COMPUTER-ASSISTED PROBLEM-BASED LEARNING FOR WORD-BASED MATHEMATICAL PROBLEM SOLVING

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Abstract

A focus on mathematical understanding and problem solving in math education has developed a need to implement alternate ways of testing to better assess the students' understanding and problem solving skills. Furthermore, previous computer-assisted problem-solving systems designed for elementary school mathematical education have focused mainly on developing students' cognitive skills and less interest is dedicated to related procedural skills. However, procedural and conceptual knowledge are highly correlated. This study proposes a computer-assisted system, whose design is based on Polya's problem-solving model. The system is designed to help low-achieving second graders in mathematics with word-based addition and subtraction questions. The emphasis of using the specific model was on dividing the problem solving procedure into stages and the concentration on the students' cognitive and procedural processes at each stage. In order to help students overcome procedural obstacles, we developed an agent oriented evaluation so as to give them a meaningful feedback to better monitor and support students learning performance.

Keywords: Problem based learning, Cognitive and procedural skills evaluation, Addition and subtraction word problems resolution, Polya's strategy.

1 Introduction

Descriptive evaluation is recognized as an effective method that requires students to write down the problem solving process so that teachers could analyze what the students do not understand and help improve their understanding. During the past 30 years, there has been an increasing emphasis on assessing problem solving by examining the cognitive processes that students use while engaged in problem solving. The multidimensionality of the problem-solving process is certainly made evident as attempts are made to look at all thinking done to solve a problem.

Descriptive assessment does not ask simple questions that could be answered with a fact. Instead, they ask students to describe their problem solving process and evaluate the students' advanced thinking skills such as reasoning skills in mathematics. Whang (Whang, 2004) suggested that descriptive assessment is one of the most effective evaluation methods because it allows teachers to know explicitly the thought process of the students.

Furthermore, computer-assisted mathematical problem solving systems are rapidly growing in educational usage. These systems help students to better cope with difficulties encountered in solving problems and give them immediate feedback. Some of these systems are based on the four problem-solving stages mentioned by Polya (Polya, 1945): (1) understanding the problem, (2) making a plan, (3) executing the plan and (4) reviewing the solution. However, these computer-assisted problem-solving systems have incorporated all the problem-solving steps within a single stage, making it difficult to diagnose stages at which errors occurred when a student encounters difficulties and imposing a too-high cognitive load on students in their problem solving (Chang et al., 2006). Moreover, these systems focus more on cognitive thinking process, as well as abstract mathematical concepts and little interest is devoted to procedural skills. However, procedural and conceptual knowledge are highly correlated (Hallet et al., 2010) (Rittle-Johnson et al., 2001). Competence in domains such as mathematics rests on children developing and linking their knowledge of concepts and procedures (Silver, 1986), therefore we propose to develop a computer-assisted problem solving system that rules on both students' conceptual and procedural skills.

The purpose of this paper is to propose a new system that is based on the four problem-solving stages mentioned by Polya. The system assists in achieving addition and subtraction problems for second grade students by assessing cognitive reasoning at each stage and related procedural skills. Moreover, a multi agent system is used to grade students' answers and displays feedback after the problem solving completion.

The paper is organized as follows: In section 2, we discuss the goals of problem based learning strategy and its merits to enhance learning, such as creative thinking, problem solving, logical thinking and decision making. Section 3 focuses mainly on the influence of computer-assisted environments on mathematics instruction and their positive impact on students' problem-solving. In section 4, we present the proposed problem solving process and we describe student-machine interfaces that are used in each stage besides embedded techniques to assist in achieving a successful outcome at each stage. Section 5 presents the student assessment approach and depicts the role of each agent in the assessment process. In section 6, we conclude on adopted strategy, and future work.

2 Problem Based Learning

A focus on mathematical understanding and problem solving in math education has developed a need to implement alternate ways of testing to better assess the students' understanding and problem solving skills. The purpose of education has shifted from simply delivering knowledge and information to student-centered education that focuses on fostering creativity and enhancing problem solving skills.

