A Gradual Semantics with Imprecise Probabilities for Support Argumentation Frameworks

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Abstract
Support Argumentation Frameworks (SAFs) are a type of the Abstract Argumentation Framework, where the interactions between arguments have a positive nature. A quantitative way of evaluating the arguments in a SAF is by applying a gradual semantics, which assigns a numerical value to each argument with the aim of ranking or evaluate them. In the literature, studied gradual semantics determine precise probability values; however, in many applications there is the necessity of imprecise evaluations which consider a range of values for assessing an argument. Thus, the first contribution of this article is an imprecise gradual semantics (IGS) based on credal networks theory. The second contribution is a set of properties for evaluating IGSs, which extend some properties proposed for precise gradual semantics. Besides, we suggest a classification of semantics considering the set of properties and evaluate our proposed IGS according to the extended properties. Finally, the practical application of the results is discussed by using an example from Network Science, i.e, PageRank. We also discuss how gradual semantics benefit PageRank research by allowing to generate contrastive explanations about the scores in a more natural way.

Keywords
Support argumentation framework, Formal argumentation, Gradual semantics, Impreciseness, PageRank

1. Introduction

An abstract argumentation framework (AAF) is generally defined as a set of arguments and a binary relation encoding disagreements – called attacks – between arguments. Other studies on argumentation (e.g. [1][2]) have demonstrated the necessity of encoding positive interactions between arguments, which are called supports. Generally, support interactions have been studied along with attack interactions in what is known as Bipolar Argumentation Frameworks (BAFs) [1]. The approach that considers only the support interaction is known as Support Argumentation Framework (SAF). SAFs have interesting applications like trust evaluation, which studies trust relationships and measures different confidence properties of entities in a society or network [3]. Another possible application is for constructing persuasive essays, which aim to make the reader to agree with an opinion that is supported with arguments, examples, or expert opinions. In [2], the authors claim that the interpretation of the support relation may be diverse. They consider three possible specializations: the deductive, the necessary, and the evidential support. Another specialization that can be studied is the causal one, which is intended to capture the following intuition: *if an argument A supports another argument B in a causal way, this means that the acceptance of A causes the acceptance of B or B is accepted as an effect of accepting A.*

An important notion in formal argumentation is the acceptability of arguments. This is assessed by applying argumentation semantics over the AAF. Many semantics have been proposed (e.g. see [4][5][6]). These semantics return sets of consistent arguments, that is, arguments that can be accepted together (these sets are known as extensions). Other family of semantics are based on numerical evaluations or rankings, these are called gradual semantics (e.g. see [7][8][9]), and they aim to assign a numerical value to the arguments in order to rank them according to their acceptability. In these gradual semantics, each argument is assigned an initial value – known as base score – and after evaluating its dialectical relations (attacks and/or supports) with other arguments, another value is obtained, this is called the strength of the argument. Numerous works about gradual evaluation methods have been proposed (see [10][11][12][9]). However, all these methods consider that both the base score and the strength are precise values, which depending on the problem modelled by a SAF could be insufficient to represent the epistemic value of the arguments.

For a better illustration of the problem, let us present the following scenario. It is based on the PageRank (PR) citation ranking [13], which is an algorithm designed to measure the ranking of web pages in Google's search

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engine. In PageRank, the web can be seen as a directed graph, where nodes represent pages and edges represent the links between pages and the method measures each node's influence. PageRank counts the number and quality of links to a page to determine a rough estimate of how important that page is. The directed graph that represents the web can be seen as a SAF and a gradual semantics as a method for calculating the PageRank. Besides, the influence between nodes can be seen as a sort of causal relation where the positive quality of a page A causes the positive quality of a page B.

The PR is a precise value fitted between 0 and 10 that is calculated based on more than one criterion such as the number and the quality of links to a page, the update frequency or the internal coherence of the page, or even on design issues. In [14], the authors suggest that the value of PR is biased measuring external characteristics and also subjective value indicators and propose to use social metrics extracted from Semantic Web resources for adjusting the link–based metrics used by PageRank algorithm. Such social metrics are represented by imprecise values that are combined with the other precise metrics to return the final PR value, which is a precise value. Even though, in [14] only the social metrics are imprecise, any other used criteria can also be represented by imprecise values and aggregating all of them in a precise value provokes loss of information. Besides, some pages ranked with the same PR do not have the same PR value. For example, a page with PR 4 might have five times more PR than another page with PR 4, but the Google score do not tell that until the log base threshold has crossed the next value marker. Thus, there is no granularity between values. This problem can be smoothed if the result is an imprecise value from which a ranking can be constructed. By using this ranking, we can know which pages are more important than others even when they are assigned with the same PR. Therefore, this ranking can also be employed for obtaining explanations about the reasons for a page be a in given position, which is an important

To the best of our knowledge, there is no a gradual semantics evaluation method that returns imprecise values for ranking arguments in SAFs. Hence, we have our first research questions: (i) *How to model a SAF in the settings of imprecise probability?*, (ii) *how to calculate the imprecise values of arguments and how to compare them in order to generate a ranking?*, and (iii) *how to generate explanations from this ranking?*

In addressing the former question, we use credal sets [15] to model the uncertainty values of arguments and credal networks theory [16] for modelling the relation between arguments. Regarding the second one, we base on credal networks theory for calculating the imprecise value, which is modeled as an interval with a lower and an upper bound. With these calculated values, three criteria are used for comparing the intervals. These are the location, the precision of the interval, and the combination of both. Thus, our approach allows a three dimensional comparison, which generates a ranking that can be interpreted depending on the application and also can be used for explaining the reasons an argument is stronger than another. In this sense, we propose the generation of explanations for contrastive questions. Bouwel and Weber [17] distinguish three types of contrastive questions: (i) *P-contrast:* why does object o have property P, rather than property P'?, (ii) *O-contrast:* why does object o have property P, while object o' has property P'?, and (iii) *T-contrast:* why does object o have property P at time t, but property P' at time t'?. In this work, we model the first two types and consider that arguments can be seen as objects and positions in the ranking as properties[1].

