Forecasting of Categorical Time Series Using Computing with Words Model

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Abstract

This paper addresses the problem of forecasting categorical time series, which have wide application in all areas of activity where judgement and expert judgement are used. One of the main problems in forecasting such series is to account for the linguistic uncertainty that arises in expert judgement, moving from qualitative to quantitative assessment and vice versa. This paper proposes a categorical time series prediction model based on computing with words. A codebook of 32, 15, 11, 9, 8, 7, 6 or 4 words is suggested for comparing categorical time series attributes. The number of words depends on the level of detail of the categorical attributes of the series. Words from codebooks are described using discrete interval type-2 fuzzy sets, which allows for the linguistic uncertainty of the categorical attributes of the series. Based on the proposed model, a fuzzy algorithm for categorical time series forecasting is developed, consisting of five steps: word model definition, fuzzy relationship definition, fuzzy relationship grouping, fuzzy forecasting, result interpretation. The quality of the proposed model is confirmed by three estimated characteristics: mean absolute prediction error; mean square prediction error; mean relative prediction error.

Keywords [1]

Time series, categorical data, discrete interval type-2 fuzzy set, uncertainty, computing with words, fuzzy prediction.

1. Introduction

Categorical Times Series (CTS) is a time series in which observations at each point in time have categorical values (nominal or ordinal). Obviously, such series are quite common in practice, although in the literature they are paid much less attention than series with numerical continuous variables. The source of CTS can be both devices and expert judgments. For example, IP addresses, web addresses, area codes are obtained from the web server; diagnoses are recorded as a result of medical examination; sequences of letters and words are processed during speech recognition, etc.

During the last twenty years of the last century a number of different approaches to CTS modelling have been proposed. Mostly these models were based on Markov chain model and discrete ARMA model. Particular attention was paid to the theory of generalized linear models, the authors of which are P. McCullagh and J. Nelder (1989) [1]. This approach was developed in the works of F. Konstantinos and B. Kedem, who proposed various models to illustrate the selection of the link function [2]. H. Pruscha and A. Göttlein devoted attention to the multivariate and the cumulative logistic regression model, with a regression term defined by a recursive scheme [3].

Parametric methods of time series analysis assume that the data are based on a stationary process. CTS is considered stationary if the marginal distribution of the data is constant over the period of time for which they were collected. The correlation between consecutive values is a function only of their distance from each other, not of their position in the series. However, there are many examples of categorical series that do not meet this definition of stationarity. A thorough analysis of non-stationarity

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of CTS is given in Heinz Kaufmann [4]. In [5] M. McGee and I. Harris presented one of the algorithms that corrects non-stationarity in categorical time series. K. Fokianos and B. Kedem proposed their own universal approach to the problem of regression modeling of CTS [2]. The main advantage of this method is that neither Markov conditions nor stationarity are assumed.

In the early 2000s, the wavelet method [6, 7] and Bayesian analysis for series clustering [8] began to be used for CTS analysis [8]. In [9] it is proposed to apply Bayesian CTS forecasting to both ordinal and nominal data. The authors of the paper consider the autoregressive process based on the Pegram operator. A new CTS clustering method was developed by the authors of the paper "Cats&Co: Categorical Time Series Coclustering" based on three-dimensional data grid models (each point is defined by three variables: sequence identifier, time value and event value) [10]. The word "Cats" in the title of the publication is an abbreviation for categorical times series.

In recent publications, the authors continue to study the problem of stationarity and likelihood maximization of CTS [11], autocorrelation functions for nominal and ordinal data [12], and the extension of basic discrete autoregressive models is proposed [13]. Classification and regression trees are also used to forecast CTS, which can work with both continuous and categorical data [14]. The advantages of these methods are that they are easy to understand and interpret, but there is a problem of overtraining the tree, excessive sensitivity to the input data, even a small change in the data can significantly change the structure of the tree. Analysis of modern research has shown that the problem of accounting for uncertainty inherent in judgments and expert assessments (linguistic uncertainty) remains insufficiently studied. The definition of fuzzy time series (FTS) and the research of their properties is carried out in Song and Chissom [15]. Shyi-Ming Chen in [16] proposed a new forecasting method based on FTS. This method proved to be more efficient than the method developed by Song and Chissom because it uses simplified arithmetic operations compared to the algorithm [15]. But the accuracy and reliability of forecasting of the developed models still need to be refined and improved.

