SUPERCONDUCTING MAGNETS EXERCISES

Ezio Todesco, S. Izquierdo Bermudez, A. Milanese
CERN TE Department
S. Farinon, INFN Genova
F. Toral, CIEMAT Madrid

CERN Accelerator School, Normal and Superconducting Magnets – St. Pölten, Austria, November 2023

PLAN

- This is a 6 hour module, spilt in two days (3+), and repeated four times
  - Each time it is given to about 25 participants
- The six-hour module is structured as follows
  - Day 1:
    - 30 minutes: recap of the main equations, plus exercise description
    - 2 h: exercise on analytical tools
    - 30 minutes correction and discussion
  - Day 2:
    - 30 minutes: tutorials about how using a finite element model, plus exercise description
    - 2 h: exercise on finite element tools
    - 30 minutes corrections and discussion

CONTENTS

- Day 1
  - Recap of the theory
  - Exercise
  - (solutions in a separate word file)
- Day 2
  - The FEMM code
  - Exercise

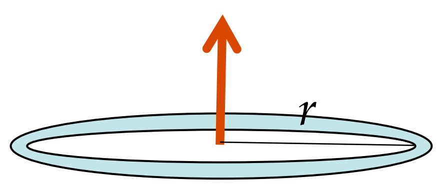
Units

- All the equations are given in the international system
  - Once you gain experience, you can use mixed variables but we suggest to start with homogeneous variables
- Length: m
  - For the dimension of magnet cross-sections, practical units are mm
  - In some codes for physics (Fluka, …) practical units are cm
- Mass: kg
- Field B: T
  - 1 tesla = 10 000 gauss; remember magnetic field on earth surface is 0.5 gauss
- Current density: A/m^2
  - Practical units are $A/mm^2 = 10^6\ A/m^2$
- Pressure: Pa
  - Practical units are MPa = 10^6 Pa

Constants

- We discuss magnets to achieve fields up to 2 T (resistive) or 10 T and beyond
  - Highest field in a accelerator dipole: order of 15 T
  - Highest field achieved in a solenoid in a static way: order of 50 T
  - Highest field in explosive devices: order of 100 T

- In the international system, the permeability of vacuum is $\mu_0=4\pi 10^{-7}$ T m / A, i.e. about 10^{-6} T m / A
  - This means that 1 A in a coil of 1 cm radius make in the centre a field of 0.5×10^{-4} T , that is close to the Earth magnetic field
  - Never forget that μ_0 is small: you need millions of ampere to make some teslas



$$B = \frac{\mu_0 I}{2r}$$

Field multipolar expansion

- Reference system is (x,y), that is the transverse plane to the beam trajectory (or to the magnet axis)
- The multipolar expansion of the field is

$$B_y + iB_x(x,y) \equiv \sum_{n=1}^{\infty} C_n \left(\frac{x+iy}{R_{ref}} \right)^{n-1} \qquad C_n = B_n + iA_n$$

  - Order n=1 is dipole, n=2 quadrupole, …
  - Where R_{ref} is usually selected as the 2/3 of the aperture radius
  - The coefficients of the multipolar expansion C_n are expresssed in T
- For a dipole, the normalized coefficients are

$$B_y + iB_x(x,y) \equiv 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_{ref}} \right)^{n-1}$$

  - The normalized multipolar coefficients b_n and a_n are adimensional

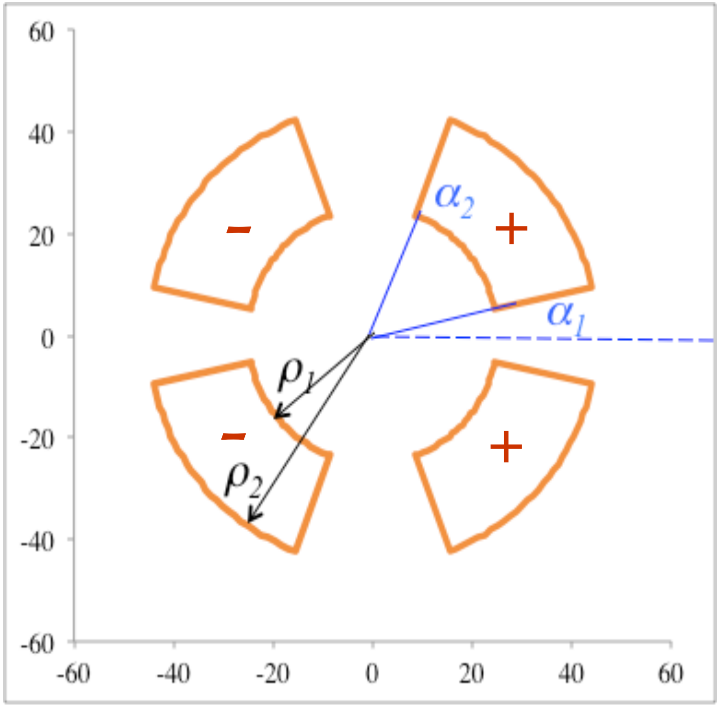
Magnetic field of a sector coil

- The multipoles of a sector coil with dipole symmetry and overall current density j are

$$C_n = -\frac{2\mu_0 R_{ref}^{n-1}}{\pi n(2-n)}\left(\sin n\alpha_2 - \sin n\alpha_1\right)\left(\rho_2^{2-n} - \rho_1^{2-n}\right) j \qquad n \neq 2$$

