SC magnet design – EM part II

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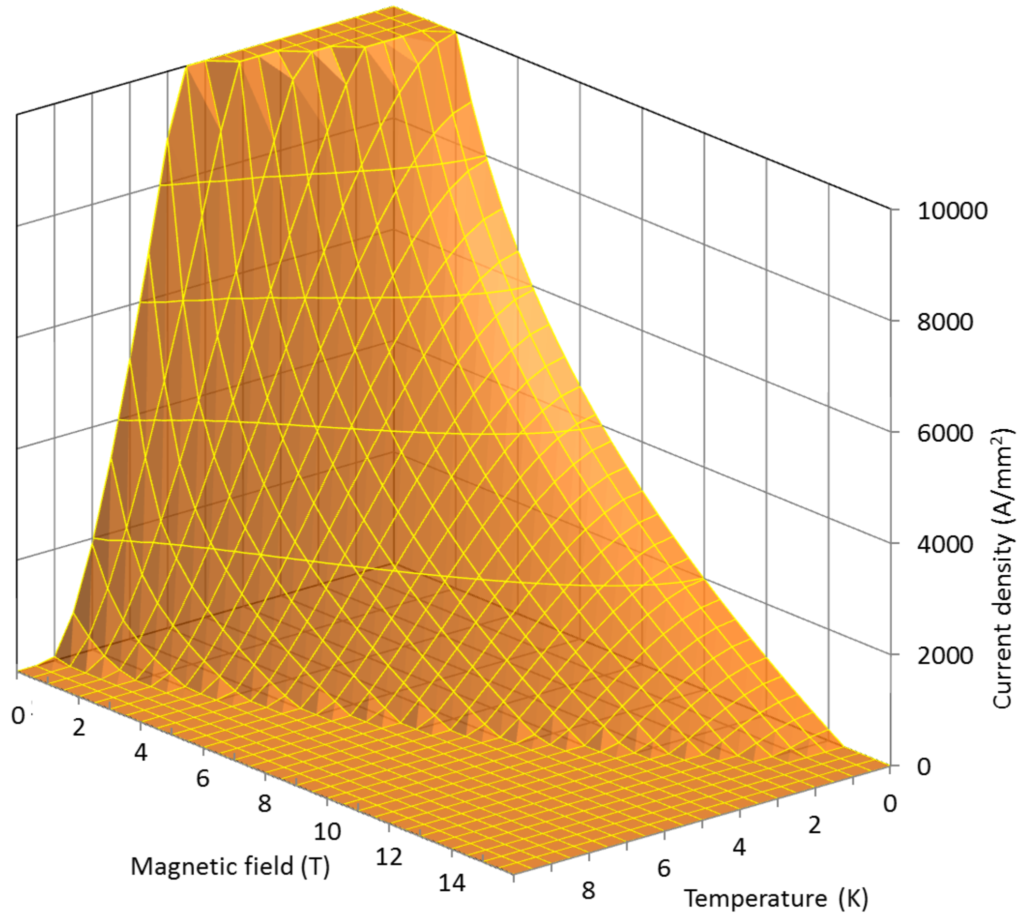
Lecture based on E. Todesco, “Masterclass -Design of superconducting magnets for particle accelerators”, https://indico.cern.ch/category/12408/

OUTLINE OF LECTURE II

- Which are the maximum performance of dipoles/quadrupoles?
  - Critical surface
  - Filling ratios
  - Peak field on coils
  - Load line
  - short sample
  - effect of iron yoke
  - current grading

CRITICAL SURFACE

- The **critical surface** defines the boundaries between superconducting state and normal conducting state in the space defined by **temperature**, magnetic **field**, and **current** densities.
- For each superconducting material, this surface, which can only be determined experimentally, can be fitted with parameterization curves



NbTi CRITICAL SURFACE APPROXIMATION

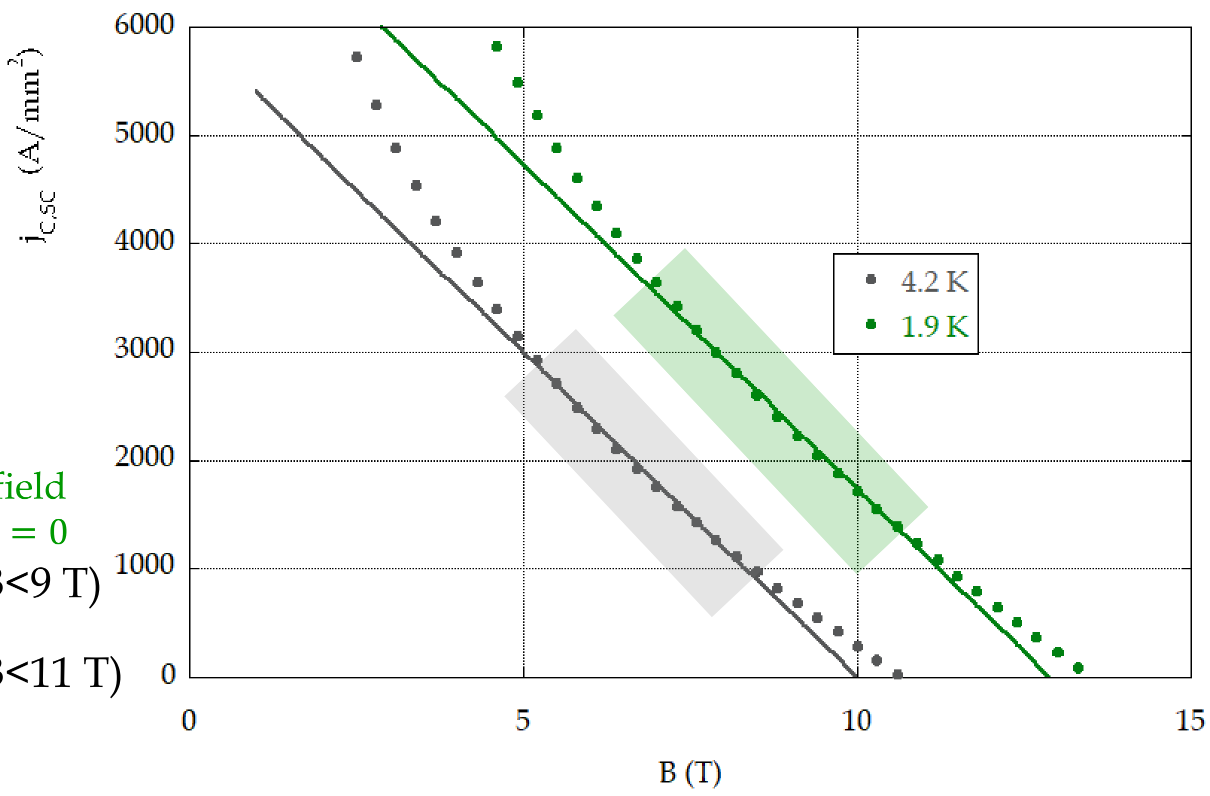
- $$\frac{J_{C,SC}(B,T)}{J_{C,ref}} = \frac{C_0}{B}\left[\frac{B}{B_{C2}(T)}\right]^{\alpha}\left(1-\frac{B}{B_{C2}(T)}\right)^{\beta}\left(1-\left(\frac{T}{T_{C0}}\right)^{1.7}\right)^{\gamma}$$
- $T_{C0} = 9.2$ K $\quad B_{C20} = 14.5$ T $\quad B_{C2}(T) = B_{C20}\left(1-\left(\frac{T}{T_{C0}}\right)^{1.7}\right)$
- $J_{C,ref} = J_C(5\,T, 4.2\,K) = 3000\frac{A}{mm^2}$
- $C_0 = 31.4\,T, \alpha = 0.63,\ \beta = 1,\ \gamma = 2.3$ (fit parameters for LHC wires)

- The critical surface can be linearly approximated as follows:

$$j_{C,SC}(B,T) = s(b(T) - B)$$

s slope, $b(T)$ field at which $j_{C,SC} = 0$

  - NbTi @ 4.2 K: s=600 A/(T mm^2), b=10 T (5 T<B<9 T)
  - NbTi @ 1.9 K: s=600 A/(T mm^2), b=12.9 T (7 T<B<11 T)



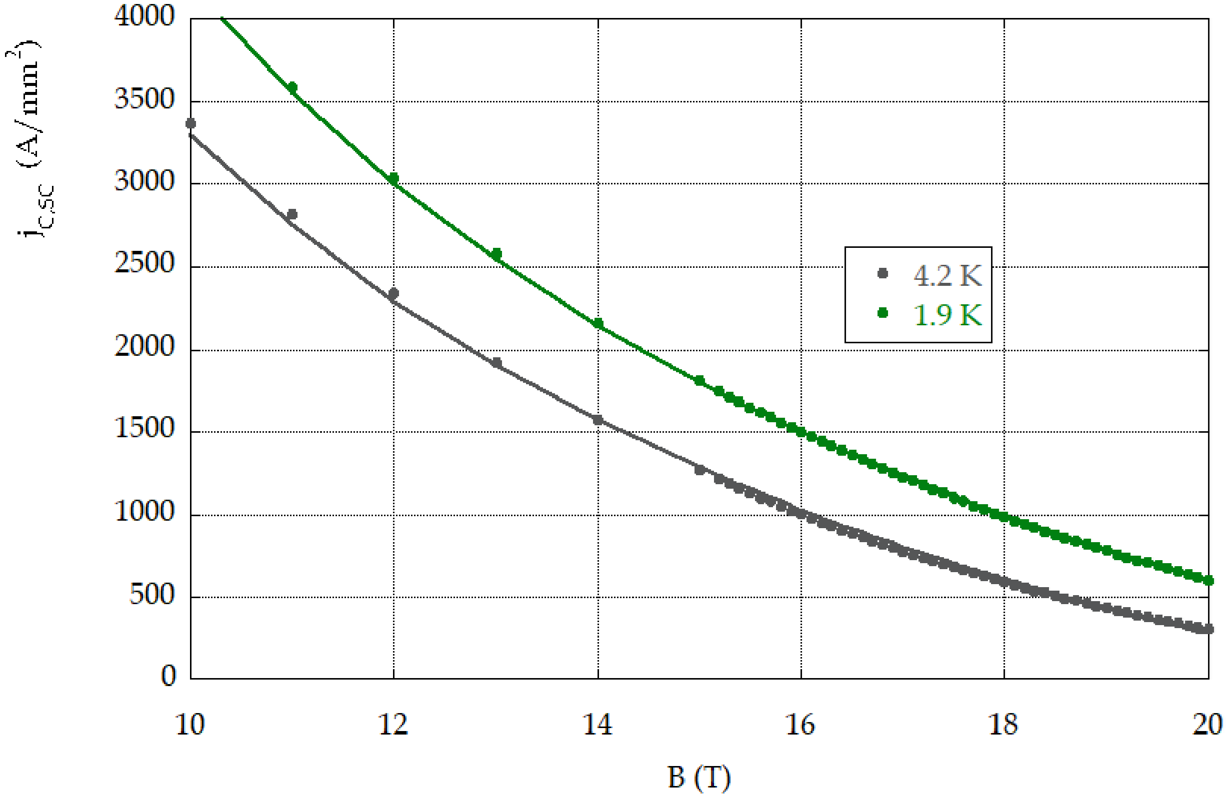
Nb_3Sn CRITICAL SURFACE APPROXIMATION

- $j_{C,sc}(B,T) = \frac{C_0}{B}\left(\frac{B}{B_{C2}(T)}\right)^{0.5}\left(1-\frac{B}{B_{C2}(T)}\right)^2\left(1-\left(\frac{T}{T_{C0}}\right)^{1.52}\right)^{\alpha}\left(1-\left(\frac{T}{T_{C0}}\right)^2\right)^{\alpha}$
- $T_{C0} = 16$ K, $B_{C20} = 29.38$ T, $\alpha = 0.96$, $B_{C2}(T) = B_{C20}\left(1-\left(\frac{T}{T_{C0}}\right)^{1.52}\right)$
- $C_0 = 178563$ AT/mm^2 $\rightarrow$ $j_{C,sc}(16\,T, 4.2\,K) = 1000$ A/mm^2