The purpose of this paper is to develop a fuzzy time series model that will allow to perform calculations with words. Based on the developed model, a fuzzy algorithm for forecasting categorical time series is proposed and the quality of forecasting is evaluated.

2. Fuzzy model for forecasting categorical time series

In this paper the theory of interval type-2 fuzzy sets (IT2 FS) is applied. It is accepted that the degree of belonging of an element of a universal set to a fuzzy subset is not defined for each element of a universal set uniquely, but there is a certain blur. That is to say, the degree of membership is fuzzy and takes values in the interval [0, 1]. In general, the fuzzy forecasting model of categorical time series (T2 FTS – type-2 fuzzy time series) can be represented as follows:

$$v^k(t) = v^j(t-1)^{\circ}R(t-n), \qquad (1)$$
$$v^k(t) \in V, v^j(t-1) \in V, \forall k,j \in \{1,\dots,m\},$$

where $v^k(t)$ – is the predicted value given as a granulated term,

$v^j(t-1)$ – last value in the T2 FTS, V – the codebook, m – number of granulated terms in the codebook, $R(t-n), n=\overline{1,N}$ – is a relation that describes the fuzzy relationship between $V(t)$ and $V(t-n)$, N – number of time series values,

° – Zadeh's Compositional Rule of Inference (MAX MIN).

A granulated term is described by a word and a discrete interval type-2 fuzzy set (DIT2FS):

$$V = \langle v_j \rangle, \qquad j \in \{1,\dots,m\}, \qquad (2)$$
$$v_j = \langle T_j, \tilde{Y}_j \rangle,$$

where v_j – granulated term,

T_j – word, $\tilde{Y}_j$ – DIT2FS, which describes the word.

3. Fuzzy algorithm for forecasting categorical time series

The proposed model (1) allowed us to develop a fuzzy algorithm for forecasting time series, which is based on the basic principles of the theory of type-2 fuzzy sets [15], computing with words [18] and the approach to forecasting fuzzy time series [19]. Consider the five main steps of the proposed algorithm.

Step 1: Identifying the word model. Generally, when interpreting numbers and predicting values, people tend to move to categories such as "a little", "a bit", "reasonably", "good", "a lot", etc. Therefore, each categorical feature in the time series is matched with a word from the codebook. This paper proposes the use of codebooks provided by J.M. Mendel. Codebooks of 32, 15, 11, 9, 8, 7, 6 and 4 words (granulated terms) are allowed. The choice of codebook depends on the level of categorical detail of the time series. Typically, granulated terms are represented as a pair - word and DIT2FS, which is defined on primary variable $X[0, 10]$.

Step 2: Identifying fuzzy relationships. After each categorical feature is matched with a codebook word, a fuzzy logic relationship matrix FR is constructed

$$FR = \begin{bmatrix} i & v_k^i \to v_j^{i+1} \end{bmatrix},\ i = \overline{1, N-1},\ v_k^i, v_j^{i+1} \in V, \tag{3}$$

where v_k^i – i-th time series value (current state),

v_j^{i+1} – i+1-th time series value (next value).

Step 3: Grouping fuzzy logic relationships. This paper uses Chen's [21] second-order grouping. To do this, the set Groups (without duplicates) of the current states v_k^i is constructed from the FR matrix. Each element of Groups is matched from the FR matrix with the set of v_j^{i+1} words (without duplicates) that were involved in the transition $v_k^i \to v_j^{i+1}$.

Step 4: Fuzzy forecasting. Fuzzy forecasting is defined using the Zadeh's Compositional Rule of Inference (MAX MIN)

$$\tilde{Y}(t) = \begin{cases} MAX\ MIN\big(Groups(v^{t-1})\big), \text{if } v^{t-1} \text{ in } Groups, \\ \tilde{Y}(t-1), \text{if } v^{t-1} \text{ not in } Groups, \end{cases} \tag{4}$$

where $\tilde{Y}(t)$ – predicted value in the form of DIT2FS,

v^{t-1} – last value of the time series, represented as a granulated term,

$\tilde{Y}(t-1)$ – last value of the time series, represented as DIT2FS.

Step 5: Interpretation of the result. The prediction result can be both a numerical value and a word from the codebook. To obtain the result in numerical form, you need to reduce the type $\tilde{Y}(t)$. Type reduction can be performed using the ECM algorithm for DIT2FS [22]. To obtain the result in the form of a word from the codebook, it is necessary to perform a similarity comparison $\tilde{Y}(t)$ with DIT2FS, which describe the words of the codebook.