- The field is given by

$$B_1 = \left[\frac{2\mu_0}{\pi}\left(\sin\alpha_2 - \sin\alpha_1\right)\right]\left(\rho_2 - \rho_1\right) j$$



Magnetic field of a sector coil

- Main field for a sector of inner radius r (ρ_1=r), width w (ρ_2-ρ_1=w) and angle from 0 to α (α_2=α), (α_1=0)

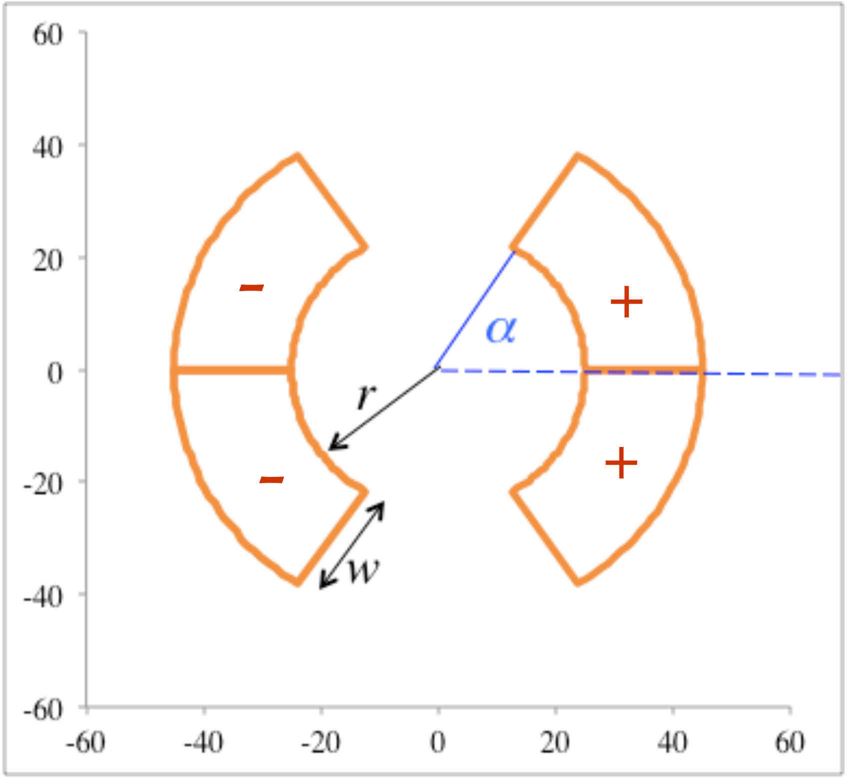
$$B_1 = \left[\frac{2\mu_0}{\pi}\sin\alpha\right]wj$$

- Definition of coil efficiency

$$B_1 \equiv \gamma_c wj$$

- For this case of 60º sector coil

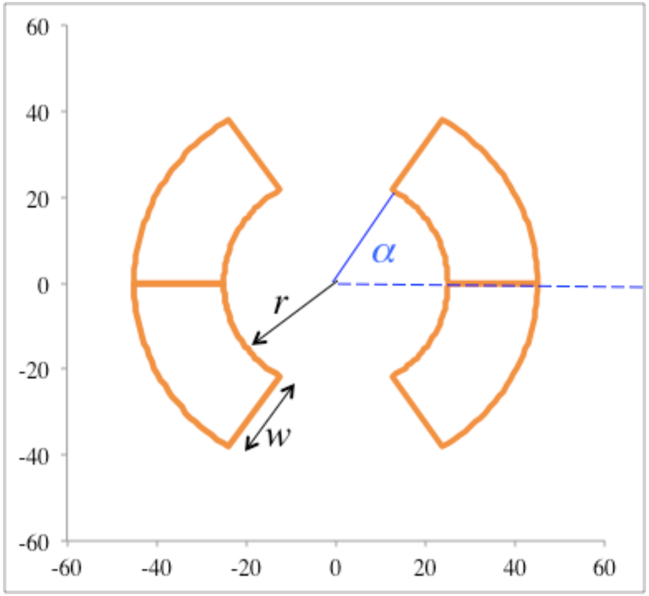
$$\gamma_c = \frac{2\mu_0}{\pi}\sin\alpha = 6.9\times10^{-7}\ \text{Tm/A}$$