- The critical surface is better approximated with a hyperbole:

$$j_{C,sc} = s\left(\frac{b(T)}{B} - 1\right)$$

  - Nb_3Sn @ 4.2 K: s=2750 A/mm^2, b=22 T
  - Nb_3Sn @ 1.9 K: s=3000 A/mm^2, b=24 T



FILLING FACTORS
(neglecting twist pitch)

- $j_{C,sc}$ is the critical current density of the superconducting filaments, but the critical current density of a coil is much lower, due to:
  - superconducting filaments are embedded in a metal matrix (usually copper)
    - if $\nu_{\mathrm{Cu/no\,Cu}}$ is the ratio between the copper and the superconductor (usually ranging from 1 to 2) then $j_{C,\mathrm{strand}} = j_{C,sc}\frac{A_{SC}}{A_{\mathrm{strand}}} = j_{C,sc}\frac{1}{\frac{A_{SC}+A_{Cu}}{A_{SC}}} = j_{C,sc}\frac{1}{1+\nu_{\mathrm{Cu/no\,Cu}}}$

  - if the strands are assembled in rectangular cables, there are voids:
    - if $\kappa_{w\text{-}c}$ is the fraction of cable occupied by strands (usually ~85%)

$$j_{C,\mathrm{cable}} = \kappa_{w-c}\cdot j_{C,\mathrm{strand}} = \frac{1}{1+\nu_{\mathrm{Cu/no\,Cu}}}\kappa_{w-c}\cdot j_{C,\mathrm{sc}}$$

  - the cables are insulated:
    - if $\kappa_{c\text{-}i}$ is the fraction of insulated cable occupied by the bare cable (~85%)

$$j_{C,\mathrm{ins.\,cable}} = \kappa_{c-i}\cdot j_{C,\mathrm{cable}} = \frac{1}{1+\nu_{\mathrm{Cu/no\,Cu}}}\kappa_{w-c}\kappa_{c-i}\cdot j_{C,\mathrm{sc}}$$

- The critical surface for j (overall current density) is $j_C(B) = \kappa j_{C,sc}(B)$

$$\kappa = \frac{1}{1+\nu_{\mathrm{Cu/no\,Cu}}}\kappa_{w-c}\,\kappa_{c-i}$$

- Examples of filling factors in dipoles

$$j_C(B) = \kappa j_{C,sc}(B)$$

$$\kappa = \frac{1}{1+\nu_{Cu/no\ Cu}}\kappa_{w-c}\,\kappa_{c-i}$$

Magnet	$\nu_{Cu/noCu}$	κ_{w-c}	κ_{c-i}	κ
Tevatron MB	1.85	0.82	0.81	0.23
HERA MB	1.88	0.89	0.85	0.26
SSC MB inner	1.5	0.84	0.89	0.30
RHIC MB	2.25	0.87	0.84	0.22
LHC MB inner	1.65	0.87	0.87	0.29
FRESCA	1.6	0.87	0.88	0.29
MSUT inner	1.25	0.85	0.88	0.33
D20 inner	0.43	0.83	0.84	0.49
FNAL HFDA	1.25	0.86	0.76	0.29

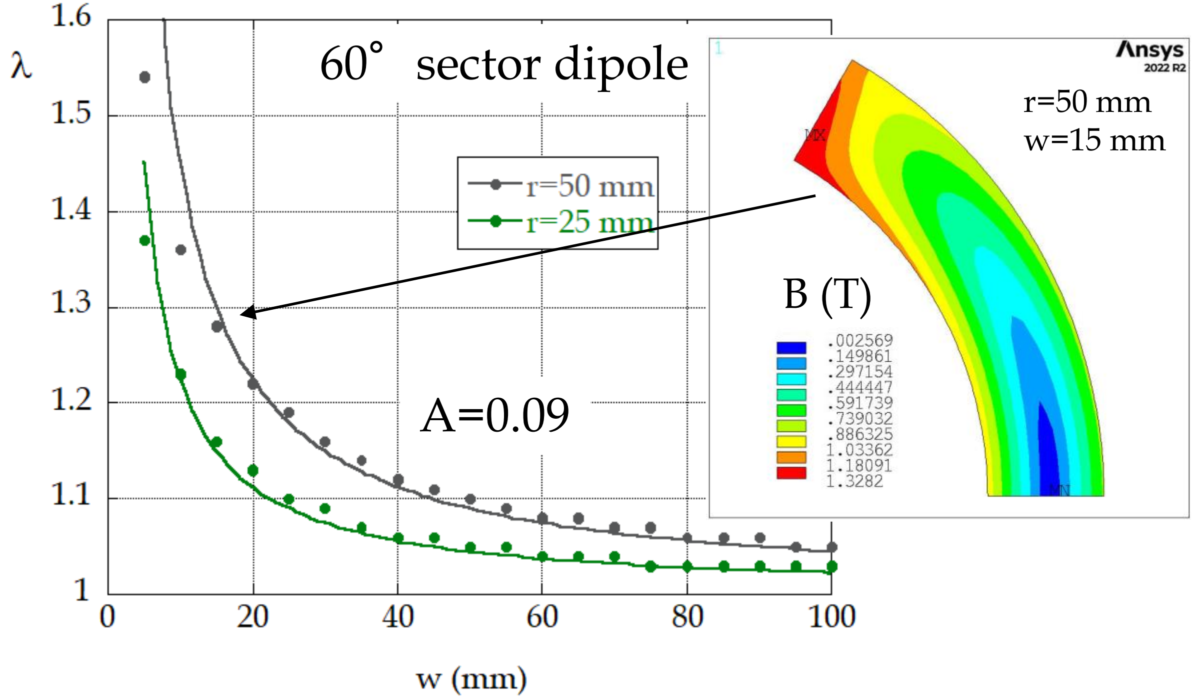
- Generally:
  - copper to superconductor ratio ranging from 1 to 2
  - void fraction ranging from 10% to 20%
  - insulation fraction from 10% to 20%

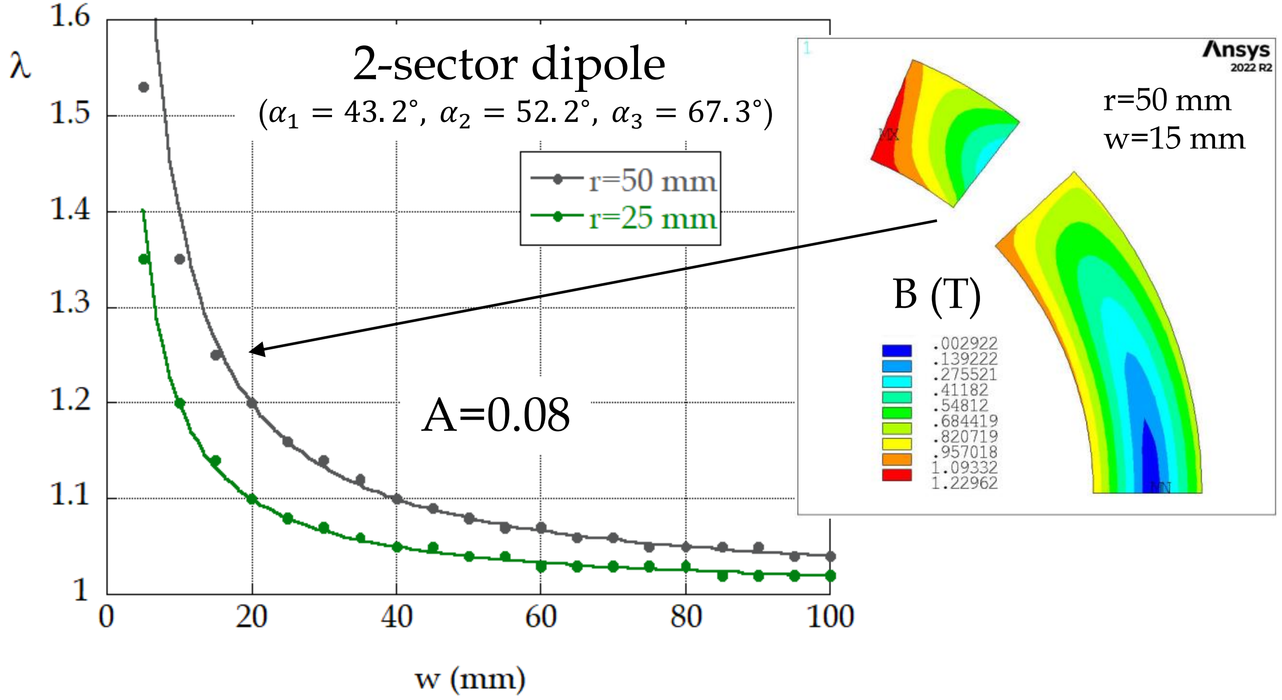
} κ ranging from 0.2 to 0.4

DIPOLES

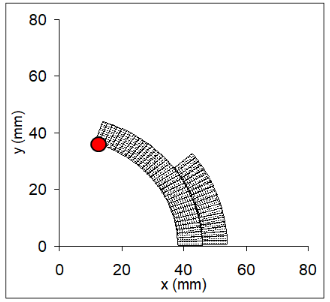
PEAK FIELD ON THE COILS

- The best performer is the cosθ distribution, where the peak field is equal to the bore field: $B_p = B_1 = \frac{\mu_0 j w}{2}$
- For sector dipoles, the ratio between peak field and bore field $\lambda \frac{B_p}{B_0}$ follow the hyperbolic fit $\lambda(w,r) \sim 1 + A\frac{r}{w}$ (r aperture radius, w coil width of sector coil)

