Superconductivity

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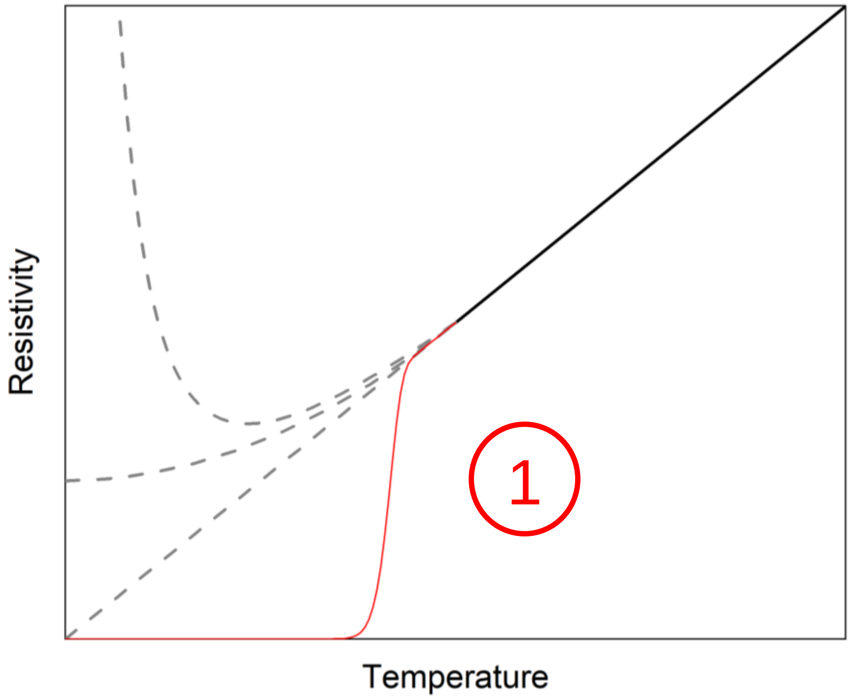
Outline

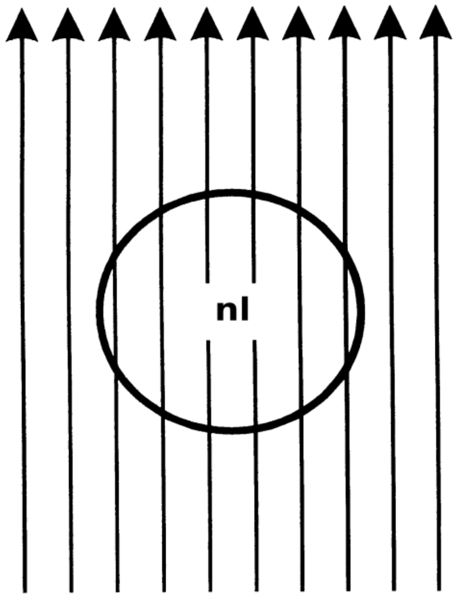
- What is superconductivity?
  - Defining properties: zero resistivity, flux expulsion
- Important properties
  - Condensation Energy, superfluid density
  - Fux(oid) quantization
- Limitations of superconductivity
  - Field (type-I and type-II superconductors)
    - Ginzburg Landau theory
  - Temperature
  - Current
- Flux pinning, critical currents
- BCS theory

What is superconductivity?

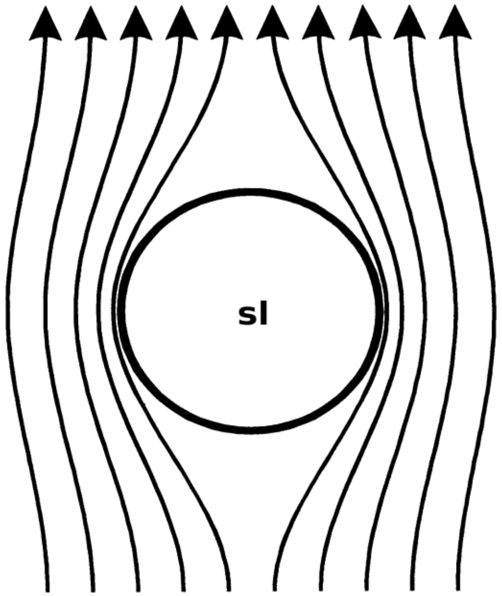
- Thermodynamic state of the electron system
- Defining properties:
  - Zero-resistivity (at least at low magnetic fields and currents)
  - Meissner effect: flux expulsion (at low magnetic fields)



Meissner Effect



normal

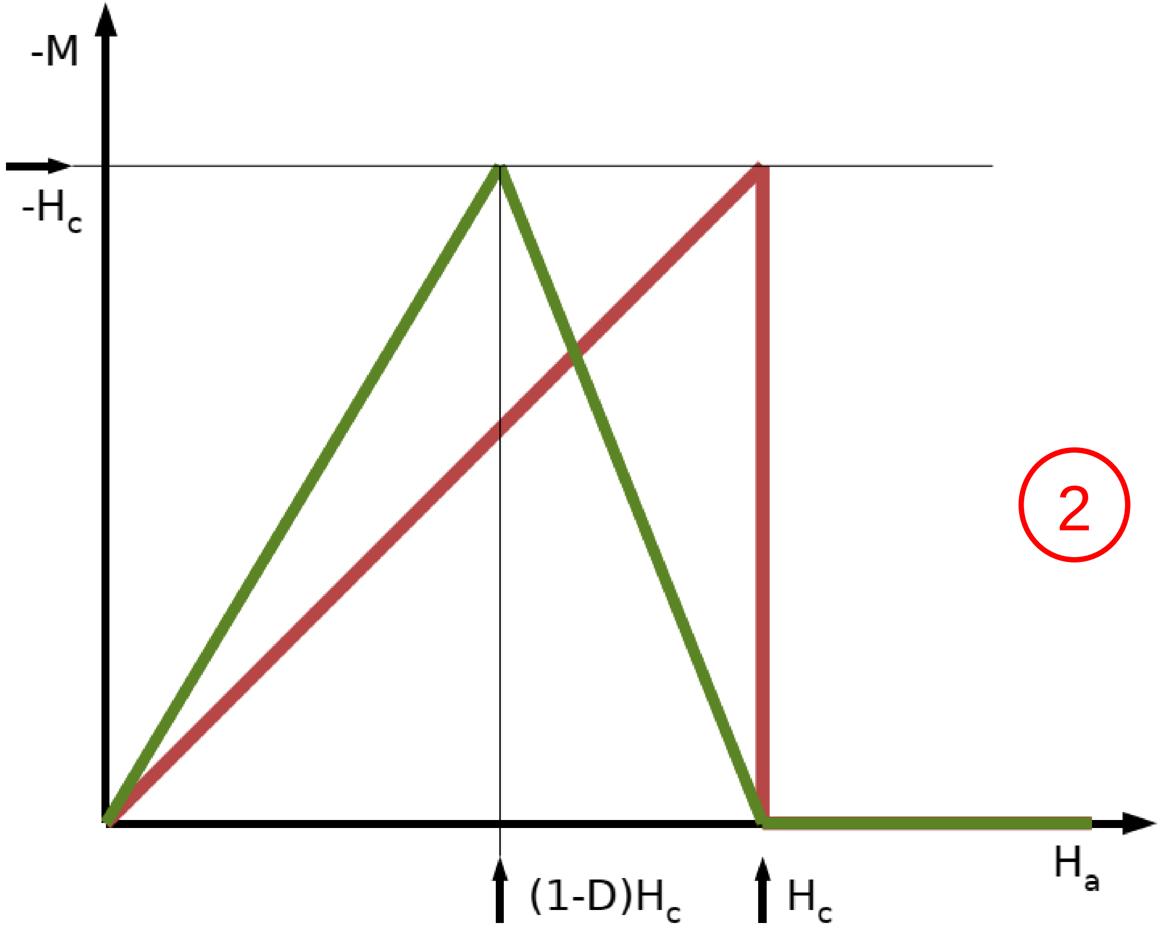


superconducting

- Perfect flux expulsion (except for a small surface layer)
- Ideal diamagnet: $\chi = -1$: $M = \chi H = -H$

