Mathematical Models and Information System for Modeling Optimum Upgrade of Fixed Capital Assets

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Abstract

This article discusses models and methods for the optimal replacement of fixed assets and introduces new algorithms for replacing assets with a limited prediction of changes in indefinite costs. The total cost is generated from the operating costs of the current asset that is being used; the cost of liquidating the asset and the cost of new assets. The variation in these costs depends on various factors such as technological changes, economic and environmental changes. Modern innovations enlarge the significance and intricacy of technological progress. The optimum substitution of assets is dissected when future direction of technical development is known over a restricted horizon. Matching of the actual and desired characteristics of existing substitution methods leads us to the idea of how to increase their effectiveness when technology changes, which translates into lower exploitative and new asset costs. We are studying renewed modes of the classical method of substituting an economic resource at an uncertain cost. We show that the modified methods provide solution substitution equal to or close to infinite horizon substitution under technological development. The considered algorithms work well for a random age apportionment of deterministic or stochastic recurrent expenses. We showcase their excellent productivity in a variety of technology advancement scenarios, resulting in reduction in operating costs and costs for new assets. Numerical research is presented and administrative decisions of the received results are considered. The information system of the mathematical model of asset replacement using the conventional economic life method and the modified economic life method is presented.

Keywords [1]
Asset replacement, optimal lifetime, technological update, capital, method, algorithm

1. Introduction

This article explores the main algorithms for asset replacement in the presence of incomplete and indefinite data on technological changes. A new mathematical model of the modified method of economic life is proposed. Based on the developed methods, an information system for modeling the optimal renewal of fixed assets was created. Significant research is dedicated to the issue of substitution of assets, in specific works [1-24]. The Infinite Horizon Method (IH) with regard to technological change has been found to be an excellent example for asset substitution.

At the moment, there are different ways to replace assets, but in practice most of them are not suitable in reality. The reason for this is the problem with limited data, time intervals and other restrictions. In this regard, in the scientific works of engineering and economics, the method of economic life (EL) is proposed (7, 21, 22). This algorithm is unique, simple and safe for real life applications and allows you to determine the optimal output when replacing a single asset. It is worth noting that in the case of an increase in the operating costs of assets, the Economic Life method produces different data than the Infinite Horizon method. As technological changes are taking place, the results produced by the Economic Life method are not practical. In order to solve this problem, the authors [10] propose a new modified method of Economic Life, while adding a new parameter that takes into account the return on

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capital. The modified Economic Life method and the Infinite Horizon method release the same outcomes, provided that technological changes affect operating costs and the cost of new assets, as presented in [10]. It also had an impact on performance indicators in the task of replacing assets at stochastic costs [12]. At present, when studying the replacement of one asset at an unknown cost, the method of minimizing the cost of an infinite horizon and the method of optimal stopping are widely used. This article proposes a modified method of Economic Life under the condition of unknown operating. An algorithm has been developed that can be used in practical life, and an information system based on this algorithm has also been implemented.

2. Methods: updating the asset with technological changes

For example, let's define a company that needs to serially substitute one asset with new assets that execute the same statements but have better substitution costs due to technological change. Let us set out this substitution operation in continuous time $0 \leq t < \infty$. Significant changes in technology and economics are expressed using the following functions:

(1) the *cost* $P(t)$ (buy price and setup cost) *of a new asse*t at time t;

(2) the *cost of exploitation and service* $A(t,u)$ for an asset acquired on time t;

(3) the *liquidation cost* $S(t,u)$ at time u of the asset acquired on time t, $0 \leq S(t,u) < P(t)$.

The variable $a = u\text{-}t$ is an asset age, $0 \leq a \leq M$, where M is maximum natural asset life. Advances in technology manifest the emergence of new assets that require less service and/or are less costly, so $P(t)$ and $A(t,u)$ lessening in t. This fact is discovered as the technological *change* (*TC*). The exploitative cost $A(t,u)$ grows with asset age $u\text{-}t$ due to natural wear, however, it may also fall due to training. The function $A(t,u)$ can express various impairment and learning speculations.

To estimate the factual cost of substitution during a final stage, the theory of substitution uses the capital reimbursement ratio. $R(r,T)$ which changes the actual value of definite costs over a specific future interval into a sequence of interchangeable yearly costs. Assuming continuous interest accrual, the yearly return on capital for the interval $[0,T]$ is

$$R(r,T) = \frac{r}{1-e^{-rT}}, \qquad (1)$$

where $r > 0$ is general industry discount rate.

