

*TOP-YUKAWA-INDUCED CORRECTIONS
TO HIGGS PAIR PRODUCTION*

Michael Spira (PSI)

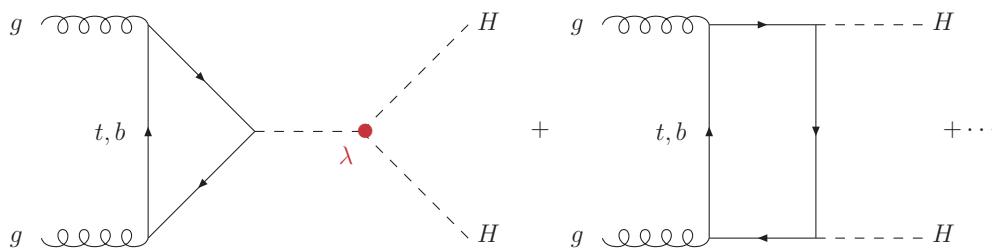
I Introduction

II $gg \rightarrow HH$

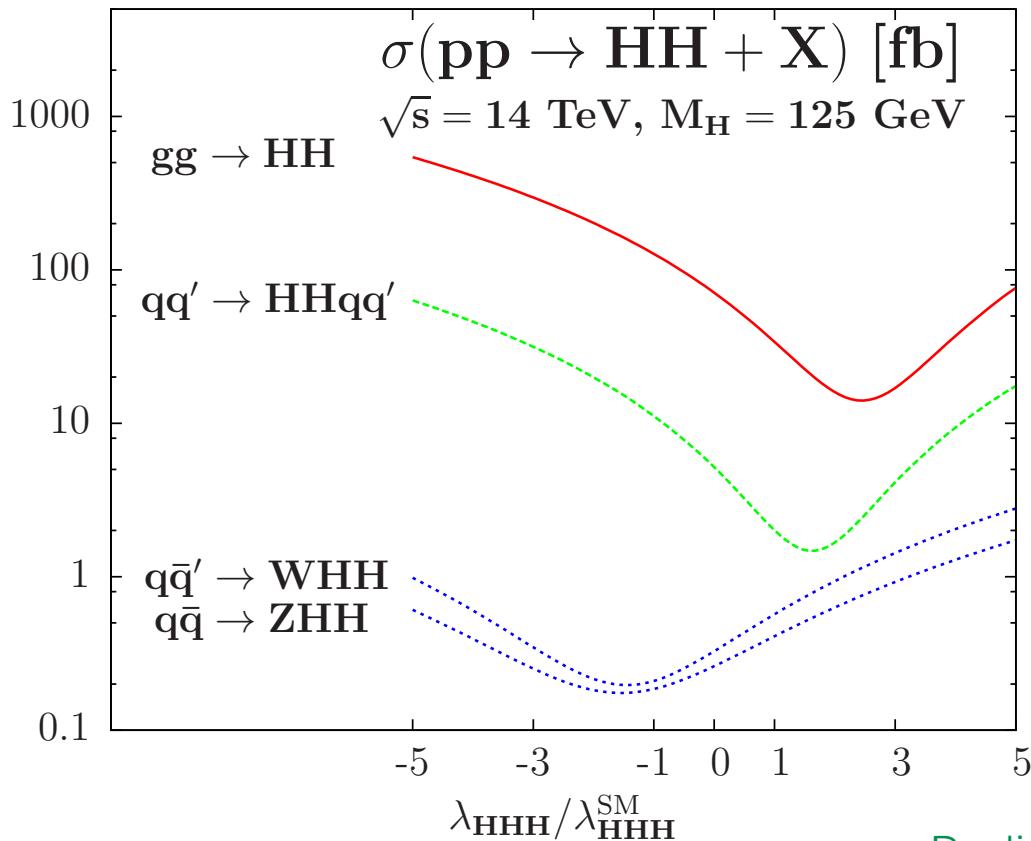
III Conclusions

in collaboration with J. Baglio, F. Campanario, S. Glaus, J. Ronca, M. Mühlleitner
and J. Schlenk