Multipoles of a sector coil

- Multipoles or a sector of inner radius r ($\rho_1=r$) , width w (ρ_2-$\rho_1=w$) and angle from 0 to α ($\alpha_2=\alpha$), ($\alpha_1=0$)

$$B_3 = -\frac{2\mu_0 R_{ref}^2}{3\pi}\sin 3\alpha\left(\frac{1}{r+w}-\frac{1}{r}\right)j$$



$$B_5 = -\frac{2\mu_0 R_{ref}^4}{15\pi}\sin 5\alpha\left(\frac{1}{(r+w)^3}-\frac{1}{r^3}\right)j$$

$$B_7 = -\frac{2\mu_0 R_{ref}^6}{35\pi}\sin 7\alpha\left(\frac{1}{(r+w)^5}-\frac{1}{r^5}\right)j$$

Effect of iron

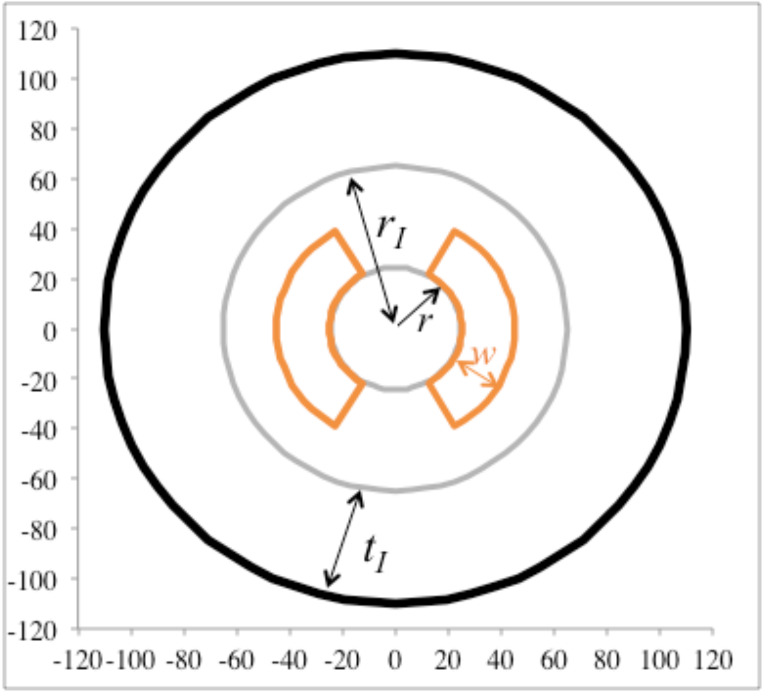
- Field increase due to non saturated iron $\Delta_I \equiv \frac{\Delta B_1}{B_1} = \frac{r(r+w)}{r_I^2}$

$$B_1 = \gamma_c (1+\Delta_I) wj$$

- Shielding condition

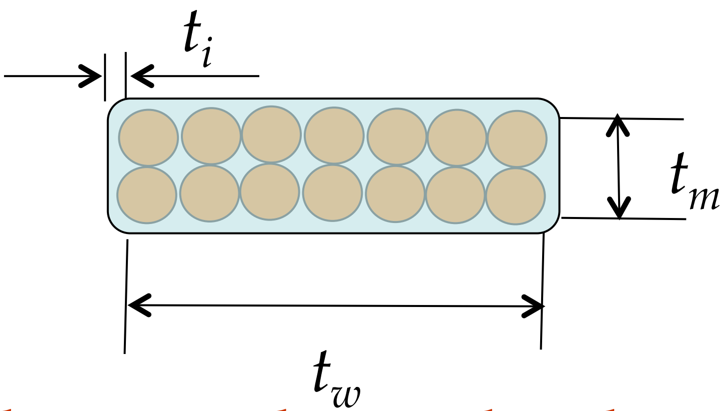
$$t_I = \frac{rB_1}{B_{sat}}$$

- $B_{sat} \approx 2$ T



Cable filling factor

- ν: copper to superconductor ratio
  - ν=1 means ν/(1+ν)=50% of Cu and 1/(1+ν)=50% of superconductor
  - ν=2 means ν/(1+ν)=66% of Cu and 1/(1+ν)=33% of superconductor
- d: strand diameter
- N_s= number of strand per cable
- t_m= mid thickness of bare cable
- t_w= width of bare cable
- t_i = insulation thickness



- Filling factor: fraction of superconductor in the insulated cable
  - (This expression neglects the twist pitch)

