Ying-Ying Li

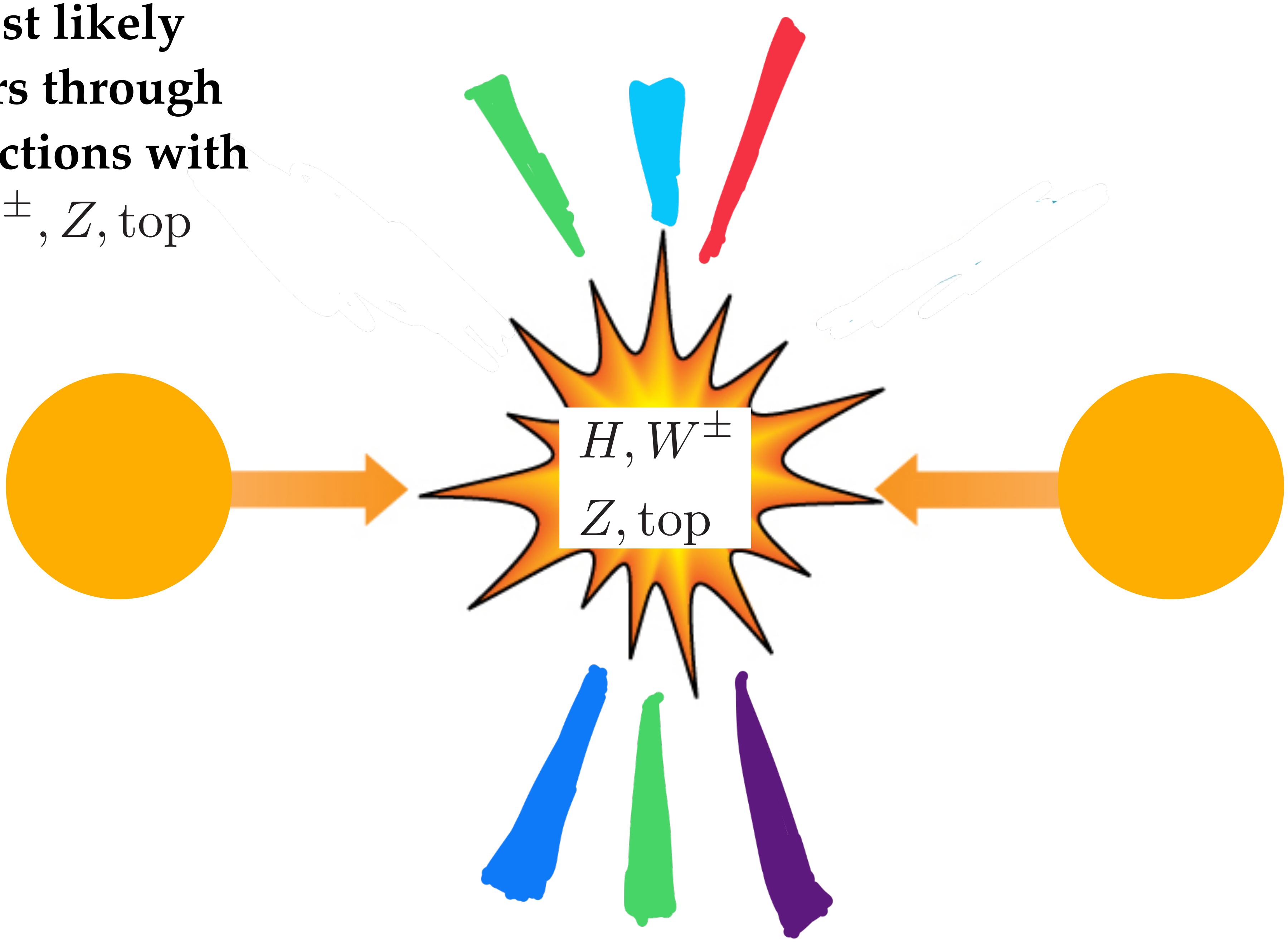
Quantum Computing for Lattice Gauge Theories

PRD.**104**, 094519,
PRL.**129**, 051601,
arXiv: 2208.10417,
in collaboration with
Marcela Carena, Erik J. Gustafson,
Henry Lamm, Wanqiang Liu

QT4HEP Nov. 1, 2022

-Now-: precision measurement

New physics most likely enters through interactions with $H, W^{\pm}, Z, \text{top}$

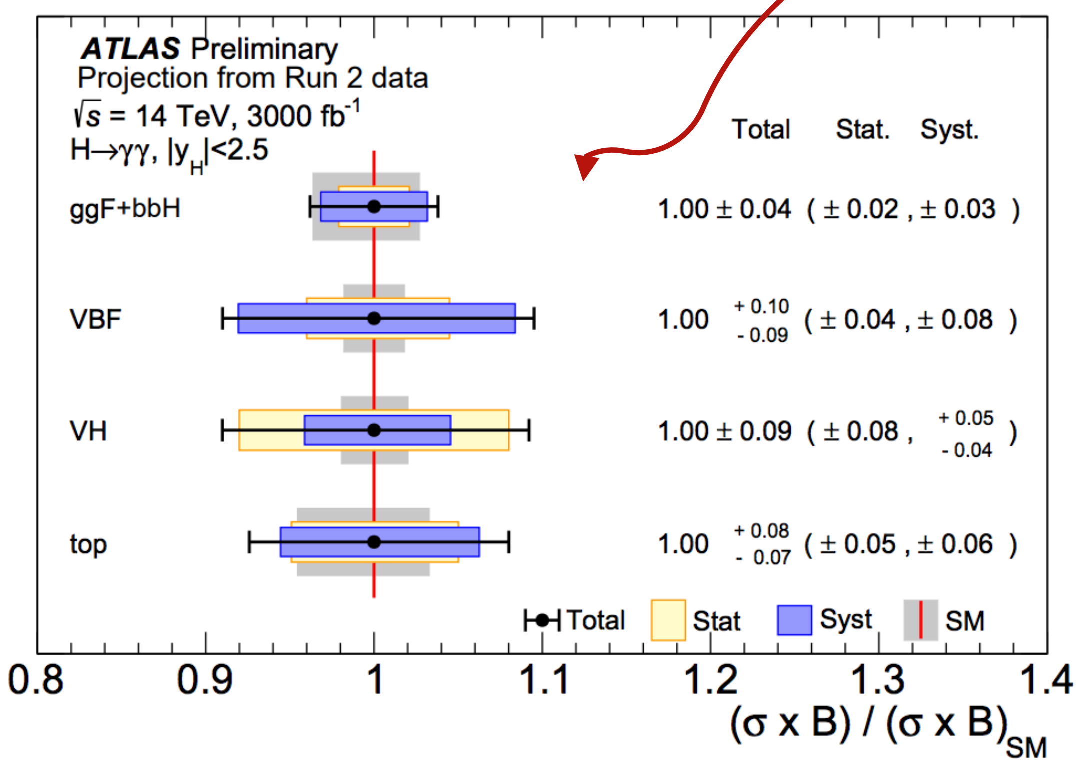


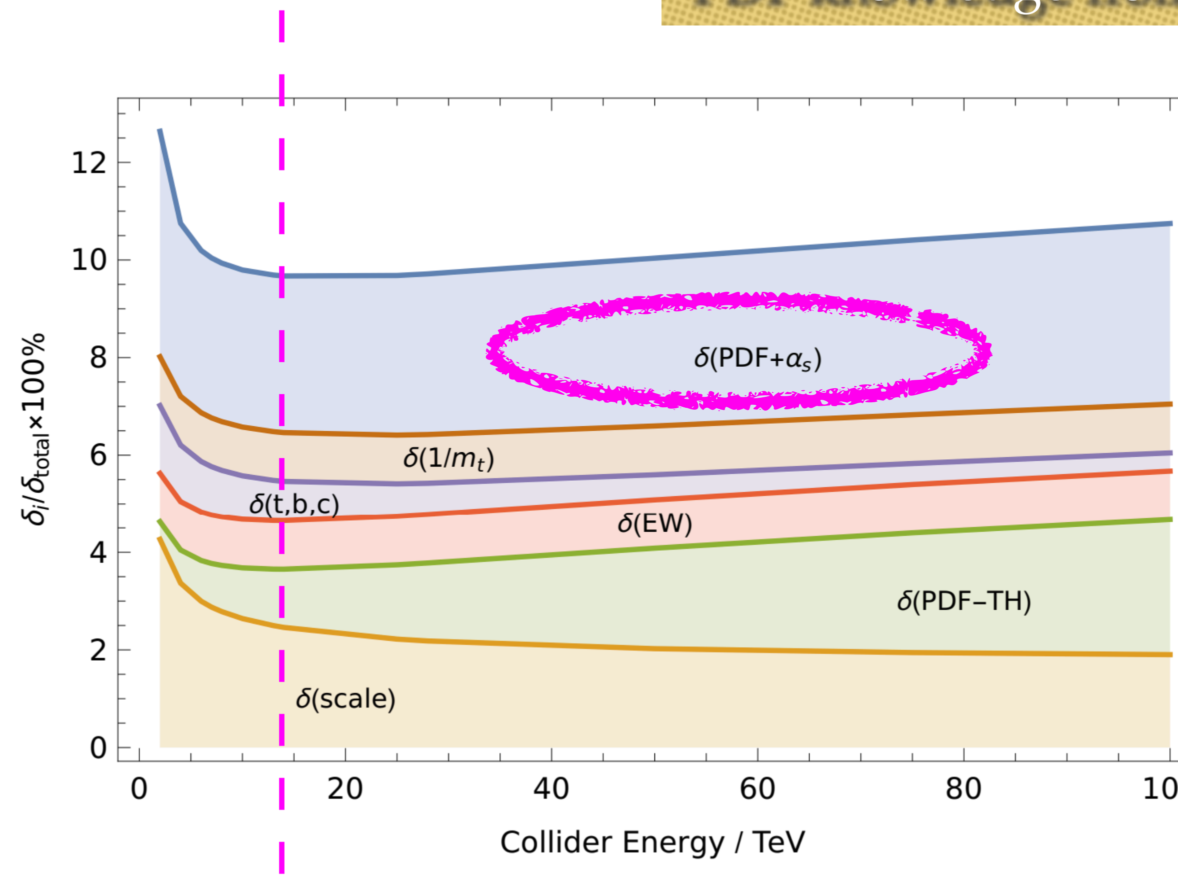
Theoretical inputs to colliders

theoretical uncertainties

real-time strong dynamics

PDF knowledge from data





[F. Dulat, et al, arXiv:1802.00827]
[M. Cepeda, et al, arXiv:1902.00134]

non perturbative, non equilibrium dynamics of QCD in QGP, parton shower, etc

Universal Quantum Computing

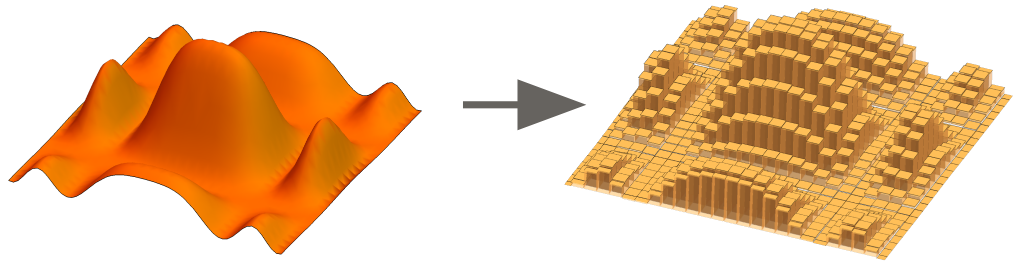
State of the art devices: $\mathcal{O}(10)$ **physical** qubits with $\mathcal{O}(10)$ gates
Error correction: Noiseless, **logical** qubits from set of physical ones

Noise limits fidelity of primitive gates to $95-99\%$ today

entering Noisy Intermediate-Scale Quantum (NISQ) era:
more than 50 well controlled qubits, not error-corrected yet.

