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Quantum SPAIN

High-dimensional Quantum Computing

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Barcelona Supercomputing Center – Centro Nacional de Supercomputación

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International Conference on Quantum Technologies for High-Energy Physics (QT4HEP22), CERN

Outlook

1. High-dimensional quantum systems
2. Quantum Information beyond qubits
3. Experimental example: generation of qutrit GHZ state
4. Algorithmic example: quantum function fitting with qudits
5. Conclusions

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High-dimensional quantum systems

Any quantum system contains more than two levels

- Quantum computation is mathematically described with the algebra of $SU(N)$
- For qubits: $SU(2^n)$
- For qudits (d-dimensional units of quantum information): $SU(d^n)$

Quantum harmonic oscillator: d levels of Fock space

Solid state physics: Fermi or Bose-Hubbard model

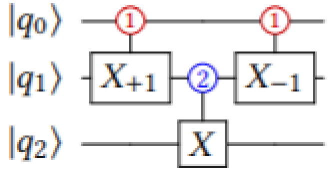
Particles with spin: $2s + 1$ levels

Other quantum numbers: e.g. color: QCD 3 levels, described by $SU(3)$ (in quantum info terms, qutrits)

Why using a 2-dimensional (qubit) quantum computer to study systems that are not 2-level?

Quantum information beyond qubits

- **High-dimensional entanglement and non-locality** is genuinely different than in 2-dim systems
  - Bi-partite non-locality tests (Bell) tested for high-dimensional systems with photons.
  - Multi-partite non-locality (Bell or GHZ) NOT TESTED YET for high-dimensional systems.



In [Quantum Computation]():

- **Quantum Error Correction**: high-dimensional QEC codes [1][2][3]
- **Quantum Circuit Simplifications**, e.g. reduction in the circuit depth of a qubit Toffoli gate by manipulating states in the 3rd level [4].
- **Qubit control improvement** by using the third level in superconducting quantum chips [5].
- **Quantum algorithm improvement?** Growing the Hilbert space in "d" instead of "n" (suitable for NISQ)

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[1] *Enhanced Fault-Tolerant Quantum Computing in d-Level Systems,* E. T. Campbell, Phys. Rev. Lett. **113**, 230501 (2014).
[2] *Magic-State Distillation in All Prime Dimensions Using Quantum Reed-Muller Codes,* E. T. Campbell, et. al. , Phys. Rev. X **2**, 041021 (2012).
[3] *Optimal quantum error correcting codes from absolutely maximally entangled states,* Z. Raissi, et. al., J. Phys. A: Math. Theor. **51** 075301 (2018)
[4] *Asymptotic improvements to quantum circuits via qutrits,* P. Gokhale, et. al., ISCA '19, 554–566 (2019).
[5] *Rabi oscillations in a large josephsonjunction qubit*, J. M. Martinis, et. al., Phys. Rev. Lett. 89, 117901 (2002).

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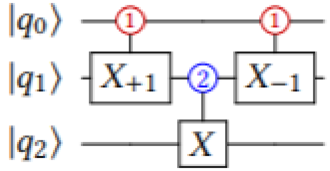

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Experimental high-dimensional quantum computation

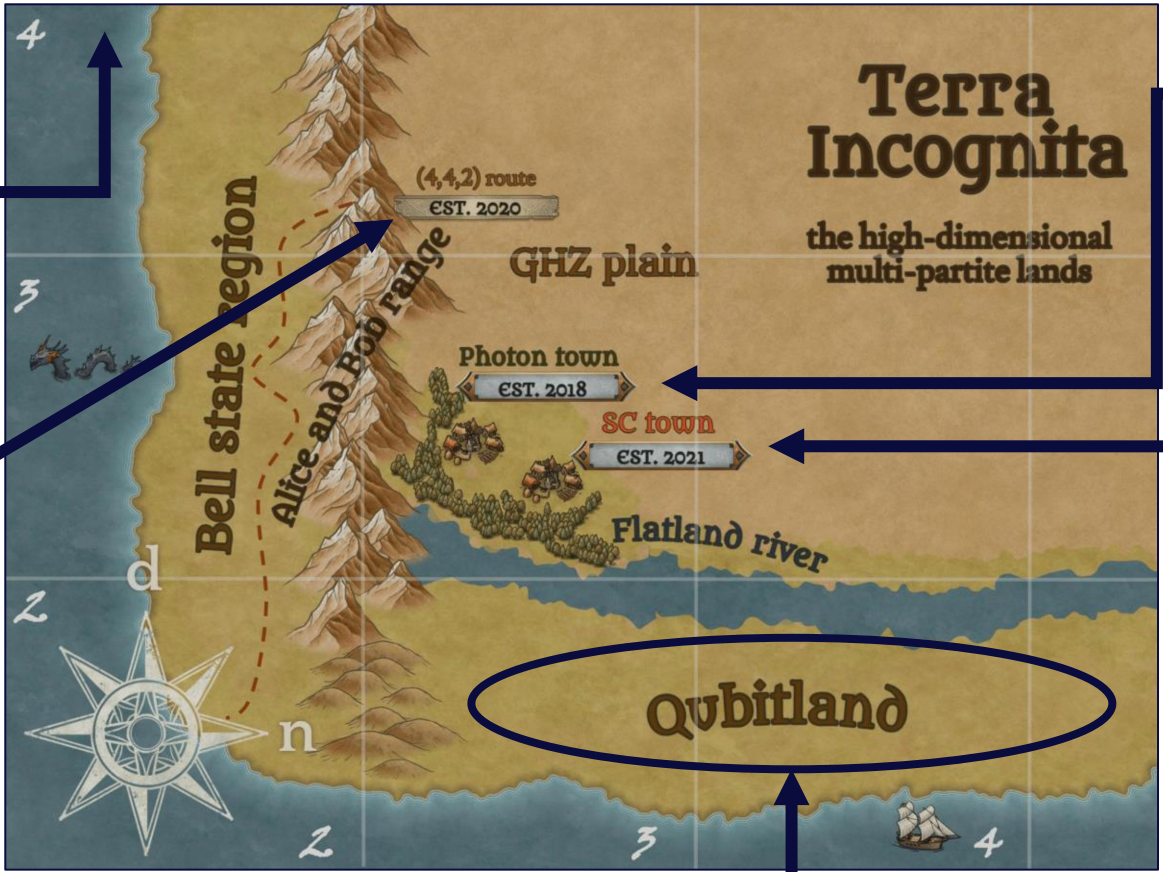
Based on: “*Experimental high-dimensional Greenberger-Horne-Zeilinger entanglement with superconducting transmon qutrits*”, Alba Cervera-Lierta, Mario Krenn, Alán Aspuru-Guzik, Alexey Galda, Phys. Rev. Applied **17**, 024062 (2022)