To depict the successive replacement of one asset by a new asset, we represent an endogenous longevity L_k of the k-th asset, k=1,2,…. Then, the time τ_k of the substittution of the k-th asset with the (k+1)-th asset is

$$\tau_k = \tau_{k-1} + L_k = \sum_{j=1}^{k} L_j, \qquad (2)$$

For clarity, imagine that the first asset is acquired at time t = 0 and will be substituted at the end of its life cycle, then $\tau_0 = 0$ and $\tau_1 = L_1$.

The asset substitution cost: The actual value of full substitution cost of the k-th asset, k=1,2,…, during your future life L_k is computed at a certain industrial discount rate r>0 as [7]

$$PW_k(L_k,\tau_k) = e^{-r(\tau_k+L_k)}\left[P(\tau_k+L_k) - S(\tau_k,\tau_k+L_k)\right] + \int_{\tau_k}^{\tau_k+L_k} e^{-ru} A(\tau_{k-1},u)\,du, \qquad (3)$$

The first part (3) is the actual value of the new asset minus the discounted liquidation value of the actual asset, and the integral is the exploitative operating costs over the future life of the current asset. The challenge is to create substitution methods that use limited data of technological change, but give the same results as for an ideal technological prediction. Accordingly, our ideal task is to optimize on an infinite horizon. Below we present the mathematical statements of the substitution methods under study.

2.1 Infinite-Horizon (IH) Substitution

The IH substitution algorithm [23, 24] proposes that the outer technological parameters P, A and S are revealed over an infinite horizon $[0,\infty)$ and defines the endless optimum sequence of serial asset lifetimes L_k, k=1,2,…, that decreases the actual value of the summary substitution cost over $[0,\infty)$:

$$PW_\infty(L_1*, L_2*,...) = \min_{L_k, k=1,...;\ 0<L_k\le M} PW_\infty(L_1, L_2,...) \ , (4)$$

$$PW_\infty(L_1, L_2,...) = \sum_{k=1}^{\infty} PW_k(L_k, \tau_k), \quad (5)$$

where PW_k is given by (3) and $\square_k$ is defined from (2).

On the contrary, the following substitution methods work in the case of a restricted technological prediction. We will consider that the technological parameters $P(t)$, $A(t,u)$, and $S(t,u)$ are noted for $0\le t \le u \le T <\infty$ at some finite interval in the future $[0, T]$, where the value T should not be less than the future obscure lifetime L_1 of the actual asset. For example, T may be the maximal natural lifetime M of assets.

2.2 Economic Life (EL) Substitution Method

The EL method defines the lifetime L_1 that decreases the *interchangeable yearly cost (EAC)* of the first asset substitution [21]

$$C_1(L_1) = R(r, L_1)PW_1(L_1, 0)\,, \qquad (6)$$

where $R(r, L_1)$ is determined by (1) and PW_1 is given by (3). By the EL algorithm, the first optimum lifetime EL_1 is defined as

$$EL_1 = \arg\min_{0<L\le M} C_1(L)\,. \qquad (7)$$

To discover the first optimum lifetime EL_1, it is sufficiently to know the cost $P(t)$ and the sequences $S(0,t)$ and $A(0,t)$ over the future interval $[1, EL_1]$. As a result, the EL method gives various optimum lifetimes EL_1, EL_2,…, for successively substitutions k=1,2,3,… of the asset. In research of engineering, searching the first optimum service life EL_1 is the most pressing issue.

The general coherence in the theory of substitution is that the EL method does not take into account technological changes. This is only partly correct. The variant (6) of the EL algorithm above supposes substitution at the end of the actual life cycle of the asset and thus actually considers a possible technological development as a shift in the value of a new asset. $P(\tau_k + L_k)$. However, the EL (6)-(7) algorithm does not account for enhancements in exploitative costs. Below we define a modified method that overcomes this shortcoming.

2.3 Modified EL Method

To cope with constant technological improvements, we perform an effective return on capital ratio.

$$\hat{R}(r, c, L) = R(r + c, L)\,, \qquad (8)$$

where c is the *cumulative TC rate*. The selection of rate c for different types of TC should be based on a matching of the actual and desired properties of replacement methods. Specifically, using $\hat{R}(r, c, L)$ instead of $R(r, L)$ in the EL algorithm noticeably raises its effectiveness. The *modified EL algorithm* defines the lifetime L_1 that decreases the corrected EAC of the first asset substitution

$$L_1 = \arg\min_{0<L\le M} \hat{C}_1(L)\,,$$

$$\hat{C}_1(L) = R(r + c, L)PW_1(L, 0)\,, \qquad (9)$$

in which $\hat{R}(r, c, L)$ is used instead of $R(r, L)$ as in (6).

