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Institute for Theoretical Particle Physics

Two-loop Yukawa corrections to double Higgs production

The 19th Workshop of the LHC Higgs Working Group | November 28, 2022

Kay Schönwald based on JHEP 08 (2022) 259
in collaboration with Joshua Davies, Go Mishima, Matthias Steinhauser and Hantian Zhang | November 28, 2022

Higgs Self Coupling

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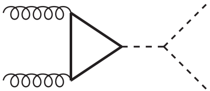
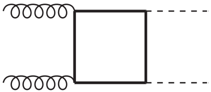
- Standard Model Higgs potential:

$$\boxed{V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4}, \text{ where } \lambda = m_H^2/(2v^2) \approx 0.13.$$

- We want to measure λ, to determine if $V(H)$ is consistent with nature.
  - $-3.3 < \lambda/\lambda_{SM} < 8.5$ [CMS '21]
- λ appears in various production channels, but gluon fusion dominates:





$gg \to HH$ Beyond LO

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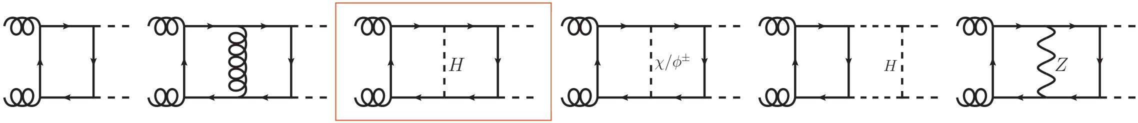
- NLO QCD corrections with full m_t-dependence
  - Numerical approach [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
  - Expansions in different regions [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13] [Gröber, Maier, Rauh '17] [Bonciani, Degrassi, Giardino, Gröber '18] [Davies, Mishima, Steinhauser, Wellmann '18,'19]
  - Numerical approach combined with expansions [Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann '19] [Bellafronte, Degrassi, Giardino, Gröber, Vitti '22]
- NNLO and N^{3}LO QCD corrections are available in large-m_t limit/expansion:
  - NNLO [de Florian, Mazzitelli '13] [Grigo, Melnikov, Steinhauser '14] [Grigo, Hoff, Steinhauser '15] [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18] [Davies, Herren, Mishima Steinhauser '19, '21]
  - N^{3}LO [Spira '16] [Gerlach, Herren, Steinhauser '18] [Banerjee, Borowka, Dhabi, Gehrmann, Ravindran '18] [Chen, Li, Shao, Wang '19]
- NLO EW corrections are partly available (leading top-Yukawa corrections)
  - small-m_t expansion [Davies, Mishima, Schönwald, Steinhauser, Zhang '22] this talk
  - large-m_t limit [Mühlleitner, Schlenk, Spira '22] see Michael Spiras talk

Electroweak Corrections

As we investigate NNLO QCD and beyond, we should consider NLO EW:

$$\mathcal{M} \sim \alpha_s \alpha_t \left(A_1 + \alpha_s A_2 + \alpha_t A_3 + \alpha_{t,\lambda,gauge} A_4 + \mathcal{O}(\alpha_s^2, \alpha_t^2, \ldots) \right)$$

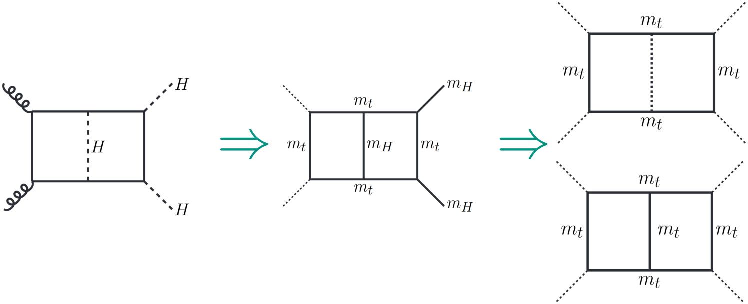


There are more scales to deal with, compared to the QCD contribution,

- start with $\alpha_s \alpha_t^2$ diagrams with internally propagating Higgs:
  - expansion parameter not small $\alpha_t = \alpha m_t^2/(2 s_W^2 m_W^2) \sim \alpha_s/2$
  - only planar integrals in this subset

Analytic High-Energy Expansion

- Integrals depend on many scales: s, t, m_H, m_t, ϵ (, m_W, m_Z)
- Use expansions to make an analytic calculation feasible.



