Sublinear-Time Computation in the Presence of an Online Adverary

BOSTON UNIVERSITY

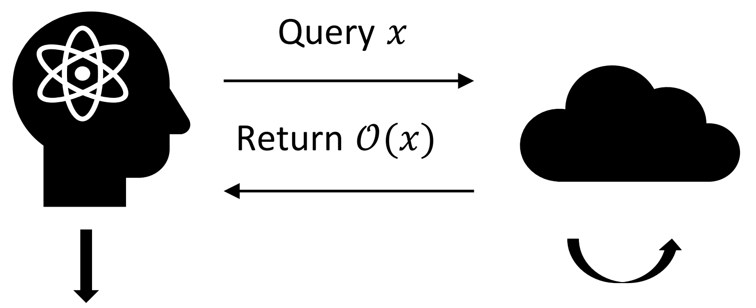
Iden Kalemaj   Sofya Raskhodnikova   Nithin Varma

University of Haifa International School

Model and Motivation

- We study sublinear computation with an online adversary.
- Adversary hides or corrupts up to t input values *after* each query is answered.
- Deeper understanding of structure of violations to fundamental properties.

At the start: $\mathcal{O}(x) = f(x)$ for all points x.



Is f in P or ε-far from P?

Modify values $\mathcal{O}(x)$ in up to t points x

[1] Blum M, Luby M, Rubinfeld R. Self-testing/correcting with applications to numerical problems. J. Comput. Syst. Sci., 1993.
[2] Alon N, Kaufman T, Kirvelevich M, Litsyn S, Ron D. Testing Reed-Muller codes. IEEE Trans. Inf. Theory, 2005.
[3] Ergun F, Kannan S, Kumar R, Rubinfeld R, Viswanathan M. Spot-checkers. J. Comput. Syst. Sci., 2000.
[4] Goldreich O, Goldwasser S, Ron D. Property testing and its connection to learning and approximation. JACM, 1998.
[5] Rubinfeld R, Sudan M. Robust characterization of polynomials with appications to program testing. SIAM J. Comput., 1996.

Online-Erasure-Resilient Testers

Theorem 1. *Linearity* and *quadraticity* can be tested with online erasures with the same query complexity as in standard property testing.

A function $f\colon\{0,1\}^d \to \{0,1\}$ is linear if it is a polynomial of degree at most 1; quadratic if polynomial of degree at most 2.

Linearity. BLR tester [1] optimal for no erasures.
Repeat $O(1/\varepsilon)$ times:

- Sample pair (x, y).
- Reject if $f(x) + f(y) \neq f(x + y)$.

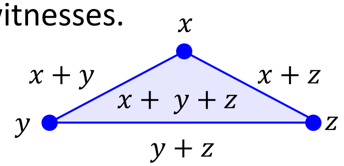
Issue with 1-online-erasure oracle: once x and y are queried, oracle erases $x + y$.

New structural result: For all even k, the fraction of k-tuples that violate linearity is at least ε.

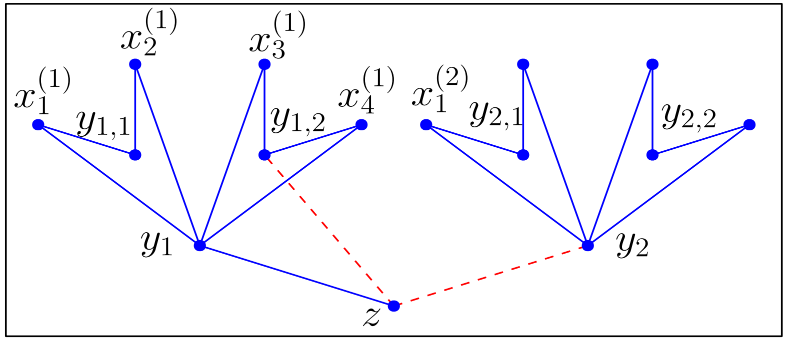
Our tester

- sample and query reserve of $O(\log t / \varepsilon)$ points.
- query sums of k elements sampled from reserve, for some even k.

Quadraticity. Tester of Alon et al. [2] looks for more complicated witnesses.



Two-Player Game



Lower Bounds

Theorem 2. For testing linearity, $\log t$ queries are required.

Theorem 3. Some properties are impossible to test even with a 1-online-erasure oracle: *sortedness* and *Lipschitz* property of sequences.

Sorted sequence: $f(x) \leq f(y)$ for all $x < y$.

If no erasures, can be tested with $O(\log n)$ queries [3] or $O(\sqrt{n})$ uniform queries.

Hard to test instances: