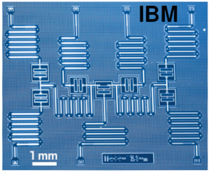
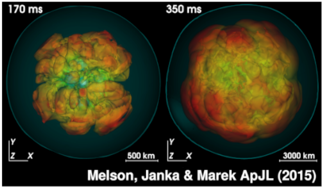
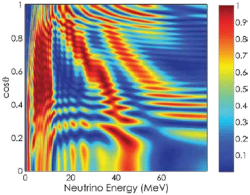
Quantum Simulation of Collective Neutrino Oscillations

Alessandro Roggero







Duan et al. (2006)

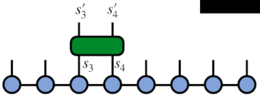
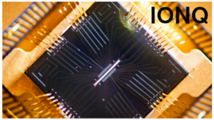
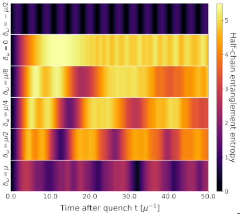


image from itensor.org





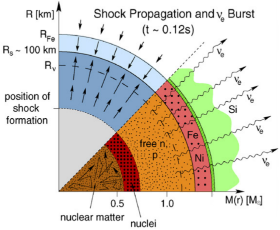
QT4HEP

CERN – 01 Nov, 2022

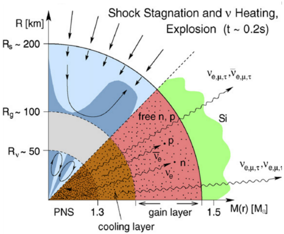
Neutrino's roles in supernovae

figures from Janka et al. (2007)

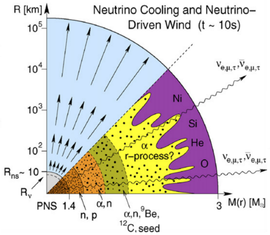
- efficient energy transport away from the shock region (burst)



- energy deposition to revive the stalled shock (explosion)



- regulation of electron fraction in ν-driven wind (nucleosynthesis)



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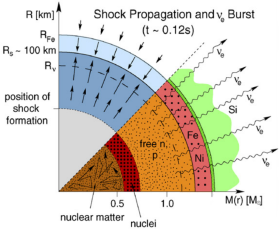

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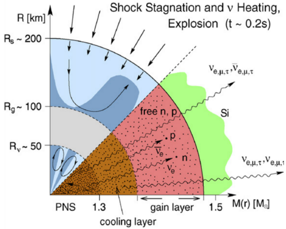

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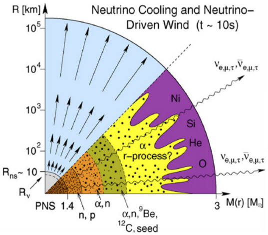
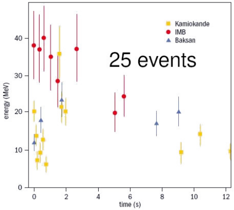
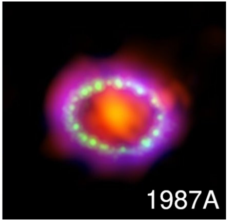


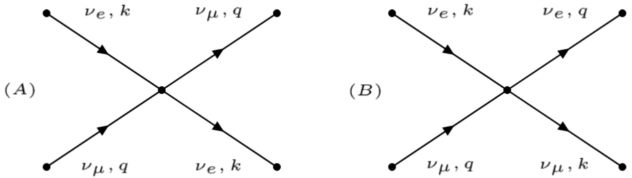
$\approx 10^{58}$ neutrinos emitted in few sec.





Neutrino-neutrino forward scattering

Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .



- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
  - total flavor is conserved

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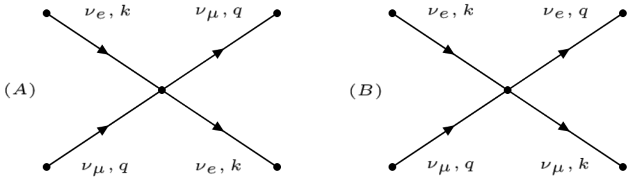

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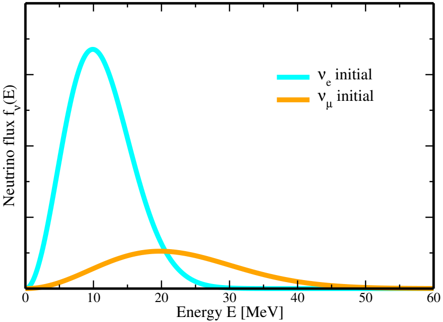
Important effect if initial distributions are strongly flavor dependent



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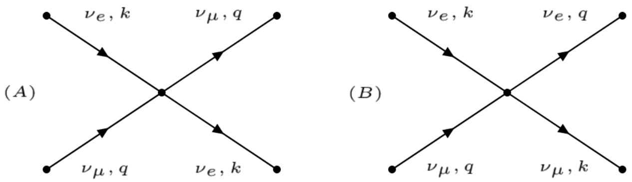

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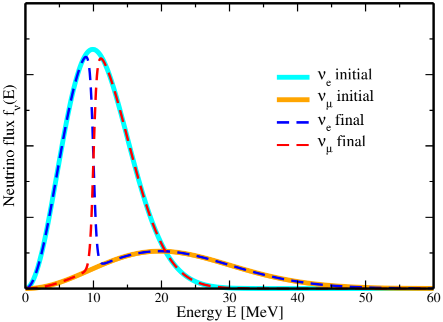

Two-flavor approximation and the iso-spin Hamiltonian

Consider two active flavors (ν_e,ν_x) and encode flavor amplitudes for a neutrino with momentum p_i into an $SU(2)$ iso-spin:

$$|\Phi_i\rangle = \cos(\eta_i)|\nu_e\rangle + \sin(\eta_i)|\nu_x\rangle \equiv \cos(\eta_i)|\uparrow\rangle + \sin(\eta_i)|\downarrow\rangle$$

A system of N interacting neutrinos is then described by the Hamiltonian

$$H = \sum_i \frac{\Delta m^2}{4E_i}\vec{B}\cdot\vec{\sigma}_i + \lambda\sum_i \sigma_i^z + \frac{\mu}{2N}\sum_{i<j}\left(1-\cos(\phi_{ij})\right)\vec{\sigma}_i\cdot\vec{\sigma}_j$$

- vacuum oscillations: $\vec{B} = (\sin(2\theta_{mix}), 0, -\cos(2\theta_{mix}))$
- interaction with matter: $\lambda = \sqrt{2}G_F\rho_e$
- neutrino-neutrino interaction: $\mu = \sqrt{2}G_F\rho_\nu$
  - dependence on momentum direction: $\cos(\phi_{ij}) = \frac{\vec{p}_i}{\|\vec{p}_i\|}\cdot\frac{\vec{p}_j}{\|\vec{p}_j\|}$

for a full derivation, see e.g. Pehlivan et al. PRD(2011)

The mean field approximation

Approximate eq. of motion

$$\frac{d}{dt}\langle\vec{\sigma}_i\rangle = F\left[\langle\vec{\sigma}_i\rangle, \langle\vec{\sigma}_i \times \vec{\sigma}_j\rangle \; \forall j \neq i\right]$$

$$\approx F\left[\langle\vec{\sigma}_i\rangle, \langle\vec{\sigma}_i\rangle \times \langle\vec{\sigma}_j\rangle \; \forall j \neq i\right]$$

$\rightarrow$ Classical evolution of polarization vectors $\vec{P}_i = \langle\vec{\sigma}_i\rangle$ in flavor space

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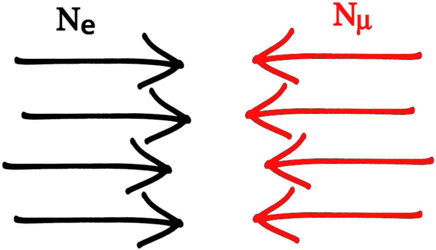
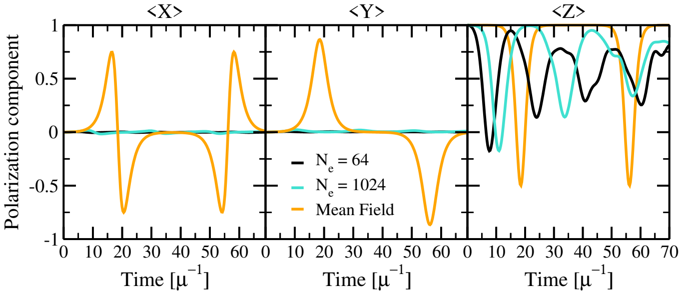




J. Martin, AR, H. Duan, J. Carlson, V. Cirigliano PRD(2022)

Quantum Computing and Quantum Simulations

R.Feynman(1982) we can use a controllable quantum system to simulate the behaviour of another quantum system

Quantum System we have control over

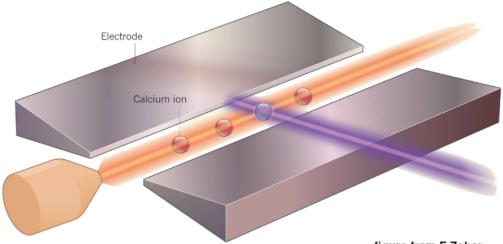
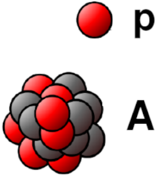


figure from E.Zohar

Quantum System we want to simulate



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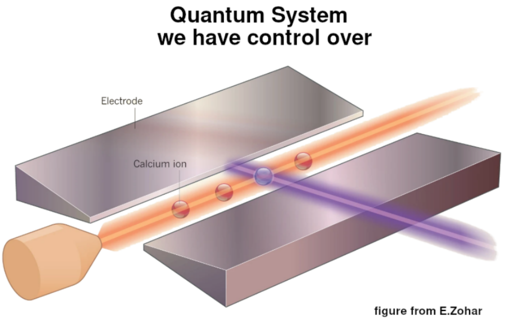
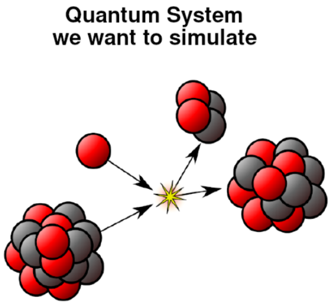
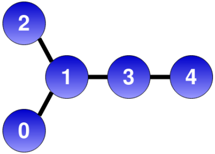




Quantum simulation of collective neutrino oscillations

$$H = \sum_i \omega_i \vec{B} \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



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- all-to-all interactions are difficult with reduced connectivity

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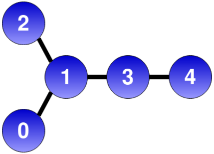

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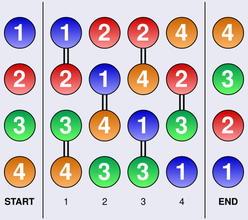
SWAP network



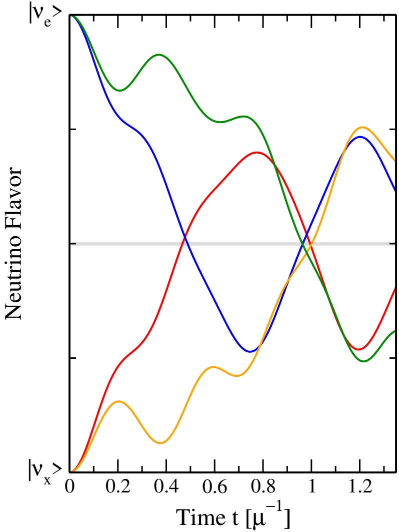
- SWAP qubits every time we apply time-evolution to neighboring terms
- in N steps we perform full evolution using only $\binom{N}{2}$ two qubit gates
  - NOTE: final order will be reversed

Kivlichan et al. PRL (2018)

B.Hall, AR, A.Baroni, J.Carlson PRD(2021)

Accuracy in flavor evolution

How's the current ($\approx$ Fall 2020) accuracy in predicting flavor evolution?