In Algeria, evaluation in new mathematics curriculum now focuses more on assessing problem solving skills and advanced mathematical thinking skills such as reasoning, communications and mathematical connection skills.
In traditional teaching, assessment of whether students had understood a mathematical problem was based on whether they could describe the correct arithmetic procedure. However, it was not enough to evaluate students' mathematics concepts and abilities of solving math problems merely depending on their writing.
Several researches also pointed out that good problem solving skills are the key to acquiring a successful solution in learning mathematics (Gagne, 1985) (Mayer, 1992).

Obviously, if a student could successfully solve a math problem by arithmetic calculation, that did not mean the student really understood it. It has been maintained in the literature that PBL positively influence learning outcomes along with learners' higher order thinking skills such as creative thinking, problem solving, logical thinking and decision making. For instance, Elshafei (Elshafei, 1999) compared the Calculus achievement levels of second graders in five different high schools and 15 different classes where PBL and traditional instructional methods were implemented. Findings indicated that students in PBL settings did prefer this method and had higher levels of achievement. In addition, it was revealed that students in PBL settings had better solutions for given problems in comparison to those who were in traditional classroom settings. Kaptan and Korkmaz (Kaptan and Korkmaz, 2000) investigated the influence of PBL on problem solving skills and self-efficacy levels of in-service teachers. Findings revealed that the experimental group which was exposed to fundamental science activities through PBL had higher self-efficacy and logical thinking scores than the control group.

The goal of PBL-based cognitive strategy instruction is to teach learners multiple cognitive and metacognitive processes and strategies to facilitate and enhance performance in academic domains (e.g., mathematical problem solving). The processes and strategies range from simple to complex depending on task difficulty and context of the task (Montague and Van Garderen, 2008). Polya's Mathematics Problem-Solving Process is one of these PBL instructional strategies. He was the first to introduce the concept of problem-solving model believing that mathematics is not all about the result.
According to Polya's problem-solving model, the proposed system is designed to guide low-achievers through the parts of the problem-solving process that they often ignore. Furthermore, the system adopts schema representations strategy (Reusser, 1996) in the "Making a plan" stage to enable students describe the solution steps in detail by ordering operands of the problem (see figure 3).

3 Relevant Research Work

Arithmetic word problems play an important role in the elementary school mathematics curricula in terms of developing general problem-solving skills (Verschaffel et al., 2007). Studies of children's solutions of addition and subtraction problems date to the early part of the last century (Arnet, 1905) (Browne, 1906). Since that time a number of researchers have investigated how children solve addition and subtraction problems. (For example, see (Carpenter et al., 1981) (Groen and Parkman, 1972) (Svenson, 1975)). With technological advancement and the arrival of the multimedia computer instruction era, the attention of more and more studies has been fastened on interactive learning methods through multimedia computers. The use of computers to implement findings from qualitative research related to problem-solving teaching strategies can furnish more effective learning opportunities for learners.
Over the last years there has been an increased research studies about the influence of computer-assisted environments on mathematics instruction (see (Huang and Ke, 2009)). We present below some of them that are interested in enhancing cognitive problem solving ability.

Chang et al. (Chang et al., 2006) proposed a computer-assisted system named MathCAL, the design of which was based on the four polya's problem-solving stages. A sample of 130 fifth-grade students (average 11 years old) completed a range of elementary school mathematical problems. The result showed that MathCAL was effective in improving the performance of students with lower problem solving ability. These assistances improved students' problem solving skills in each stage.

Huang et al. (Huang et al., 2012) developed a computer-assisted mathematical problem-solving system in the form of a network instruction website to help low-achieving second- and third-graders in mathematics with word-based addition and subtraction questions. According to Polya's problem-solving model, the proposed system was designed to guide these low-achievers through the parts of the problem-solving process. They found that the mathematical problem solving ability of experiment group students was significantly superior to that of control group students. Most of the participants were able to continue the practice of solving word-based mathematical questions, and their willingness to use the system was high. Their findings indicate that the computer-assisted mathematical problem solving system can serve effectively as a tool for teachers engaged in remedial education.

