Quark Mass Effects in Gluon Fusion at NNLO in QCD

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Institute for Theoretical Particle Physics and Cosmology

RWTH AACHEN UNIVERSITY

TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

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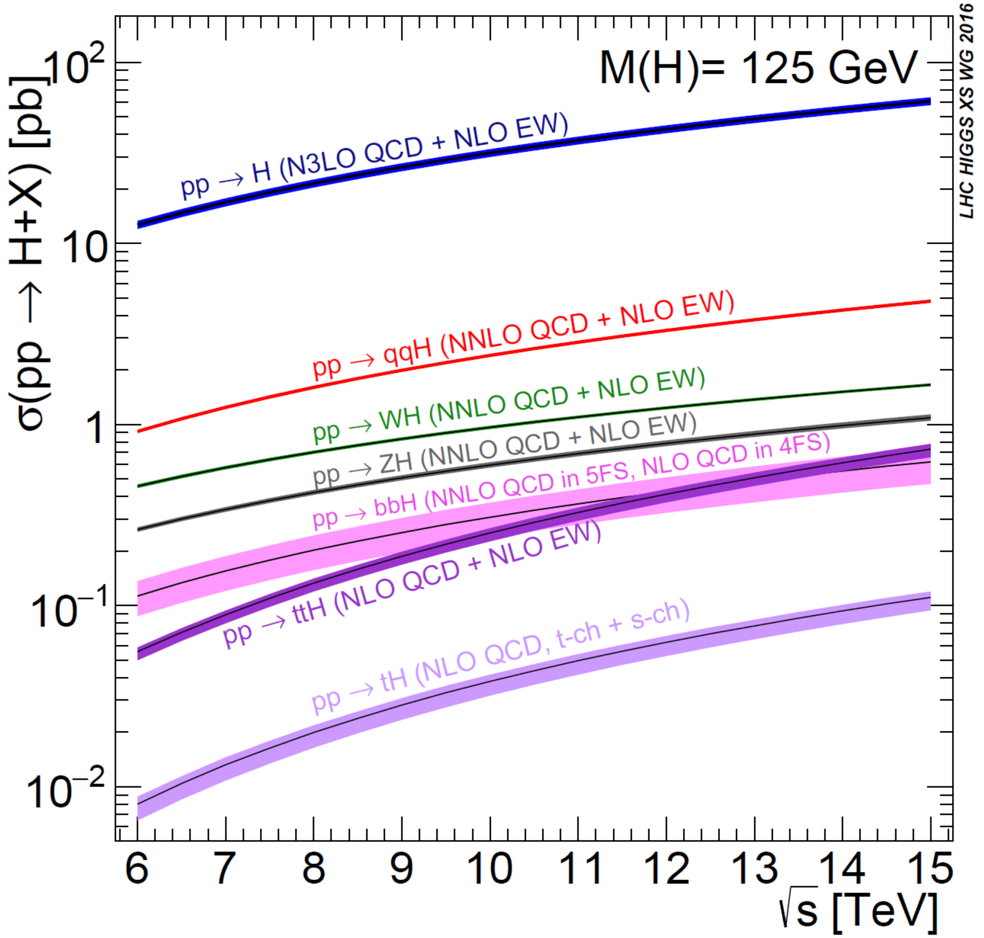
MAX-PLANCK-INSTITUT FÜR PHYSIK

Motivation

- Gluon fusion is the predominant Higgs-boson production mode at the LHC
- Higgs-boson plays unique role in the SM:
  - Only scalar particle
  - Only particle with Yukawa interactions to fermions



Handbook of LHC Higgs cross sections:
4. Deciphering the nature of the Higgs sector
Report of the LHC Higgs Cross Section Working Group `16

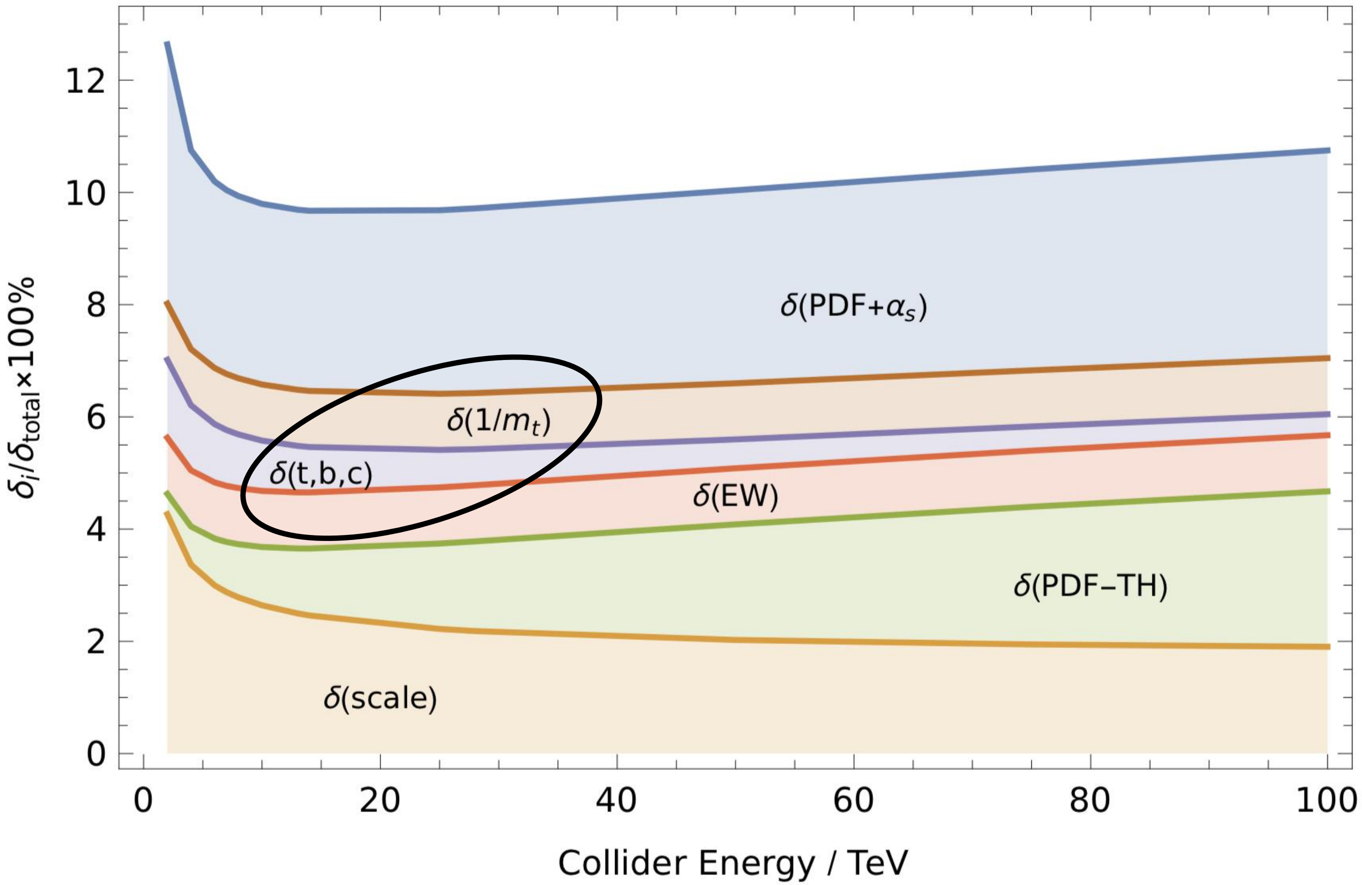
LHC @13 TeV

$$\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ \mathrm{(theory)} \pm 1.56\,\mathrm{pb}\ (3.20\%)\ (\mathbf{PDF}+\alpha_s)$$

Theory uncertainties

- δ(scale) and δ(PDF-TH) due to missing higher-order terms in $\hat{\sigma}$ and PDFs Anastasiou, et al. `15
- δ(trunc) has been removed Mistlberger `18
- δ(EW) was addressed recently
  Bonetti, Melnikov, Tancredi `18
  Anastasiou, del Duca, et al. `19
  Becchetti, Bonciani, et al. `21
- δ(t,b,c) and $\delta(1/m_t)$ related to quark mass effects

Higgs Physics at the HL-LHC and HE-LHC
Report from Working Group 2 on the Physics of the HL-LHC, and Perspectives at the HE-LHC `19



δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
$+0.10$ pb -1.15 pb	±0.18 pb	±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
$+0.21\%$ -2.37%	$\pm0.37\%$	$\pm1.16\%$	$\pm1\%$	$\pm0.83\%$	$\pm1\%$

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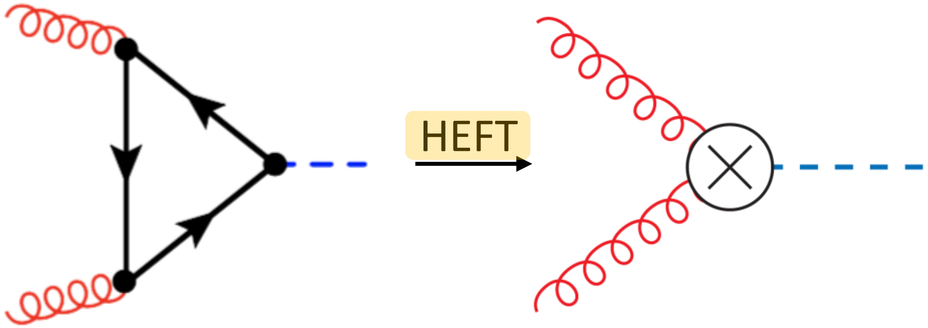
Contributions to σ_{tot}