In order to evaluate gradual semantics, some properties have been defined and studied (see [18] for a survey). However, none of these properties can be used to evaluate IGSs. Thus, the next research questions addressed in this article are: (iv) *How the properties defined for precise evaluation methods can be extended for imprecise gradual semantics?* and (v) *Do the proposed IGS fulfil the suggested properties? which of them?*.

The remainder of this paper is structured as follows. Next section gives a brief overview on credal networks and SAFs. In Section 3, we introduce an imprecise gradual semantics based on credal networks theory. In Section 4, we present how contrastive explanations are constructed. We study the properties of imprecise gradual semantics in Section 5 and present a classification of semantics in Section 6. A theoretical evaluation of the proposed semantics is presented in Section 7. A discussion about the proposal is presented in Section 8. Finally, Section 9 is devoted to conclusions and future work.

2. Background

In this section, we revise concepts of credal networks and SAFs.

2.1. Credal Networks

Before presenting credal networks, let us define credal sets (from Levi's credal sets [15]). Let $\mathbb{X} = \{X_1, ..., X_n\}$ be a set of probabilistic variables, a credal set defined by probability distributions $p(X)$ is denoted by $K(X)$ and $\mathbb{K} = \{K(X_1), ..., K(X_n)\}$ denotes a finite set of credal sets of the variables of $\mathbb{X}$. In this work, we assume that the cardinality of the credal sets of $\mathbb{K}$ is the same (let us denote it by m) and is determined by the number of agents. We also assume that $p_i(X)$ denotes the suggested

[1] Here, we use the position in the ranking; however, any other property can be used to require and generate an explanation.

probability of the agent i w.r.t. variable X such that $1 \leq i \leq m$ and $X \in \mathbb{X}$.

A credal network is a graphical model that associates nodes and variables with sets of probability measures [19]. A credal network consists of a directed acyclic graph, where each node in the graph is associated with a random variable X and the parents (i.e., the variables corresponding to the immediate predecessors of X according to the graph) of X are denoted by $pa(X)$. Each variable X is associated with a (conditional) credal set $K(X \mid pa(X)) = \{p_1(X \mid pa(X)), ..., p_m(X \mid pa(X))\}$. Inference is performed by applying Bayes rule to each measure in a joint credal set. The goal is to combine these credal sets into a set of joint distributions. Next, let us show how this combination will be done in order to obtain the lower and upper bounds from the credal sets of a credal network.

Given a random variable X and its credal set $K(X)$, the lower and upper bounds for variable X are determined as follows:

$$\underline{P}(X) = inf\{p(X) \mid p(X) \in K(X)\} \qquad (1)$$
$$\overline{P}(X) = sup\{p(X) \mid p(X) \in K(X)\}$$

2.2. Support Argumentation Framework (SAF)

In a SAF, arguments are abstract entities that have a base score expressed by a numerical value which is generally in the interval $[0, 1]$. The value 0 means that the argument is worthless whereas 1 means that the argument is very strong. Thus, the base score on a set of arguments $\mathtt{ARG}$ is a function $\tau : \mathtt{ARG} \longrightarrow [0, 1]$.

Definition 1. ***(SAF) [20]*** *A SAF is an ordered tuple* $\mathcal{S} = \langle \mathtt{ARG}, \mathcal{R}^+, \tau \rangle$*, where* $\mathtt{ARG}$ *is a non empty finite set of arguments,* τ *is a base score function on* $\mathtt{ARG}$ *and* $\mathcal{R}^+ \subseteq \mathtt{ARG} \times \mathtt{ARG}$ *is a support relation. For* $A, B \in \mathtt{ARG}$*, the notation* $(A, B) \in \mathcal{R}^+$ *means that* A *supports* B*.*

Regarding gradual semantics, it is a function that assigns to each argument in a SAF a value between 0 and 1. Thus, for all $A \in \mathtt{ARG}$, $\sigma(A)$ denotes the image of argument A and it is called the strength degree of A.

Definition 2. ***(Gradual semantics)*** *Let* $\mathcal{S} = \langle \mathtt{ARG}, \mathcal{R}^+, \tau \rangle$ *be a SAF. A gradual semantics is a function* $\sigma(A) : \mathtt{ARG} \longrightarrow [0, 1]$*.*

3. Imprecise Gradual Semantics

In this section, we introduce an imprecise gradual semantics based on credal networks theory. We present a credal SAF, in which, we use credal sets to model the degrees of belief about arguments. This means that each argument in a SAF has associated a credal set, which contains a set of probability distributions. Such distributions can represent different concepts. For example, in the scenario of the page rank, the probabilities in a credal set represent the values of the criteria used to calculate the PageRank.

3.1. Credal SAF

Before presenting the concept of credal SAF, we present the imprecise strength definition. Like in precise SAFs, in the imprecise context, an imprecise gradual semantics is in charge of calculating the strength of each argument in the SAF from their support relations. Thus, for any $A \in \mathtt{ARG}$, the imprecise strength of A is given by the function $\sigma_I(A)$, where $\sigma_I : \mathtt{ARG} \rightarrow [0, 1] \times [0, 1]$. The first number of the interval represents the lower bound and the second the upper bound. It also holds that the lower bound is less or equal than the upper bound.

Let us recall that the support relation in our approach can be interpreted as a causality relation that exists between arguments. Thus, an argument in a causality relation can play two different roles, it can either be caused or be the cause, this means that we can have caused arguments (this set is denoted by $\mathtt{ARG}_{\leftarrow}$), arguments that cause other ones (this set is denoted by $\mathtt{ARG}_{\rightarrow}$), and arguments that have no causality relation with the rest (this set is denoted by $\mathtt{ARG}_{\circ}$). We characterize these sets as follows. Given a set of arguments $\mathtt{ARG}$ and a support relation $\mathcal{R}^+$:

a) $\mathtt{ARG} = \mathtt{ARG}_{\leftarrow} \cup \mathtt{ARG}_{\rightarrow} \cup \mathtt{ARG}_{\circ}$;
b) $\mathtt{ARG}_{\leftarrow} = \{B | (A, B) \in \mathcal{R}^+\}$, $\mathtt{ARG}_{\rightarrow} = \{A | (A, B) \in \mathcal{R}^+\}$, and $\mathtt{ARG}_{\circ} = \{C | C \in \mathtt{ARG} - (\mathtt{ARG}_{\leftarrow} \cup \mathtt{ARG}_{\rightarrow})\}$;
c) $\mathtt{ARG}_{\leftarrow}$ and $\mathtt{ARG}_{\rightarrow}$ are not necessarily pairwise disjoint; however, $(\mathtt{ARG}_{\leftarrow} \cup \mathtt{ARG}_{\rightarrow}) \cap \mathtt{ARG}_{\circ} = \emptyset$;

We can now define a credal SAF, where arguments are assigned with credal sets, from which the imprecise base score of each argument can be obtained.

Definition 3. ***(Credal SAF)*** *An imprecise SAF based on credal sets is a tuple* $\mathcal{S}_{CS} = \langle \mathtt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle$ *where (i)* $\mathtt{ARG} = \mathtt{ARG}_{\leftarrow} \cup \mathtt{ARG}_{\rightarrow} \cup \mathtt{ARG}_{\circ}$*; (ii)* $\mathcal{R}^+ \subseteq \mathtt{ARG} \times \mathtt{ARG}$ *is the support relation between arguments; (iii)* $f_K : \mathtt{ARG} \rightarrow \mathbb{K}^U$ *is a function that attributes a credal set to each argument, where* $\mathbb{K}^U$ *is the set of all possible credal sets; and (iv)* $\tau_I : \mathtt{ARG} \rightarrow [0, 1] \times [0, 1]$ *is a function for any* $A \in \mathtt{ARG}$ *that is called imprecise base score of* A*. This is obtained by applying Equation (1) to* $f_K(A)$*.*

Example 1. *Let* $\mathcal{S}_{CS}^{PR}$ *be the credal SAF for the scenario of the PageRank:* $\mathcal{S}_{CS}^{PR} = \langle \mathtt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle$ *, where* $\mathtt{ARG} = \{A, B, C, D, E, F\}$*,* $\mathcal{R}^+ = \{(A, B), (B, C), (C, B), (C, E), (A, E), (D, C), (C, F), (F, D), (F, E)\}$*, and both* f_K *and* τ_I *are shown in Figure 1 next to each argument.* f_K *is represented by a vector and* τ_I *by an* *interval. Figure 1 shows the graph of the credal SAF $\mathcal{S}_{CS}^{PR}$. The nodes represent the pages and the edges the support relation. The probability values of the credal sets correspond to (i) the quantity and quality of the supporting links, (ii) the update frequency, (iii) the internal coherence of the page, and (iv) the design issues. The imprecise base score are obtained from these credal sets by applying Equation (1).*

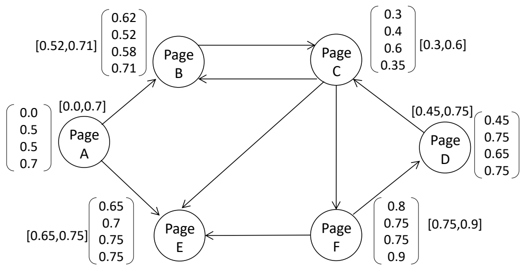

Figure 1: Credal SAF $\mathcal{S}_{CS}^{PR}$ for the PagerRank scenario. Intervals next to each credal set represent the imprecise base score of each argument.

Let us now better explain the correspondence between arguments and credal sets. Arguments in $\texttt{ARG}_\circ$ have associated only one credal set because they are not caused by any other argument. Arguments in $\texttt{ARG}_\leftarrow$ have associated an initial credal set and one conditional credal set that has to be calculated based on the supporters. Finally, some arguments in $\texttt{ARG}_\rightarrow$ have associated only one credal set and the others have also a conditional credal set. This happens because some causing arguments are also caused ones. Formally:

1. $\forall A \in \texttt{ARG}_\circ$, there is a credal set $K(A)$, that is, a non conditional credal set;
2. $\forall A \in \texttt{ARG}_\leftarrow$, there is a credal set $K(A)$ and a conditional credal set $K(A \mid pa(A))$;
3. $\forall A \in \texttt{ARG}_\rightarrow - (\texttt{ARG}_\rightarrow \cap \texttt{ARG}_\leftarrow)$, there is a credal set $K(A)$.

Example 2. *(Cont. Example 1) In the credal SAF $\mathcal{S}_{CS}^{PR}$, we have that $\texttt{ARG}_\circ = \emptyset$, $\texttt{ARG}_\leftarrow = \{B, C, D, E, F\}$, and $\texttt{ARG}_\rightarrow = \{A, B, C, D, F\}$. Every argument x has a credal set $K(x)$ (for $x \in \{A, B, C, D, E, F\}$) that can be used to obtain its imprecise base score. Those arguments that besides have a conditional credal set are B, C, D, E, and F. Thus, $K(B \mid A, C)$ is the conditional credal set of B, $K(C \mid B, D)$ is for the conditional credal set of C, $K(D \mid F)$ is the conditional credal set of D, $K(E \mid A, C, F)$ is the conditional credal set of E, and $K(F \mid C)$ is the conditional credal set of F.*

3.2. Calculating the Imprecise Strength

This section shows how the imprecise strength of arguments is calculated. Let us recall that such strength is represented by an interval. In the same manner as the imprecise base score, the interval is the result of assessing the credal set associated to each argument, which is calculated considering the support relations. In the case of causality, these relations are expressed as conditional ones between arguments.