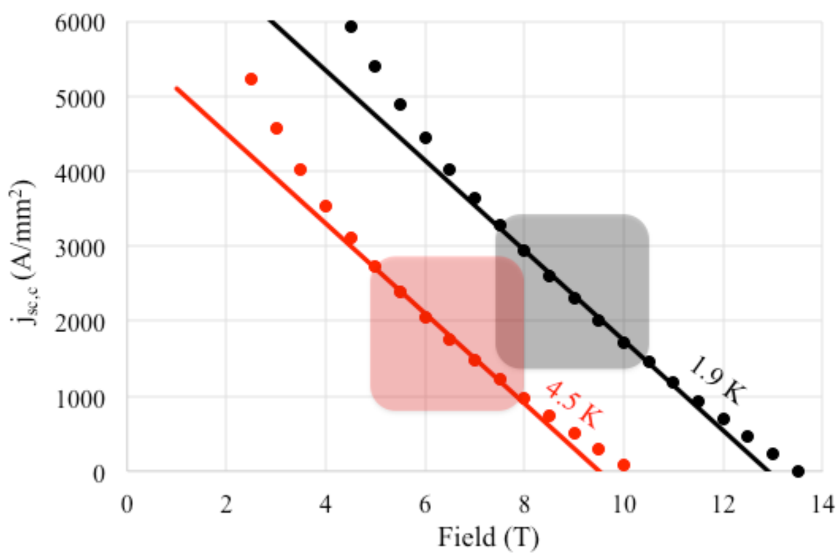
$$\kappa \equiv \frac{1}{1+\nu}\frac{\pi N_s d^2}{4}\frac{1}{(t_m+2t_i)(t_w+2t_i)}$$

Critical surface linearization

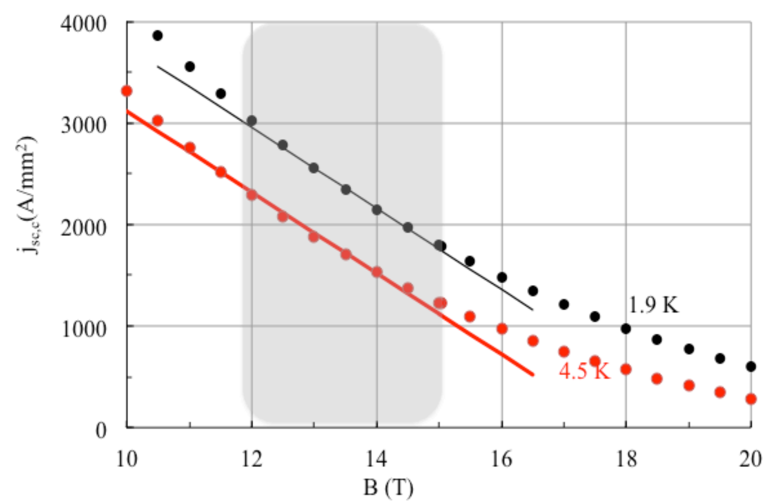
- Example of linearization of the critical surface of low temperature superconductors

$$j_{sc}(B) = s(b - B)$$

- Nb-Ti: s=600 A/(T mm^2)=600×10^6 A/(T m^2) and b=12.9 T at 1.9 K and b=9.5 T at 4.5 K
- For Nb_3Sn s=400 A/(T mm^2)=400×10^6 A/(T mm^2) and b=19.4 T at 1.9 K and b=17.8 T at 4.5 K



Linearization for Nb-Ti critical surface



Linearization for Nb_3Sn critical surface

Short sample conditions

- Short sample field

$$B_{ss} = b\frac{ks\gamma_c(1+\Delta_I)w}{1+\lambda ks\gamma_c(1+\Delta_I)w}$$

  - γ_χ (T m/A) coil efficiency (see slide 8)
  - w (m) width of the coil (see slide 8)
  - Δ_I (adim) iron contribution (see slide 10)
  - κ (adim) fraction of superconductor in the insulated cable (see slide 11)
  - s (A/T m), b (T) slope and intercept of critical surface (see slide 12)
  - λ ratio peak field in the coil / bore field
- Can be written as
  - with X (adim) defined as