4. Description of input data for fuzzy time series modeling of the second type

A classic example of CTS is the data set on sleep in newborns [5, 17]. A pediatric neurologist evaluated the baby's electroencephalogram (EEG) every minute for about two hours. The neurologist classified the infant's sleep state as one of the following: qt – quiet sleep, trace alternant; qh – quiet sleep, high voltage; tr – transitional sleep; al – active sleep, low voltage; ah – active sleep, high voltage; aw – awake. For further modeling the data set given by Stoffer, Tyler, and Wendt [17] (2000) was used.

Thus, we have 107 observations with five unique values (the state "awake" is not represented in this set, but it is clear that such a state is possible and should be taken into account during modeling). The frequency of values is distributed as follows: ah – 32; al – 27; qh – 3; qt – 33; tr – 12.

The most common way to analyze CTS involves assigning numerical integer values to categories. Let's order the six sleep states of the infant from the least active to the most active state and replace them with numerical equivalents from 1 to 6: qt – 1; qh – 2; tr – 3; al – 4; ah – 5; aw – 6. Figure 1 shows the graph of the obtained series.

5. Description of the experiment's results

When comparing categorical assessments obtained, for example, by experts, with integer numerical values, linguistic uncertainty arises, which needs to be eliminated. This problem is considered in the theory of fuzzy sets, where the degree of membership of an element of a universal set to a fuzzy subset can be any real number from the interval [0, 1]. The results of the experiment on the implementation of the fuzzy algorithm for forecasting time categorical series are given below.

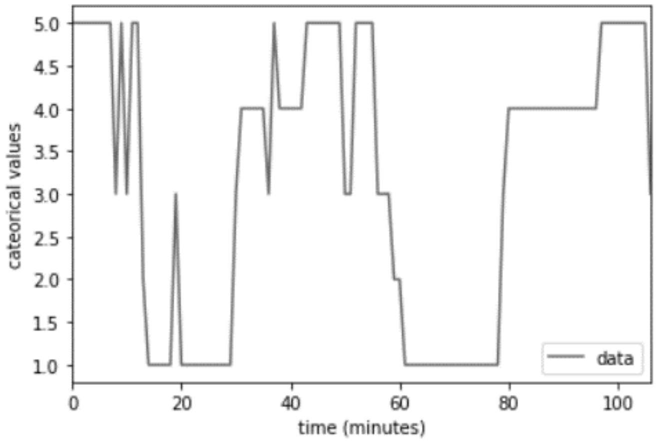

Figure 1: Graph of categorical time series obtained by replacing categories with numerical values

Step 1: Identifying the word model

The six-word codebook was used to represent the categorical features [qt, qh, tr, al, ah, aw]: "Unimportant", "More or less unimportant", "Moderately unimportant", "More or less important", "Moderately important", "Very important". Each categorical feature was matched with a word from the codebook:

- qt – "Unimportant";
- qh – "More or less unimportant";
- tr – "Moderately unimportant";
- al – "More or less important";
- ah – "Moderately important";
- aw – "Very important".

The uncertainty fingerprints of the words and their lower and upper membership function parameters are shown in Table 1 and Figure 2 respectively.

Table 1

FOU data for the 6 words

Word	LMF						UMF				
	MF type	a	b	c	d	h	MF type	a	b	c	d
qt	trapmf	0	0	0.09	1	1	trapmf	0	0	0.55	4.61
qh	trapmf	2.79	3.21	3.21	3.71	0.34	trapmf	0.42	2.25	4.0	5.41
tr	trapmf	2.79	3.34	3.34	3.67	0.35	trapmf	1.59	2.75	4.35	6.26
al	trapmf	5.79	6.28	6.28	6.67	0.33	trapmf	3.38	5.5	7.25	9.02
ah	trapmf	6.29	6.67	6.67	7.17	0.39	trapmf	4.59	5.9	7.25	8.5
aw	trapmf	8.68	9.91	10	10	1	trapmf	7.37	9.36	10	10

Step 2: Identifying fuzzy relationships

After each infant's sleep state is matched with a word from the codebook, a fuzzy logical relationship matrix is constructed $v_k^i \rightarrow v_j^{i+1}$ (Table 2), where v_k^i is the infant's sleep state at time *i* (current state), and v_j^{i+1} is the infant's sleep state at time $i + 1$ (next value).