Meissner Effect

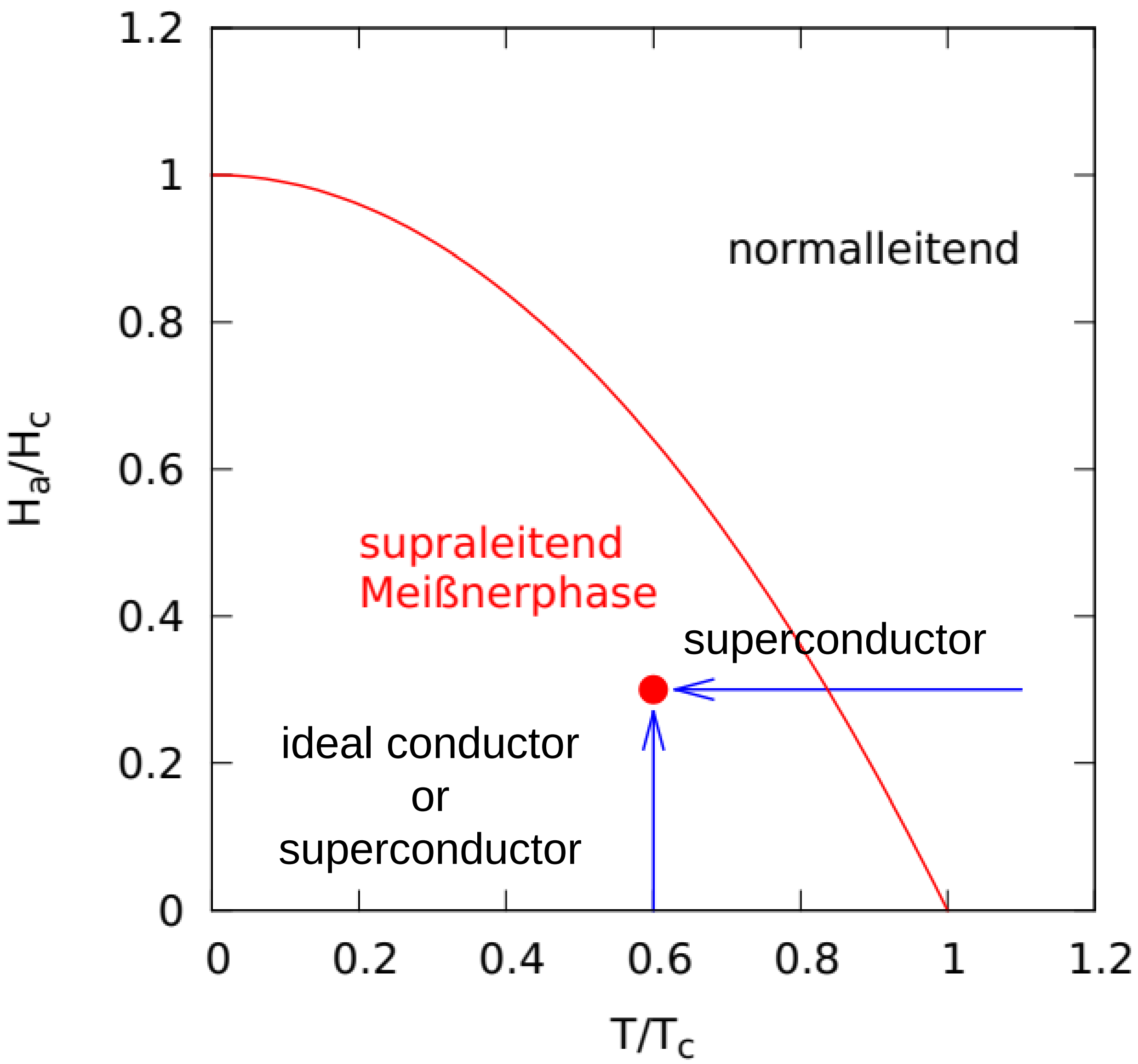
$$B = \mu_0(H + M) = |M = -H| = 0$$



Superconductivity disappears at a certain field:

2 Thermodynamical critical field: H_c

Meissner Effect



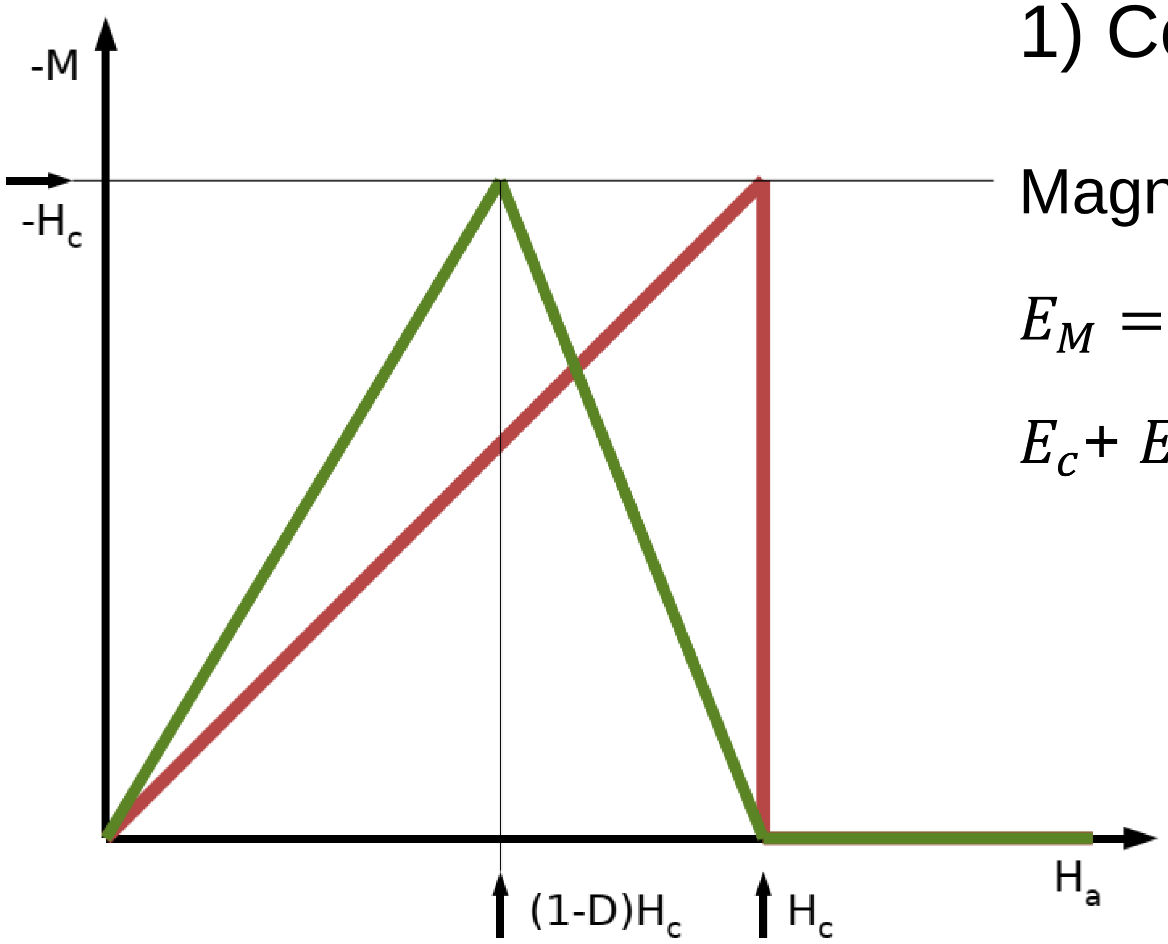
$$H_c = H_c(0\ K)\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

Phenomenological observation, however in agreement with theory (within a few percent)

Independent of history:
thermodynamic state!