Approach "A":

- $s, t \gg m_t^2 \sim (m_H^{\text{int}})^2, (m_H^{\text{ext}})^2$

✓ reduces to the set of QCD master integrals

✗ hard to apply to full EW corrections

Approach "B":

- $s, t \gg m_t^2 \sim (m_H^{\text{int}})^2 \gg (m_H^{\text{ext}})^2$

✓ can be applied to all EW corrections

✗ needs the calculation of new master integrals

Analytic High-Energy Expansion

- The amplitudes are generated by `QGRAF` [Nogueira '93] , `q2e/exp` [Harlander, Seidensticker, Steinhauser '97] , `FORM` [Ruijl, Ueda, Vermaseren '17] and integration-by-parts reduction with `FIRE6` [Smirnov, Chuharev '19] .

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- We derive a system of coupled differential equations for the 140 master integrals with `LiteRed` [Lee '12] :

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- Higher order m_t^2/s corrections can be obtained by inserting a power-log ansatz:

$$I_n = \sum_{i=-2}^{0}\sum_{j=-1}^{60}\sum_{k=0}^{i+4} C_{(n)}^{ijk}(s,t)\epsilon^i (m_t^2)^j \ln^k(m_t^2)$$

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- Boundary conditions are calculated in asymptotic limit $m_t \to 0$ by Mellin-Barnes method and analytic summation with the help of the packages `MB` [Czakon '05] , `HarmonicSums`, `Sigma` and `EvaluateMultiSums` [Ablinger, Blümlein, Schneider '07] .

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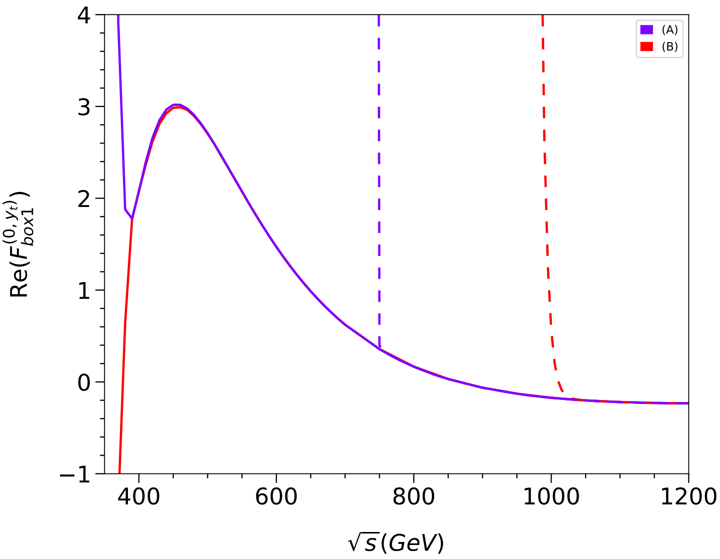
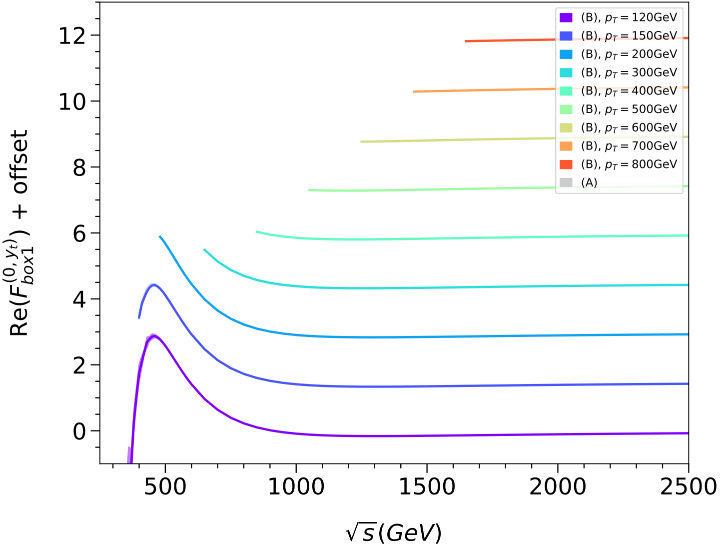
- The convergence of the expansion is improved by applying Padé approximations at the form factor level.



