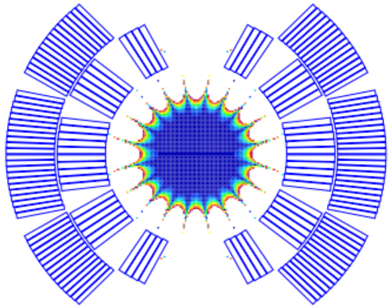
What to measure

What do we want to measure

Beam-line magnets

- Integrated field/gradient (all that matters for the beam in 1st approx.)
- Average magnetic axis i.e. locus where $\boldsymbol{B} \equiv 0$
  (NB: for lenses n≥2, there are (n-1) solutions!. Can be defined for dipoles e.g. B_{10}=0 in LHC)
- Average field direction (= phase of main harmonic)
  errors couple betatron oscillations in the horizontal and vertical planes
- Field quality i.e. harmonics *or* field/gradient uniformity



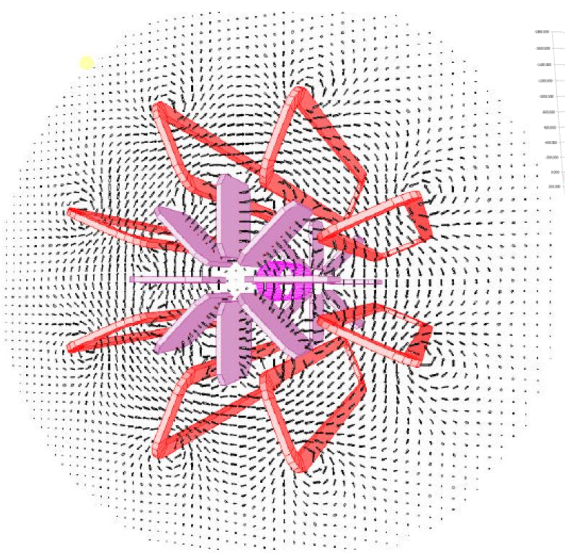


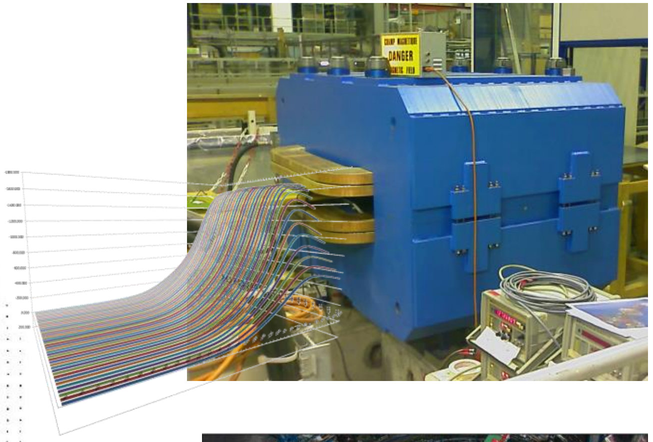
All of the above = *f*(*I*) for non-linear control, local = *f*(*z*) for manufacturing quality, high-order beam dynamics

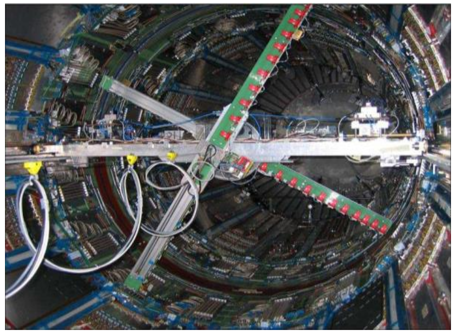
Harmonic expansion: full information in 2D case only (integral/central field)

Experimental magnets

- 3D vector field maps needed by tracking codes for experimental magnets, spectrometers and certain beam line magnets (short, strongly bent, large β swings)
- “universal” but costly representation



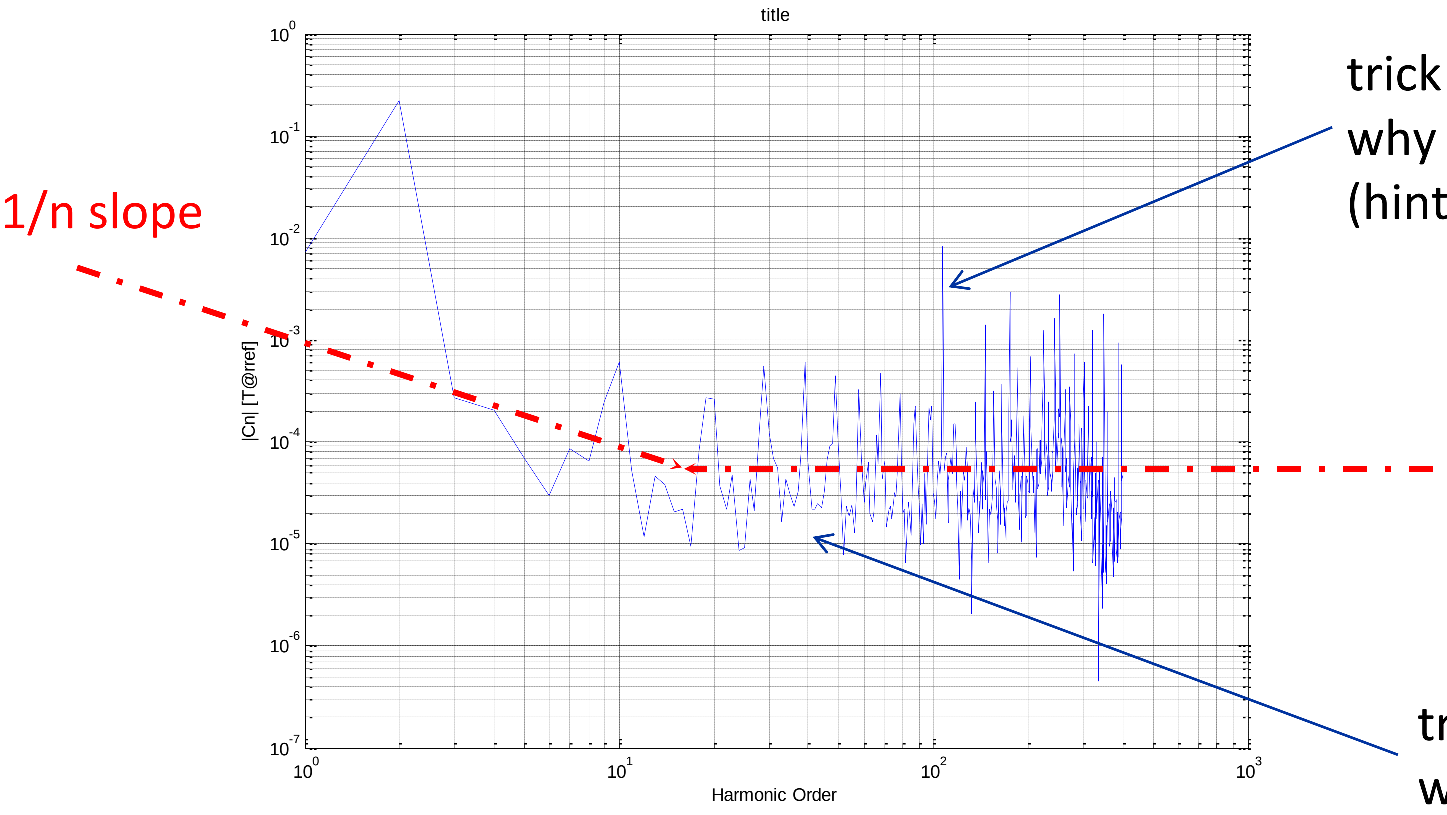




ATLAS field map & mapper

Which harmonics to measure

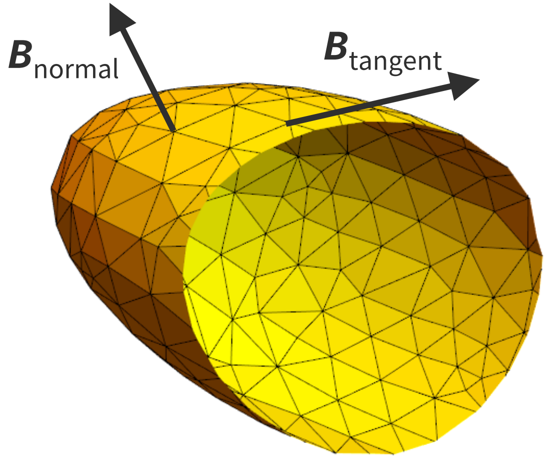
- **Typical needs** for the beam: C1 (orbit), C2 (focusing, betatron tuning and coupling), C3 (chromaticity correction); C3~C6 (in LHC) for lattice correction; B4 for Landau damping; B3/B4 for excitation of instabilities e.g. extraction
- **Higher orders** may be needed to compute correctly field or gradient uniformity if the quality is bad
- **Where do we stop ?**: Cauchy estimate for holomorphic functions helps: $\mathbf{|C_n| < const. \cdot r_{ref}^{-n}}$



Example 1024-harmonic spectrum of a CLIC DBQ

FEM/BEM processing of boundary data

- Back to Maxwell: Unique solution of Laplace equation in a source-free domain + Neumann (normal field) or Dirichlet (tangential) boundary conditions (if: simply connected domain, smooth boundary)
- **Reduce problem dimensionality**: scan only the boundary, fit a BEM model to get interior values at arbitrary resolution (no volume mesh required !)



- Back to Maxwell: Maximum Principle: extremal values always on the boundary → metrological advantage of interpolating the interior

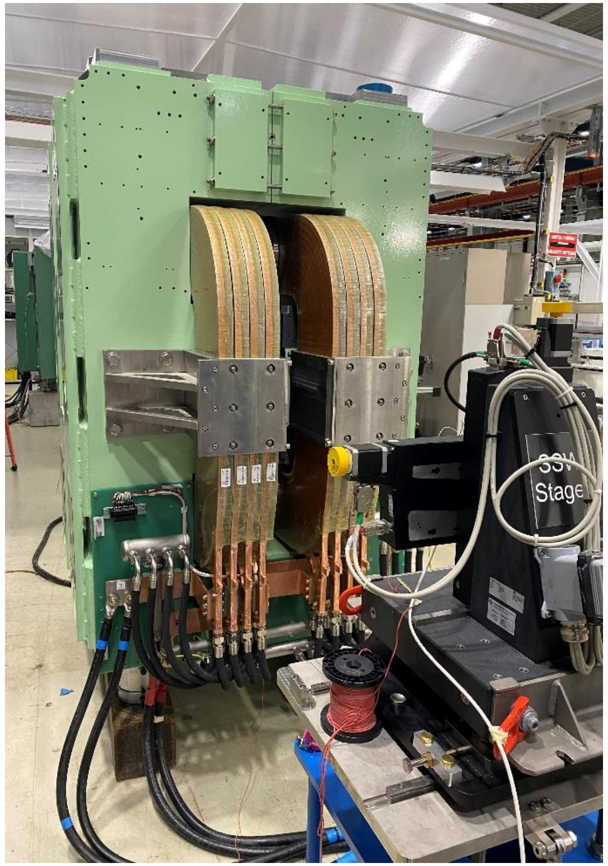
2D example, cylindrical symmetry:

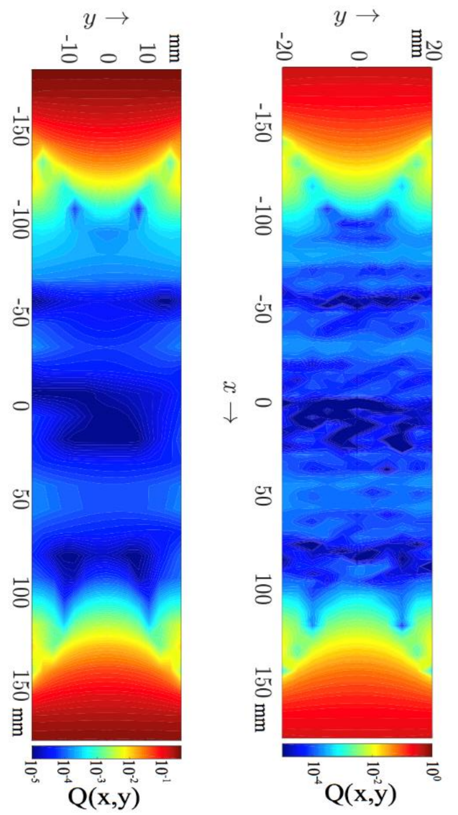
$$B_n = C_n \left(\frac{z}{r_0}\right)^{n-1} \Rightarrow$$

Measurement uncertainty at $|z|=r_0$

$$\sigma(B_n(z)) = \sigma_B \left(\frac{z}{r_0}\right)^{n-1}$$

- R&D challenges: automated meshing and setup of BEM model; hybrid boundary conditions (e.g. integrals from flux measurements); incorporate arbitrarily scattered/multiple/interior points