"Entangland"

Experimental generation of high-dimensional states is hard

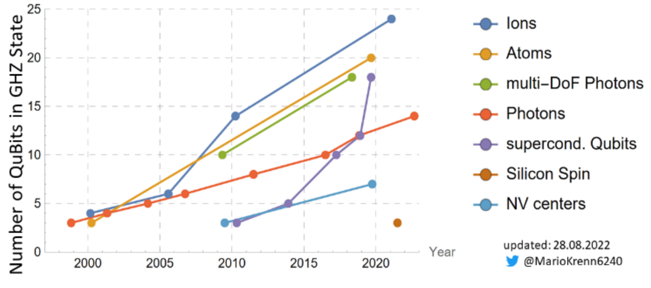
Generation and confirmation of a (100 × 100)-dimensional entangled quantum system
Mario Krenn et. al., PNAS **111** (17) 6243-6247 (2014)

Experimental creation of multi-photon high-dimensional layered quantum states
Xiao-Min Hu et. al., npj Quantum Information **6**, 88 (2020)



Experimental Greenberger–Horne–Zeilinger entanglement beyond qubits
Manuel Erhard et. al., Nature Photonics **12**, pages759–764 (2018)

Experimental high-dimensional Greenberger-Horne-Zeilinger entanglement with superconducting transmon qutrits
Alba Cervera-Lierta, Mario Krenn, Alán Aspuru-Guzik, Alexey Galda, Phys. Rev. Applied 17, 024062 (2022).



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Quantum non-locality in high-dimensions

Quantum foundations: non-locality tests (GHZ contradictions)

	Theory	***Experiment***
Qubits	*Bell's theorem without inequalities*, D. M. Greenberger, A. Horne, A. Zeilinger American Journal of Physics **58**, 1131 (1990)	*Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement* J-W Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, A. Zeilinger Nature **403**, 515–519 (2000)
Qudits	*Rotational covariance and Greenberger-Horne-Zeilinger theorems for three or more particles of any dimension* J. Lawrence, Phys. Rev. A 89, (2014)	**NOT YET!**

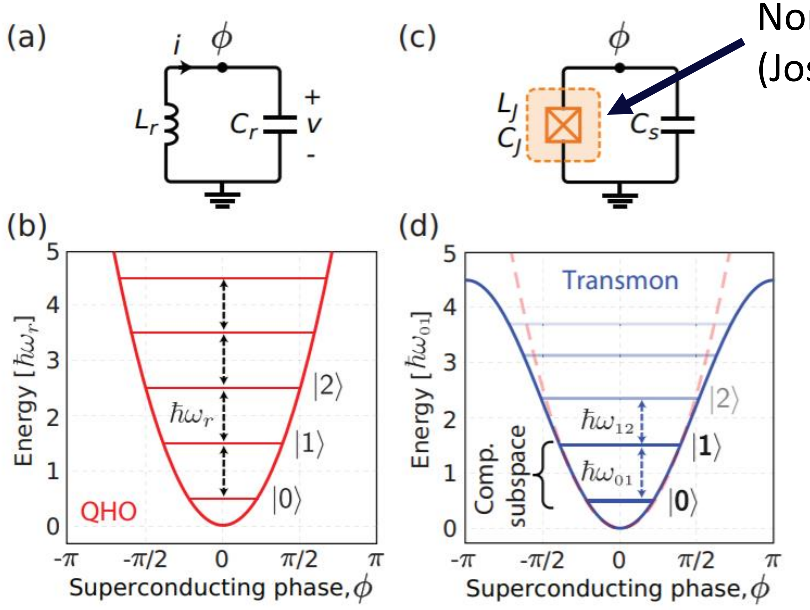
For that, we need to generate a GHZ state first

With photons, it takes way too long to perform a GHZ-test afterwards (due to photon detection and generation)

Superconducting circuits are faster, can we generate a qudit GHZ state with them?