What do we learn?

(from these basic properties)



1) Condensation energy E_c:

Magnetization energy:

$$E_M = -\int_0^H \mu_0 M dH = \mu_0 \int_0^H H dH = \frac{\mu_0 H^2}{2}$$

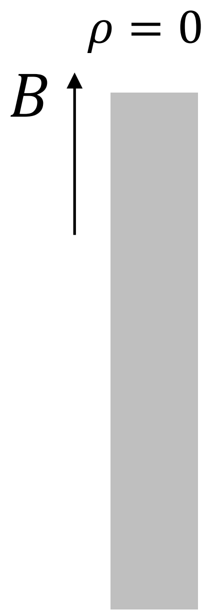
$E_c + E_M = 0$ at $H = H_c$ $\rightarrow$

$$\boldsymbol{E_c = -\frac{\mu_0 H_c^2}{2}}$$

What do we learn?

2) London penetration depth of the magnetic field λ_L:

Graphically: Long cylindrical, ideal conductor. Parallel magnetic field is applied. Induction voltage $U = -\frac{d\phi}{dt}$ induces a screening current. Field decreases towards the center.



Formally: Newton's law: $m_s\dot{\vec{v}} = q_s\vec{E} \ \rightarrow \ \dot{\vec{j}} = \frac{n_s q_s^2}{m_s}\vec{E}$…1st London equation.

$\vec{\nabla} \times \dot{\vec{j}} = \frac{n_s q_s^2}{m_s}\vec{\nabla} \times \vec{E} \rightarrow \vec{\nabla} \times \vec{\nabla}\dot{\vec{H}} = -\Delta\dot{\vec{H}} = -\frac{n_s q_s^2}{m_s}\mu_0 \ \dot{\vec{H}}$

2nd London equation: $\Delta\vec{H} = \frac{\mu_0 n_s q_s^2}{m_s}\vec{H} =: \frac{1}{\lambda_L^2}\vec{H}$ (Meissner effect)

1 dimensional: $\frac{d^2H}{dx^2} = \frac{1}{\lambda_L^2}H$; particular solution: $H = H_0 e^{-\frac{x}{\lambda_L}}$

Characteristic shielding length of the magnetic field: $\lambda_L = \sqrt{\frac{m_s}{\mu_0 n_s q_s^2}}$

What do we learn?

3) Canonical momentum (within London theory)

Skipping the time derivative earlier (in $\vec{\nabla} \times \dot{\vec{j}} = -\frac{n_s q_s^2}{m_s} \mu_0 \dot{\vec{H}}$) leads to

$$-\mu_0 \lambda_L^2 \vec{\nabla} \times \vec{j} = \mu_0 \vec{H} = \vec{\nabla} \times \vec{A} \quad \text{or} \quad \vec{\nabla} \times (\mu_0 \lambda_L^2 \vec{j} + \vec{A}) = 0$$

One possible choice: $\mu_0 \lambda_L^2 \vec{j} + \vec{A}$ (London gauge)

Canonical momentum of a charged particle: $\vec{p} = m\vec{v} + q\vec{A} = q(\frac{m}{nq^2}\vec{j} + \vec{A})$

For a superconductor within London theory: $\overrightarrow{p_s} = q_s(\mu_0 \lambda_L^2 \vec{j} + \vec{A})$

Meissner effect: $\overrightarrow{p_s} = 0$ (momentum that is conserved in a magnetic field)

Fluxoid quantization

New ingredient: All superconducting particles are described by the same wave function ψ!

Momentum operator: $-i\hbar\vec{\nabla}$ $\left(-i\hbar\vec{\nabla}\psi = \overrightarrow{p_s}\psi\right)$, $\overrightarrow{p_s} = \hbar\overrightarrow{k_s}$

ψ is a complex valued function: $\psi = |\psi|e^{i\theta}$, $\vec{\nabla}\psi = e^{i\theta}\vec{\nabla}|\psi| + |\psi|e^{i\theta}i\vec{\nabla}\theta = \psi i\vec{\nabla}\theta$

$$-i\hbar\vec{\nabla}\psi = \hbar\psi\vec{\nabla}\theta = \overrightarrow{p_s}\psi \rightarrow \boldsymbol{\vec{\nabla}\theta = \frac{1}{\hbar}\overrightarrow{p_s}}$$

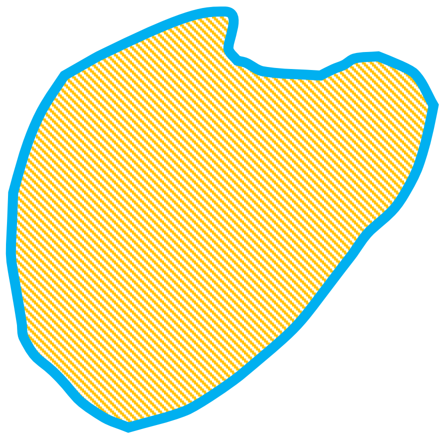
$$\theta(\overrightarrow{r_2}) - \theta(\overrightarrow{r_1}) = \frac{1}{\hbar}\int_{\overrightarrow{r_1}}^{\overrightarrow{r_2}} q_s(\mu_0\lambda_L^2\vec{J} + \vec{A})\mathrm{d}\vec{r}$$

Closed path: $\frac{q_s}{\hbar}\oint(\mu_0\lambda_L^2\vec{J} + \vec{A})\mathrm{d}\vec{r} = n2\pi$

Fluxoid quantization

$$\oint (\mu_0 \lambda_L^2 \vec{j} + \vec{A}) \mathrm{d}\vec{r} = n \frac{h}{q_s}$$

Stokes theorem: $\oint \vec{A} \mathrm{d}\vec{r} = \iint_F \vec{\nabla} \times \vec{A} \mathrm{d}\vec{f} = \iint_F \vec{B} \mathrm{d}\vec{f} = \phi_i$

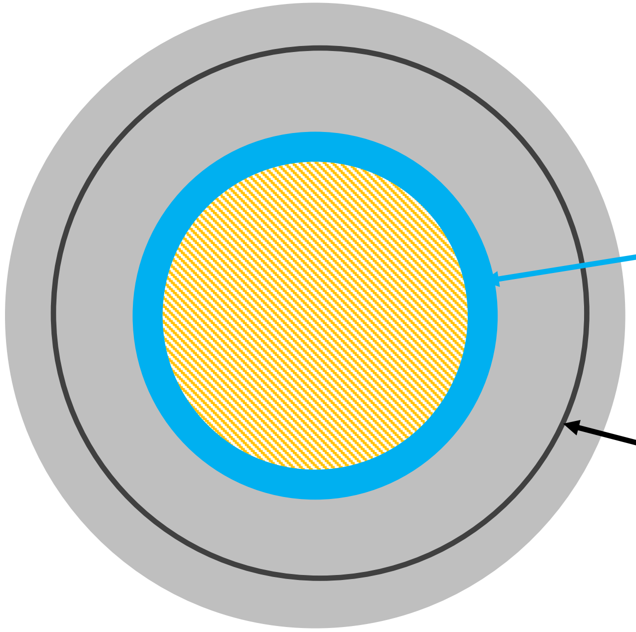


Fluxoid quantization:

$$\phi_i + \frac{m_s}{n_s q_s^2} \oint \vec{j} \mathrm{d}\vec{r} = n \frac{h}{q_s}$$

Flux quantization

$$\phi_i + \frac{m_s}{n_s q_s^2} \oint \vec{j} \mathrm{d}\vec{r} = n \frac{h}{q_s} := n\phi_0$$



Region of shielding currents (fluxoid qantization)