BEM reconstruction

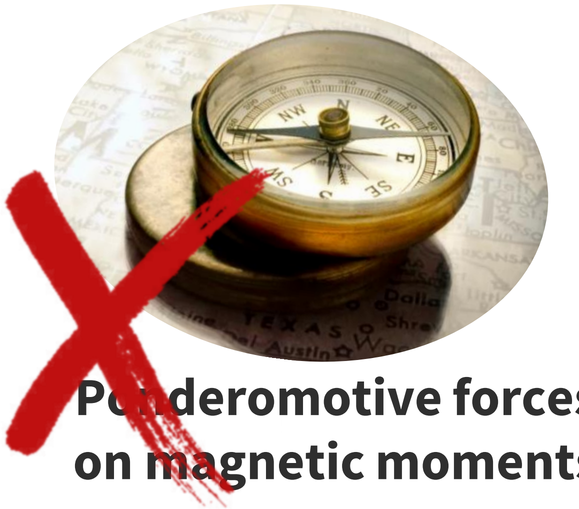
Full grid

Credit: M. Liebsch

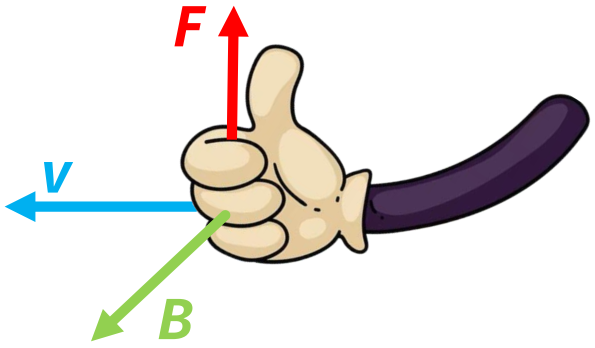
Measurement methods

How to measure

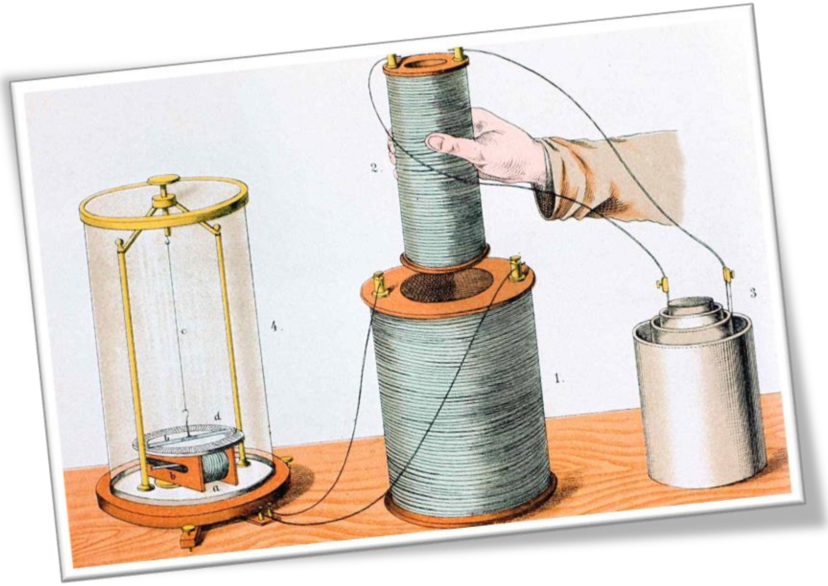
Magnetic field sensing principles



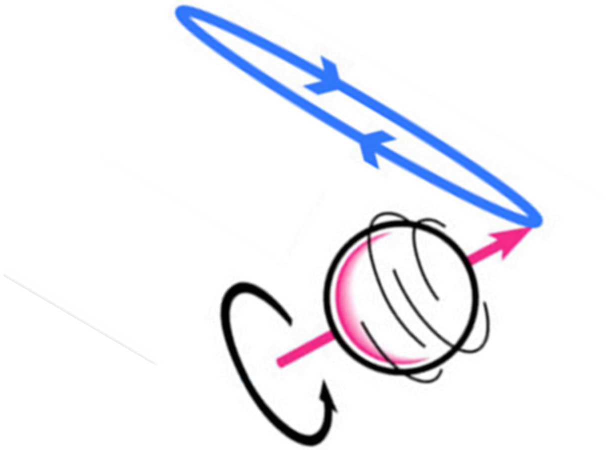
Ponderomotive forces on magnetic moments



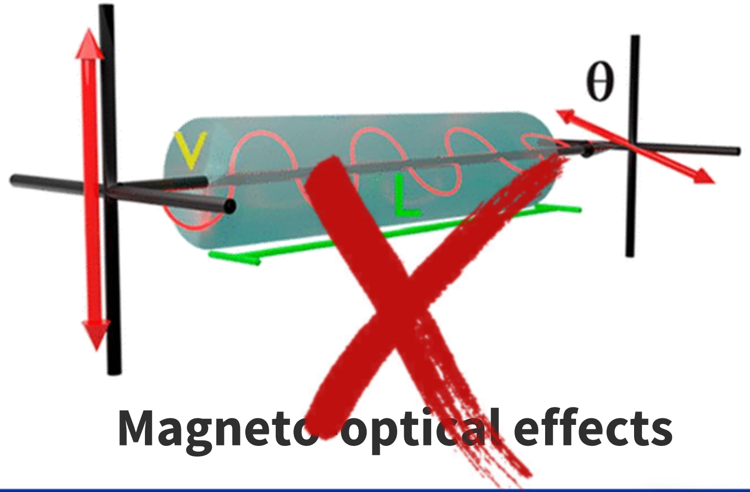
Ponderomotive forces on currents (Lorentz force)



Electromotive force (Faraday's induction law)



Magnetic resonance methods



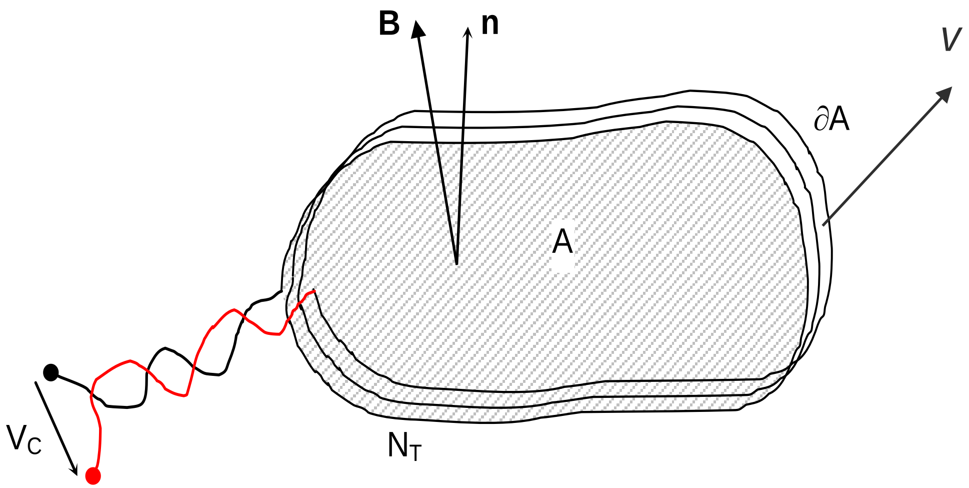
Magneto-optical effects

Induction sensors

$$-V_c = \frac{\partial \Phi}{\partial t} = N_\mathrm{t} \frac{d}{dt} \iint_{\mathcal{A}} \boldsymbol{B} \cdot \boldsymbol{n} \, dA =$$

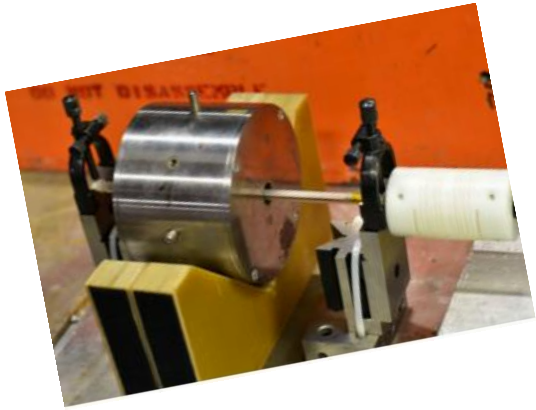
$$N_\mathrm{t} \iint_{\mathcal{A}} \frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{n} \, dA + N_\mathrm{t} \oint_{\partial \mathcal{A}} \boldsymbol{v} \times \boldsymbol{B} \, d\ell =$$

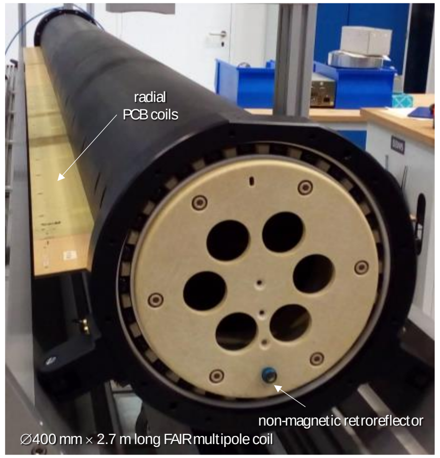
- $A_c \frac{\partial B}{\partial t}$ — Fixed coil in a time-changing field (fluxgate, AC stretched wire)
- $A_c B \frac{\partial \theta}{\partial t}$ — Coil rotating in a DC field
- $A_c \frac{\partial B}{\partial x} \frac{\partial x}{\partial t}$ — Coil translating in a DC field with a gradient
- $l_w B \frac{\partial x}{\partial t}$ — Wire translating in a DC field (classical DC stretched wire)

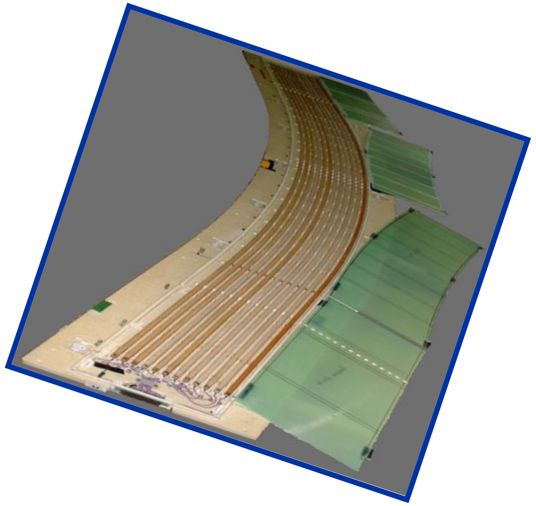


Assume: infinitely thin, geometrically coincident windings

Induction coils







Lucio Fiscarelli this CAS

- Workhorse of CERN instrumentation park; **most accurate and cost-effective method**
  ideal for integral field; coil “bucking” → immunity from mechanical imperfections; harmonic expansion contains all results of interest
- Sensitive to the **flux;** the field must be derived
- Inherently **linear** (but : finite acquisition Z_{in} , frequency response)
- Fixed-coil S/N improve with dΦ/dt; bandwidth of rotating coil is limited by mechanics
- **Voltage integration** → noise reduced proportionally to frequency; integrator drift is major issue
  Direct post-processing of the voltage also possible, relies on accurate speed control and measurement
- Length, width, number of turns, rotation radius … must be **adapted to the magnet.** Difficult for bent dipoles.
  no off-the-shelf solutions; industrial PCB designs possible; optimization requires in-house winding and high-quality electromechanical components

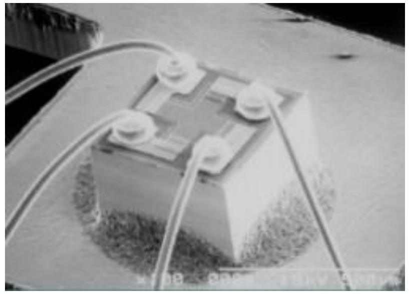
Lorentz force-based sensors

$$\frac{\boldsymbol{F}}{q} = \boldsymbol{E} + \boldsymbol{B} \times \boldsymbol{v}$$

$$\frac{\partial \boldsymbol{F}}{\partial l} = \boldsymbol{B} \times \boldsymbol{I}$$ Vibrating wire

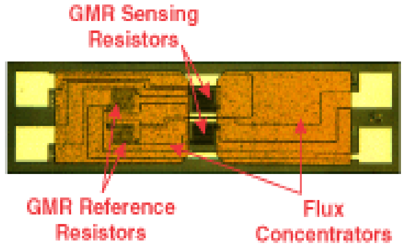
Carlo Petrone
Hands-on

$$V_H = k_H B\ I$$ Hall effect sensor



Melvin Liebsch
this CAS

$$\frac{\Delta\rho}{\rho} = kB^2$$ Magneto-resistor
(non-linear, hardly used in our field)

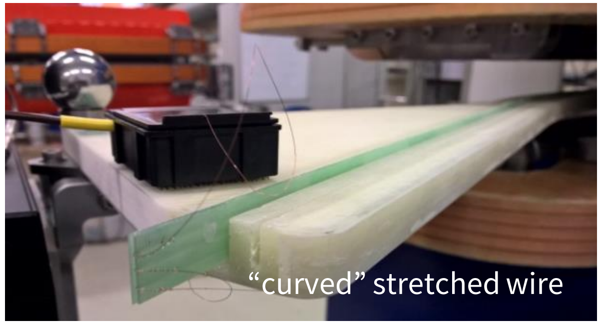


- Point-like probes directly sensitive to a **single field component** (with higher order correction terms)
- Mechanical/galvanomagnetic phenomena → stronger non-linearity → **repeated calibration**

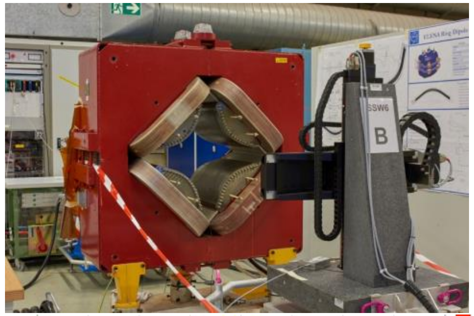
Stretched-wire and Hall probe systems

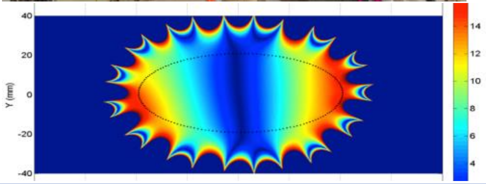
Stretched-wire systems

- adapt to any (straight) gap size and length
- inductive (DC/AC/pulsed-mode magnet, translating/rotating wire) and ponderomotive implementations (AC vibrating wire)
- longitudinal center + pitch and yaw (counter-directional wire movements)
- sub-µm axis localization (vibrating mode at resonance)
- reference for integrated field strength, axis and direction in high-field magnets (1-turn, variable-geometry coil)


“curved” stretched wire

Credit: Carlo Petrone



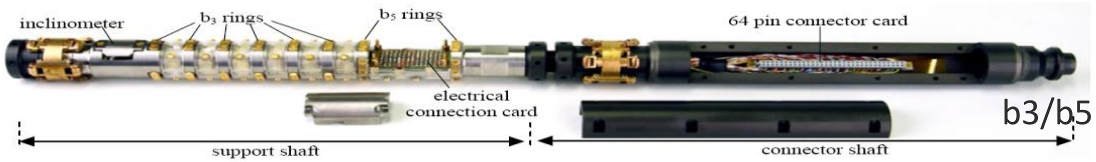


rotating wire in elliptical gap

Hall probe systems

- Commercially available
- << 1 mm→ high-resolution field maps
- kHz bandwidth possible with appropriate design
- non-linear sensitivity to in- and out-of-plane ***B***
- offset and sensitivity drift with T → frequent recalibration/thermal compensation/stabilization