I INTRODUCTION



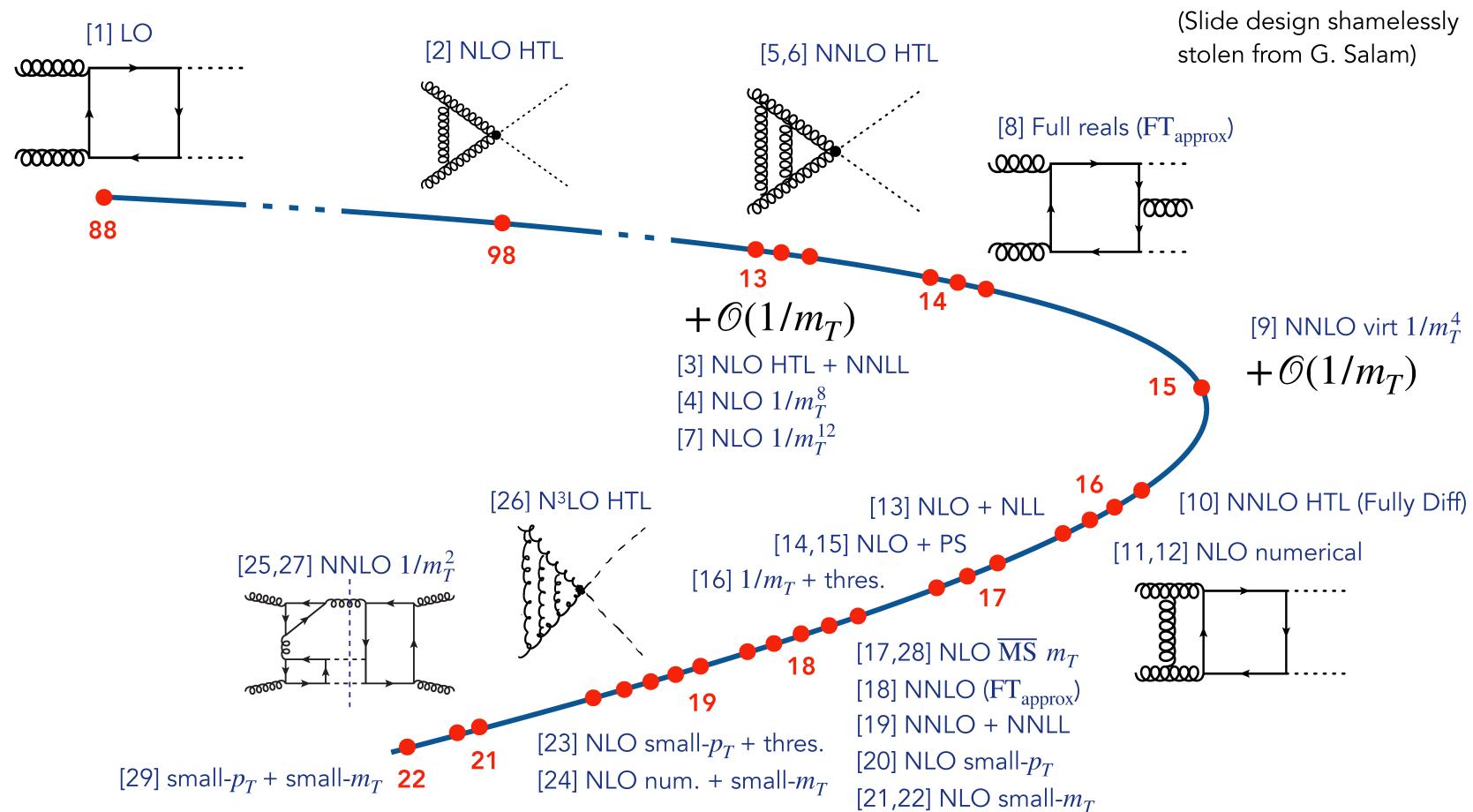
- third generation dominant: $t(b)$



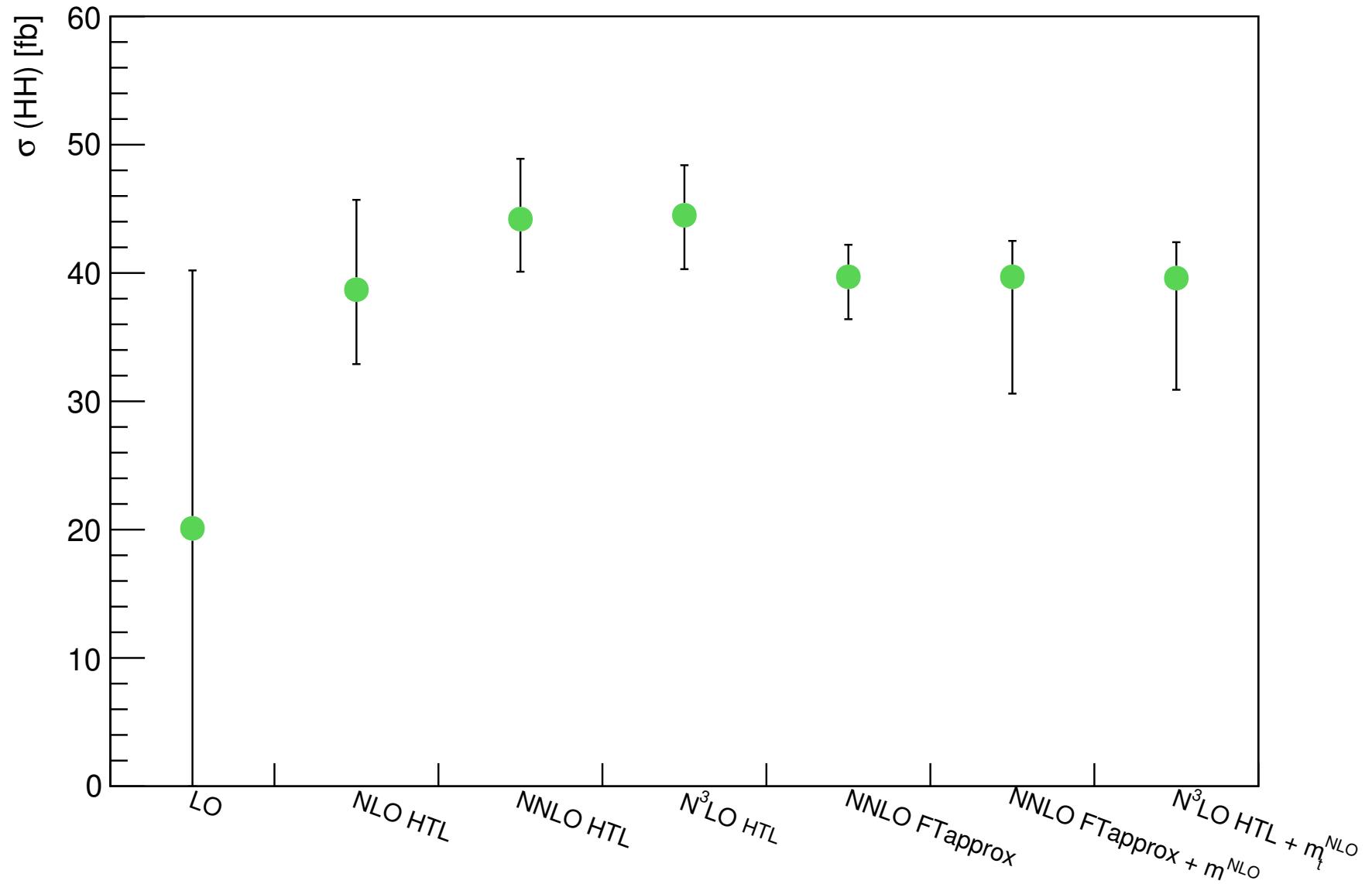
$gg \rightarrow HH :$

$$\frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

An approximate history (30 years in 30 seconds)



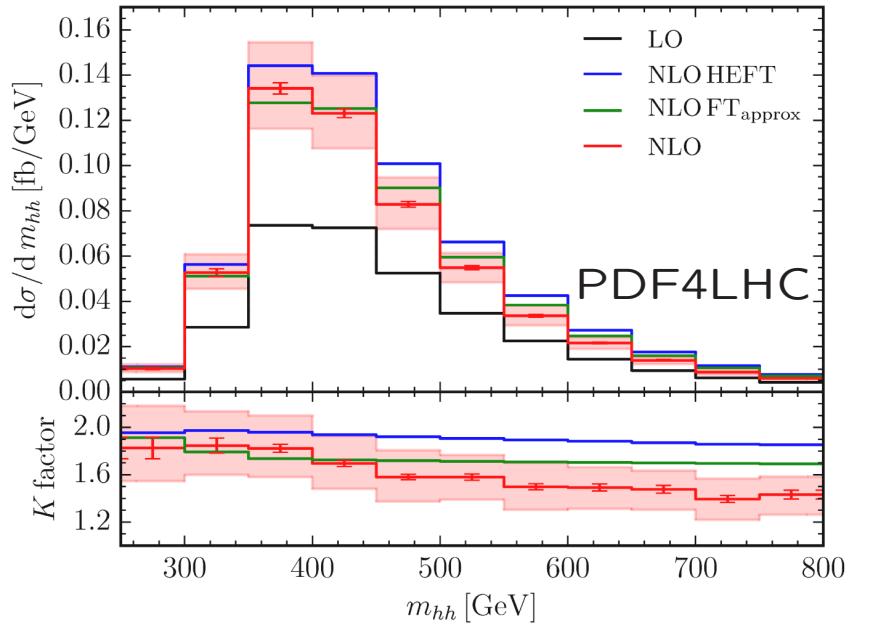
[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22;



Full NLO calculation: top only, numerical integration

Borowka <i>et al.</i>	Baglio <i>et al.</i>
tensor reduction	no tensor reduction
sector decomposition	IR, end-point subtraction
contour deformation	IBP, Richardson extrapolation
$m_t = 173$ GeV	$m_t = 172.5$ GeV

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke
Baglio, Campanario, Glaus, Mühlleitner, Ronca, S., Streicher