Panaoura (Panaoura, 2012)] investigated the improvement of students' mathematical performance by using a mathematical model through a computerized approach. He developed an intervention program and 11 years students worked independently on a mathematical model in order to improve their self-representation in mathematics, to self-regulate their performance and consequently to improve their problem solving ability. The emphasis of using the specific model was on dividing the problem solving procedure into stages. The use of the computer offered the opportunity to give students general comments, hints and feedback without the involvement of their teachers. Students had to communicate with a cartoon animation presenting a human being who faced difficulties and cognitive obstacles during problem solving procedure. Experiments involved 255 students (11 years old), who constituted the experimental and the control group. Results confirmed that providing students with the opportunity to self-reflect on their learning behavior when they encounter obstacles in problem solving is one possible way to enhance students' self-regulation and consequently their mathematical performance.

Leh and Jitendra (Leh and Jitendra, 2012) compared the effectiveness of computer-mediated instruction and teacher-mediated instruction on the word problem-solving performance of students struggling in mathematics. Both conditions integrated cognitive modeling that focused on the problem structure using visual representations with critical instructional elements specifically targeting the needs of at-risk students. Participants were 25 third-grade students with mathematics difficulties. But results indicated no statistically significant between-condition differences at posttest and on a 4-week retention test of word problem-solving.

Based on Polya's four problem-solving steps (understanding the problem, devising a plan, carrying out the plan and looking back), Ma and Wu (Ma and Wu, 2000) designed a set of interesting, active learning materials for teaching. Research outcomes indicated both students' learning interest and achievement had improved. Chang (Chang, 2004) incorporated strategies such as key-point marking, diagram illustration and answer review in the problem-solving process and developed a process-oriented, computer-aided mathematics problem solving system. The system was applied mathematical questions (mainly elementary-level arithmetic computation) with fifth graders as the subjects of the empirical study. Results showed that the system was effective in enhancing low-achievers' problem-solving ability.

Summary of the examples cited above evince that computer-assisted mathematics-problem-solving systems have a positive impact on children's problem-solving ability. However all these systems focus on cognitive skills used in problem solving and no interest is devoted to related problem solving procedural skills. Here our focus is mainly on both cognitive and procedural skills training using polya's problem solving strategy and multi agent systems to analyze second grade students' word based problem solving ability. Precisely, we address the problem of how to evaluate the students' skills objectively so as to provide them the best feedback.

4 System Design and Framework Outline

The purpose of this paper is to propose a computer-assisted Learning system that is based on the mathematical problem-solving process proposed by Polya. It guides the second grade students to think and solve the mathematical problems by using a graphical representation in the making plan stage consisting of two operand nodes; one operator node and one result node (see Fig. 3). Each operand and result node comes with two attributes, label and value, representing the meaning of the node and its numerical value, respectively. The values for the two operand nodes and the operator node correspond to the two operands and an operator in the mathematical expression. The value at the result node is the result of the expression.

The schema representation is very helpful for conceptualizing the semantics of the problem [3]. The problems would be divided into four types based on the classification of Vergnaud (Vergnaud, 1982): (1) PUT-TOGETHER, (2) CHANGE-GET-MORE, (3) CHANGE-GET-LESS, and (4) COMPARE (See table 1).
Because all problems involve three quantities and any of these quantities can be unknown, there are three possible problem subtypes within each main problem type. Two of these require subtraction of the two given numbers in the problem and one requires addition of the two givens.

Addition situations	**Subtraction situations**
Change-Get-More Missing end Ali had 3 marbles. Then Omar gave him 5 more marbles. How many marbles does Ali have now? Missing Change Ali had 3 marbles. Then Omar gave him some more marbles. Now Ali has 8 marbles. How many marbles did Omar give him? Missing start Ali had some marbles. Then Omar gave him 5 more marbles. Now Ali has 8 marbles. How many marbles did Ali have in the beginning?	**Change-Get-Less** Missing end Ali had 8 marbles. Then he gave 5 marbles to Omar. How many marbles does Ali have now? Missing change Ali had 8 marbles. Then he gave some marbles to Omar. Now Ali has 3 marbles. How many marbles did he give to Omar? Missing start Ali had some marbles. Then he gave 5 marbles to Omar. Now Ali has 3 marbles. How many marbles did Ali have in the beginning?
Put-Together Missing all Ali has 3 marbles. Omar has 5 marbles. How many marbles do they have altogether? Missing first part Ali and Omar have 8 marbles altogether. Omar has 3 marbles. How many marbles does Ali have? Missing second part Ali and Omar have 8 marbles altogether. Ali has 3 marbles. How many marbles does Omar have?	**Compare** Missing difference Ali has 8 marbles. Omar has 5 marbles. How many more marbles does Ali have than Omar? Missing big Ali has 3 marbles. Omar has 5 more marbles than Ali. How many marbles does Omar have? Missing small Ali has 8 marbles. He has 5 more marbles than Omar. How many marbles does Omar have?