3D/3D hall probe mapper

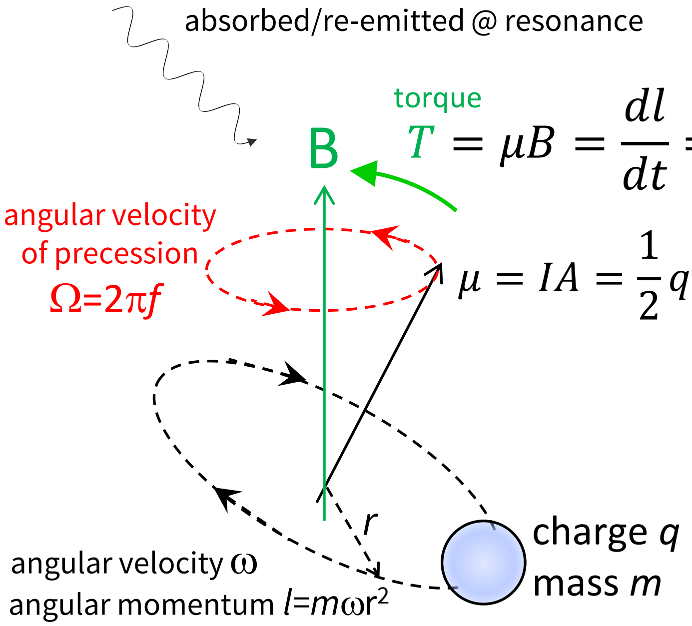


b3/b5 probe

Magnetic resonance sensors

Melvin Liebsch this CAS

- Resonant absorption/re-emission of RF waves in a sample within a <u>uniform field</u>
  (field gradient spreads the resonance, impact depends on sample size and shape)
- Transducer sensitive to ||***B***||
- Proton γ depends on fundamental constants → **metrological standard**
  (impact of temperature, shape, orientation and chemical nature of the sample: < 10^{-6} for NMR, 10^{-4}~10^{-3} for EPR)`



torque

$$T = \mu B = \frac{dl}{dt} = \Omega l$$

$$\mu = IA = \frac{1}{2} q \omega r^2 = \frac{q}{2m} l$$

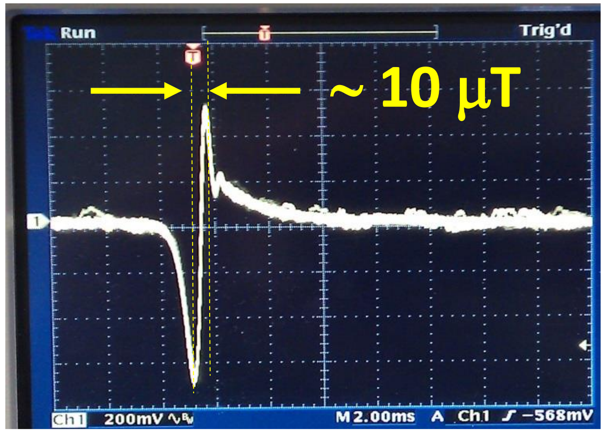
γ gyromagnetic ratio

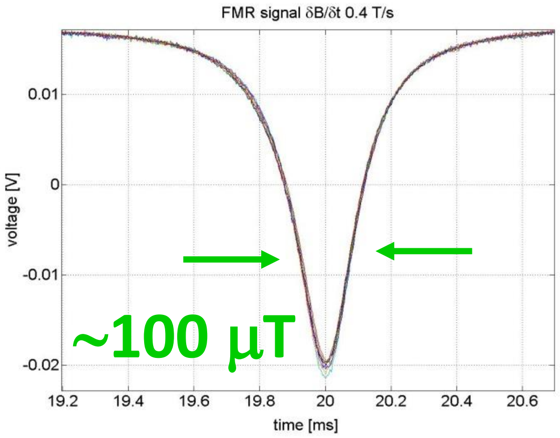
$$\frac{f}{B} = g \frac{q}{4\pi m} = \begin{cases} H^+ \text{(proton)} & 42.577 \\ \text{free electron} & 28\,015.737 \end{cases} \quad MHz\,T^{-1}$$

g factor
g_e=2.002, g_p=5.6

NMR (Nuclear Magnetic Resonance)
γ known to better than 1 ppm

EPR/ESR (Electron Paramagnetic/Spin Resonance),
FMR (FerriMagnetic Resonance)
γ depends upon chemical composition, T, axis …





Assume: classical treatment; B ⊥ μ

Method selection criteria

"hard" criteria

1) **Compatibility** with field level/gradients (could not work at all!)
2) **Transverse size** (it must fit, and should reach as wide as possible)
   - local ripple close to the pole may degrade the accuracy of harmonics
   - extrapolation further from the axis can be applied, at a cost

"soft" criteria

3) **Bandwidth**
   - sensitivity will drop above cutoff frequency
   - additional errors e.g. from inductive cable loops
4) **Longitudinal size**
   - the integral can be computed by scanning longitudinally (time-consuming)
   - de-convolution of longitudinal scans done with a longer probe → low-pass filter, noise
5) **Accuracy**
   - uncertainty can be reduced by repetition, changing orientation, cross-checks …
6) **Result format**: harmonics vs. map (1D/2D/3D)
   - can be translated into one another, with caveats
7) **Practical considerations**:
   cost, measurement time, output signal level, cabling length, commensurate size of sensors and magnet, availability of trained personnel …

Measurement accuracy

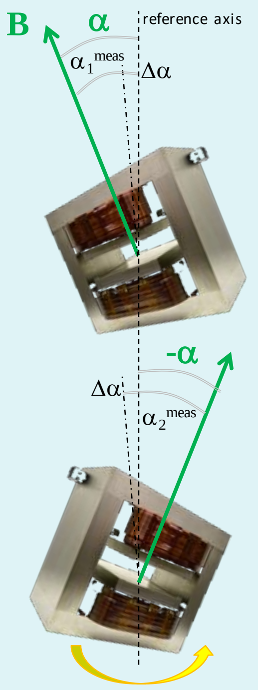
The **standard uncertainty** of an instrument is a function of the **operating conditions** (field range/frequency, gradient, temperature …)

Further improvement possible, based on the **time and effort** taken

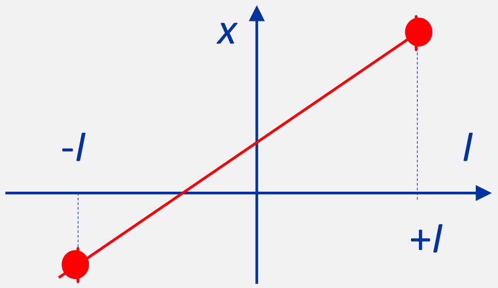
- Repeat to get rid of random errors: $\sigma(\langle x \rangle) = \frac{\sigma(x)}{\sqrt{n}}$
  diminishing returns for large *n*
- Oversample
  time domain: MHz sample rate even for sub-kHz bandwidth → much improved voltage integration drift correction:
  angular domain: oversample the flux when rotating a coil to reduce aliasing error in FFT
- Flip and repeat to estimate and subtract systematic errors
  either the magnet or the instrument, as is more practical
- Reverse polarity to recover ambient or intrinsic offsets
  e.g. remanent field
- Redundant takes will always give you confidence !

$$\begin{cases} \alpha_1^{meas} = +\alpha - \Delta\alpha \\ \alpha_2^{meas} = -\alpha - \Delta\alpha \end{cases}$$

$$\begin{cases} \alpha = \frac{\alpha_1^{meas} - \alpha_2^{meas}}{2} \\ \Delta\alpha = -\frac{\alpha_1^{meas} + \alpha_2^{meas}}{2} \end{cases}$$



Field angle calibration

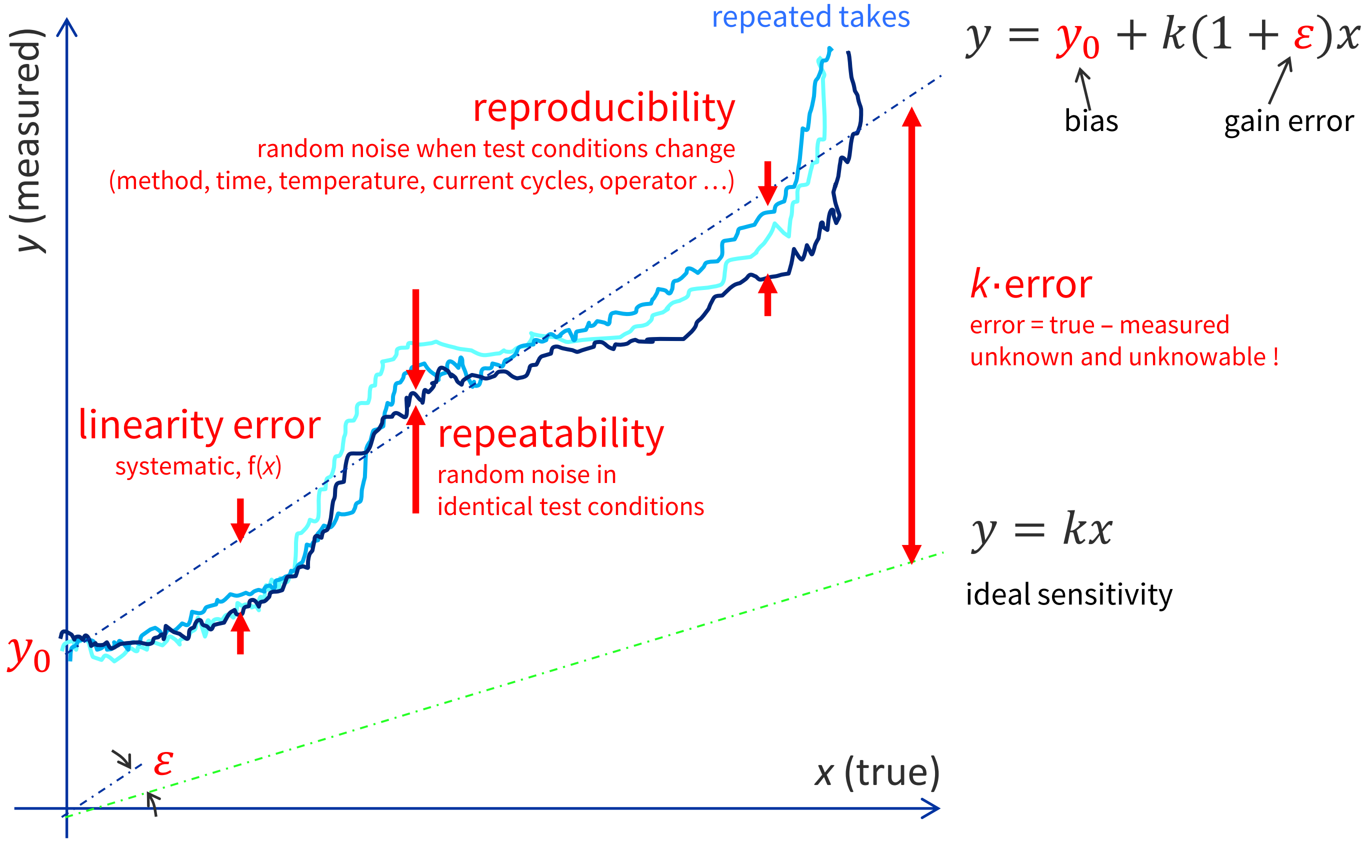


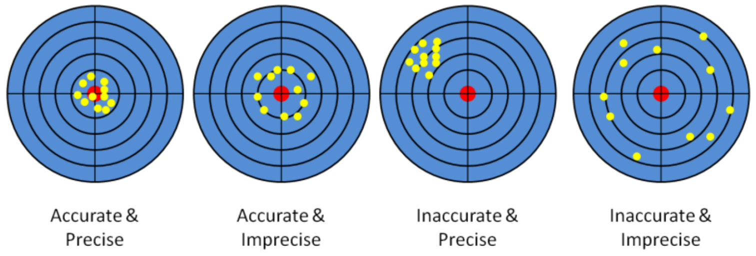
$$x(0) = \frac{x(I) + x(-I)}{2}$$

Metrological aspects

Units, standards and calibration

Measurement terminology





qualitative

- **precision:** measure of dispersion
- **accuracy:** closeness of agreement between measure (average) and true value
- **resolution:** smallest detectable change

rigorous

- **estimated standard deviation σ** measure of dispersion ("*noise*") obtained from a finite sample : $\sigma^2 = \frac{1}{n}\sum(x-\bar{x})^2$

  NB: normally distributed sources of dispersion add quadratically

- **standard measurement uncertainty *u*** dispersion of an averaged measurement $u(\bar{x}) = \pm\frac{\sigma}{\sqrt{n}}$

calibration = finding the transfer function of an instrument by comparison with another, of known accuracy

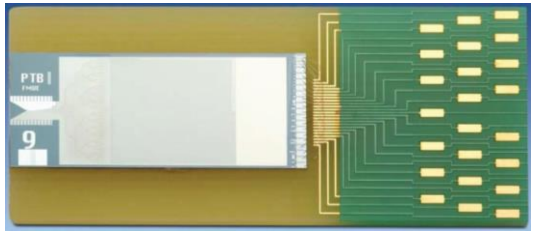
Reference: ISO GUM (*Guide to the Expression of Uncertainty in Measurement)*

Measurement reference standards

Measurement standard: artifact, device or process that embodies the definition of a physical measurement unit

- **primary standard**: lowest-uncertainty, at the top of the reference chain, said to *realize* a unit
- **secondary standard**: calibrated w.r.t. a primary, said to *represent* a unit; easier dissemination, higher uncertainty
- **working standard**: certified standard on user's premises, traceable to a primary standard

Fun fact: international standards and procedures are defined by consensus (democratic process !)