Integration path inside the superconductor:

$\vec{j} \approx 0$: Flux quantization: $\phi_i = n\phi_0$

Flux quantum: $\boldsymbol{\phi_0 = \frac{h}{2e}}$ (experimental value)

→ paired electons (holes)

London Theory

$\rightarrow m_s = 2m_e, e_s = 2e, n_s = \frac{n}{2}$ (n...density of condensed charge carriers)

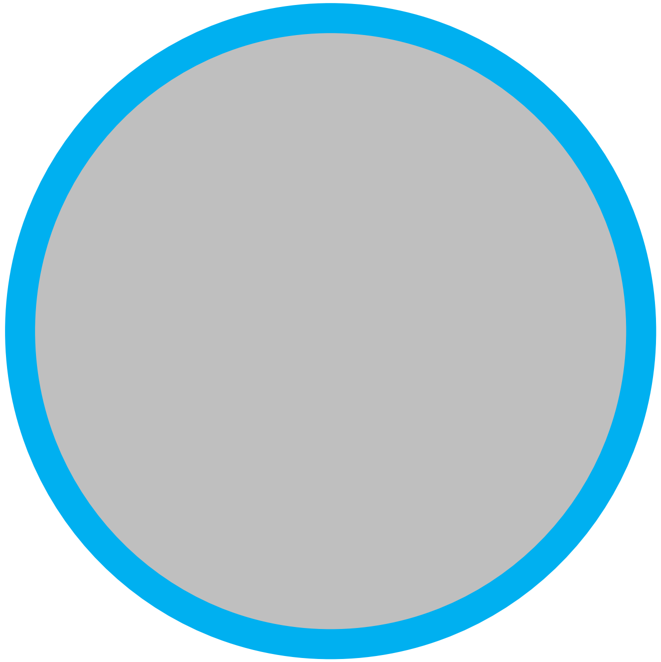
London penetration depth: $\lambda_L = \sqrt{\frac{m_e}{\mu_0 n e^2}}$

Superfluid density: $n \propto \frac{1}{\lambda_L^2}$

Shortcomings of London theory:

- Local theory (point particles, e.g. $\overrightarrow{p_s} = q_s(\mu_0 \lambda_L^2 \vec{J} + \vec{A})$)
- Superfluid density is assumed as constant.

Type-II superconductors

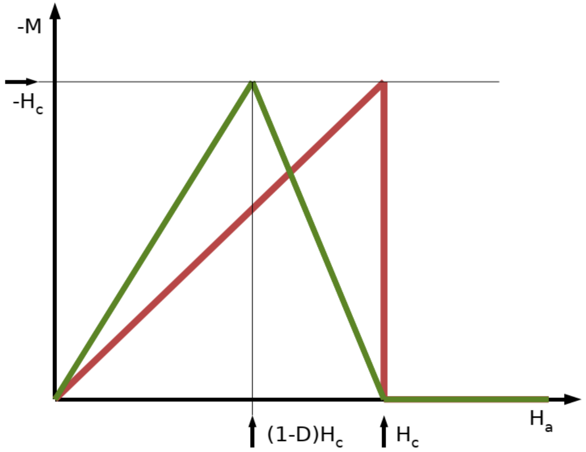


$$\phi_s + \frac{m_s}{n_s q_s^2} \oint \vec{j} d\vec{r} = n\phi_0$$

Meissner state $n = 0$:
Flux penetrates only at the surface, shielding currents given by fluxoid qantization.

High energy cost for magnetization:

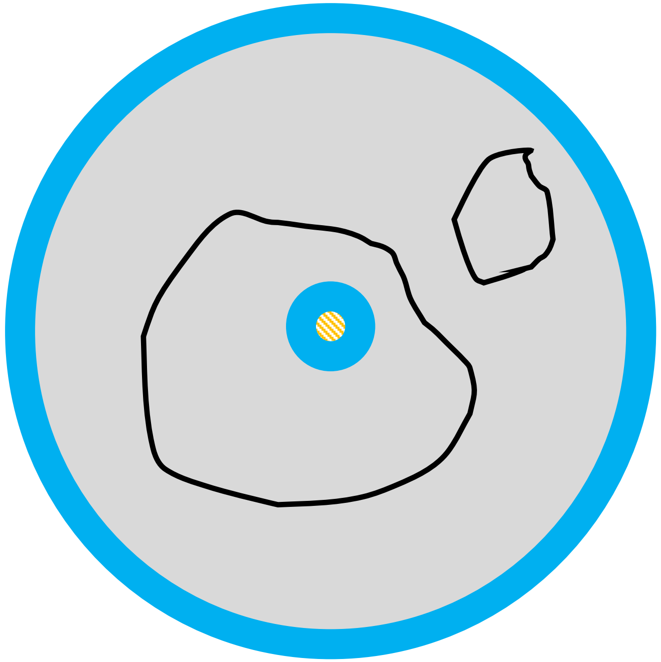
$$E_M = -\int_0^H \mu_0 M dH$$



What about $n > 0$?

$$\phi_s + n\phi_0 + \frac{m_s}{n_s q_s^2} \oint \vec{j} d\vec{r} = n\phi_0$$

Type-II superconductors

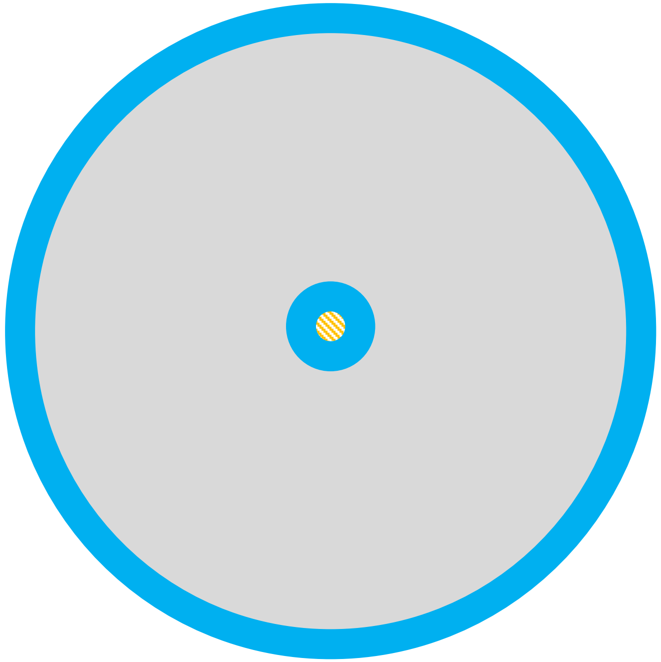


$$\phi_s + \phi_0 + \frac{m_s}{n_s q_s^2} \oint \vec{j} \mathrm{d}\vec{r} = \phi_0$$

e.g., $n = 1$:

- Ad another flux quantum in the from of a vortex.
- Field of vortex is generated by currents fulfilling fluxoid quantization.
- Opposite orientation than surface currents.
- Flux (fluxuid) quantization is fulfilled everywhere.
- Seems to work!

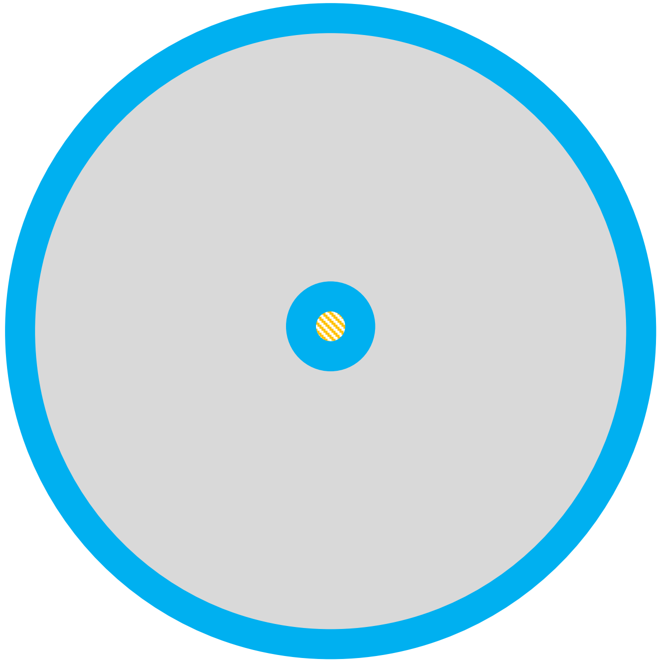
Type-II superconductors



Does it happen?

