Mapping properties of heterogeneous ontologies

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Abstract. State of the art formalisms for distributed ontology integration provide ways to express semantic relations between concepts belonging to different ontologies. However, the extensive usage of multiple distributed ontologies requires the capability for expressing different forms of mappings, which extend the semantic relations between concepts studied so far. In this paper, we propose an extension of the formalism of Distributed Description Logic (DDL) to represent mappings between concepts as well as mappings between relations, and an effective decision procedure for reasoning with multiple ontologies bridged with these mappings.

1 Introduction

In the extensive usage of ontologies envisaged by the Semantic Web there is a compelling need for expressing mappings between heterogeneous ontologies. These mappings are of many different forms and involve the different components of ontologies.

Most of the formalisms for distributed ontology integration based on the p2p architecture, usually called *mapping languages* [11], provide ways to express semantic relations between concepts of different ontologies. Only few mapping languages allow also to express semantic relations between roles [6, 4]. These approaches define mappings between ontologies expressed in first order logics. From a theoretical point of view this is not a problem, even if ontologies are nowadays expressed in OWL and their syntax/semantics is based on Description Logics (DL). In fact, DLs can be translated in a fragment of first order logic [3], and we can therefore use the above mentioned formalism as mapping languages between ontologies. However, this translation makes the reasoning task more complex, and does not allow to reuse (or build on top of) the existing efficient reasoning tools for Description Logics. For this reason, we believe that is preferable to extend a DL-native formalism such as for instance DDL [13] and in $\mathcal{E}$-connection [10, 8] to express new forms of mappings, and in particular mappings between roles. In [7] we have extended the formalism of Distributed Description Logic (DDL) with primitives to specify two different sorts of mappings: homogeneous mappings, that is mappings between concepts and mappings between roles, and heterogeneous mappings, that is mappings between concepts and roles. In [7] neither an axiomatic characterization on the effects of mappings, nor a decision procedure to check concept satisfiability in presence of these extended mappings are produced.

The first goal of this paper is to study in depth the DDL obtained by considering homogeneous mappings. We have decided to focus first on homogeneous mappings because they are a very intuitive and popular form of mappings. The chapter on Ontology

Mappings of the OWL Web Ontology Language Guide [1], for instance, highlights the importance of expressing mappings between concepts as well as mappings between roles. Moreover, state of the art ontology mapping tools (see e.g., [12]) represent mappings between roles in addition to mappings between concepts of different ontologies. This highlights the need of expressive mapping languages able to represent semantic relations between concepts and semantic relations between roles of different ontologies.

The second goal of this paper is to define an effective decision procedure to compute subsumption between classes and roles in ontologies connected by homogeneous mappings, where the ontologies are expressed in the $\mathcal{SHIQ}$ language. This extension is done in the style of DDL: first we define a bridge operator, which expresses how subsumption migrates from an ontology O_1 to an ontology O_2 by means of the mappings, and then we use this operator to sketch how the distributed tableaux algorithm for DDL described in [13] can be extended to support reasoning with homogeneous mappings.

2 Role mapping primitives

Before introducing a new primitive, there are a number of questions that need to be answered. The first and more important question is that of whether such a primitive is expressible as a combination of already existing primitives, in our case mappings between concepts. In the following we show that this is not the case and we base our argument on the semantics of mappings described in [4].

Given two concepts C and D belonging to two ontologies O_1 and O_2 respectively, the mapping "C and D are equivalent" is satisfied when the first order formula $\forall x(C(x) \equiv D(x))$ is satisfied, where $C(x)$ and $D(x)$ are the translation in first order logic of the concepts C and D, defined as in [3]. Similarly, given two roles R and S belonging to O_1 and O_2, respectively, the semantics of the mapping "R and S are equivalent" is expressed by the first order formula: $\forall x(\forall y(R(x,y) \equiv S(x,y)))$ It is easy to see that in first order logic the second implication cannot be written in terms of the first. This does not change even if we adopt a semantics based on distributed domains such as the one proposed in [6]. This imply that we need a primitive that allows one to express the fact that a role R in an ontology is more/less general than a role R in another ontology.