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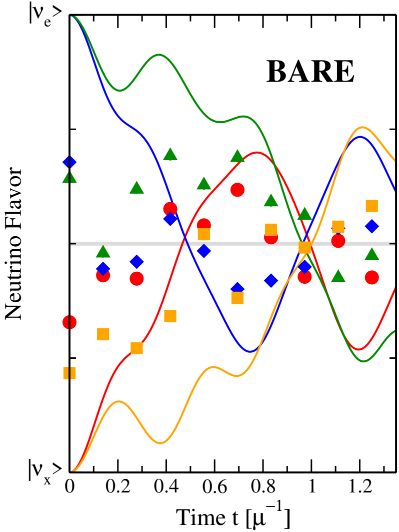

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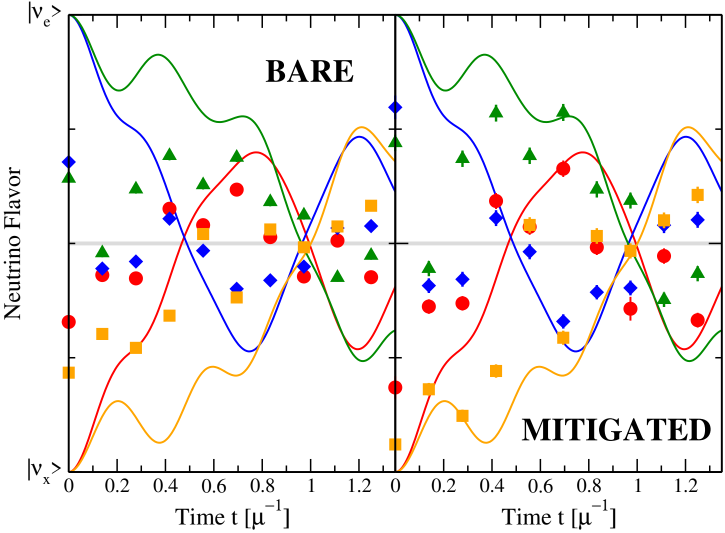

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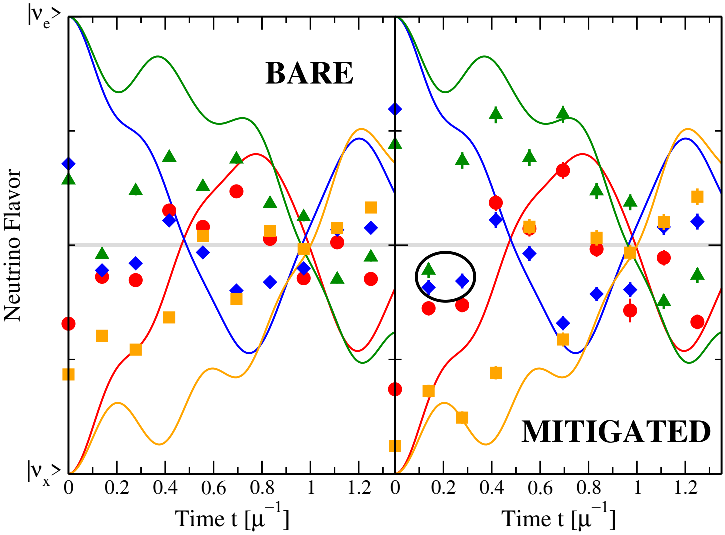

Fidelity of quantum hardware is improving fast

The device used for the previous results was Vigo with a QV of 16

$QV = 2^n \approx$ we can run n full layers on n qubits with fidelity $\geq 66\%$

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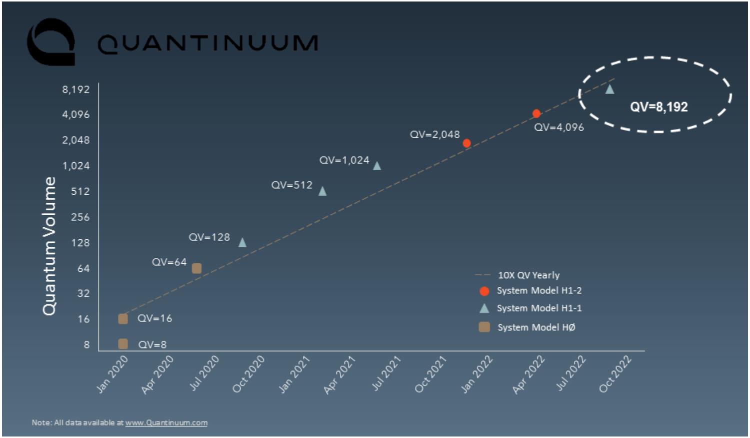