Quantum Computing

galactic algorithm to simulate quantum field theory

Discretization  *KS Hamiltonian*

Digitization $|q\rangle^N \to |G\rangle$ *truncation, discrete subgroup, …*

Initialization $\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$ *ground/thermal/particle state prep*

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$ *Trotterization, variational approach, Taylor series, …*

Evaluation $\langle\mathcal{O}\rangle$ *Parton distribution function, particle decay, …*

Error mitigation

See references in [M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

Discretization of space

J. Kogut and L. Susskind [Phys. Rev. D **11**, 395]

For pure SU(N) gauge theory,

$$H_{\rm co} = \frac{1}{2} \int \mathrm{d}^d x \,\mathrm{Tr} \left[\mathbf{E}^2(\mathbf{x}) + \mathbf{B}^2(\mathbf{x})\right]$$

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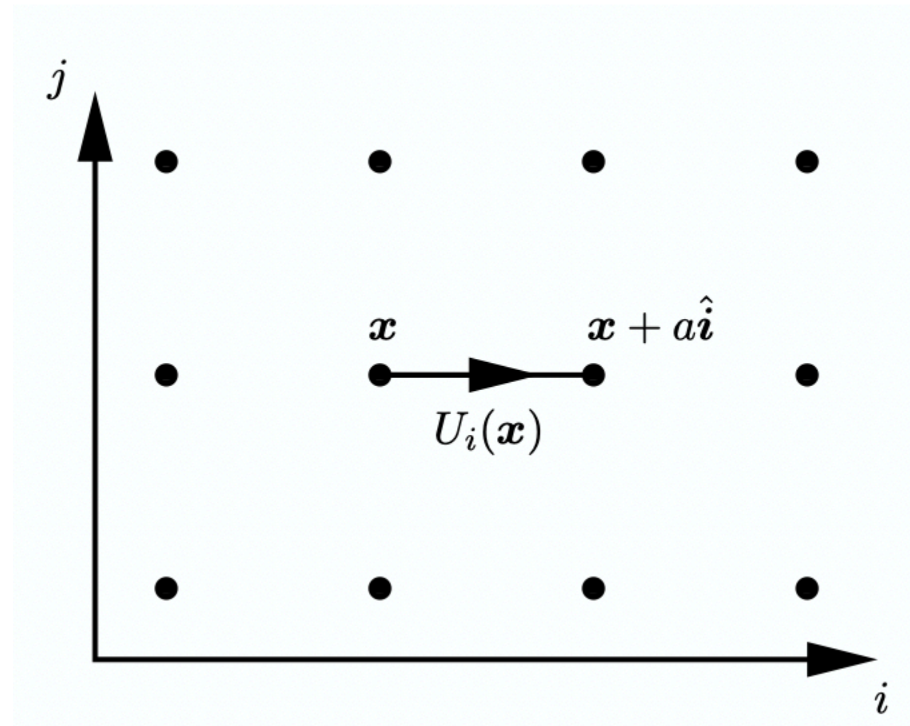


$$U_i(x) = e^{ig\int_a^0 dt A_i(x+t\hat{i})}$$

On a lattice, to build a discrete theory with exact gauge invariance

Wilson loop

$$P_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\left\{\square_{ij}\right\}$$

$$P_{ij}(x) \;=\; 1 - \frac{1}{N}\mathrm{ReTr}\exp\left\{ig\oint_{\square} A\cdot dx\right\}$$

$$E_i^L \to L_i$$

$$H_{KS} = K_{KS} + V_{KS},$$

$$K_{KS} = \sum_{\mathbf{x},i}\frac{g_t^2}{a}\,\mathrm{Tr}\,L_i^2(\mathbf{x})$$

$$V_{KS} = -\sum_{\mathbf{x},i<j}\frac{2}{g_s^2 a}\,\mathrm{Re\,Tr}\,P_{ij}(\mathbf{x})$$

Discretization of space

[J. Carlsson, et al, hep-lat/0105018]

$$P_{ij}(x) \;=\; 1 - \frac{1}{N}\mathrm{Re}\mathrm{Tr}\exp\left\{ig\oint_{\square} A\cdot dx\right\} \;\approx\; \frac{g^2a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{g^2a^6}{12N}\mathrm{Tr}\left\{F_{ij}(x)(D_i^2+D_j^2)F_{ij}(x)\right\} + \ldots$$

deviations from the continuum, starts from a^2 error, classical computational resources proportional to a^{-k} for Wilson action

$$R_{ij}(x) = 1 - \frac{1}{N}\mathrm{Re}\mathrm{Tr}\left\{\square_{i\times 2j}\right\} \;=\; \frac{4g^2a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{4g^2a^6}{24N}\mathrm{Tr}\left\{F_{ij}(x)(4D_i^2+D_j^2)F_{ij}(x)\right\} + \ldots$$

deviations from the continuum starts from a^2g^2 at quantum level

Discretization of space

$$H_I = K_I + V_I$$

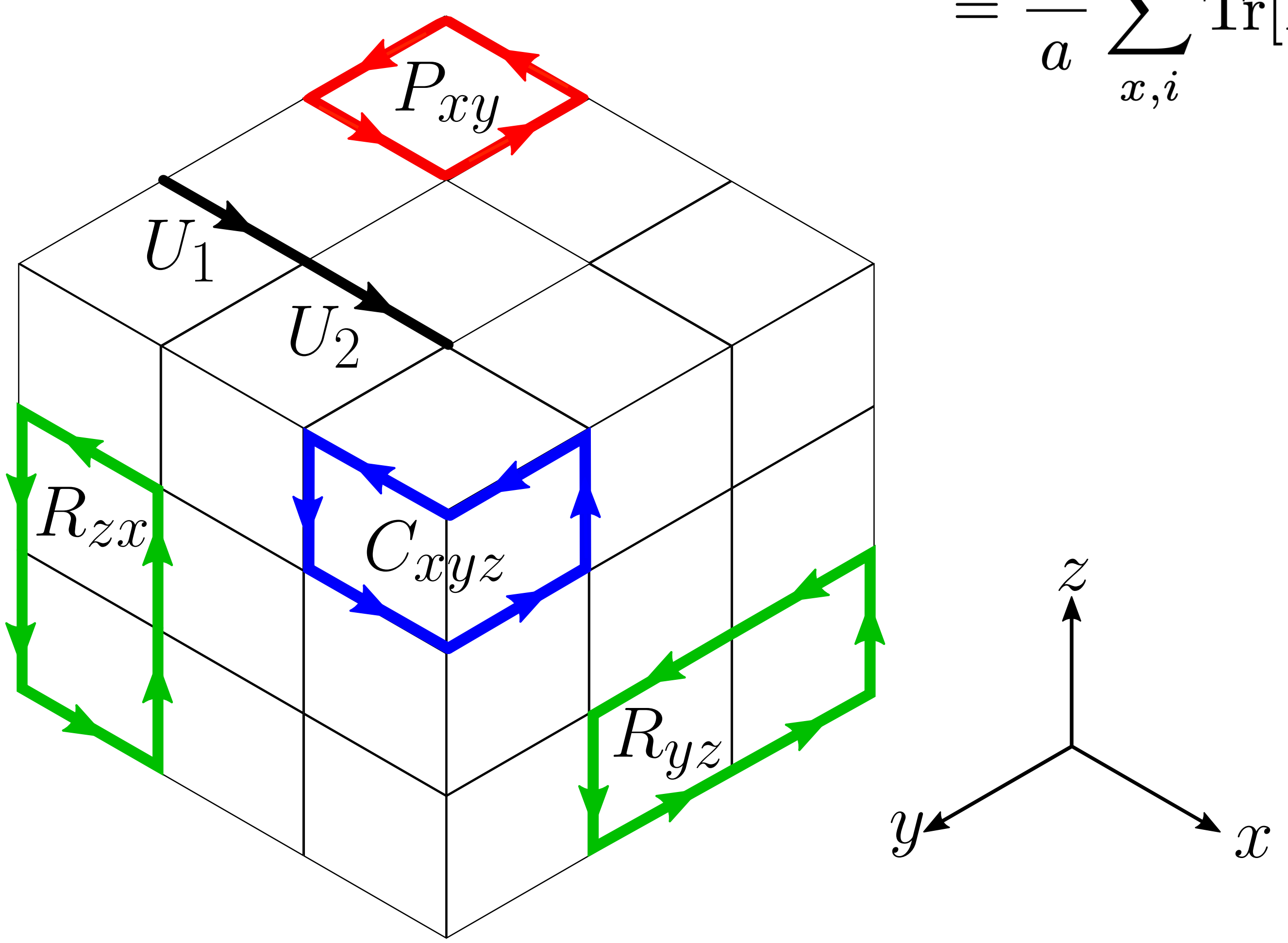
$$V_I = \beta_{V0} V_{KS} + \beta_{V1} V_{\text{rect}} \, .$$

$$K_I = \beta_{K0} K_{KS} + \beta_{K1} K_{2L}$$

$$V_{\text{rect}} = \frac{2}{a g_s^2} \sum_{\mathbf{x}, i<j} \text{Re}\,\text{Tr}\left[R_{ij}(\mathbf{x}) + R_{ji}(\mathbf{x})\right]$$

$$K_{2L} = \frac{g_t^2}{a} \sum_{\mathbf{x},i} \text{Tr}\left[L_i(\mathbf{x}) U_i(\mathbf{x}) L_i(\mathbf{x}+a\mathbf{i}) U_i^\dagger(\mathbf{x})\right]$$

$$= \frac{g_t^2}{a} \sum_{x,i} \text{Tr}[R_i(\mathbf{x}) L_i(\mathbf{x}+a\mathbf{i})]$$



[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Discretization of space

[M. Alford, et al, hep-lat/9507010]
[…, …]

With improved action, for Euclidean spacetime at the same error level, simulations can be done with a lattice spacing of at least 2 larger

Qubits required

$$N_q \sim \left(\frac{L}{a}\right)^d$$

Only count qubits:
saving us at least 3 years for 3+1d,
assuming number of qubits increases by a factor of 2 each year on hardware

circuits for improved Hamiltonian need to be designed
classical field to quantum operator

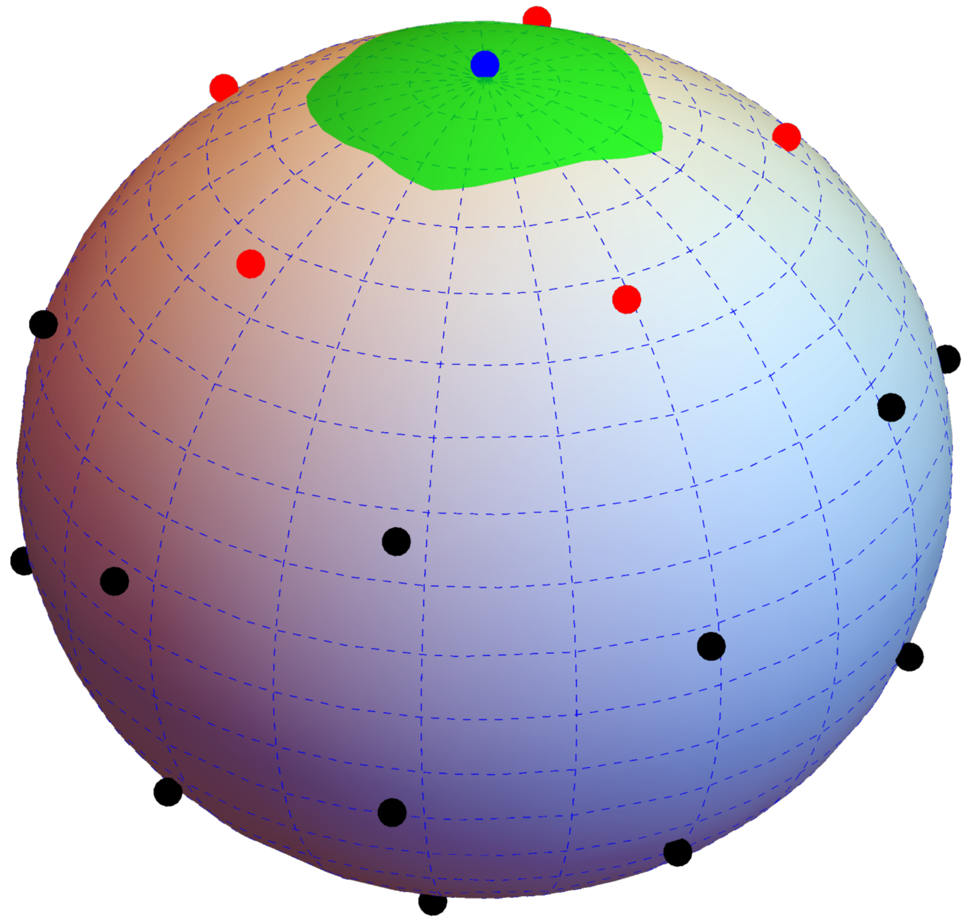
[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Digitization and Propagation

digitize gluon:

qubit regularization, quantum link models,

discrete subgroups, etc



Digitization

$G-$register : $U_i(x)$ → $|g\rangle$

D4 group: 4 rotations, 4 reflections

group element g: $|abc\rangle$

$$\left[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right]^a \left[\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}\right]^{2b+c}$$

Digitization and Propagation

$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_{KS}} e^{-i\delta t V_{KS}}\right]^{t/\delta t}$$

$$P_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\left\{\square_{ij}\right\}$$

Propagation

[H. Lamm, et al, arXiv:1903.08807]