3. Resutls: comparative algorithm analysis

Calculations of the optimal service life of the asset were carried out using two methods: the replacement method is active excluding the technological update; a method of replacing an asset, taking into account the technological update. The method of replacing an asset without taking into account technological renewal corresponds to the method of economic life, in turn, the method of substitution an asset with regard to technological renewal corresponds to the modified method of economic life.

Formula of the Economic Life method:

$$EAC(L)=\frac{d(1+d)^{L}}{(1+d)^{L}-1^{L}}\left[\frac{P(\tau_0)}{(1+d)^{L-\tau_0}}-\frac{S(\tau_0,L)}{(1+d)^{L-\tau_0}}+\sum_{j=1}^{L}\frac{A(\tau_0,j)}{(1+d)^{j-\tau_0}}\right]. \tag{10}$$

describes the effective annual equivalent cost (EEAC)

The formula for the modified Economic Life method is:

$$EEAC(L)=\frac{d(1+d)^{L}}{(1+d)^{L}-q^{L}}\left[\frac{P(\tau_0)a^{L-\tau_0}}{(1+d)^{L-\tau_0}}-\frac{S(\tau_0,L)}{(1+d)^{L-\tau_0}}+\sum_{j=1}^{L}\frac{A(\tau_0,j)}{(1+d)^{j-\tau_0}}\right]. \tag{11}$$

The modified method of economic life is identical to the original method of economic life at a = q = 1, that is, excluding technological improvements.

i = 1,2,..- discrete time (i = 0 is the current year);

P(i) is the cost of acquiring (installing) a new asset in year i;

A(i,j) - cost of exploitation and service (O&M) of an asset of age j established in year i;

S(i,j) - liquidation value of an asset of age j established in year i;

A - annual factor of change in the purchase price P of a new asset;

q is the yearly change in O&M value A of the new asset;

L1 = L - unknown lifetime of the current asset;

τ_0 - the current moment of installation of the current (first) asset;

d is the given annual discount rate;

EEAC is the effective annual present value equivalent of the total value of current assets.

Having analyzed the possibility of substitution the asset during this year, it is enough to compare the two values of EEAC: EEAC(τ_0) for this year and EEAC(τ_0+1) for the coming year. Replacement decision:

- If EEAC(τ_0+1) > EEAC(τ_0), then the asset is due to be substituted this year.
- If EEAC(τ_0+1) ≤ EEAC(τ_0), then the asset continues to operate this year.

Comparing the results of calculations for the two methods of asset replacement, it can be seen that the calculation of the year of asset replacement using the Asset Replacement Method without taking into account technological renewal does not correspond to the real life of the asset and is far from reality. With the Technology Update Asset Replacement Method, the calculation of the year of asset replacement is close to real data.

4. Information system for modeling asset replacement

The information system was developed using the PHP scripting language and the MySQL database management system. The information system is designed to calculate the optimal life of an asset using asset replacement methods; creating an asset database.

In the course of the study, an analysis was made of existing methods, models of asset replacement, their advantages and disadvantages. This information system implements two methods for replacing an asset: the method for replacing an asset without taking into account technological updates; a method of replacing an asset, taking into account the technological update. Information system capabilities:

1. Create a new asset database.
2. Calculation of the optimal time to replace an asset using the asset replacement method without taking into account technological upgrades.
3. Calculation of the optimal time to replace an asset using the asset replacement method, taking into account the technological update.

Algorithm for calculating the optimal time to replace an asset using the asset replacement method without taking into account technological renewal (Fig. 1). Algorithm for calculating the optimal time to replace an asset using the asset replacement method, taking into account technological updates (Fig. 2).

