A Report on Sequential KR-Approaches as Cognitive Logic

Kai Sauerwald*, Eda Ismail-Tsaous and Nina Thorwart

FernUniversität in Hagen, Hagen, Germany

Abstract

We present an approach for employing KR methods sequentially and evaluating them according to their predictive power for human reasoning. The approach uses epistemic spaces and allows the injection of cognitive aspects into the approach. We report two instantiations of the general approach in which the epistemic states are ranking functions. The first is based on belief merging, and the second instantiation is based on belief revision. Both instantiations also use cognitively inspired formal approaches to construct meaningful internal representations. We also report the evaluation of these instantiations on an experimental dataset about human reasoning. The results suggest that KR approaches may benefit from augmentation with cognitively inspired processes.

Keywords

cognitive logic, epistemic space, prediction, sequential, human reasoning, merging, revision

1. Introduction

Knowledge representation and reasoning (KR) has a long tradition as a subfield of artificial intelligence that takes inspiration from human reasoning capabilities. In this context, KR approaches can be understood as analogues, metaphors or models of specific human mental tasks. An example is belief merging, the process where an agent combines multiple pieces of information with the same significance consistently. Another example is belief revision, the process of incorporating new information into the agent's belief set such that the result is consistent, whenever that is possible. When KR approaches, like merging or revision, are considered metaphors for cognitive processes, the question arises on how these formal approaches compare to human reasoners. However, because of their origins, KR approaches are often developed and evaluated from a formal or philosophical perspective. Experimental results that evaluate the extent to which KR approaches comply with human reasoning are less common.

In this paper, we report two recent evaluations of sequential versions of belief merging and belief revision that are enhanced by cognitive theories and show that they fit in a common framework. The starting point of this paper is the assumption that humans process information sequentially. We present a simplistic framework that models the comprehension of information as a process:

- First, given information is transformed to an internal representation A.
- Second, by using a KR method, the representation A is subsequently integrated into the agent's existing knowledge representation.

Clearly, the analogy to a human reasoner can be drawn easily: First, humans perceive information via sensors, which transform the information into a neural representation. Second, this representation is then passed to the brain, where it is processed and incorporated for future use. Recently, the performance of both sequentially used merging operators [1] and sequentially used belief revision operators [2] in predicting human reasoning has been evaluated. In this paper, we see that both approaches fit well to the framework we sketched above. Furthermore, we compare their results and discuss further implications on the general approach. The purpose of this line of work is to gain an understanding of how cognitive processes and evaluations can be employed to advance KR approaches.

Joint Workshop on Knowledge Diversity and Cognitive Aspects of KR (KoDis/CAKR) co-located with the 21st International Conference on Principles of Knowledge Representation and Reasoning (KR2024), November 2–8, 2024 Hanoi, Vietnam

*Corresponding author.
kai@kai-sauerwald.de (K. Sauerwald)
https://kai-sauerwald.de/ (K. Sauerwald)
0000-0002-1551-7016 (K. Sauerwald); 0009-0007-2810-8952 (E. Ismail-Tsaous)

In summary, our main contributions are:

- A formal framework that works analogously to how humans may conceive and process information, the *sequential approach.*
- We show that recent approaches fit into this framework.
- A discussion of recent empirical evaluations on the predictive power for human propositional reasoning.

In the next section, we start with providing the background.

2. Background on Logic, Epistemic Spaces and Lists

In this section we present basic notions and concepts from logic, ranking theory and order theory that are used in this article.

Classical Propositional Logic. Let Σ be a propositional signature, i. e., a non-empty finite set of propositional variables. With $\mathcal{L}$ we denote the propositional language over Σ, using the connectives $\wedge$ (and), $\vee$ (or), $\neg$ (negation) and the connectives $\rightarrow$ (implication), $\leftrightarrow$ (bi-implication). As semantics of these connectives we have the standard Boolean truth-functionally semantics and use its model-theoretic representation. The set of propositional interpretations, also called *worlds*, is denoted by Ω. With $\models$ we denote the model relation, i. e., $\omega \models \varphi$ indicates that ω is a model of φ. We let $\mathrm{Mod}(\varphi)$ be the set of models of φ. We overload the symbol $\models$ and write $a \models b$ for $a, b \in \mathcal{L}$, when $\mathrm{Mod}(a) \subseteq \mathrm{Mod}(b)$. These notions are extended to sets of formulas in the usual way, e. g., for $L \subseteq \mathcal{L}$ we define $\mathrm{Mod}(L) = \bigcap_{a \in F} \mathrm{Mod}(a)$, and $L \models b$ holds exactly when $\mathrm{Mod}(L) \subseteq \mathrm{Mod}(b)$. For $L \subseteq \mathcal{L}$ we define $Cn(L) = \{\beta \mid L \models \beta\}$ and $L + \alpha = \mathrm{Cn}(L \cup \{\alpha\})$. We say L is deductively closed if $L = \mathrm{Cn}(L)$ and $\mathcal{L}^{\mathrm{Bel}}$ is the set of all deductively closed sets.

Lists. We deal with (finite) lists of elements in this article. For a set X and $x_1, \ldots, x_n \in X$ we denote with $[x_1, \ldots, x_n]$ the list containing $x_1, \ldots, x_n$ where x_1 is the first element, x_2 the second element, etc. With $\mathbb{L}[X] = \{\ [x_1, \ldots, x_n] \mid n \in \mathbb{N}, x_1, ..., x_n \in X\}$ we denote the set of all finite lists over X.

Epistemic Spaces. In this work, we model agents by the means of logic. Deductively closed sets of formulas, which we denote from now as *belief sets*, represent deductive capabilities; agents are assumed to be perfect reasoners. The interpretations represent worlds that the agent is capable to imagine. The following notion describes the space of epistemic possibilities of an agent's mind in a general way.

Definition 2.1 ([3]; adapted)**.** *A tuple* $\mathbb{E} = \langle \mathcal{E}, \mathrm{Bel} \rangle$ *is called an* epistemic space *if* $\mathcal{E}$ *is a non-empty set and* $\mathrm{Bel} : \mathcal{E} \rightarrow \mathcal{L}^{\mathrm{Bel}}$.

We call the elements of $\mathcal{E}$ *epistemic states*, and for each $\Psi \in \mathcal{E}$, we use $\mathrm{Mod}(\Psi)$ as shorthand for $\mathrm{Mod}(\mathrm{Bel}(\Psi))$. Definition 2.1 differs from the definition given by Schwind et al. [3] insofar as we do *not* exclude inconsistent belief sets and forbid emptiness of $\mathcal{E}$. The rationale is that it was recently shown to be a necessary extension to fully capture AGM revision within epistemic spaces [4].