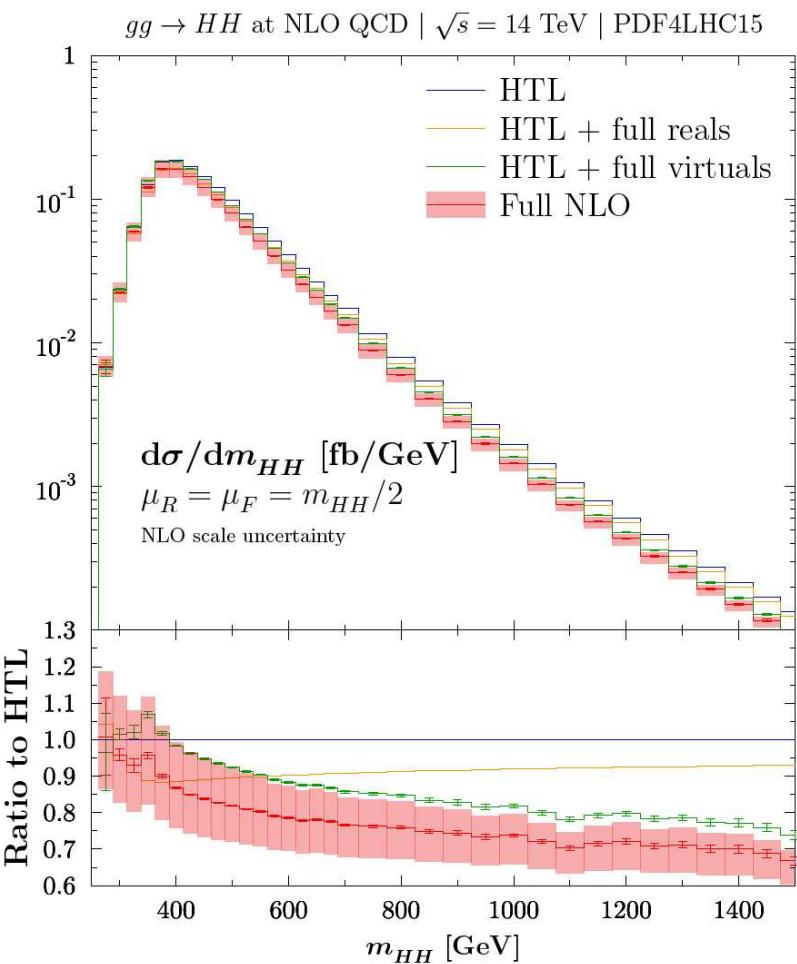
Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke

$$\sigma_{NLO} = 32.91(10)^{+13.8\%}_{-12.8\%} \text{ fb}$$

$$\sigma_{NLO}^{HTL} = 38.75^{+18\%}_{-15\%} \text{ fb}$$

$$m_t = 173 \text{ GeV}$$

⇒ -15% mass effects on top of LO

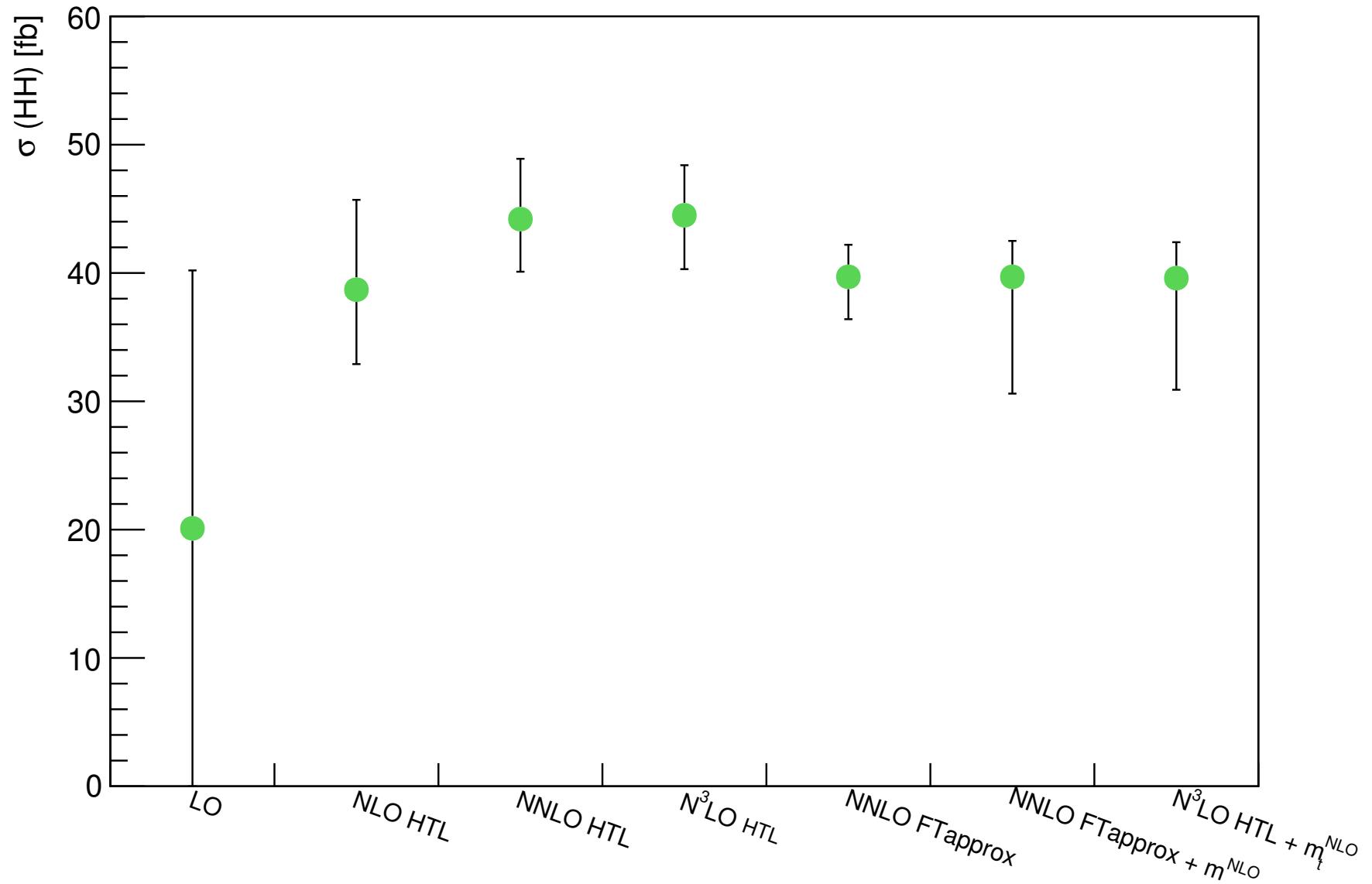


Baglio, Campanario, Glaus,
Mühlleitner, Ronca, S., Streicher

$$32.81(7)^{+13.5\%}_{-12.5\%} \text{ fb}$$

$$38.66^{+18\%}_{-15\%} \text{ fb}$$

$$172.5 \text{ GeV}$$



uncertainties due to m_t

- use m_t , $\overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=300 \text{ GeV}} = 0.02978(7)^{+6\%}_{-34\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=400 \text{ GeV}} = 0.1609(4)^{+0\%}_{-13\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=600 \text{ GeV}} = 0.03204(9)^{+0\%}_{-30\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=1200 \text{ GeV}} = 0.000435(4)^{+0\%}_{-35\%} \text{ fb/GeV}$$

- bin-by-bin interpolation:

$$\sigma(gg \rightarrow HH) = 32.81^{+4\%}_{-18\%} \text{ fb}$$

final combined ren./fac. scale and m_t scale/scheme unc. @ NNLO_{FTapprox}:

$$\sqrt{s} = 13 \text{ TeV} : \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$

$$\sqrt{s} = 14 \text{ TeV} : \quad \sigma_{tot} = 36.69^{+6\%}_{-23\%} \text{ fb}$$

$$\sqrt{s} = 27 \text{ TeV} : \quad \sigma_{tot} = 139.9^{+5\%}_{-22\%} \text{ fb}$$

$$\sqrt{s} = 100 \text{ TeV} : \quad \sigma_{tot} = 1224^{+4\%}_{-21\%} \text{ fb}$$

Is this everything?



Is this everything?

No. . .

Is this everything?

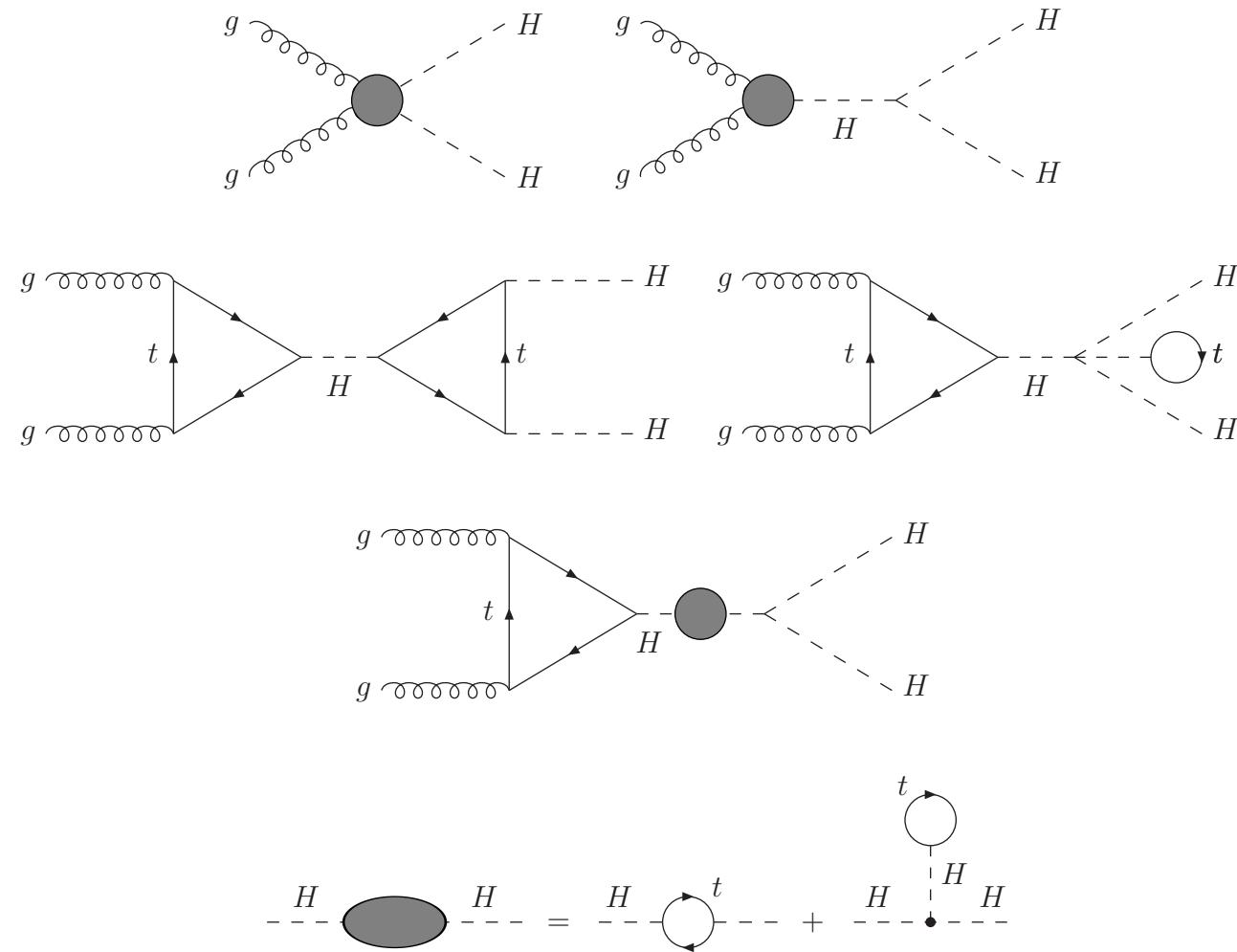
No. . .

electroweak corrections. . .