Table 1. Classification of word problems.

4.1 Students' problem solving process

Students' problem-solving guidance process is shown in Figure 1. Firstly, the adequate problem is provided by the system among other problems stored in the problem solving information database. In stage 1, the assessing module for understanding the problem proposes responses to be tagged by the student and other techniques allowing to rule about comprehension of the problem. The plan making stage enables the student to build a schema representation of his solution. Stage 3offers, according to the operation type, a calculation interface and finally a multi questionnaire is proposed to the student to validate his solution. Problem information is provided at each stage of the problem solving process and assessment of each stage is recorded in the student tracking database. The system displays feedback messages after the whole problem solving completion.

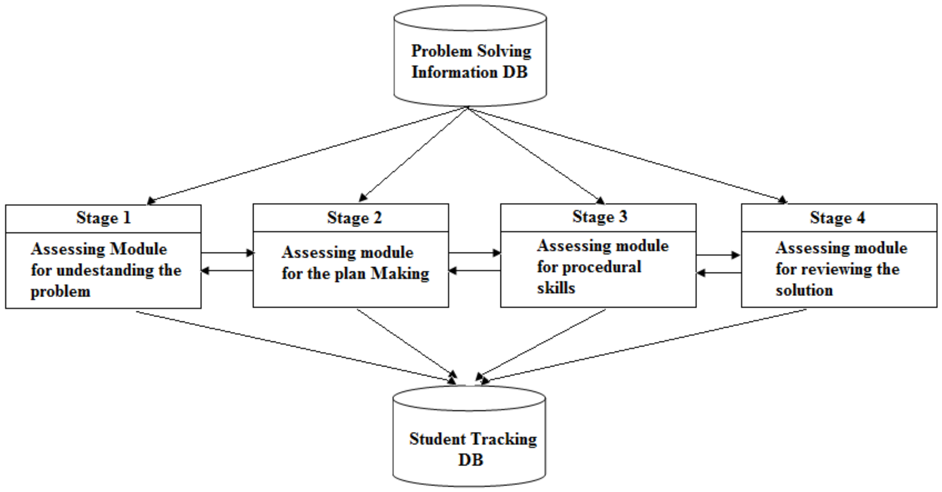

Figure 1. *Students' problem solving Process.*

4.2 The stage of understanding the problem

In this stage, the system offers to students the possibility to circle important words in the problem. Furthermore, students have to distinguish between what is known and what is requested in the problem by selecting adequate responses. For illustration, figure 2 displays the problem in the problem frame and check boxes for needed answers.