$48.58\,\text{pb} =$	$16.00\,\text{pb}$	$(+32.9\%)$	(LO, rEFT)
	$+\,20.84\,\text{pb}$	$(+42.9\%)$	(NLO, rEFT)
	$-\;\;2.05\,\text{pb}$	(-4.2%)	((t,b,c), exact NLO)
	$+\;\;9.56\,\text{pb}$	$(+19.7\%)$	(NNLO, rEFT)
	$+\;\;0.34\,\text{pb}$	$(+0.7\%)$	(NNLO, $1/m_t$)
	$+\;\;2.40\,\text{pb}$	$(+4.9\%)$	(EW, QCD-EW)
	$+\;\;1.49\,\text{pb}$	$(+3.1\%)$	(N^3LO, rEFT)

"Born-improved" total cross section:

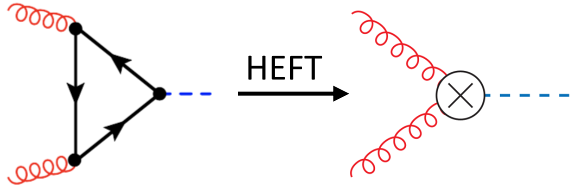
$$\sigma^{\text{HO}}_{\text{HEFT}} = \left(\frac{\sigma^{\text{HO}}}{\sigma^{\text{LO}}}\right)_{M_t\to\infty} \sigma^{\text{LO}}$$

- Gluon-fusion is induced by quark loops
  - NLO result available for arbitrary quark masses — Graudenz, Spira, Zerwas `93
  - Radiative corrections beyond NLO restricted to top-loop induced terms
- Dominant effect of top-loop induced terms can be accounted for in HEFT approximation

Anastasiou, Melnikov `02
Harlander, Kilgore `02
Ravindran, Smith, van Neerven `03
Marzani, Ball, Del Duca, et al. `08
Harlander, Mantler, Marzani, et al. `09
Pak, Rogal, Steinhauser `09

HEFT

- Introduce effective Higgs-gluon vertex
  - → reduce number of loops by one
  - → reduce number of scales by one

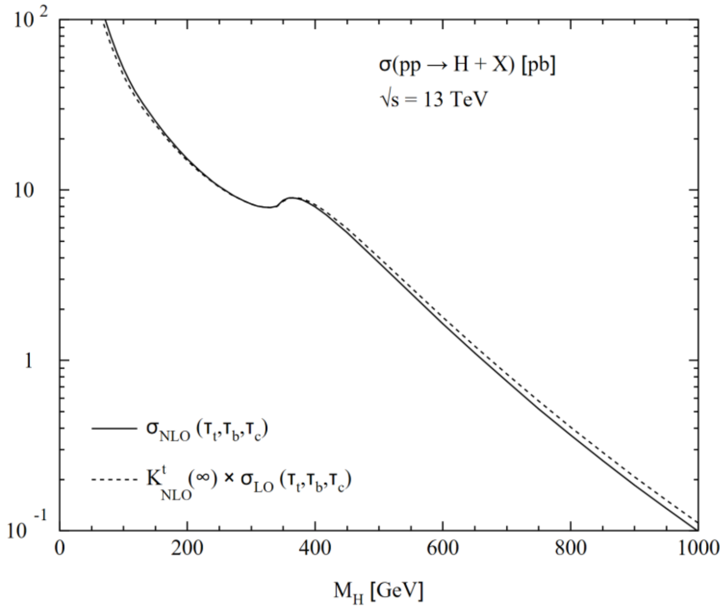


- Very good agreement with exact result at NLO
  → Remarkable, because
  - M_t being the largest scale is invalid over large range of $\sqrt{\hat{s}}$
  - $M_t \to \infty$ is applied to more than 50% of total cross section
  - HEFT fails to capture top-mass effects for partonic quark channels

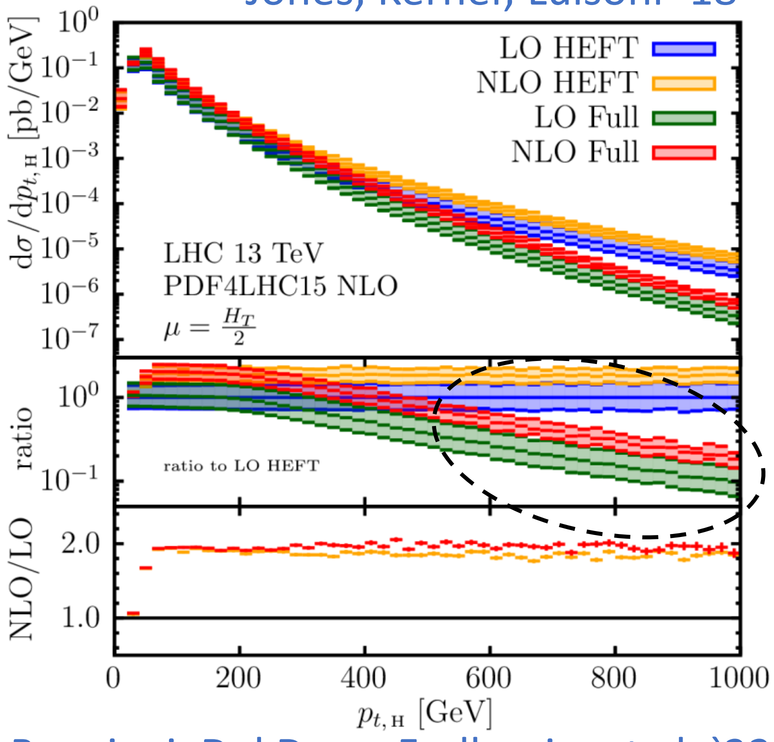
- Qualitative explanation:
  - Suppression of large-$\hat{s}$ region by PDFs
  - Dominance of the soft region

- Only estimate of top-mass effects beyond HEFT at NNLO based on combination of $1/M_t$-expansion with leading terms in large-$\hat{s}$ limit
  → Eliminate this uncertainty with exact calculation of top-quark mass effects

Spira `16



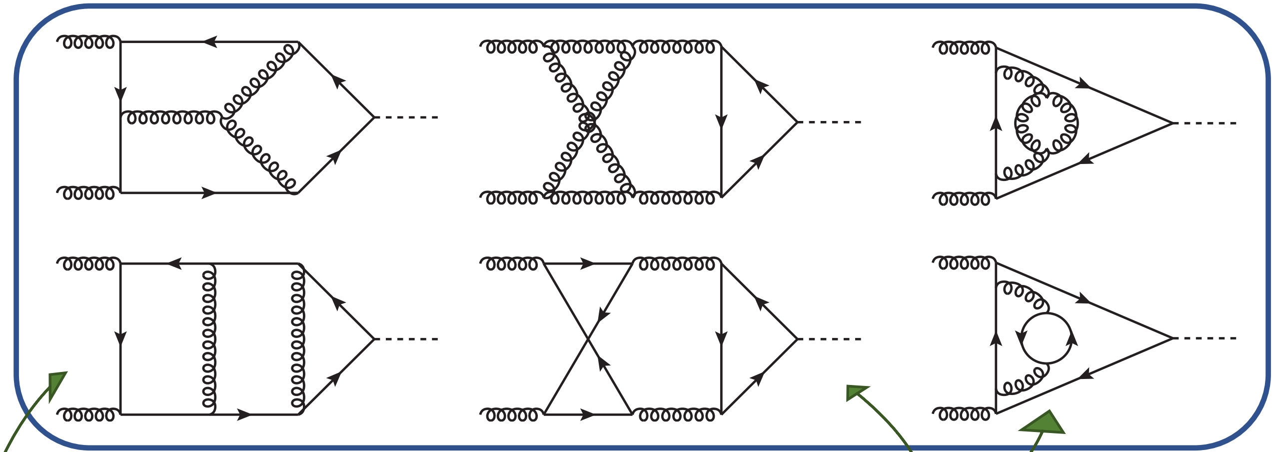
Jones, Kerner, Luisoni `18



(also Bonciani, Del Duca, Frellesvig, et al. `22)