Superconducting transmon ~~qubits~~ qudits

Non-linear inductance
(Josephson Junction)



Quantum Harmonic Oscillator   Quantum anharmonic Oscillator

Philip Krantz et. al., Applied Physics Reviews **6**, 021318 (2019)

Transmons contain more than two energy levels.

By finding the proper frequency transitions we can address and manipulate high-dimensional states.

We have to calibrate and define “manually” all qudit quantum gates (unless the quantum computer software includes high-dimensional quantum computation, which is rare at the moment).

With Trapped Ions:
“A universal qudit quantum processor with trapped ions”
M. Ringbauer, M. Meth, L. Postler, R. Stricker, R. Blatt, P. Schindler, T. Monz, Nature Physics **18**, 1053–1057 (2022).

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A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, Phys. Rev. Applied **17**, 024062 (2022).

Pulse Schedule quantum circuit

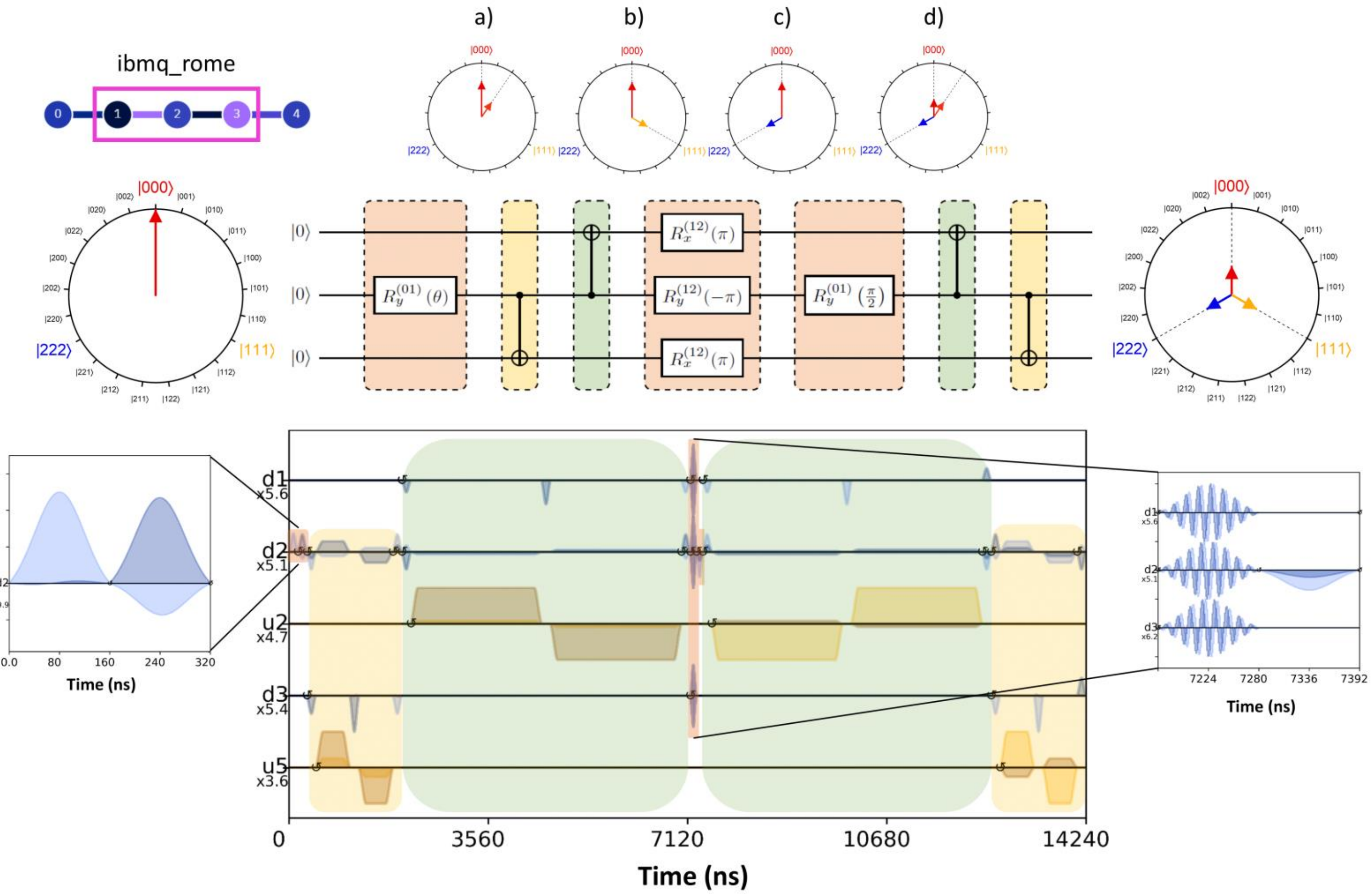
Qutrit GHZ generation

Qubit gates: $R_y^{(01)}$ and CNOT (default calibration by IBM).

Qutrit gates: $R_\alpha^{(12)}(\theta) = e^{-i\frac{\theta}{2}\sigma_\alpha^{(12)}}$ (manual calibration and definition).

We used Qiskit pulse software to program directly on the quantum chip hardware.

$$|GHZ\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$$



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A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, Phys. Rev. Applied **17**, 024062 (2022).