The second question we have to answer is why we provide an expressive mapping language and a distributed decision procedure instead of encoding mappings as DL assertions over a unique top global ontology. The main reasons to propose a "distributed" semantics for DDL, which is based on multiple domains connected via domain relations (rather than a single domain) concerns the capability of DDL to capture the properties of *localized inconsistency* and *directionality* as shown in [13]. According to localized inconsistency, the inconsistency in one component ontology should not automatically propagate to all other ontologies that are integrated in a distributed system. According to the directionality property, semantic mappings have a direction from a source ontology to a target ontology, and support knowledge propagation only in that direction. The interested reader can refer to [13] for a detailed discussion which we omit for lack of space. In addition to these motivations which are common to all our work on DDL, there are also specific reasons to consider explicitly mappings between roles, instead of encoding them in DL assertions. As we show more in detail in Section 6, encoding mappings between roles in DL assertions requires the usage of the role composition operator, which is, in the general case, undecidable.

3 A language for mappings

Description Logic (DL) has been advocated as the suitable formal tool to represent and reason about ontologies. Distributed Description Logic [13] is a *natural* generalization of the DL framework designed to formalize multiple ontologies interconnected by semantic mappings. In DDL, ontologies correspond to description logic theories (T-boxes), while semantic mappings correspond to collections of *bridge rules* ($\mathfrak{B}$). In the following we recall the definitions of a DDL able to to express mappings between concepts as well as mappings between roles.

Given a non empty set I of indexes, used to identify ontologies, let $\{\mathcal{D}L_i\}_{i\in I}$ be a collection of description logics[1]. For each $i \in I$ let us denote a T-box of $\mathcal{D}L_i$ as $\mathcal{T}_i$. In this paper, we assume that each $\mathcal{D}L_i$ is weaker or at most equivalent to $\mathcal{SHIQ}$.

We call $\mathbf{T} = \{\mathcal{T}_i\}_{i\in I}$ a family of T-Boxes indexed by I. Intuitively, $\mathcal{T}_i$ is the description logic formalization of the i-th ontology. To make every description distinct, we will prefix it with the index of ontology it belongs to. For instance, the concept C that occurs in the i-th ontology is denoted as $i : C$. Similarly, $i : C \sqsubseteq D$ denotes the fact that the axiom $C \sqsubseteq D$ is being considered in the i-th ontology.

Semantic mappings between different ontologies are expressed via *bridge rules*. The bridge rules we consider in this paper are defined as follows:

Definition 1 (Bridge rules). *A bridge rule from i to j is an expression of the form:*

1. $i : X \xrightarrow{\sqsupseteq} j : Y$ *(*onto-bridge rule*)*
2. $i : X \xrightarrow{\sqsubseteq} j : Y$ *(*into-bridge rule*)*

where X and Y are either concepts of $\mathcal{D}L_i$ and $\mathcal{D}L_j$, or roles of $\mathcal{D}L_i$ and $\mathcal{D}L_j$. The expression $i : X \xrightarrow{\equiv} j : Y$ denotes the combination of onto and into bridge rules.

Definition 2 (Distributed T-box). *A distributed T-box (DTB) $\mathfrak{T} = \langle\{\mathcal{T}_i\}_{i\in I}, \mathfrak{B}\rangle$ consists of a collection of T-boxes $\{\mathcal{T}_i\}_{i\in I}$, and a collection of bridge rules $\mathfrak{B} = \{\mathfrak{B}_{ij}\}_{i\neq j\in I}$ between them.*

In [7] the bridge rules defined above are called *homogeneous bridge rules*, as they map homogeneous components of distributed ontologies (concepts with concepts and roles with roles). In that paper we introduce also heterogeneous bridge rules, that is mappings between different components of ontologies. For instance, mappings of concepts into (onto) roles and vice-versa. In this paper we concentrate on homogeneous bridge rules as a first step towards a DDL able to represent and reason with an extensive family of mappings.