10^{-7}

10^{-6}

10^{-3}

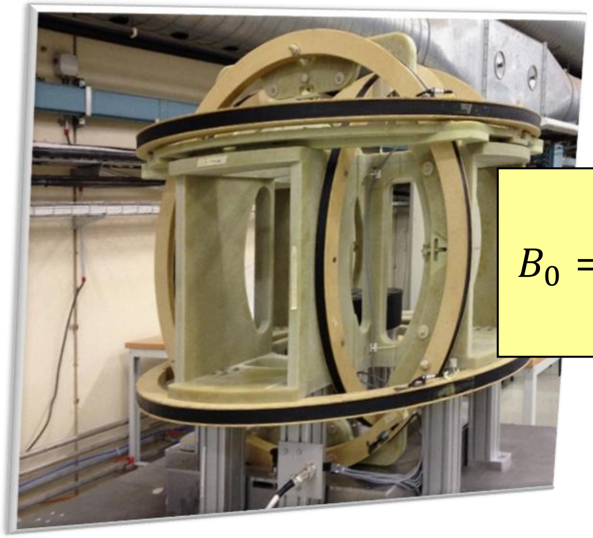
Calibration chain traceability

- unbroken chain of comparisons, each with a stated uncertainty, from a measurement to a primary standard
- Fundamental concept to certify (also legally) a measurement
- based on formal "good practices": maintaining systematic records, databases, documented procedures …

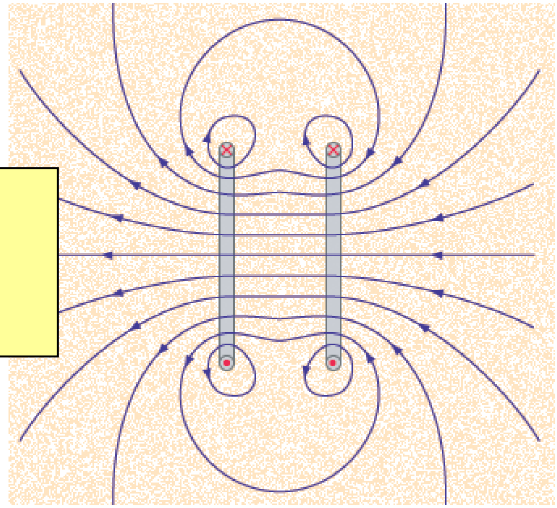
Magnetic reference standards

Realization of the Tesla

1. **calculable source** (iron-free solenoid, Helmholtz coils …)
   severe limits to accuracy and field level (winding geometry, Lorentz forces, heating)



$$B_0 = \sqrt[3]{\frac{4}{5}}\frac{\mu_0 n I}{R}$$

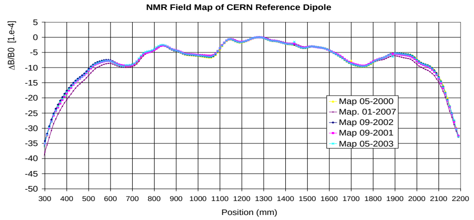


2. any other sufficiently uniform and stable source, measured with a **reference NMR sample**
   e.g. sphere of pure $H_2 0$ at 25°C + γ_p from CODATA

BIPM, SI Brochure, *Mise en pratique* for the definition of the ampere and other electric units in the SI, 2019

Realization of the Weber

1. **realize the Tesla** over a known area



2. realize the **Volt·second**
   standard lab equipment

3. realize a multiple of the flux quantum ***Φ_0=h/2e***
   severe limits to field level

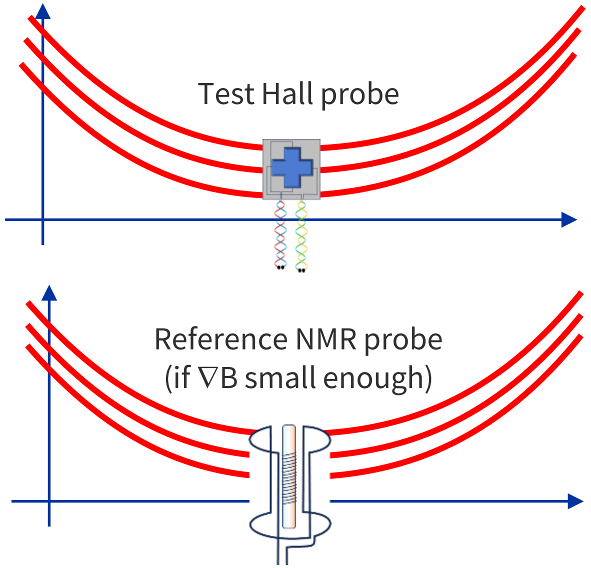
- partial embodiment of unit's definition
- no standard for other quantities of interest (field components, direction, axis…)

Calibration methods – point-like probes

- General method: compare to reference measurement at multiple field levels over the full range
- Many different schemes possible:

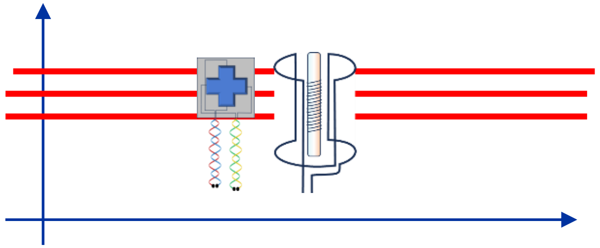
Sequential

- Measure at same position, multiple field levels
- Positioning accuracy, field reproducibility
- Relaxed field uniformity



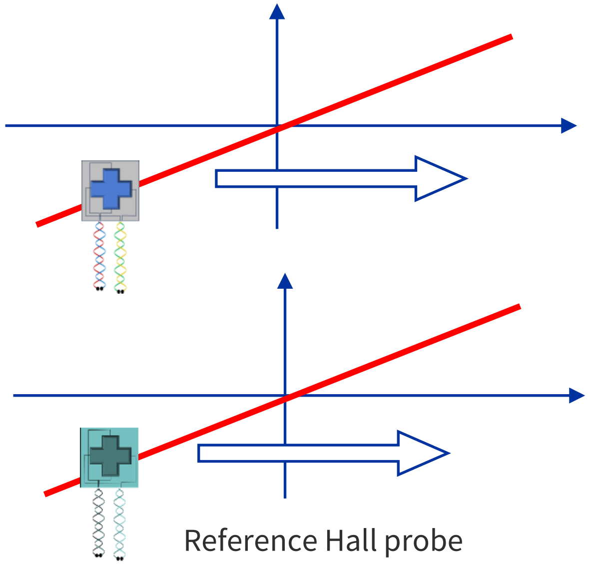
Synchronous

- Measure test and reference probes next to each other at multiple field levels
- Field uniformity and stability
- Relaxed position accuracy



In-situ

- Measure sequentially a field map (1D ,2D or 3D) covering the whole B range
- Match via least-squares
- Positioning accuracy, field reproducibility



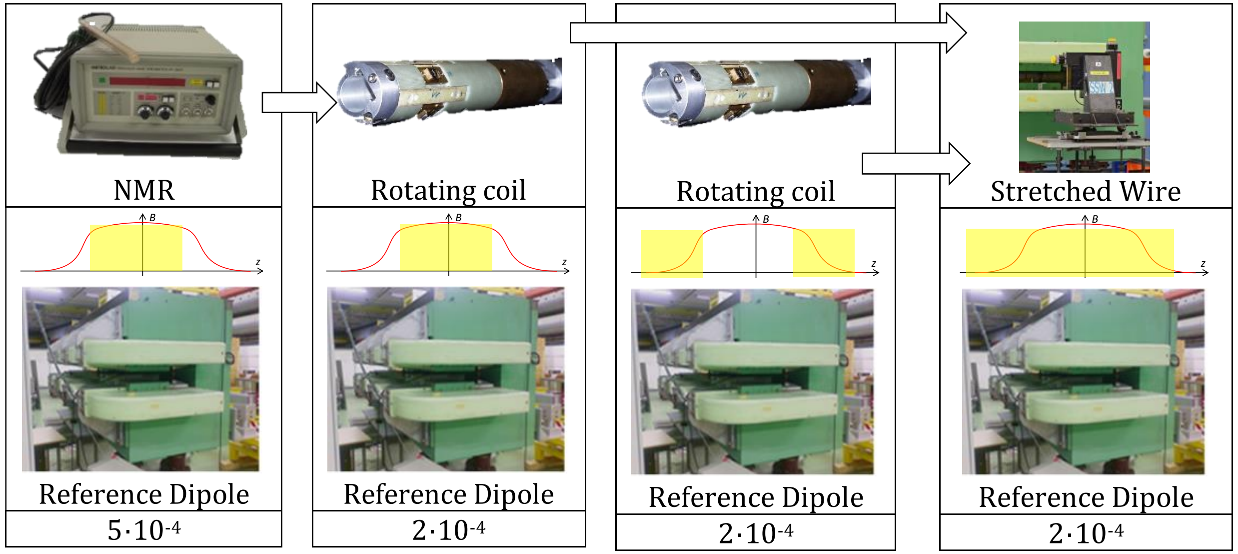
Calibration chain for a Stretched Wire system

Comparative calibration

- No metrological reference for integral/average field
- Use NMR where ∇B sufficiently low
- Complement with additional probe (coil, Hall)

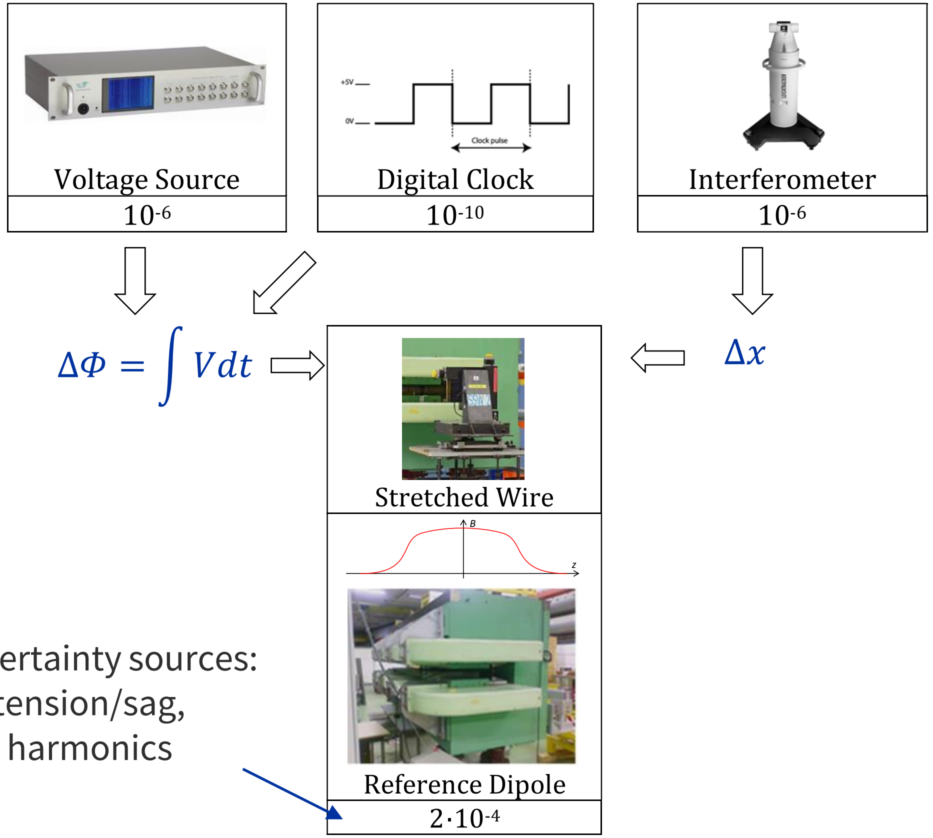
$$\int B dl = k \frac{\Delta\Phi}{\Delta x}$$

$k \approx 1$
calibration factor



Ab-initio calibration

- Uncertainty propagation thru subsystems
- SSW takes on functional reference role

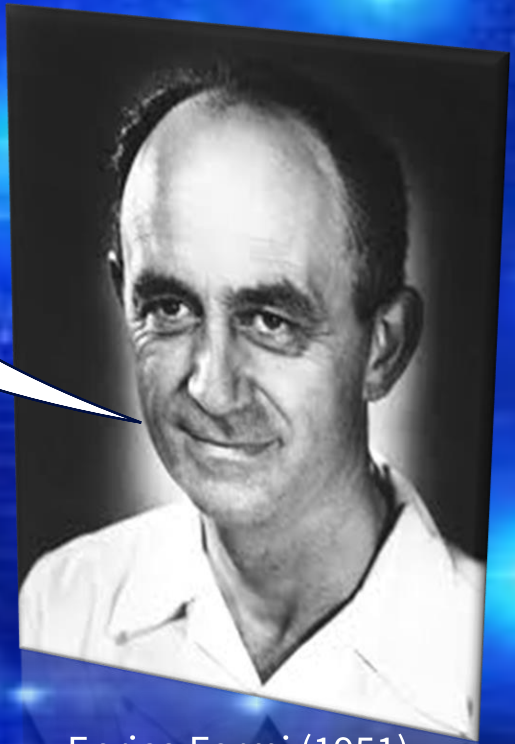


Other uncertainty sources:
wire tension/sag,
field harmonics

Conclusions

- **Magnetic measurements** are an essential part of the magnet qualification process, complementing computer simulations and beam-based measurements
- No single instrument or technique can cover all requirements
- Multiple instruments are complementary; overlap provides estimation of absolute uncertainty
- Very little available commercially – be prepared for R&D
- Precise mechanics, quality materials and sturdy benches are the foundation of good instruments
- Stability comes from **mass**: **ponderal**, **thermal** and **electrical**
- Many trade offs: bandwidth for sensitivity, time for accuracy, time for spatial resolution …
- Different strategies to reduce **errors**:
  - optimize critical parameters at design time e.g. rotating coil radius
  - repeat to average away random errors
  - compare instruments and exploit symmetries to calibrate systematic errors

"Before I came here, I was confused on this topic. Having heard this talk I am still confused, but on a higher level."