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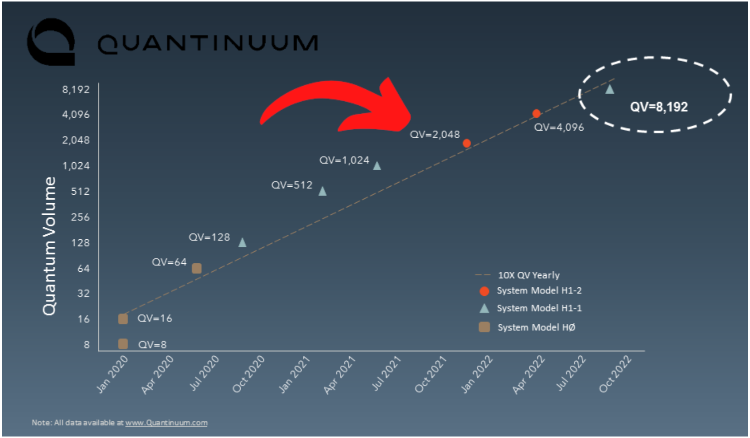
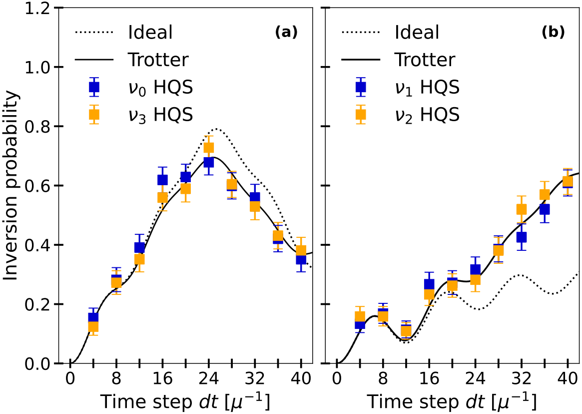


Recent progress in porting the scheme to trapped ions

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

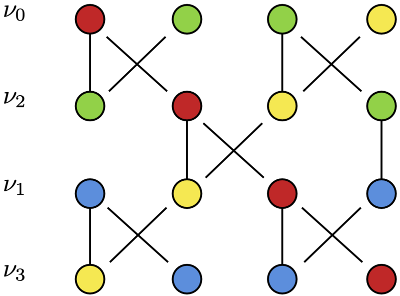
$N = 4$ neutrinos, one time step

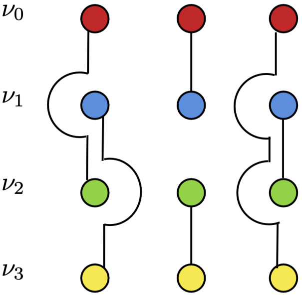


Practical advantages of trapped ion devices

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

- all-to-all connectivity allows a reduction in circuit depth and the possibility of exploring different orderings for the decomposition



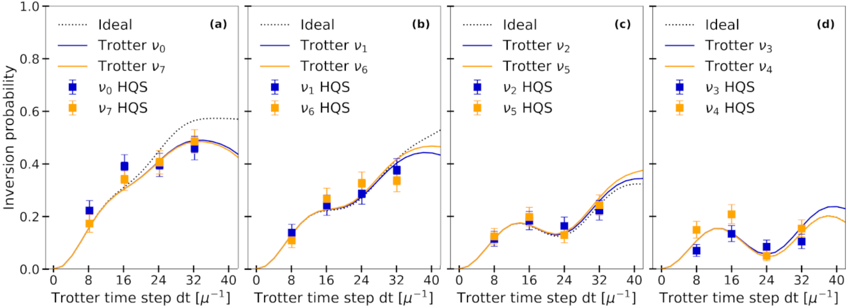


- removing SWAPs allows for a big **reduction in number of rotations**
- **very low infidelities**: $\approx 5 \times 10^{-5}$ one-qubit, $\approx 3 \times 10^{-3}$ two-qubit