Tomography and Entanglement Witness

Verification of the genuine multi-partite high-dimensional entanglement

Fidelity

To compute the fidelity w.r.t. the GHZ state, we only need to measure the diagonal terms + 3 off-diagonal terms of the density matrix:

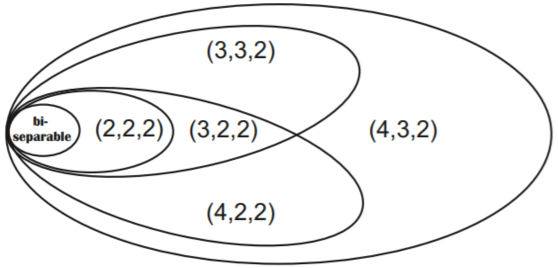
$$F_{exp} = \frac{1}{3}\left(\sum_{i=0}^{2}\langle iii|\rho|iii\rangle + 2\sum_{\substack{i,j=0\\ i<j}}^{2}\mathrm{Re}\langle iii|\rho|jjj\rangle\right)$$

We have to project into the $\sigma_{x,y}^{(ij)}$ basis to compute the expectation values.

$$\begin{aligned}\mathrm{Re}\left(\langle ijk|\rho|lmn\rangle\right) = \frac{1}{8}\big(&\langle\sigma_x^{(il)}\sigma_x^{(jm)}\sigma_x^{(kn)}\rangle\\ &-\langle\sigma_y^{(il)}\sigma_y^{(jm)}\sigma_x^{(kn)}\rangle\\ &-\langle\sigma_y^{(il)}\sigma_x^{(jm)}\sigma_y^{(kn)}\rangle\\ &-\langle\sigma_x^{(il)}\sigma_y^{(jm)}\sigma_y^{(kn)}\rangle\big)\end{aligned}$$

(And similar for the imaginary part)

Entanglement Witness



M. Huber, J. I. de Vicente, Phys. Rev. Lett. **110**, 030501 (2013)

The maximal fidelity achievable by a state with Schmidt Rank (3,3,2) is 66%.

If we measure a fidelity > 66%, we have a genuine 3-dimensional 3-partite entangled state.

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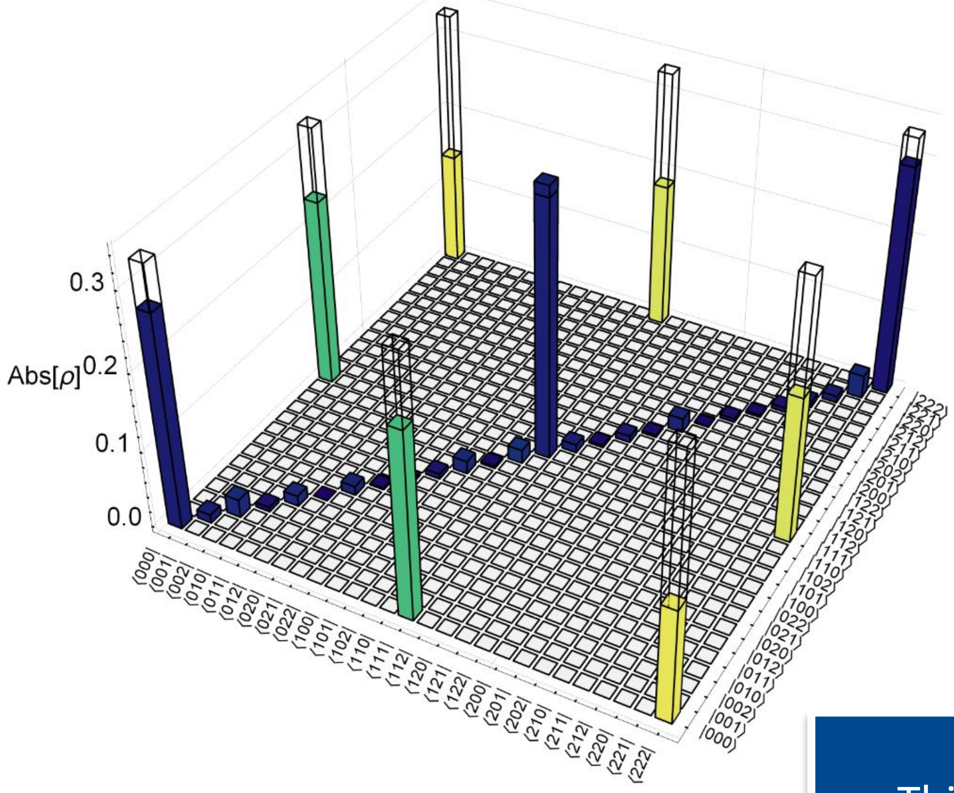
A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, Phys. Rev. Applied **17**, 024062 (2022).

Results

1st generation of a qutrit GHZ state with a non-photonic platform



$$F_{raw} = 0.69 \pm 0.01$$

$$F_{mit} = 0.76 \pm 0.01$$

Post-processing needed:
- SPAM errors (state preparation and measurement) mitigation.
- Relative phase between (01) and (12) subspace measurement and correction (due to the gates calibration used).

This experiment took ~1 min, ~30,000 times faster than the photonic one, and it's implemented in a programable device.

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A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, Phys. Rev. Applied **17**, 024062 (2022).

Algorithmic high-dimensional quantum computation

Based on: Berta Casas Font Master Thesis (2022) and publication in preparation.