Bridge rules from i to j express relations between i and j viewed from the *subjective* point of view of the j-th ontology. Bridge rules between concepts have been introduced

[1] We assume familiarity with Description Logic and related reasoning systems, described in [2].

and studied in [13]. Intuitively, the concept-into-concept bridge rule $i : A \xrightarrow{\sqsubseteq} j : B$ states that, from the j-th point of view the concept A in i is less general than its local concept B. Similarly, the concept-onto-concept bridge rule $i : A \xrightarrow{\sqsupseteq} j : B$ expresses the fact that, according to j, A in i is more general than B in j. Therefore, bridge rules from i to j provide the possibility of translating into j's ontology (under some approximation) the concepts of a foreign i's ontology. Note, that since bridge rules reflect a subjective point of view, bridge rules from j to i are not necessarily the inverse of the rules from i to j, and in fact bridge rules from i to j do not force the existence of bridge rules in the opposite direction. Thus, the bridge rule

$$i : \mathsf{Article} \xrightarrow{\sqsupseteq} j : \mathsf{ConferencePaper}$$

expresses the fact that, according to ontology j, the concept Article in ontology i is more general than its local concept ConferencePapers. Bridge rules on roles formalize the analogous intuition for roles. Thus, the bridge rule:

$$i : \mathsf{marriedTo} \xrightarrow{\sqsubseteq} j : \ \mathsf{partnerOf}$$

says that according to ontology j, the relation marriedTo in ontology i is less general than its own relation partnerOf.

The semantic of DDL, which is a customization of Local Models Semantics [5, 6], assigns to each ontology $\mathcal{T}_i$ a *local interpretation domain*. The first component of an interpretation of a DTB is a family of interpretations $\{\mathcal{I}_i\}_{i \in I}$, one for each T-box $\mathcal{T}_i$. Each $\mathcal{I}_i$ is called a *local interpretation* and consists of a *possibly empty* domain $\Delta^{\mathcal{I}_i}$ and a valuation function $\cdot^{\mathcal{I}_i}$, which maps every concept to a subset of $\Delta^{\mathcal{I}_i}$, and every role to a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$. Interpretations with empty domains are necessary to provide a semantics for *partially inconsistent* distributed T-boxes (see [13]). The second component of the semantics is a family of domain relations. Domain relations define how the different T-box interact and are necessary to define the satisfiability of bridge rules.

Definition 3 (Domain relation). *A domain relation r_{ij} from $\Delta^{\mathcal{I}_i}$ to $\Delta^{\mathcal{I}_j}$ is a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$. We use $r_{ij}(d)$ to denote $\{d' \in \Delta^{\mathcal{I}_j} \mid \langle d, d' \rangle \in r_{ij}\}$; for any subset D of $\Delta^{\mathcal{I}_i}$, we use $r_{ij}(D)$ to denote $\bigcup_{d \in D} r_{ij}(d)$; for any $R \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$ we use $r_{ij}(R)$ to denote $\bigcup_{\langle d,d' \rangle \in R} r_{ij}(d) \times r_{ij}(d')$.*

A domain relation r_{ij} represents a possible way of mapping the elements of $\Delta^{\mathcal{I}_i}$ into its domain $\Delta^{\mathcal{I}_j}$, seen from j's perspective. For instance, if $\Delta^{\mathcal{I}_1}$ and $\Delta^{\mathcal{I}_2}$ are the representation of time as Rationals and as Naturals, r_{ij} could be the round off function, or some other approximation relation. This function has to be conservative w.r.t., the order relations defined on Rationals and Naturals. Domain relations are used to interpret bridge rules according with the following definition.