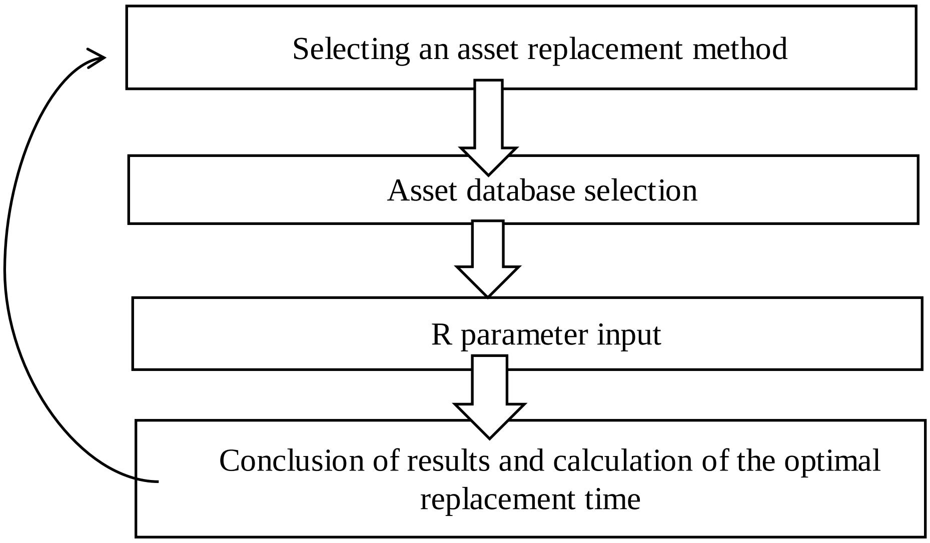


Figure 1. Calculation algorithm for the replacement method without technological update

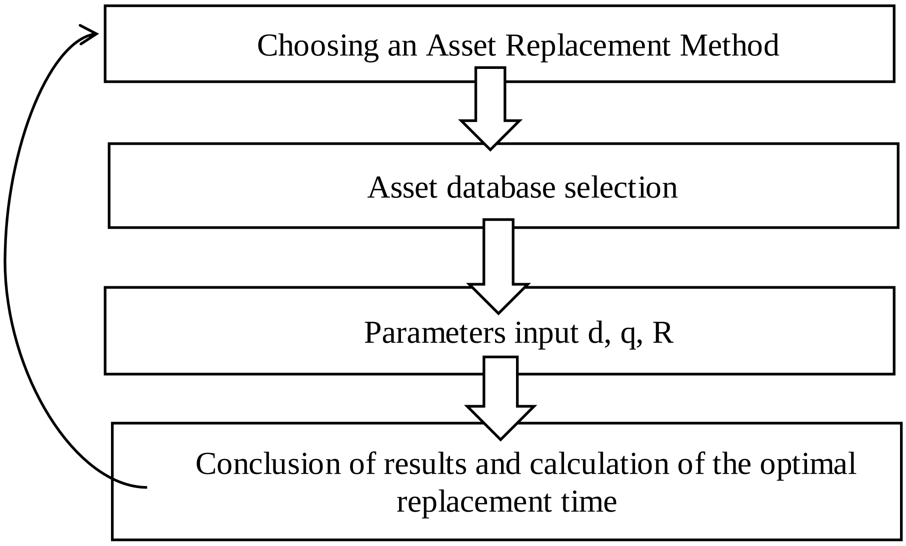


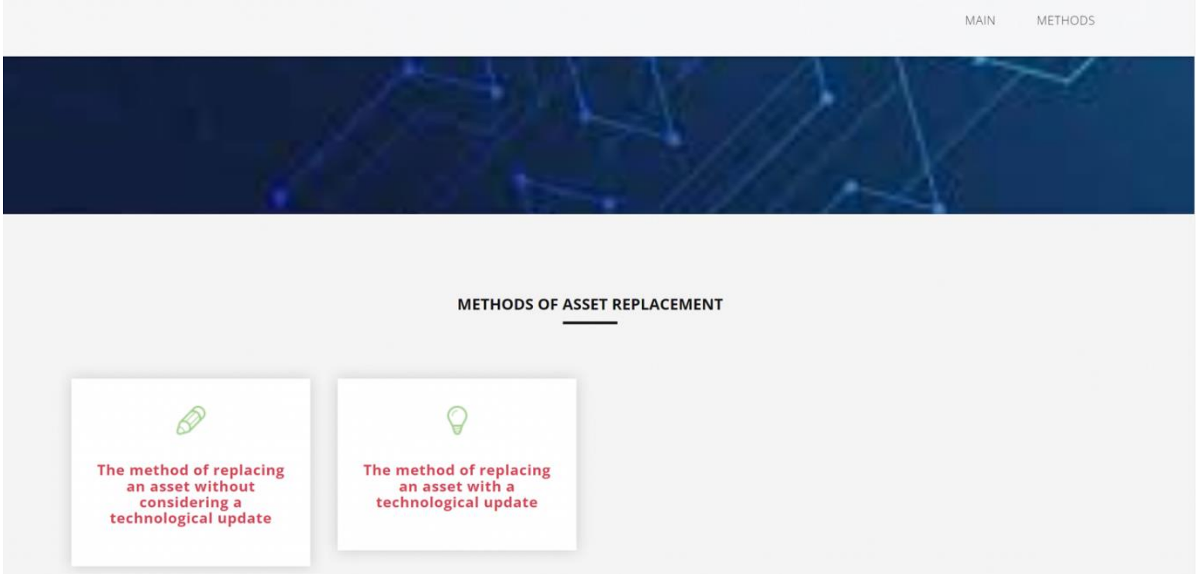
Figure 2. Calculation algorithm for the replacement method, taking into account the technological update