Step 3: Grouping fuzzy logic relationships

The relationship groups derived from Table 2 are shown in Table 3.

Step 4: Fuzzy forecasting

Forecasting the next infant's sleep state is based on operations on the FOUs of the current state and the corresponding group from Table 3.

Step 5: Interpretation of the result

To match the forecast value to a categorical attribute, we use Jaccard's similarity index: select the attribute that will have the greatest similarity between the FOU that describes that attribute and the resulting DITFS.

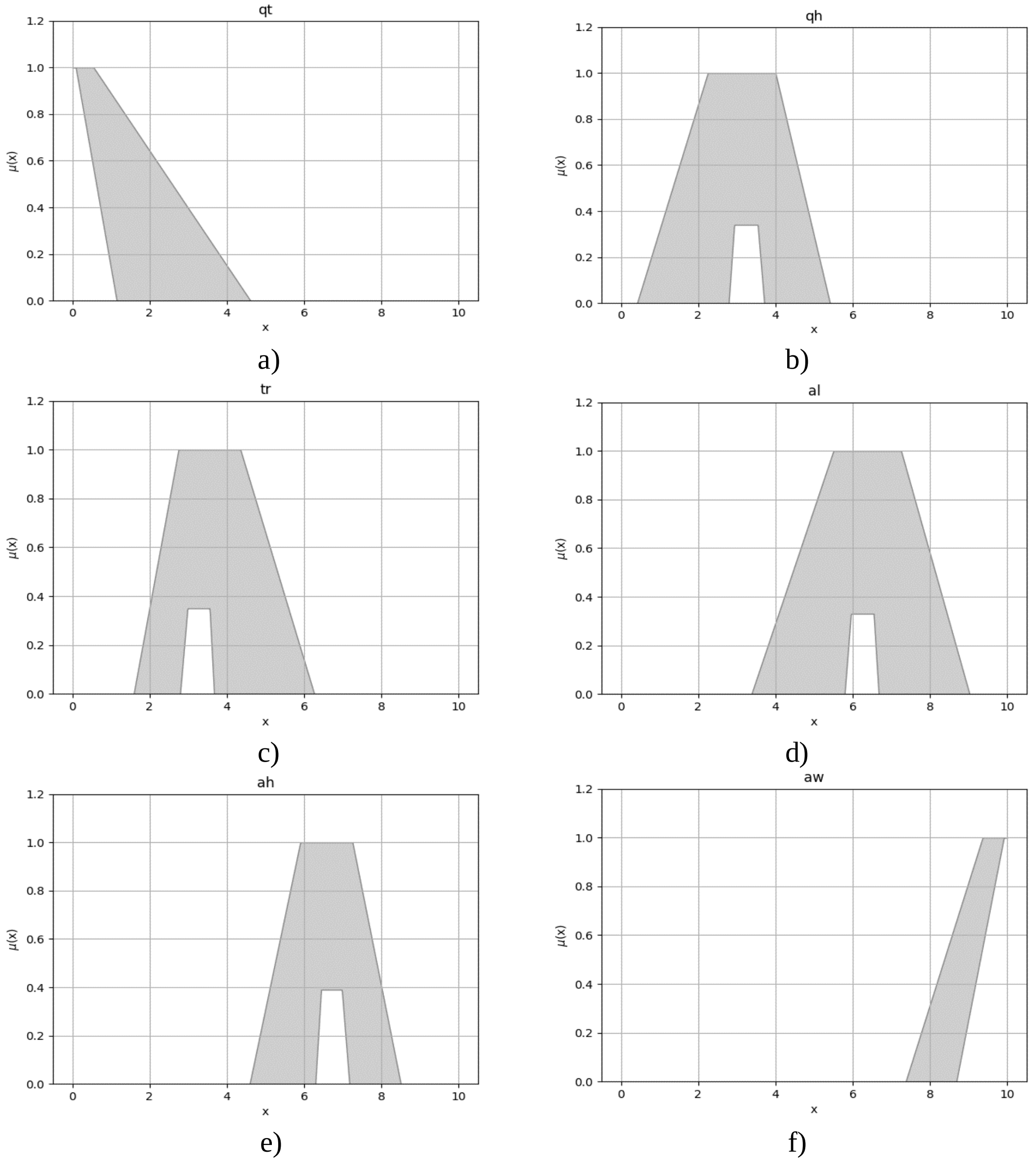

Figure 2: FOU Plots for the 6 words: a) – qt, b) – qh, c) – tr, d) – al, e) – ah, f) – aw