Recent progress in porting the scheme to trapped ions II

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

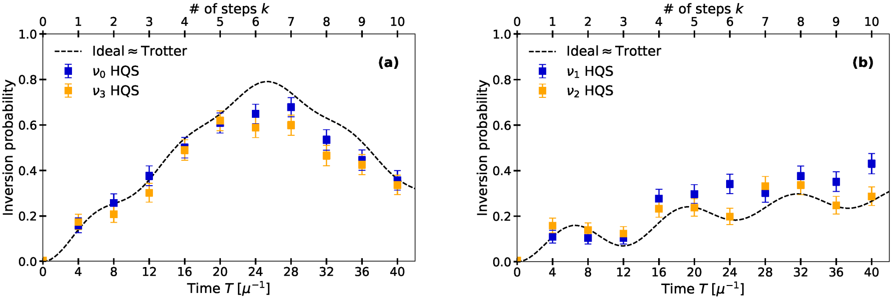
$N = 8$ neutrinos, one time step



Recent progress in porting the scheme to trapped ions III

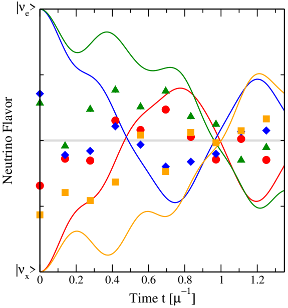
V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

$N = 4$ neutrinos, multiple time steps

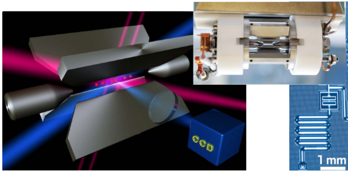


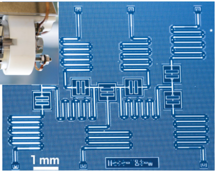
Last two points required: ≈ 350 two-qubit gates over 8 qubits

Current limitations of digital quantum simulations



current and near term digital quantum devices have limited fidelity and might not scale much beyond $N = \mathcal{O}(10)$ neutrinos in next years





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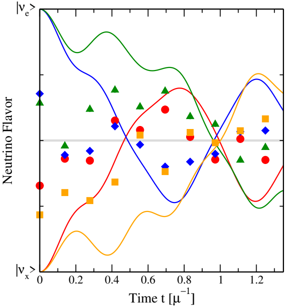

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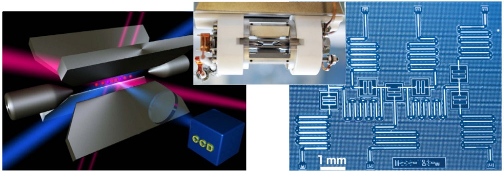


Possible paths to scalability in the meantime

- Analog Quantum Simulators

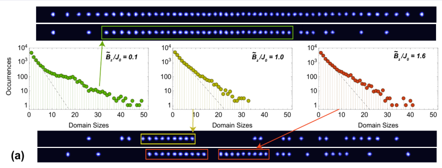


figure from Zhang et al Nature(2017)

- Describe low entanglement states with Tensor Networks

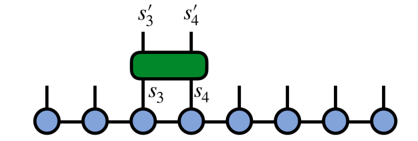
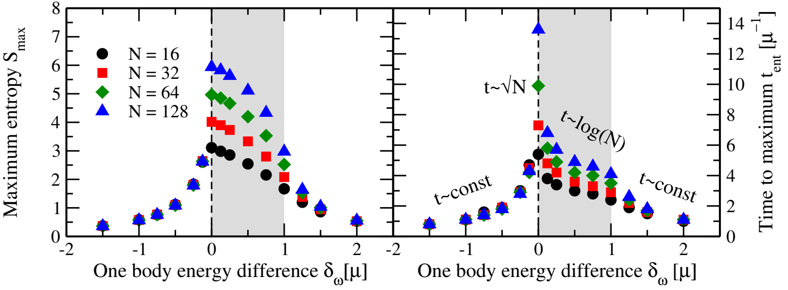


image from itensor.org

Collective oscillations and entanglement scaling

AR, PRD 104, 103016 (2021) & PRD 104, 123023 (2021)



Why is this interesting?