$$X \equiv ks\gamma_c(1+\Delta_I)w$$

$$B_{ss} = b\frac{X}{1+\lambda X}$$

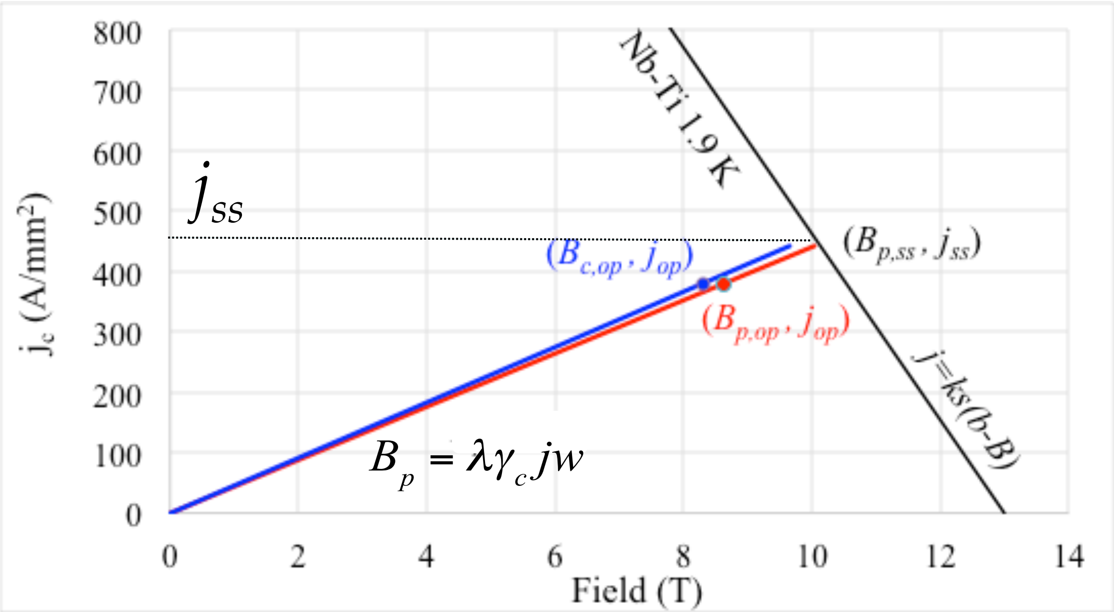
Margin on the loadline

- Loadline fraction

$$f \equiv \frac{j_{op}}{j_{ss}} \approx \frac{B_{op}}{B_{ss}}$$

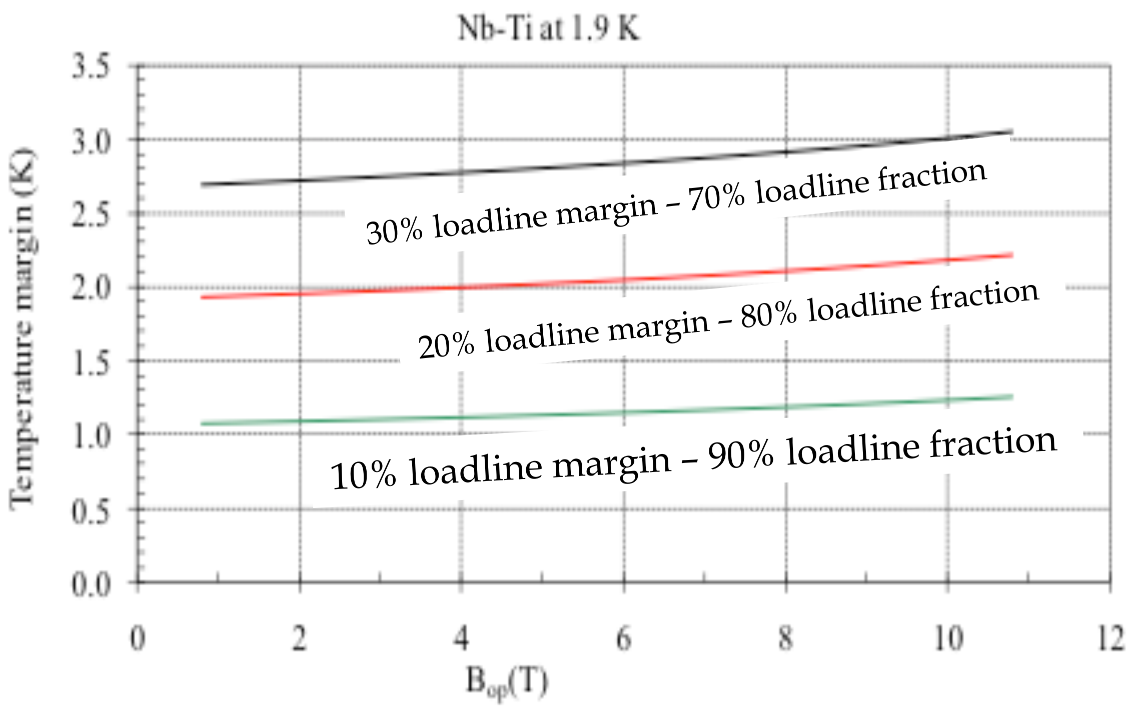
- Loadline margin

$$1 - f$$



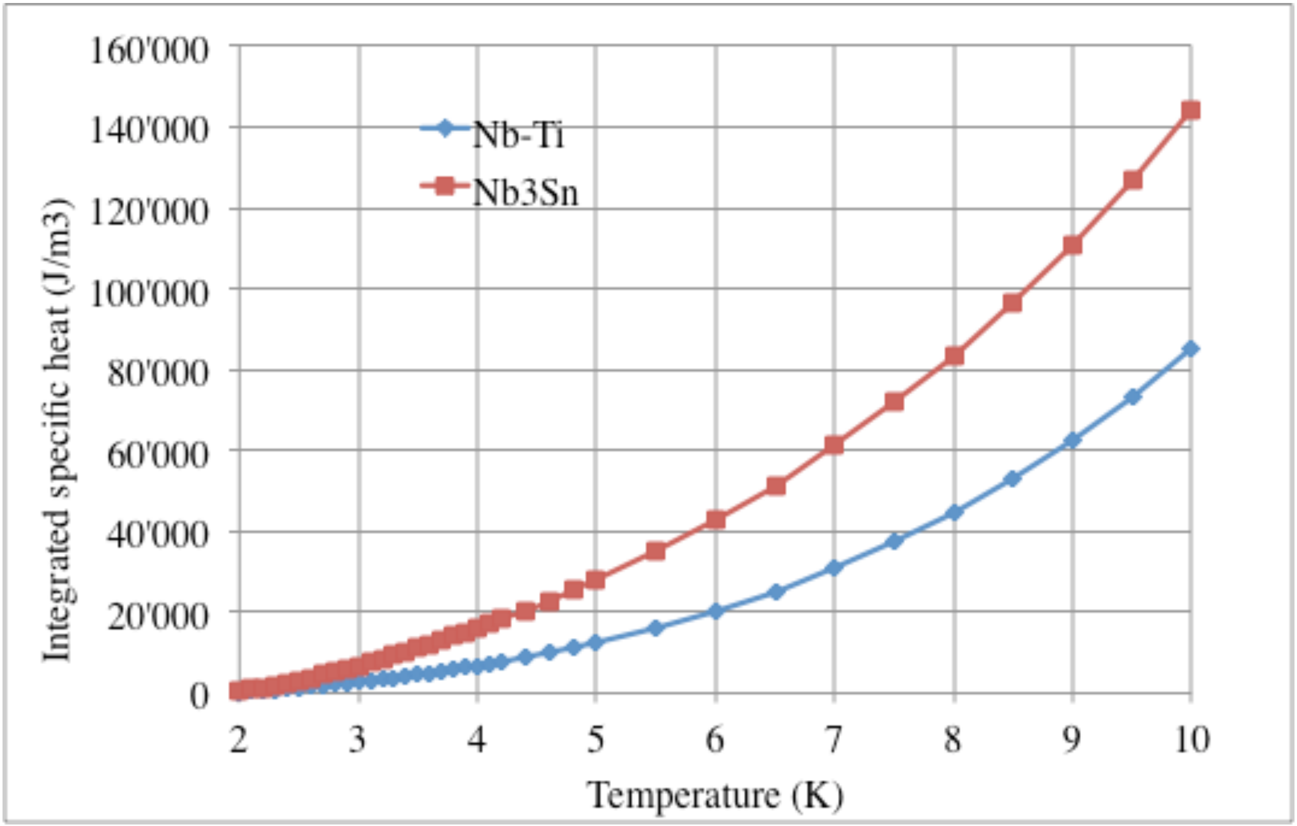
Temperature margin

- Temperature margin: how much you have to heat the superconductor to cross the critical surface with your operational point
  - In Nb-Ti at 1.9 K, about 1 K for each 10% of loadline margin