Ranking Functions. *Pre-ranking functions* are functions with type $\kappa : \Omega \rightarrow \mathbb{N}_0$. An ordinal conditional function (OCF), or short *ranking function*, is a pre-ranking function $\kappa : \Omega \rightarrow \mathbb{N}_0$ such that $\kappa(\omega) = 0$ for at least one $\omega \in \Omega$ [5]. Intuitively, ranking functions describe degrees of implausibility, i. e., if ω has a rank of 0, this means that ω is considered maximally plausible, whereas interpretations with larger ranks are considered more implausible. With $\mathbb{K}$ we denote the set of all ranking functions (over Ω). We let $\mathrm{Bel}_{\mathbb{K}} : \mathbb{K} \rightarrow \mathcal{L}^{\mathrm{Bel}},\ \mathrm{Bel}_{\mathbb{K}}(\kappa) = \{\varphi \in \mathcal{L} \mid \kappa^{-1}(0) \subseteq \mathrm{Mod}(\varphi)\}$ the function that assigns to each ranking function κ the set of formulas complying with the most plausible interpretations with respect to κ. With $\mathbb{E}_{\mathbb{K}} = \langle \mathbb{K}, \mathrm{Bel}_{\mathbb{K}} \rangle$ we denote the epistemic space, where the epistemic states are ranking functions from $\mathbb{K}$. Since we are only considering this epistemic space, we will write Bel instead of $\mathrm{Bel}_{\mathbb{K}}$ in the following.

3. Background on Cognitive Science

Classical logic has several limitations for describing how humans reason. One of the main observations is that human reasoning does not comply with the standard semantics of classical propositional logic [6]. We consider some basic observations and explanations from cognitive science.

Experiments show that almost all humans infer β without hesitation when $\alpha \rightarrow \beta$ and α are given [7, 8]. A robust finding is that some reasoners do not infer $\neg\alpha$ from $\alpha \rightarrow \beta$ and $\neg\beta$ [7, 9, 10]. Instead, many conclude that "nothing" follows [11]. Some subjects endorse that from $\alpha \rightarrow \beta$ and β follows α, respectively that from $\alpha \rightarrow \beta$ and $\neg\alpha$ follows $\neg\beta$ [8, 12, 13]. The following two principles are considered as explanation for these observations.

Principle of the Biconditional Interpretation of Conditionals. It is hypothesized that conditionals are sometimes interpreted as biconditionals. This complies with the fact that conditional statements are often used to express biconditional relations in everyday life [14].

Principle of Preferred Interpretations. The idea is that a conditional $\alpha \rightarrow \beta$ has three possible readings. The first reading is $\alpha \wedge \beta$ (*conjunctive interpretation*), the second reading is $\neg\alpha \wedge \neg\beta$ (*biconditional interpretation*) and the third reading is $\neg\alpha \wedge \beta$ (*conditional interpretation*). Because not all readings are equally obvious to human reasoners, it is hypothized that the more mental effort is invested, the more readings of $\alpha \rightarrow \beta$ are employed by the agent to draw conclusions [12]. Whereby the order of the reading as given above is respected.

In general, reasoning tasks involving sentences with negations are considered to require more effort [15]. For reasoning with disjunctions humans also show various patterns that diverge from classical propositional logic [16–18]. The principle of truth from mental model theory provides an explanation for these phenomena.

Principle of Truth. Mental model theory (MMT) assumes that humans build (multiple) mental models about an imagined object or situation. The *principle of truth* [19] states that humans prefer to build mental models that include only what is coherent and what is surely known ("true"), and omitting the properties and features that do not hold. Therefore, humans may consider certain possibilities, but not necessarily all possibilities, when reasoning and, thus, may arrive at results that deviate from classical logic, such as those mentioned in this section.

4. General Sequential Approach

Our model of how agents process information is based on the following assumptions:

(A1) Subjects process new information sequentially,

(A2) Classical (propositional) logic[1] $\mathcal{L}$ is an adequate basic theory of reasoning,

(A3) $\mathcal{L}$ is adequate to represent input of new information, and

(A4) Grasping of information might be imperfect or influenced by cognitive biases.

When an agent approaches a (mental) task, e. g., wants to make conclusions according to several pieces of information, it processes them one by one. In the sequential approach, the premises in a task are therefore modelled as a list $[\varphi_1, \ldots, \varphi_n]$. We assume that the integration of each piece of new information can be modelled by a two-step process. In the first step, conceiving a piece of information, which is represented by a formula φ_i, yields an internal representation φ_i^*. Formally, this is captured by the following:

Definition 4.1. *Let* $\mathbb{A}$ *be a set. An* $\mathbb{A}$-perception *is a function* $* : \mathcal{L} \rightarrow \mathbb{A}$ *that assigns to every propositional formula* $\varphi \in \mathcal{L}$ *an element* $\varphi^* \in \mathbb{A}$.

[1]For the cases we consider in this paper, propositional logic should be sufficient. Clearly, for other tasks one might need to consider more expressive classical logics, like first-order predicate logic.

$$
\begin{tikzcd}
\kappa_0 \arrow[r] & \kappa_1 = \kappa_0 \,\square\, \varphi_1^* = \kappa_0 \boxast [\varphi_1] & \cdots & \kappa_{n-1} \arrow[r] & \kappa_n = \kappa_{n-1} \,\square\, \varphi_n^* = \kappa_0 \boxast [\varphi_1, \ldots, \varphi_n] \\
\varphi_1^* \arrow[u] & \varphi_1 \arrow[l] & \cdots & \varphi_n^* \arrow[u] & \varphi_n \arrow[l]
\end{tikzcd}
$$

Figure 1: Illustration of the mechanics of a *sequential approach* when starting with a ranking function $\kappa_0 = \kappa$ and processing information $[\varphi_1, \ldots, \varphi_n]$.

The name "perception" is chosen here to highlight that we employ the function from Definition 4.1 to model the subjective processing of information, which might be influenced by cognitive biases (A4).

The second step consists of combining the current epistemic state Ψ and the representation φ_i^* to a new epistemic state.

Definition 4.2. *Let $\mathbb{A}$ be a set and let $\mathbb{E} = \langle \mathcal{E}, \mathrm{Bel} \rangle$ be an epistemic space. An* $\mathbb{E}$-$\mathbb{A}$ combinator *is a function $\square : \mathcal{E} \times \mathbb{A} \to \mathcal{E}$ that assigns to every epistemic state $\Psi \in \mathcal{E}$ and every $A \in \mathbb{A}$ an epistemic state $\Psi \,\square\, A \in \mathcal{E}$.*

Consequently, a sequential operator is parametrized by an $\mathbb{A}$-perception and an $\mathbb{E}$-$\mathbb{A}$ combinator. In summary, processing $[\varphi_1, \ldots, \varphi_n]$ is the sequential application of the procedure sketched above (see also Figure 1).