- Energy E_v of this vortex (length l):
  - Average field: $\phi_0 \approx \overline{B_v} \lambda_L^2 \pi \rightarrow \overline{B_v} \approx \frac{\phi_0}{\lambda_L^2 \pi}$
  - $E_v \approx \frac{\overline{B_v}^2}{2\mu_0} \lambda_L^2 \pi l = \frac{\phi_0^2}{2\mu_0 \lambda_L^2 \pi} l$
- Change of magnetization energy density in the Meissner state ($E_M = -\int_0^H \mu_0 M dH$) by increasing magnetic field by $dH = \frac{\phi_0}{R_S^2 \pi}$: $dE_M = HdH = \frac{H\phi_0}{R_S^2 \pi}$
- Change of magnetization energy of the sample: $dE_M^S = dE_M R_S^2 \pi l = H\phi_0 l$
- Adding a vortex is energetically favorable for
- $H_v \phi_0 > \frac{\phi_0^2}{2\mu_0 \lambda_L^2 \pi}$

Type-II superconductors



It does happen if $H_v < H_c$: Type-II superconductor

Problem: Phase of wave function in the very center of the vortex! ψ and n have to be zero there!

Not describable within London theory!

$\Longrightarrow$ Ginzburg-Landau theory

Ginzburg-Landau theory

- Based on Landau's theory of order phase transition.
- Valid near the transition (T_c)
- Order parameter identified with $|\psi|^2$.
- Energy functional:

$$F_{GL}(T,A) = F_n + \alpha|\psi|^2 + \beta|\psi|^4 + \frac{1}{2m_s}\left|(-i\hbar\vec{\nabla} - q_s\vec{A})\psi\right|^2 + \frac{B^2}{2\mu_0}$$

- Optimization with respect to ψ and A leads to the two Ginzburg Landau equations

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m_s}\left(-i\hbar\vec{\nabla} - q_s\vec{A}\right)\psi = 0$$

$$\vec{j} = \frac{q_s\hbar}{2m_s i}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - \frac{q_s^2}{m_s}|\psi|^2\vec{A}$$

Ginzburg-Landau theory

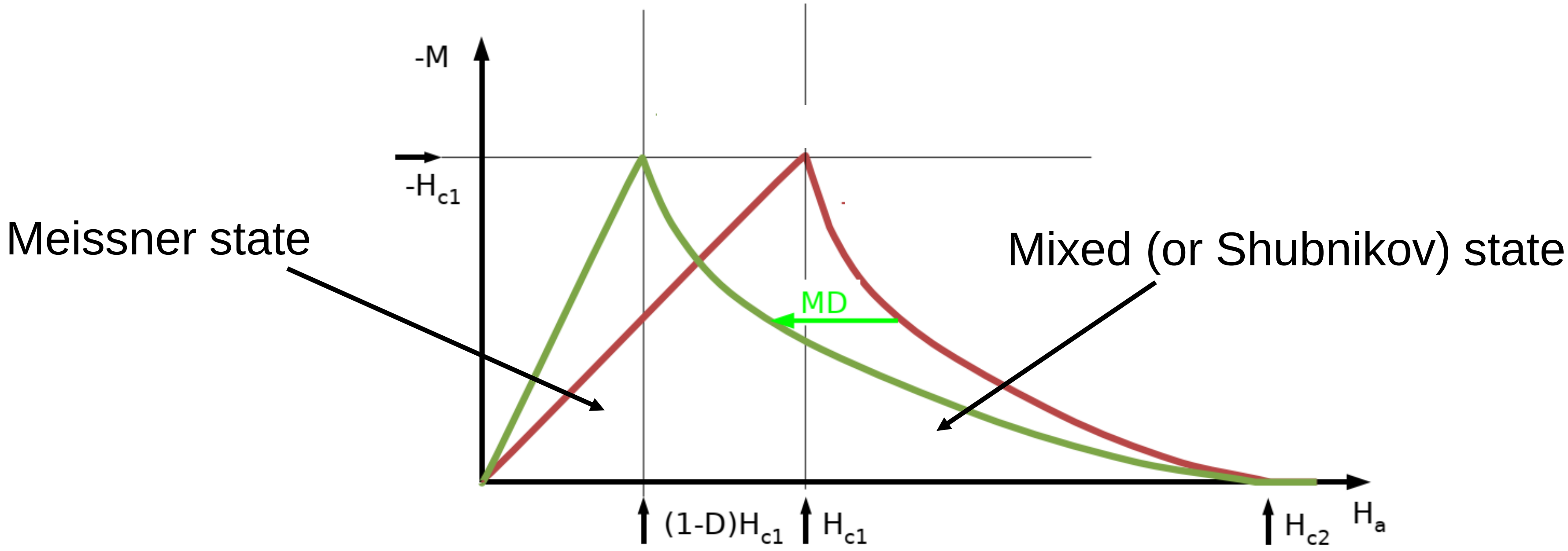
Solution:

- Two characteristic length scales
  - Magnetic penetration depth λ (in general $\neq \lambda_L$)
  - GL coherence length ξ (in general $\neq \xi_{BCS}$ or ξ_0)
    Variation length of the superconducting order parameter $|\psi|^2$
- $|\psi|^2$ is the (local) density of condensed charge carriers. Equilibrium value: $n_s = |\psi_0|^2$;
- $\lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}}$; superfluid density: $n_s \propto \frac{1}{\lambda^2}$ (cf. London theory)
- Ginzburg Landau parameter $\kappa = \frac{\lambda}{\xi}$
  - $\kappa < \frac{1}{\sqrt{2}}$: Type-I superconductor
  - $\kappa > \frac{1}{\sqrt{2}}$: Type-II superconductor

Ginzburg-Landau theory

Reversible (thermodynamic) magnetic properties are entirely described by λ and ξ.

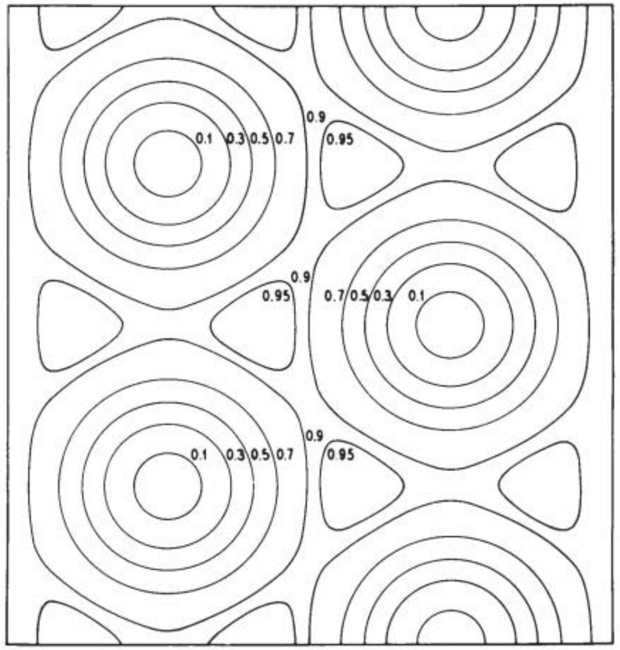
- Thermodynamic critical field: $B_c = \frac{\phi_0}{2\pi\sqrt{2}\lambda\xi}$
- Lower critical field: $B_{c1} = \frac{\phi_0}{4\pi\lambda^2}\ln\kappa$
- Upper critical field: $B_{c2} = \frac{\phi_0}{2\pi\xi^2}$