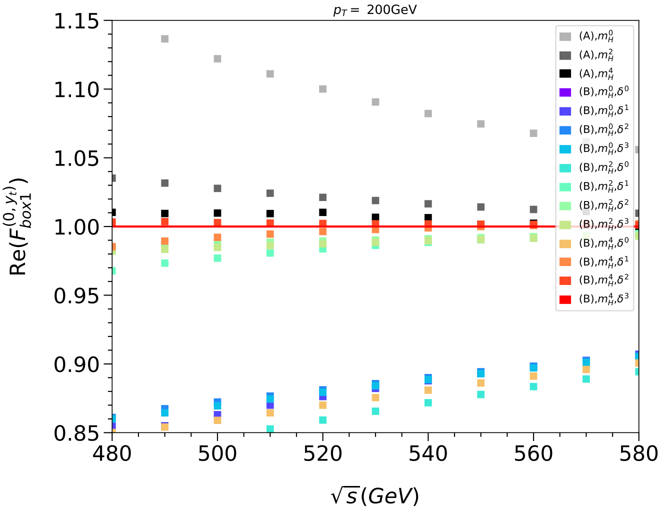
Expansions at $\cos(\theta) = 0$:

- dashed lines: naive expansion up to $(m_t^2/s)^{58}$
- solid lines: Padé improved approximations
- both expansions agree well down to $\sqrt{s} \sim 400\text{GeV}$



Form Factors at Fixed p_T

- deep expansions of the MIs allow high order Padé approximations
- expansions in approaches "A" and "B" agree for p_T values as small as 120 GeV



Convergence of the Expansions:

- Benchmark expansion: $\mathcal{O}\left((m_H^{\text{ext}})^4 \delta^3 m_t^{116}\right)$, with $\delta = 1 - m_H^{\text{int}}/m_t$
- Both expansions converge well and to the same result.

Conclusion

Conclusions:

First step towards electroweak corrections to double Higgs production:

- more difficult than the QCD contribution (extra internal scales)
- expansions make analytic calculation feasible

High-energy expansion:

- Padé-based approximation to improve expansion
- good description of (partial) form factors for $p_T \gtrsim 120\text{GeV}$
- two different expansion methods, which give equivalent results

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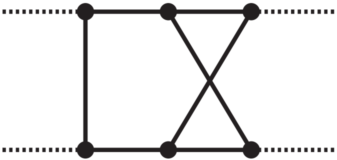
Outlook:

- Apply calculation strategy to the full electroweak corrections.
  $\Rightarrow$ This will include also non-planar sectors.
- Explore complementary expansions to cover the whole kinematic range.



Backup

High-Energy Expansion and Padé Approximation

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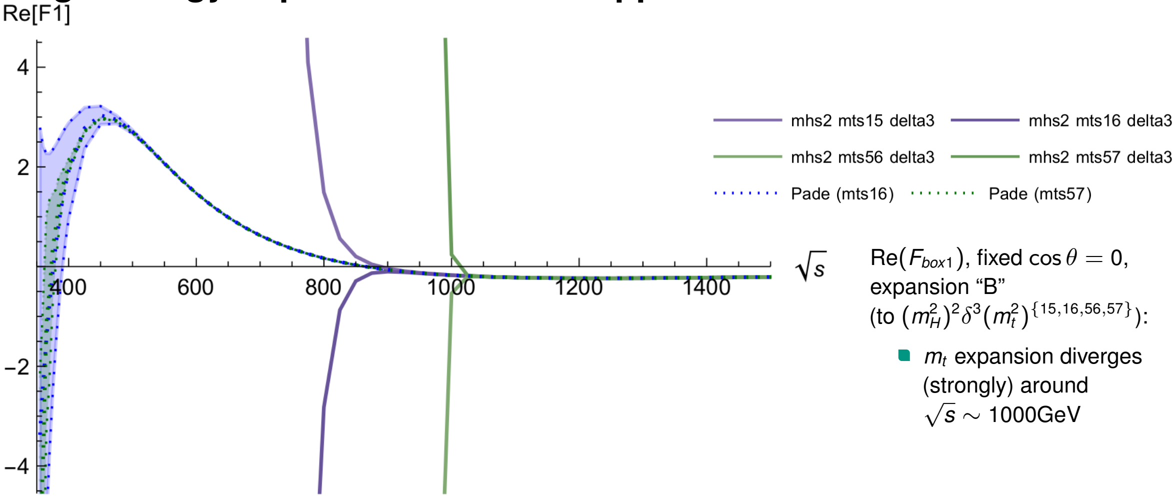
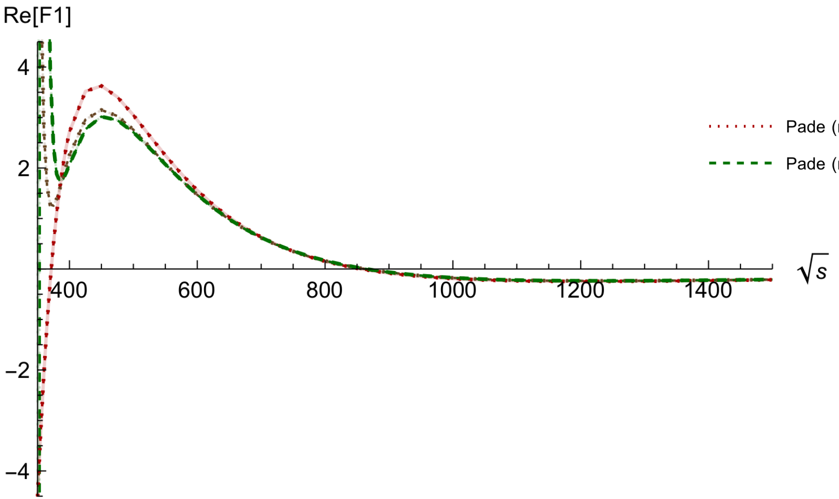


Re(F_{box1}), fixed $\cos\theta = 0$, expansion "B" (to $(m_H^2)^2\delta^3(m_t^2)^{\{15,16,56,57\}}$):

- m_t expansion diverges (strongly) around $\sqrt{s} \sim$ 1000GeV

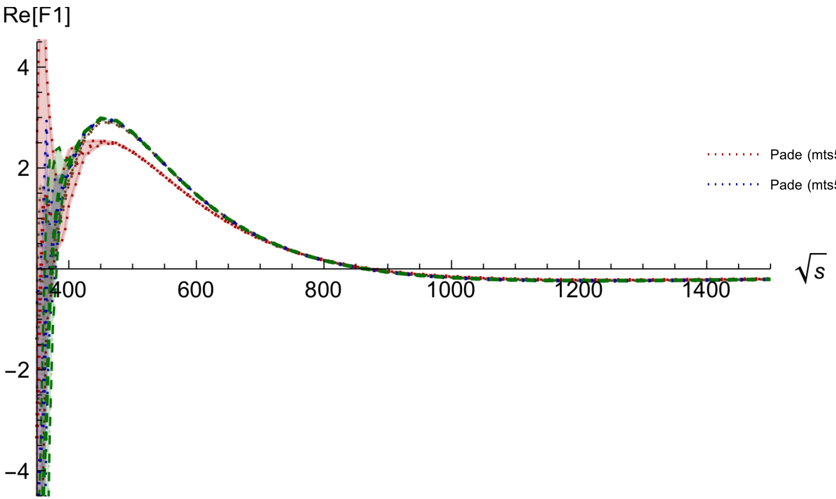
Convergence of Asymptotic Expansion (“A”)



$\mathrm{Re}(F_{box1})$, fixed $\cos\theta = 0$, expansion “A” Padé (to $(m_H^2)^{\{0,1,2\}}$):

- $(m_H^2)^1$ and $(m_H^2)^2$ terms differ by at most 5% for $\sqrt{s} \geq 400\mathrm{GeV}$

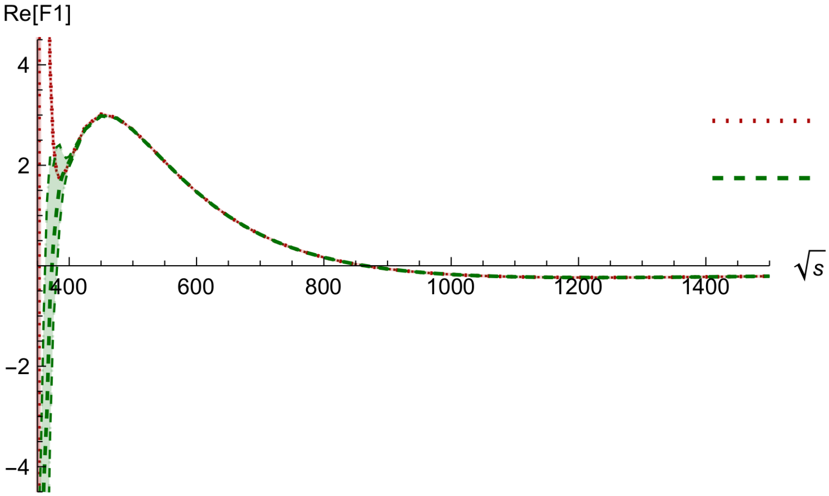
Convergence of δ Expansion (“B”)