Definition 4.3. *Let $\mathbb{A}$ be a set and let $\mathbb{E} = \langle \mathcal{E}, \mathrm{Bel} \rangle$ be an epistemic space. Let $\square$ be an $\mathbb{E}$-$\mathbb{A}$ combinator, and let $*$ be an $\mathbb{A}$-perception. The* sequential operator $\boxast : \mathcal{E} \times \mathbb{L}[\mathcal{L}] \to \mathcal{E}$ based on $\langle \square, * \rangle$ *is defined by:*

$$
\begin{aligned}
\Psi \boxast [\varphi_1] &= \Psi \,\square\, \varphi_1^* \\
\Psi \boxast [\varphi_1, \ldots, \varphi_n] &= (\Psi \,\square\, \varphi_1^*) \boxast [\varphi_2, \ldots, \varphi_n]
\end{aligned}
$$

We make the following additional assumption when working with the framework in this paper:

(A5) Individuals' epistemic states can be represented by ranking functions from $\mathbb{K}$.

The rationale for this is that ranking functions are a well-established representation formalism, which is widely accepted in KR due to its properties. For instance, it has been shown that ranking functions are expressive enough for (iterated) belief revision [3, 20]. Hence, in the remaining parts of this paper we consider only the epistemic space $\mathbb{E}_{\mathbb{K}}$. In the next two sections, we present instantiations for $\langle \square, * \rangle$.

5. Sequential Merging and Cognitive Ranking Constructions

We will now consider the work by Ismail-Tsaous et al. [1] and show that their approach is an instance of the sequential approach from Section 4. Ismail-Tsaous et al. [1] evaluate the accuracy of operators that sequentially apply belief merging to ranking functions in predicting the answers given by human reasoners. A specific feature is the usage of functions that construct ranking functions from propositions based on cognitive theories. We start by considering the background of merging.

5.1. Merging Operators

Merging operators [21] aggregate multiple pieces of information from different sources such that every source has the same priority [22]. Merging operators (for ranking functions) are functions that map a list of ranking functions to a ranking function [23].

Definition 5.1 ([23])**.** *A* merging operator *(for ranking functions) is a function $\Delta : \mathbb{L}[\mathbb{K}] \to \mathbb{K}$.*

Ismail-Tsaous et al. [1] consider six merging operators. For reasons of space, we only list these operators here and refer the interested reader to their publication [1] or to the work by Meyer [23] for further details. The *(basic) minimum merging operator* $\Delta_{\min}$ assigns the smallest rank over all sources to an interpretation. The *(basic) maximum merging operator* $\Delta_{\max}$ assigns the highest rank over all sources to an interpretation. The *(basic) majority operator* Δ_{Σ} sums the ranks over all sources and normalizes the result. For these basic operators there also exist corresponding refined operators; the *refined minimum operator* Δ_{Rmin}, the *generalized maximum operator* Δ_{Gmax} and the *refined majority operator* $\Delta_{\mathrm{R}\Sigma}$, which follow a similar idea as the corresponding basic operators but take commensurability between sources into account by respecting the individual scales of each source. For illustrative purposes, consider the maximum merging operator $\Delta_{\max}$, which is given by

$$\Delta_{\max}(E)(\omega) = \max\{\kappa_1(\omega), \dots, \kappa_n(\omega)\} - \min\{\max\{\kappa_1(\omega'), \dots, \kappa_n(\omega')\} \mid \omega' \in \Omega\}$$

whereby $E = [\kappa_1, \dots, \kappa_n] \in \mathbb{L}[\mathbb{K}]$ is a list of ranking functions and $\omega \in \Omega$ an interpretation. The maximum operator represents a cautious approach, which favours the weakest belief among all sources.

5.2. Ranking Construction Functions to Model Cognitive Bias

For each input formula, Ismail-Tsaous et al. [1] construct ranking functions that can be delegated to a merging operator. Formally, they use an abstract notion that map formulas to ranking functions.

Definition 5.2. *A* ranking construction function *is a function* $\kappa^\bullet : \mathcal{L} \to \mathbb{K}$ *that assigns to every formula* φ *a ranking function* κ^φ *such that* $\varphi \in \mathrm{Bel}(\kappa^\varphi)$ *holds.*

The specific ranking construction functions take inspiration from the psychological observations and theories given in Section 3. We describe the different functions in the following.

Fully Explicit Models. The fully explicit model ranking function assigns models of φ the rank 0, and all other interpretations the rank[2] $|\Omega| - 1$:

$$\kappa^{\varphi}_{\text{FEM}}(\omega) = \begin{cases} 0 & \text{if } \omega \models \varphi \text{ or if } \varphi \text{ is inconsistent} \\ |\Omega|\text{-}1 & \text{otherwise} \end{cases}$$

This models the situation where no cognitive bias is applied and reasoning is close to classical logic.

Biconditional Interpretation. The second approach is based on the principle that states that people sometimes interpret conditionals as biconditionals [14]. If no conditional is given, then the ranking function is constructed as in the fully explicit models case.

$$\kappa^{\varphi}_{\text{BI-FEM}} = \begin{cases} \kappa^{\alpha \leftrightarrow \beta}_{\text{FEM}} & \text{if } \varphi = \alpha \to \beta \\ \kappa^{\varphi}_{\text{FEM}} & \text{otherwise} \end{cases}$$

Principle of Preferred Interpretations. The third construction approach is motivated by the principle of preferred interpretations for conditionals [12]. Recall that this principle hypothesizes that humans apply to conditionals specific readings, which was modelled as follows:

$$\kappa^{\varphi}_{\text{PI-FEM}} = \begin{cases} \kappa^{\varphi}_{\text{PI}} & \text{if } \varphi = \alpha \to \beta \\ \kappa^{\varphi}_{\text{FEM}} & \text{otherwise} \end{cases} \quad \text{with } \kappa^{\varphi}_{\text{PI}}(\omega) = \begin{cases} |\Omega|\text{-}1 & \text{if } \omega \not\models \varphi \text{ and } \varphi \text{ is consistent} \\ 2 & \text{if } \omega \models \varphi \text{ and } \omega \models \neg\alpha \wedge \beta \quad \text{(conditional)} \\ 1 & \text{if } \omega \models \varphi \text{ and } \omega \models \neg\alpha \wedge \neg\beta \quad \text{(biconditional)} \\ 0 & \text{otherwise} \quad \text{(conjunctive)} \end{cases}$$

Principle of Truth. The fourth function is inspired by the *principle of truth* from mental model theory. Recall that the principle of truth predicts that reasoners give preference to explicitly given information. Here, propositional interpretations are used as representations for mental models [1]. The principle of truth is implemented in such a way that certain models of an input formula φ are considered more plausible than other models of φ. For instance, for a disjunction $\varphi = \alpha \vee \beta$ the models of $\alpha \wedge \beta$ are