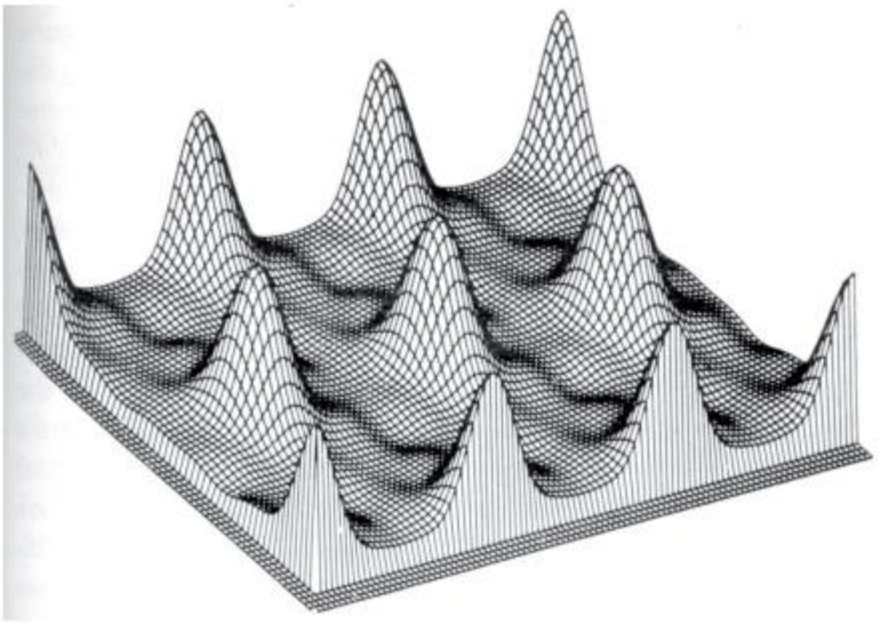
Mixed state

- Flux penetrates in the form of vortices each carrying the elementary flux quantum ϕ_0
- They arrange in a hexagonal lattice.
- The (average) magnetic field B is proportional to the number of vortices.

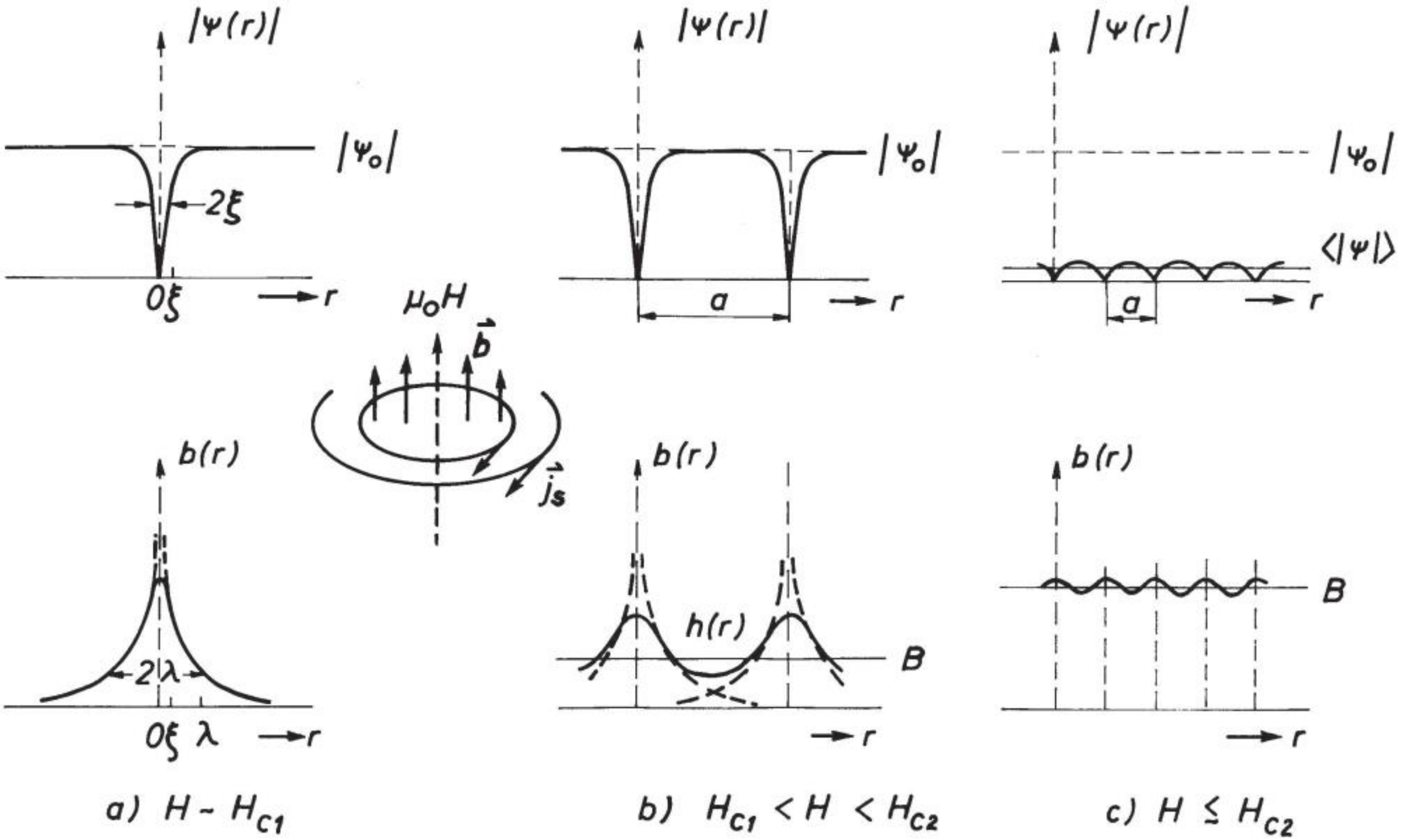
Order parameter $|\psi|^2$



Local magnetic field



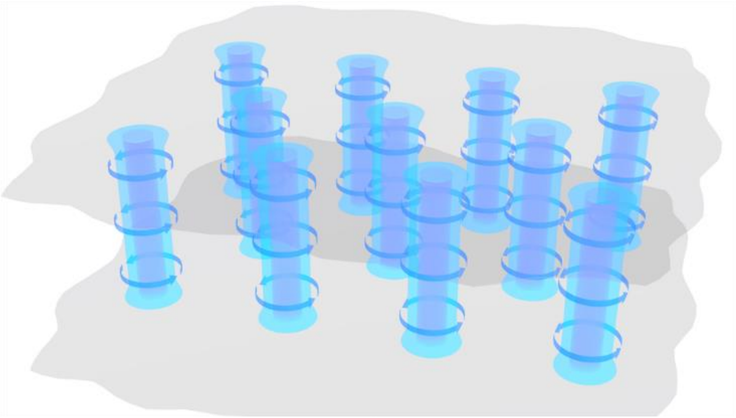
Mixed state

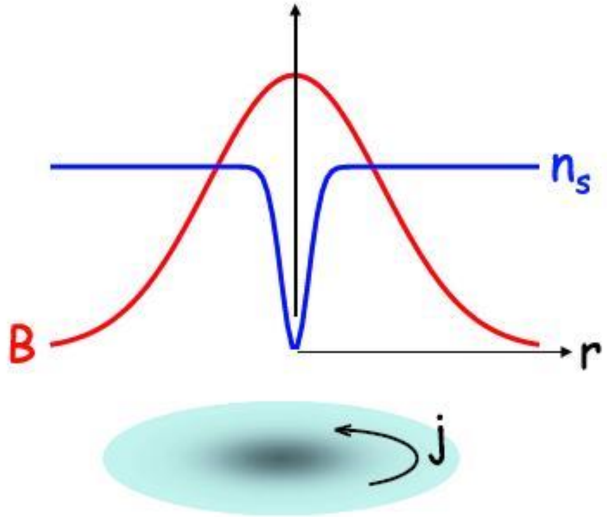


Simplified (London-like) picture of single vortices

- Useful at low fields $B < 0.5B_{c2}$ better $B < 0.2B_{c2}$
- Normal conducting core ($|\psi|^2 = 0$) with radius ξ.
- Undisturbed superconductivity outside core ($|\psi|^2 = |\psi_0|^2$)