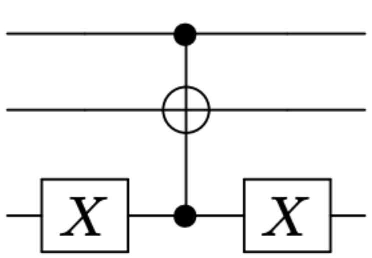
$$\mathfrak{U}_\times |g\rangle |h\rangle = |g\rangle |gh\rangle$$

$$\mathfrak{U}_{-1} |g\rangle = |g^{-1}\rangle$$

$$\mathfrak{U}_{\mathrm{Tr}}(\theta) |g\rangle = e^{i\theta\, \mathrm{Re\,Tr}\, g} |g\rangle$$

$$\mathfrak{U}_F \sum_{g\in G} f(g) |g\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij} |\rho, i, j\rangle$$

$\mathfrak{U}_{-1}$ gate for D4 group

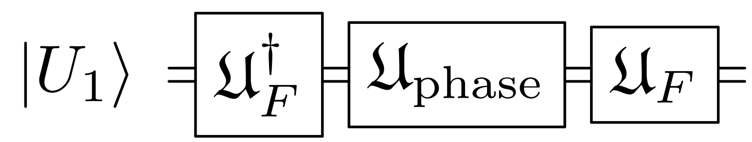


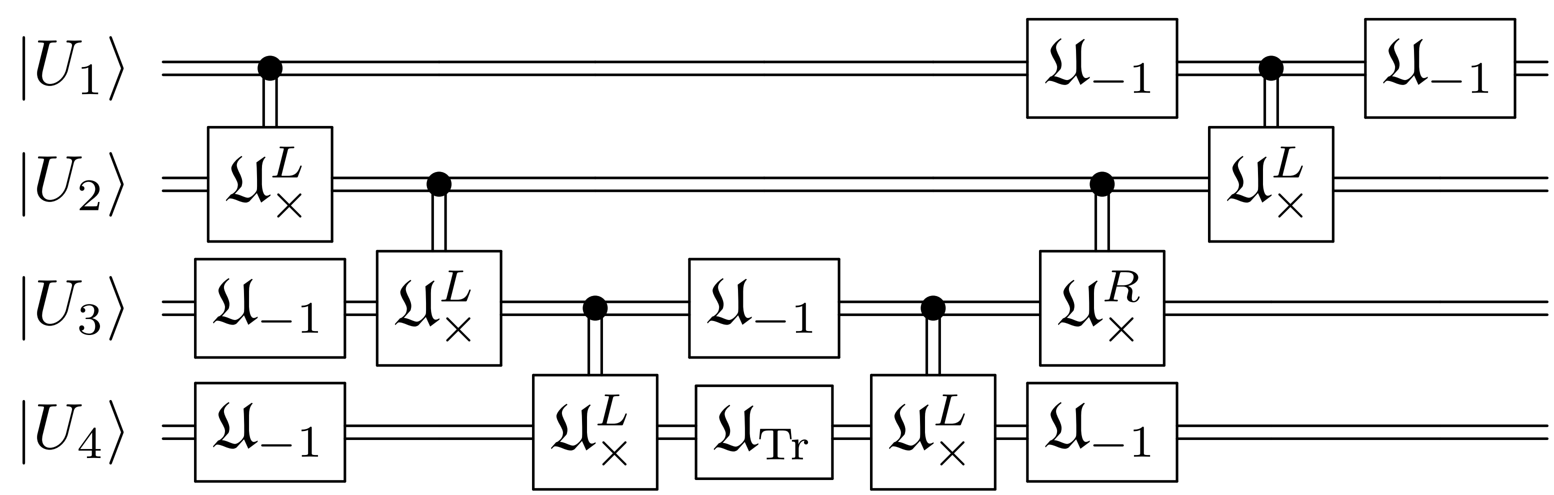
Implementation with quantum logic gate

Propagation

$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_{KS}} e^{-i\delta t V_{KS}}\right]^{t/\delta t}$$

$$\frac{g_t^2}{a}\,\mathrm{Tr}\,L_i^2(\mathbf{x})$$

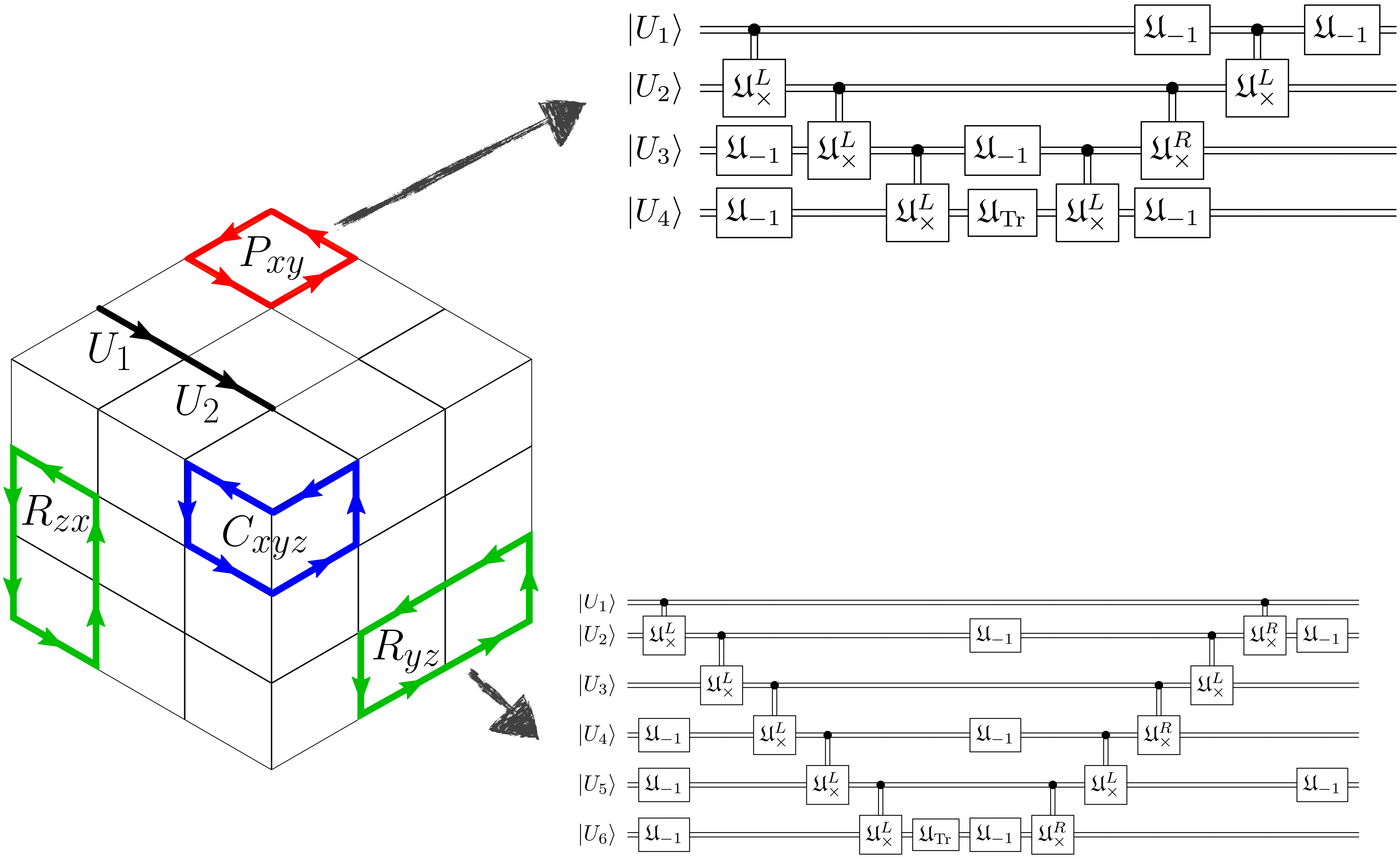




$\mathcal{U}_{V_{KS}}$ assuming linear register connectivity

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Propagation for Improved Hamiltonian



[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Propagation for Improved Hamiltonian

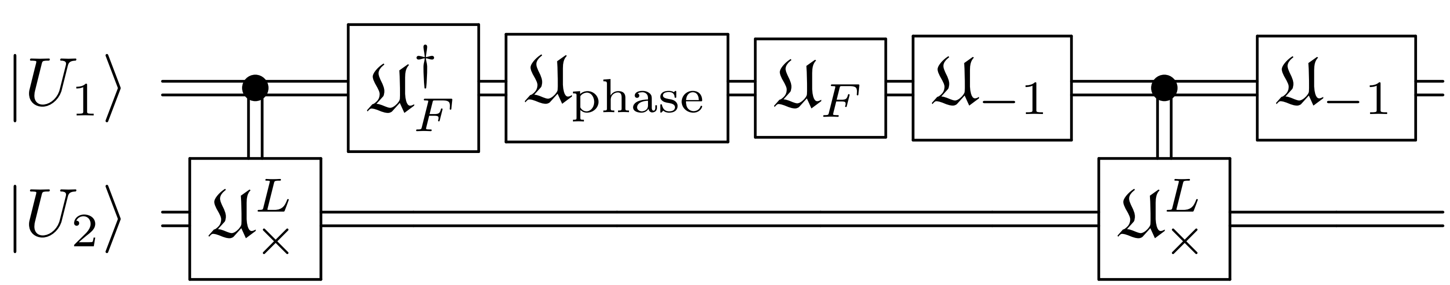
$$\hat{K}_{2L} = \frac{g_t^2}{a} \sum_{x,i} \mathrm{Tr}[\hat{R}_i(\mathbf{x})\hat{L}_i(\mathbf{x}+a\mathbf{i})]$$

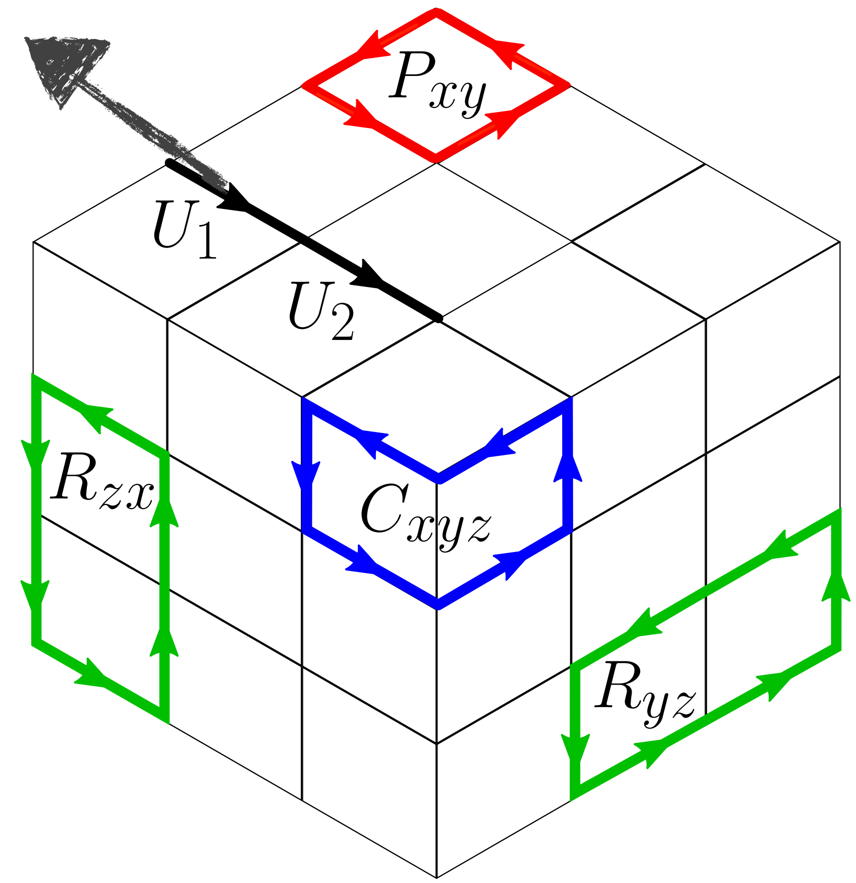
$$\mathrm{Tr}(\hat{R}_1\hat{L}_2) = \mathrm{Tr}[\hat{L}_2^2 + \hat{R}_1^2 - (\hat{L}_2 - \hat{R}_1)^2]/2$$

$$\mathcal{U}_{K_{2L}} \equiv e^{i\theta\,\mathrm{Tr}(\hat{L}_2 - \hat{R}_1)^2}$$