4.1 Description of the program and interface

The main page of the program is shown in Figure 3. There are two buttons in the main menu: MAIN, METHODS. When you press the MAIN button, the page goes to the menu of the main program window. In the METHODS menu, there are two methods of asset replacement (Fig. 4): The method of replacing an asset without considering a technological update. The method of replacing an asset with a technological update. We will perform calculations using the asset replacement method with taking into account the technological update for the X-ray equipment. By selecting the Asset replacement method button taking into account technological update, the program will calculate the time of asset replacement with technological update parameters (Fig. 6). Next, select a table from the database using which we will calculate. Let's select the table X-ray equipment (Fig. 7). The program window opens with the table data. Next, we enter the parameters: the annual discount rate, the annual factor of change in the operating cost of a new asset, the cost of a new asset and click the Calculate button (Fig. 8). The next window displays the results where the effective annual equivalent values are calculated. And displays the year of replacement of the asset (Fig. 9). Based on the results, the X-ray equipment should be replaced in 2019 as the EAC(L) value increases in subsequent years.

INFORMATION SYSTEM FOR MODELING THE TECHNOLOGICAL RENEWAL OF ENTERPRISES

Figure 3. Main page



METHODS OF ASSET REPLACEMENT

The method of replacing an asset without considering a technological update

The method of replacing an asset with a technological update

Figure 4. Asset replacement method without taking into account technological upgrade

METHODS OF ASSET REPLACEMENT

The method of replacing an asset without considering a technological update

The method of replacing an asset with a technological update

Figure 5. The method of replacing an asset, taking into account the technological update

5. Conclusions

We focus on practical situations where the new asset and operating costs vary but are known over a short forecast horizon or even can only be estimated at some points. Our methodology is based on the assumption that the current cost dynamics will continue, at least for some time. This allows us to predict future cost dynamics based on the few discrete cost measures available and build a simple asset replacement algorithm that provides a cost-effective management decision on when to replace assets.

The algorithm of the Modified method of Economic Life is proposed. A simple information system based on this method has been developed to determine the optimal replacement period for an asset in case of uncertain technological changes.