6. Software description

Software was developed based on the developed categorical time series forecasting algorithm. The programming language Python and the libraries numpy and matplotlib were used to implement the software. Description of the modules of the developed software:

1. words_model.py – module that defines 32-, 15-, 11, -9-, 8-, 7-, 6- and 4-word codebooks. Each codebook is presented as a dictionary. An example of a 4-word codebook presentation is shown in Figure 3.
2. std_mfs.py – module in which standard membership functions (trapmf, trimf, gaussmf, smf, zmf, sigmf) are defined.
3. dit2fou.py, dit2ft.py and dit2lv.py – modules which contain a class describing the FOU DIT2FS, a term and a linguistic variable respectively. The class structure of the modules is shown in the class diagram (Figure 4).

4. similarity_measure.py – module in which functions are defined to calculate the similarity relationship between FOUs of two DITFSs A and B on primary variable X. The following functions are defined in the module: Jaccard Similarity Measure; Zhao, Xiao, Li and Deng's Similarity Measure, Yang and Lin's Similarity Measure.

```
words_4 = {
  'x': (0, 10.01, 0.001),
  'words': {
      'Unimportant':
          {'umf': ('trapmf', 0, 0, 0.55, 4.61),
           'lmf': ('trapmf', 0, 0, 0.09, 1.15, 1),
           'centroid_mean': 1.103},
      'More or less unimportant':
          {'umf': ('trapmf', 0.42, 2.25, 4.00, 5.41),
           'lmf': ('trapmf', 2.79, 3.21, 3.21, 0.34, 3.71),
           'centroid_mean': 3.036},
      'More or less important':
          {'umf': ('trapmf', 3.38, 5.50, 7.25, 9.02),
           'lmf': ('trapmf', 5.79, 6.28, 6.28, 0.33, 6.67),
           'centroid_mean': 6.241},
      'Very important':
          {'umf': ('trapmf', 7.37, 9.36, 10, 10),
           'lmf': ('trapmf', 8.68, 9.91, 10, 10, 1),
           'centroid_mean': 9.295},
  } }
```

Figure 3: 4-word codebook view

Table 2
Fuzzy logic relationships of infant sleep states

$i, v_k^i \to v_j^{i+1}$	$i, v_k^i \to v_j^{i+1}$	$i, v_k^i \to v_j^{i+1}$	$i, v_k^i \to v_j^{i+1}$	$i, v_k^i \to v_j^{i+1}$	$i, v_k^i \to v_j^{i+1}$
1, ah → ah	19, qt → tr	37, tr → ah	55, ah → ah	73, qt → qt	91, al → al
2, ah → ah	20, tr → qt	38, ah → al	56, ah → tr	74, qt → qt	92, al → al
3, ah → ah	21, qt → qt	39, al → al	57, tr → tr	75, qt → qt	93, al → al
4, ah → ah	22, qt → qt	40, al → al	58, tr → tr	76, qt → qt	94, al → al
5, ah → ah	23, qt → qt	41, al → al	59, tr → qh	77, qt → qt	95, al → al
6, ah → ah	24, qt → qt	42, al → al	60, qh → qh	78, qt → qt	96, al → al
7, ah → ah	25, qt → qt	43, al → ah	61, qh → qt	79, qt → tr	97, al → ah
8, ah → tr	26, qt → qt	44, ah → ah	62, qt → qt	80, tr → al	98, ah → ah
9, tr → ah	27, qt → qt	45, ah → ah	63, qt → qt	81, al → al	99, ah → ah
10, ah → tr	28, qt → qt	46, ah → ah	64, qt → qt	82, al → al	100, ah → ah
11, tr → ah	29, qt → qt	47, ah → ah	65, qt → qt	83, al → al	101, ah → ah
12, ah → ah	30, qt → tr	48, ah → ah	66, qt → qt	84, al → al	102, ah → ah
13, ah → qh	31, tr → al	49, ah → ah	67, qt → qt	85, al → al	103, ah → ah
14, qh → qt	32, al → al	50, ah → tr	68, qt → qt	86, al → al	104, ah → ah
15, qt → qt	33, al → al	51, tr → tr	69, qt → qt	87, al → al	105, ah → ah
16, qt → qt	34, al → al	52, tr → ah	70, qt → qt	88, al → al	106, ah → tr
17, qt → qt	35, al → al	53, ah → ah	71, qt → qt	89, al → al	
18, qt → qt	36, al → tr	54, ah → ah	72, qt → qt	90, al → al	