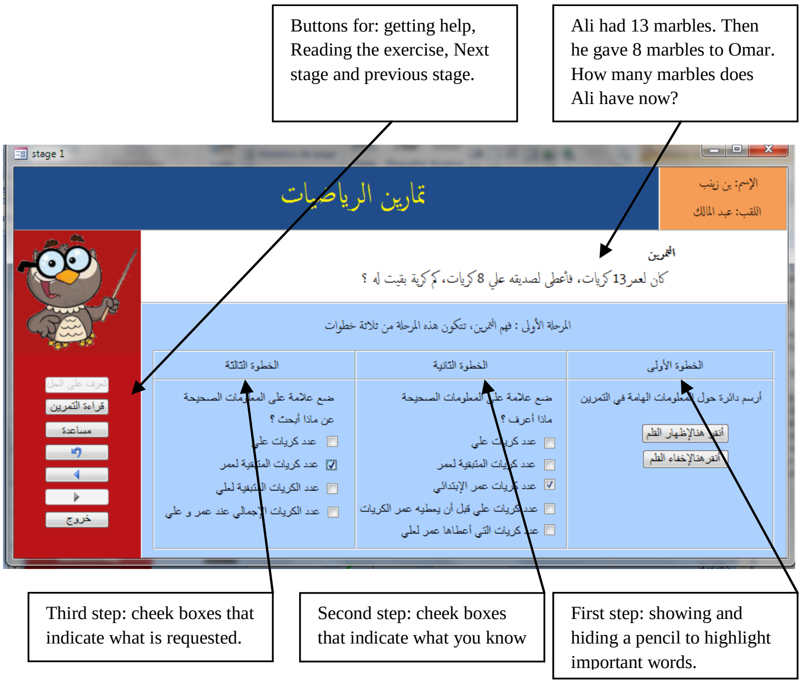

Figure 2. The screenshot of the first stage (Understanding the problem)

4.3 The stage of making the plan

The guiding process of plan elaboration is divided in four steps as illustrated in figure 3 so as to help students express graphically the problem solving. The first step consists of identifying the first operand and its value among a list of operands; the student selects the appropriate one and enters its value. The second step proposes an operator choice between addition and subtraction. In the third step, the student selects the second operand and its value and finally the fourth step requires only a result label. According to the problem missing part a result label may either be requested in first or third step.

In the example of figure 3, the student has chosen the second label for the first operand and enters the value 13, for the second operand he chosen also the second label with a value of 8 and tries subtraction between these two operands. Labels are presented so as to know if student understands the meaning of the operator.

After the student has pressed the ‘‘next’’ button, the system compares the solution plan made by the student with the solution plan built into the system, the suggestions regarding the student’s problem solving are stored in the student tracking database and displayed after the student completes the problem solving.

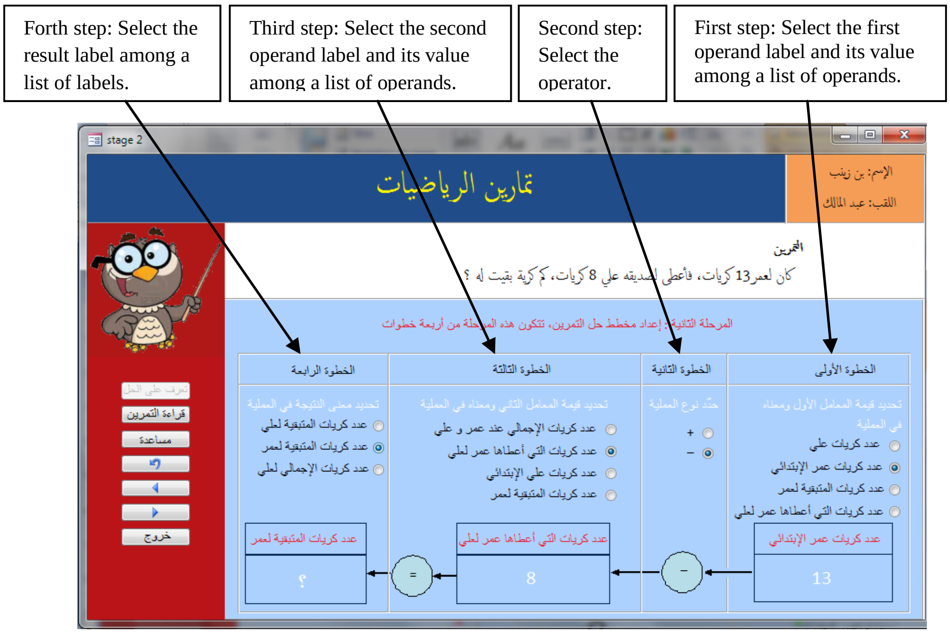

Figure 3.　　*The screenshot of the second stage (Making the plan)*

4.4　The stage of plan execution

As shown in figure 4, the system provides a graphic preview for all of the addition and subtraction worksheets in a vertical problem format where large cases are reserved for digits of operands and little cases are used if regrouping is required for subtraction (exchanging one of the tens for 10 units or one of the hundreds for 10 tens) or for addition when process involved a carryover number. In this stage, the system evaluates student's procedural knowledge and uses a multi agent evaluation approach. Three steps are required at most to complete this stage, each step aims at manipulate ones, tens and hundreds. The student has to fill in the large cases (ones, tens and hundreds) of result and little cases if regrouping is required for subtraction or if addition needs carrying over. After finishing calculation of all columns, the system assesses student's answer and feedback is stored. A last stage is needed to validate student's problem solution.