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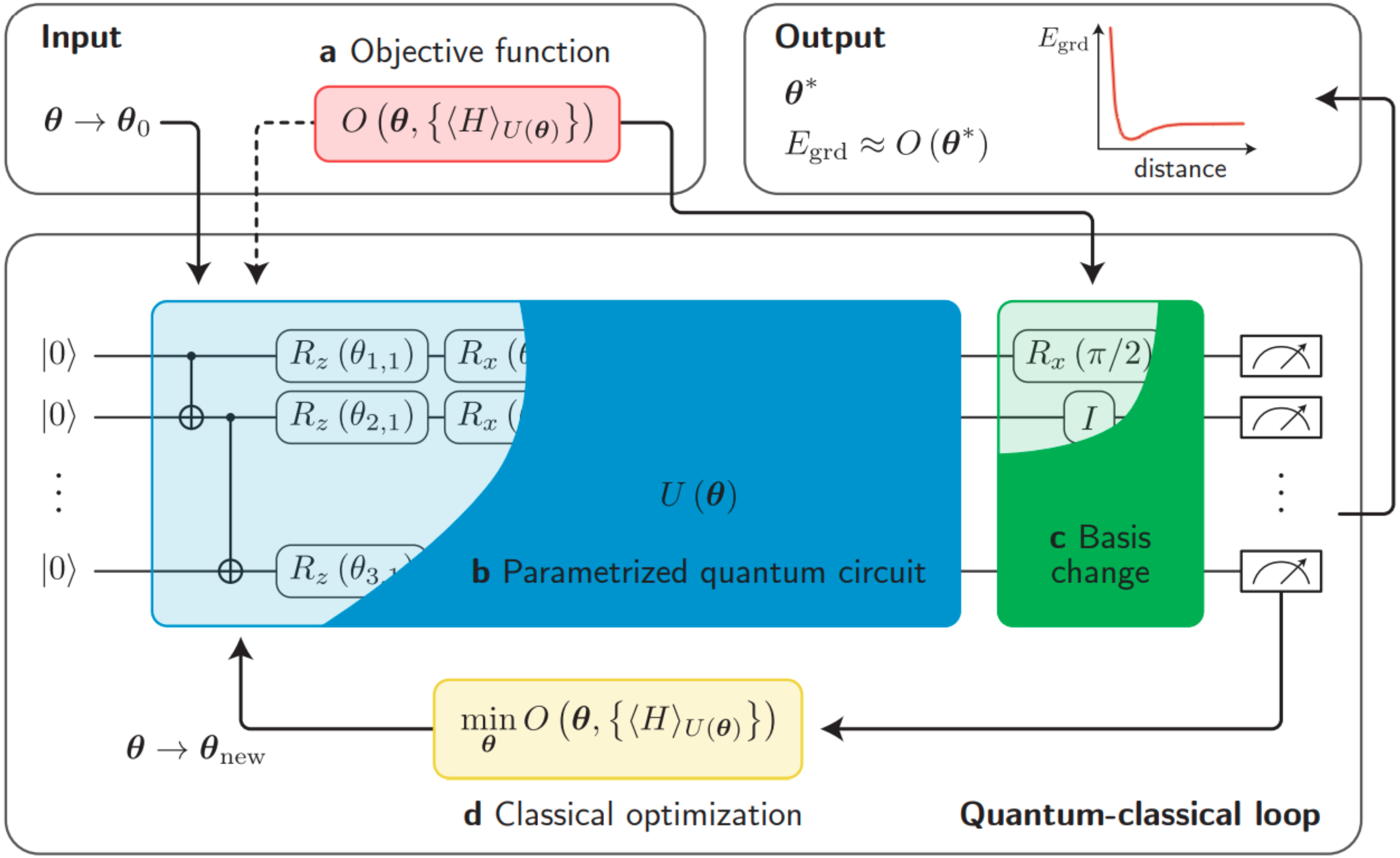
Variational Quantum Algorithms



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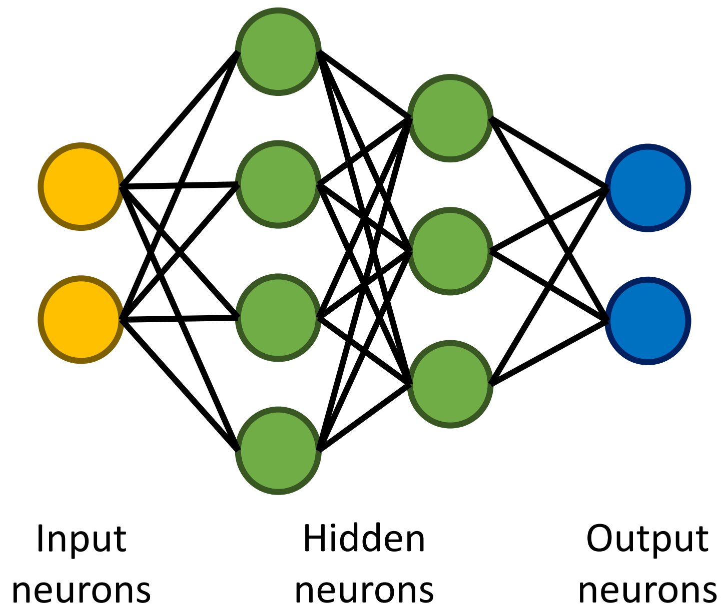
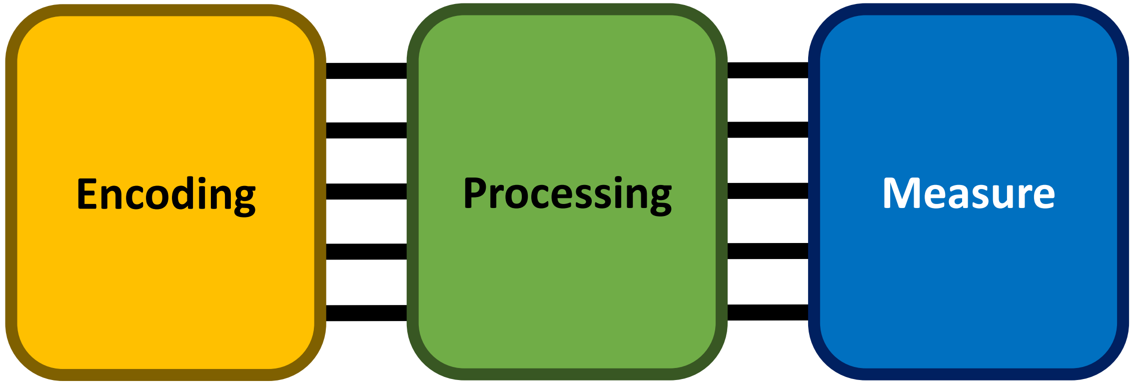
From classical to quantum NN