Definition 4. ***(Calculation of the imprecise strength)*** *Let $\mathcal{S}_{CS} = \langle \texttt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle$ be a credal SAF and $A \in \texttt{ARG}$ an argument, the imprecise strength of A, that is $\sigma_I(A) = [\underline{P}(A), \overline{P}(A)]$, is obtained as follows:*

- If $pa(A) = \emptyset$, then $\overline{P}(A)$ and $\underline{P}(A)$ are obtained by applying Equation (1) to $f_K(A)$

- If $pa(A) \neq \emptyset$, then

1. Obtain the conditional credal set for A (that is, $K(A \mid pa(A))$) by applying the Bayes rule in the following way:

$$P(A \mid pa(A)) = \frac{P(A) \times P(pa(A) \mid A)}{P(pa(A))}$$

2. Calculate $\overline{P}(A)$ and $\underline{P}(A)$ by applying Equation (1) to the resultant conditional credal set $K(A \mid pa(A))$.

Once we have the interval that represents the imprecise strength, we need a way to compare such intervals. For evaluating the ordering of the intervals we will base on the approach of [21], which considers the precision of the intervals (denoted by PREC), the location of the intervals (denoted by LOCA), or the combination of both (denoted by COMB). Thus, given an argument A whose associated interval is $I = [\underline{P}(A), \overline{P}(A)]$, the evaluating criteria are calculated as follows: $\texttt{PREC}(I) = 1 - (\overline{P}(A) - \underline{P}(A))$, $\texttt{LOCA}(I) = \frac{\overline{P}(A) + \underline{P}(A)}{2}$, and $\texttt{COMB}(I) = \texttt{PREC}(A) \times \texttt{LOCA}(A)$.

When we compare two intervals, we can use their precision, their location, or the combination of both. Even though these criteria are represented by a precise value; actually, an argument is stronger if its precision is high or its location is close to 1. This allows to compare arguments in more than one way and gives flexibility to the approach, which is important depending on the context or domain of the application. For example, in the PageRank scenario, let us suppose that $\sigma_I(A') = [0.3, 0.35]$ and $\sigma_I(B') = [0.6, 1.0]$. Thus, we have $\texttt{PREC}(\sigma_I(A')) = 0.95$, $\texttt{PREC}(\sigma_I(B')) = 0.6$, $\texttt{LOCA}(\sigma_I(A')) = 0.325$, $\texttt{LOCA}(\sigma_I(B')) = 0.8$, $\texttt{COMB}(\sigma_I(A')) = 0.31$, and $\texttt{COMB}(\sigma_I(B')) = 0.48$. We can notice that A' has better precision than B' whereas B' has better location than A'. If we only consider the precision criteria, we can say that A' has a better PR score than B' and if we consider location or the combined measure, we can say that B' has a better PR score than A'. The question is: *which is a better criteria to be used in this context?* even with a better location, the range of values of B' makes difficult to classify it in a given PR. This means that a good location with low imprecision shows a high uncertainty degree, which in this context is not desirable. Thus, although the location of A' is not good, its high precision helps to better determine its PR.

Now, imagine another scenario where the imprecise strength of an argument C' is the interval $[0.4, 0.8]$ and the imprecise strength of another argument D' is $[0.5, 0.7]$. We have that $\texttt{PREC}(\sigma_I(C')) = 0.6$, $\texttt{PREC}(\sigma_I(D')) = 0.8$, $\texttt{LOCA}(\sigma_I(C')) = 0.5$, $\texttt{LOCA}(\sigma_I(D')) = 0.5$, $\texttt{COMB}(\sigma_I(C')) = 0.3$, and $\texttt{COMB}(\sigma_I(D')) = 0.4$. If only location is considered, we could say that both have the same strength; however, we can use precision for breaking the tie and determine which is stronger. The point is that although the criteria for comparing intervals (other criteria can also be considered) are represented by precise values, this does not diminish the quality of the information when expressed with intervals.

Example 3. *(Cont. Example 2). Let us recall that in the credal SAF* $\mathcal{S}^{PR}_{CS}$*, we have that* $\texttt{ARG}_\circ = \emptyset$*, which means that there is no argument that has no support or does not support another argument. Also, notice that only argument* A *does not have any parent, that is, it is not supported by any other argument. Regarding the rest of arguments, all of them have at least one parent, which means that the conditional credal sets for them have to be calculated. Table 1 presents the values necessary for calculating the imprecise strength of the arguments.*

After the calculations, we have the following imprecise strengths: $\sigma_I(A) = [0.0, 0.7]$*,* $\sigma_I(B) = [0.23, 0.81]$*,* $\sigma_I(C) = [0.2, 0.66]$*,* $\sigma_I(D) = [0.67, 0.94]$*,* $\sigma_I(E) = [0.64, 0.8]$*, and* $\sigma_I(F) = [0.5, 0.74]$*. The precision, location, and combined values for each interval are presented in Table 2. These evaluation criteria give us three ways for comparing pages. We can assume that the value of the decimal gives the PR of a page. Thus, if we use precision for assigning the PR to the pages, E is the page with the highest PR (that is 8). We can also observe that D and F share the same PR (that is, 7); however, since F is more precise then it is more relevant than D, which may impact on which page will be showed first and therefore on the visits to such pages. If we use location, D is the page with the highest PR (that is 8), and if we use the combined value, E has PR 8 and it is the best ranked page. Let us note that the benefit of using imprecise evaluation for PR gives the option of obtaining different rankings which may reflect the preferences of users. For example demanding users may use the combined value because they want to obtain pages with both good location and precision. Other users may want to get well located pages disregarding the precision or vice-versa. Besides, precision and location are not the only ways for comparing intervals, so this gives a range of possibilities for modelling users preferences and therefore turn PR result more customizable.*

Table 1
Values for calculating the imprecise strength. The names of the probabilities are shown in the top row. The rest of rows show the values of the corresponding credal sets.

P(A, C | B)	P(A, C)	P(B, D | C)	P(B, D)	P(F | D)	P(A, C, F | E)	P(A, C, F)	P(C | F)
0.3	0.45	0.5	0.74	0.55	0.55	0.57	0.5
0.48	0.45	0.55	0.66	0.5	0.5	0.55	0.65
0.45	0.55	0.65	0.59	0.62	0.6	0.62	0.6
0.6	0.52	0.7	0.79	0.4	0.7	0.65	0.62

Table 2
Precision, location, and combined values of the arguments for the PageRank scenario. For the ranking: the values in bold represent the best measures and the underlined, the worst ones.

