

Design Optimization, Kinematics & Dynamics

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CERN Academic Training: Robotics

Content

- **1. What is Robotics?**
- **2. Dynamics**
- **3. Kinematics**
- **4. Design Optimization**

What is Robotics?

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Dynamics

• The equation of motion

RP measurements on old LHC TDE w/ CERNbot

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1

 -3 -2.5 -1.5

 -1

 -2

 -0.5

 $\overline{0}$ 0.5

Dynamics

Dynamics

Example: Compliant Control

clideo.com

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Kinematics

Direct Kinematics

$$
\mathbf{z} = \mathbf{f}(\mathbf{q}%
$$

$$
\dot{\mathbf{z}} = \frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial t} = \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}
$$
 Jacobian

Inverse **Kinematics**

 $\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{-1}\dot{\mathbf{z}}$

Conventions

$z = f(q)$

 $\dot{\mathbf{z}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$

 $\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{-1}\dot{\mathbf{z}}$

Solution for \dot{q} Solution for q

Def. of Error

Final Algorithm

Orientation

2 **Difficulties**:

- 1. Rotations are not commutative!
- 2. Representation Singularities!

Conventions:

- **Euler**: z -> x -> z
- **Kardan**: $z \rightarrow y \rightarrow x$ (yaw pitch roll)
- **Axis-Angle**: unit vector defines axis + angle
- **Quaternions:**

$i^2 = i^2 = k^2 = iik = -1$

Conventions

 $z = f(q)$

 $\dot{\mathbf{z}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$

Solution for \dot{q}

 $\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{-1}\dot{\mathbf{z}}$

Solution for q

Def. of Error

Final Algorithm

Inverse Kinematics

Jacobian Matrix

 $z = f(q)$ $\dot{\mathbf{z}} = \frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial t} = \left(\frac{\partial \dot{\mathbf{f}}(\mathbf{q})}{\partial \dot{\mathbf{q}}}\right) \dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$ • **Analytic Jacobian** • **Geometric Jacobian** • Contains representation • No representation singularities! singularities! • Derivative can be avoided, by deriving the Jacobian via the kinematic chain!

- Moore-Penrose pseudo inverse
- Minimizing velocities

Conventions $z = f(q)$ $\dot{\mathbf{z}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$ $\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{-1}\dot{\mathbf{z}}$ Solution for \dot{q} \bullet Solution for q Def. of Error Final Algorithm

Optimization Based 1st Order Redundancy Resolution

min $\frac{1}{2}\dot{\mathbf{q}}^T \mathbf{W} \dot{\mathbf{q}}$
s.t. $\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} - \dot{\mathbf{z}} = 0$ $\dot{\mathbf{q}} = \mathbf{W}^{-1} \mathbf{J}^T \left(\mathbf{J} \mathbf{W}^{-1} \mathbf{J}^T \right)^{-1} \dot{\mathbf{z}}$

- Minimizing joint velocities
- Yields feasible solutions
- Corresponds to Moore-Penrose pseudo-inverse

Artificial potential energy:

Inverse Kinematics

Conventions $z = f(q)$ $\dot{\mathbf{z}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$ $\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{-1}\dot{\mathbf{z}}$ Solution for \dot{q} \bullet Solution for q Def. of Error Final Algorithm

Optimization Based 1st Order Redundancy Resolution

$$
\min_{\dot{\mathbf{q}}} \quad \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{W} \dot{\mathbf{q}} + \nabla h(\mathbf{q})^T \dot{\mathbf{q}} \text{s.t.} \quad \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}} - \dot{\mathbf{z}} = 0
$$

$$
\frac{d}{dt}h(\mathbf{q}) = \frac{dh(\mathbf{q})}{d\mathbf{q}}\frac{d\mathbf{q}}{dt} = \nabla h(\mathbf{q})^T\dot{\mathbf{q}}
$$

$$
\boxed{\dot{\mathbf{q}} = \mathbf{J}^{\dagger} \dot{\mathbf{z}} + \underbrace{(\mathbf{I} - \mathbf{J}^{\dagger} \mathbf{J}) \mathbf{W}^{-1} \nabla h(\mathbf{q})}_{\mathbf{N}}}
$$

- Minimizing joint velocities
- Yields feasible solutions
- Possibility of Different Objectives

Optimization Based 1st Order Redundancy Resolution

 $\dot{\mathbf{q}} = \mathbf{J}^{\dagger} \dot{\mathbf{z}} + \underbrace{(\mathbf{I} - \mathbf{J}^{\dagger} \mathbf{J})} \mathbf{W}^{-1} \nabla h(\mathbf{q})$

Conventions

$\mathbf{z} = \mathbf{f}(\mathbf{q})$ $\dot{\mathbf{z}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$ $\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{-1}\dot{\mathbf{z}}$ Solution for \dot{q} Solution for q \bullet Def. of Error