- Currents outside the core build the field in accordance with fluxoid quantization





http://www.oettinger-physics.de/vortex.html

Currents in superconductors

- Thermodynamic limit: depairing current density.
- Kinetic energy of the charge carriers exceeds the condensation energy.
- $E_c = \frac{B_c^2}{2\mu_0} = \frac{1}{2\mu_0}\frac{\phi_0^2}{8\pi^2\lambda^2\xi^2}$
- $E_{kin} = \frac{n_s m_e v^2}{2} = |j = n_s e v| = \frac{n_s m_e j^2}{2n_s^2 e^2} = \left|n_s = \frac{m_e}{\mu_0\lambda^2 e^2}\right| = \frac{\mu_0\lambda^2 j^2}{2}$
- $E_c = E_{kin}$: $\frac{\phi_0^2}{8\pi^2\mu_0\lambda^2\xi^2} = \mu_0\lambda^2 j^2 \rightarrow j = \frac{\phi_0}{2\sqrt{2}\pi\mu_0\lambda^2\xi}$
- Ginzburg-Landau theory: $J_d = \frac{\phi_0}{3\sqrt{3}\mu_0\pi\lambda^2\xi}$

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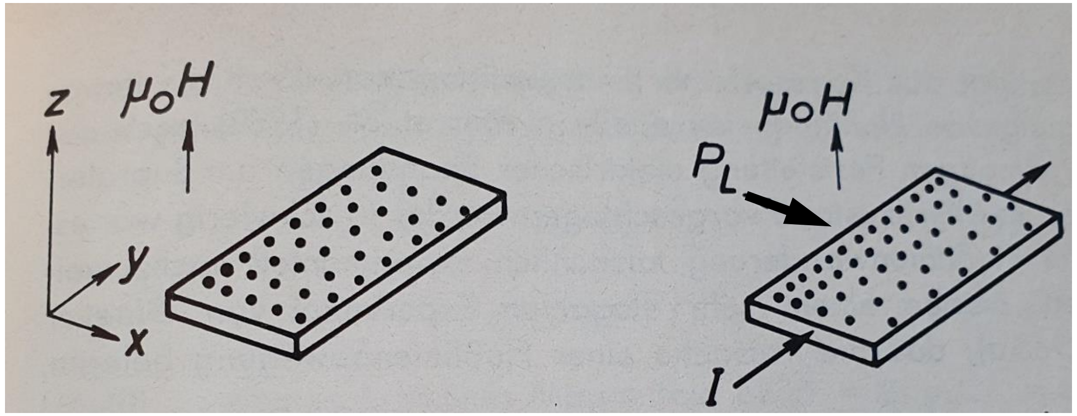
Currents in type-II superconductors

- Three thermodynamic limitations:
  1. Temperature (T_c)
  2. Magnetic field (B_c, B_{c2})
  3. Current density (J_d)



- Currents are not necessarily loss free in the mixed state
- Lorentz force acts on the superconducting condensate: $\overrightarrow{F_L} = \vec{J} \times \vec{B}$
- Losses due to the moving vortices (acceleration of normal electrons in the core)
- Flux pinning: loss free currents.
- Limit: Maximum pinning force: $\vec{F}_{p,max} := -\overrightarrow{J_c} \times \vec{B}$ “critical state”
- $(J \perp B) : J_c = \frac{F_{p,max}}{B}$: critical current density. $J > J_c$: dissipative currents

Currents in type-II superconductors

Supercritical currents:

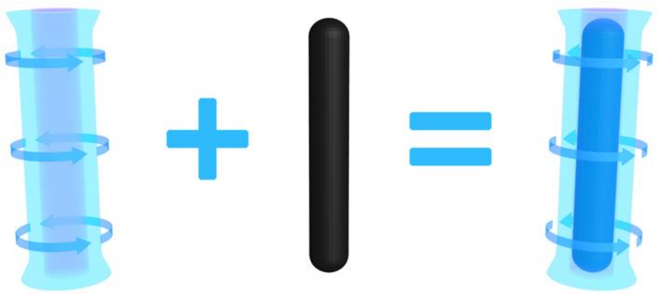
$$I < I_c = \iint_S \overrightarrow{J_c} d\vec{f}$$

Inside of superconductor is free of current: Flux and current always penetrate from the borders of the superconductor. Current free regions inside the superconductor. (Bean model: $J = \pm J_c$ or zero).

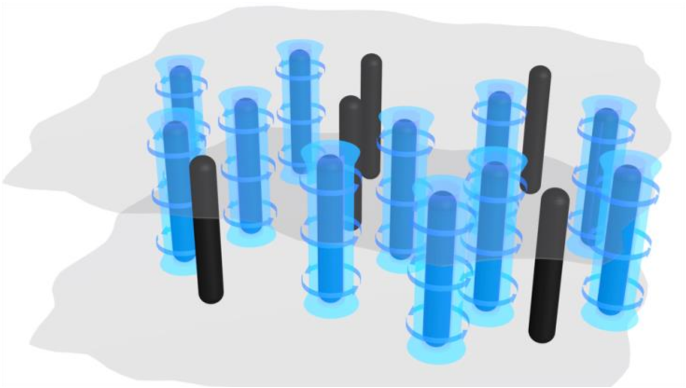
Critical current density: flux pinning

- Energy of vortex core per meter: $E_{\text{core}} = E_c \xi^2 \pi = \frac{\phi_0^2}{16\pi\mu_0\lambda^2}$

$$f_p^{\max} = \frac{E_{\text{core}}}{\xi} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2\xi}$$



- Critical state: $F_p = F_L = |J_c \times B\,|$
- Highest possible pinning force per vortex and unit length: cylindrical defect with $r_D \geq \xi$
- Force balance for one vortex ($B \perp J_c$): $f_L = f_p$

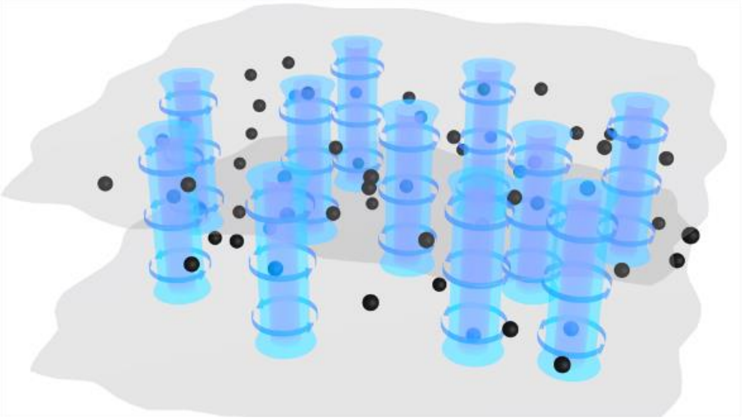


$$f_L = \iint F_L \mathrm{d}A = \iint J_c \times B \mathrm{d}A = J_c \phi_0 \leq f_p^{\max} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2\xi}$$

- $J_c^{max} = \frac{f_p^{max}}{\phi_0} = \frac{\phi_0}{16\pi\mu_0\lambda^2\xi} = \frac{3\sqrt{3}}{16} J_d \approx 0.32 J_d$
- Loss free currents are always limited by pinning, however, J_d sets the scale.