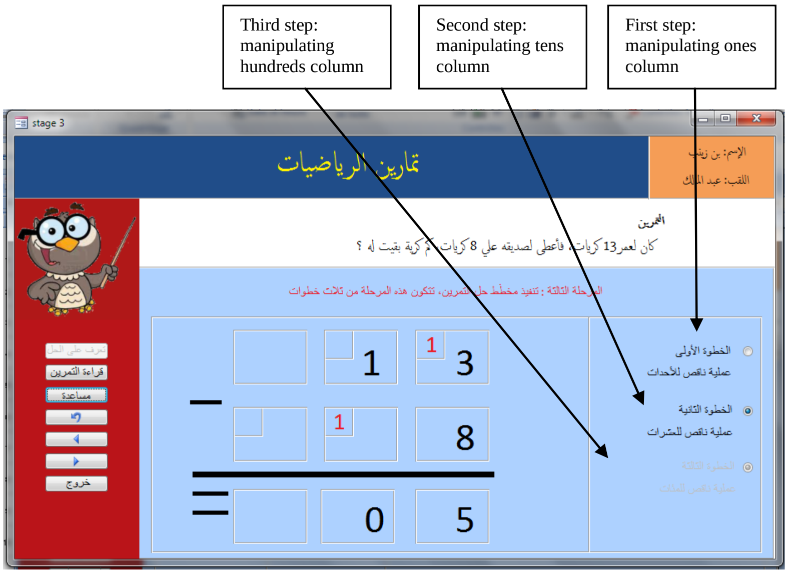

Figure 4.　　*The screenshot of the third stage (Executing the plan)*

4.5 The stage of reviewing the solution

In this stage the student answers questions as shown in figure 5. In order to validate the solution given at previous stage, the system proposes some questions that are related to the problem and student must answer by true or false. After completing this stage, the student presses the Evaluation button which triggers the system to evaluate the results, and messages appear that indicate whether any mistake was made. Also, the correct problem-solving steps are displayed simultaneously on the same stage interface of student’s answers.

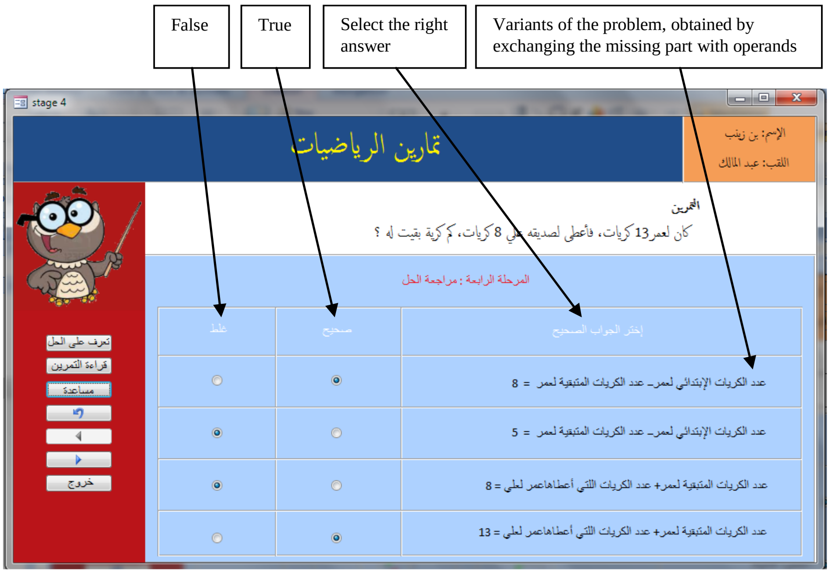

Figure 5. The screenshot of the fourth stage (Reviewing the solution)

5 Student Assessment

One of the most common strategies for studying the problem-solving process involves the use of an analytic scoring scale. Analytic scoring is an evaluation method that assigns point values to various dimensions of the problem-solving episode (Charles et al., 1987).

The grading rubric graded the solutions in 4 specific areas. A total of 10 maximum possible points were awarded to each problem by assigning 2 maximum possible points for understanding the problem, 3 maximum possible points for making a plan stage, 3 maximum possible points for procedural skills and finally 2 maximum possible points for reviewing the solution. Detail of stages’ scoring is given in table 2.