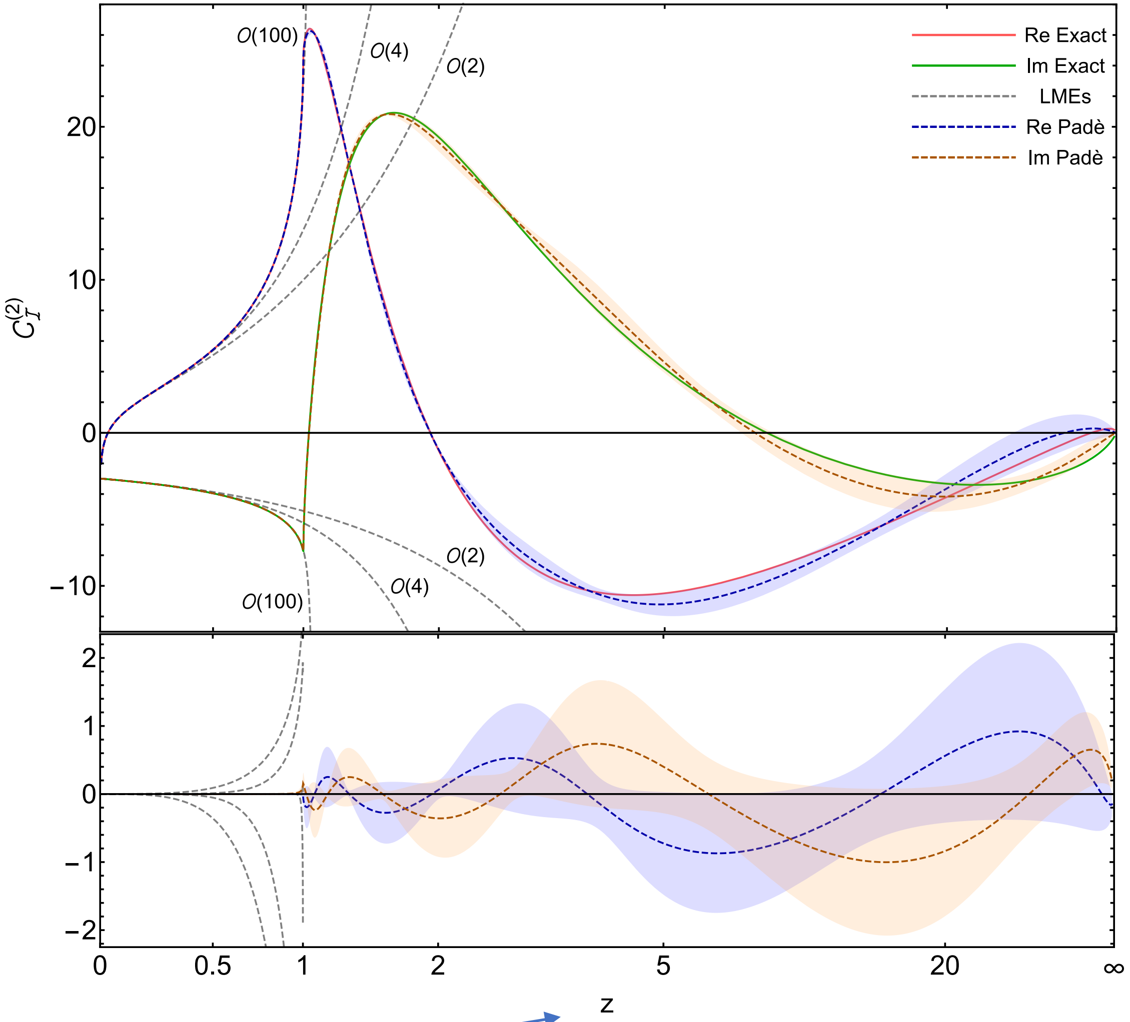
Ingredients for gluon fusion at NNLO

Ingredients – Double Virtual



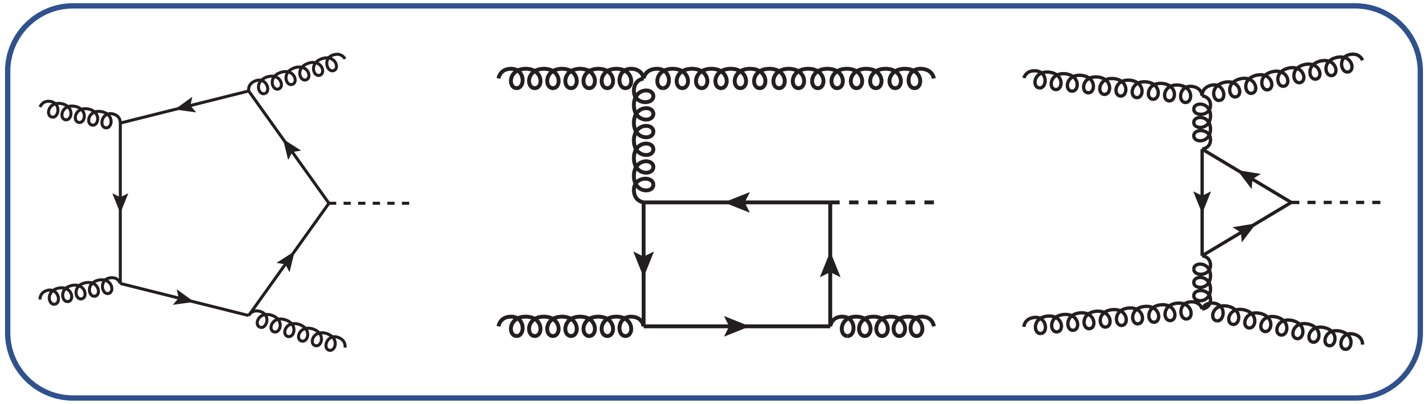
Light-fermion contribution (analytically):
Harlander, Prausa, Usovitsch `19
Leading color contribution (analytically):
Prausa, Usovitsch `20

Padé approximation: Davies, Gröber, Maier et al. `19
Numerically exact: Czakon, MN `20



$z = m_H^2/4m_q^2$

Ingredients – Double Real



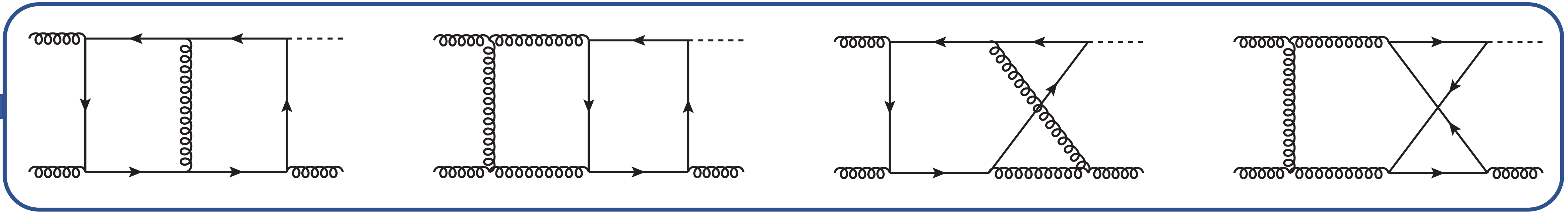
+ quark channels with possibly different quark flavors

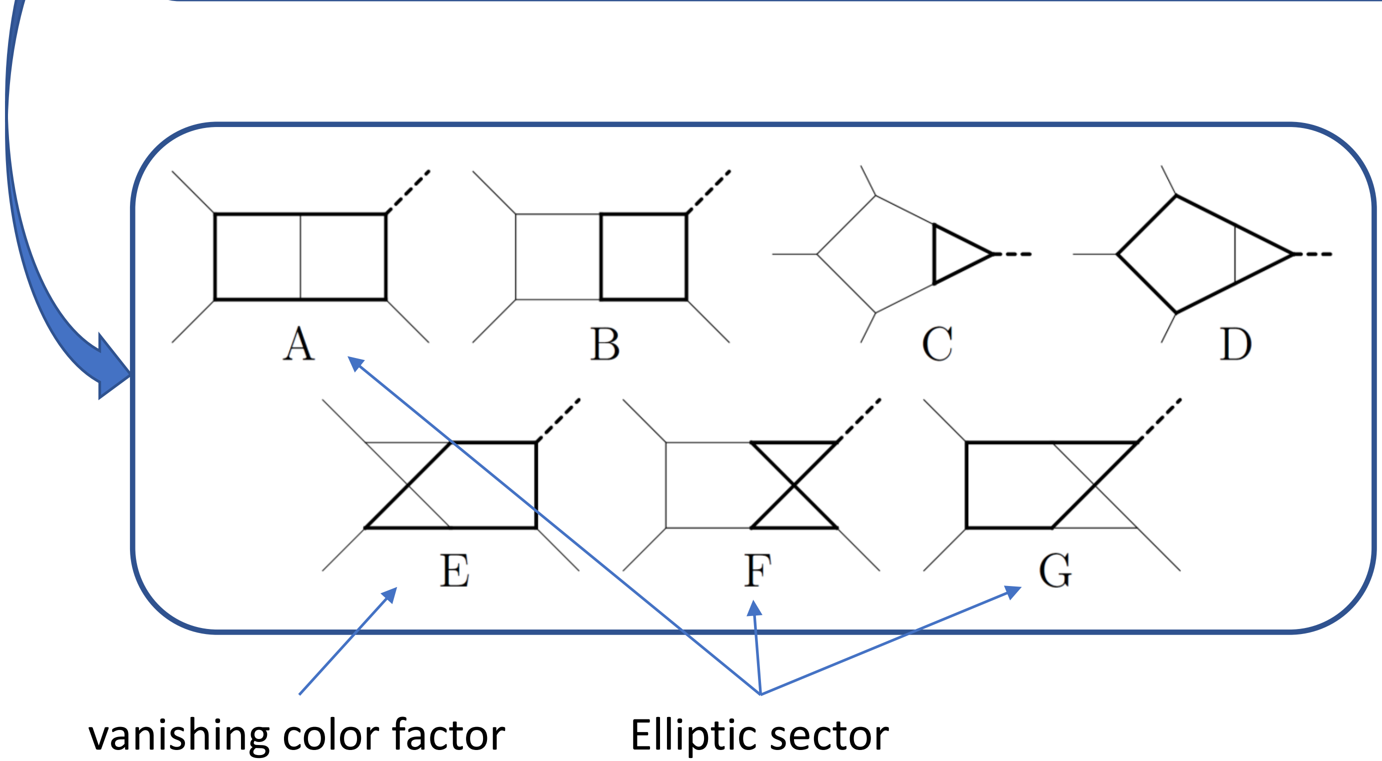
Evaluation:

Analytically: Del Duca, Kilgore, Oleari, et al. `01
OpenLoops 2: Buccioni, Lang, Lindert, et al. `19

Analytically (more compact): Budge, Campbell, De Laurentis, et al. `20

Ingredients – Real-Virtual

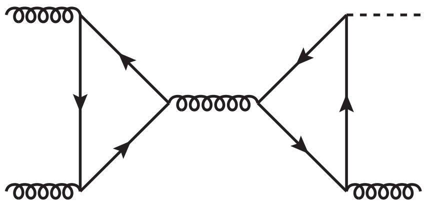




A,B,C,D: Bonciani, Del Duca, Frellesvig, et al. `16
F: Bonciani, Del Duca, Frellesvig, et al. `19
G: Frellesvig, Hidding, Maestri, et al. `19

→ leading to H+jet at NLO with top and bottom massive:
Bonciani, Del Duca, Frellesvig, et al. `22

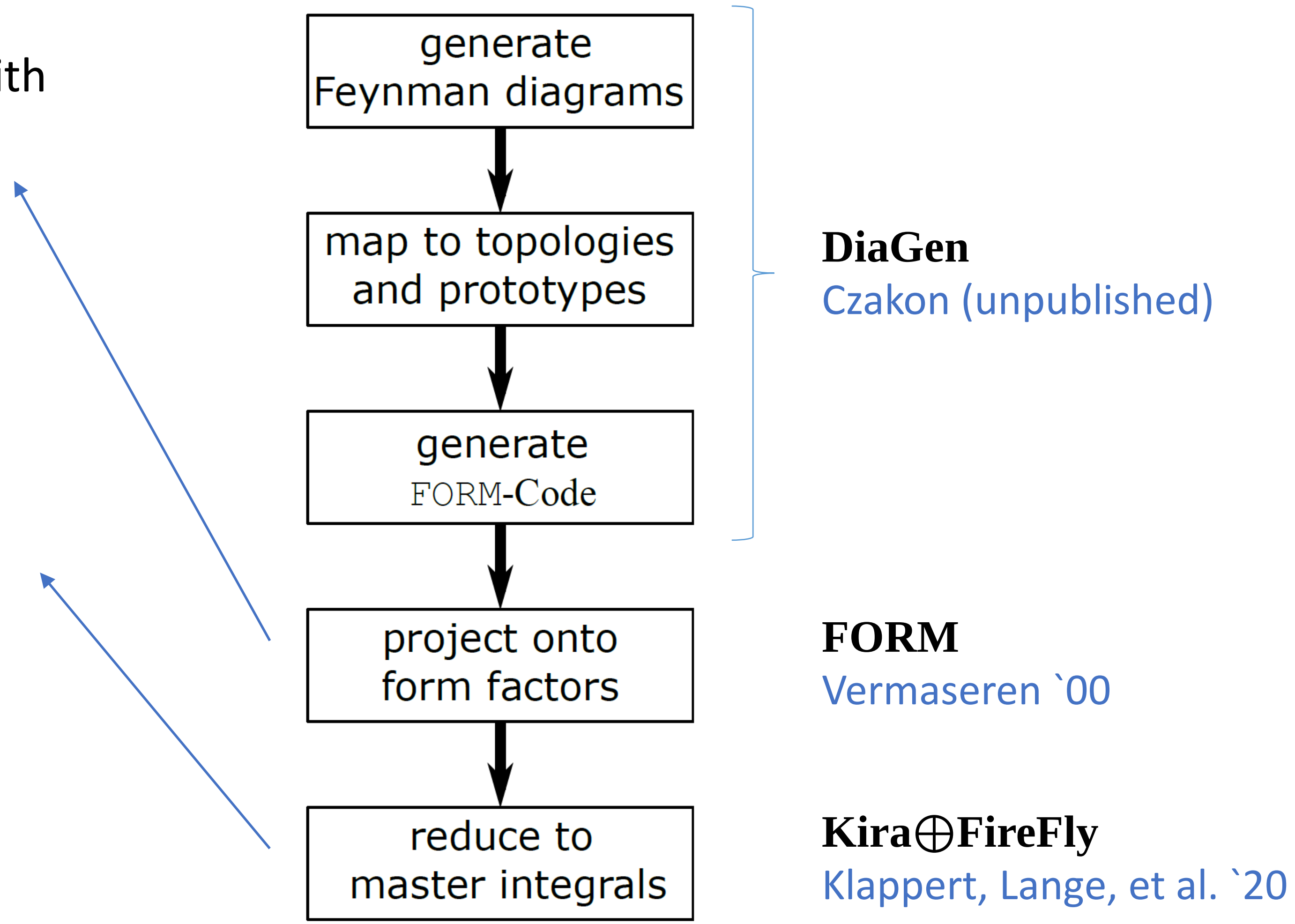
Contributions with two closed fermion chains are always factorizable:



Workflow of the computation

- Get rid of tensor/colour structure to end up with a linear combination of scalar integrals with rational function coefficients in front

- Reduce the scalar integrals to a linearly independent set of master integrals (MI) (447 master integrals for $gg \to Hg$)
- Reduction is highly non-trivial since rational coefficients depend on 5 variables!
  → Use finite fields to reconstruct symbolic coefficients from numerical probes of the system of equations



Computation of the MIs

Parametrization

- Variables: $\hat{s}$, $\hat{t}$, $\hat{u}$, m_H^2, m_t^2
- Introduce dimensionless variables and fix ratio m_t^2/m_H^2
  - z parametrizes soft limit
  - λ parametrizes collinear limit

z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$

m_t^2/m_H^2 = 23/12

Range of parameters:

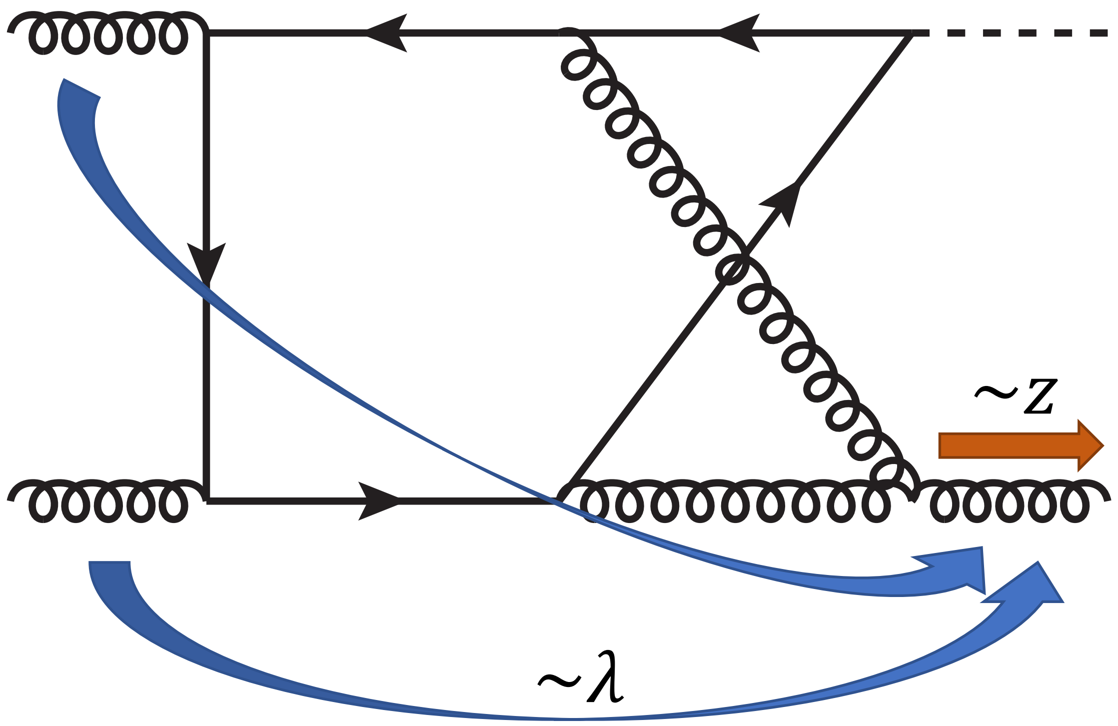
- $\lambda \in (0,1)$
- $z \in (0,1)$

$\hat{t}/\hat{s}$ = $z\,\lambda$

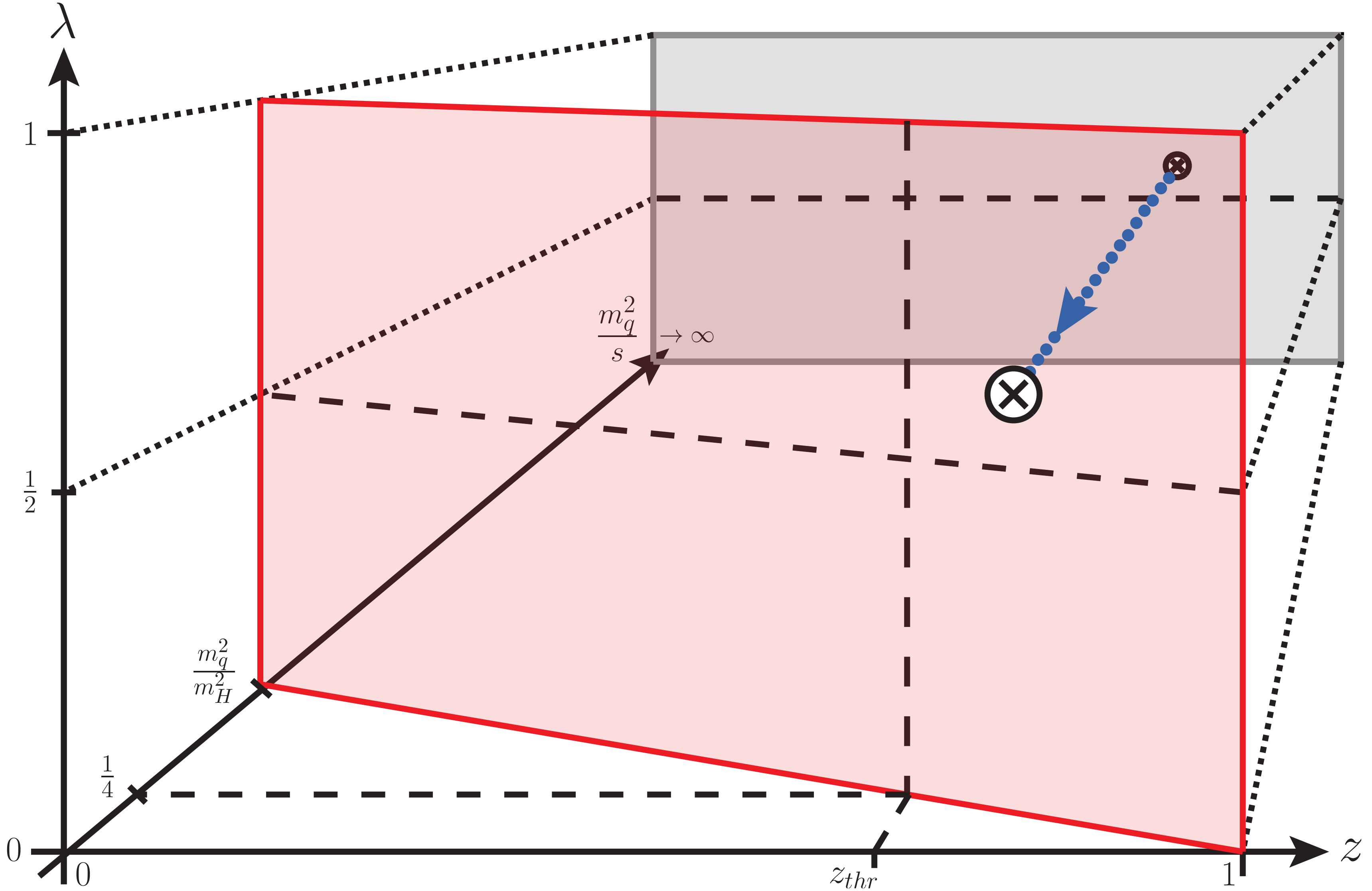
$\hat{u}/\hat{s}$ = z (1-λ)

z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$



Evolution of differential equations



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

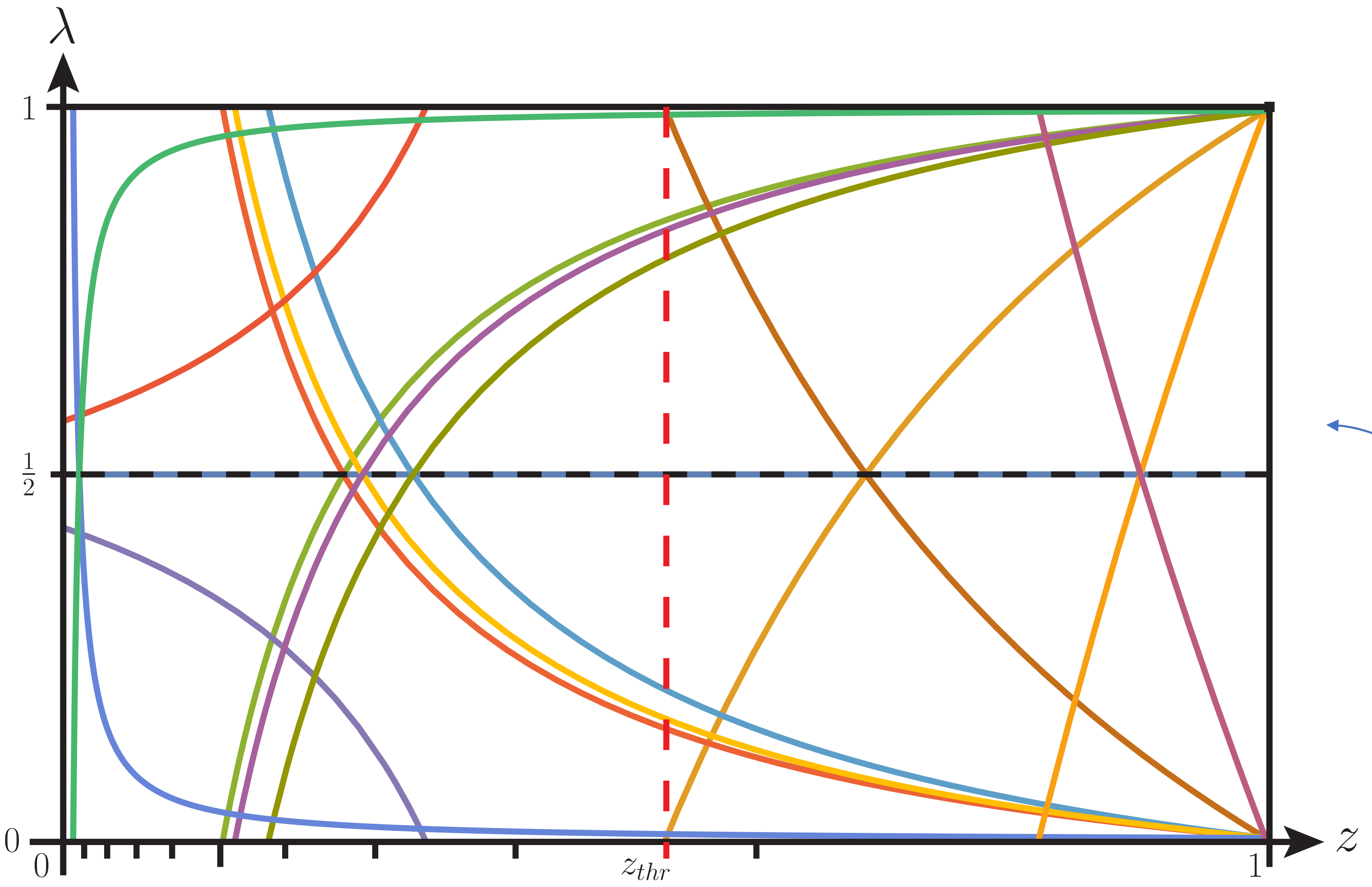
Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

$z_{thr} = 1 - \frac{m_H^2}{4m_q^2}$

For $m_t^2/m_H^2 = 23/12$:
$z_{thr} = 20/23 \approx 0.87$

Evolution in the (z,λ)-plane



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

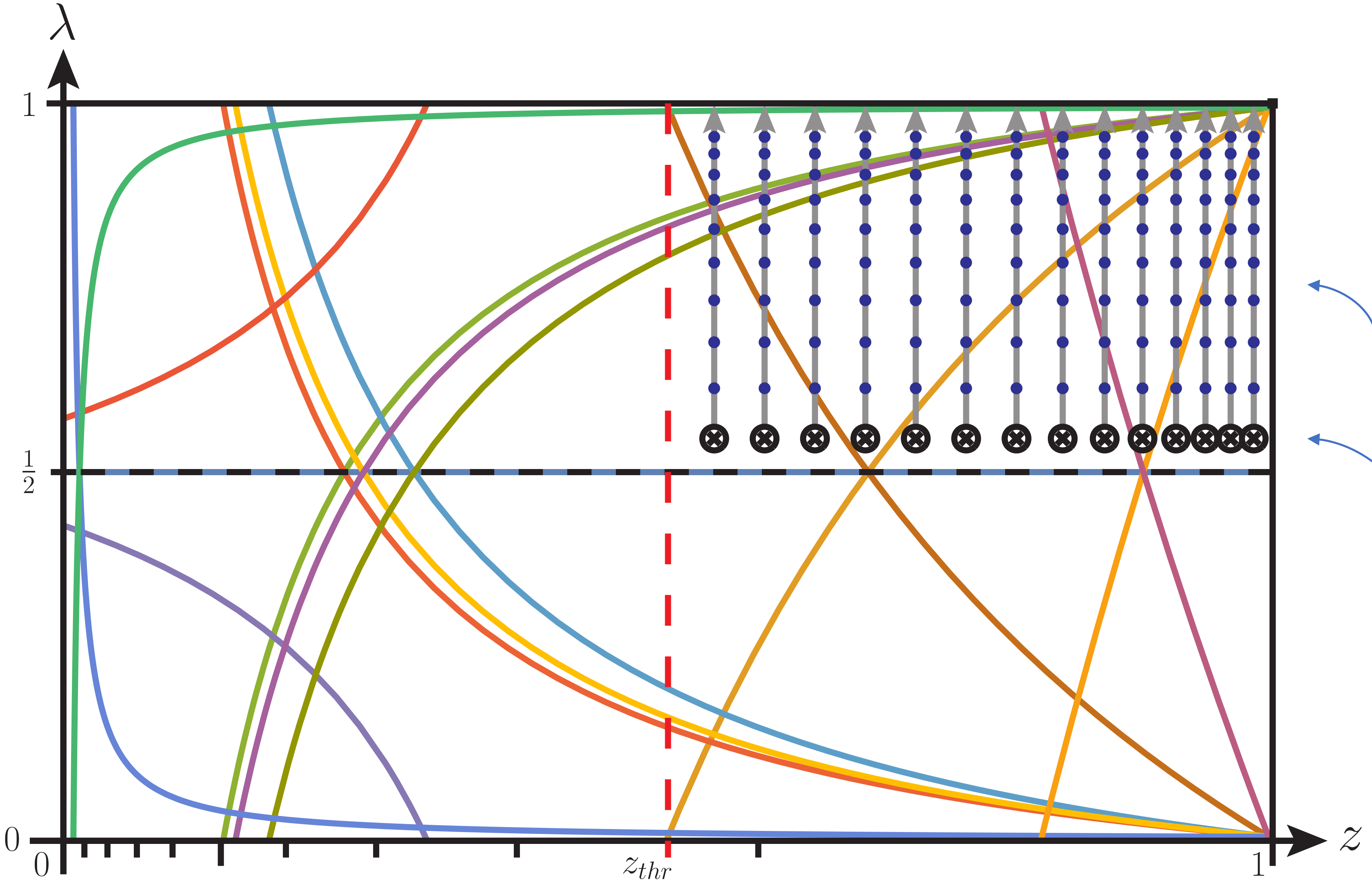
$m_t^2/m_H^2 = 23/12$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Poles of differential equations in λ