Enthalpy margin

- Enthalpy margin: how much energy density you need to increase the temperature above the temperature margin
  - Depends on coil property, below the plot for a typical Nb3Sn and Nb-Ti coil

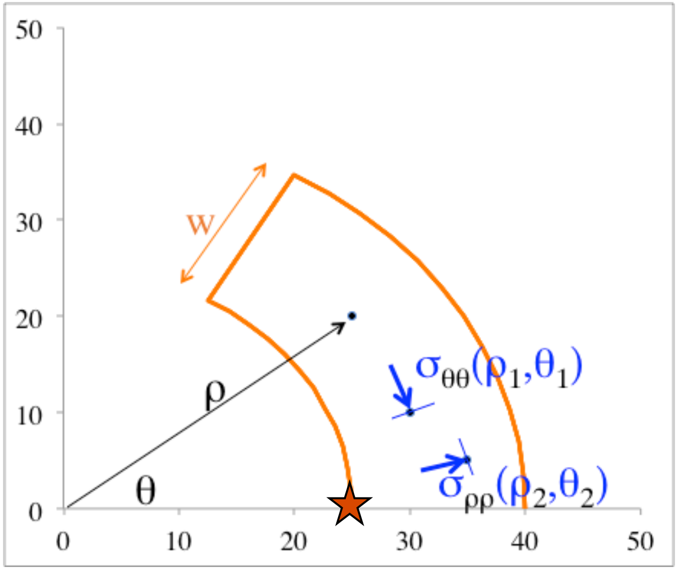


Forces and stress

- Stress due to the accumulation of forces in the midplane for a sector coil of aperture α on the bore radius

$$\sigma_{\theta\theta}(r,0) = \left(\cos\alpha - 1\right) Bjr$$

  - *B*: bore field (T)
  - *j*: overall current density (A/m²)
  - *r*: aperture radius (m)
  - σ will be in Pa