Furthermore, one of interesting research issue in Computer-Assisted Instruction Systems is Intelligent Agents. Some Intelligent Agents are used for helping students doing science experiments in Virtual Experiment Environment (Huang et al., 1999) (Kuo et al., 2001); the others are used to help student solving problems with a kind of knowledge structure and Knowledge Map (Kuo et al., 2002). In this paper, intelligent agents are designed to diagnose students' learning status in problem solving system.

Stage	***Understanding the problem***	***Making a plan***	***Executing the plan***	***Reviewing the solution***
Criteria	Complete understanding of the problem with recognition of what is given and what is requested in the problem with correct identification of important words in the problem.	Accuracy of setting the plan and degree of describing and interpreting the operands and their values, the operation and the missing value.	Accuracy of computation with correct manipulation of numbers (adding/subtracting of units, tens and hundreds) and right use of regrouping and carrying over concepts.	Correct validation of problem solution by selecting the correct problem solution variants. Variants are obtained by Combining addition and subtraction of problem operands and missing part.
Rubrics and scores	• What is known: 0.7 pt • What is requested: 0.7 pt • Identifying the important words: 0.6 pt	• First operand label identification: 0.5 • First operand value: 0.5 • Second operand label identification: 0.5 • Second operand value: 0.5 • Identification of the operator: 0.5 • Identification of label of missing: 0.5	• Manipulation of units: 1pt • Manipulation of tens: 1pt • Manipulation of hundreds: 1pt In case of manipulating numbers without hundreds, the rating score of this category is affected equally to units and tens manipulations.	• First problem solution variant: 0.5 pt. • Second problem solution variant: 0.5 pt. • Third problem solution variant: 0.5 pt. • Fourth problem solution variant: 0.5 pt.

Table 2. Assessment grading rubric

5.1 Stage assessor agents

At the end of each problem solving stage, and when the student presses the next stage button the stage assessor agent is started. The evaluations for stages: understanding the problem, making a plan and reviewing the solution is the same and consists of some operations to be designed in an Intelligent Agent: *1. Load stage solution of the problem*; *2. Evaluate student's answers*: comparing student's answers with stage solution; *3.generate feedback and calculate stage score*: feedback is generated according to mastered concepts and developed skills and score is awarded. All stage solving informations are stored in the student tracking database.

5.2 Procedural skills assessment agents

Assessment of plan execution stage is different from the other stages assessment and requires a multi agent assessment to diagnose student's addition and subtraction skills. For this purpose, four agents are needed: ones agent, tens agent, hundreds agent and assessment agent, each of the first three agents is responsible of evaluating one column operation and skills related to both addition and subtraction (if regrouping is required for subtraction or if addition needs carrying over). The assessment agent consults the problem solving information database to know the number of columns (units, tens and hundreds) used in the problem solution, afterward he creates the agents needed for evaluation. After evaluating his column, each agent sends a report to the assessment agent which assesses the whole student's procedural skills (see figure 6). The motivation for applying this assessment method is to understand all computing details used by the student to complete addition and subtraction operations.

5.2.1 The ones agent

The ones assessment agent checks the digits' computation of the ones column and verifies if techniques related to operator type are used.
If needed operation is addition, the agent carries out the following operations:

- Calculates the sum of digits of the ones column,
- Verifies if the result units digit is correct,
- Checks if the student has not miss the carry over number if the operation requires so, and sends a message to tens agent to inform him that carrying over is required after computing ones column.
- Generates a report according to results and sends it as evaluation grid to assessment agent.

If the problem solution requires a subtraction operation, the agent completes the following actions:

- Calculates the difference between digits of the ones column,
- Verifies if the result units digit is correct,
- Checks if the student has used the regrouping technique if the subtraction requires so, and sends a message to tens agent to inform him that regrouping is used to compute ones column.
- Generates the feedback according to results and sends it as evaluation grid (See tables 3 and 4) to assessment agent. The 3 maximum possible points that can be awarded in this stage are divided depending on whether the solution of the problem requires one, two or three columns.