5. forecasting_categorical.py – module containing a basic function for predicting a categorical time series. The function signature forecasting:

forecasting(, data=None,*
categories=None,
model_words='words_32',
sm_method='jaccard',
steps=1)

data – dataset,

categories – a list of categorical features from the dataset to be associated with codebook words, *model_words – codebook type, default is 32-word codebook*

sm_method – method for determining the similarity between FOUs DIT2FSs, the default is Jaccard Similarity Measure,

steps – number of foreseeable values, default is 1.

6. main.py – entry point to the application.

Table 3

Groups of fuzzy relationships of infant sleep states

Number	Group
Group 1	ah → {ah, qh, tr, al}
Group 2	tr → {al, ah, qh, tr, qt}
Group 3	qh → {qh, qt}
Group 4	qt → {qt, tr}
Group 5	al → {ah, al, tr}

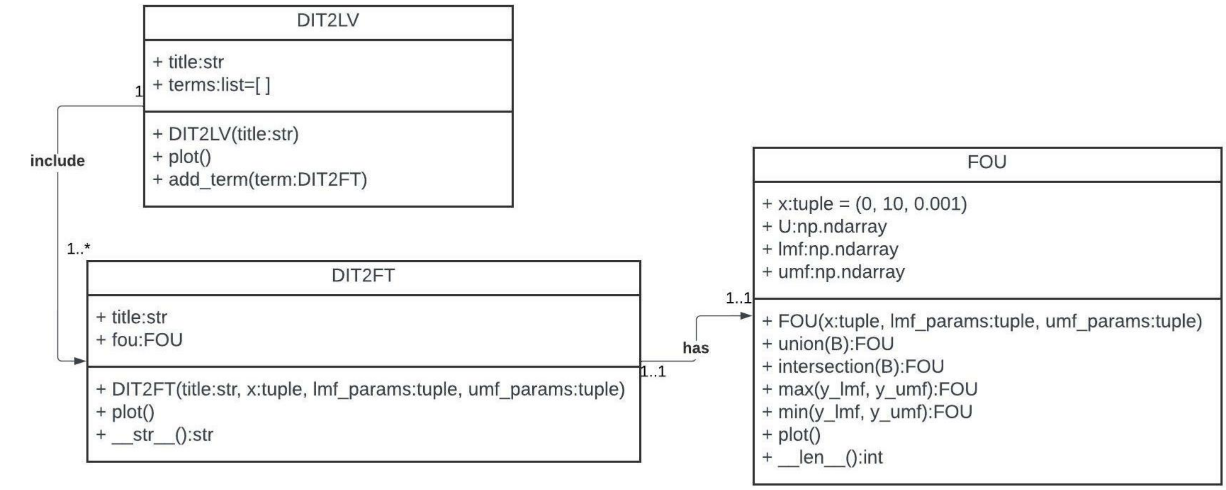

Figure 4: Class diagram FOU, DIT2FT, DIT2LV

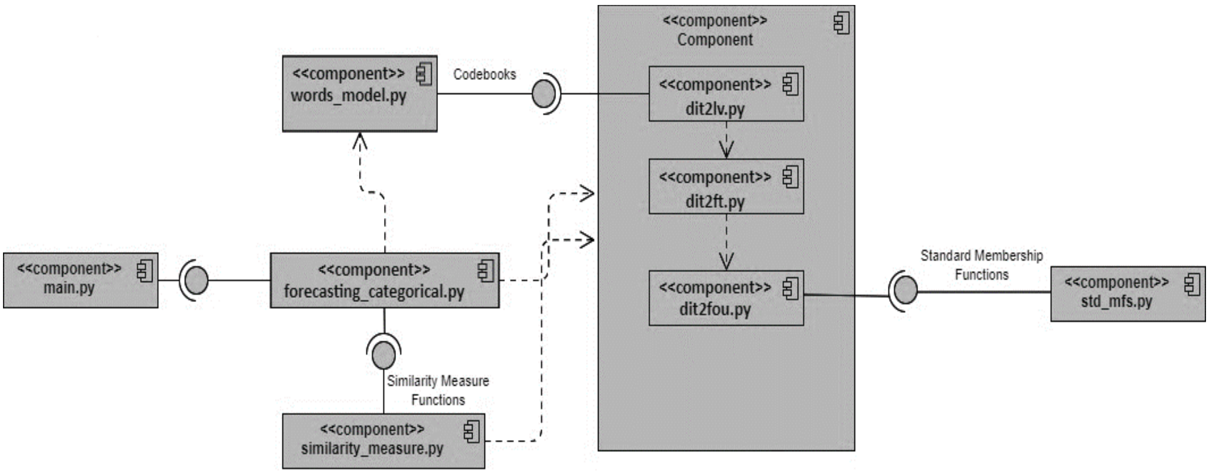

Figure 5: Diagram of the components of the developed software

A diagram of how the modules interact is shown in Figure 5. To use the developed software, run the script from the main.py module, specifying the necessary prediction parameters. An example of the Python code from the main.py module for the test case described in the next section is shown in Listing main.py (Figure 6).

7. Results and Discussion

To evaluate the obtained forecasting result, we again resort to replacing categorical values with their numerical equivalents. Figure 3 shows three rows of data: Real, Prediction and Error (Error = Real

– Prediction, mean(Error) = –0,315, std(Error) = 0.912). Д To test the algorithm, a test set of 77 observations was taken. The graph of the fuzzy time series simulation result is shown in Figure 7.

```python
import forecasting_categorical


def read_data(file_name):
    with open(file_name, encoding='utf-8') as file:
        return [line.strip() for line in file]


if __name__ == '__main__':
    data = read_data('sleep.txt')
    categories = ['qt', 'qh', 'tr', 'al', 'ah', 'aw']
    results = forecasting_categorical.forecasting(data=data[:-5],
                                                  categories=categories,
                                                  model_words=f'words_{len(categories)}',
                                                  sm_method='jaccard',
                                                  steps=5)
    for item in results:
        print(item)
```

Figure 6: Listing of main.py

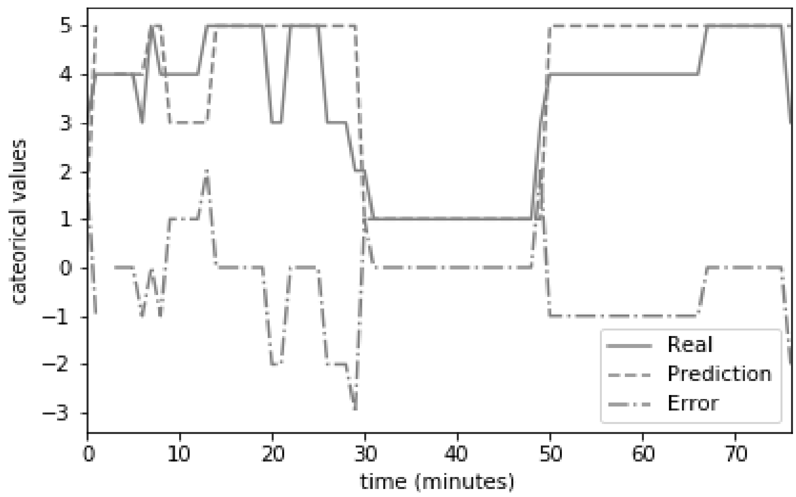

Figure 7: Graph of the fuzzy time series simulation result

The main estimated characteristics of the forecast model quality are the following indicators (5)-(7):

1) mean absolute error (MAE):

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|\varepsilon_t| = 0{,}605, \tag{5}$$

2) root mean squared error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}\varepsilon_t^2}{n-1}} = 0{,}959, \tag{6}$$

3) mean absolute percentage error (MAPE):

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|\varepsilon_t|}{y_t}\times 100\% = 18{,}179\%, \tag{7}$$

where ε_t – error, difference between predicted and true values in t period,
y_t – the actual (true) value, n – number of observations.
These indicators can be further used to compare forecasting models and choose the best one.

Thus, a forecast mathematical model of a fuzzy time series is developed, which is based on the basic principles of the theory of type-2 fuzzy sets and computing with words. A fuzzy algorithm for forecasting categorical time series based on the developed model is proposed. In the future, the developed model can be used to solve practical problems of computing with words in decision-making.

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