Re(F_{box1}), fixed $\cos\theta = 0$, expansion “B” Padé (to $(m_H^2)^2\delta^{\{0,1,2,3\}}$):

- δ^2 and δ^3 terms differ by at most 0.5% for $\sqrt{s} \geq 400$GeV

Comparison of “A”, “B” Expansions



Re(F_{box1}), fixed $\cos\theta = 0$, best “A” and “B” Padé

- “A”, “B” differ by at most 2% for $\sqrt{s} \geq 400$GeV,
- 0.1% for $\sqrt{s} \geq 500$GeV

Padé-Improved High-Energy Expansion

The master integrals for both methods are computed as an expansion in $m_t \ll s, |t|$.

The expansions diverge for $\sqrt{s} \sim 750\text{GeV}$ ("A"), $\sqrt{s} \sim 1000\text{GeV}$ ("B").

The situation can be improved using Padé Approximants:

- Approximate a function using a rational polynomial

$$f(x) \approx \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m},$$

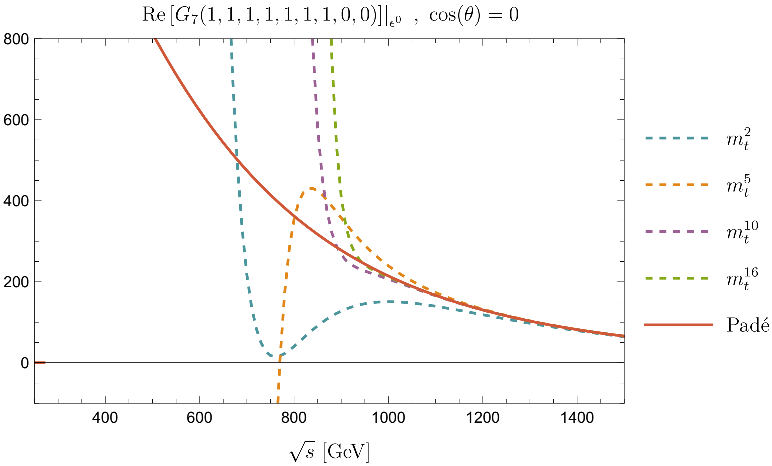
where a_i, b_j coefficients are fixed by the series coefficients of $f(x)$.

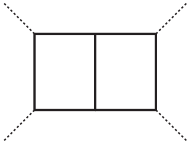
We compute a set of various Padé Approximants:

- combine to give a central value and error estimates
- a deeper input expansion $\Rightarrow$ larger $n + m$ $\Rightarrow$ smaller error
- here, m_t^{120} expansion allows for very high-order Padé Approximants

Master Integrals Results

Padé Improvement

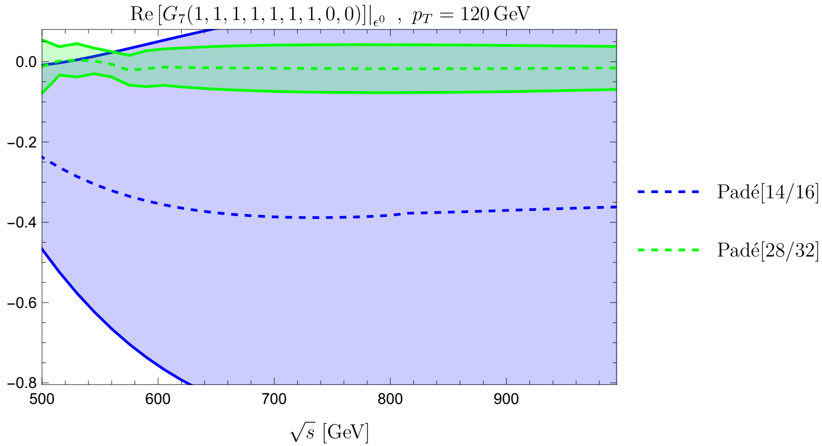




$$\cos(\theta) = \frac{s + 2t - 2m_h^2}{s\sqrt{1 - 4m_h^2/s}}$$

- Fixed order m_t expansions diverge at $\sqrt{s} \sim 1000\,\text{GeV}$.
- The Padé approximation extends the range of validity.