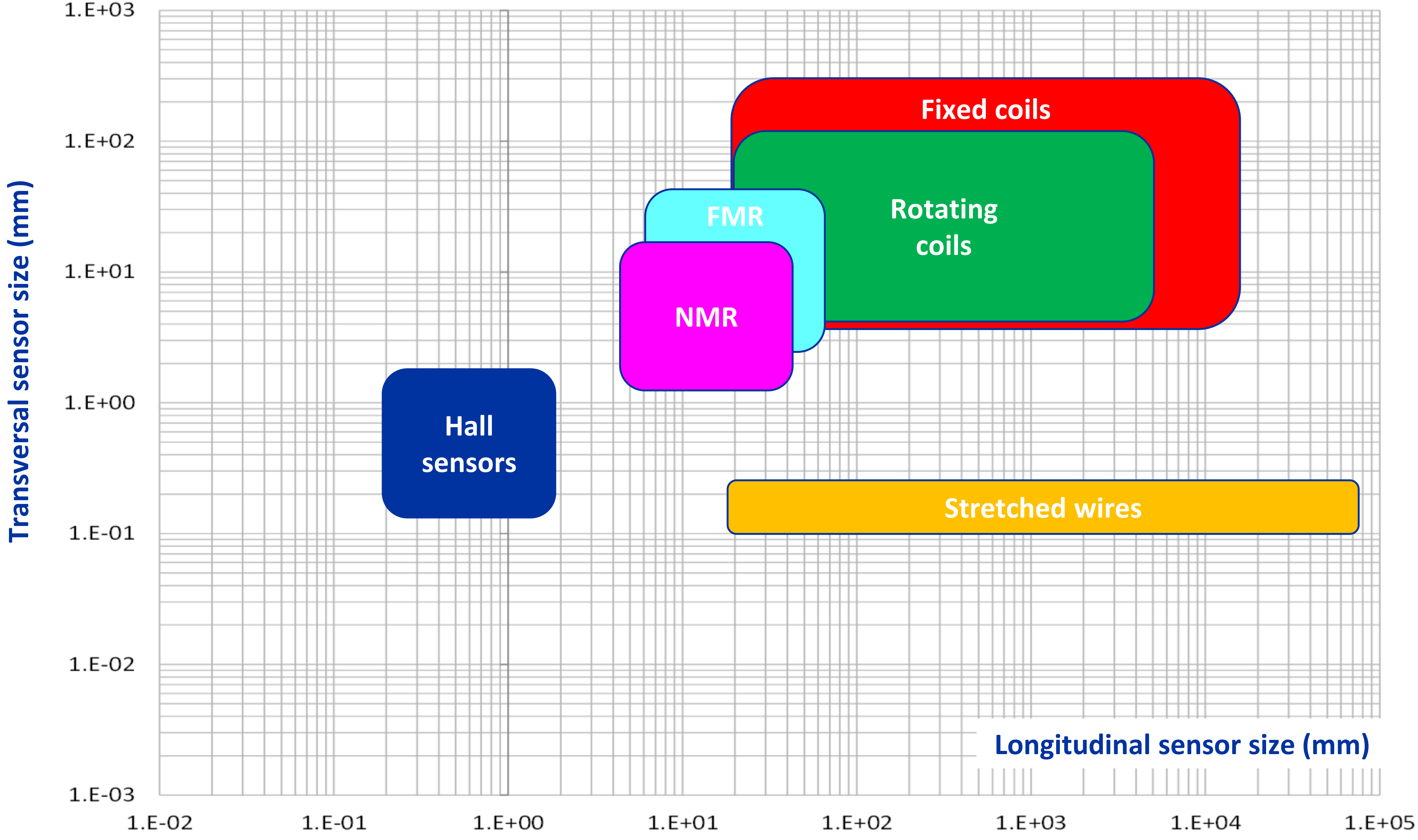


Enrico Fermi (1951)

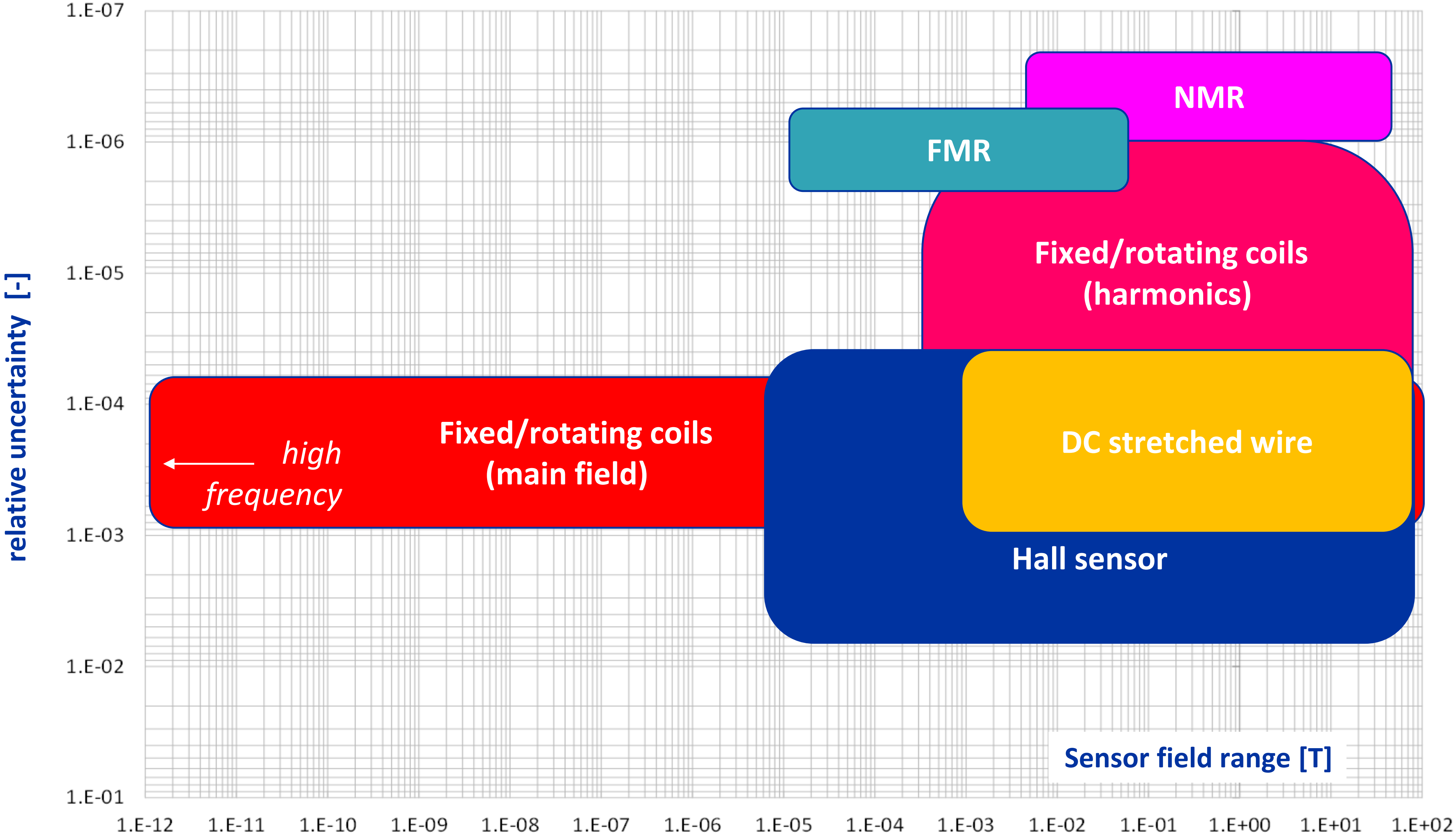
Thank you for your attention

Additional slides

Typical transversal vs. longitudinal size



Typical accuracy vs. field range



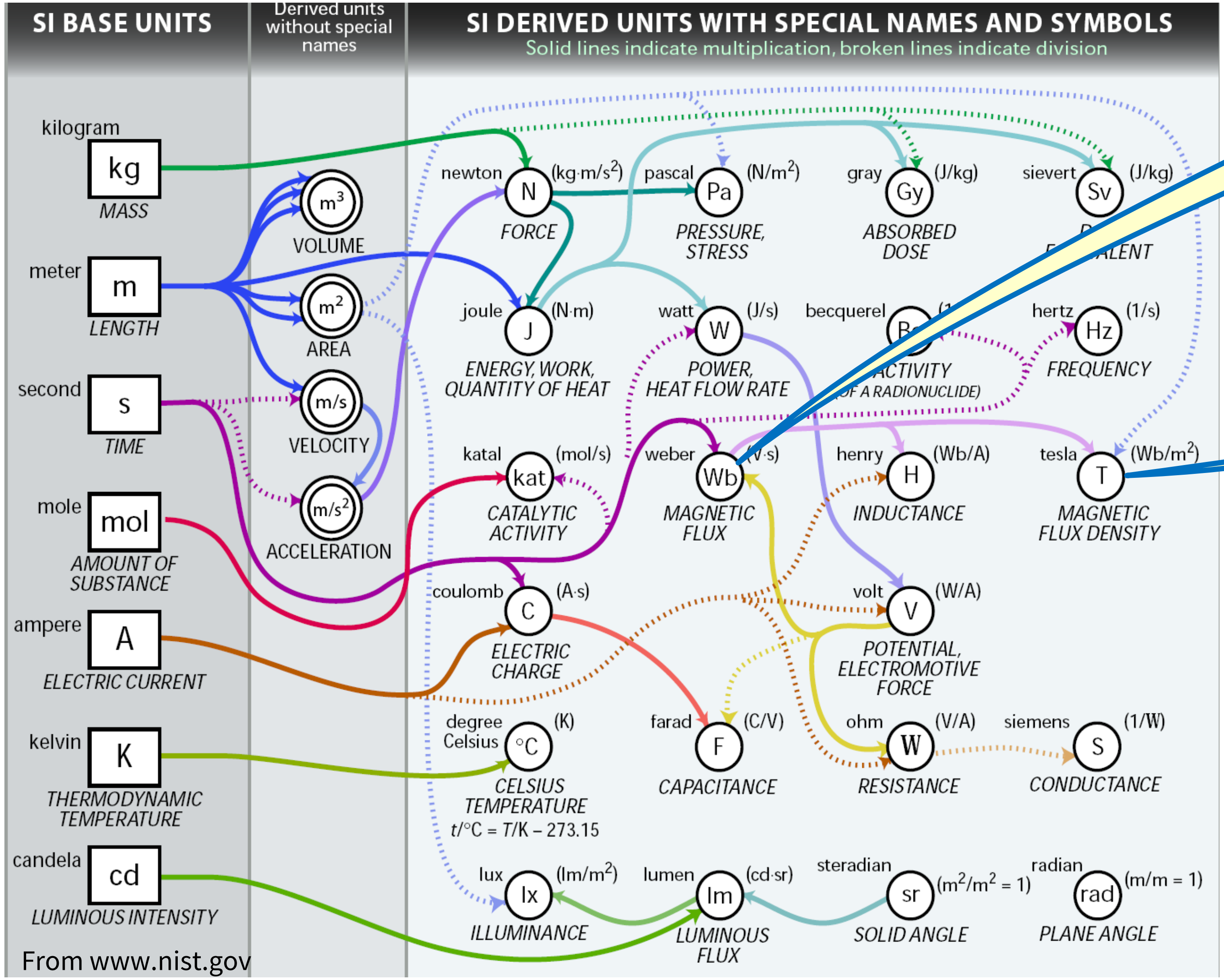
SI (Système International)

$[\Phi] = [M]^1\ [L]^2\ [T]^{-2}\ [I]^{-1}$

$Wb = Vs = kg\ m^2/As^2 = J/A = T\ m^2$

$[B] = [M]^1\ [L]^0\ [T]^{-2}\ [I]^{-1}$

$T = Wb/m^2 = kg/As^2 = N/Am = Vs/m^2$



From www.nist.gov

Both are derived units depending in a complex way upon base units
→ direct *realization* is difficult !

Voltage	**[V] =**	$[M]^1\ [L]^2\ [T]^{-3}\ [I]^{-1}$	= W/A = Tm²/s
Capacity	**[C] =**	$[M]^{-1}\ [L]^{-2}\ [T]^4\ [I]^2$	= F = C/V
Resistance	**[R] =**	$[M]^1\ [L]^2\ [T]^{-3}\ [I]^{-2}$	= Ω =V/A
Resistivity	**[ρ] =**	$[M]^1\ [L]^3\ [T]^{-3}\ [I]^{-2}$	= Ωm=Vm/A
Magnetic Induction	**[B] =**	$[M]^1\ [L]^0\ [T]^{-2}\ [I]^{-1}$	= T = kg/As²= N/Am = Wb/m²= Vs/m²
Magnetic Flux	**[Φ] =**	$[M]^1\ [L]^2\ [T]^{-2}\ [I]^{-1}$	= Wb = T m² = J/A = Vs
Inductance	**[L] =**	$[M]^1\ [L]^2\ [T]^{-2}\ [I]^{-2}$	= H = Wb/A= J/A²
Magnetic Field	**[H] =**	$[M]^0\ [L]^{-1}\ [T]^0\ [I]^1$	= A/m
Volume Magnetization	**[M] =**	$[M]^0\ [L]^{-1}\ [T]^0\ [I]^1$	= A/m
Mass Magnetization	**[M’] =**	$[M]^{-1}\ [L]^2\ [T]^0\ [I]^1$	= Am/kg
Permeability	**[μ] =**	$[M]^1\ [L]^1\ [T]^{-2}\ [I]^{-2}$	= Tm/A = N/A² = H/m = Wb / Am
Magnetic Moment	**[m] =**	$[M]^1\ [L]^1\ [T]^{-2}\ [I]^{-2}$	= J/T=A m²

CGS (Gaussian) unit system

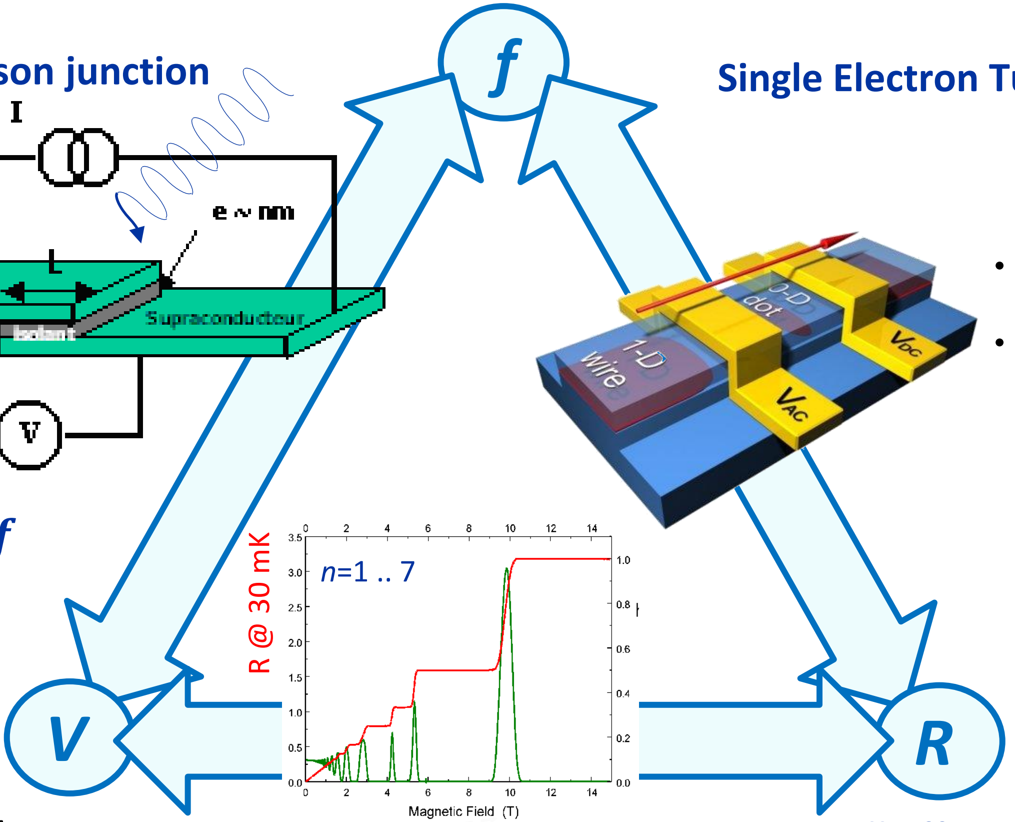
- based on the concept of magnetic poles, still used in certain domains (e.g. geomagnetism)
- identical numerical values for *B* and *H* in air
- assume unit dimensionless μ_0 and ε_0 and include 4π factors to simplify calculations (convenient before computers)
- SI based on the concept of current sources, prevalent in the literature after 1980s
- CGS and SI units may have different dimensions and can't always be considered as multiples of each other