Final Algorithm

Numeric Stabilization

 $\mathbf{q}(t) = \int_0^t \dot{\mathbf{q}}(\tau) d\tau + \mathbf{q}(0)$

 $\dot{\mathbf{e}} = \dot{\mathbf{z}}_d - \mathbf{J}\dot{\mathbf{q}}$

Integration Methods, e.g. Euler will drift

Define an Error

$$
\dot{\mathbf{q}} = \mathbf{J}^{\dagger} (\dot{\mathbf{z}}_d + \mathbf{K} \mathbf{e}) + (\mathbf{I} - \mathbf{J}^{\dagger} \mathbf{J}) \nabla h(\mathbf{q})
$$

$$
\mathbf{J} \dot{\mathbf{q}} = \dot{\mathbf{z}}_d + \mathbf{K} \mathbf{e} + \underbrace{\mathbf{J} (\mathbf{I} - \mathbf{J}^{\dagger} \mathbf{J})}_{0} \nabla h(\mathbf{q})
$$

$$
\mathbf{0} = \dot{\mathbf{e}} + \mathbf{K} \mathbf{e}.
$$

Deriving the differential Equation for e

Diff. Equation asymptotically stable iff $K > 0!$

Leads to Closed Loop Inverse Kinematics - CLIK

Conventions

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Solution for \dot{q}

Solution for q

Def. of Error \bullet

Final Algorithm

Inverse Kinematics

Definition of an Error

Position

 $\mathbf{e}_p = \mathbf{r}_{Ed} - \mathbf{r}_E(\mathbf{q})$

Orientation

$$
f_{\rm{max}}
$$

 $\Delta \mathbf{R} = \mathbf{R}_{IE,d} \mathbf{R}_{EI}(\mathbf{q})$

$$
\mathbf{e}_o = \frac{1}{2} \left[\widetilde{\mathbf{n}}(\mathbf{q}) \mathbf{n}_d + \widetilde{\mathbf{s}}(\mathbf{q}) \mathbf{s}_d + \widetilde{\mathbf{a}}(\mathbf{q}) \mathbf{a}_d \right] \qquad \text{with} \qquad \mathbf{R} = [\mathbf{n} \ \mathbf{s} \ \mathbf{a}]
$$

- Orientation Error derived from Axis/Angle representation
- Using this error directly in feedback-loop is a simplification, but stability can be easily proven with Ljapunov

Robotic Manipulator for FCC

Kinematics Library in C++ (IK for highly redundant systems):

- Redundancy resolution for higher dexterity
- Null space projections for task priority assignment

Alejandro Diaz Rosales, Laura Rodrigo Perez

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4 very positive for solution 3 positiv for solution 2 negativ for solution Restrictions L_1 very negativ for solution

Tunnel & Environment Tasks General

B

 $\overline{}$ III

 5.5

Design Process

Requirements:

Space:

- Reach points I-IV (workspace of 3.35x5.50x10^5m)
- Pack up in limited space (2.9x0.55m) while moving along tunnel axis
- Avoid Obstacles

Gamper, H.; Gattringer, H.; Müller, A. and Di Castro, M. (2021). Design Optimization of a Manipulator for CERN's Future Circular Collider (FCC). In Proceedings of the 18th International Conference on Informatics in Control, Automation and Robotics, ISBN 978-989-758-522-7

 $\mathrm{^{r}IV}$

 $\frac{8}{2}$

- Min. payload (~15kg)
- Max. robot weight (~300kg)

Optimization Goals:

- Min. the Degree of Freedom (DoF)
- Min. the Robot Link Lengths
- Min. kinematic and dynamic perf. criteria

Design Optimization

Example: FCC Robot

Figure 10: Optimization results FCC-ee (collision objects)

Figure 11: Optimization results FCC-hh (collision objects)

Design Process

Optimal Solution:

- Optimal Geometric Parameters (link lengths)
- Optimal Topology (11 DoF, Joint Configuration)

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Figure 13: Prototype in FCC-ee - folded configuration

Figure 14: Prototype in FCC-ee cross-section

Figure 15: Prototype in FCC-hh cross-section

Example: Cavity Design Optimization

- Visual inspection of inner surface after assembly
- Small allowed robot space
- Big workspace for inspection
- No restrictions on robot design (topology, geometry)

Gamper, H.; Luthi, A.; Gattringer, H.; Müller, A. and Di Castro, M.; Design Optimization of Quality Inspection Robots for Particle Accelerator Components, In Proceedings of the ECCOMAS Multibody Dynamics Conference, 2021

Design Process Gamper, H.; Luthi, A.; Gattringer, H.; Müller, A. and Di Castro, M.;

Example: Cavity Design Optimization

Design Optimization of Quality Inspection Robots for Particle Accelerator Components, In Proceedings of the ECCOMAS Multibody Dynamics Conference, 2021

Controls Electronics & Mechatronics

Thank you for your attention!