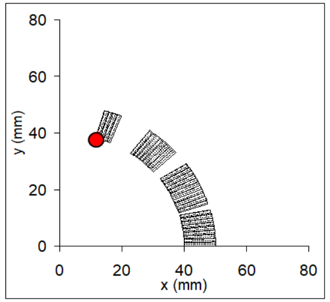




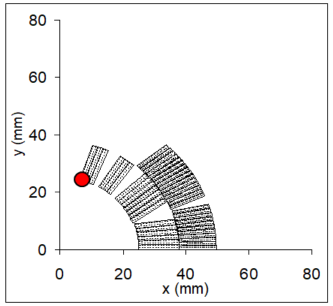
- The location of the peak is always on the border of the coils



Tevatron main dipole – location of the peak field

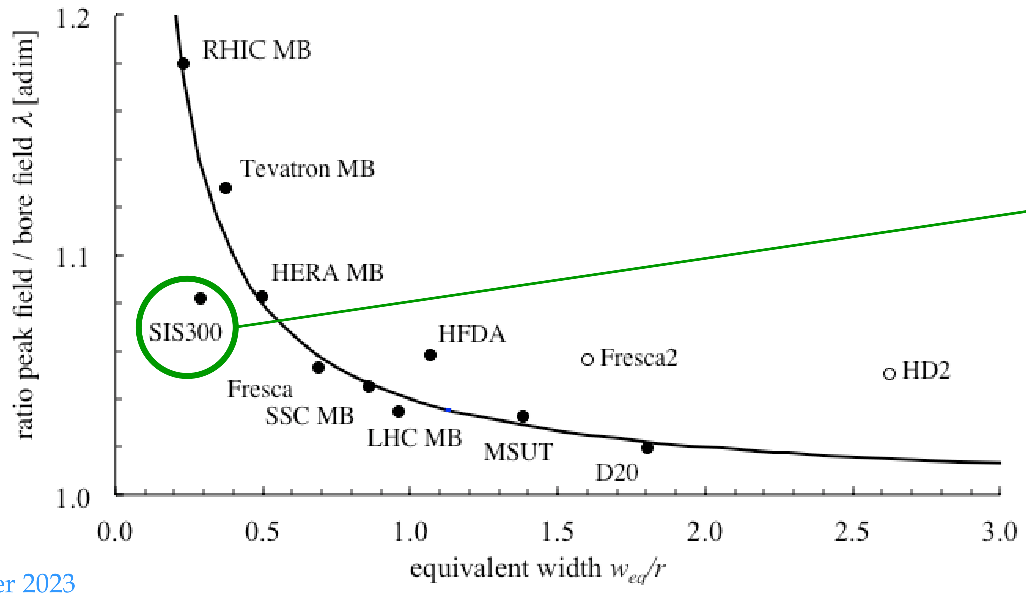


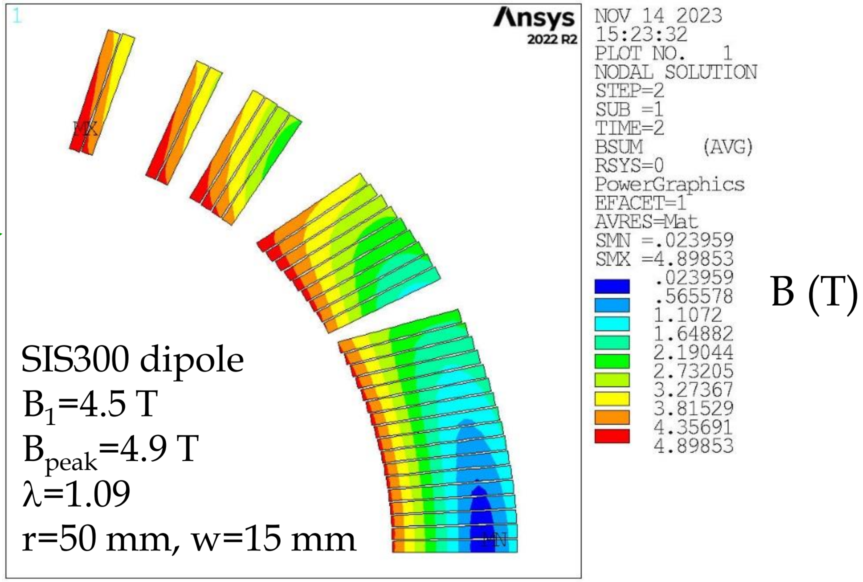
RHIC main dipole – location of the peak field



LHC main dipole – location of the peak field

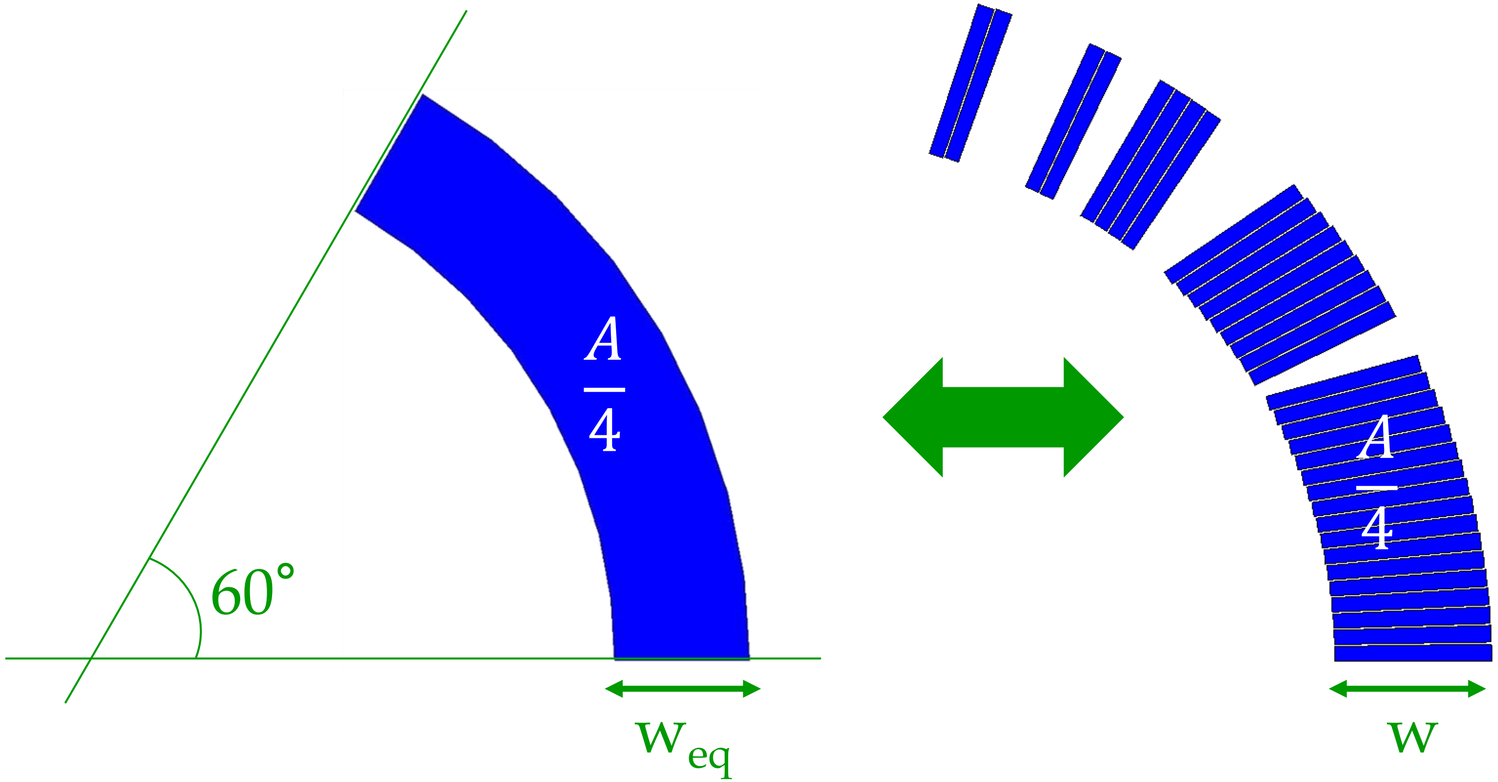
- The hyperbolic fit stands also for real magnets:



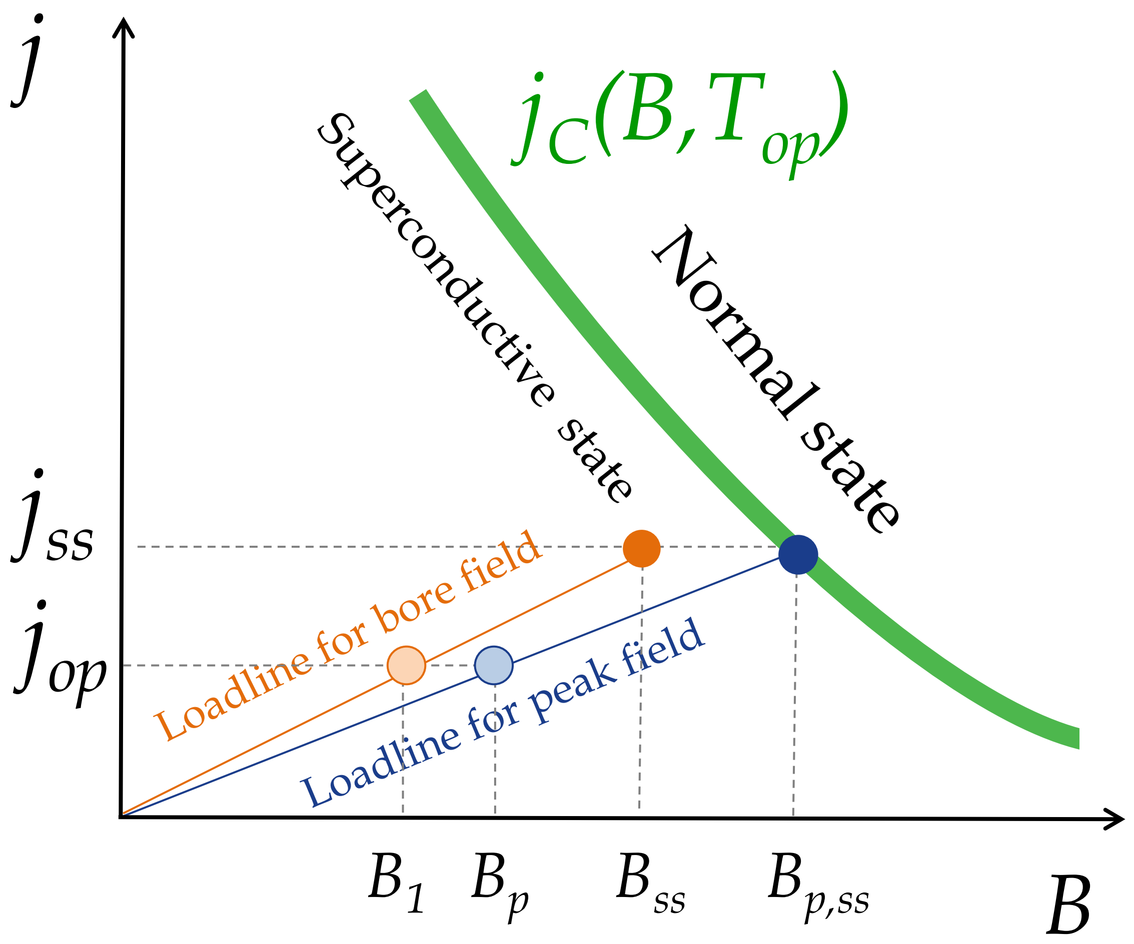


EQUIVALENT WIDTH

- Any kind of dipole can be reduced to a 60° sector of thickness w_{eq} coil by equating the 2 total areas:
- $A = \frac{2\pi}{3}\left[(r + w_{eq})^2 - r^2\right]$, A is the total conductor area of the dipole
- $w_{eq} = r\left(\sqrt{1 + \frac{3A}{2\pi r^2}} - 1\right)$