Critical current density: flux pinning

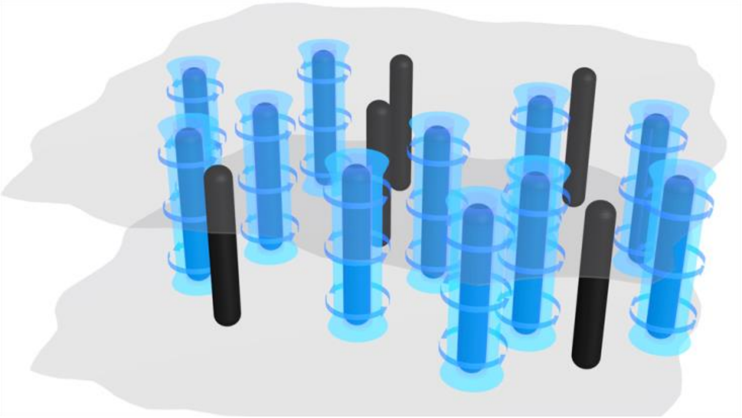
The critical current density denpends on the defect structure and is hence an extrinsic property.



- Quantitative predictions are very difficult
- Useful: scaling laws

$$F_p = J_c B \propto b^p (1-b)^q \text{ with } b = \frac{B}{B_{c2}}$$

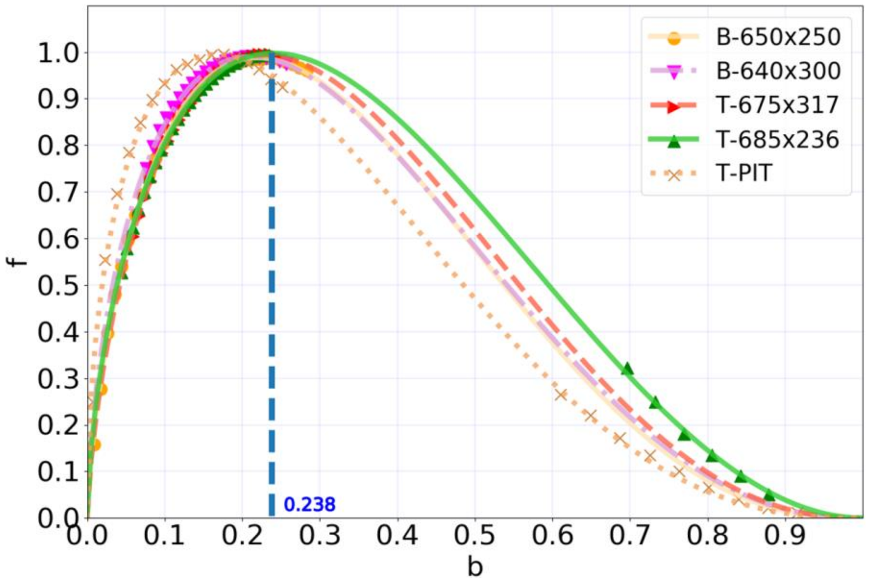
  - $p = 0.5$ planar defects (grain boundaries)
  - $p = 1$ spheric defects (artificial pinning)
  - q is expected to be 1 or 2, but is often higher.



Pinning contributions

$F_p = J_c B \propto b^p(1-b)^q$ with $b = \frac{B}{B_{c2}}$

- $p = 0.5$, $q = 2$ (grain boundaries): peak at $b = 0.2$
- $p = 1$, $q = 2$ spheric defects (artificial pinning): peak at $b = 0.3$
- Attempts to separate different pinning contributions.



Ortino et al., *Supercond. Sci. Technol*. **34** (2021) 035028

The BCS explanation

- Electrons (holes) pair to form **bosonic particles**

(pairing electrons have opposite spin and momentum $-\vec{k}\uparrow, \vec{k}\downarrow$)

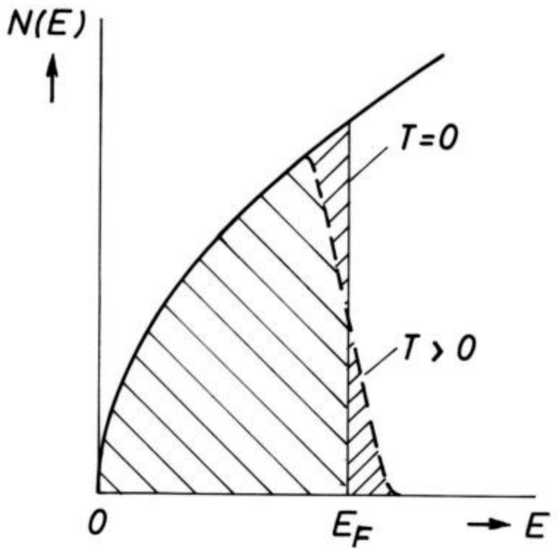
- Pairing due to an attractive interaction via virtual phonons.
- Cooper pairs immediately condense into **one ground state**.
- Elementary excitations: breaking pairs
- Breaking a Cooper pair requires a **minimum energy of 2*Δ***

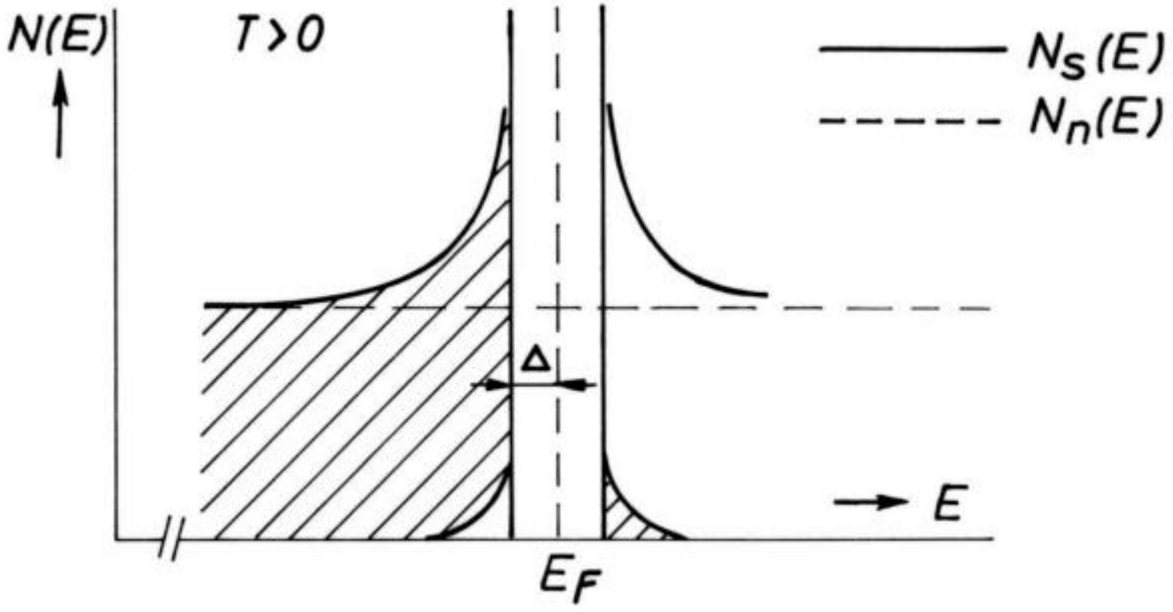
Δ…energy gap

- Copper pairs are **mobile.**
- They cannot transfer moment (energy) to the lattice

BCS predictions

- Energy gap in the density of states at the Fermi level





- $k_B T_c = 1.13\hbar\omega_D e^{-\frac{1}{N(E_F)V}}$
- $2\Delta(0) = 3.54 k_B T_c$
- Isotope effect….
- Ginzburg Landau equations can be derived from BCS theory