$$[\mathcal{U}_{K_{2L}}, \hat{U}_1\hat{U}_2] = 0$$

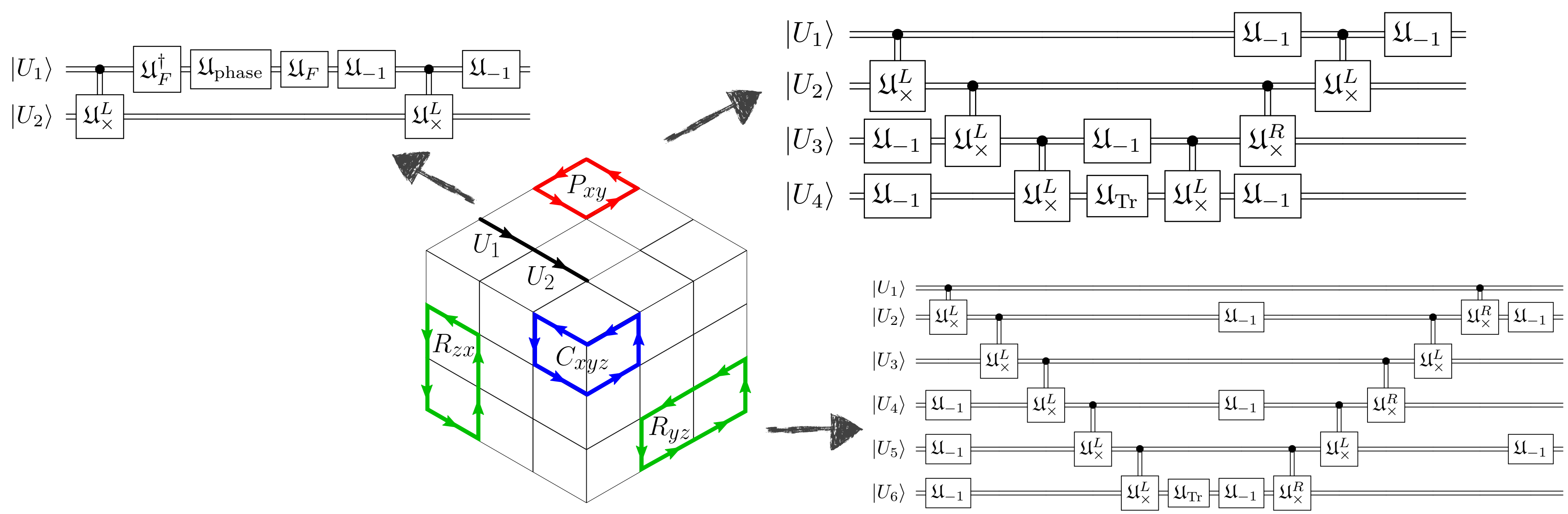
$$\langle U_1', U_2' | \mathcal{U}_{K_{2L}} | U_1, U_2 \rangle = \delta_{U_1'U_2', U_1U_2} \langle U_1' | e^{i\theta\,\mathrm{Tr}\,\hat{L}_1^2} | U_1 \rangle$$





[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Resource for Improved Hamiltonian

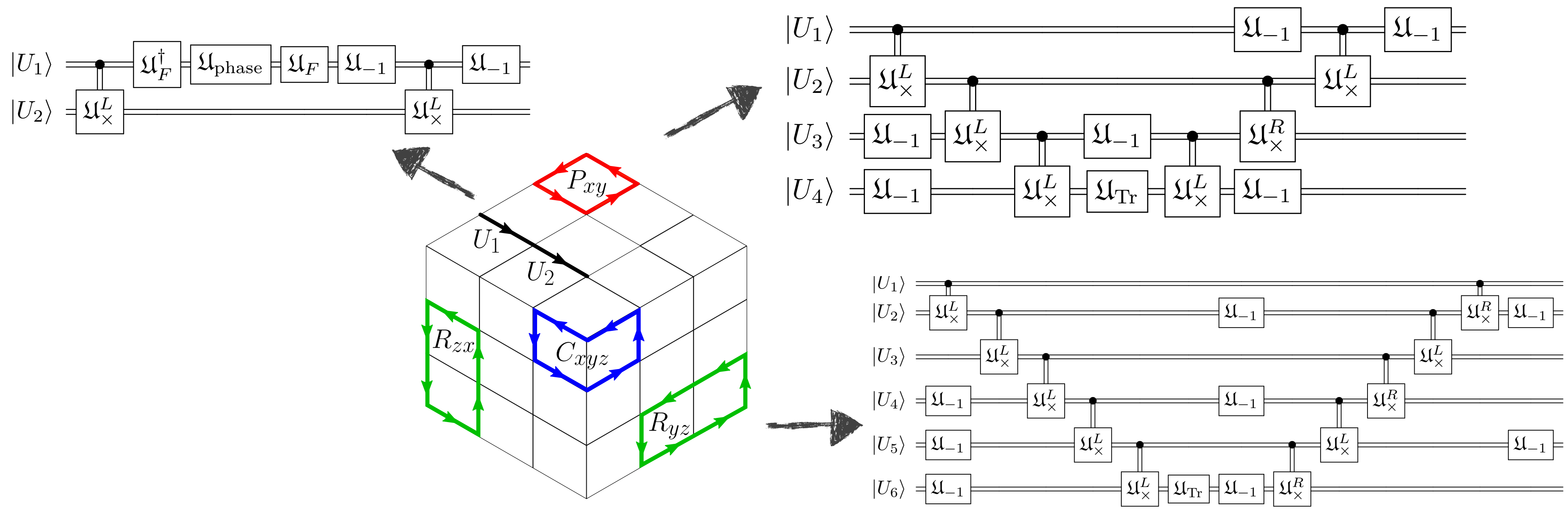


Gate	$N[\hat{K}_{KS}+\hat{V}_{KS}]$	$N[\hat{K}_{2L}+\hat{V}_{rect}]$
$\mathfrak{U}_F$	2	2
$\mathfrak{U}_{phase}$	1	1
$\mathfrak{U}_{Tr}$	$\frac{d-1}{2}$	$d-1$
$\mathfrak{U}_{-1}$	$3(d-1)$	$2+8(d-1)$
$\mathfrak{U}_{\times}$	$6(d-1)$	$4+20(d-1)$

- # of Gates here for a single trotter is increasing only multiplicatively, could be compensated by the decreasing of links.
- Larger trotter steps, instead could be used for improved Hamiltonian.

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Resource for Improved Hamiltonian



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So far, circuits for improved Hamiltonian are designed, reducing the number of qubits required, with comparable or less quantum gates.

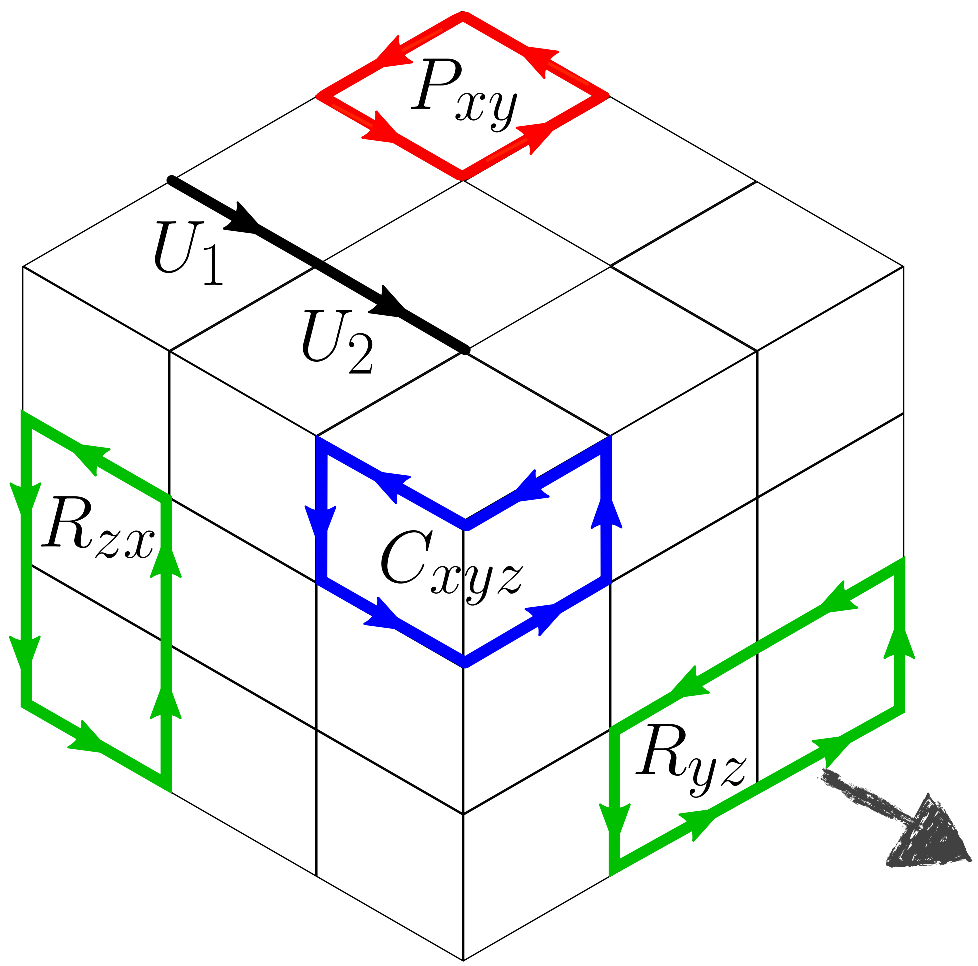
Demonstration

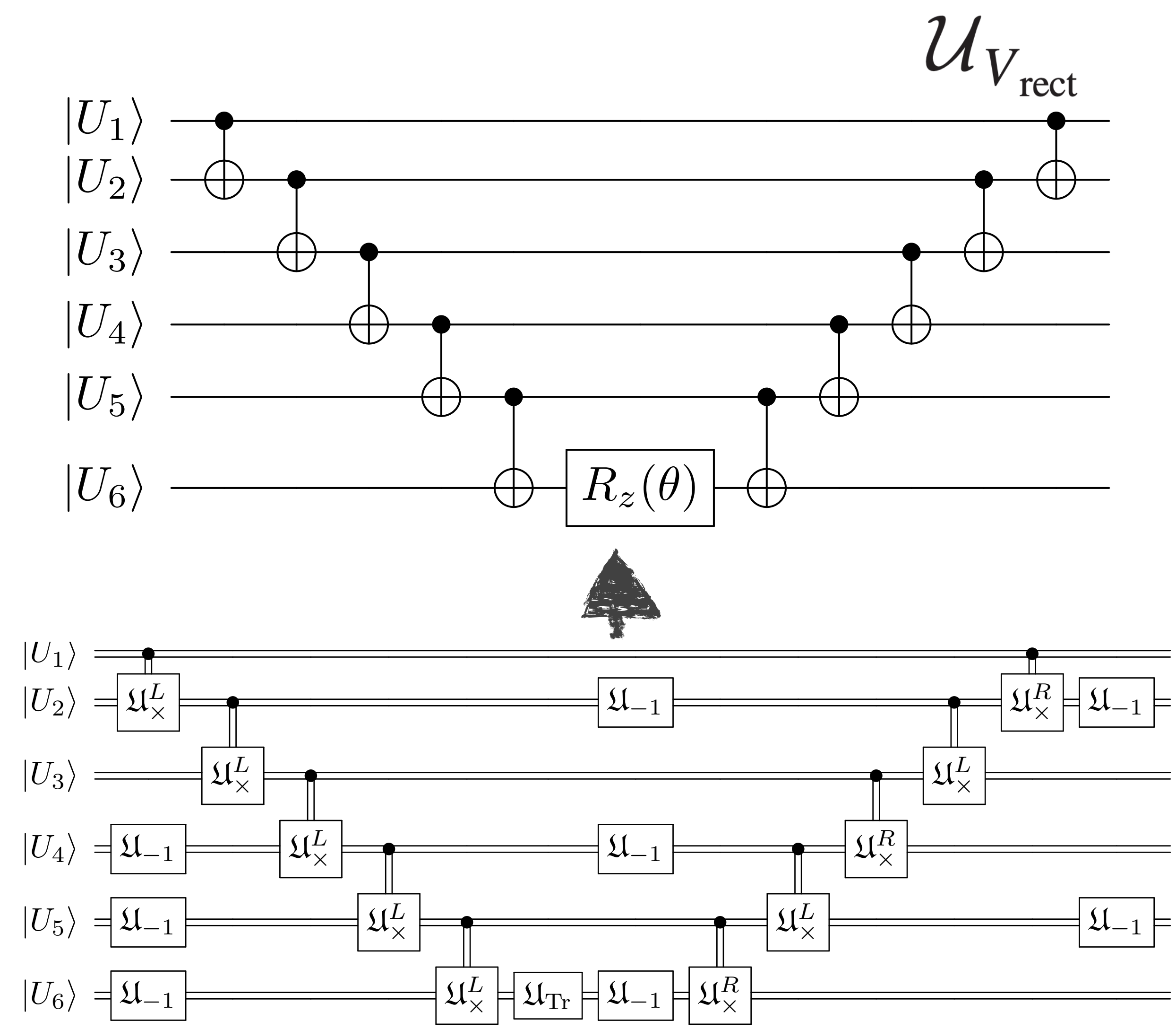
[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Demonstration for Improved Hamiltonian

$$\mathbb{Z}_2 \qquad \begin{array}{l} 1 \to |0\rangle \\ -1 \to |1\rangle \end{array}$$