Evolution in the (z,λ)-plane



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

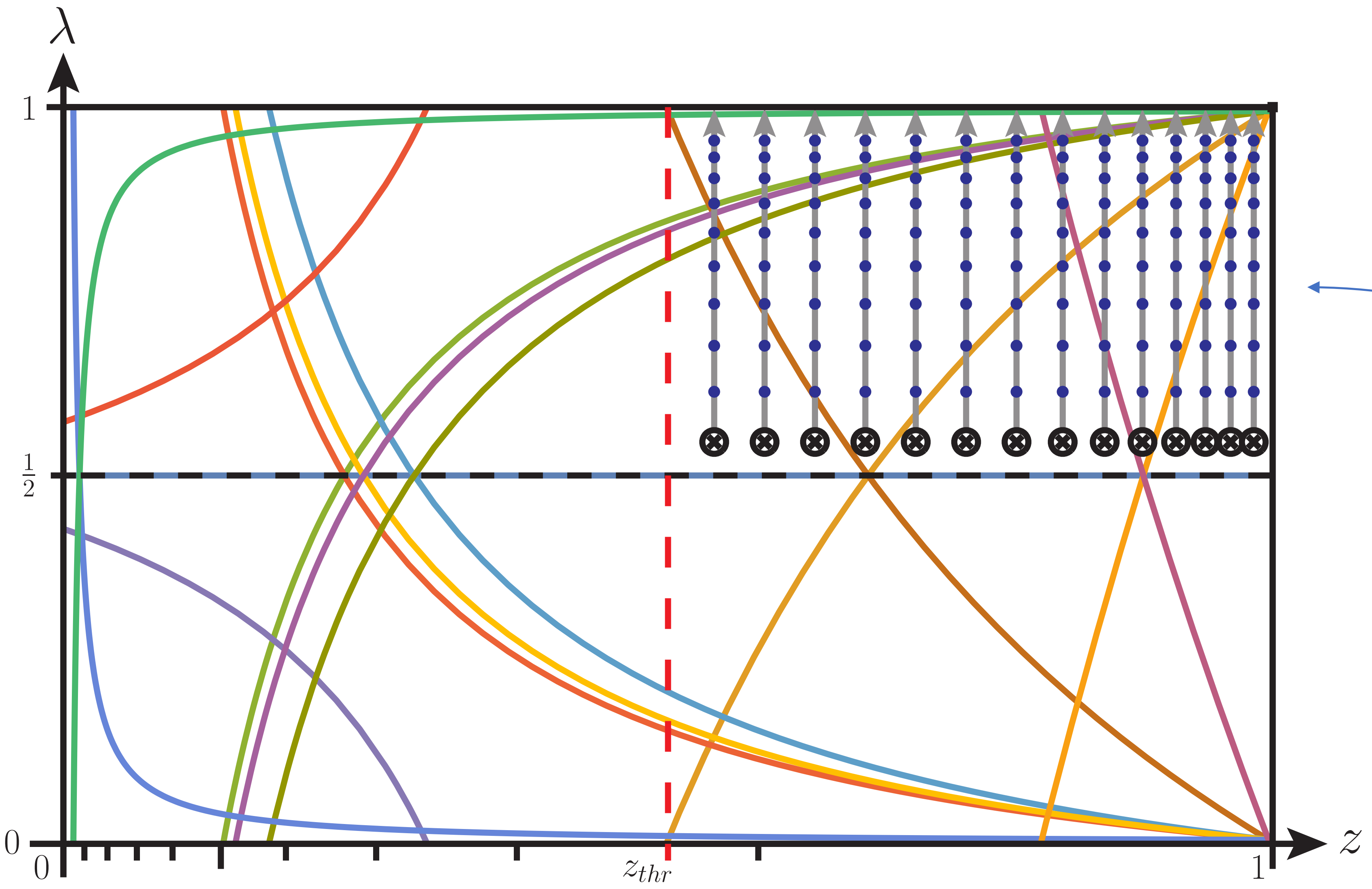
Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Collect numerical samples for MI along straight integration contours

Boundaries for numerical integration in the mass

Evolution in the (z,λ)-plane



$z = 1\text{-}m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

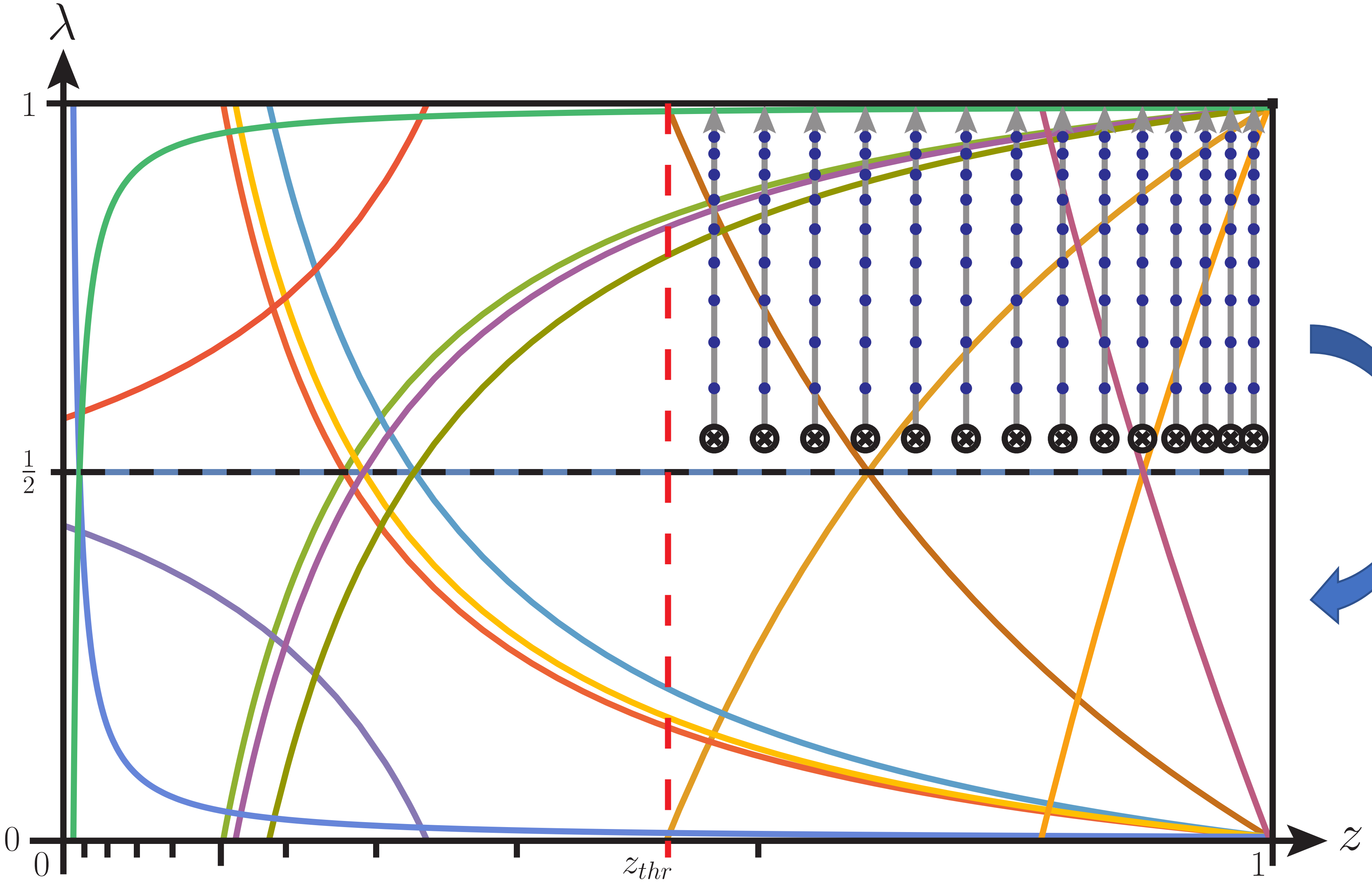
$m_t^2/m_H^2 = 23/12$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