Quantity	CGS	→	SI
magnetic moment, *m*	**emu** (erg G^{-1}) [Am^2]	10^{-3}	[Am^2]
volume magnetization, *M*	(emu cm^{-3}) [Am^{-1}]	10^3	[Am^{-1}]
magnetic field, *H*	**Oersted** [Am^{-1}]	$\frac{10^3}{4\pi} \approx 79.6$	[Am^{-1}]
magnetic induction, *B*	**Gauss** (G)[kg $A^{-1}s^{-2}$]	10^{-4}	**Tesla** (T)[kg $A^{-1}s^{-2}$]
volume permeability, μ_0	1 [-]	$4\pi 10^{-7}$ [H m^{-1}]	[Hm^{-1}]
volume susceptibility, χ	(emu cm^{-3} Oe^{-1}) [-]	4π [-]	[-]
Constitutive relation	**B=H+4πM**		**B**=μ_0(**H**+**M**)
Lorentz force	$\boldsymbol{F} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c}\right)$		$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

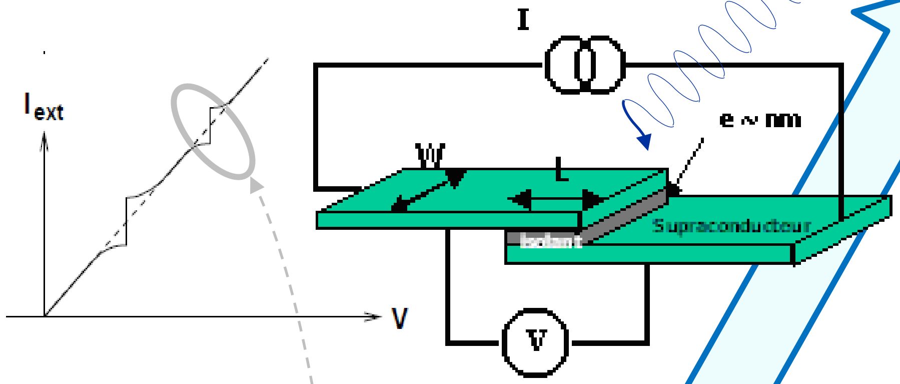
Jackson, *Classical Electrodynamics, Wiley*

Quantum Metrological Triangle

- Super- and semi-conducting cryogenic devices → **quantum standard *realization*** of ***V, I*** and ***R*** *(~1970s)*
- Metrological triangle + high precision frequency measurement → experimental verification of the consistency of e K_J and R_K



Superconducting Josephson junction



$$V = n\frac{h}{2e}f$$

$$K_J = \Phi_0^{-1} = 483\,598\ \text{GHz/V}$$

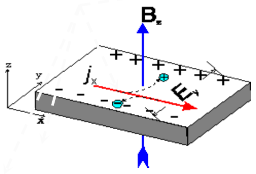
Φ_0

- Cooper pairs tunnel across an insulating barrier
- under microwave RF, the V/I curve will exhibit Shapiro steps at integer multiples of Φ_0
- arrays of 10^4 elements with f=100 GHz → 10 V reference
- reproducibility 10^{-7}, already used as a stable reference

Single Electron Tunneling pump

$$I = fe$$

- V_{RF} driven devices able to pump one electron/cycle due to Coulomb blockade
- Ex: tunneling across an insulating barrier @ 10T, f~GHz → nA currents with few 10^{-7} accuracy



Quantum Hall effect device

- observed in 2D e gas at low T, high Bv (e.g. the drain-source channel of a silicon MOSFET)
- resistance quantized in terms of h,e
- reproducibility ~10^{-10}.

$$V = n\frac{h}{e^2}I$$

$$R_K \approx 25\,812.807\ \Omega$$

Redefinition of SI base units (20.05.2018)

time (1960)

$$\begin{cases} \Delta\nu_{\mathrm{Cs}} \triangleq 9\,192\,631\,770\ \mathrm{Hz} \\ 1\ \mathrm{s} \triangleq \dfrac{1}{\Delta\nu_{\mathrm{Cs}}} \end{cases}$$

hyperfine transition of ^{133}Cs
uncertainty 10^{-17}

length (1983)

$$\begin{cases} c \triangleq 299\,792\,458\ \mathrm{m/s} \\ 1\ \mathrm{m} \triangleq c \cdot 1\ \mathrm{s} \end{cases}$$

Now flux can be realized from fundamental constants

mass (1901)

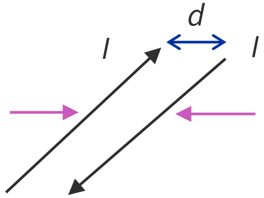
Pt-Ir prototype
poor stability 10^{-8}

→ **mass** (2019)

$$\begin{cases} h \triangleq 6.626\,070\,15 \cdot 10^{34}\ \mathrm{kg\ m^2/s} \\ 1\ \mathrm{kg} \triangleq \dfrac{h\Delta\nu_{\mathrm{Cs}}}{c^2} \end{cases}$$

$$\boldsymbol{\Phi_0 = \frac{h}{2e}}$$

current (1948)



→ **current** (2019)

$$\begin{cases} e \triangleq 1.602\,176\,634 \cdot 10^{-19} As \\ 1\ \mathrm{A} \triangleq e\Delta\nu_{\mathrm{Cs}} \end{cases}$$

poorest uncertainty
of electrical quantities
~10^{-7}

$$\frac{F}{\ell} = \mu_0 \frac{I^2}{d}$$

$$\mu_0 \triangleq 4\pi \cdot 10^{-7} H/m$$

μ_0 is now a measured constant

fine structure constant

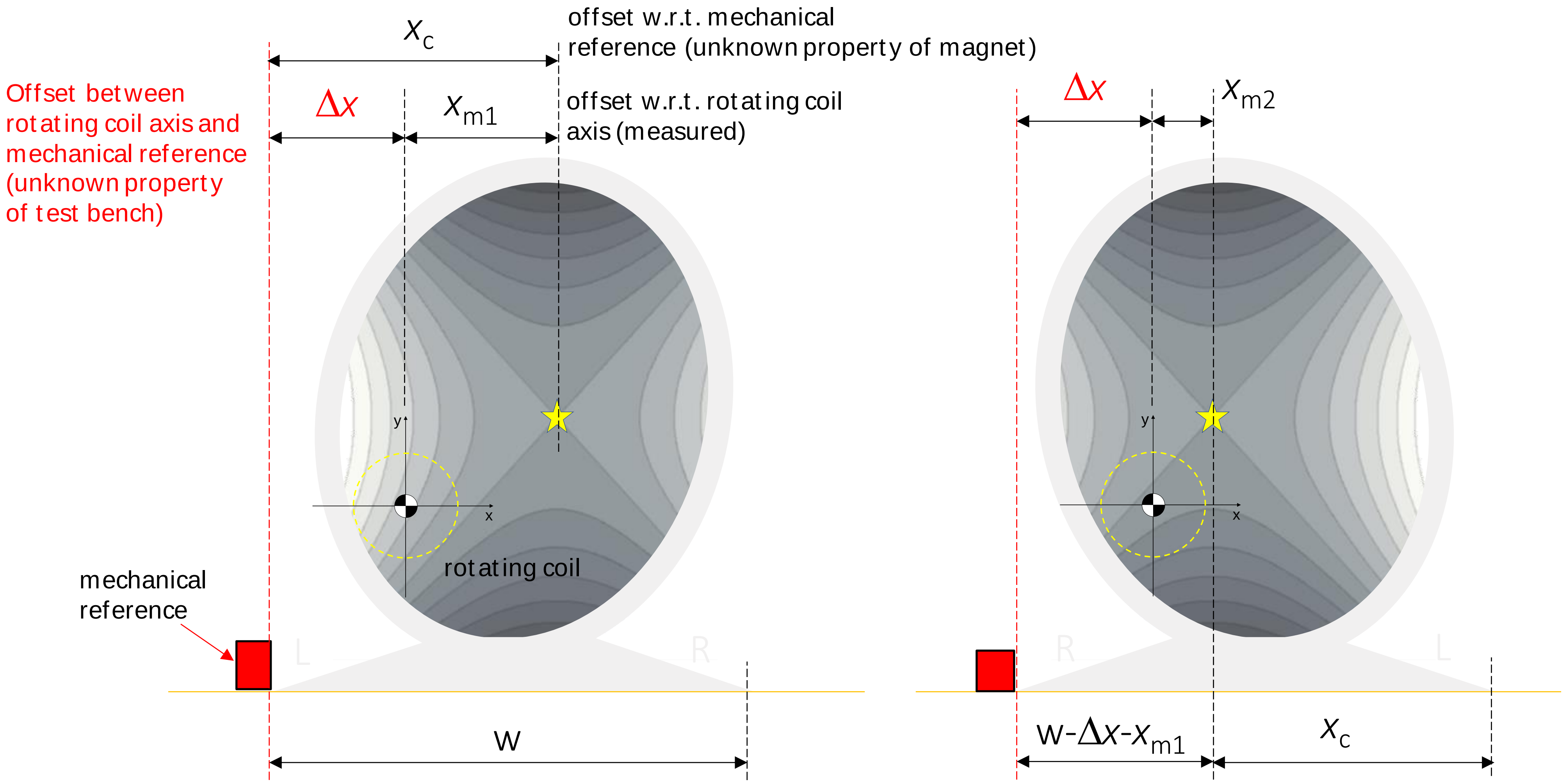
Standard uncertainty

$$\mu_0 = \alpha \frac{2h}{c^2} \approx 4\pi(1 + 2.0(2.3) \cdot 10^{-10}) \cdot 10^{-7}\ \frac{\mathrm{H}}{\mathrm{m}}$$

Fiducialization methods

Calibration of systematic geometric errors for magnetic axis

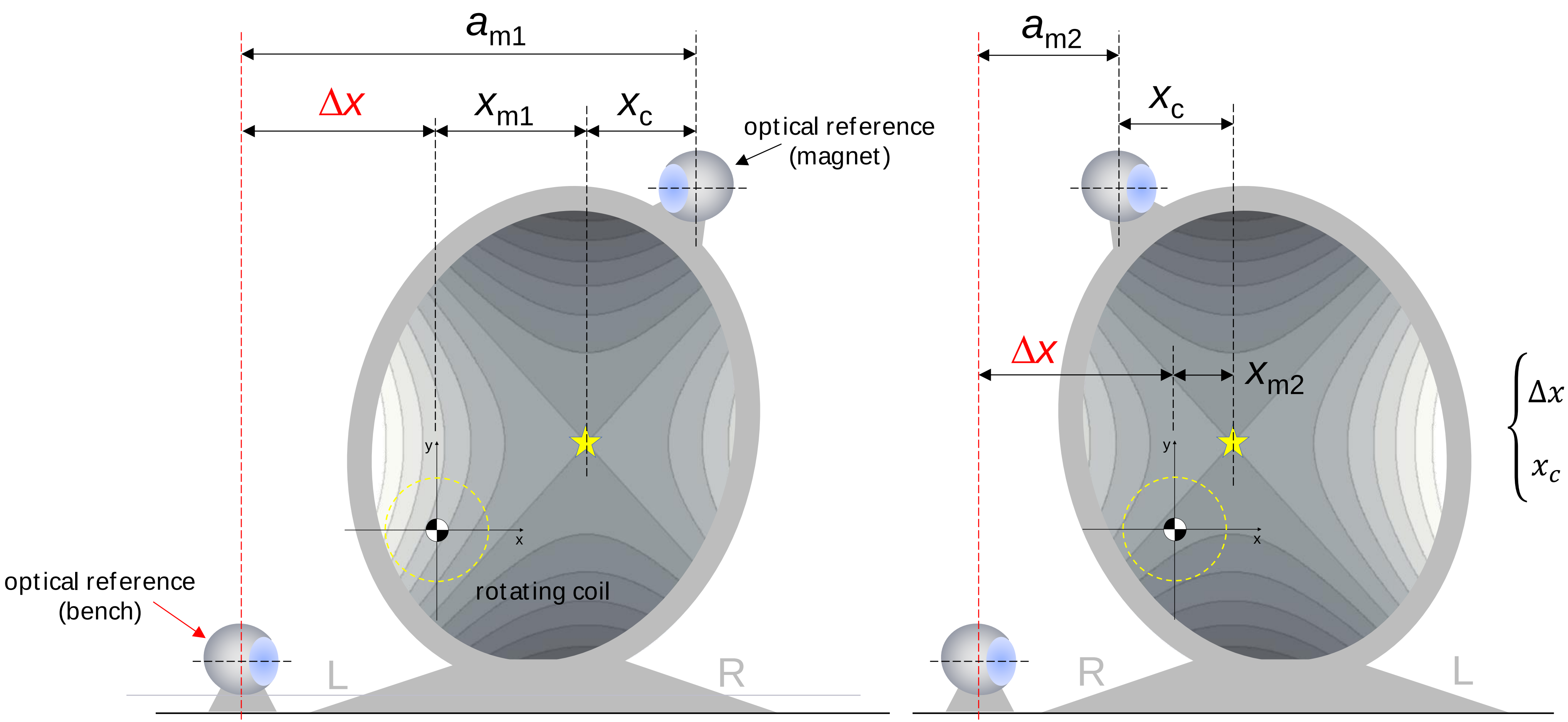
Horizontal magnetic axis – mechanical reference



$$\Delta x = \frac{w}{2} - \frac{x_{m1} + x_{m2}}{2}$$

$$x_c = \frac{w}{2} + \frac{x_{m2} - x_{m1}}{2}$$

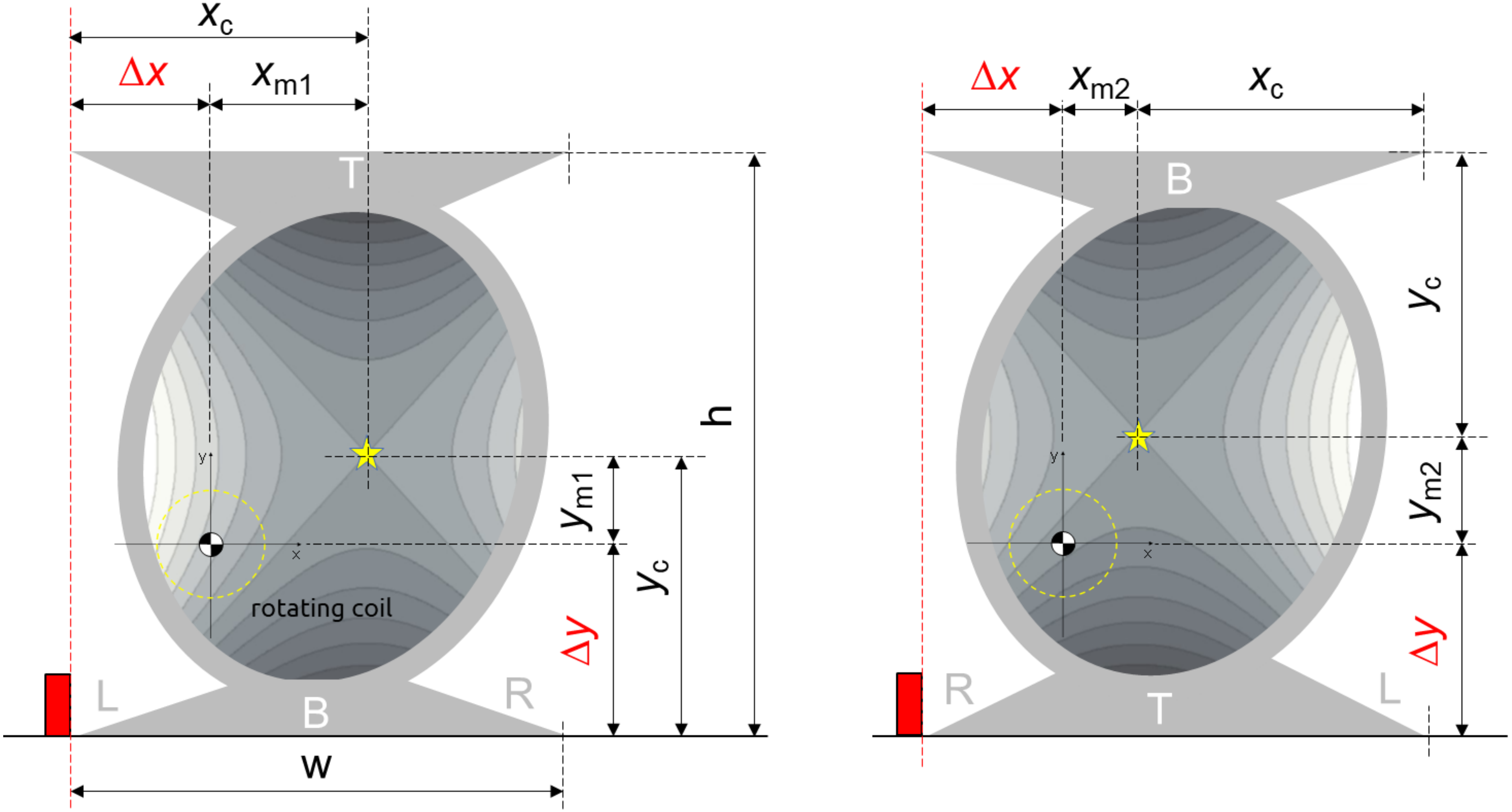
Horizontal magnetic axis– optical reference



$$\begin{cases} a_{m1} = \Delta x + x_{m1} + x_c \\ a_{m2} = \Delta x + x_{m2} - x_c \end{cases}$$

$$\begin{cases} \Delta x = \dfrac{a_{m1} + a_{m2}}{2} - \dfrac{x_{m1} + x_{m2}}{2} \\ x_c = \dfrac{a_{m1} - a_{m2}}{2} - \dfrac{x_{m1} - x_{m2}}{2} \end{cases}$$

Horizontal & vertical axis – mechanical references



$$\begin{cases} \Delta x = \dfrac{w}{2} - \dfrac{x_{m1} + x_{m2}}{2} \\ \Delta y = \dfrac{h}{2} - \dfrac{y_{m1} + y_{m2}}{2} \end{cases}$$

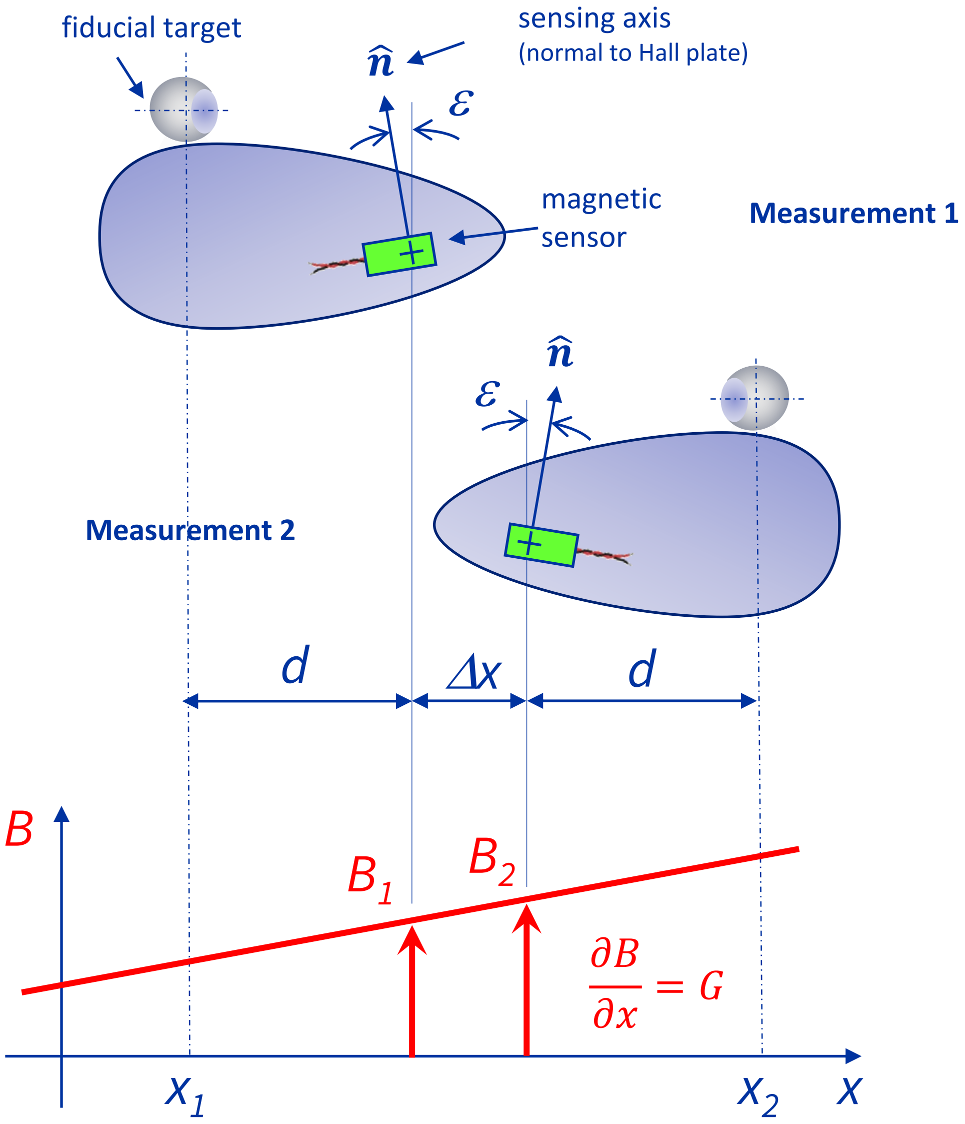
$$\begin{cases} x_c = \dfrac{w}{2} + \dfrac{x_{m2} - x_{m1}}{2} \\ y_c = \dfrac{h}{2} + \dfrac{y_{m2} - y_{m1}}{2} \end{cases}$$

Turn magnet by 180° around longitudinal axis
repeat magnetic measurement

Fiducialization methods

Calibration of systematic geometric errors for point-like probe assemblies

1) Turnaround in a linear gradient, B || sensing axis



- For Hall probe: field normal to Hall sensor, alignment error ε → negligible 1-cosε readout error
- For NMR: insensitive to direction
- Two measurements at 180° (arbitrary offset Δx) provide the distance sensor-fiducial

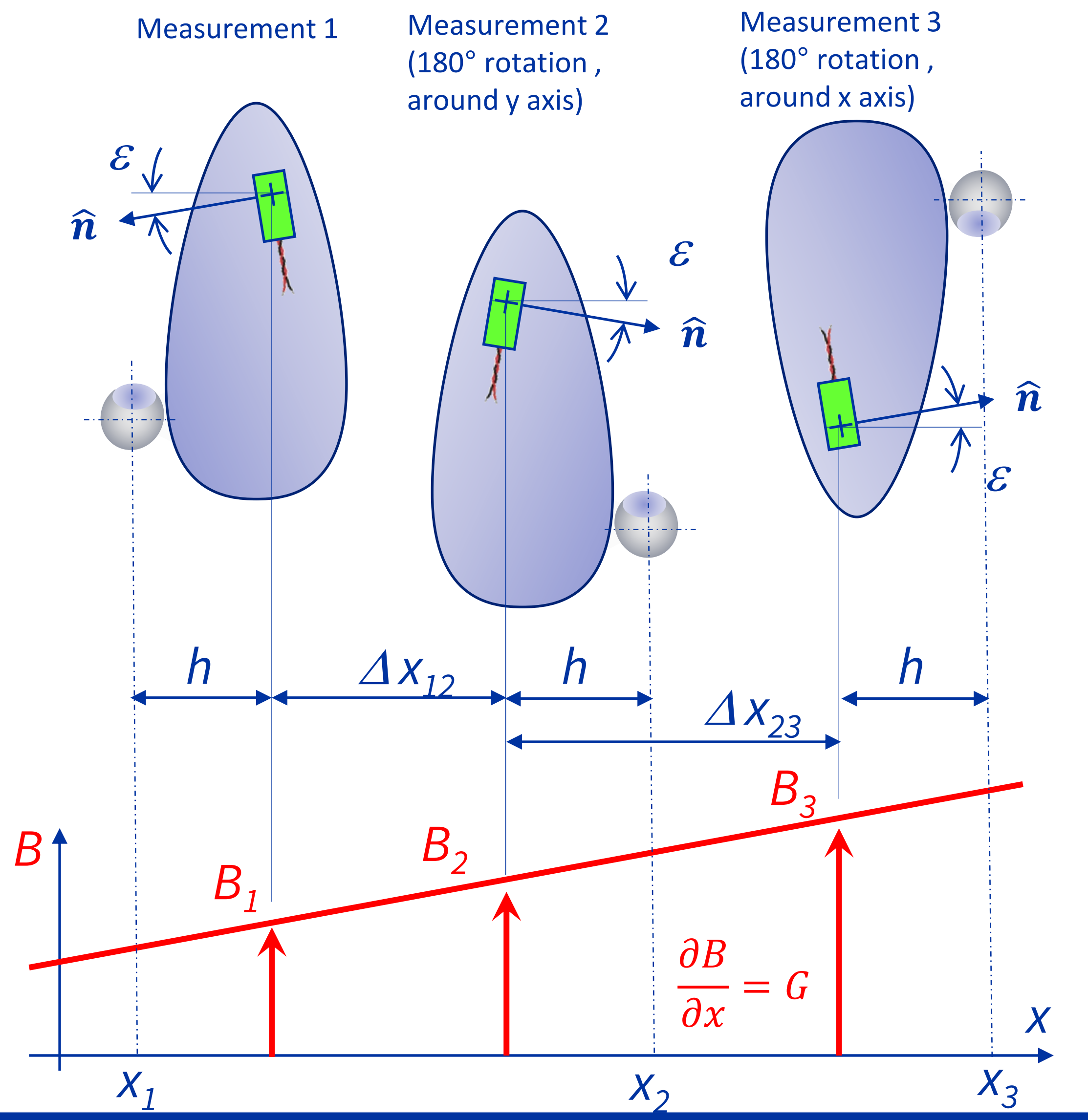
$$\begin{cases} B_1^{\text{meas}} = \cos\varepsilon\, B_1 \approx (1-\frac{\varepsilon^2}{2})B_1 \\ B_2^{\text{meas}} = \cos\varepsilon\, B_2 \approx (1-\frac{\varepsilon^2}{2})B_2 \end{cases} \Rightarrow$$

- An additional measurement at 180° provides also G

$$\begin{cases} \Delta x = \frac{(B_2 - B_1)}{G} \approx \frac{B_2^{\text{meas}} - B_1^{\text{meas}}}{G} \\ x_2 - x_1 = \Delta x + 2d \end{cases} \Rightarrow d = \frac{x_2 - x_1}{2} + \frac{B_2^{\text{meas}} - B_1^{\text{meas}}}{2G}$$

$$d = \frac{x_1(B_3 - B_2) - x_2(B_3 - B_1) + x_3(B_2 - B_1)}{2(B_2 - B_3)}, \qquad G = \frac{B_2 - B_3}{x_2 - x_3}$$