- entanglement scaling provides general criterion for appearance of collective modes in full many-body treatment
- entropy scaling as $\log(N) \Rightarrow$ large ab-initio simulations possible
- MPS method fails when entanglement too large $\Rightarrow$ we can use this to detect interesting regimes to study on quantum simulators!

Summary and perspectives

- collective neutrino oscillations are an interesting **strongly coupled** many-body system driven by the **weak interaction** with possible important impact on flavor dynamics in extreme environments
- even the basic 2-flavor model for collective oscillations poses a challenging many-body problem well suited to quantum technologies
  - Hamiltonian is two-local but all-to-all $\rightarrow$ best suited for trapped-ions
- first calculations on small scale digital devices show promise in studying flavor evolution and achievable fidelity is advancing at a rapid pace ($N = 12$ only 2 weeks ago [Illa & Savage arXiv:2210.08656])
- analog trapped ion devices are an ideal platform to study mid-size systems as the interactions can be embedded in a natural way
- tensor network methods can help push the boundary of classical simulations and identify interesting regimes to study with simulators

Thanks to my collaborators

- Joseph Carlson (LANL)
- Alessandro Baroni (LANL→ORNL)
- Benjamin Hall (MSU)
- Valentina Amitrano (UniTN/TIFPA)
- Piero Luchi (UniTN/TIFPA)
- Francesco Turro (UniTN/TIFPA)
- Luca Vespucci (UniTN/TIFPA)
- Francesco Pederiva (UniTN/TIFPA)

Error mitigation with zero-noise extrapolation

Li & Benjamin PRX(2017), Temme, Bravy, Gambetta PRL(2017), Endo,Benjamin,Li PRX(2018)

Zero noise extrapolation

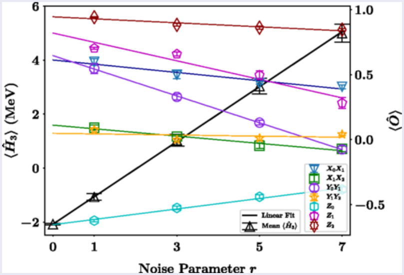
For small enough noise we can write

$$M(\epsilon) = M_0 + \epsilon M_1 + \frac{\epsilon^2}{2} M_2 + \dots$$

Using two points $\epsilon_2 = \eta \epsilon_1$ we have

$$M_0 \approx M(\epsilon_1) - \frac{M(\epsilon_1) - M(\epsilon_2)}{\eta - 1}$$

picture from Dumitrescu et al. PRL(2018)



- for moderate ϵ other parametrizations (like exp) might be more useful

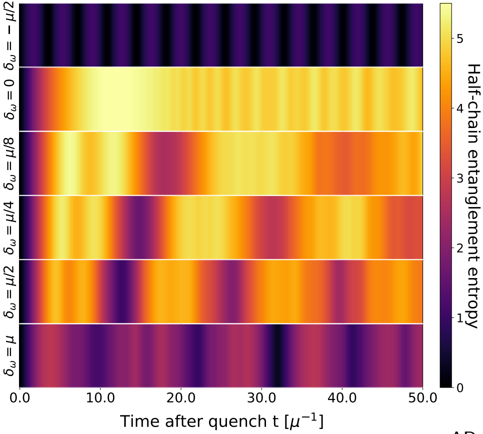
$$M(\epsilon) = M_0 e^{-\alpha\epsilon} \Rightarrow M_0 \approx M(\epsilon_1) \left(\frac{M(\epsilon_2)}{M(\epsilon_1)} \right)^{\frac{\epsilon_1}{\epsilon_1 - \epsilon_2}}$$

In that case it is very beneficial to ensure $M(\epsilon \to \infty) \to 0$ (mitigated B)

Collective oscillations with MPS

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\{1,\ldots,N/2\}}\sigma_i^z - \sum_{i\in\{N/2+1,\ldots,N\}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\ ,$$

MF predicts no evolution, MPS has oscillations for $0 \leq \delta_\omega/\mu \lesssim 1$



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