Definition 4 (Satisfiability of bridge rules). *The domain relation r_{ij} satisfies a bridge rule $\mathfrak{b}$ w.r.t., $\mathcal{I}_i$ and $\mathcal{I}_j$, in symbols $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle \models \mathfrak{b}$, according with the following:*

1. $\langle \mathcal{I}_i, \mathcal{I}_j, r_{ij} \rangle \models i : X \xrightarrow{\sqsubseteq} j : X$, *if* $r_{ij}(X^{\mathcal{I}_i}) \subseteq Y^{\mathcal{I}_j}$

2. $\langle \mathcal{I}_i, \mathcal{I}_j, r_{ij} \rangle \models i : Y \xrightarrow{\sqsupseteq} j : Y$, *if* $r_{ij}(X^{\mathcal{I}_i}) \supseteq Y^{\mathcal{I}_j}$

where X *and* Y *are either two concept expressions or two role expressions.*

Definition 5 (Distributed interpretation). *A* distributed interpretation $\mathfrak{I} = \langle \{\mathcal{I}_i\}_{i \in I}, \{r_{ij}\}_{i \neq j \in I} \rangle$ *of a DTB* $\mathfrak{T}$ *consists of local interpretations* $\mathcal{I}_i$ *for each* $\mathcal{T}_i$ *on local domains* $\Delta^{\mathcal{I}_i}$*, and families of domain relations* r_{ij} *between local domains.*

Definition 6 (Satisfiability of a Distributed T-box). *A distributed interpretation* $\mathfrak{I}$ *satisfies the elements of a DTB* $\mathfrak{T}$ *according to the following clauses: for every* $i, j \in I$

1. $\mathfrak{I} \models i : A \sqsubseteq B$, *if* $\mathcal{I}_i \models A \sqsubseteq B$
2. $\mathfrak{I} \models \mathcal{T}_i$, *if* $\mathfrak{I} \models i : A \sqsubseteq B$ *for all* $A \sqsubseteq B$ *in* $\mathcal{T}_i$
3. $\mathfrak{I} \models \mathfrak{B}_{ij}$, *if* $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle$ *satisfies all the bridge rules in* $\mathfrak{B}_{ij}$
4. $\mathfrak{I} \models \mathfrak{T}$, *if for every* $i, j \in I$, $\mathfrak{I} \models \mathcal{T}_i$ *and* $\mathfrak{I} \models \mathfrak{B}_{ij}$

Definition 7 (Distributed Entailment and Satisfiability). $\mathfrak{T} \models i : C \sqsubseteq D$ *(read as "*$\mathfrak{T}$ *entails* $i : C \sqsubseteq D$*") if for every* $\mathfrak{I}$, $\mathfrak{I} \models \mathfrak{T}$ *implies* $\mathfrak{I} \models i : C \sqsubseteq D$. $\mathfrak{T}$ *is* satisfiable *if there exists a* $\mathfrak{I}$ *such that* $\mathfrak{I} \models \mathfrak{T}$. *Concept* $i : C$ *is* satisfiable *with respect to* $\mathfrak{T}$ *if there is a* $\mathfrak{I}$ *such that* $\mathfrak{I} \models \mathfrak{T}$ *and* $C^{\mathcal{I}_i} \neq \emptyset$.

4 The effects of mappings

In the previous section we have defined a declarative language (of mappings) which allows to state set-theoretic relations between the extensions of the concepts and roles in different ontologies (T-boxes). Mappings can be thought of as inter-theory axioms, which constrain the possible models of the theories representing the different ontologies. Thus, a set of mappings between two ontologies allows to combine the knowledge contained in the two ontologies in order to derive new knowledge. In this section we discuss the main effects of mapping in terms of the new ontological knowledge they allow to infer.

We characterize the effect of bridge rules in any distributed T-box, in two steps: first we characterize the mappings of a simple DTB of the form $\langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle$, composed of two T-boxes $\mathcal{T}_1$ and $\mathcal{T}_2$ and a set of bridge rules $\mathfrak{B}_{12}$ from $\mathcal{T}_1$ to $\mathcal{T}_2$. This allows us to define an operator $\mathfrak{B}_{12}(.)$ that encodes the propagation of knowledge provided by $\mathfrak{B}_{12}$. Following [13], the axiomatization of the general case $\mathfrak{T} = \langle \{\mathcal{T}_i\}_{i \in I}, \mathfrak{B} \rangle$ can be obtained as a fixed-point application of the operators $\mathfrak{B}_{ij}$ for each $i \neq j \in I$. This methodology is standard, and is not included here for lack of space.