- two works: top-Yukawa-induced as first step
 - (i) HTL for $ggH(H)$ coupling + full corrections to HHH vertex
Mühlleitner, Schlenk, S.
 - (ii) analytical results for $ggHH$ coupling in the HEL
Davies, Mishima, Schönwald, Steinhauser, Zhang

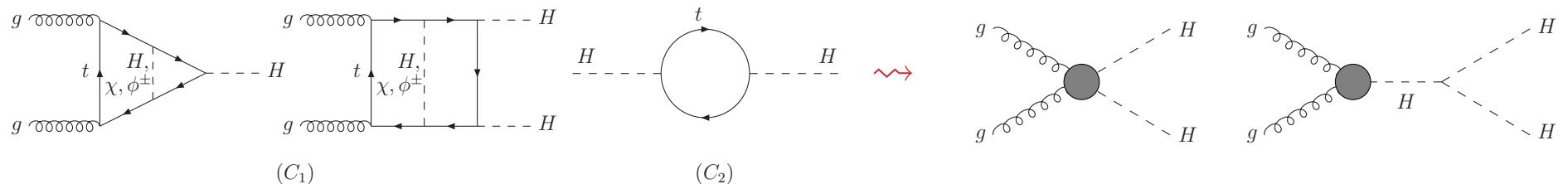
Top-induced elw. corrections

Mühlleitner, Schlenk, S.



(i) effective $ggH(H)$ couplings:

$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \log \left(1 + C_2 \frac{H}{v} \right)$$



- $C_1 = 1 - 3x_t$: genuine vertex corrections

$$[x_t = G_F m_t^2 / (8\sqrt{2}\pi^2)]$$

Djoaudi, Gambino
Chetyrkin, Kniehl, Steinhauser

- $C_2 = 1 + 7x_t/2$: universal corrections

Kniehl, Spira
Kwiatkowski, Steinhauser

$$\begin{aligned} \mathcal{L}_{eff} &= \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \left\{ (1 + \delta_1) \frac{H}{v} + (1 + \eta_1) \frac{H^2}{2v^2} + \mathcal{O}(H^3) \right\} \\ \delta_1 &= \frac{x_t}{2} \sim 0.16\% \quad \eta_1 = 4x_t \sim 1.2\% \quad [\leftarrow \text{tiny}] \end{aligned}$$

[HEL: Davies, Mishima, Schönwald, Steinhauser, Zhang]

(ii) effective $HHH(H)$ couplings:

- effective Higgs potential:

Coleman, Weinberg

$$V_{eff} = V_0 + V_1$$

$$V_0 = \mu_0^2 |\phi|^2 + \frac{\lambda_0}{2} |\phi|^4$$

$$V_1 = \frac{3\bar{m}_t^4}{16\pi^2} \Gamma(1+\epsilon) (4\pi^2)^\epsilon \left(\frac{1}{\epsilon} + \log \frac{\bar{\mu}^2}{\bar{m}_t^2} + \frac{3}{2} \right)$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$\bar{m}_t = m_t \left(1 + \frac{H}{v} \right)$$

- renormalize bare μ_0, λ_0 parameters:

$$\mu_0^2 = -\frac{\lambda_0}{2} v^2 + \delta\mu^2 \quad M_{H0}^2 = \lambda_0 v^2 = \lambda v^2 + (\delta\lambda)v^2 = \lambda v^2 + \delta M_H^2$$

$$\delta\mu^2 = -\frac{3m_t^4}{4\pi^2 v^2} C_\epsilon \left\{ \frac{1}{\epsilon} + \log \frac{\bar{\mu}^2}{m_t^2} + 1 \right\}$$

$$\delta M_H^2 = -\frac{3m_t^4}{2\pi^2 v^2} C_\epsilon \left\{ \frac{1}{\epsilon} + \log \frac{\bar{\mu}^2}{m_t^2} \right\}$$