[2] $|\Omega|$ denotes the cardinality of Ω.

considered less plausible than the models of $\alpha \wedge \neg\beta$ or $\neg\alpha \wedge \beta$. This results in the following ranking construction function:

$$\kappa^{\varphi}_{\text{MM}}(\omega) = \begin{cases} |\Omega|\text{-}1 & \text{if } \omega \not\models \varphi \text{ and } \varphi \text{ is consistent} \\ 2 & \text{if } \omega \models \varphi, \omega \not\models \alpha \wedge \beta \text{ and } (\varphi = \alpha \rightarrow \beta \text{ or } \varphi = \alpha \leftrightarrow \beta) \quad \text{((bi)conditionals)} \\ 1 & \text{if } \omega \models \varphi, \omega \not\models \alpha \wedge \beta \text{ and } \varphi = \alpha \vee \beta \quad \text{(disjunctions)} \\ 1 & \text{if } \omega \models \varphi, \omega \not\models \neg\alpha \wedge \neg\beta \text{ and } \varphi = \neg(\alpha \wedge \beta) \quad \text{(negations)} \\ 0 & \text{otherwise} \quad \text{(plausible mental models)} \end{cases}$$

For the details we refer to Ismail-Tsaous et al. [1]. Because $\kappa^{\varphi}_{\text{MM}}$ works for all formulas, one blends $\kappa^{\varphi}_{\text{MM}}$ with constructions for the biconditional interpretation and the principle of preferred interpretation; leading to the following two variations:

$$\kappa^{\varphi}_{\text{BI-MM}} = \begin{cases} \kappa^{\alpha\leftrightarrow\beta}_{\text{MM}} & \text{if } \varphi = \alpha \rightarrow \beta \\ \kappa^{\varphi}_{\text{MM}} & \text{otherwise} \end{cases} \qquad \kappa^{\varphi}_{\text{PI-MM}} = \begin{cases} \kappa^{\varphi}_{\text{PI}} & \text{if } \varphi = \alpha \rightarrow \beta \\ \kappa^{\varphi}_{\text{MM}} & \text{otherwise} \end{cases}$$

5.3. Instantiation as Sequential Operator

Ismail-Tsaous et al. [1] combine a merging operator Δ and a ranking construction operator $\kappa^{\bullet}$ to a binary operator $\mathbin{\triangle\!\!\!\!\triangle} : \mathbb{K} \times \mathcal{L} \to \mathbb{K}$ defined by $\kappa' \mathbin{\triangle\!\!\!\!\triangle} \varphi = \Delta([\kappa', \kappa^{\varphi}])$. Analogous to Definition 4.3, this can be extended to lists of ranking functions. Moreover, it fits easily into the framework presented in Section 4. When $\mathbb{A} = \mathbb{K}$ is set as the set of all ranking functions, we can define $\kappa \square \kappa' = \Delta([\kappa, \kappa'])$ as the merge of the two ranking functions, which is hence a $\mathbb{K}$-$\mathbb{K}$ combinator in the sense of Definition 4.2. Ranking construction functions are $\mathbb{K}$-perceptions in the sense of Definition 4.1.

Since there are many merging operators and ranking construction operators, we yield multiple operators $\mathbin{\triangle\!\!\!\!\triangle}$; each is a combination of one of the merging operators $\Delta_{\min}$, $\Delta_{\max}\Delta_{\Sigma}$, Δ_{Rmin}, Δ_{Gmax} or $\Delta_{\text{R}\Sigma}$ and one of the ranking construction operators $\kappa^{\bullet}_{\text{FEM}}$, $\kappa^{\bullet}_{\text{BI-FEM}}$, $\kappa^{\bullet}_{\text{PI-FEM}}$, $\kappa^{\bullet}_{\text{MM}}$, $\kappa^{\bullet}_{\text{BI-MM}}$ or $\kappa^{\bullet}_{\text{PI-MM}}$. Hence, we have 36 sequential operators based on such a combination.

6. Sequential Revision and Cognitive Formula Alterations

We will now consider the work by Thorwart [2] and show that the approach by Thorwart is an instance of the sequential approach from Section 4. Thorwart [2] evaluates the accuracy in predicting the answers given by human reasoners. In her approach belief revision operators are applied sequentially on formulas constructed from formulas according to the principles described in Section 3.

6.1. AGM Revision

Revision operators incorporate new beliefs into an agent's belief set, consistently, whenever this is possible. We use an adaptation of the postulates for revision by Alchourrón, Gärdenfors and Makinson [24] (AGM) for epistemic states [25], which is inspired by the approach of Katsuno and Mendelzon [26].

Definition 6.1. *Let $\mathbb{E} = \langle \mathcal{E}, \text{Bel} \rangle$ be an epistemic space. An (AGM) belief revision operator $\mathbb{E}$ is a function $\circ : \mathcal{E} \times \mathcal{L} \to \mathcal{E}$ such that the following postulates are satisfied [25]:*

(R1) $\alpha \in \text{Bel}(\Psi \circ \alpha)$

(R2) $\text{Bel}(\Psi \circ \alpha) = \text{Bel}(\Psi) + \alpha$ *if* $\text{Bel}(\Psi) + \alpha$ *is consistent*

(R3) *If* α *is consistent, then* $\text{Bel}(\Psi \circ \alpha)$ *is consistent*

(R4) *If* $\alpha \equiv \beta$, *then* $\text{Bel}(\Psi \circ \alpha) = \text{Bel}(\Psi \circ \beta)$

(R5) $\text{Bel}(\Psi \circ (\alpha \wedge \beta)) \subseteq \text{Bel}(\Psi \circ \alpha) + \beta$

(R6) *If* $\text{Bel}(\Psi \circ \alpha) + \beta$ *is consistent, then* $\text{Bel}(\Psi \circ \alpha) + \beta \subseteq \text{Bel}(\Psi \circ (\alpha \wedge \beta))$

The postulates (R1)–(R6) are known for establishing minimal change on the prior beliefs when revising an epistemic state. In the remaining parts of this paper we sometimes write *revision operator* instead of *AGM revision operator*.

Thorwart considers multiple approaches to revision, especially to revision on epistemic states. The central idea of revising epistemic states is that not only $\mathrm{Bel}(\Psi)$ is considered, but also additional information is taken into account and how this extra information evolves. Usually, it is assumed that such additional information contains at least a plausibility order $\leq_\Psi$ over the interpretations, where lower positions mean higher plausibility. This coincides with how ranking functions are interpreted. Of course, after performing a revision of Ψ by φ, the follow-up epistemic state $\Psi \circ \varphi$ also has a plausibility ordering $\leq_{\Psi\circ\varphi}$. There are different revision operators, which make different claims about how $\leq_\Psi$ and $\leq_{\Psi\circ\varphi}$ relate with respect to φ. The operators considered by Thorwart [2] are *natural revision* $\circ^{\text{nat}}$ [27], *lexicographic revision* $\circ^{\text{lex}}$ [28], *restrained revision* $\circ^{\text{res}}$ [29], *Darwiche-Pearl revision* $\circ^{\text{DP}}$ [25] and *reinforcement revision* $\circ^{\text{ref}}$ [30]. Again, for reasons of space, we refer to the literature and in particular to Fermé and Hansson [31] for an introduction and overview.