Classical



Quantum

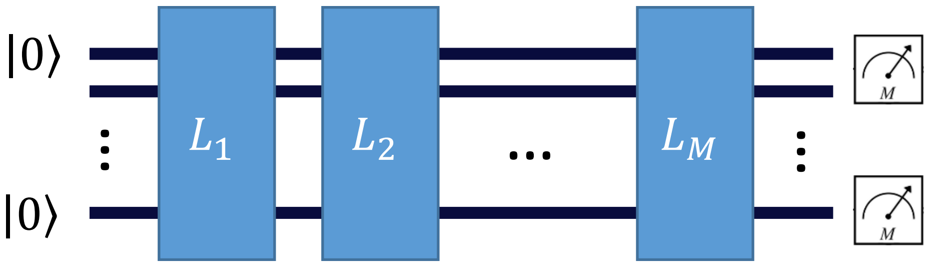


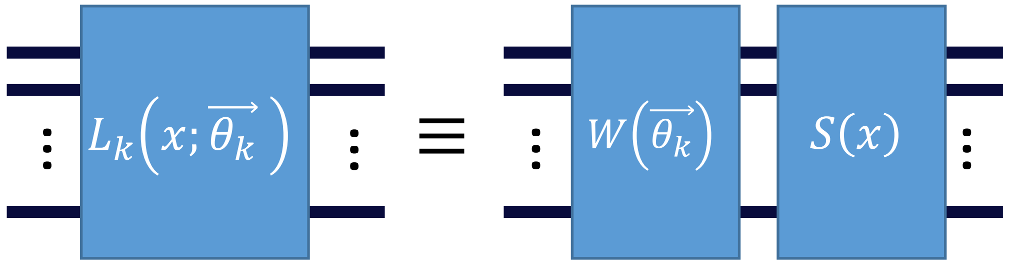
K Mitarai, M Negoro, M Kitagawa, K Fujii Phys. Revs A 98 (3), 032309 (2018)
E. Farhi and H.Neven, arXiv:1802.06002 (2018)
M. Schuld and N. Killoran, Phys. Rev. Lett. 122, 040504 (2019)
M. Schuld, A. Bocharov, K. M. Svore, and N. Wiebe, Phys. Rev. A 101, 032308 (2020)

Fourier series in a quantum circuit

Expectation values can be described with Fourier series

Data re-uploading strategy: define an encoding layer and repeat it multiple times along the circuit.





$W\left(\overrightarrow{\theta_k}\right)$: general unitary gate of dimension $N = d^n$
(n qudits of dimension d)

Encoding:

Reabsorbed in $W\left(\overrightarrow{\theta_k}\right)$

$$S(x) = e^{ixH} = A^{\dagger} R(x) A$$

$$R(x) = diag\left(e^{ix\lambda_1}, e^{ix\lambda_2}, \dots, e^{ix\lambda_N}\right)$$

$\lambda_1, \lambda_2, \dots, \lambda_N$ are the eigenvalues of H

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"Data re-uploading for a universal quantum classifier", A. Pérez-Salinas et. al., Quantum **4**, 226 (2020).

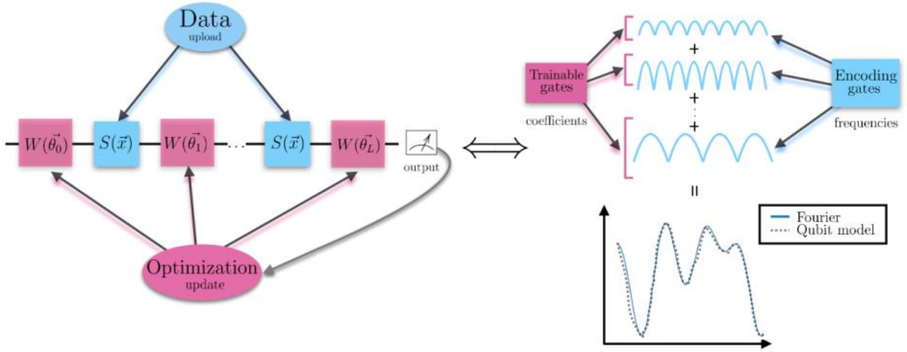
Fourier series in a quantum circuit

Expectation values can be described with Fourier series

Matrix components of W

$$\langle M\rangle = \sum_{\omega\geq 0} \sum_{\substack{\mathbf{k},\mathbf{k}'=1\\ \Omega_{kk'}=\omega}}^{N} e^{ix\omega} \underbrace{\left(\sum_{i=1}^{N} M_i a^{(L)}_{ik_L} a^{*(L)}_{ik'_L}\right) a^{(0)}_{k_1 1} a^{*(0)}_{k'_1 1} \prod_{p=2}^{L} a^{(p-1)}_{k_p k_{p-1}} a^{*(p-1)}_{k'_p k'_{p-1}}}_{c_\omega} + h.c.$$

ω are the Fourier frequencies, constructed with the combination of the encoding Hamiltonian H eigenvalues.