- radiatively corrected Higgs self-couplings:

$$\begin{aligned}\lambda_{HHH}^{eff} &= 3\frac{M_H^2}{v} + \Delta\lambda_{HHH}, & \lambda_{HHHH}^{eff} &= 3\frac{M_H^2}{v^2} + \Delta\lambda_{HHHH} \\ \Delta\lambda_{HHH} &= -\frac{3m_t^4}{\pi^2 v^3}, & \Delta\lambda_{HHHH} &= -\frac{12m_t^4}{\pi^2 v^4}\end{aligned}$$

$$\lambda_{HHH}^{eff} = 3\frac{M_H^2}{v} - \frac{3m_t^4}{\pi^2 v^3} \approx \textcolor{red}{0.91} \times 3\frac{M_H^2}{v}$$

$$\begin{aligned}\sigma_{\text{LO}} &= \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \\ \frac{d\mathcal{L}^{gg}}{d\tau} &= \int_\tau^1 \frac{dx}{x} g(x, \mu_F) g\left(\frac{\tau}{x}, \mu_F\right) \\ \hat{\sigma}_{LO} &= \frac{G_F^2 \alpha_s^2(\mu_R)}{512(2\pi)^3} \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} [|C_\Delta F_\Delta + F_\square|^2 + |G_\square|^2]\end{aligned}$$

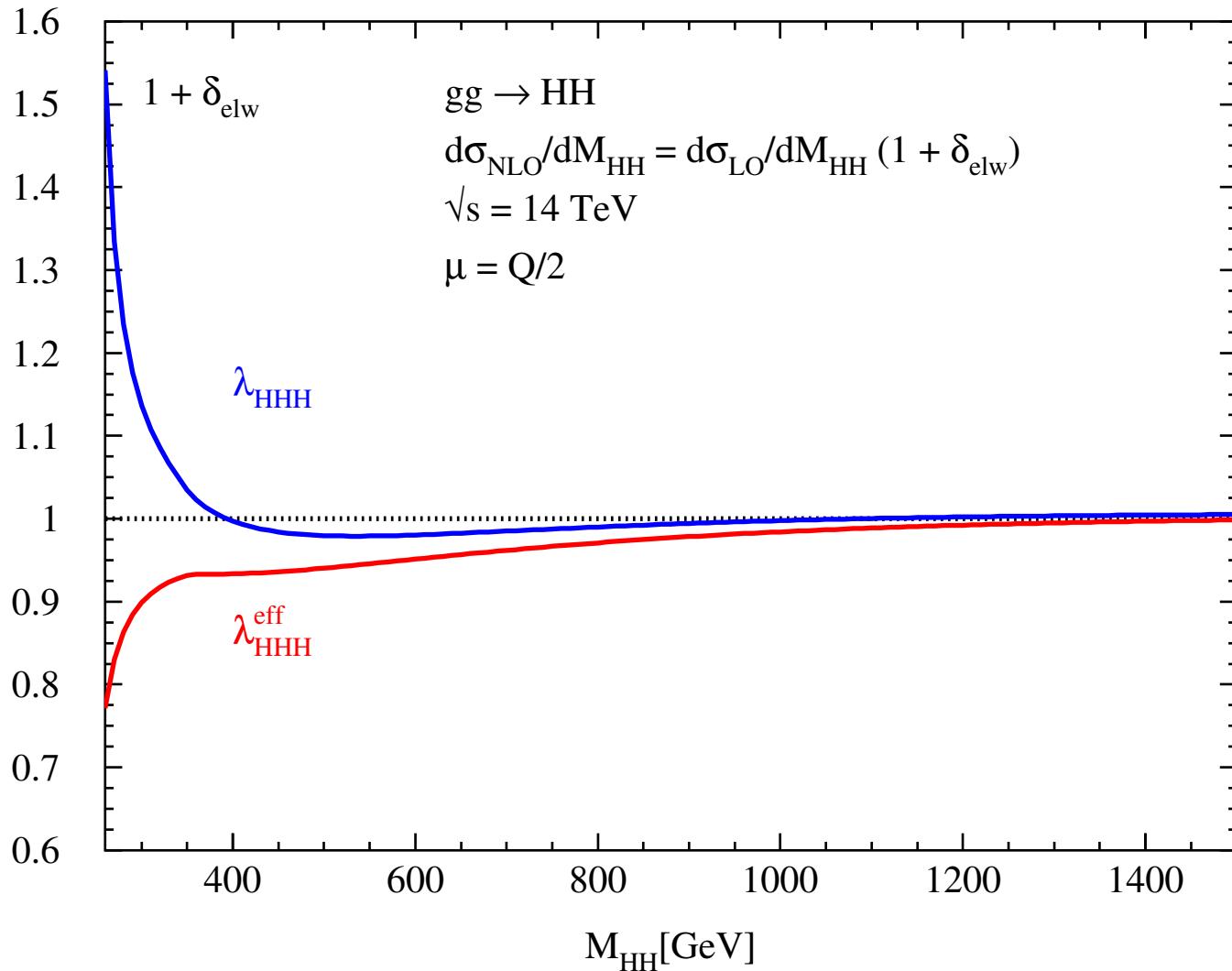
$$\hat{t}_\pm = -\frac{1}{2} \left[Q^2 - 2M_H^2 \mp Q^2 \sqrt{1 - 4 \frac{M_H^2}{Q^2}} \right]$$

$$\lambda_{HHH} = 3 \frac{M_H^2}{v} \qquad C_\Delta = \frac{\lambda_{HHH} v}{(Q^2 - M_H^2)}$$

$$\textsf{HTL}: F_\Delta \rightarrow 2/3, \qquad F_\square \rightarrow -2/3, \qquad G_\square \rightarrow 0$$

$$\begin{aligned}C_\Delta F_\Delta &\rightarrow C_\Delta F_\Delta (1 + \Delta_\Delta) \\ F_\square &\rightarrow F_\square (1 + \Delta_\square)\end{aligned}$$

$$\begin{aligned}\Delta_\Delta &= \delta_1 + \Delta_{HHH} \\ \Delta_\square &= \eta_1\end{aligned}\qquad\qquad\qquad [\text{only } \delta_1, \eta_1 \text{ in HTL!}]$$



$$\begin{aligned}
 \sigma &= 1.002 \times \sigma_{LO} & (\lambda_{HHH}) \\
 \sigma &= 0.938 \times \sigma_{LO} & (\lambda_{HHH}^{\text{eff}})
 \end{aligned}$$