6.2. Instantiation as Sequential Operator

In Thorwart's work [2], for each input formula another formula is constructed and then delegated to a revision operator. Clearly, such a processing of formulas can be understood as a $\mathcal{L}$-perception in the sense of Definition 4.1. In the following, we present the $\mathcal{L}$-perception functions Thorwart [2] considers.

Fully Explicit Models. First, we consider the situation where no cognitive bias is applied and reasoning works closely to classical logic. This is modelled by the identity function $*[\text{FEM}] : \mathcal{L} \to \mathcal{L}$ with $\varphi^{*[\text{FEM}]} = \varphi$.

Biconditional Interpretation. This is the principle that states that people sometimes interpret conditionals as biconditionals [14]. It is modelled by a function $*[\text{BI}] : \mathcal{L} \to \mathcal{L}$ with

$$\varphi^{*[\text{BI}]} = \begin{cases} \alpha \leftrightarrow \beta & \text{if } \varphi = \alpha \to \beta \\ \varphi & \text{otherwise} \end{cases} .$$

Principle of Preferred Interpretations. The third construction approach is motivated by the principle of preferred interpretations for conditionals [12]. Recall that this principle hypothesizes that humans apply to conditionals specific readings. Note that the most preferred reading of a conditional $a \to b$ is the conjunctive reading $a \wedge b$. Thorwart [2] models this by a function $*[\text{PI}] : \mathcal{L} \to \mathcal{L}$ with

$$\varphi^{*[\text{PI}]} = \begin{cases} \alpha \wedge \beta & \text{if } \varphi = \alpha \to \beta \\ \varphi & \text{otherwise} \end{cases} .$$

Principle of Truth. To incorporate mental model theory, Thorwart [2] considers multiple cases[3]. As an example, we select the interpretation of disjunctions by exclusive disjunctions. It has been observed that humans sometimes treat disjunctions as exclusive disjunction, which can also be explained by the principle of truth (cf. Section 3). Thorwart [2] models this by a function $*[\text{ED}] : \mathcal{L} \to \mathcal{L}$ with

$$\varphi^{*[\text{ED}]} = \begin{cases} (\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta) & \text{if } \varphi = \alpha \vee \beta \\ \varphi & \text{otherwise} \end{cases} .$$

6.3. Instantiation as Sequential Operator

In the approach by Thorwart [2] a revision operator $\circ$ and a $\mathcal{L}$-perception[4] $*$ are combined to a binary operator $\circledast : \mathbb{K} \times \mathcal{L} \to \mathbb{K}$ defined by $\kappa \circledast \psi = \kappa \circ \psi^*$. Clearly, this fits into the framework presented in

[3]These cases go beyond the framework considered here. We leave it open for another paper to consider the implementation of the other ideas suggested in that work.

[4]The notion used by Thorwart [2] translates to this notion here.

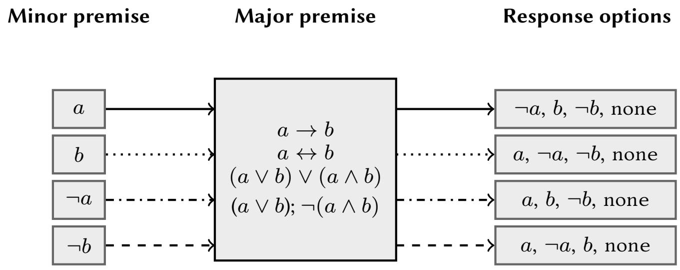

Figure 2: Premises and response choice combinations in the data set. The a and b in the figure stand for propositional atoms. Each task is a combination of a minor and a major premise and four response options. The presented response options are always the ones that do not contain the minor premise. The different arrows indicate which response options are associated with a specific minor premise. Each minor premise is combined once with one of four possible major premises, including either a statement with a conditional, a biconditional, an inclusive disjunction or an exclusive disjunction, yielding 16 tasks in total. In tasks with an exclusive disjunction the major premise consists of two statements, here separated by a semicolon [1].

Section 4. When $\mathbb{A} = \mathcal{L}$ is set as the set of all ranking functions, we can define $\kappa \,\square\, \varphi = \kappa \circledast \varphi$ as the revision of κ by φ, which is hence an $\mathbb{E}_{\mathbb{K}}$-$\mathcal{L}$ combinator in the sense of Definition 4.2.

The different revisions operators and $\mathcal{L}$-perceptions can be combined to multiple operators $\circledast$. The resulting operators are each a combination of one of the revision operators $\circ^{\text{nat}}$, $\circ^{\text{lex}}$, $\circ^{\text{red}}$, $\circ^{\text{DP}}$ or $\circ^{\text{ref}}$, and one of the $\mathcal{L}$-perceptions $*[\text{FEM}]$, $*[\text{BI}]$, $*[\text{PI}]$ or $*[\text{ED}]$. Hence, there are 20 different operators based on such a combination.

7. Experimental Dataset and Modelling

The works by Ismail-Tsaous et al. [1] and by Thorwart [2] both use data from an experiment on human reasoning by Ragni et al. (see [1] for details) to evaluate their sequential approaches. In the following, we outline the experimental design, the obtained data, and how they are connected with the sequential approach presented in this paper.

7.1. The Experiment

The experiment was designed as a survey with *single-choice tasks*. Subjects were presented multiple abstract propositional statements (i. e., premises) and response options in natural language and were asked afterwards to select exactly one answer from the response options that follows from all the given premises. Premises were always one minor premise, a singular literal, and a major premise, which was either one implication of the form $a \to b$, one bi-implication of the form $a \leftrightarrow b$, an inclusive disjunction in the form $(a \vee b) \vee (a \wedge b)$, or an exclusive disjunction represented by two statements, where the first was of the form $a \vee b$ and the second of the form $\neg(a \wedge b)$. In the next section, we refer also to task groups, where a group is defined by their major premise, so that there are *conditional* tasks, *biconditional* tasks, *inclusive disjunction* tasks and *exclusive disjunction* tasks. There were always four response options, three of the offered responses were statements and the fourth one the option “none”, denoting that none of the three statements follows from the premises. The tasks were designed in such a way that at most one of the possible answers was also an implication of the given premises in classical propositional logic. For instance, in a task with the two premises “There is a square.” and “If there is a circle, there is a square.”, the subjects were asked to choose from the following possible answers: “There is a circle.”, “There is no circle.”, “There is no square.” or “None of these answers follow from the premises.”. Figure 2 shows all task types. The experiment was conducted online via Amazon Mechanical Turk with participants who were not trained in logic. The cleaned data set $\mathbb{D}_{\text{Ex}}$ consists of 1097 records from 35 subjects and 16 unique tasks, each presented twice with different content[5].