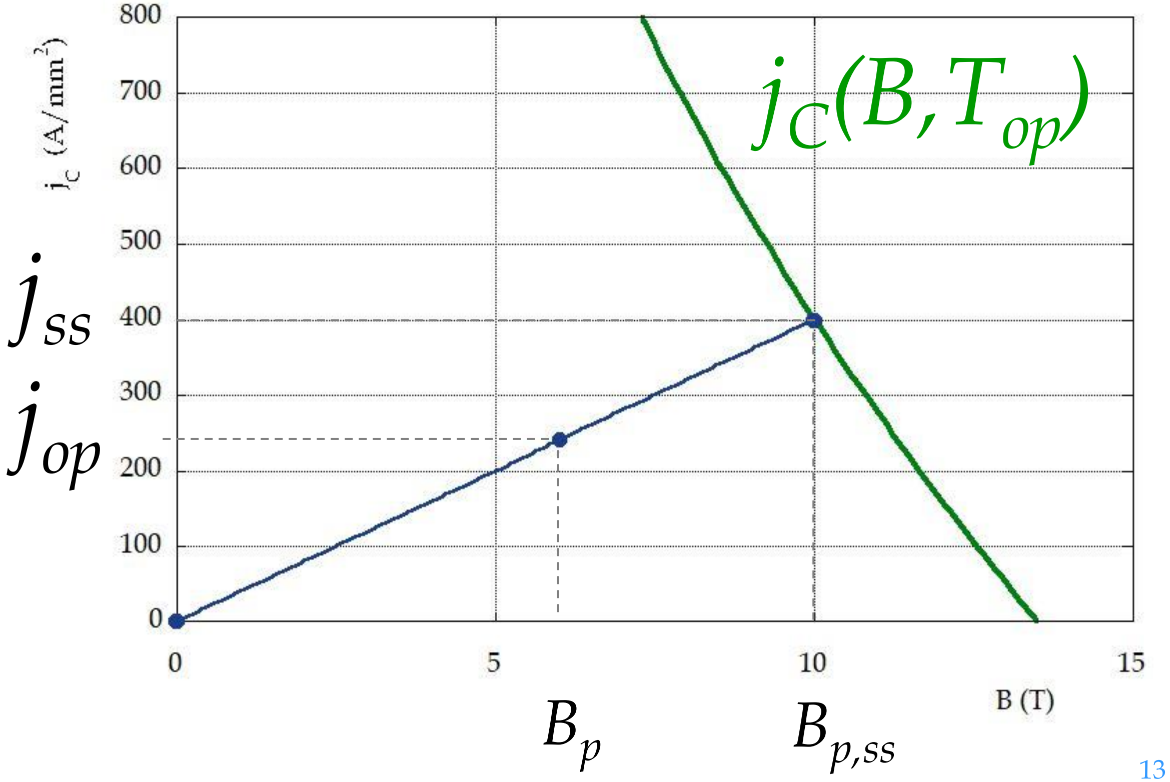


- A dipole coil can be characterized by 2 lines:
- $B_1 = \gamma_c j w$ and $B_p = \lambda B_1 = \lambda \gamma_c j w$
- In the (j_{sc}, B) plane they can be represented by two lines called loadlines
- The intersection of the load line for the peak field and the critical current density is the so-called short sample
- The short sample is the maximum theoretical performance of the magnet



LOADLINE MARGIN

- Obviously, a magnet cannot operate on (or too close to) the critical surface
  - Loadline fraction: ratio between operational current and short sample current
  - Loadline margin: 1- loadline fraction
- Example:
  - Let's consider a dipole operating at T_{op} and j_{op}=240 A/mm^2 with a peak field B_p=6 T
  - The short sample is j_{ss}=400 A/mm^2 and $B_{p,ss}$=10 T
  - The loadline fraction is $\frac{j_{op}}{j_{ss}} = \frac{B_p}{B_{p,ss}}$ =0.6
  - The loadline margin is 1-0.6=40%



LOADLINE MARGIN

- The loadine margin concept:
  - has no physical meaning and is difficult to generalize (only magnets of the same class have comparable margins)
  - is widely used and so far has not yet been replaced by any other criterion.
- Examples:
  - Main superconducting magnets have a 10-30% loadline margin
  - Correctors have about 50% margin

Loadline margin of the main dipoles in four accelerators

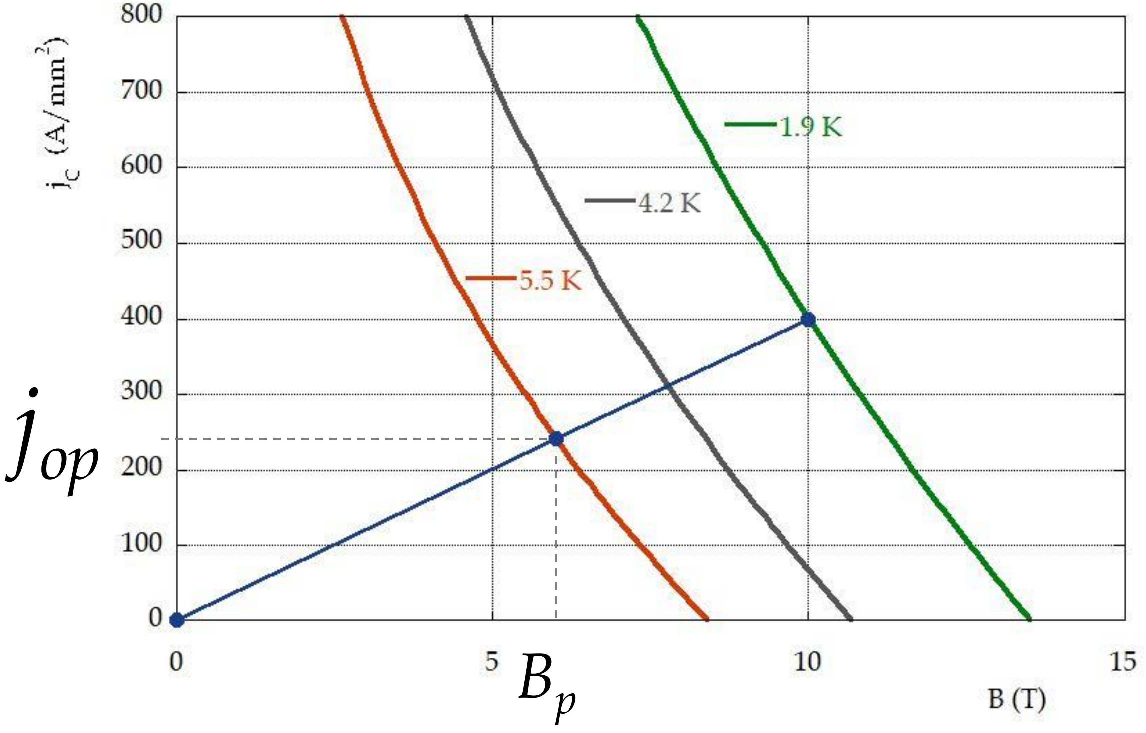
	Nominal			Actual		
	Temp. (K)	Field (T)	Margin	Temp. (K)	Field (T)	Margin
Tevatron	4.6	4.3	4%	4.6	4.2	6%
Hera	4.6	4.7	23%	3.9	5.3	23%
RHIC	4.5	3.5	30%	4.5	3.5	30%
LHC	1.9	8.3	14%	1.9	*7.8**	*19%*

TEMPERATURE MARGIN

- The temperature margin is a more physical quantity
- It is the increase in temperature that would lead to the magnet transition:

$$j_C(B_p, T_{op}+\Delta T) = j_{op}$$

- In the example:
  - T_{op} =1.9 K
  - $j_C(B_p, 5.5\ K) = j_{op}$
  - ΔT=3.6 K



- The temperature margin can be translated in energy density required to reach the critical surface

SHORT SAMPLE FIELD IN Nb-Ti DIPOLES

- We consider a sector coil of width w
  - The equation for the bore field is $B_1 = \gamma_c j w$, with γ_c=6.5×10^{-7} (T m/A) for a coil with one wedge setting to zero b_3 to b_7
  - The equation for the peak field is $B_p = \lambda B_1 = \lambda \gamma_c j w$, with $\lambda \sim 1 + A\frac{r}{w}$ (A=0.08)
  - The critical surface of NbTi can be linearly approximated as $j_{C,SC} = s(b - B)$
  - The overall critical current density is $j_C = \kappa j_{C,SC} = \kappa s(b - B)$
  - The short sample is defined as the intersection between the lines:

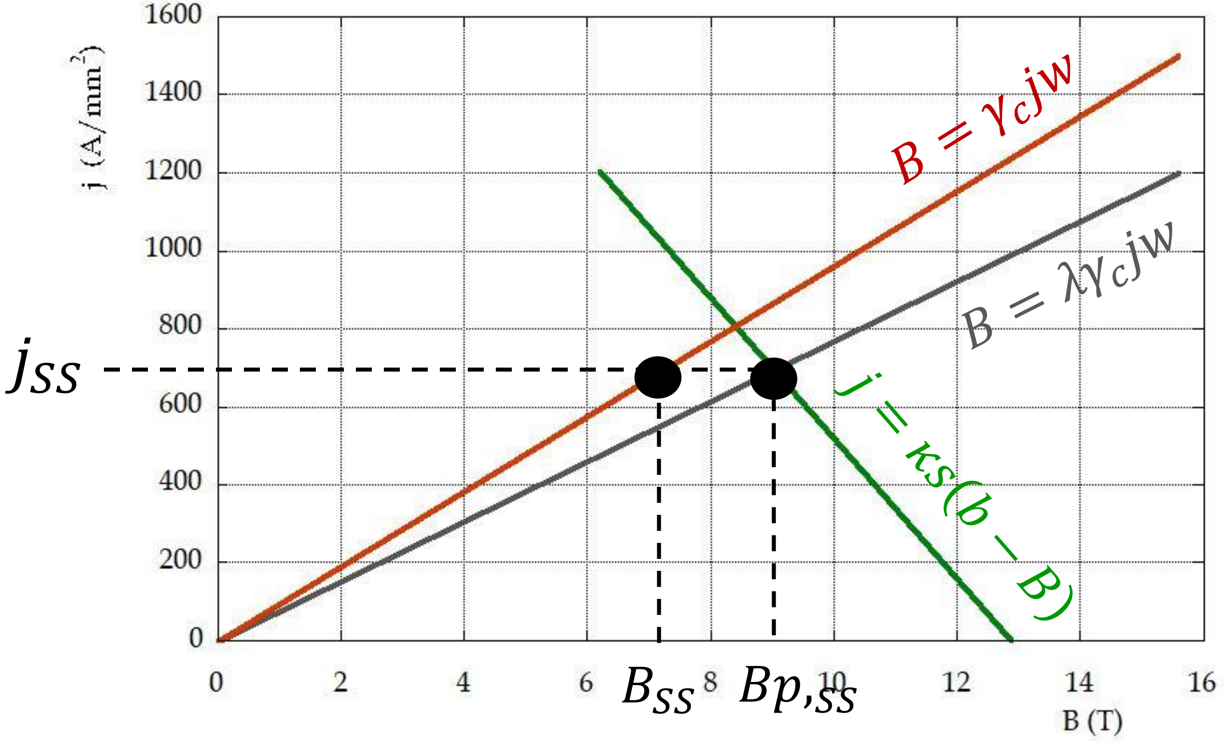
    $B_{p,SS} = \lambda \gamma_c j_{SS} w$ and $j_{SS} = \kappa s\left(b - B_{p,SS}\right)$

- Short sample limits are:

$$j_{SS} = \frac{\kappa s b}{1+\lambda\gamma_c \kappa s w}, \qquad B_{p,SS} = \frac{\kappa s b \lambda \gamma_c w}{1+\lambda\gamma_c \kappa s w}$$

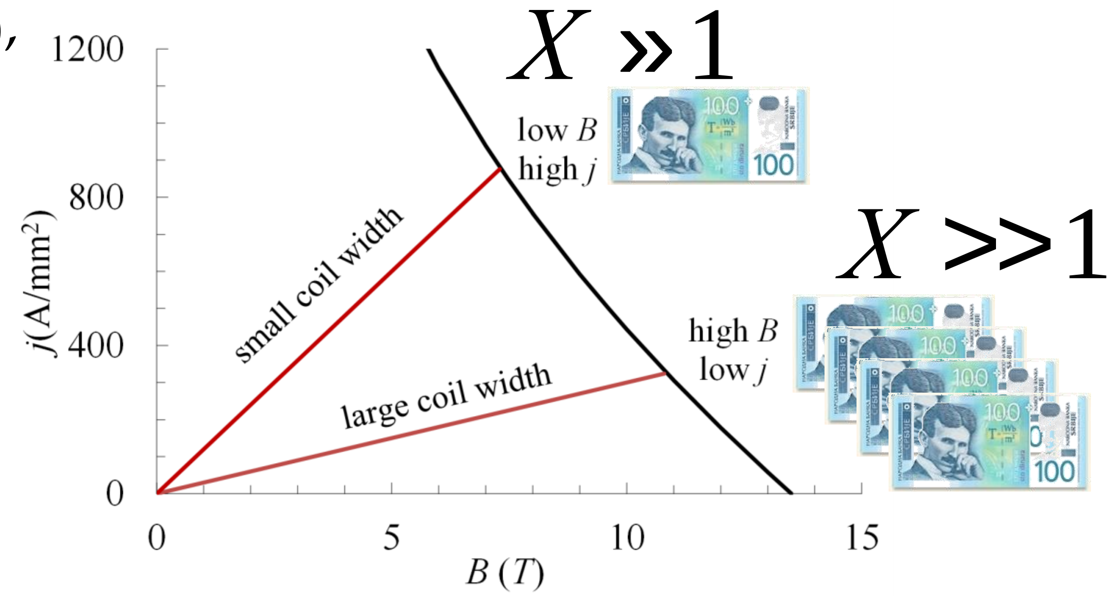
- The bore short sample field is:

$$B_{SS} = \frac{B_{p,SS}}{\lambda} = \frac{\kappa s b \gamma_c w}{1+\lambda\gamma_c \kappa s w}$$



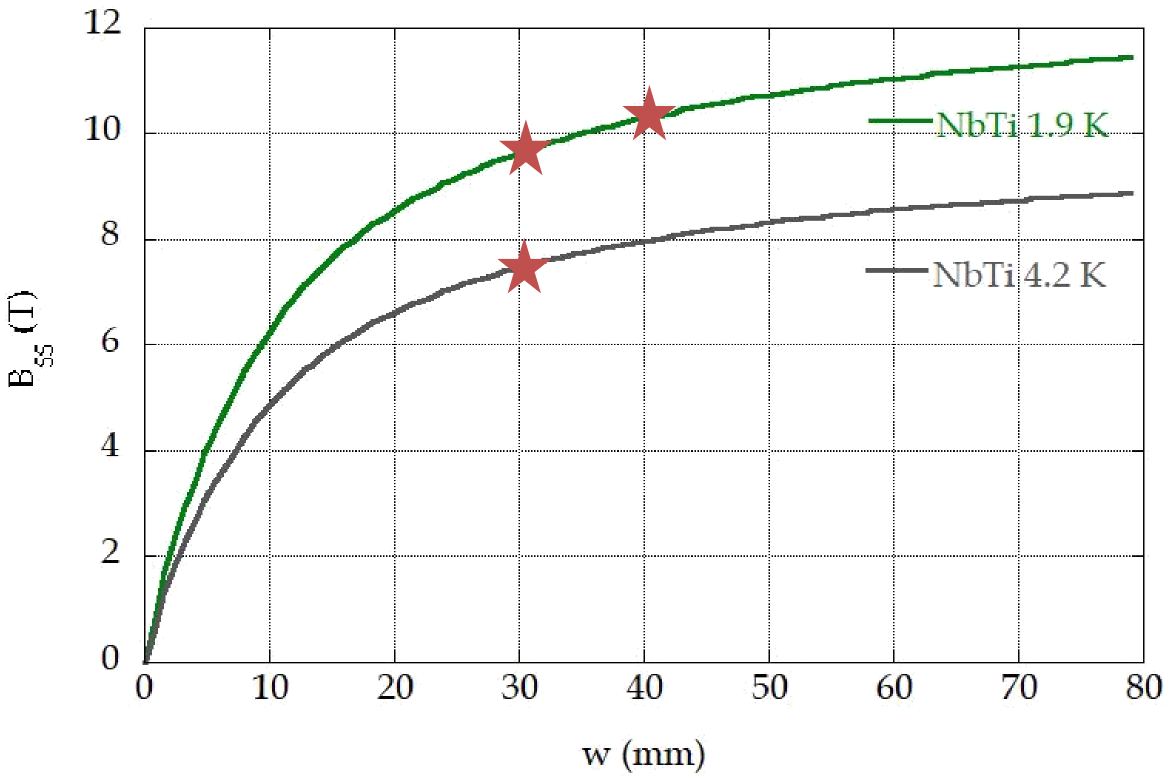
SHORT SAMPLE FIELD IN Nb-Ti DIPOLES

- Final equation for short sample field can be generalized as:
- $B_{SS} = \frac{\kappa s b \gamma_c w_{eq}}{1+\lambda\gamma_c \kappa s w_{eq}}$ and $j_{SS} = \frac{\kappa s b}{1+\lambda\gamma_c \kappa s w_{eq}}$
  - Superconductor parameters
    s, b (linear fit of j_C curve)
  - cable parameters
    κ (global filling factor)
  - primary geometrical parameters
    r, w_{eq} (aperture radius and coil width)
  - derived geometrical parameters
    $\gamma_c, \lambda = 1 + A\frac{r}{w}$
- The relevant quantity is $X = \kappa s \gamma_c w_{eq}$ (adimensional),
  so that $B_{SS} = \frac{bX}{1+\lambda X}$

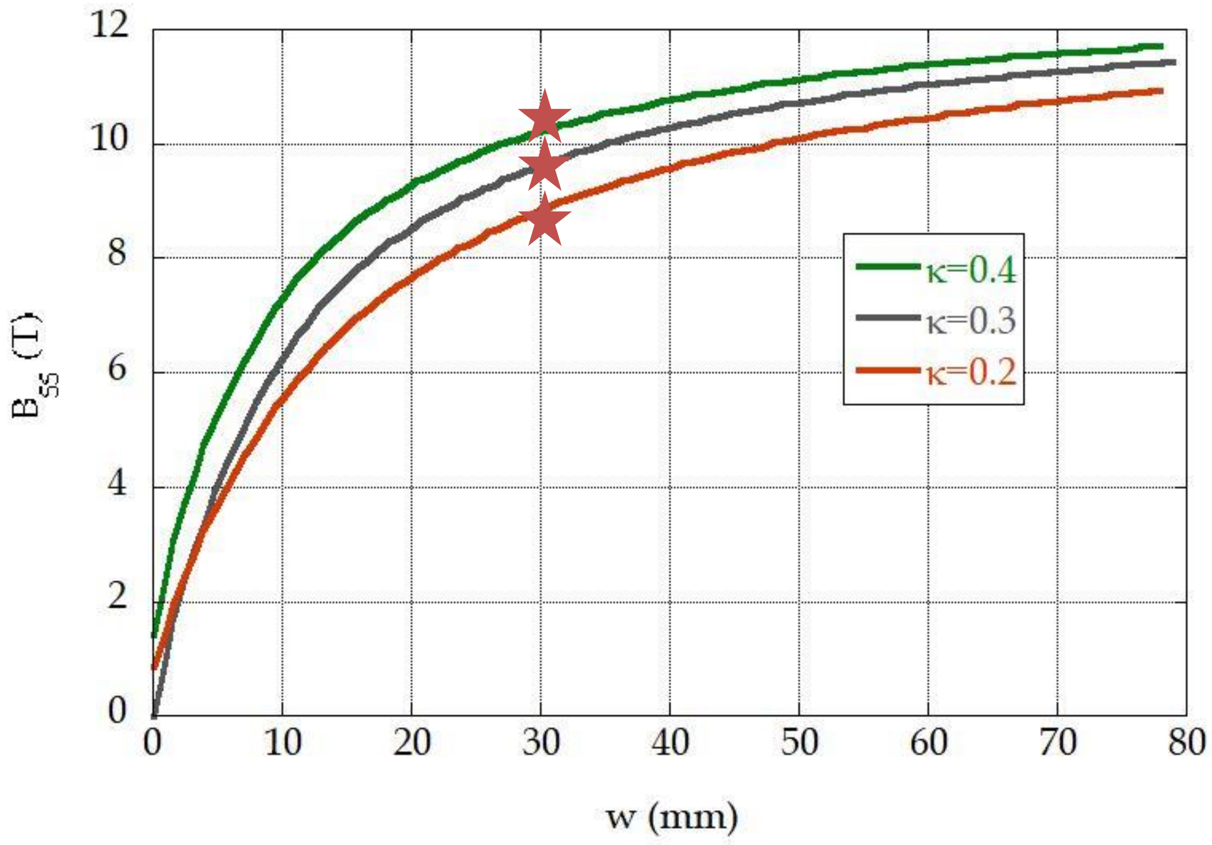


SENSITIVITY ANALYSIS: coil width w

- How efficient is increasing w to increase $B_{SS} = \frac{bX}{1+\lambda X}$?
- example: 60° degree sector, r=25 mm, κ=0.3
  - if w→∞, then λ→1, X →∞ and B_{SS}→b
  - If w>30 mm, the increase in w has minimal benefit on B_{SS}:
    - B_{SS}(40 mm)/ B_{SS}(30 mm)=1.07
  - the benefit of reducing T_{op} is more relevant:
    - B_{SS}(1.9 K)/ B_{SS}(4.2 K)=1.3