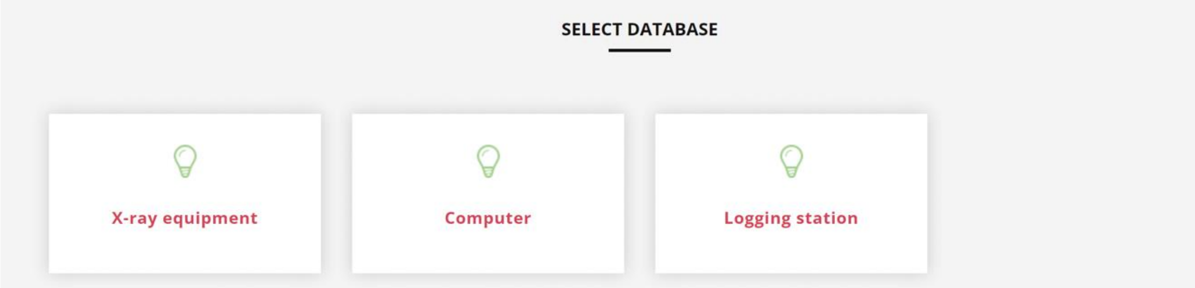
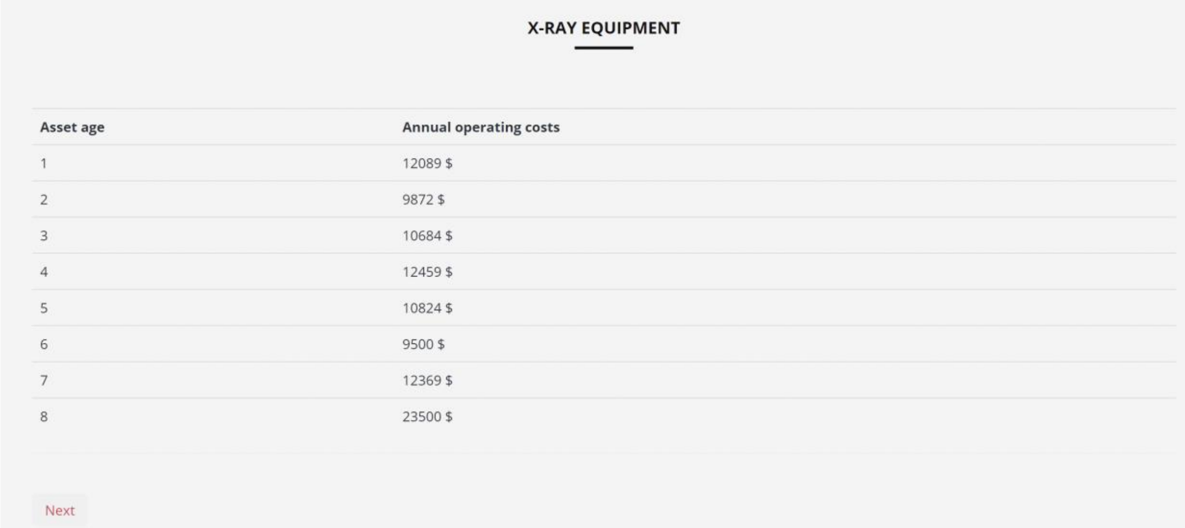
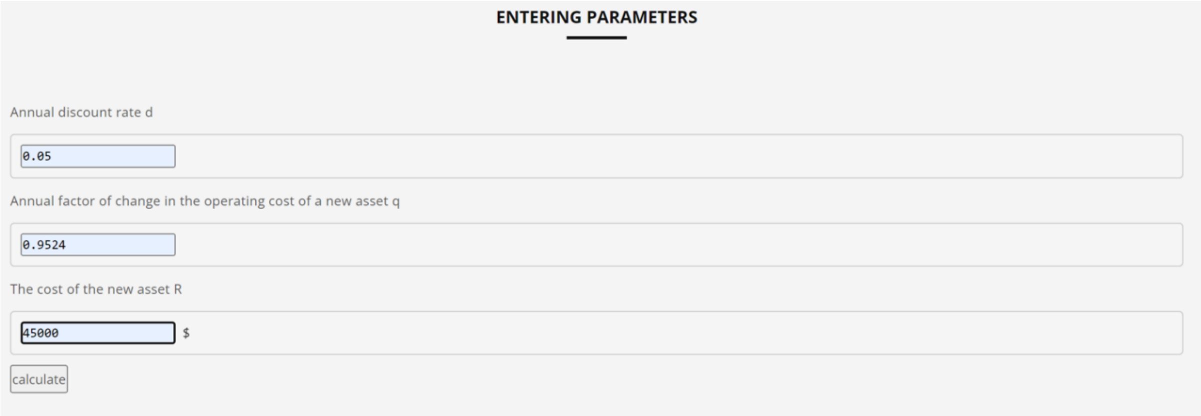


Figure 6. Database selection



Asset age	Annual operating costs
1	12089 $
2	9872 $
3	10684 $
4	12459 $
5	10824 $
6	9500 $
7	12369 $
8	23500 $

Figure 7. Table data



Annual discount rate d

0.05

Annual factor of change in the operating cost of a new asset q

0.9524

The cost of the new asset R

45000 $

Figure 8. Entering parameters

6. Acknowledgement

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RESULTS

Asset age	Year	Annual operating costs	Effective Annual Equivalent Present Value EAC(L)
1	2013	12089 $	57090.14
2	2014	9872 $	32959.95
3	2015	10684 $	25180.57
4	2016	12459 $	21707.39
5	2017	10824 $	19330.91
6	2018	9500 $	17555.07
7	2019	12369 $	16642.06
8	2020	23500 $	17125.07

Solution: The device must be replaced in 2019

Figure 9. Output of the result according to the method, taking into account the technological update

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