[5]The encoded dataset is available here: https://e.feu.de/ecsqaru2023data

Operator Group		Number of Correct Predictions and Accuracy for Task Groups									
		All Tasks		Conditional		Biconditional		Incl. Disjunct.		Excl. Disjunct.	
		n = 1097		n = 275		n = 275		n = 274		n = 273	
		CP	Accuracy	CP	Accuracy	CP	Accuracy	CP	Accuracy	CP	Accuracy
Classical	Logic	844	76.9 %	160	58.2 %	252	91.6 %	187	68.2 %	245	89.7 %
Merging	PI	879	80.1 %	195	70.9 %	252	91.6 %	187	68.2 %	245	89.7 %
Merging	MM	859	78.3 %	175	63.6 %	252	91.6 %	187	68.2 %	245	89.7 %
Merging	FEM	844	76.9 %	160	58.2 %	252	91.6 %	187	68.2 %	245	89.7 %
Merging	BMmin	771	70.3 %	195	70.9 %	252	91.6 %	187	68.2 %	137	50.2 %
Merging	PFmin	737	67.2 %	160	58.2 %	145	52.7 %	187	68.2 %	245	89.7 %
Merging	MMmin	629	57.4 %	160	58.2 %	145	52.7 %	187	68.2 %	137	50.2 %
Revision	BI	879	80.1 %	195	70.9 %	252	91.6 %	187	68.2 %	245	89.7 %
Revision	FEM	844	76.9 %	160	58.2 %	252	91.6 %	187	68.2 %	245	89.7 %
Revision	PI	794	72.4 %	110	40.0 %	252	91.6 %	187	68.2 %	245	89.7 %
Revision	ED	755	68.8 %	160	58.2 %	252	91.6 %	98	35.8 %	245	89.7 %

Figure 3: General predictive performance. The number n in each column denotes the number of task records and "CP" stands for *Correct Predictions*.

7.2. Formalization and Predictions by Operators

The following definition captures the schematics of each task record formally.

Definition 7.1 (Task record). *A* task record *is a tuple* $R = \langle L, \{\varphi_1, \varphi_2, \varphi_3\}, r\rangle$ *where*

- $L \subseteq \mathbb{L}(\mathcal{L})$ *is the list of given premises,*
- $\varphi_1, \varphi_2, \varphi_3 \in \mathcal{L}$ *are the offered answers*[6], *where* $\varphi_1, \varphi_2, \varphi_3$ *are pairwise non-equivalent, and*
- $r \in \{\varphi_1, \varphi_2, \varphi_3, none\}$ *is the participant's response.*

We model the processing of a task record $R = \langle L, \{\varphi_1, \varphi_2, \varphi_3\}, r\rangle$ as sequential process, whereby $L = [\psi_1, \ldots, \psi_n]$. In each step i, the participant constructs an internal representation ψ_i^* for the presented premise ψ_i. The prior state represented by the ranking function κ_{i-1} and the newly perceived information ψ_i^* is combined to the new state $\kappa_i = \kappa_{i-1} \circledast \psi_i$. The given premises in L are processed sequentially. The final ranking function is $\kappa_n = \kappa_0 \circledast L$.

We say that our pipeline predicts the participant's choice, if r is believed in κ_n, i. e., $r \in \mathrm{Bel}(\kappa_\mathrm{n})$. To express our assumption that participants have no bias or prior information, we choose the uniform ranking function, i. e., $\kappa_\mathrm{uni}(\omega) = 0$ for all $\omega \in \Omega$, as the participants' initial epistemic state. The following definition captures the notion of a (correct) prediction formally.

Definition 7.2 (Predicts). *We say a* sequential operator $\circledast$ predicts a task record $R=\langle L,\{\varphi_1, \varphi_2, \varphi_3\},r\rangle$ *when the following holds:*

- *If* $r \in \{\varphi_1, \varphi_2, \varphi_3\}$, *then* $r \in \mathrm{Bel}(\kappa_0 \circledast L)$ *holds.*
- *If* $r = \mathrm{none}$, *then* $\varphi_1, \varphi_2, \varphi_3 \notin \mathrm{Bel}(\kappa_0 \circledast L)$ *holds.*

Next, we observe that, on $\mathbb{D}_\mathrm{Ex}$, many considered operators make the same predictions.

7.3. Equivalences of Sequential Operators

When considering a concrete data set $\mathbb{D}$, certain operators can exhibit the same prediction behaviour on this dataset. This is captured formally, by saying that such operators are equivalent with respect to $\mathbb{D}$.

Definition 7.3 (Equivalent with respect to $\mathbb{D}$, $\simeq_\mathbb{D}$). *Let* $\circledast$ *and* $\circledast'$ *be sequential operators and let* $\mathbb{D}$ *be a finite set of task records. We say* $\circledast$ is equivalent to $\circledast'$ with respect to $\mathbb{D}$, *written* $\circledast \simeq_\mathbb{D} \circledast'$, *if for all* $R \in \mathbb{D}$ *holds that* $\circledast$ *predicts* R *if and only if* $\circledast'$ *predicts* R.

[6]Because none is always offered, we do not mention this option explicitly.

In what remains of this section, we consider equivalence of sequential operators from Section 5 and Section 6 with respect to the dataset $\mathbb{D}_{\text{Ex}}$.

Sequential Merging Approach. For the sequential approach based on belief merging presented in Section 5, Ismail et al. [1] observe that the predictions of the operators are mainly influenced by the underlying ranking construction functions. We consider more formally, which $\mathbb{D}_{\text{Ex}}$ are present. Recall that in Section 5 sequential operators where based on $\langle \Delta, \kappa^\bullet \rangle$, where Δ is a merging operator and $\kappa^\bullet$ is a ranking construction operator.