- How efficient is increasing κ to increase $B_{SS}=\frac{bX}{1+\lambda X}$?
- example: 60° degree sector, r=25 mm, κ=0.2-0.4, Nb-Ti @1.9 K
  - If w=30 mm:
    - $B_{SS}(0.4) - B_{SS}(0.3) = 0.6$ T
    - $B_{SS}(0.3) - B_{SS}(0.2) = 0.8$ T



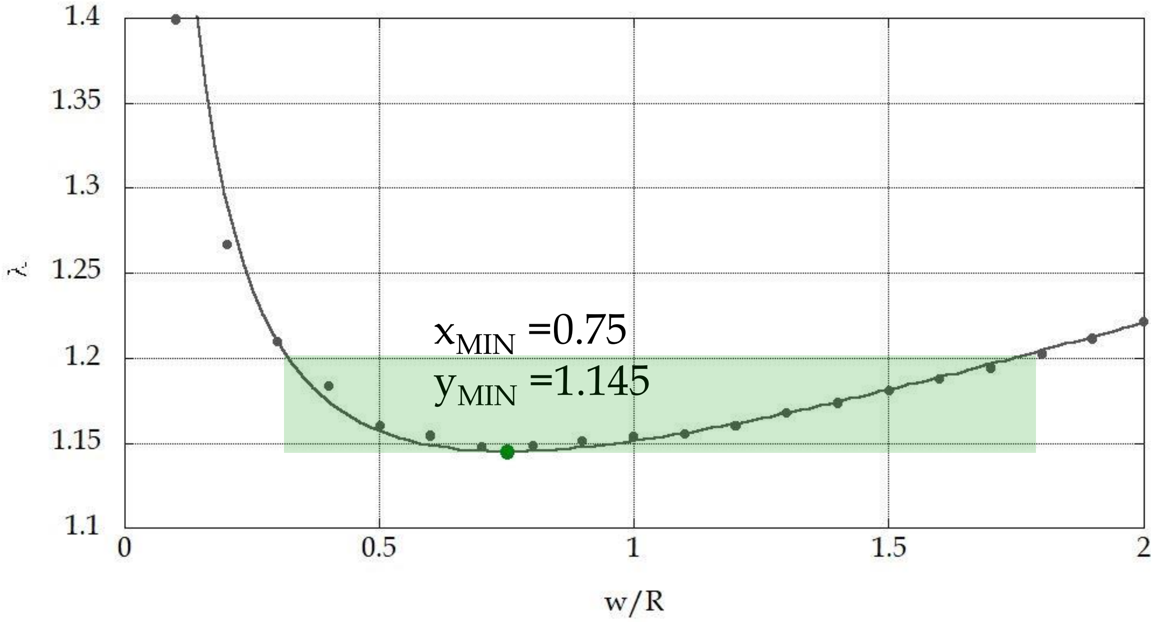
QUADRUPOLES

QUADRUPOLE SHORT SAMPLE GRADIENT

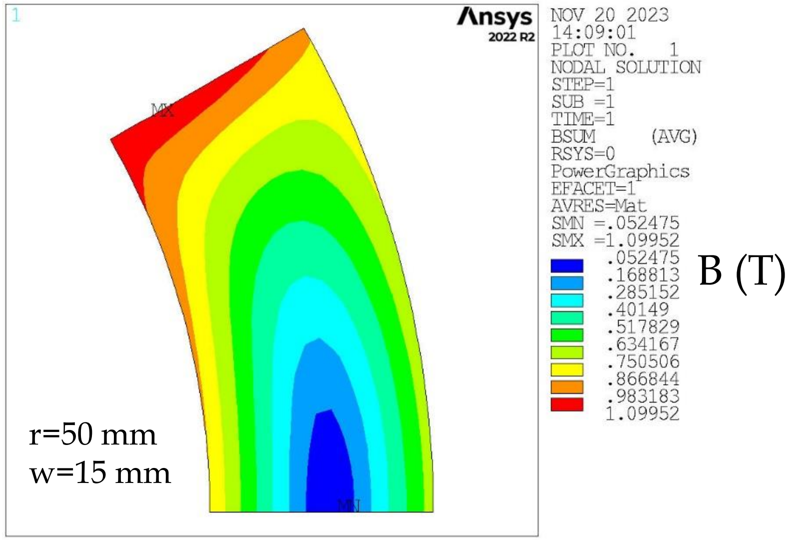
- The same approach can be used for a quadrupole
  - The loadline is given by $G = \gamma_C j \ln\left(1 + \frac{w}{R}\right)$, with $\gamma_c = \frac{2\mu_0}{\pi} \sin 2\alpha = 6.9 \cdot 10^{-7}$ Tm/A (sector quad $\alpha = 30^{o}$)
  - The peak field is given by $B_p = \lambda R G$, with λ to be found and R aperture radius (magnetic field in the center of a quadrupole is 0)
  - As done for dipoles, with a linear surface one has
    - $j_{SS} = \frac{\kappa s b}{1+\lambda R \gamma_c \kappa s \ln\left(1+\frac{w}{R}\right)}$,
    - $B_{p,SS} = \frac{\kappa s b \lambda R \gamma_c \ln\left(1+\frac{w}{R}\right)}{1+\lambda R \gamma_c \kappa s \ln\left(1+\frac{w}{R}\right)}$
    - $G_{SS} = \frac{\kappa s b \gamma_c \ln\left(1+\frac{w}{R}\right)}{1+\lambda R \gamma_c \kappa s \ln\left(1+\frac{w}{R}\right)}$

PEAK FIELD ON THE COILS

- For sector quadrupoles, the ratio between peak field and bore field $\lambda = \frac{B_p}{B_0}$ follow the fit $\lambda(w,r) \sim 1 + C_1 \frac{r}{w} + C_2 \frac{w}{r}$ (r aperture radius, w coil width of sector coil)
- the two constants can be derived from the position of the minimum (x_{MIN}, y_{MIN}) $\lambda(w,r) \sim 1 + \frac{y_{MIN}-1}{2x_{MIN}} \left(x_{MIN}^2 \frac{r}{w} + \frac{w}{r} \right)$
- to be noted that if 0.4<w/R<1.8 then 1.15< λ <1.2