$\mathfrak{U}_F$	H
$\mathfrak{U}_{\mathrm{phase}}$	$R_z(\theta)$
$\mathfrak{U}_{\mathrm{Tr}}$	$R_z(\theta)$
$\mathfrak{U}_{-1}$	$\mathbb{1}$
$\mathfrak{U}_{\times}$	CNOT





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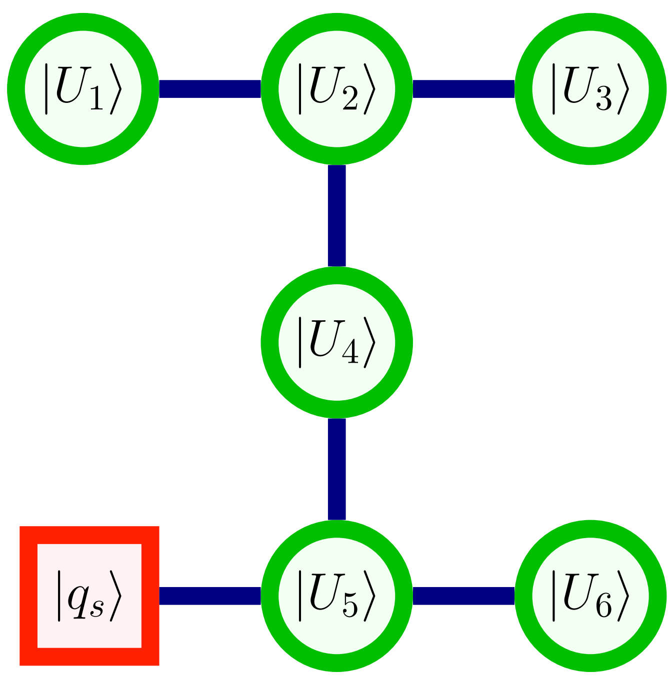
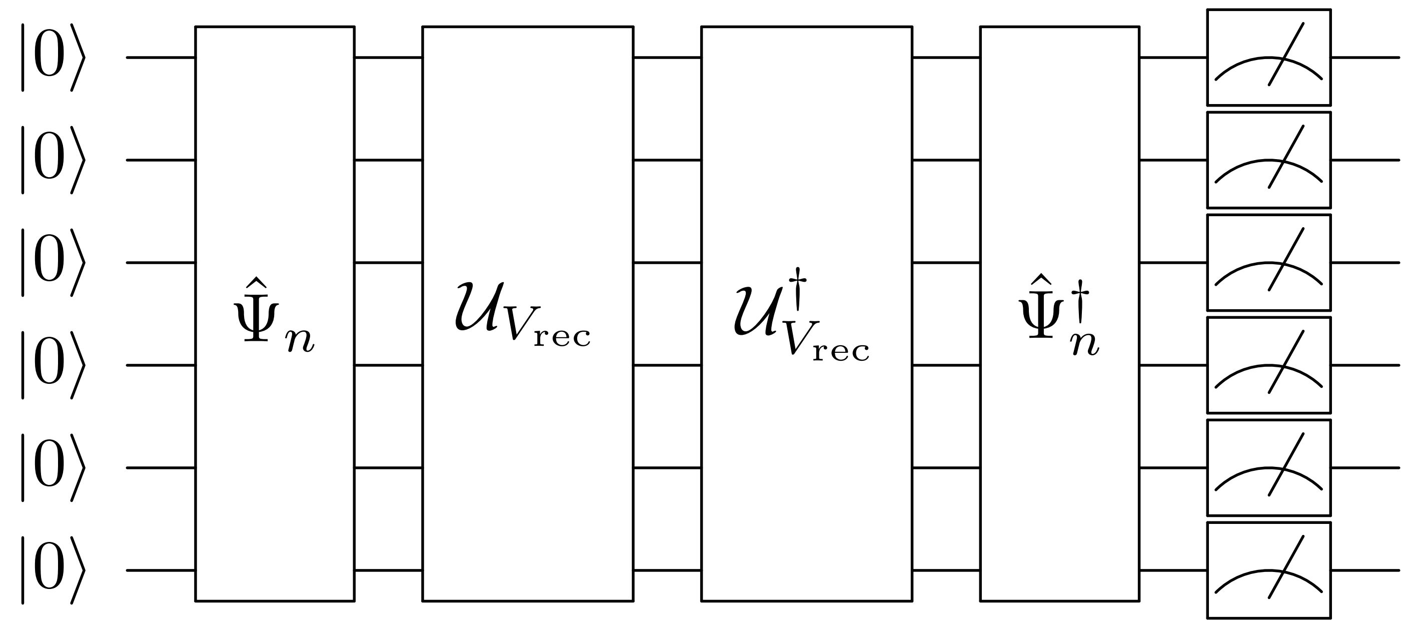
Demonstration for Improved Hamiltonian



ibm_perth device



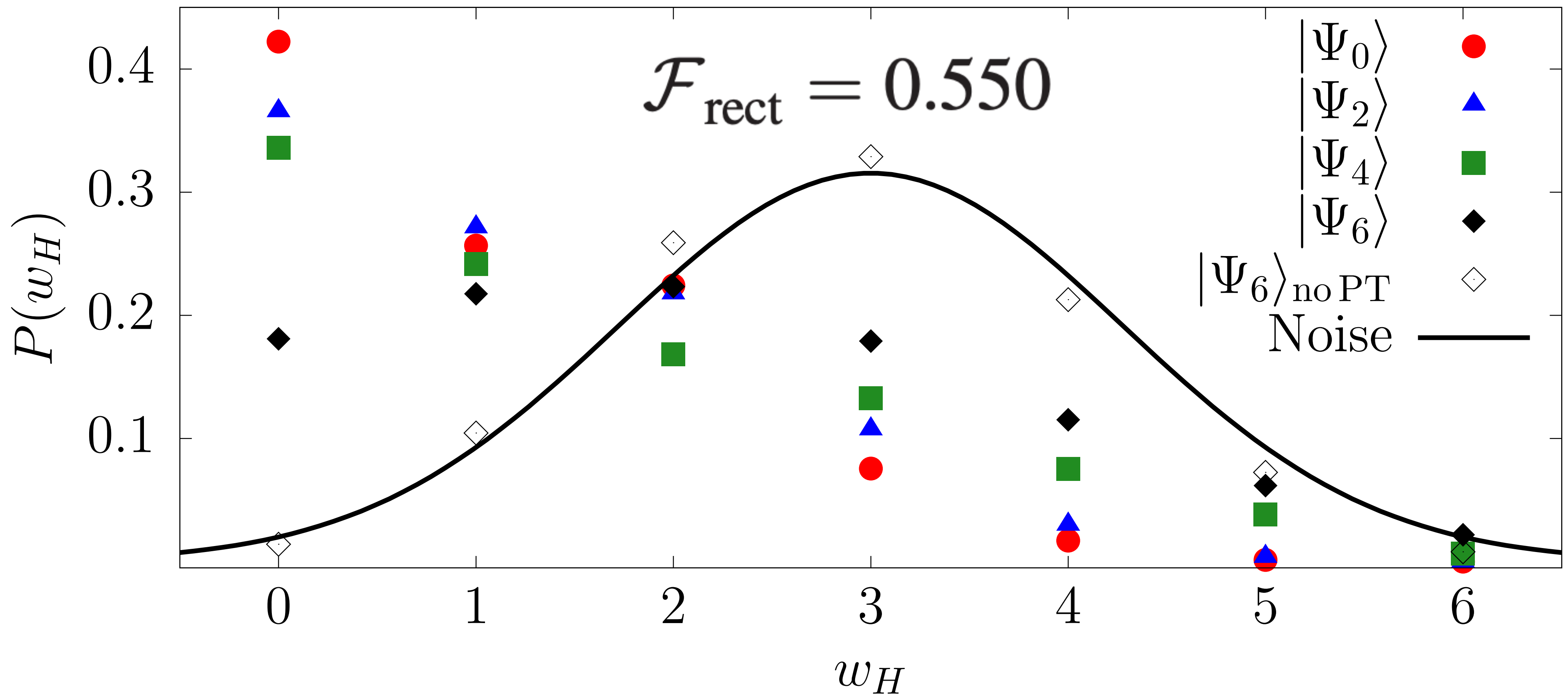
$$\hat{\Psi}_n = \prod_{m\leq n} H^{\otimes}_m$$

$$\left[\prod_i (\sigma_i^{b_i})^{\otimes}\right] \mathrm{CNOT} \otimes \mathbb{1}_4 \left[\prod_i (\sigma_i^{a_i})^{\otimes}\right] = \mathrm{CNOT} \otimes \mathbb{1}_4$$

w_H :number of states measured in the 1 state

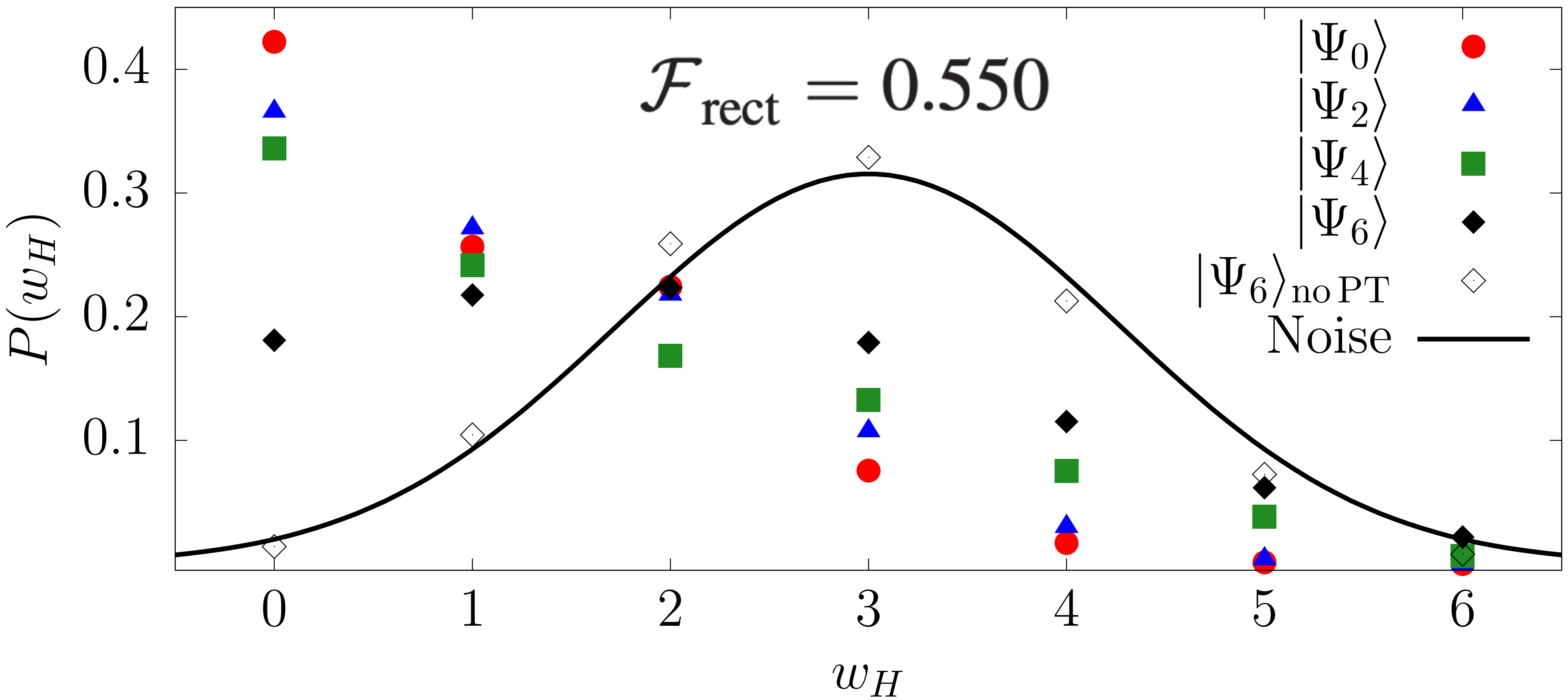
[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Demonstration for Improved Hamiltonian