Padé Improvement



$$p_T^2 = \frac{tu - m_h^4}{s}$$

- Lower order Padé approximantions cannot reach low values of p_T.
- For QCD corrections expansions up to m_t^{32} were available: $p_T \gtrsim 150\,\mathrm{GeV}$
- With expansions up to m_t^{120} we reach: $p_T \gtrsim 120\,\mathrm{GeV}$.
- Error estimate from Padé approximations is reliable.

Padé Improvement

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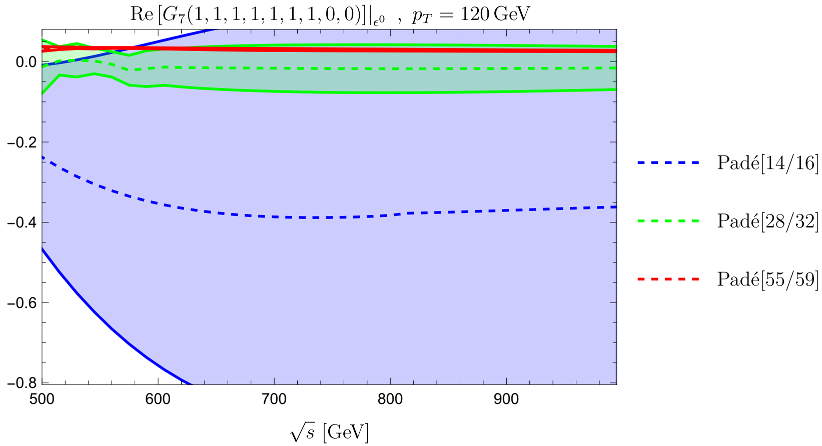

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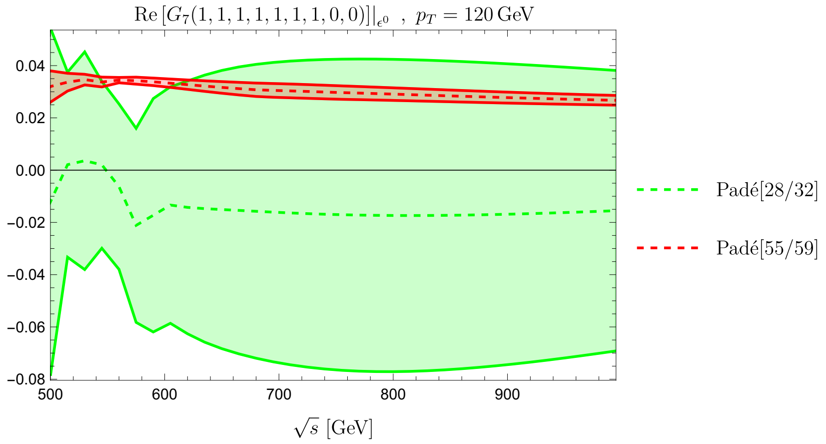

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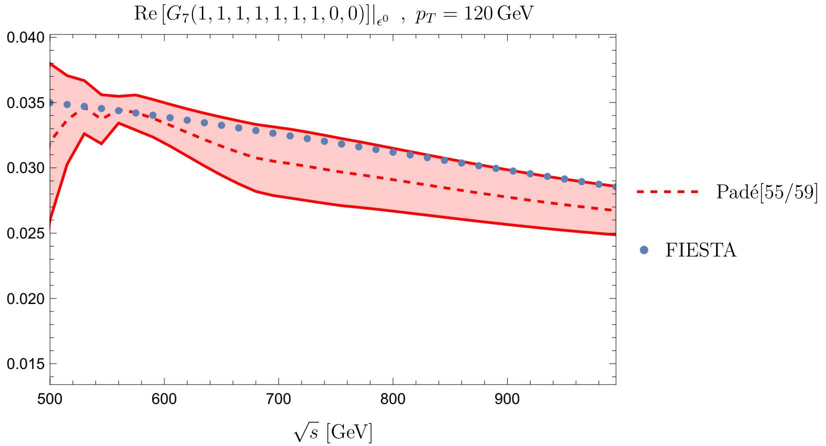

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