	A	B	C	D	E	F
PREC	<u>0.3</u>	0.42	0.54	0.73	**0.84**	0.76
LOCA	<u>0.35</u>	0.52	0.43	**0.81**	0.72	0.62
COMB	<u>0.105</u>	0.22	0.23	0.59	**0.61**	0.47

4. Generating Contrastive Explanations

In this section, we present how to generate explanations for contrastive questions. These kinds of questions can be answered with a contrastive explanation that compares the properties of the intervals associated to arguments. Producing this kind of explanation benefits from our approach by enriching the returned information to the user.

For generating the explanations, we will consider the ranking based on the combined value; thus, we generate the explanations based the criteria precision and location. Given an argument A, the contrastive questions are expressed in the following way:
-*P-contrast*: $\texttt{WHY}(A, p_{\sigma_I}(A), \texttt{pos})$ (*Why is argument* A *in position* $p_{\sigma_I}(A)$*, rather than in position* $\texttt{pos}$*?*)
-*O-contrast*: $\texttt{WHY}(A, p_{\sigma_I}(A), B, p_{\sigma_I}(B))$ (*Why is argument* A *in position* $p_{\sigma_I}(A)$ *whereas argument* B *in position* $p_{\sigma_I}(B)$*?*)
where $p_{\sigma_I}(A)$ and $p_{\sigma_I}(B)$ are functions that return a position of argument A and argument B, respectively, under an imprecise strength function σ_I and $\texttt{pos}$ is an expected position. This position can be based on the ranking constructed using the $\texttt{COMB}$. The resultant contrastive explanations can be seen as sequences of observations that constitute beliefs for the agent.

For contrastive question $\texttt{WHY}(A, p_{\sigma_I}(A), \texttt{pos})$, we consider the case when $p_{\sigma_I}(A) > \texttt{pos}$. Algorithm 1 shows how the explanation is generated. The algorithm takes as input a credal SAF $\mathcal{S}_{CS}$ and an imprecise strength function σ_I and returns a set of beliefs $\texttt{EXP}$. The beliefs that can be generated are: (i) $better\_loca\_prec(x, y)$, which means that *argument x is more precise and better located than argument y*; (ii) $better\_loca(x, y)$, which means that *argument x is better located than argument y*; and (iii) $better\_prec(x, y)$, which means that *argument x is more precise than argument y*.

For contrastive question $\texttt{WHY}(A, p_{\sigma_I}(A), B, p_{\sigma_I}(B))$, we consider the case when $p_{\sigma_I}(A) < p_{\sigma_I}(B)$. Algorithm 2 shows how the explanation is generated.

Example 4. *(Cont. Example 3) Let's consider Table 2, which shows the values for precision, location, and the combination of both for the scenario of PageRank. The ranking based on the combined value is the following: E, D, F, C, B, A, where E is the best ranked argument and A the worst one. Let us now show two explanations: before the P-contrast question $\texttt{WHY}(A, 6, 3)$, we have $\texttt{EXP} = \{better\_loca\_prec(B, A), better\_loca\_prec(C, A), better\_loca\_prec(F, A)\}$ and before the O-contrast question $\texttt{WHY}(E, 1, D, 2)$, we have $\texttt{EXP} = better\_loca(E, D)$.*

Algorithm 1 Explanation for the P-contrast question $\texttt{WHY}(A, p_{\sigma_I}(A), \texttt{pos})$

Require: $\mathcal{S}_{CS} := \langle \texttt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle, \sigma_I$
Ensure: $\texttt{EXP}$
$\texttt{ARG\_PREV} := \{B \in \texttt{ARG} \mid p_{\sigma_I}(B) < p_{\sigma_I}(A)$ and $p_{\sigma_I}(B) \geq \texttt{pos}\}$
$\texttt{EXP} := \emptyset$
for all $B \in \texttt{ARG\_PREV}$ **do**
  if $\texttt{PREC}(\sigma_I(B)) > \texttt{PREC}(\sigma_I(A))$ and $\texttt{LOCA}(\sigma_I(B)) > \texttt{LOCA}(\sigma_I(A))$ **then**
    $\texttt{EXP}' := better\_loca\_prec(B, A)$
  else
    if $\texttt{PREC}(\sigma_I(B)) > \texttt{PREC}(\sigma_I(A))$ **then**
      $\texttt{EXP}' := better\_prec(B, A)$
    end if
  else
    $\texttt{EXP}' := better\_loca(B, A)$
  end if
  $\texttt{EXP} := \texttt{EXP} \cup \texttt{EXP}'$
end for

5. Axioms for IGSs

In this section, we extend some properties studied in [18] for the imprecise context, that is, for IGSs. We study the behaviour of these properties considering the intervals and the evaluation criteria: precision and location.

Before presenting the axioms, let us make some assertions about the notation:

Algorithm 2 Explanation for the O-contrast question $\texttt{WHY}(A, p_{\sigma_I}(A), B, p_{\sigma_I}(B))$

Require: $\mathcal{S}_{CS} := \langle \texttt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle, \sigma_I$
Ensure: $\texttt{EXP}$
if $\texttt{PREC}(\sigma_I(A)) > \texttt{PREC}(\sigma_I(B))$ and $\texttt{LOCA}(\sigma_I(A)) > \texttt{LOCA}(\sigma_I(B))$ **then**
  $\texttt{EXP} := better\_loca\_prec(A, B)$
else
  if $\sigma_I^{\texttt{PREC}}(A) > \sigma_I^{\texttt{PREC}}(B)$ **then**
    $\texttt{EXP} := better\_prec(A, B)$
  end if
else
  $\texttt{EXP} := better\_loca(A, B)$
end if

- When we say that two intervals are equal, we mean that both the lower and the upper bounds are the same. Formally, given two intervals $[\underline{P}(A), \overline{P}(A)]$ and $[\underline{P}(B), \overline{P}(B)]$ for arguments A and B, respectively. When we say that $[\underline{P}(A), \overline{P}(A)] = [\underline{P}(B), \overline{P}(B)]$, it means that $\underline{P}(A) = \underline{P}(B)$ and $\overline{P}(A) = \overline{P}(B)$.
- We use $\top$ and $\perp$ for denoting $[1, 1]$ and $[0, 0]$, respectively. Thus, when we say that $[\underline{P}(A), \overline{P}(A)] < \top$, we mean that $\underline{P}(A) < 1$ and $\overline{P}(A) \leq 1$ and when we say $[\underline{P}(A), \overline{P}(A)] > \perp$, we mean that $\underline{P}(A) \geq 0$ and $\overline{P}(A) > 0$. Recall that it holds that $\underline{P}(A) \leq \overline{P}(A)$.