[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Demonstration for Improved Hamiltonian

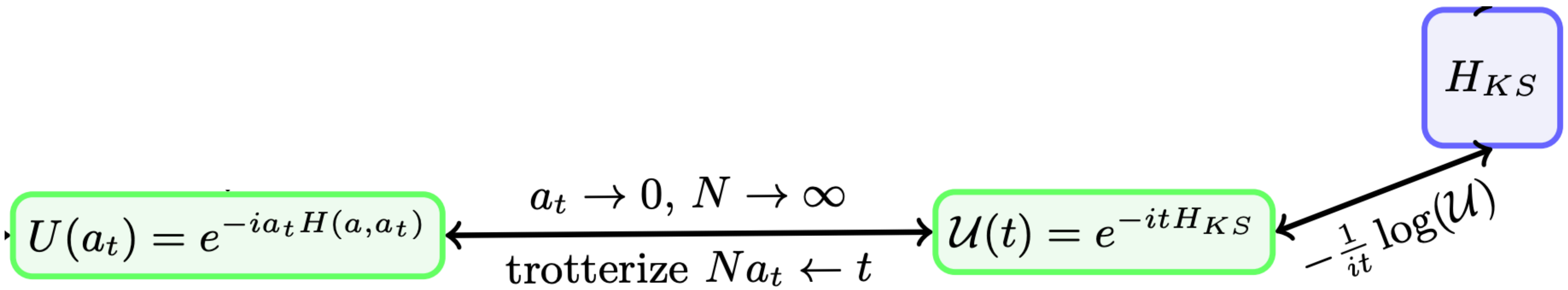


$$\mathcal{F}_\delta \approx 0.25$$

demonstration of improved Hamiltonian is allowed in the near future

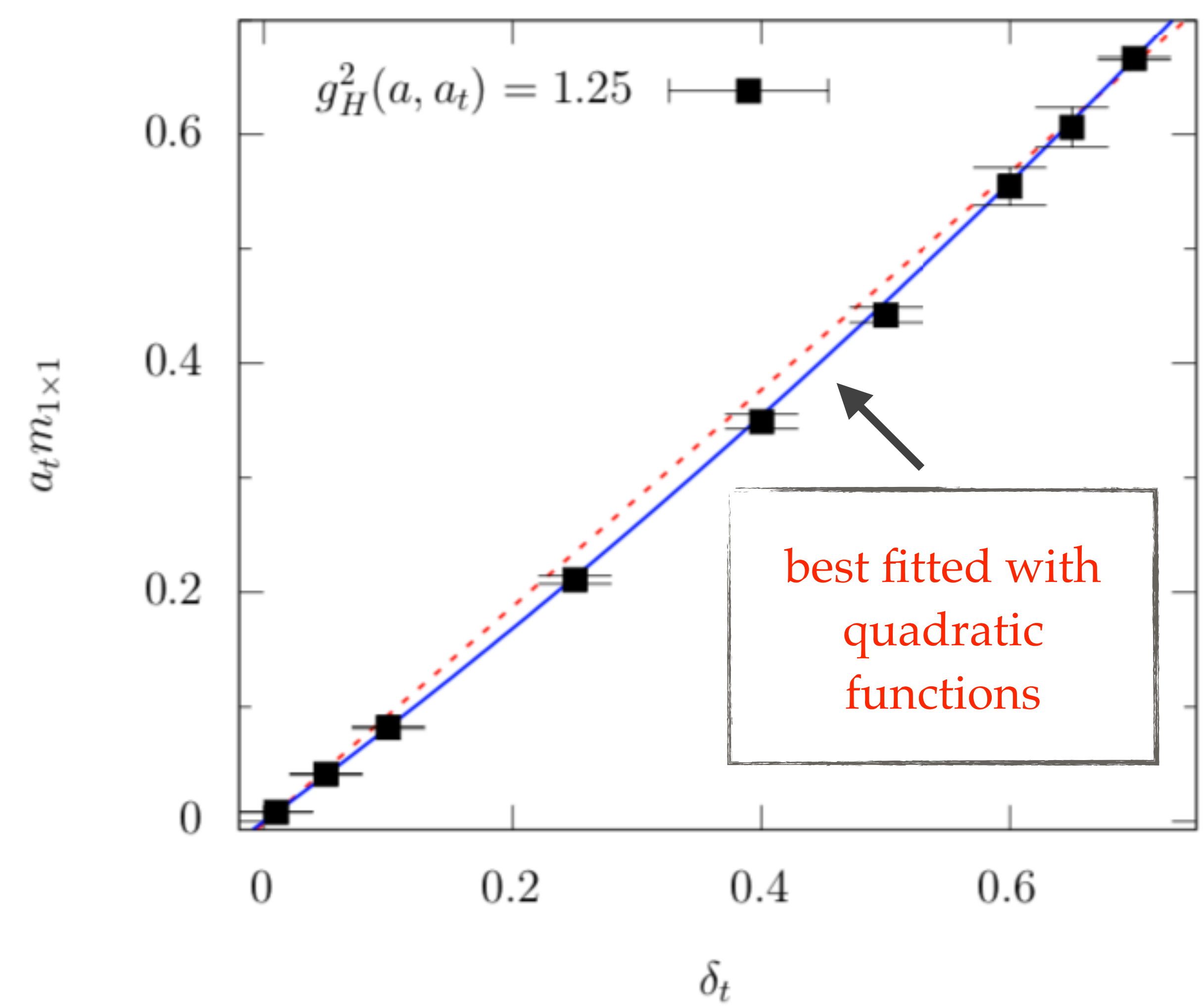
[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Propagation– Discretization in Time



$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx [e^{ix_1 H_V} e^{ix_2 H_K} e^{ix_3 H_V} e^{ix_4 H_K} \cdots]^N + O(a_t^p)$$

Anisotropic lattices $\xi = \frac{a}{a_t}$, introducing finite temporal lattice spacing renormalized via scale setting

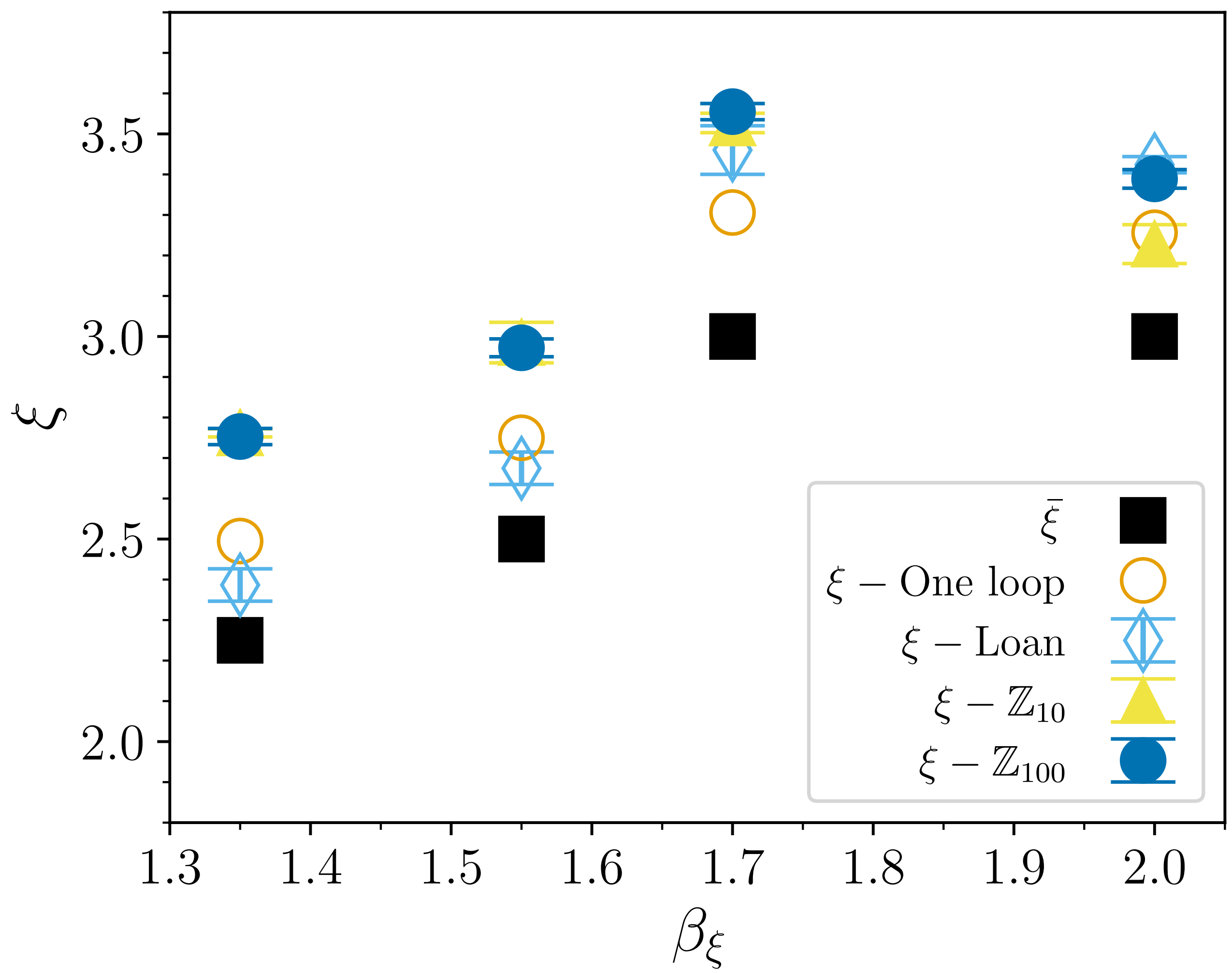


[M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

Propagation– Discretization in Time

Anisotropic Parameter Renormalization

- numerical results is pretty tedious—saving measurement on the Euclidean side
- Preferred for analytical continuation
- Determine the fixed anisotropic trajectory
- Continuous group agrees quite well with their discrete subgroups



β_ξ	N_s	N_t	$\bar{\xi}$	$\xi_{1-\text{loop}}$	ξ	
					$\mathbb{BI}$	$SU(2)$ [111]
$D=3$						
2.00	36	72	2.00	2.097	2.099(1)	⋯
2.00	12[a]	60[a]	4.00	4.278	⋯	4.35(19)
2.65	16[a]	64[a]	4.00	4.207	⋯	4.22(11)
3.00	36	72	1.33	1.351	1.369(19)	⋯
4.00	24[a]	96[a]	4.00	4.136	⋯	4.08(9)
$D=4$						
3.0	36	72	1.33	1.351	1.36(1)	⋯

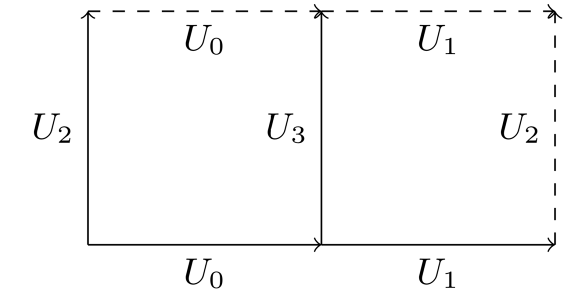
[M. Carena, E. J. Gustafson, H. Lamm, **YYL**, W. Liu, arXiv:2208.10417]