- 302 integration contours at different z
- Collected more than 1.5×10^6 samples

Evolution in the (z,λ)-plane



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

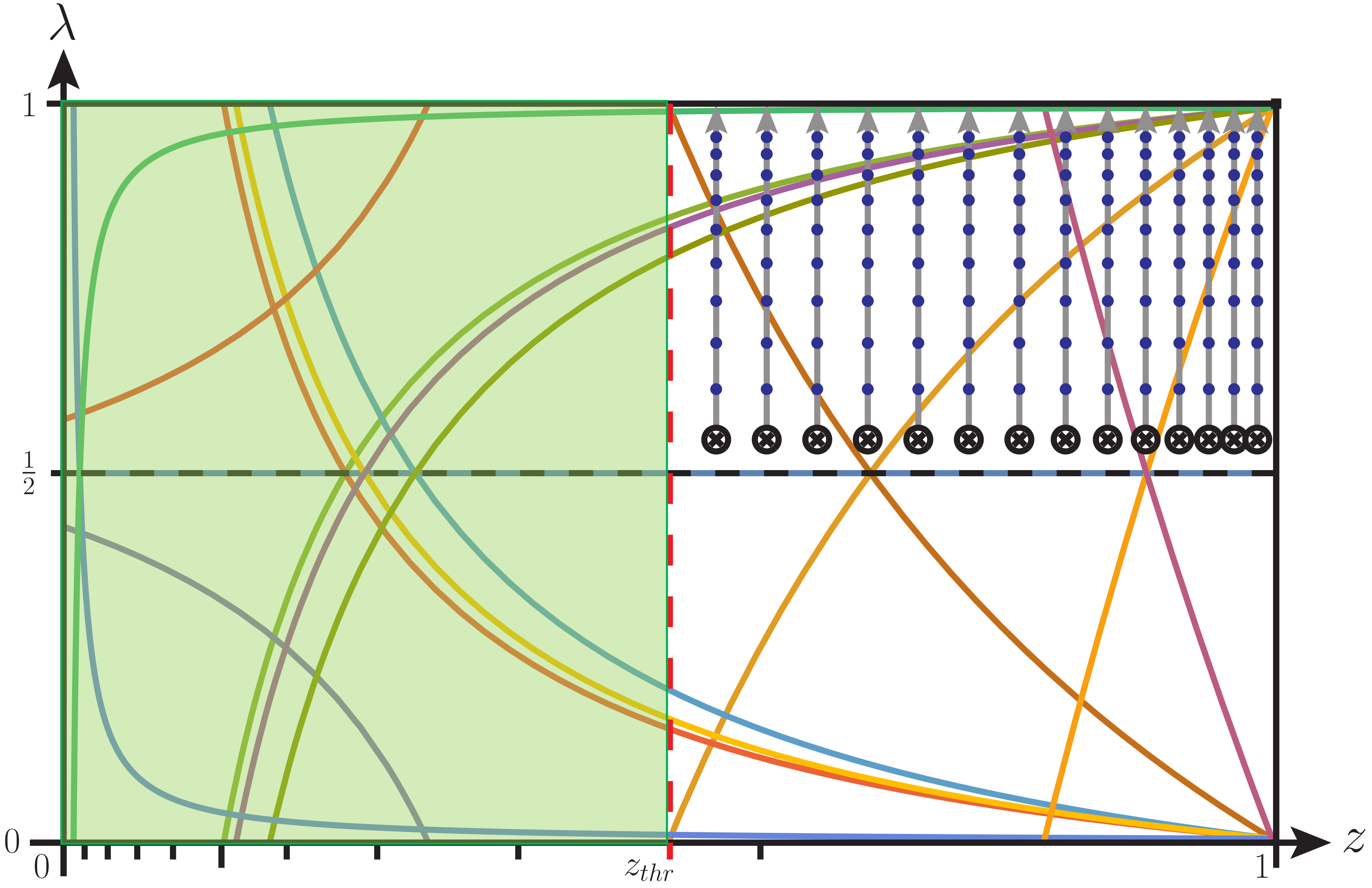
$m_t^2/m_H^2 = 23/12$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Exploit symmetry of the problem

Evolution in the (z,λ)-plane



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Region below threshold covered by LME

LME
$\mathcal{O}((1/m_q^2)^{40})$

Subtraction of IR limits

Subtraction for gg → gH

z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$

m_t^2/m_H^2 = 23/12

- Interested in finite results
  - Soft and collinear divergences
  - Amplitudes have to be regulated

Higgs-gluon form factor in HEFT is exact at leading order $\left|F^{(1)}_{\text{exact}}\right|^2 = \left|F^{(1)}_{\text{HEFT}}\right|^2$

Free of divergences!

- Directly evaluate difference between HEFT and exact result:

NLO: $$\langle M^{(1)}_{\text{exact}}|M^{(1)}_{\text{exact}}\rangle\big|_{\text{regulated}} \equiv \langle M^{(1)}_{\text{exact}}|M^{(1)}_{\text{exact}}\rangle - \left[\langle M^{(1)}_{\text{HEFT}}|M^{(1)}_{\text{HEFT}}\rangle\right]$$

NNLO: $$\langle M^{(1)}_{\text{exact}}|M^{(2)}_{\text{exact}}\rangle\big|_{\text{regulated}} \equiv \langle M^{(1)}_{\text{exact}}|M^{(2)}_{\text{exact}}\rangle - \left[\langle M^{(1)}_{\text{HEFT}}|M^{(2)}_{\text{HEFT}}\rangle + \frac{8\pi\alpha_s}{\hat{t}}\left\langle P^{(0)}_{gg}\left(\frac{\hat{s}}{\hat{s}+\hat{u}}\right)\right\rangle\langle F^{(1)}|(F^{(2)}_{\text{exact}} - F^{(2)}_{\text{HEFT}})\rangle\right]$$

Splitting-function regulates residual soft and collinear divergences

- Better numerical stability!
- Appearance of divergences delayed!
  - Simple counterterms!

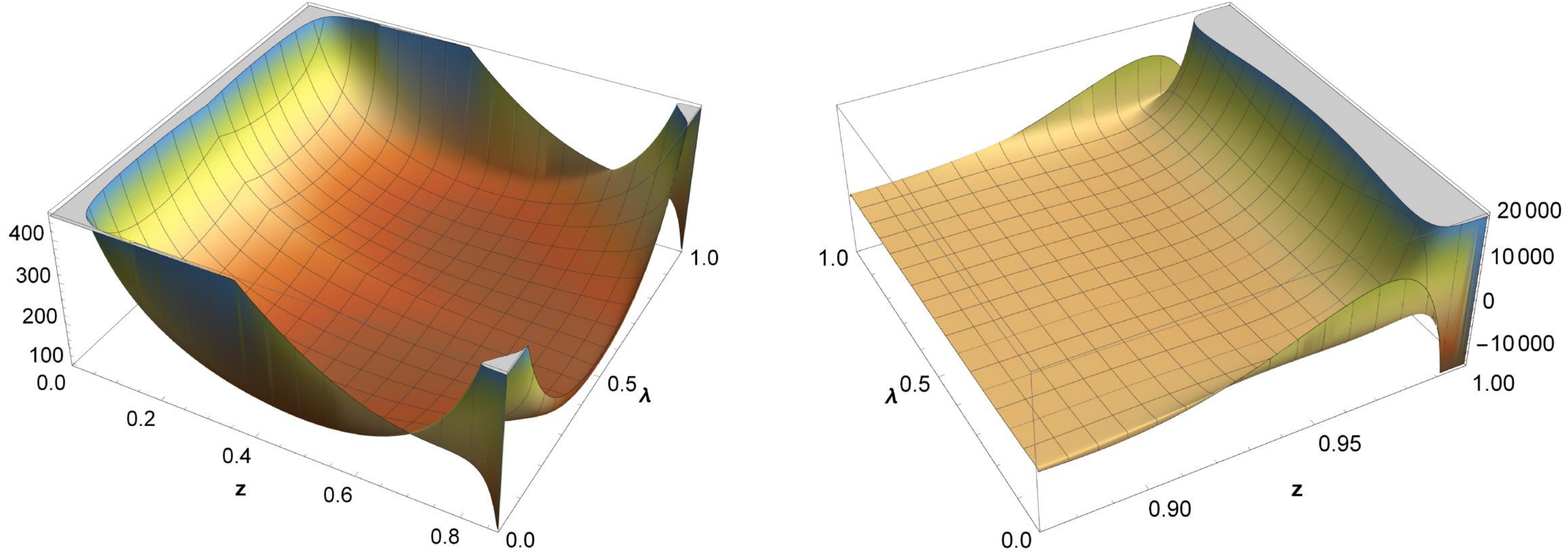
One of the reasons for the smallness of top-quark mass effects beyond HEFT