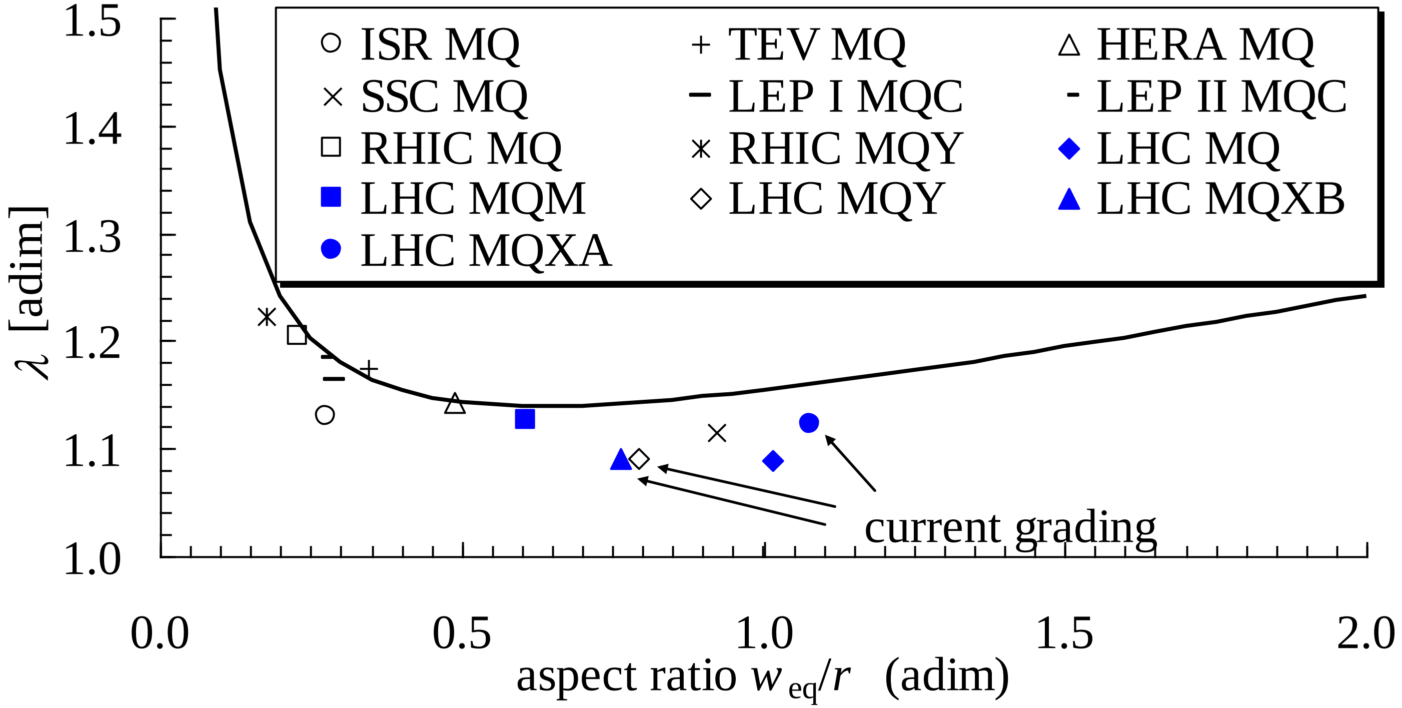


45° sector dipole



PEAK FIELD ON THE COILS

- as in dipoles, also for quadrupoles the fit stands also for real magnets



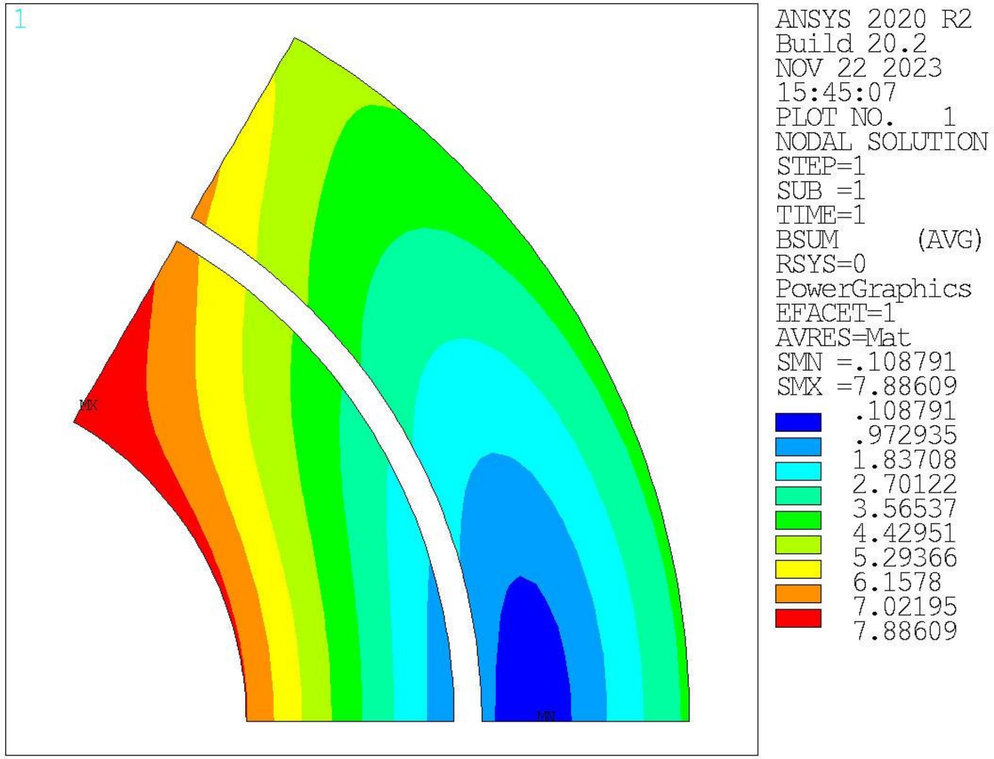
CURRENT GRADING

GRADING TECHNIQUE

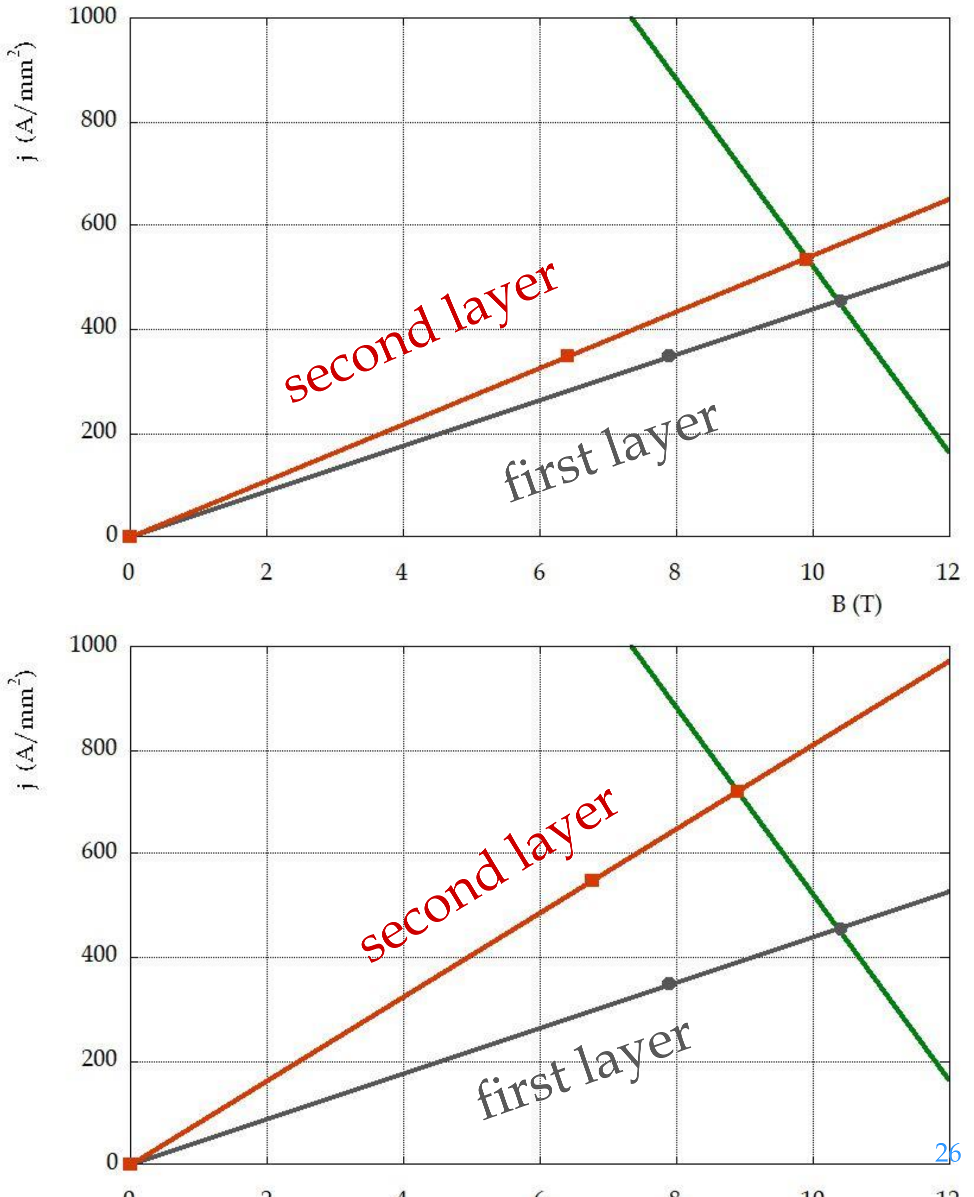
- The map of the field inside a coil is strongly non-uniform.
  Two grading possibilities:
  - In the outer layer the peak field can be lower than in the inner layer, therefore larger current density can be used, and thinner coil
  - The same current density is kept, but lower performance material is used for the lower field blocks, saving money

- Example: 2 concentric 60° sector dipole
  - Peak field 1st layer: 7.9 T
  - Peak field 2nd layer: 6.4 T
  - A_{cond} 1st layer: 2040 mm^2
  - A_{cond} 2nd layer: 3110 mm^2

R=25 mm
w=15 mm
j=350 A/mm^2
B_0=7.3 T

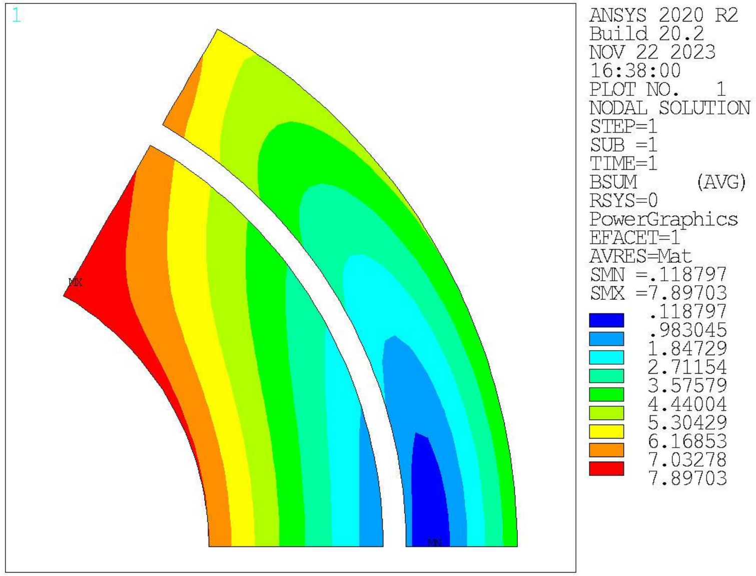


- Example: 2 concentric 60° sector dipole
  - Peak field 1st layer: 7.9 T → 24% LL margin
  - Peak field 2nd layer: 6.4 T → 35% LL margin
    A_{cond} 2nd layer: 3110 mm^2
  - We can change the slope of the second layer so to balance the 2 margins
  - By increasing j from 350 to 550 A/mm^2 in the 2nd layer and correspondingly decreasing w in the same proportion from 15 to 15x350/550=9.5 mm:
  - Peak field 1st layer: 7.9 T → 24% LL margin
  - Peak field 2nd layer: 6.8 T → 24% LL margin
    A_{cond} 2nd layer: 1860 mm^2 (40% less conductor)

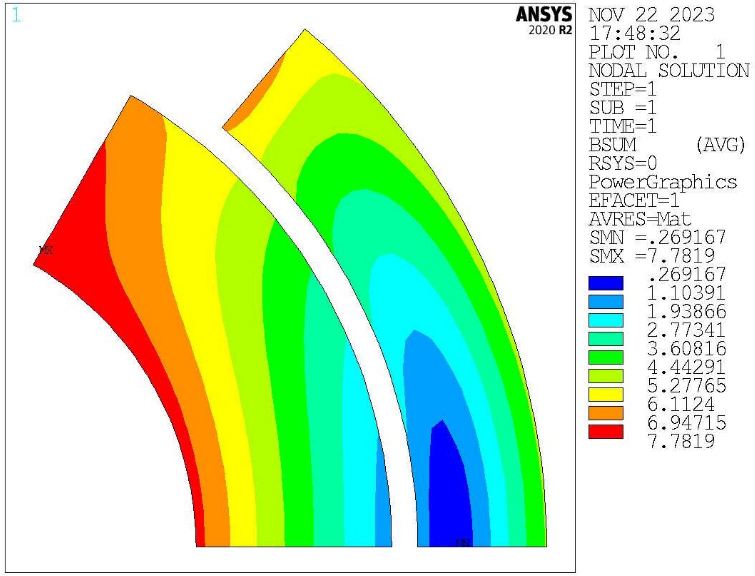


GRADING TECHNIQUE

- Graded option
  - 1st layer: w=15 mm, j=350 A/mm^2, B_p= 7.9 T → 24% LL margin
  - 2nd layer: w=9.5 mm, j= 550 A/mm^2, B_p= 6.8 T → 24% LL margin
    A_{cond} 2nd layer: 1860 mm^2 (40% less conductor)



- Graded option with smaller outer angle
  - 1st layer: w=15 mm, j=350 A/mm^2, B_p= 7.9 T → 24% LL margin
  - 2nd layer: w=10 mm, j= 600 A/mm^2,B_p= 6.5 T → 24% LL margin
    A_{cond} 2nd layer: 1640 mm^2 (47%less conductor)