Proposition 7.4 ([1]). *Each sequential operator based on $\langle \Delta, \kappa^\bullet \rangle$, where $\Delta \in \{\Delta_{\min}, \Delta_{\max}, \Delta_\Sigma, \Delta_{\text{Rmin}}, \Delta_{\text{Gmax}}, \Delta_{\text{R}\Sigma}\}$ and $\kappa^\bullet \in \{\kappa^\bullet_{\textit{FEM}}, \kappa^\bullet_{\textit{BI-FEM}}, \kappa^\bullet_{\textit{PI-FEM}}, \kappa^\bullet_{\textit{MM}}, \kappa^\bullet_{\textit{BI-MM}}, \kappa^\bullet_{\textit{PI-MM}}\}$ fall into the equivalence classes of $\simeq_{\mathbb{D}_{\text{Ex}}}$ as follows:*

- *[Merging FEM] Operators that are based on $\kappa^\bullet_{\textit{FEM}}$ are in the same class.*
- *[Merging PI] Operators that are based on $\kappa^\bullet_{\textit{PI-FEM}}$,$\kappa^\bullet_{\textit{PI-MM}}$,$\kappa^\bullet_{\textit{BI-FEM}}$, or $\kappa^\bullet_{\textit{BI-MM}}$, except for $\langle \Delta_{min}, \kappa^\bullet_{\textit{PI-FEM}} \rangle$, $\langle \Delta_{min}, \kappa^\bullet_{\textit{PI-MM}} \rangle$ and $\langle \Delta_{min}, \kappa^\bullet_{\textit{BI-MM}} \rangle$, are in the same class.*
- *[Merging MM] All operators that are based on the ranking construction operator $\kappa^\bullet_{\textit{MM}}$ are in the same class, except for the operator $\langle \Delta_{min}, \kappa^\bullet_{\textit{MM}} \rangle$.*
- *[Merging PFmin] $\langle \Delta_{min}, \kappa^\bullet_{\textit{PI-FEM}} \rangle$ is a singleton class.*
- *[Merging MMmin] $\langle \Delta_{min}, \kappa^\bullet_{\textit{MM}} \rangle$ and $\langle \Delta_{min}, \kappa^\bullet_{\textit{PI-MM}} \rangle$ form a class.*
- *[Merging BMmin] $\langle \Delta_{min}, \kappa^\bullet_{\textit{BI-MM}} \rangle$ is a singleton class.*

Sequential Revision Approach. For the sequential approach by revision presented in Section 6, Thorwart [2] observed that only the used perception function influences the prediction in $\mathbb{D}_{\text{Ex}}$. We state this more formally. Recall that in Section 5 sequential operators where based on $\langle \circ, * \rangle$, where $\circ$ is a revision operator and $*$ is a $\mathcal{L}$-perception.

Proposition 7.5 ([2]). *Each sequential operator based on $\langle \circ, * \rangle$, where $\circ \in \{\circ^{nat}\circ^{lex}, \circ^{red}, \circ^{DP}, \circ^{ref}\}$ and $* \in \{*[FEM], *[BI], *[PI], *[ED]\}$ fall into the equivalence classes of $\simeq_{\mathbb{D}_{\text{Ex}}}$ as follows:*

- *[Revision FEM] All operators based on $*[FEM]$ fall into the same class.*
- *[Revision BI] All operators based on $*[BI]$ fall into the same class.*
- *[Revision PI] All operators based on $*[PI]$ fall into the same class.*
- *[Revision ED] All operators based on $*[ED]$ fall into the same class.*

In the next section, we analyse and compare the accuracy of the predictions made by merging operators and revision operators.

8. Experimental Results

Ismail-Tsaous et al. [1] and Thorwart [2] evaluate the respective sequential approaches on the dataset $\mathbb{D}_{\text{Ex}}$ introduced in Section 7. Recall that, as discussed in Section 7, multiple operators exhibit the same prediction behaviour on the considered dataset $\mathbb{D}_{\text{Ex}}$. Because of that, we refer only to the groups of operators given in Proposition 7.4 and Proposition 7.5 instead of individual operators.

We consider the results of the two approaches on the aggregate level, i.e., the predictive performance in the mean over all participants. Figure 3 summarizes the results given by Ismail-Tsaous et al. [1] and by Thorwart [2]. First, observe that in both approaches the choice of used underlying belief revision operator, respectively belief merging operator, have only little influence on the predictive performance. Thus, operator(group)s that use fully explicit models (FEM) have the same performance as classical logic. Operator(group)s that apply cognitive biases to conditionals, i.e., treat conditionals as biconditionals or take inspiration from the principle of preferred interpration, perform best overall and in each task group. One exception is the group [Revision PI], where this is not the case. While the combination of mental model approaches with belief merging leads to results better than classical logic, all remaining operator groups perform worse than just using classical logic for the predictions.

9. Conclusions and Future Work

In this paper, we introduced a sequential approach to study the predictive power and cognitive adequacy of KR approaches for human propositional reasoning. We first briefly considered the background of principles from cognitive sciences. Then, we presented the general sequential approach and two instantiations of this approach by Ismail-Tsaous et al. [1] and by Thorwart [2]. We reported how both approaches employ cognitive principles to model the human perception and understanding of presented information. We summarized and compared the experimental results of both approaches and identified common observations. Our work demonstrates how understanding cognitive processes and empirical evaluations can be used as a tool for the improvement and evaluation of KR approaches, leading to cognitive logics [32].

For future work on the sequential approach, we will investigate refinements to improve the predictive power of the operators. One very striking point is that the experimental results exhibit that the KR methods used in both approaches, belief merging and belief revision, seem to have little influence on the resulting predictions. However, as Thorwart [2] explains, the considered data set is not rich enough so that any considered operators could make a difference, i.e., there were not "enough" steps in the process so that considered operators could show any effect. Hence, further experiments and evaluations will be necessary before coming to a final conclusion in this matter. However, we are convinced that KR-approaches can benefit from taking inspiration from theories and experimental results of cognitive psychology.

Acknowledgements. We thank the anonymous reviewers for their valuable hints and comments that helped us to improve this paper. The research reported here was partially supported by the Deutsche Forschungsgemeinschaft (DFG, grant 424710479, project "Predictive and Interactive Management of Potential Inconsistencies in Business Rules", MIB).

References

[1] E. Ismail-Tsaous, K. Sauerwald, M. Ragni, G. Kern-Isberner, C. Beierle, On the cognitive logic of human propositional reasoning: Merging ranking functions, in: Z. Bouraoui, S. Vesic (Eds.), Proceedings of the 17th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2023), volume 14294 of *Lecture Notes in Computer Science*, Springer, 2023, pp. 224–238. doi:10.1007/978-3-031-45608-4_18.

[2] N. Thorwart, Evaluating Revision Operators as Explanation for Human Reasoning over Two Variables, mathesis, FernUniversität in Hagen, 2024.

[3] N. Schwind, S. Konieczny, R. Pino Pérez, On the Representation of Darwiche and Pearl's Epistemic States for Iterated Belief Revision, in: G. Kern-Isberner, G. Lakemeyer, T. Meyer (Eds.), Proceedings of the 19th International Conference on Principles of Knowledge Representation and Reasoning (KR 2022), 2022, pp. 320–330. doi:10.24963/kr.2022/32.