Subtraction for gg → gH

z = 1-$m_H^2/\hat{s}$
λ = $\hat{t}/(\hat{t}+\hat{u})$
m_t^2/m_H^2 = 23/12

$$\langle M^{(1)}_{\text{exact}}|M^{(2)}_{\text{exact}}\rangle\big|_{\text{regulated}} \equiv \langle M^{(1)}_{\text{exact}}|M^{(2)}_{\text{exact}}\rangle - \left[\langle M^{(1)}_{\text{HEFT}}|M^{(2)}_{\text{HEFT}}\rangle + \frac{8\pi\alpha_s}{\hat{t}}\left\langle P^{(0)}_{gg}\left(\frac{\hat{s}}{\hat{s}+\hat{u}}\right)\right\rangle\langle F^{(1)}|(F^{(2)}_{\text{exact}} - F^{(2)}_{\text{HEFT}})\rangle\right]$$

- Real part of the regulated quantity at $\mu_R = m_H/2$:



- Integrate in λ and convolute with PDFs to obtain contribution to σ_{tot}
- Subtraction term and other contributions are computed with Monte Carlo methods using **Stripper** Czakon (unpublished)

Results

- Effects of a finite top-quark mass on the total hadronic Higgs-boson production cross section for the LHC
  - PDF set: NNPDF31_nnlo_as_0118
  - $\mu_R = \mu_F = m_H/2$
  - M_H = 125 GeV ⇒ $M_t \approx$ 173.055 GeV

channel	$\sigma_{\mathrm{HEFT}}^{\mathrm{NNLO}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma_{\mathrm{exact}}^{\mathrm{NNLO}} - \sigma_{\mathrm{HEFT}}^{\mathrm{NNLO}})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma_{\mathrm{exact}}^{\mathrm{NNLO}}/\sigma_{\mathrm{HEFT}}^{\mathrm{NNLO}} - 1)$ [%]
$\sqrt{s} = 8$ TeV				
gg	$7.39 + 8.58 + 3.88$	$+0.0353$	$+0.0879 \pm 0.0005$	$+0.62$
qg	$0.55 + 0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01 + 0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39 + 9.15 + 4.18$	-0.0873	$+0.0667 \pm 0.0007$	-0.10
$\sqrt{s} = 13$ TeV				
gg	$16.30 + 19.64 + 8.76$	$+0.0345$	$+0.2431 \pm 0.0020$	$+0.62$
qg	$1.49 + 0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02 + 0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30 + 21.15 + 9.70$	-0.3029	$+0.1815 \pm 0.0023$	-0.26

Results

- Effects of a finite top-quark mass on the total hadronic Higgs-boson production cross section for the LHC
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  - $\mu_R = \mu_F = m_H/2$
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channel	$\sigma^{\mathrm{NNLO}}_{\mathrm{HEFT}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma^{\mathrm{NNLO}}_{\mathrm{exact}}-\sigma^{\mathrm{NNLO}}_{\mathrm{HEFT}})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma^{\mathrm{NNLO}}_{\mathrm{exact}}/\sigma^{\mathrm{NNLO}}_{\mathrm{HEFT}}-1)$ [%]		
$\sqrt{s}=8\,\mathrm{TeV}$						
gg	$7.39+8.58+3.88$	$+0.0353$	$+0.0879\pm0.0005$	$+0.62$	(= 0.18	+ 0.44)
qg	$0.55+0.26$	-0.1397	-0.0021 ± 0.0005	-18	(= -17.5	- 0.5)
qq	$0.01+0.04$	$+0.0171$	-0.0191 ± 0.0002	-4	(= +34	- 38)
total	$7.39+9.15+4.18$	-0.0873	$+0.0667\pm0.0007$	-0.10	(= -0.42	+ 0.32)
$\sqrt{s}=13\,\mathrm{TeV}$						
gg	$16.30+19.64+8.76$	$+0.0345$	$+0.2431\pm0.0020$	$+0.62$	(= 0.08	+ 0.54)
qg	$1.49+0.84$	-0.3696	-0.0115 ± 0.0010	-16	(= -15.5	- 0.5)
qq	$0.02+0.10$	$+0.0322$	-0.0501 ± 0.0006	-15	(= +27	- 42)
total	$16.30+21.15+9.70$	-0.3029	$+0.1815\pm0.0023$	-0.26	(= -0.64	+ 0.38)

Comparison with HEFT $\oplus$ $(1/M_t^n)$

- The impact of subleading terms in $1/M_t^2$ is determined with SusHi Harlander, Liebler, Mantler `16
  - Include terms up to $1/M_t^4$ at NLO and NNLO and match with high-energy limit
  - Total result of $1/M_t$ approximation very close to exact result

channel	$\sigma_{\mathrm{HEFT}}^{\mathrm{NNLO}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma_{\mathrm{subl.}}^{\mathrm{NNLO}}-\sigma_{\mathrm{HEFT}}^{\mathrm{NNLO}})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma_{\mathrm{subl.}}^{\mathrm{NNLO}}/\sigma_{\mathrm{HEFT}}^{\mathrm{NNLO}}-1)$ [%]
$\sqrt{s}=8\,\mathrm{TeV}$				
gg	$7.39+8.58+3.88$	-0.0104	$+0.1088$	$+0.50$
qg	$0.55+0.26$	-0.1265	$+0.0142$	-14
qq	$0.01+0.04$	$+0.0025$	-0.0076	-10
total	$7.39+9.15+4.18$	-0.1344	$+0.1153$	-0.09
$\sqrt{s}=13\,\mathrm{TeV}$				
gg	$16.30+19.64+8.76$	-0.1145	$+0.3127$	$+0.44$
qg	$1.49+0.84$	-0.3348	$+0.0482$	-12
qq	$0.02+0.10$	$+0.0036$	-0.0246	-17
total	$16.30+21.15+9.70$	-0.4457	$+0.3363$	-0.23

Summary

✓ The hadronic Higgs production cross section including the full top-quark mass dependence was computed!

✓ Slight decrease relative to HEFT at NNLO

- -0.26% at 13 TeV
- -0.10% at 8 TeV

✓ The result confirms and eliminates the uncertainty estimate from the lack of knowledge of the exact top-quark mass effects!

✓ Amplitudes for $\mathrm{gg} \to \mathrm{gH}$ (and quark channels) have been computed numerically for a physical top quark mass by supplementing a deep LME with a dense grid of phase space samples above threshold

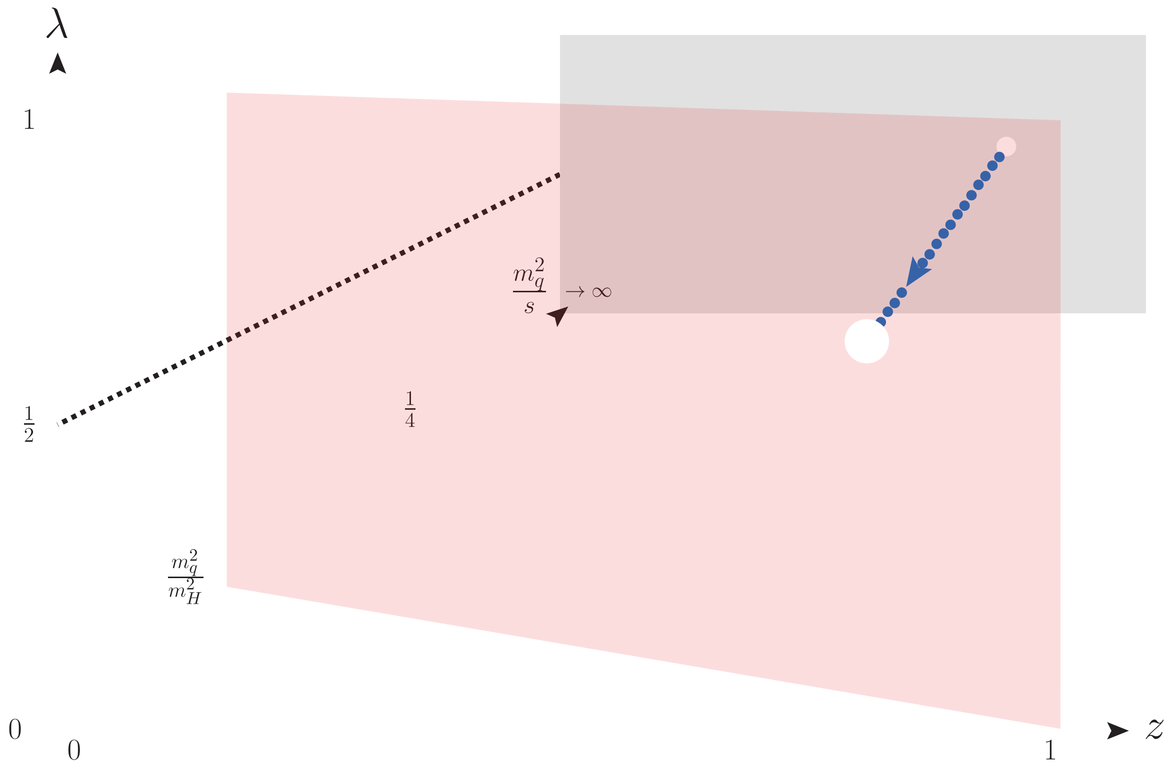
- Computation can be repeated for different values of m_q^2 and m_H^2
- Now straightforward for $m_t^2/m_H^2 \sim 23/12$

➢ Same techniques can be applied to compute bottom quark mass effects…

Summary

➢ Same techniques can be applied to compute bottom quark mass effects…

- Large hierarchy between m_b^2 and m_H^2 can lead to numerical instabilities when solving the differential equations
- Boundaries at $m_q^2 \to \infty$ not optimal



Corrections to gg → H at three loops for two different massive quark flavors unknown