E.g. for one qubit quantum circuit with $H = \sigma_z/2$ (eigenvalues $\pm 1/2$)

$$\omega = -L, -L+1, \ldots L-1, L$$

For $N = d^n$, the Fourier series degree grows linearly with N and L. Also, each c_ω can be generated by a binomially growing number of matrix combinations.

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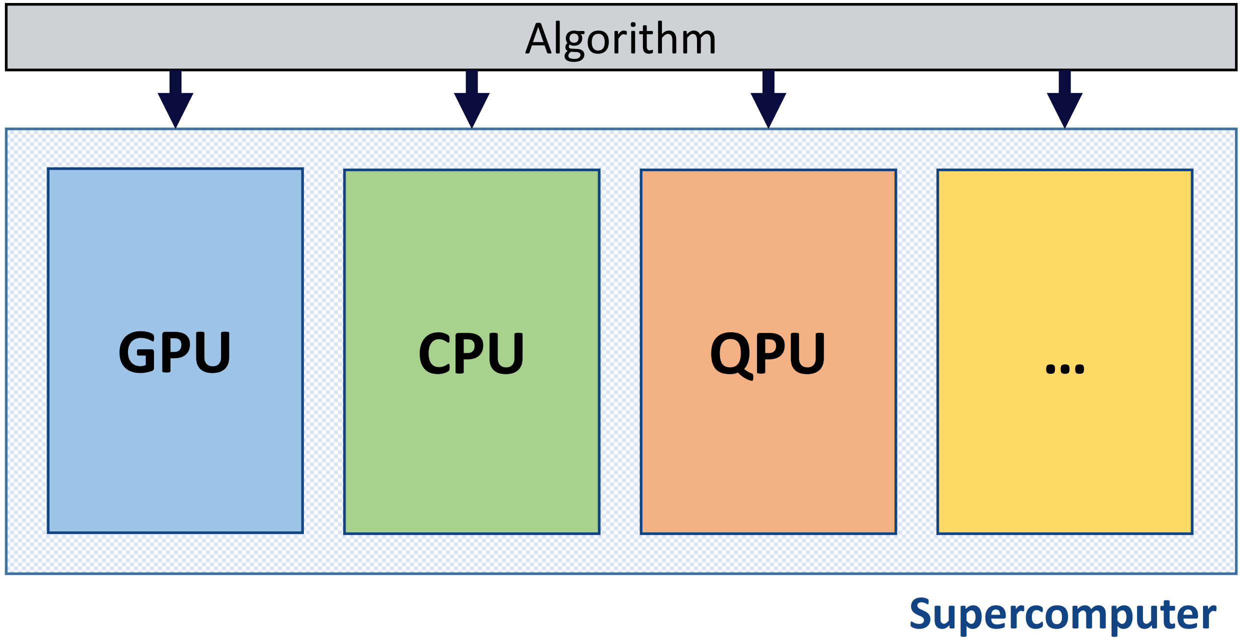
B. Casas, A. Cervera-Lierta, in preparation.
Z. Yu, H. Yao, M. Li, X. Wang, arXiv:2205.07848 [quant-ph] (2022)
M. Schuld, R. Sweke, J. J. Meyer, Phys. Rev. A, 103 032430 (2021)

Towards the Quantum-HPC integration

~~**Supercomputers vs Quantum Computers**~~ → **Supercomputers will include Quantum Computers**

Supercomputing centers are the natural place to allocate and operate quantum computers (as they do for other types of computers).

Quantum Processing Units (QPU) will become another supercomputer partition, suitable to solve certain kind (or some part) of problems.



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Quantum Spain

The definitive boost to the quantum computing ecosystem in Spain

A collaboration of 27 research institutions in Spain.

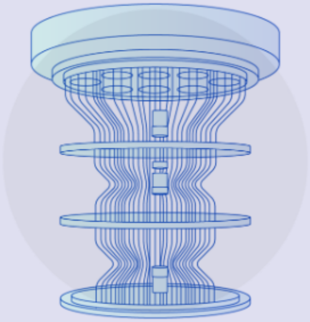
Install a superconducting circuits quantum computer at BSC-CNS and **offer its access** to all the society (as we do for supercomputers).

https://quantumspain-project.es

@QuantumSpain_ES

quantum-spain-project

Quantum computer



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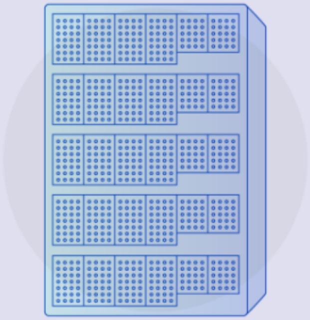
Adquisition and installation of a quantum computer based on superconducting qubits technology.