The first effect of mappings is a general property which allows to identify the ontologies affected by given mappings. This property states that a mapping from a T-box $\mathcal{T}_1$ to a T-box $\mathcal{T}_2$ only affects the target ontology $\mathcal{T}_2$:

Proposition 1. $\langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle \models 1 : X \sqsubseteq Y$ *if and only if* $\mathcal{T}_1 \models X \sqsubseteq Y$.

The proof of this property is a trivial generalization of the one contained in [13]. According to Proposition 1 mappings go from a source ontology to a target ontology, and support knowledge propagation in this direction only.

In addition to this property, the effects of the bridge rules introduced in this paper can be divided in three main classes: (i) Propagation of the concept hierarchy; (ii) Propagation of the role hierarchy and of certain role properties; and (iii) Propagation of the role domain and of the range restriction. In the remaining of the section we describe these three forms of effects in detail.

4.1 Propagation of the concept hierarchy

The propagation of the concept hierarchy is forced by mappings between concepts and is widely described in [13]. It is summarized by the following operator

$$\mathfrak{C}_{12}(\mathcal{T}_1) = \left\{ G \sqsubseteq \bigsqcup_{k=1}^{n} H_k \middle| \begin{array}{l} \mathcal{T}_1 \models A \sqsubseteq \bigsqcup_{k=1}^{n} B_k, \\ 1 : A \xrightarrow{\sqsupseteq} 2 : G \in \mathfrak{B}_{12}, \\ 1 : B_k \xrightarrow{\sqsubseteq} 2 : H_k \in \mathfrak{B}_{12}, \text{ for } 1 \leq k \leq n \end{array} \right\}$$

where A, G, B_k and H_k are concepts. The simplest version of this operator states that if $\mathcal{T}_1 \models A \sqsubseteq B$ and if the mappings $1 : A \xrightarrow{\sqsupseteq} 2 : G$ and $1 : B \xrightarrow{\sqsubseteq} 2 : H$ hold between $\mathcal{T}_1$ and $\mathcal{T}_2$, then $G \sqsubseteq H$ is added to $\mathcal{T}_2$.

4.2 Propagation of the role hierarchy and of role properties

The first effect of mappings between roles concern the propagation of the role hierarchy. Let us start with an example: Suppose that ontology O_1 contains the role axioms

$$\mathsf{IsTheBossOf} \equiv \mathsf{HasBoss}^{-} \qquad (1)$$

$$\mathsf{IsTheBossOf} \sqsubseteq \mathsf{WorksWith} \qquad (2)$$

$$\mathsf{WorksWith} \equiv \mathsf{WorksWith}^{-} \qquad (3)$$

Suppose also that ontology O_2 contains the role IsSecretaryOf, representing the relation between the secretary and his/her boss, and the role KnowsTheNameOf representing the relations between two persons where the first knows the name of the second. Assume now that the roles of O_1 and O_2 are connected by the following mappings

$$O_1 : \mathsf{HasBoss} \xrightarrow{\sqsupseteq} O_2 : \mathsf{IsTheSecretaryOf} \qquad (4)$$

$$O_1 : \mathsf{WorkWith} \xrightarrow{\sqsubseteq} O_2 : \mathsf{KnowsTheNameOf} \qquad (5)$$

In this scenario we expect to be able to infer in O_2 the axiom IsTheSecretaryOf $\sqsubseteq$ KnowsTheNameOf about the role hierarchy. In fact, if x is the secretary of y, then by mapping (4) y is the boss of x. Using the role hierarchy in O_1, and in particular the fact (2), x works with y, which means that x knows the name of y because of mapping (5).