  - For a 60° sector coil cos(60°)=1/2 and

$$\sigma_{\theta\theta}(r,0) = -\frac{1}{2} Bjr$$

Magnetic pressure, stored energy and forces in the heads

- Magnetic pressure or energy density of magnetic field
  - Note that this is a pressure (N/m^2)
  - Indeed, it is also an energy density (N.m/m^3)= J/m^3

$$\sigma_B = \frac{B^2}{2\mu_0}$$

- Stored energy in a magnet
  - Can be approximated by
  - B: bore field
  - r: aperture radius
  - w: coil width
  - l_m: magnet length

$$U = \int_V \frac{\left[B(x,y,z)\right]^2}{2\mu_0} dx\, dy\, dz$$

$$U \approx \frac{B^2}{2\mu_0}\pi(r+w)^2 l_m$$

- Forces in magnet heads

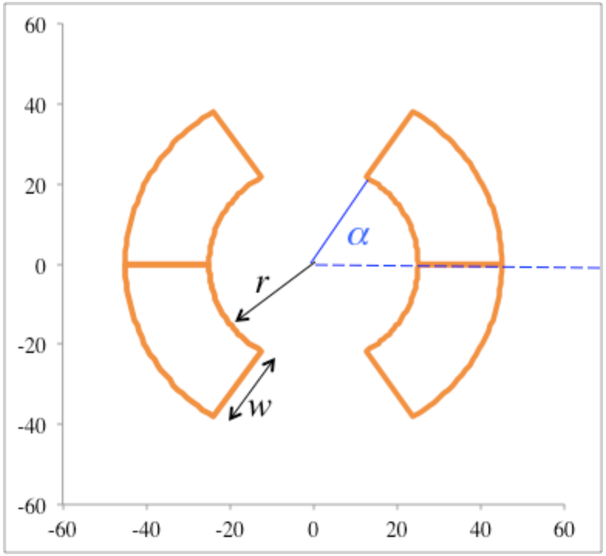
$$F = \frac{U}{l_m} \approx \frac{B^2}{2\mu_0}\pi(r+w)^2$$

CONTENTS

- Day 1
  - Recap of the theory
  - Exercise
  - (solutions in a separate word file)

Day 1 - exercise 1: electromagnetic design

- We consider a dipole made as a 60° sector coil, with 50 mm aperture (diameter) and 15 mm width cable.