[4] K. Sauerwald, Credibility-limited revision for epistemic spaces, in: G. Sanfilippo, V. Martinez, S. Destercke (Eds.), Proceedings of the 16th International Conference on Scalable Uncertainty Management (SUM 2024), Lecture Notes in Artificial Intelligence, Springer, 2024. To appear.

[5] W. Spohn, Ordinal conditional functions: a dynamic theory of epistemic states, in: W. Harper, B. Skyrms (Eds.), Causation in Decision, Belief Change, and Statistics, volume II, Kluwer Academic Publishers, 1988, pp. 105–134. doi:10.1007/978-94-009-2865-7_6.

[6] S. Khemlani, P. Johnson-Laird, Why machines don't (yet) reason like people, KI - Künstliche Intelligenz 33 (2019) 219–228. doi:10.1007/s13218-019-00599-w.

[7] K. Oberauer, Reasoning with conditionals: A test of formal models of four theories, Cognitive Psychology 53 (2006) 238–283. doi:10.1016/j.cogpsych.2006.04.001.

[8] M. Ragni, H. Dames, P. Johnson-Laird, A meta-analysis of conditional reasoning, in: Proceedings of the 17th International Conference on Cognitive Modeling (ICCM 2019), Applied Cognitive Science Lab, Penn State, 2019, pp. 151–156.

[9] P. Wason, Reasoning about a rule, The Quarterly journal of experimental psychology 20 (1968) 273–281. doi:10.1080/14640746808400161.

[10] M. Ragni, I. Kola, P. Johnson-Laird, On selecting evidence to test hypotheses: A theory of selection tasks, Psychological Bulletin 144 (2018) 779–796. doi:10.1037/bul0000146.

[11] R. Byrne, P. Johnson-Laird, 'If' and the problems of conditional reasoning, Trends in Cognitive Sciences 13 (2009) 282–287. doi:10.1016/j.tics.2009.04.003.

[12] P. Barrouillet, N. Grosset, J. Lecas, Conditional reasoning by mental models: Chronometric and developmental evidence, Cognition 75 (2000) 237–266. doi:10.1016/s0010-0277(00)00066-4.

[13] M. Oaksford, N. Chater, Bayesian rationality, Oxford University Press, 2007. doi:10.1093/acprof:oso/9780198524496.001.0001.

[14] N. Verschueren, W. Schroyens, W. Schaeken, G. D'Ydewalle, Why do participants draw non-valid inferences in conditional reasoning?, Current psychology letters 6 (2001). doi:10.4000/cpl.199.

[15] S. Khemlani, I. Orenes, P. Johnson-Laird, The negations of conjunctions, conditionals, and disjunctions, Acta Psychologica 151 (2014) 1–7. doi:10.1016/j.actpsy.2014.05.004.

[16] P. Johnson-Laird, M. Lotstein, R. Byrne, The consistency of disjunctive assertions, Memory & Cognition 40 (2012) 769–778. doi:10.3758/s13421-012-0188-2.

[17] A. Quelhas, P. Johnson-Laird, The modulation of disjunctive assertions, The Quarterly Journal of Experimental Psychology 70 (2017) 703–717. doi:10.1080/17470218.2016.1154079.

[18] A. Quelhas, C. Rasga, P. Johnson-Laird, The analytic truth and falsity of disjunctions, Cognitive Science 43 (2019). doi:10.1111/cogs.12739.

[19] P. Johnson-Laird, R. Byrne, Conditionals: A theory of meaning, pragmatics, and inference, Psychological Review 109 (2002) 646–678. doi:10.1037/0033-295x.109.4.646.

[20] K. Sauerwald, C. Beierle, Iterated Belief Change, Computationally, in: G. Kern-Isberner, G. Lakemeyer, T. Meyer (Eds.), Proceedings of the 19th International Conference on Principles of Knowledge Representation and Reasoning (KR 2022), 2022, pp. 410–414. doi:10.24963/kr.2022/42.

[21] S. Konieczny, R. Pino Pérez, On the logic of merging, in: A. G. Cohn, L. K. Schubert, S. C. Shapiro (Eds.), Proceedings of the 6th International Conference on Principles of Knowledge Representation and Reasoning (KR 1998), Morgan Kaufmann, 1998, pp. 488–498.

[22] S. Chopra, A. Ghose, T. Meyer, Non-prioritized ranked belief change, Journal of Philosophical Logic 32 (2003) 417–443. doi:10.1023/A:1024888511283.

[23] T. Meyer, Merging epistemic states, in: R. Mizoguchi, J. Slaney (Eds.), Proceedings of the 6th Pacific Rim International Conference on Artificial Intelligence (PRICAI 2000), volume 1886 of *Lecture Notes in Computer Science*, 2000, p. 286–296. doi:10.1007/3-540-44533-1_31.

[24] C. E. Alchourrón, P. Gärdenfors, D. Makinson, On the logic of theory change: Partial meet contraction and revision functions, J. Symb. Log. 50 (1985) 510–530. doi:10.2307/2274239.

[25] A. Darwiche, J. Pearl, On the logic of iterated belief revision, Artificial Intelligence 89 (1997) 1–29. doi:10.1016/S0004-3702(96)00038-0.

[26] H. Katsuno, A. O. Mendelzon, Propositional knowledge base revision and minimal change, Artificial Intelligence 52 (1992) 263–294. doi:10.1016/0004-3702(91)90069-V.

[27] C. Boutilier, Iterated revision and minimal change of conditional beliefs, Journal of Philosophical Logic 25 (1996) 263–305. doi:10.1007/BF00248151.

[28] A. C. Nayak, Iterated belief change based on epistemic entrenchment, Erkenntnis 41 (1994) 353–390. doi:10.1007/BF01130759.

[29] R. Booth, T. A. Meyer, Admissible and restrained revision, J. Artif. Intell. Res. 26 (2006) 127–151. doi:10.1613/jair.1874.

[30] Y. Jin, M. Thielscher, Iterated belief revision, revised, Artificial Intelligence 171 (2007) 1–18. doi:10.1016/j.artint.2006.11.002.

[31] E. L. Fermé, S. O. Hansson, Belief Change - Introduction and Overview, Springer Briefs in Intelligent Systems, Springer, 2018. doi:10.1007/978-3-319-60535-7.

[32] M. Ragni, G. Kern-Isberner, C. Beierle, K. Sauerwald, Cognitive logics - features, formalisms, and challenges, in: G. D. Giacomo, A. Catalá, B. Dilkina, M. Milano, S. Barro, A. Bugarín, J. Lang (Eds.), Proceedings of the 24nd European Conference on Artificial Intelligence (ECAI 2020), volume 325 of *Frontiers in Artificial Intelligence and Applications*, IOS Press, 2020, pp. 2931–2932. doi:10.3233/FAIA200459.