To the Continuum

$$a \to 0, a_t \to 0$$

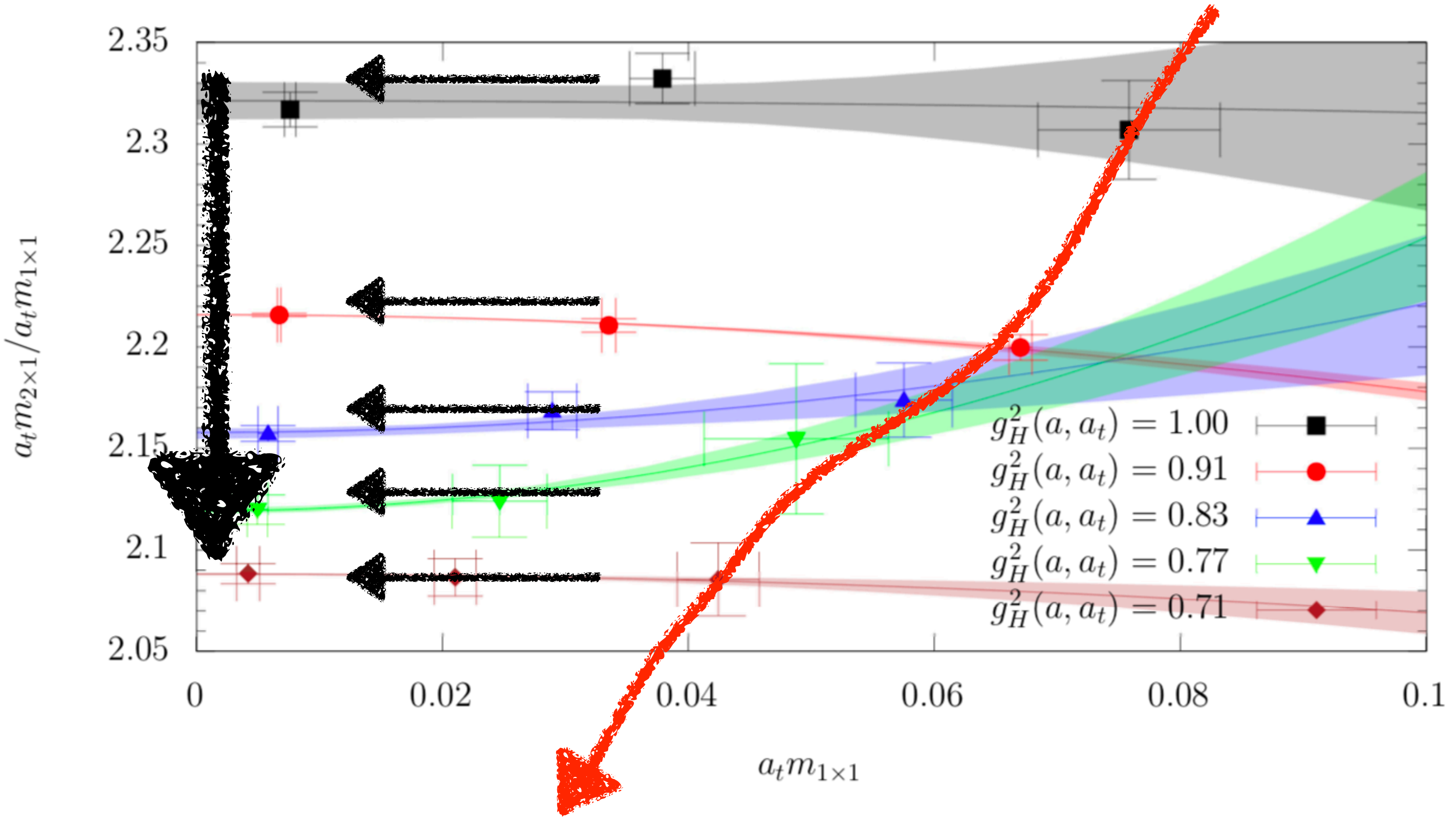
To the Continuum

D4 group



Eco-trajectory: extrapolation to the continuum at fixed $\xi = a/a_t$

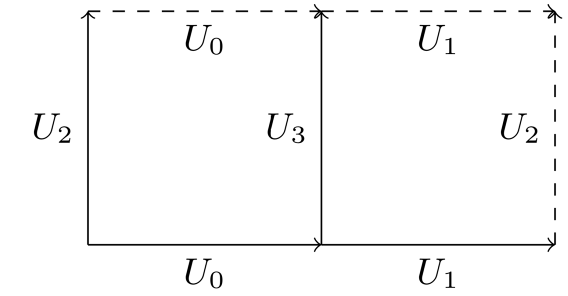
weak coupling input: $\xi^{-1} \sim \delta_t$, if $\delta_t \ll 1$



[M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

To the Continuum

D4 group



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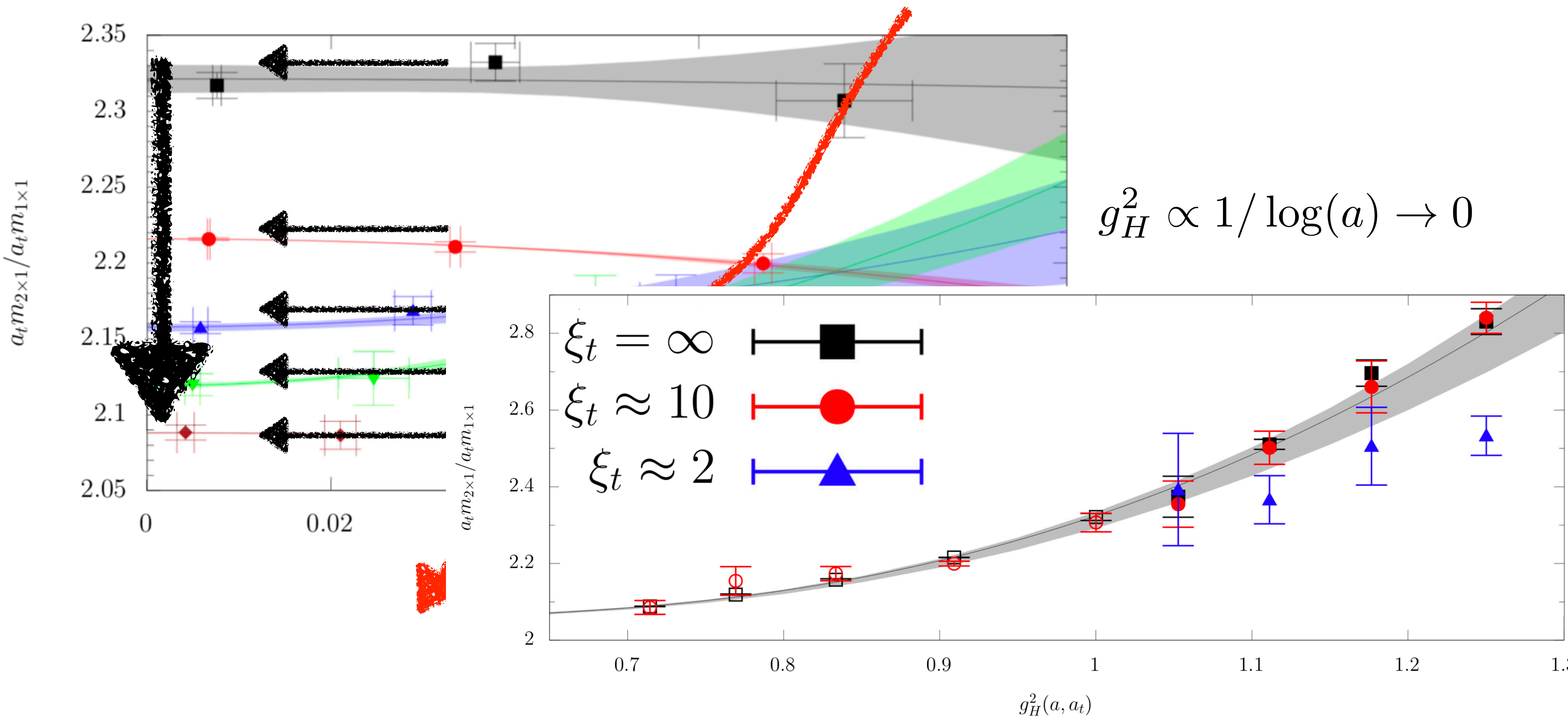


$g_H^2 \propto 1/\log(a) \to 0$

[M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

Conclusions

- Techniques on real-time simulation of lattice field theory:

  improved Hamiltonian: matrix elements for the improved terms,
  circuits designed, fidelity for current device.

  scale setting, fixed anisotropic trajectory.

- More to explore

  demonstration of different Hamiltonians, renormalization in error mitigation, discrete group validity regime, …

For High Energy Physics

- All of these techniques could be used for simulations of high energy processes in the future!!

Thank you

BACK UP

Discretization of space

$$\hat{K}_{2L} = \frac{g_t^2}{a} \sum_{x,i} \mathrm{Tr}[\hat{R}_i(\mathbf{x}) \hat{L}_i(\mathbf{x} + a\mathbf{i})]$$

?

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

Discretization of space

$$\mathrm{Tr}(\hat{R}_1\hat{L}_2) = \mathrm{Tr}[\hat{L}_2^2 + \hat{R}_1^2 - (\hat{L}_2 - \hat{R}_1)^2]/2$$

$$\mathcal{U}_{K_{2L}} \equiv e^{i\theta\,\mathrm{Tr}(\hat{L}_2 - \hat{R}_1)^2}$$

$$[\mathcal{U}_{K_{2L}}, \hat{U}_1\hat{U}_2] = 0$$

$$\langle U_1', U_2' | \mathcal{U}_{K_{2L}} | U_1, U_2 \rangle = \delta_{U_1'U_2', U_1U_2} \langle U_1' | e^{i\theta\,\mathrm{Tr}\,\hat{L}_1^2} | U_1 \rangle$$

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601]

$$\langle U_1', U_2'| \mathcal{U}_{K_{2L}} |U_1, U_2\rangle = \delta_{U_1'U_2', U_1 U_2} \mathcal{A}(U_1, U_2, U_1').$$

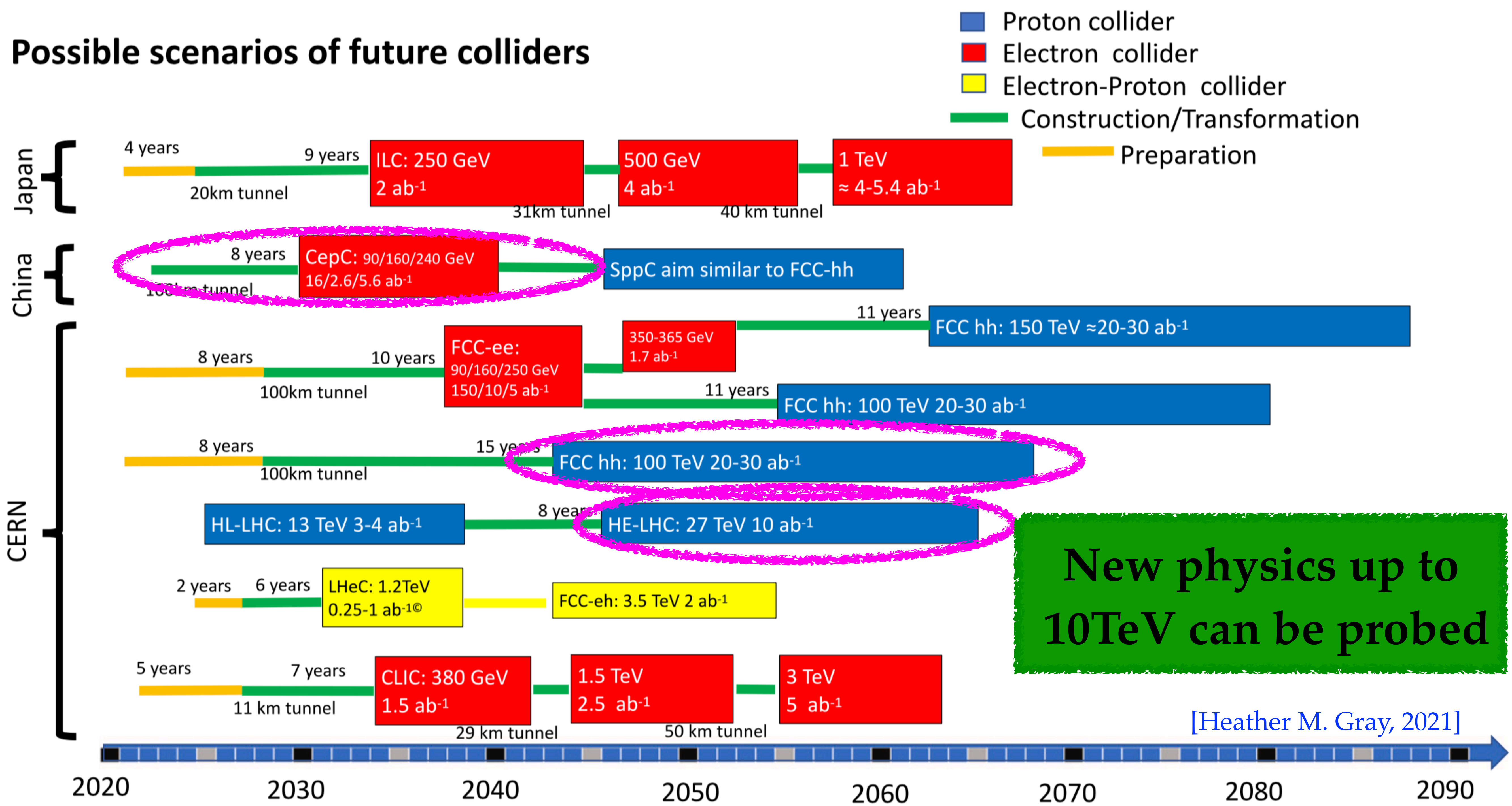
$$\mathcal{A}(U_1, U_2, U_1') = \int dU_2' \langle U_1', U_2'| \mathcal{U}_{K_{2L}} |U_1, U_2\rangle \, .$$

$$\mathcal{A}(U_1, U_2, U_1') = \int dV \, \langle U_1', U_2| \, e^{i\alpha_c \hat{L}_2^c} \mathcal{U}_{K_{2L}} |U_1, U_2\rangle \, .$$

$$\int dV e^{i\alpha_a \hat{L}_2^a} = |G| \, |J_2 = 0\rangle \, \langle J_2 = 0| = |G| \hat{P}_{J_2=0}.$$

$$\begin{aligned} \mathcal{A}(U_1, U_2, U_1') &= \langle U_1', J_2 = 0| \, \mathcal{U}_{K_{2L}} \, |U_1, J_2 = 0\rangle \\ &= \langle U_1'| \, e^{i\theta \, \mathrm{Tr} \, \hat{R}_1^2} \, |U_1\rangle \, , \end{aligned}$$