We can now begin with the axioms. The first one is about **minimality**. For the precise case, this axiom ensures that if an argument does not have any support, its strength is equal to its base score. In the imprecise case, we compare the interval of the imprecise base score with the interval of the imprecise strength. When there is no support for an argument both its lower and the upper bounds have to remain the same to satisfy minimality. Regarding precision and location, these are not considered because two different intervals may result in the same precision (or location) value, which does not mean that minimality was satisfied. Since intervals are compared element by element, we call this axiom of absolute.

Axiom 1. ***(Absolute Minimality)*** *An imprecise gradual semantics satisfies absolute minimality iff for any imprecise SAF $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$, for any argument $A \in \texttt{ARG}$, if $\mathcal{R}^+(A) = \emptyset$ then $\tau_I(A) = \sigma_I(A)$.*

The following axiom, called **strengthening**, has to do with the role of supports. It states that a support strengthens its target by increasing its strength. In this case, we can use the evaluation criteria in order to compare the intervals. Thus, in terms of intervals, we can say that the more precise the interval of an argument, the stronger the argument and the closer to 1 the location of the interval is the stronger its argument is. Besides, when the interval of an argument is already [1,1], the supports are useless.

Axiom 2. ***(Strengthening)*** *An imprecise gradual semantics satisfies strengthening iff for any imprecise SAF* $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$*, for any argument* $A \in \texttt{ARG}$*, if* $\texttt{crit}(\tau_I(A)) < 1$ *and* $\exists B \in \mathcal{R}^+(A)$ *such that* $\texttt{crit}(\tau_I(B)) > \texttt{crit}(\tau_I(A))$ *then* $\texttt{crit}(\tau_I(A)) < \texttt{crit}(\sigma_I(A))$ *(for* $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$*).*

The next axiom is **strengthening soundness**, it states that the only way of increasing the strength of an argument is by supporting it with an acceptable argument. In this case, we also use precision and location because these are criteria for evaluating the behavior of intervals, this means, that they allow us to measure if an interval is stronger than other.

Axiom 3. ***(Strengthening soundness)*** *An imprecise gradual semantics satisfies strengthening soundness iff for any imprecise* $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$*, for any argument* $A \in \texttt{ARG}$*, if* $\texttt{crit}(\sigma_I(A)) > \texttt{crit}(\tau_I(A))$ *then* $\exists B \in \mathcal{R}^+(A)$ *such that* $\texttt{crit}(\sigma_I(B)) > 0$ *(for* $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$*).*

The next axiom is about **equivalence**, the idea is that arguments with equal conditions in terms of supporters and base score have the same strength. In the imprecise context, we consider that we can have two types of equivalence: (i) the first type considers that two arguments have the same interval as imprecise base score and (ii) the second one considers that two arguments have the same precision or/and location values, which not necessarily means that both arguments have the same imprecise base score. For example, assume that $[0.3, 0.35]$ and $[0.5, 0.55]$ be the imprecise base score of arguments A and B, respectively. The precision value is the same for both intervals: it is 0.95. Now, assume that $[0.6, 1]$ and $[0.7, 0.9]$ be the imprecise base score of arguments A' and B', respectively. The location value is the same of both intervals: it is 0.8. Thus, even different intervals may have the same precision or location value, which has to be reflected in the equivalence property. We call the case (i) absolute equivalence and the case (ii) just equivalence.

Axiom 4. ***((Absolute) equivalence)*** *An imprecise gradual semantics satisfies absolute equivalence (resp. equivalence) iff for any imprecise SAF* $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$*, for all argument* $A, B \in \texttt{ARG}$*, if* $\tau_I(A) = \tau_I(B)$ *(resp.* $\texttt{crit}(\tau_I(A)) = \texttt{crit}(\tau_I(B))$*), there exists a bijective function* f *from* $\mathcal{R}^+(A)$ *to* $\mathcal{R}^+(B)$ *such that* $\forall C \in \mathcal{R}^+(A), \sigma_I(C) = \sigma_I(f(C))$ *(resp.* $\texttt{crit}(\sigma_I(C)) = \texttt{crit}(\sigma_I(f(C)))$*) then* $\sigma_I(A) = \sigma_I(B)$ *(resp.* $\texttt{crit}(\sigma_I(A)) = \texttt{crit}(\sigma_I(B))$*, for* $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$*).*

The following axiom, called **dummy**, states that arguments with strength 0 have no impact on the arguments they support. In the context of impreciseness, we can have two evaluations for dummy: (i) the first one considers that a dummy argument has associated the interval $[0, 0]$ and (ii) the second one considers that the value of precision (location or measure) of the dummy argument is zero. As in the case of equivalence, we call the former absolute dummy and the latter just dummy.

Axiom 5. ***((Absolute) dummy)*** *An imprecise gradual semantics satisfies absolute dummy (resp. dummy) iff for any imprecise SAF* $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$*, for all argument* $A, B \in \texttt{ARG}$*, if* $\tau_I(A) = \tau_I(B)$ *(resp.* $\texttt{crit}(\tau_I(A)) = \texttt{crit}(\tau_I(B))$*),* $\mathcal{R}^+(A) = \mathcal{R}^+(B) \setminus \{C\}$ *and* $C \in \mathcal{R}^+(B)$ *with* $\sigma_I(C) = [0, 0]$ *(resp.* $\texttt{crit}(\sigma_I(C)) = 0$*) then* $\sigma_I(B) = \sigma_I(A)$ *(resp.* $\texttt{crit}(\sigma_I(B)) = \texttt{crit}(\sigma_I(A))$*, for* $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$*).*

The following axiom has to do with the number of supporters and their quality. It states that the more quantity of acceptable supporters an argument has, the stronger the argument is. In the context of impreciseness, the quality is related to the precision and location of the supporters. Thus, only when a supporter is more precise or has a better location, it has a positive impact on the strength of the supported argument.