IV CONCLUSIONS

- scale and scheme uncertainties due to m_t relevant for large momenta
- Higgs pair production: m_t effects on top of LO $\sim -15\%$ for σ_{tot} [larger for distributions]
- uncertainties due to factorization/renormalization scale and m_t scale/scheme choice @NNLO_{FTapprox} $\lesssim 25\%$
- combined uncertainties available for λ dependence, too.
- top-induced electroweak corrections: small for total cxn, larger for distributions
- uncertainties due to unknown full elw. corrections $\sim 10 - 20\%$

BACKUP SLIDES

$$\sigma_{\mathsf{NLO}}(pp \rightarrow HH + X) = \sigma_{\mathsf{LO}} + \Delta\sigma_{\mathsf{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\begin{aligned}\sigma_{\mathsf{LO}} &= \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\mathsf{LO}}(Q^2 = \tau s) \\ \Delta\sigma_{\mathsf{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\mathsf{LO}}(Q^2 = \tau s) \ C \\ \Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\mathsf{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\ &\quad \left. + d_{gg}(z) + 6[1+z^4+(1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\} \\ \Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\mathsf{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s(1-z)^2} + d_{gq}(z) \right\} \\ \Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\mathsf{LO}}(Q^2 = z\tau s) \ d_{q\bar{q}}(z)\end{aligned}$$

$$C \rightarrow \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \quad d_{gg} \rightarrow -\frac{11}{2}(1-z)^3, \quad d_{gq} \rightarrow \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27}(1-z)^3$$

- m_t scale/scheme uncertainties at LO:

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=300 \text{ GeV}} = 0.01656^{+62\%}_{-2.4\%} \text{ fb/GeV}$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=400 \text{ GeV}} = 0.09391^{+0\%}_{-20\%} \text{ fb/GeV}$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=600 \text{ GeV}} = 0.02132^{+0\%}_{-48\%} \text{ fb/GeV}$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=1200 \text{ GeV}} = 0.0003223^{+0\%}_{-56\%} \text{ fb/GeV}$$

$$\Delta_{HHH} = \Delta_{vertex} + \Delta_{self} + \Delta_{CT}$$

$$\begin{aligned}\Delta_{vertex} &= \frac{m_t^4}{v^2 M_H^2} \frac{8}{(4\pi)^2} \left\{ B_0(Q^2; m_t, m_t) + 2B_0(M_H^2; m_t, m_t) \right. \\ &\quad \left. + \left(4m_t^2 - \frac{Q^2 + 2M_H^2}{2} \right) C_0(Q^2, M_H^2, M_H^2; m_t, m_t, m_t) \right\} + \frac{T_1}{v M_H^2}\end{aligned}$$

$$\Delta_{self} = \frac{\Sigma_H(Q^2)}{Q^2 - M_H^2} + \frac{1}{2} \Sigma'_H(M_H^2)$$

$$\Delta_{CT} = \frac{\delta M_H^2}{Q^2 - M_H^2} + \frac{\delta \lambda_{HHH}}{\lambda_{HHH}}$$

$$\Sigma_H(Q^2) = 3 \frac{T_1}{v} + 6 \frac{m_t^2}{(4\pi)^2 v^2} \left\{ 2A_0(m_t) + (4m_t^2 - Q^2) B_0(Q^2; m_t, m_t) \right\} + \mathcal{O}(m_t^0)$$

$$\Sigma'_H(Q^2) = 6 \frac{m_t^2}{(4\pi)^2 v^2} \left\{ (4m_t^2 - Q^2) B'_0(Q^2; m_t, m_t) - B_0(Q^2; m_t, m_t) \right\} + \mathcal{O}(m_t^0)$$

$$\frac{T_1}{v} = -12 \frac{m_t^2}{(4\pi)^2 v^2} A_0(m_t)$$

$$\frac{\delta \lambda_{HHH}}{\lambda_{HHH}} = \frac{\delta M_H^2}{M_H^2} + \frac{1}{2} \frac{\Sigma_W(0)}{M_W^2}$$

$$\frac{\Sigma_W(0)}{M_W^2} = 2 \frac{T_1}{v M_H^2} + \frac{2m_t^2}{(4\pi)^2 v^2} \left\{ B_0(0; m_t, 0) + 2B_0(0; m_t, m_t) + m_t^2 B'_0(0; m_t, 0) \right\} + \mathcal{O}(m_t^0)$$

$$\delta M_H^2 = -\Sigma_H(M_H^2)$$