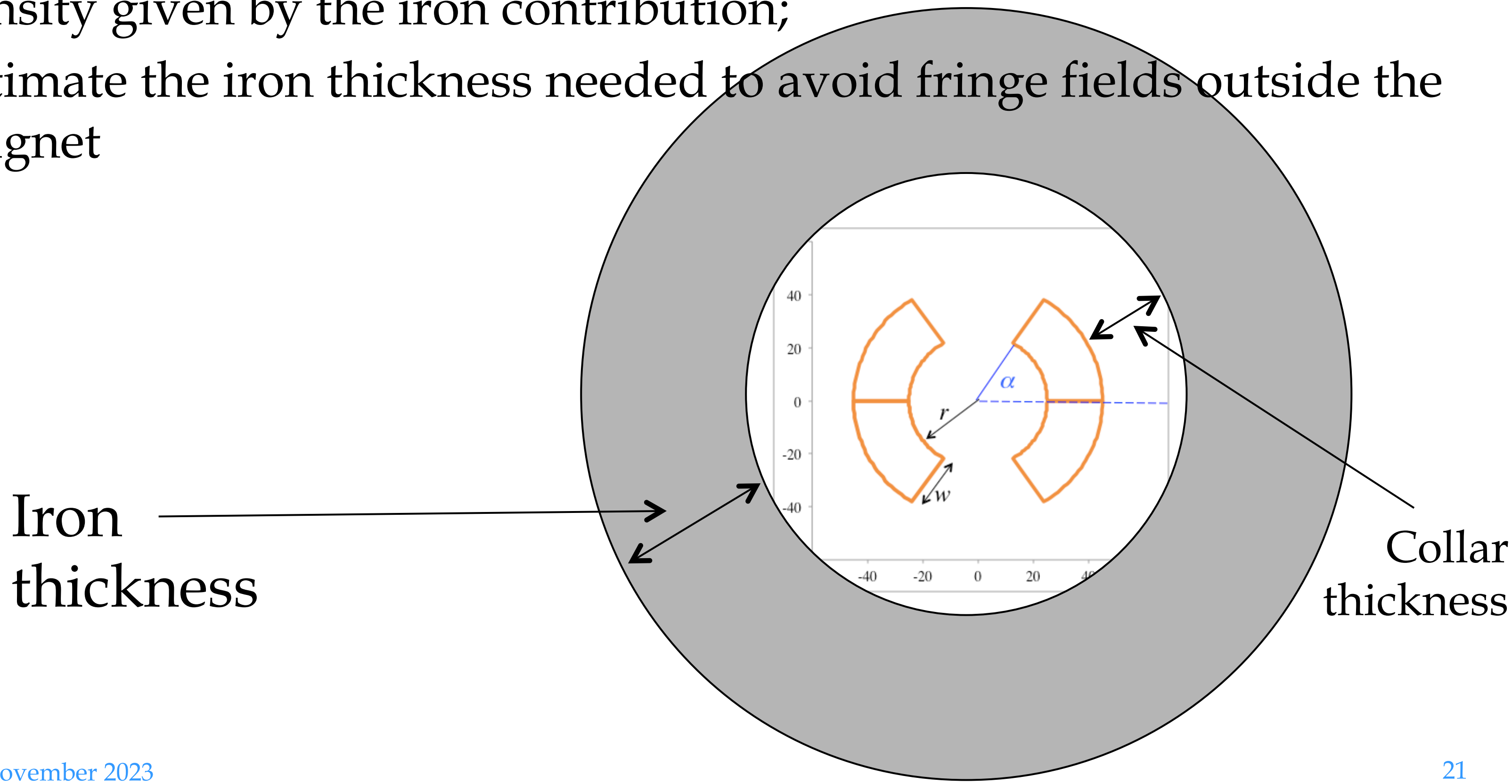


- <u>Exercise 1</u>:
  - Compute the overall current density needed to have a 5 T bore field , and compute the field harmonics b_3, b_5 and b_7, using 17 mm as reference radius ;

Note: computing the field harmonics is the most complex equation – I suggest you to do it at the end, when you have completed excercise 6

Day 1 – exercise 2: iron

- Exercise 2: iron
  - Estimate the relative field increase due to the presence of non-saturated iron assuming 25 mm thick collars;
  - Keeping the same 5 T field, estimate the reduction of the current density given by the iron contribution;
  - Estimate the iron thickness needed to avoid fringe fields outside the magnet



Day 1 – exercise 3: cable filling factor and short sample

- Exercise 3: cable filling factor and short sample
  - Considering a cable with 36 strands of 0.825 mm diameter, copper to superconductor ratio of 1.95, bare cable mid-thickness of 1.480 mm and width of 15.100 mm, and insulation thickness of 0.13 mm, compute the fraction of superconductor in the insulated cable (filling factor)
  - In this case, compute the current density in the superconductor to have 5 T dipole field with iron

Day 1 – exercise 4: short sample conditions and margins

- Exercise 4: short sample conditions and margins
  - Assuming that the conductor is Nb-Ti, and that the peak field/bore field is 1.05, compute the loadline margin at 4.5 K and at 1.9 K, with iron;
  - Assuming an operational temperature of 1.9 K, give an estimate of the temperature margin and the enthalphy margin using the plots given in slides 15 and 16;

Day 1 – exercise 5 and 6: mechanics and protection

- Exercise 5: mechanics
  - Compute the accumulation of azimuthal stress in the midplane on the bore, for the case without iron and with iron ;
  - Compute the magnetic pressure and the forces on the coil heads ;

- Exercise 6: protection
  - Compute the ratio between magnet stored energy and insulated coil volume; check if this is lower than 0.05 J/mm^3 ;
  - Compute the current density in the copper at the beginning of quench; check if this is lower than 1000 A/mm^2 ;

Day 1 – bonus exercise

- Bonus exercise

  - Compare the volumetric energy density (J/m^3) of a 5 T field to the volumetric energy density of gasoline and to the volumetric energy density of a battery for electric car ; which field is needed to have the same volumetric energetic density as gasoline ?
  - Assuming that Nb-Ti cost is 200$/kg, stainless steel is 10 $/kg and iron is 3 $/kg, compute the cost of the raw materials – take as density of Nb-Ti 5700 kg/m^3

Day 1 – Further development

- Sensitive analysis for the previous case :
  - What is the sextupole for a 61 degrees sector ?
  - What is the increase of the short sample field if the copper fraction is decreased from 1.95 to 1.5 ?
  - What is the increase of the short sample field if the iron is placed on directly the outer layer of the coil ?

Day 1 – Further development (for the enthusiast)

- Double layer case:
  - Implement the equation of page 7 in a spreadsheet for the case of a dipole based two layers of width 7.5 mm width each: find the angles of the inner and of the outer layer that allow cancel both b_3 and b_5 ;
  - For the same configuration, compute the current density needed to have the bore field of 5 T in the case without iron; compare with the result of tje pervious exercise