Access



Spanish Supercomputing Network

Provide the access to the quantum computer and High Performance Computing devices for quantum technologies.

Research

Universities and Research centers

Development and research of novel quantum algorithms and their applications in AI.

Training

TalentQ program

Training programs, workshops, schools, seminars and other initiatives to bring quantum computation to all backgrounds.

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España digital 2026

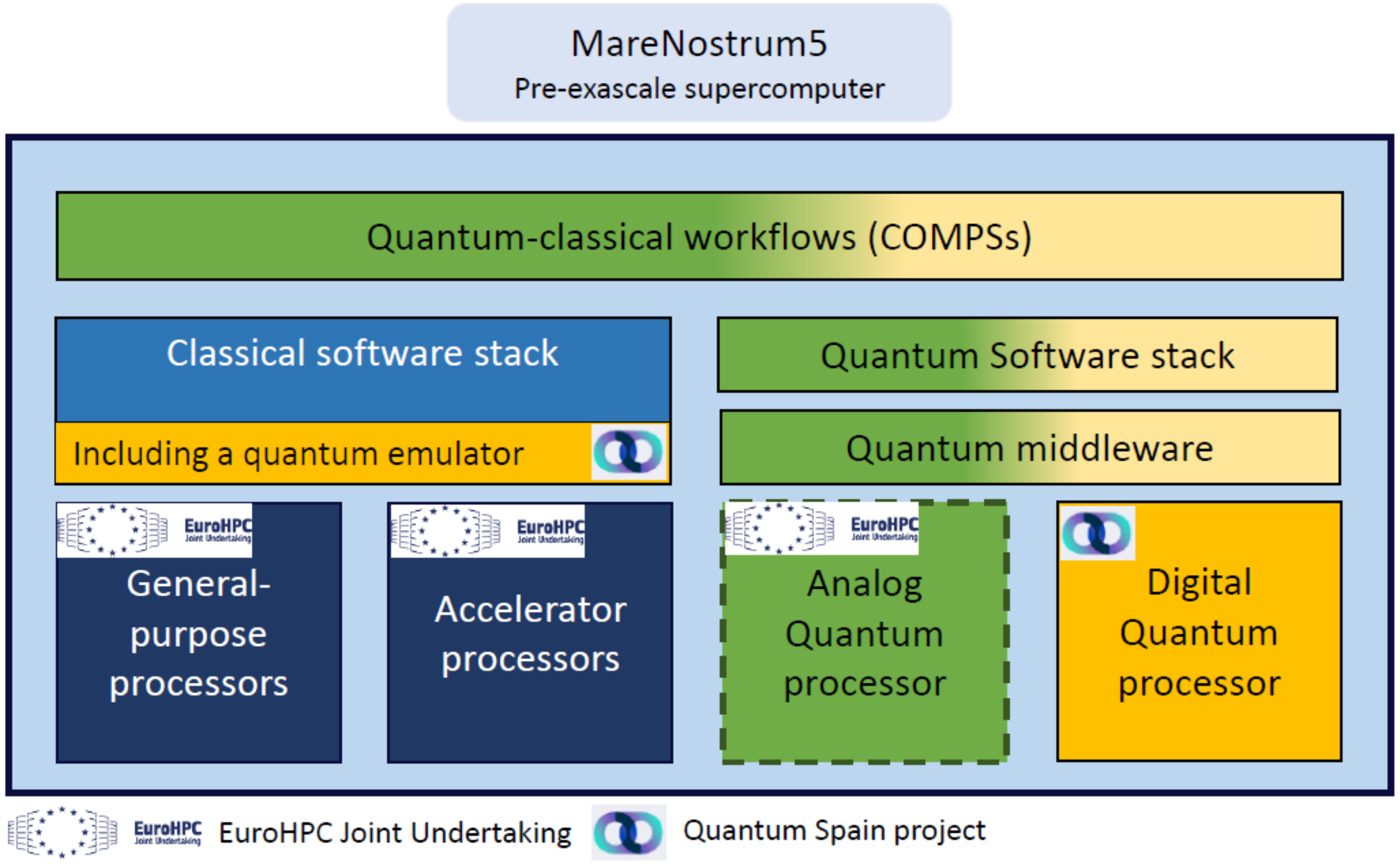
EuroQCS-Spain

One of the six projects selected to become a node of the
European Quantum Computing and Simulation Infrastructure

Supercomputer: MareNostrum5

Quantum computing partitions:
- Digital Quantum Computer (from the Quantum Spain project)
- Analog Quantum Computer [coherent quantum annealer] (from EuroHPC-JU)

HPC-QC Integration: using COMPSs workflows and developed by BSC-CNS.



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- Quantum systems are naturally high-dimensional (multi-level).
- Instead of using a qubit (two-level) quantum computer, why not exploiting the other quantum levels of the hardware?
- This requires a programming effort (low-level programming) or a software improvement (in-build high-dimensional quantum libraries).
- The pre-factor improvement introduced by the d-level systems might prove valuable for the NISQ era, where we do not have many qubits quantum chips but qubits already contain these higher levels naturally.
- High-dimensional quantum algorithms might show different properties and advantages than qubit-based ones (quantum error correction, better exploration of the Hilbert space, high-dimensional entanglement and non-locality, …)

We have now programable quantum devices.
We must exploit all kinds of quantum phenomena!

Quantum SPAIN

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Thanks for your attention

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