Axiom 6. ***(Counting)*** *An imprecise gradual semantics satisfies counting iff for any imprecise SAF* $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$*, for all argument* $A, B \in \texttt{ARG}$*, if* $\tau_I(A) = \tau_I(B)$*,* $\sigma_I(B) < \top$ *and* $\mathcal{R}^+(A) = \mathcal{R}^+(B) \cup \{C\}$ *with i.e.,* $\sigma(C) > \bot$*, then* $\texttt{crit}(\sigma_I(A)) > \texttt{crit}(\sigma_I(B))$ *(for* $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$*).*

6. Semantics Classification

In this section, we present a taxonomy of IGSs according the fulfilment of the axioms.

In the presented axioms, we can notice that there are different criteria that are considered for comparing the intervals. Let us recall that the fact that one interval is more precise than other does not mean that it is better located and vice-versa. What we can say is that these criteria have an impact on how a semantics fulfils an axiom. We can have that a semantics fulfils an axiom when the criteria precision is used and when the criteria location is not used and vice-verse. We can have semantics that fulfil an axiom when the combination of precision and location is used. Furthermore, there may be semantics that fulfil an axiom when both precision and location are applied. Considering these aspects, we can classify an IGS as follows:

1. **Absolute semantics**: An IGS is absolute when the semantics satisfies absolute minimality, absolute equivalence, and absolute dummy.

2. **One-criterion semantics**: An IGS is one-criterion when the all the axioms that it fulfils are satisfied in only one criterion: either precision or location.
3. **Two-criteria semantics**: An IGS is two-criteria when the axioms that it fulfils are satisfied in both precision and location.
4. **Combined semantics**: An IGS is combined when the axioms it fulfils are satisfied in the combination of both criteria, that is, precision and location.

Note that absolute, one-criterion, and two-criteria IGSs are disjoint sets. From the above classification, we can conclude the following theorem.

Theorem 1. *Given an IGS σ_I:*
1. If σ_I satisfies absolute equivalence (resp. absolute dummy), then σ_I also satisfies equivalence (resp. dummy).
2. If σ_I is an absolute semantics, then σ_I will be also a combined semantics.

Proof 1. *Let $\mathcal{S}_{CS} = \langle \texttt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle$ be a credal SAF.*
1. If σ_I satisfies absolute equivalence, this means that $\forall A, B \in \texttt{ARG}$, $\sigma_I(A) = \sigma_I(B)$. This in turn means that $\underline{P}(A) = \underline{P}(B)$ and $\overline{P}(A) = \overline{P}(B)$. Since the intervals are the same, when we apply the precision, location, or combination criterion, the result is the same. Thus, we have $\texttt{crit}(\sigma_I(A)) = \texttt{crit}(\sigma_I(B))$ (for $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$), which means that σ_I satisfies equivalence. The same reasoning applies for absolute dummy.
2. If σ_I is absolute, this means that $\underline{P}(A) = \underline{P}(B)$ and $\overline{P}(A) = \overline{P}(B)$. Since the intervals are the same, when we apply the combination criterion, the result is the same. So, σ_I is a combined semantics.

7. Theoretical Evaluation

In this section, we evaluate the proposed imprecise gradual semantics by checking which properties it fulfils and which it does not.

The first theorem states that the properties that are fulfilled by the proposed gradual semantics are absolute minimality and strengthening soundness. In the case of absolute minimality, since no equation has to be applied for calculating a conditional credal set, the credal sets for calculating the imprecise base score and the imprecise strength are the same, so the lower and upper bounds are also the same. Regarding strengthening soundness, the only way for an interval become more precise or better located is by improving the associated credal set and this can only happen when it has supporters.

Theorem 2. *Given an imprecise SAF $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$. The imprecise gradual semantics based on credal network theory fulfils absolute minimality and strengthening soundness.*

Proof 2. *Let $\mathcal{S}_{CS} = \langle \texttt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle$ be a credal SAF.*
*- **For absolute minimality**: Let $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$ be the respective imprecise SAF of $\mathcal{S}_{CS}$. This means that $\forall A \in \texttt{ARG}, \tau_I$ is obtained by applying Equation (1) over f_K. When we say that $\mathcal{R}^+(A) = \emptyset$, this means that $A \in \texttt{ARG}_\circ$ or $pa(A) = \emptyset$. According to Definition 4, for obtaining $\sigma_I(A)$, Equation (1) has to be applied to f_K. Since Equation (1) is applied to the same credal set, this means that $\tau_I(A) = \sigma_I(A)$.*
*- **For strengthening soundness**: By Reductio ad absurdum. Let us assume that $\nexists B \in \mathcal{R}^+(A)$ such that $\texttt{crit}(\sigma_I(B)) > 0$ (for $\texttt{crit} \in \{\texttt{PREC}, \texttt{LOCA}, \texttt{COMB}\}$). This means that Equation (1) has to be applied to f_K, which in turn means that $\texttt{crit}(\sigma_I(A)) = \texttt{crit}(\tau_I(A))$. This contradicts the premise of the axiom.*

The proposed imprecise gradual semantics bases its calculations on credal sets, which contain the probability values necessary for the inference by applying Bayes rule. We can notice that when we apply Equation (1) to two different credal sets we can obtain the same imprecise base score or imprecise strength; however, it does not ensure that after the inference with another credal set, the resultant intervals will be the same. For example, let $K(A) = \{0.4, 0.76, 0.56, 0.87\}$ and $K(B) = \{0.75, 0.4, 0.87, 0.6\}$ be two credal sets whose lower and upper bounds are $[0.4, 0.87]$; however, if we aggregate each of them with a third credal set by applying Bayes rule, the result will be different, even considering that this third credal set and the conditionals are the same for both. Thus, the only way to guarantee the same result is by using the same credal sets in all the inference process. Therefore, we can say that considering that all the credal sets have the same values, some axioms can be fulfilled. In the case of absolute equivalence, the credal sets of the equivalente arguments, the credal sets of their parents, and their conditional credal sets have to be the same. In the case of absolute dummy, the credal sets of one or more of their parents are the same because they all have zero as probability values.