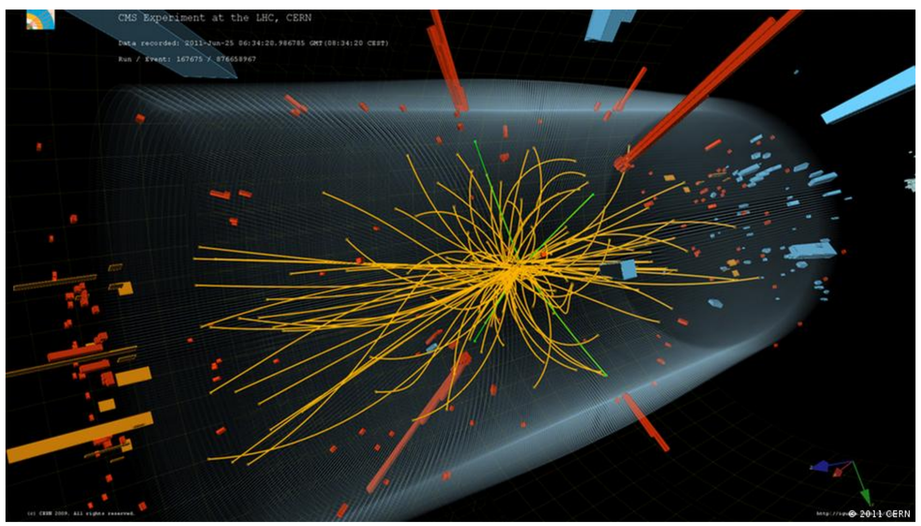
-Now-: precision measurement



-Now-: precision measurement

New physics up to TeV can be probed

HL-Large Hadron Colliders (LHC)



HL-LHC CMS & ATLAS (3 ab^{-1}) in %									
$\mu^{\gamma\gamma}_{ggh}$	4.20 4.52	$\mu^{\gamma\gamma}_{VBF}$	12.8 8.93	$\mu^{\gamma\gamma}_{Wh}$	13.9 14.1	$\mu^{\gamma\gamma}_{Zh}$	23.3 16.5	$\mu^{\gamma\gamma}_{tth}$	9.40 8.92
μ^{ZZ}_{ggh}	4.00 4.64	μ^{ZZ}_{VBF}	13.4 11.8	μ^{ZZ}_{Wh}	47.8 43.8	μ^{ZZ}_{Zh}	78.6 83.3	μ^{ZZ}_{tth}	24.6 19.7
μ^{WW}_{ggh}	3.70 6.16	μ^{WW}_{VBF}	7.30 6.68	μ^{WW}_{Wh}	13.8 –	μ^{WW}_{Zh}	18.4 –	μ^{WW}_{tth}	9.70 114
$\mu^{\tau\tau}_{ggh}$	5.50 8.79	$\mu^{\tau\tau}_{VBF}$	4.40 8.06					$\mu^{\tau\tau}_{tth}$	14.9 73.3
$\mu^{\mu\mu}_{ggh}$	13.8 18.5	$\mu^{\mu\mu}_{VBF}$	54.0 36.1						
$\mu^{Z\gamma}_{ggh}$	– 33.3	$\mu^{Z\gamma}_{VBF}$	– 68.2						
μ^{bb}_{ggh}	24.7 –			μ^{bb}_{Wh}	9.40 10.1	μ^{bb}_{Zh}	6.5 5.85	μ^{bb}_{tth}	11.6 14.8

[Jorge de Blas, et al, arXiv:1907.04311]

$$\sim \mathcal{O}(10\%)$$

$$
\begin{aligned}
\left[E_i^a(\boldsymbol{x}), A_j^b(\boldsymbol{y})\right] &= i\delta_{ij}\delta_{ab}\delta(\boldsymbol{x}-\boldsymbol{y}) \\
\left[A_i^a(\boldsymbol{x}), A_j^b(\boldsymbol{y})\right] &= \left[E_i^a(\boldsymbol{x}), E_j^b(\boldsymbol{y})\right] = 0.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_l &= \frac{1}{a}\int_{-a/2}^{a/2} dt A_i(\boldsymbol{x} + t\hat{\boldsymbol{i}}), \\
&= \frac{1}{a}\int_{-a/2}^{a/2} dt \left[A_i(\boldsymbol{x}) + t\partial_i A_i(\boldsymbol{x}) + \frac{1}{2}t^2\partial_i^2 A_i(\boldsymbol{x}) + \mathcal{O}(a^3)\right] \\
&= A_i(\boldsymbol{x}) + \frac{a^2}{24}\partial_i^2 A_i(\boldsymbol{x}) + \frac{a^4}{1920}\partial_i^2 A_i(\boldsymbol{x}) \ldots
\end{aligned}
$$

$$
\mathcal{E}_i^{(1)\alpha}(\boldsymbol{x}) = -\frac{a^{d-1}}{e}\left[E_i^\alpha(\boldsymbol{x}) - \frac{a^2}{24}\partial_i^2 E_i^\alpha(\boldsymbol{x})\right]
$$

$$
\begin{aligned}
[\hat{L}_i^a(\mathbf{x}), \hat{U}_i(\mathbf{x})] &= \lambda_a \hat{U}_i(\mathbf{x}), \\
[\hat{L}_i^a(\mathbf{x}), \hat{L}_i^b(\mathbf{x})] &= -if_{abc}\hat{L}_i^c(\mathbf{x}),
\end{aligned}
$$

Discretization of space

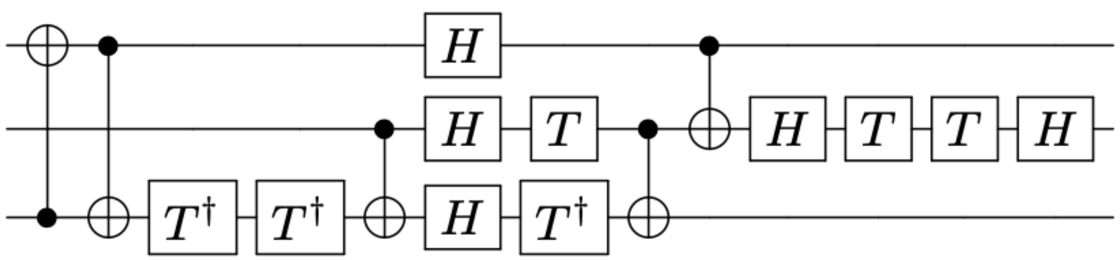
$$\left[E_i^a(\boldsymbol{x}), A_j^b(\boldsymbol{y})\right] = i\delta_{ij}\delta_{ab}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$\left[A_i^a(\boldsymbol{x}), A_j^b(\boldsymbol{y})\right] = \left[E_i^a(\boldsymbol{x}), E_j^b(\boldsymbol{y})\right] = 0.$$

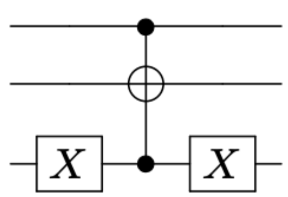
$$[\hat{L}_i^a(\mathbf{x}), \hat{U}_i(\mathbf{x})] = \lambda_a \hat{U}_i(\mathbf{x}),$$
$$[\hat{L}_i^a(\mathbf{x}), \hat{L}_i^b(\mathbf{x})] = -i f_{abc} \hat{L}_i^c(\mathbf{x})$$

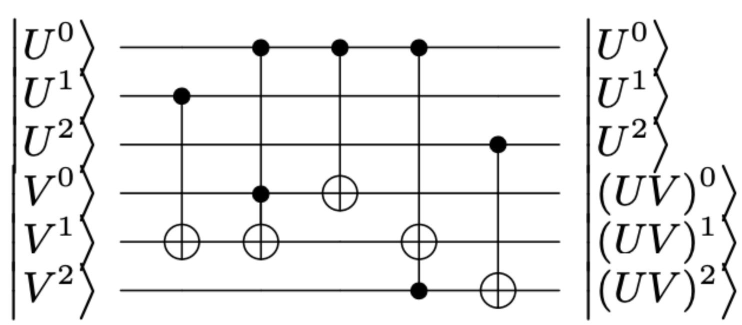
Gate counting for D4

TABLE I. Gate requirements for the propagation of a lattice with N_P plaquettes and L links, for a time T with time-steps of size Δt

Gate	Number
$\mathfrak{U}_F$	$2L\frac{T}{\Delta t}$
$\mathfrak{U}_{\text{phase}}$	$L\frac{T}{\Delta t}$
$\mathfrak{U}_{-1}$	$6N_P\frac{T}{\Delta t}$
$\mathfrak{U}_{\times}$	$6N_P\frac{T}{\Delta t}$
$\mathfrak{U}_{\text{Tr}}$	$N_P\frac{T}{\Delta t}$







[H. Lamm, et al, arXiv:1903.08807]

Discretization of space

[J. Carlsson, et al, hep-lat/0105018]

$$[L_i^a(\mathbf{x}), U_i(\mathbf{x})] = T_a U_i(\mathbf{x})$$

$$[L_i^a(\mathbf{x}), L_i^b(\mathbf{x})] = -i f_{abc} L_i^c(\mathbf{x})$$

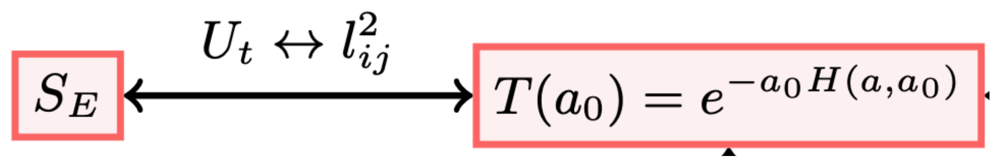
$$[R_i^a(\mathbf{x}), U_i(\mathbf{x})] = U_i(\mathbf{x}) T_a$$

$$[R_i^a(\mathbf{x}), R_i^b(\mathbf{x})] = i f_{abc} R_i^c(\mathbf{x})$$

$$[L_i^a(\mathbf{x}), R_i^b(\mathbf{x})] = 0$$

Anisotropic Parameter Renormalization

Moreover, one-loop calculation of the renormalization effects on the Euclidean side



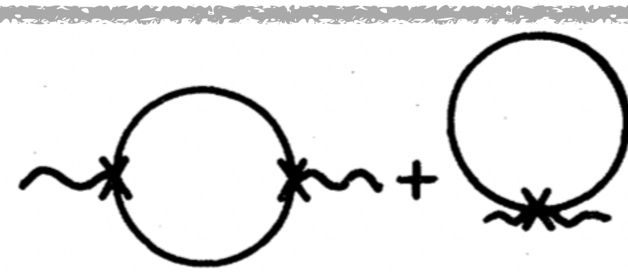
SU(N) For $\xi \to \infty$ [C. J. Hamer, Phys. Rev. D **53**, 7316, …]

For finite ξ in $3+1$ [F. Karsch, Nuclear Physics B205 (1982) 285-300, …]

Background field method at one loop order [R. Dashen and D. Gross, Phys. Rev. D **23**, 2340]

$$U_{x,x+\mu} = e^{ieg_E \alpha_\mu(x)} U^{(0)}_{x,x+\mu}, \quad U^{(0)}_{x,x+\mu} = e^{iea_\mu A_\mu(x)}$$

$$S_{gf} = a^{D-1} a_\tau \sum_x \mathrm{Tr} \left(\sum_\mu \overline{D^{(0)}_\mu} \alpha_\mu(x) \right)^2 \qquad S_{gh} = 2a^{D-1} a_\tau \sum_{x,\mu} \mathrm{Tr}[(D^{(0)}_\mu \phi(x))^\dagger (D^{(0)}_\mu \phi(x))]$$



α_μ/ϕ running in the loop

Independent of regularization $\Delta S_{\text{eff}} = S_{\text{eff}}^{(\xi=1)} - S_{\text{eff}}^{(\xi\neq1)} = 0$