5.2.2 The tens/hundreds agent

This agent performs the same operations as the previous agent and makes sure that student adds the carry over number to the sum of digits in tens (hundreds) column if needed operation is addition. In the subtraction case, the agent checks up if the student adds the regrouping number to the subtracted number of tens (hundreds) column. Finally, he sends details of evaluation to the assessment agent.

5.2.3 The assessment agent

After receiving reports from the three other agents, the assessment agent proceeds to the assessment of the student's procedural skills. The scores are awarded according to previous agents' evaluation grids. Results of assessment are displayed and stored in the student tracking database.

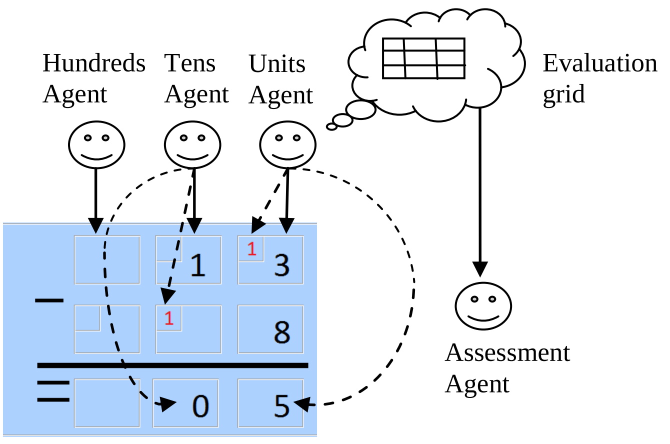

Figure 6. Agent oriented assessment of procedural skills.

Skill	*Needed in computation?*	*Performed?*	*Awarded score*
Alignment of digits on worksheet	Yes	Yes	0.1
Arrangement of numbers on subtraction worksheet	Yes	Yes	0.1
Subtracting numbers less than 10	No	No	
Subtracting numbers greater than 10	Yes	No	0
Management of regrouping technique	Yes	No	0

Table 3. Example of the ones agent's evaluation grid (case of subtraction).

Skill	*Needed in computation?*	*Performed?*	*Awarded score*
Alignment of digits on worksheet	Yes	Yes	0.1
Subtracting numbers less than 10	Yes	Yes	0.2
Subtracting numbers greater than 10	Yes	No	0
Increment by 1 the value of tens to subtract	Yes	No	0
Management of regrouping technique	No	No	

Table 4. Example of the tens agent's evaluation grid (case of subtraction).

6 Conclusion

Mathematics education in Algeria lacked practical and effective descriptive methods that could be readily used in the schools and students who experienced mathematical learning difficulties became unable to attend expanded mathematical curricula. Our study was designed to offer one possible solution to the problem. We developed a Computer-assisted problem-based learning system to help low-achieving elementary students improve their ability to solve basic word-based addition and subtractions questions, and enhance their willingness to continue the learning. Our system is based on Polya's four problem-solving steps; the emphasis of using this model was on dividing the problem solving procedure into stages so as to diagnose stages at which errors occurred when a student encounters difficulties. Furthermore, we focus on remedial computation of addition and subtraction operations by developing an agent based evaluation approach that can furnish a meaningful feedback to be effective in improving the performance of students with lower problem solving ability. Many researches proposed problem-solving assistance to help students with their problem solving, and one of the most suggested problem-solving assistances is visualization. As far as that goes this study continues the convention of using such assistance, it included a few major differences from previous studies. The first is the use of multiple-choice question in the *"Understanding the problem*" stage where student have to identify what is known and what is requested in the problem. The second is using of schema representations strategy in the "Making a plan" stage to enable students describe the solution steps in detail by ordering operands of the problem. The third is the graphic presentation of addition and subtraction worksheets so as to enable students to perform regrouping if subtraction is needed to solve the problem or carrying over if addition is requested. These improvements allowed us to objectively evaluate students' cognitive and procedural skills.
Future research is planned to empirically demonstrate the effectiveness of our system on elementary school mathematical problems that involve the operations of addition and subtraction. Another improvement concerns student modeling so as to better monitor and support his learning.

In summary, this study improves assistance used in previous computer-assisted systems to help low-achieving second-graders in mathematics by combining schema representation, graphical worksheets, and other assistance in the problem-solving procedure.

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