Definition 5. ***(Equality in credal sets)** Let $K(A)$ and $K(B)$ be two credal sets. We say that $K(A)$ and $K(B)$ are equal when $\forall i p_i(A) = p_i(B)$ for $1 \leq i \leq m$, where m is the total number of elements of the credal sets.*

Theorem 3. *Given an imprecise SAF $\mathcal{S}_I = \langle \texttt{ARG}, \mathcal{R}^+, \tau_I \rangle$. Let $A, B \in \texttt{ARG}$ be two arguments that are absolute equivalent. The IGS based on credal network theory fulfils absolute equivalence when $\forall A, B$:*

1. *$\mathcal{R}^+(A) = \mathcal{R}^+(B)$;*
2. *$K(A)$ and $K(B)$ are equal;*
3. *$pa(A)$ and $pa(B)$ are the same;*
4. *$pa(A) \mid A$ and $pa(B) \mid B$ are the same.*

Proof 3. *Let $\mathcal{S}_{CS} = \langle \texttt{ARG}, \mathcal{R}^+, f_K, \tau_I \rangle$ be a credal SAF.*

Let $K(A)$ and $K(B)$ be the credal sets for arguments A and B, respectively. Let us assume that $K(A) = K(B)$. After applying Equation (1) to both, we can say that $\tau_I(A) = \tau_I(B)$. Following the premise of the axiom, we know that both arguments have the same supports. Let us also assume that (i) the credal sets of such supports are also equal and (ii) the calculated conditional credal have the same values. This means that after applying Equation (1) to credal sets and conditional credal sets, the value of the imprecise strength will be the same: $\sigma_I(A) = \sigma_I(B)$.

Regarding the other axioms, their fulfilment can not be guaranteed due to nature of the inference, which is based on the Bayes rule. For instance, for counting axiom, the amount of supporters do not mean that the supported will increase its strength, it depends on the quality of the values of all the supporters together with the conditional credal set of the parents of the supported argument given the supported one.

8. Discussion

In this section, we discuss our approach by comparing it with credal networks. We make such comparison because we base on credal networks for the calculation of the imprecise strength and it is important to highlight what differences gradual semantics from it. We are not going to compare our approach directly with related work because, to the best of our knowledge, there is no an IGS nor a set of properties for evaluating it. Besides, we compare our proposed explanation generation with some related work.

A credal network is a method for information fusion where the inference derives the probability of one or more random variables taking a specific value or set of values. On the other hand, gradual semantics aims to calculate the strength value of the arguments of an AAF with the aim of ranking or ordering them. The strength value and the ranking can be used to make decisions or determine how acceptable each argument is. For example, imagine a scenario of a negotiation persuasive dialogue where two agents (proponent and opponent) want to convince the other to accept a proposal [22]. In this scenario, the agents exchange rhetorical arguments that represent threats and rewards. Both agents generate such arguments and have to decide which of them to send to his respective opponent. The calculation of the strength of such arguments can help the agents to make such decision. On the other hand, in the scenario of the PageRank, the strength value of each page can be seen as a measure of how acceptable each page is. The more acceptable, the more PR value the page has.

Since, the support relation that we are tackling in our approach is the causal one, we use credal networks theory to calculate the strength of arguments; however, the intended meaning of gradual semantics complements the interpretability of the resultant values. In the case of SAFs, the calculated strength may reflect how supported an argument is. This can be interpreted in two ways: (i) an argument is strong because it has many supports and/or (ii) an argument, even with few supports, is strong because its supports are strong enough. This interpretation can be the base for generating explanations about the behaviour of the elements of the graph and for analyse such behaviour in the light of other interactions.

Our approach also complement the calculations by applying the location and precision criteria in order to rank the resultant intervals. The use of more than one dimension for comparing the intervals, gives flexibility to the approach and allows to use one or more of them depending on the domain of application. We have proposed to use such criteria; however, any other criteria can be used with this aim.

To the best of our knowledge there are few works that study explainability in gradual semantics. Albini et al. [23] generate three types of explications for PR. They use a QBAF for modelling the problem and generate contrastive explanations as well. In their case, they focus on answering 'What are the links that make pages A and B have different scores?'. We can note that the contrast focus is different and they do not consider the ranking generated by the gradual semantics. In [24], the authors focus on explaining which arguments are responsible of causing the change in the strengths of other arguments. Thus, the explanations are sets of arguments. Even though, they also study explainability in QBAFs, their explanations are qualitative.

9. Conclusions and Future Work

This work presented an IGS for SAFs considering that the support relation is causality. We use credal sets to model the probability values of each argument and credal networks theory for calculating the imprecise strength, which is represented by an interval. For ranking the interval, we use two criteria, the location and the precision of the interval. From the resultant ranking, we propose to generate contrastive explanations about the positions of the arguments.

In order to evaluate our proposed IGS, we study and propose a set of axioms that describe the behavior of IGSs. We demonstrated that our approach fulfils absolute minimality and strengthening soundness and fulfils absolute equivalence and absolute dummy under some circumstances. Besides, we propose a classification of IGSs based on the fulfilment of axioms.

In general, in formal argumentation explanations have a qualitative nature, that is, they are based on sets of arguments, even though when they are based on Quantitative Bipolar Argumentation Frameworks (QBAFs) (e.g., see [24]). There exist few works for quantitative explanations (e.g. in [23]), which evidences a need for further study this topic.

Regarding future work, we plan to extend our study of properties for IGSs for AAFs and BAFs. We also plan to study other methods for calculating imprecise strength, for example, using Cognitive Fuzzy Maps. Furthermore, we want to further study the generation of richer explanations.

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