- Cosθ option of the 16 T dipole in the framework of the EuroCirCol Project
- DOI:10.1109/TASC.2016.2642982

Strand diameter H.F./L.F. (mm)	1.1/0.71
Strand number H.F./L.F.	22/36
Bare cable inner thickness H.F./L.F. (mm)	1.892/1.204
Bare cable outer thickness H.F./L.F. (mm)	2.072/1.320
Bare cable width H.F./L.F. (mm)	13.2/13.3

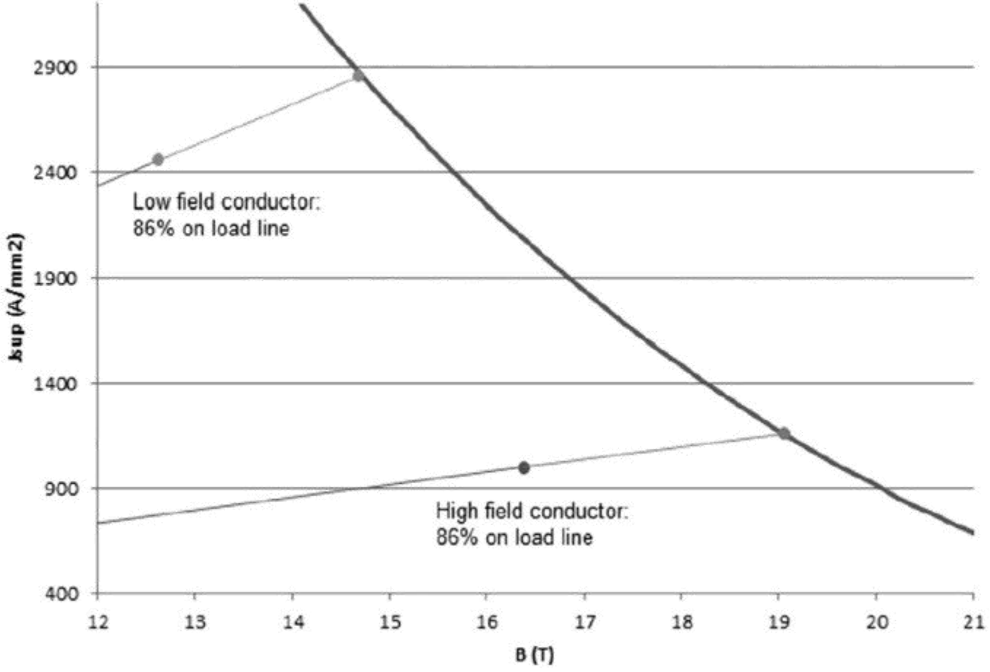


Fig. 1. Critical current density on non-copper fraction at the operating temperature of 1.9 K. The rows indicate the load lines of the peak field points in low field (upper row) and high field (lower line) conductors. Dots show the operating points at 86% on the load lines.

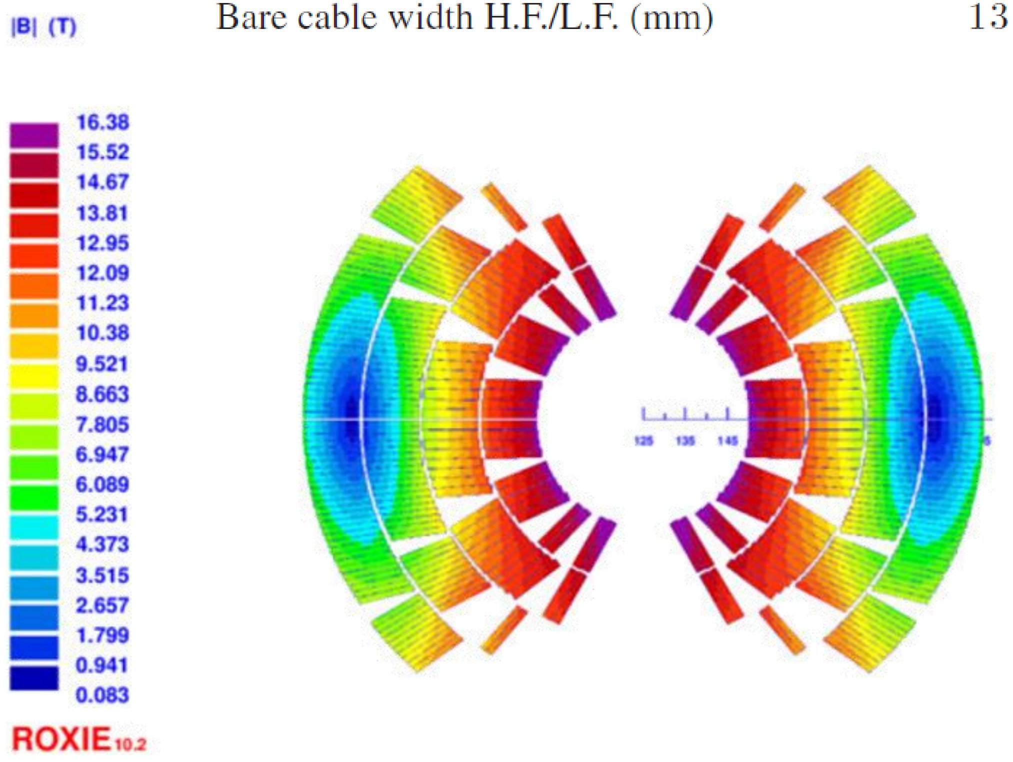


Fig. 2. Field distribution on one coil cross section (double aperture configuration) for the updated cosine-theta solution at Bo = 16 T.

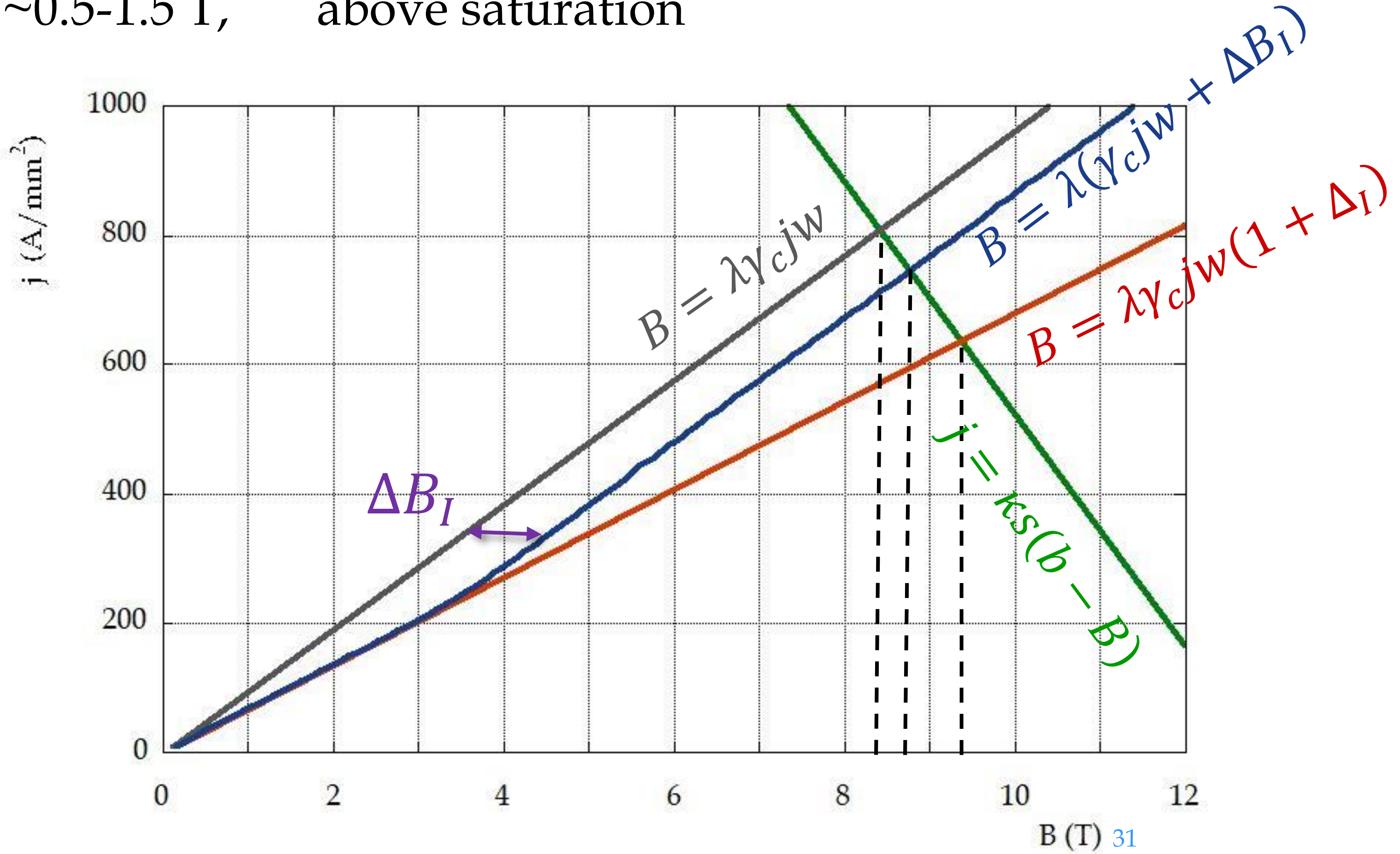
IRON YOKE EFFECT

IRON YOKE ROLE

- An iron yoke usually surrounds the collared coil – it has several functions
  - Keeps the return magnetic flux, avoiding fringe fields
  - could contribute to the mechanical structure
  - Considerably enhance the field for a given current density
    - The increase is relevant (10-30%), getting higher for thin coils
    - This allows using lower current density, reducing stress and easing protection
  - Increase the short sample field
    - The increase is small (a few percent) for "large" coils, but can be considerable for small widths
    - This action is effective when we are far from reaching the asymptotic limit of *b* (thin coils)

IRON YOKE CONTRIBUTION TO SHORT SAMPLE

- there are 2 regimes:
  - $B_{1I} = B_1(1+\Delta_I)$, $\Delta_I = \frac{R(R+w)}{R_I^2}$, below saturation ($B_1\Delta_I < \Delta B_I$)
  - $B_{1I} = B_1 + \Delta B_I$, ΔB_I=const.~0.5-1.5 T, above saturation
- below saturation:
  - $B_{SS} = \frac{bX\Delta_I}{1+\lambda X\Delta_I}$, $X = \kappa s \gamma_c w_{eq}$
    the increase of B_1 always leads
    the increase of B_{SS}
- above saturation:
  - $B_{SS} = \frac{bX+\Delta B_I}{1+\lambda X}$, $X = \kappa s \gamma_c w_{eq}$
    the value of B_{SS} is increased
    by a fixed amount



THANKS FOR THE ATTENTION