2) Turnaround in a linear gradient, B ⊥ sensing axis



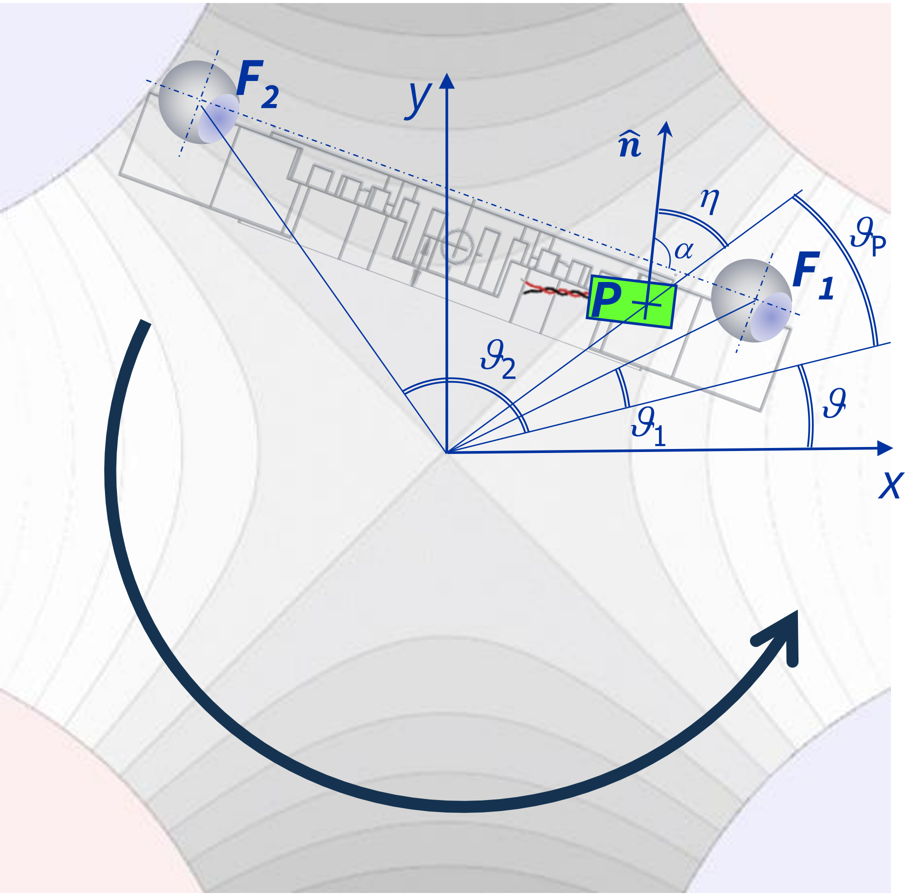
- For Hall probe: in-plane field
- For NMR: n.a.
- Three measurements at 180° (around y/z)

$$\begin{cases} B_1^{meas} = -\sin\varepsilon\, B_1 \approx -\varepsilon B_1 \\ B_2^{meas} = -\sin\varepsilon\, B_2 \approx -\varepsilon B_2 \\ B_3^{meas} = +\sin\varepsilon\, B_3 \approx +\varepsilon B_3 \end{cases}$$

$$\begin{cases} \Delta x_{12} = \dfrac{B_2 - B_1}{G}, & x_2 - x_1 = \Delta x_{12} + 2h \\ \Delta x_{23} = \dfrac{B_3 - B_2}{G}, & x_3 - x_2 = \Delta x_{23} \end{cases}$$

$$\begin{cases} \varepsilon = \dfrac{B_3^{meas} + B_2^{meas}}{G(x_3 - x_2)} \\ h = \dfrac{x_2 - x_1}{2} + \dfrac{B_2^{meas} - B_1^{meas}}{B_3^{meas} + B_2^{meas}} \dfrac{x_3 - x_2}{2} \end{cases}$$

3) Rotation in a linear gradient



Rotate the sensor assembly in a known 2D quadrupole field:

$$\begin{cases} B_x = Gy \\ B_y = Gx \end{cases} \Longrightarrow \begin{cases} B_\rho = G\rho \sin 2\vartheta \\ B_\tau = G\rho \cos 2\vartheta \end{cases}$$

Ideal case: rotation axis ≡ magnetic axis

$$\boldsymbol{F_1}: \begin{cases} x_1 = \rho_1 \cos(\vartheta + \vartheta_1) \\ y_1 = \rho_1 \sin(\vartheta + \vartheta_1) \end{cases}$$

Fiducial coordinates measured during rotation → FFT
1st harmonic amplitudes ρ_1, ρ_2 and phases ϑ_1, ϑ_2

$$\boldsymbol{F_2}: \begin{cases} x_2 = \rho_2 \cos(\vartheta + \vartheta_2) \\ y_2 = \rho_2 \sin(\vartheta + \vartheta_2) \end{cases}$$

$$\boldsymbol{P}: \begin{cases} x_P = \rho_P \cos(\vartheta + \vartheta_P) \\ y_P = \rho_P \sin(\vartheta + \vartheta_P) \end{cases}$$

Position of the sensor (unknown)

$$\hat{\boldsymbol{n}} = \cos\eta\, \hat{\boldsymbol{\rho}} + \sin\eta\, \hat{\boldsymbol{\tau}}$$

Hall probe sensing direction (rotating unit vector)

$$B_{\mathrm{Hall}}(\vartheta) = \boldsymbol{B} \cdot \hat{\boldsymbol{n}} = G\rho_P \sin(2\vartheta + \eta)$$

Measured field

FFT ($B_{meas}(\vartheta)$) → 2nd harmonic: amplitude $\boldsymbol{\rho_P}$ and phase $\boldsymbol{\eta}$

Sensing direction w.r.t. line **F_1-F_2**: $\alpha = \eta + \vartheta_P - \vartheta_1 - \beta, \quad \beta = \sin^{-1} \dfrac{\rho_2 \sin(\vartheta_2 - \vartheta_1)}{\sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos(\vartheta_2 - \vartheta_1)}}$

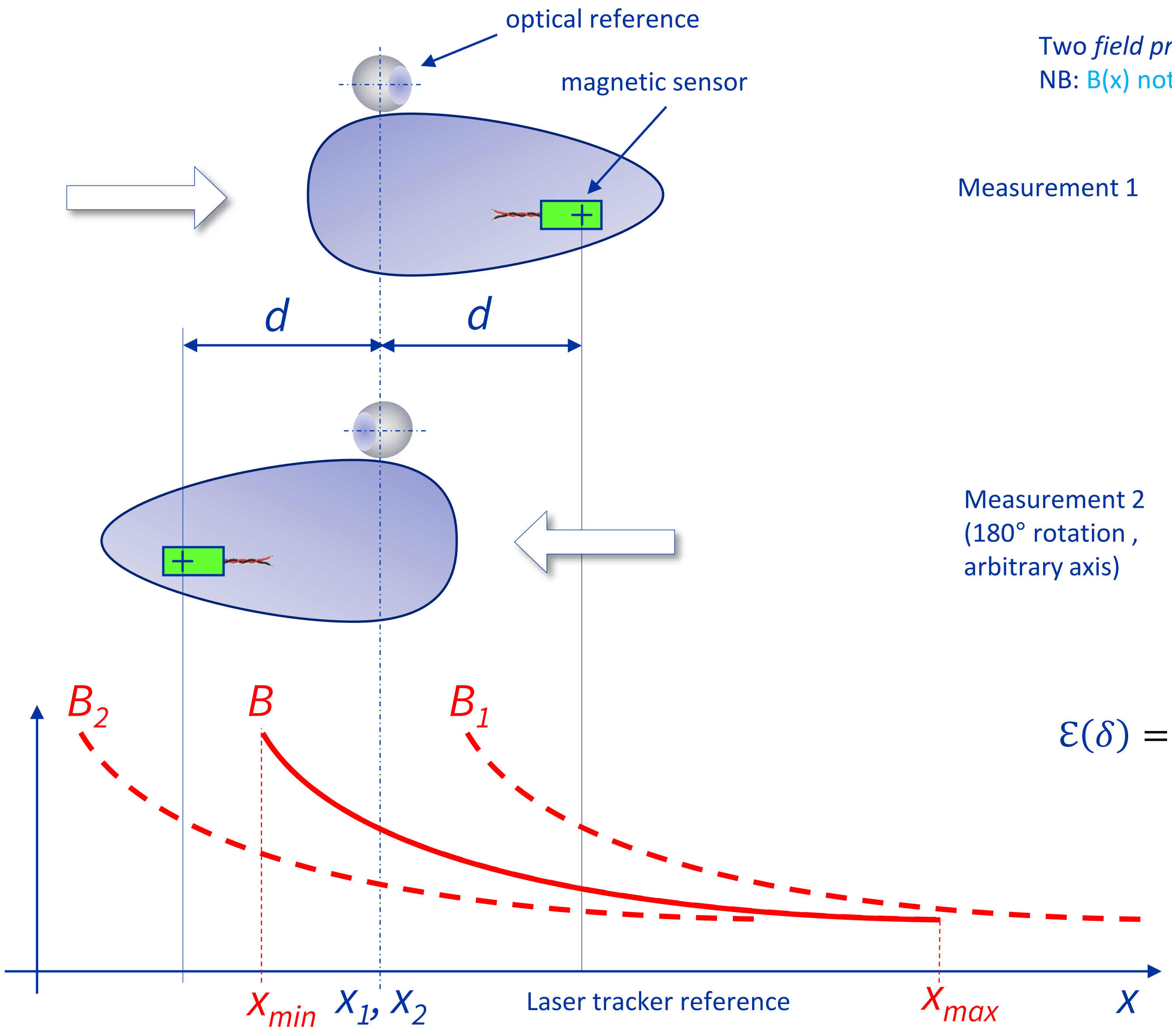
Sensor position **P-F_1**: $\begin{cases} \parallel \mathbf{F_1 - F_2} = \rho_1 \sin\beta - \rho_P \sin(\vartheta_P - \vartheta_1 + \beta) \\ \perp \mathbf{F_1 - F_2} = \rho_1 \cos\beta - \rho_P \cos(\vartheta_P - \vartheta_1 + \beta) \end{cases}$

Non-ideal case: rotation axis (x_0, y_0) ≢ magnetic axis

DC component of **$F_{1,2}$** and 1st harmonic of $B_{meas}(\vartheta)$ encode (x_0, y_0)

$$B(\vartheta) = G\rho_P \sqrt{1 + \frac{\rho_0^2}{\rho_P^2} + 2\frac{\rho_0}{\rho_P} \cos\left(\vartheta + \vartheta_P + \tan^{-1}\frac{y_0}{x_0}\right)}$$

4) Turnaround in a non-linear gradient



Two *field profile* measurements with the probe in opposite orientations:
NB: B(x) not necessarily known in advance

Measurement 1

$$\begin{cases} B_1(t) \\ x_1(t) \end{cases} \Rightarrow B_1 = f(x_1)$$

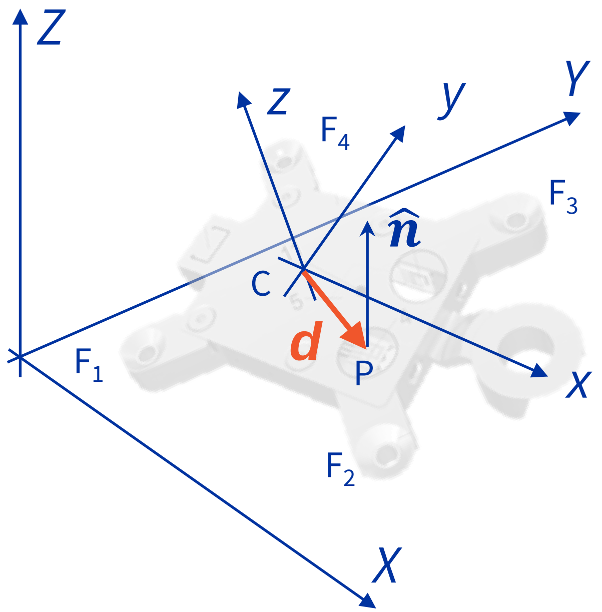
Measurement 2
(180° rotation ,
arbitrary axis)

$$\begin{cases} B_2(t) \\ x_2(t) \end{cases} \Rightarrow B_2 = g(x_2)$$

$$\mathcal{E}(\delta) = \int_{x_{\min}}^{x_{\max}} \big(f(x-\delta) - g(x+\delta)\big)^2 dx, \qquad d = \arg\min \mathcal{E}(\delta)$$

If B(x) is known → **best fit can recover probe calibration** $k = B/V_{Hall}$
Efficient combination of calibration and fiducialization from two sweeps

5) Best-fit to known field map



$\boldsymbol{R} = \{X, Y, Z\}$ (Lab) reference (attached to field map)

$\boldsymbol{r} = \{x, y, z\}$ (Local) housing reference (attached to fiducials F_i)

$\boldsymbol{R} = \boldsymbol{C} + [\Phi]\,\boldsymbol{r}$ Reference transformation

$\boldsymbol{B} = \boldsymbol{f}(\boldsymbol{R})$ Vector field map in Lab reference (e.g. Fourier-Bessel series)

$\boldsymbol{d} = \boldsymbol{d}_0 + \boldsymbol{\Delta d}$ Probe position in local reference: nominal + (small) unknown component

Measure the field at multiple positions/orientation of the housing: *j*=1..*M*

$\boldsymbol{P}_j = \boldsymbol{C}_j + [\Phi]_j\,\boldsymbol{d}$ Probe position in Lab reference (j^{th} measurement)

Hall probe

$$\hat{\boldsymbol{n}} = [\varepsilon]\hat{\boldsymbol{n}}_0 = \begin{bmatrix} 1 & -\varepsilon_x & -\varepsilon_y \\ \varepsilon_x & 1 & 0 \\ \varepsilon_y & 0 & 1 \end{bmatrix} \hat{\boldsymbol{n}}_0$$

Unit vector normal to Hall probe (sensing axis) in local reference nominal + two (small) unknown rotations

$\hat{\boldsymbol{N}}_j = [\Phi]_j \hat{\boldsymbol{n}}$ Sensing axis in Lab reference

$B_j^{\text{meas}} = B_H(\boldsymbol{P}_j) = \boldsymbol{f}(\boldsymbol{P}_j) \cdot \hat{\boldsymbol{N}}_j$ Measured field / mapped field

$$\mathcal{E}(\boldsymbol{\delta}) = \left\| \frac{\boldsymbol{\delta}}{\sigma_d} \right\|^2 + \sum_{j=1}^{M} \left(\frac{B_j - \boldsymbol{f}\left(\boldsymbol{C}_j + [\Phi]_j(\boldsymbol{d}_0 + \boldsymbol{\delta})\right) \cdot [\Phi]_j[\varepsilon]\hat{\boldsymbol{n}}}{\sigma_j} \right)^2$$

$$\boldsymbol{\Delta d} = \arg\min \mathcal{E}(\boldsymbol{\delta})$$

NMR probe

$$\mathcal{E}(\boldsymbol{\delta}) = \left\| \frac{\boldsymbol{\delta}}{\sigma_d} \right\|^2 + \sum_{j=1}^{M} \left(\frac{B_j - \left\| \boldsymbol{f}\left(\boldsymbol{C}_j + [\Phi]_j(\boldsymbol{d}_0 + \boldsymbol{\delta})\right) \right\|}{\sigma_j} \right)^2$$

3D field asymmetry → NMR direction-insensitivity is not relevant
B(x) is known → **best fit can also recover probe calibration** $k=B/V_{Hall}$