More in general, we can say that if $R \sqsubseteq S$ is a fact of the T-box $\mathcal{T}_1$, then the effect of the bridge rules $R : 1 \xrightarrow{\sqsupseteq} T : 2$ and $S : 1 \xrightarrow{\sqsubseteq} U : 2$ is that $T \sqsubseteq U$ and $T^{-} \sqsubseteq U^{-}$ are also facts in $\mathcal{T}_2$. Formally, we describe this effect by means of the following operator:

$$\mathfrak{R}_{12}(\mathcal{T}_1) = \left\{ T^{(-)} \sqsubseteq U^{(-)} \middle| \begin{array}{l} \mathcal{T}_1 \models R \sqsubseteq S, \\ 1 : R \xrightarrow{\sqsupseteq} 2 : T \in \mathfrak{B}_{12}, \\ 1 : S \xrightarrow{\sqsubseteq} 2 : U \in \mathfrak{B}_{12} \end{array} \right\}$$

where each R, T and S, U is either a role or an inverse role, and the notation $T^{(-)} \sqsubseteq U^{(-)}$ denotes either $T \sqsubseteq U$ or $T^- \sqsubseteq U^-$.

Additional effects of mappings between roles concern the propagation of the properties of relations across different ontologies. Let us consider first the property of symmetry. If R is a symmetric relation and $\mathsf{Symm}(R) \in \mathcal{T}_1$, then the mappings $1 : R \xrightarrow{\sqsubseteq} 2 : S$ and $1 : R \xrightarrow{\sqsupseteq} 2 : S$ force S also to be a symmetric relation in $\mathcal{T}_2$. This effect is already described by the operator $\mathfrak{R}_{12}$ defined above. In fact $\mathsf{Symm}(R)$ is equivalent to $R \sqsubseteq R^-$, and using $\mathfrak{R}_{12}$ with the two bridge rules between R and S above, one can infer $S \sqsubseteq S^-$ which states the symmetry of S in $\mathcal{T}_2$. If R is a transitive relation in $\mathcal{T}_1$, then the mappings $1 : R \xrightarrow{\sqsubseteq} 2 : S$ and $1 : R \xrightarrow{\sqsupseteq} 2 : S$ alone do not force S to be transitive in $\mathcal{T}_2$, and additional constraints are needed to enforce it. Since the propagation of transitivity seems a very intuitive requirement we are considering appropriate restrictions of the domain relation in order to obtain it from the bridge rules above. Finally, note that if the languages of $\mathcal{T}_1$ and $\mathcal{T}_2$ support the role union operator, then we obtain an extra propagation effect which is the analogous of the general concept propagation rule. Suppose that $\mathcal{T}_1 \models R \sqsubseteq S_1 \sqcup \cdots \sqcup S_n$, then the mappings $1 : R \xrightarrow{\sqsupseteq} 2 : T$ and $1 : S_k \xrightarrow{\sqsubseteq} 2 : U_k$ for $1 \leq k \leq n$ impose that $T \sqsubseteq U_1 \sqcup \cdots \sqcup U_n$ holds in $\mathcal{T}_2$.

4.3 Propagation of the role domain and of the range restriction

The simplest effect of the combination of mappings between roles and mappings between concepts is the propagation of domain and range restriction. Again, consider an example obtained by extending the scenario presented above with the axiom

$$\exists \mathsf{HasBoss}.\top \sqsubseteq \mathsf{Subaltern} \tag{6}$$

in ontology O_1 and the following mapping between concepts to the set of mappings:

$$O_1 : \mathsf{Subaltern} \xrightarrow{\sqsubseteq} O_2 : \mathsf{Employee} \tag{7}$$

In this case we expect that the subsumption $\exists \mathsf{IsTheSecretaryOf}^-.\top \sqsubseteq \mathsf{Employee}$ holds in O_2. Indeed, if x is the secretary of y, then by mapping (4) y is the boss of x, and by axiom (6) x is a subaltern. Thus, mapping (7) tells us that x is an employee.

More in general, if we assume that $\mathsf{Domain}(R) \sqsubseteq C$ (resp. $\mathsf{Range}(R) \sqsubseteq C$) is a fact in $\mathcal{T}_1$, then the mappings $1 : R \xrightarrow{\sqsupseteq} 2 : S$ and $1 : C \xrightarrow{\sqsubseteq} 2 : D$ force $\mathsf{Domain}(S) \sqsubseteq D$ (resp. $\mathsf{Range}(S) \sqsubseteq D$) in $\mathcal{T}_2$. The formalization of this effect is described by the operator $\mathfrak{CR}_{12}$:

$$\mathfrak{CR}_{12}(\mathcal{T}_1) = \left\{ \exists S^{(-)}.\,(\neg \textstyle\bigsqcup_{h=1}^{p} G_h) \sqsubseteq (\bigsqcup_{k=1}^{m} H_k) \left| \begin{array}{l} \mathcal{T}_1 \models \exists R^{(-)}.\,(\neg \bigsqcup_{h=1}^{p} A_h) \sqsubseteq (\bigsqcup_{k=1}^{m} B_k), \\ 1 : R \xrightarrow{\sqsupseteq} 2 : S \in \mathfrak{B}_{12}, \\ 1 : A_h \xrightarrow{\sqsubseteq} 2 : G_h \in \mathfrak{B}_{12}, \text{for } 1 \leq h \leq p \\ 1 : B_k \xrightarrow{\sqsubseteq} 2 : H_k \in \mathfrak{B}_{12}, \text{for } 1 \leq k \leq m \end{array} \right. \right\}$$

where A_h, G_h, B_k and H_k are concepts, and R and S are roles or inverse roles. As before the notation $R^{(-)}$ and $S^{(-)}$ stands for either R and S, or their inverse.

Notice that, range and domain propagation is a special case of the propagation formalized by $\mathfrak{CR}_{12}$. Indeed $\mathsf{Domain}(R) \sqsubseteq C$, corresponds to the axiom $\exists R.\top \sqsubseteq C$. By considering $p = 0$, $n = 1$, $H_1 = C$, and the mapping $1 : C \xrightarrow{\sqsubseteq} 2 : D$ we can obtain $\exists S.\top \sqsubseteq D$ from $\exists R.\top \sqsubseteq C$. [2] Similarly, the statement $\mathsf{Range}(R) \sqsubseteq C$ is equivalent to the axioms $\exists R^- \sqsubseteq C$ and it is translated into $\exists S^- \sqsubseteq D$ by the $\mathfrak{CR}_{12}$ operator.

4.4 Soundness and completeness

In this section we extend the proof given in [14] to consider bridge rules on roles, and their combined effect with bridge rules on concepts [3] Given a set of bridge rules $\mathfrak{B}_{12}$ from $\mathcal{D}L_1$ to $\mathcal{D}L_2$, we have defined three different operators taking as input a T-box in $\mathcal{D}L_1$ and producing a T-box in $\mathcal{D}L_2$. For the sake of presentation we assume that the set $\mathfrak{B}_{12}$ is closed under the operator of role inverse. That is, for each into or onto bridge rule between roles of the form $1 : R \xrightarrow{*} 2 : T$, we assume that the corresponding into or onto bridge rule $1 : R^- \xrightarrow{*} 2 : T^-$ also belongs to $\mathfrak{B}_{12}$. This allows us to drop the $^{(-)}$ notation in the bridge rules.

Given a set of bridge rules $\mathfrak{B}_{12}$ from $\mathcal{D}L_1$ to $\mathcal{D}L_2$, the operator $\mathfrak{B}_{12}(\cdot)$, taking as input a T-box in $\mathcal{D}L_1$ and producing a T-box in $\mathcal{D}L_2$, is defined as $\mathfrak{B}_{12}(\cdot) = \mathfrak{C}_{12}(\cdot) \cup \mathfrak{R}_{12}(\cdot) \cup \mathfrak{CR}_{12}(\cdot)$

Theorem 1. *Let $\mathfrak{T}_{12} = \langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle$ be a distributed T-box. where $\mathcal{T}_1$ and $\mathcal{T}_2$ are expressed in $\mathcal{SHIQ}$ descriptive language. Then:*

$$\mathfrak{T}_{12} \models 2 : X \sqsubseteq Y \Longleftrightarrow \mathcal{T}_2 \cup \mathfrak{B}_{12}(\mathcal{T}_1) \models X \sqsubseteq Y \tag{8}$$

Proof (of Theorem 1). As far as the soundness, it is enough to prove that if $\mathfrak{I} \models \mathfrak{T} = \langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle$, then $\mathcal{I}_2 \models \mathfrak{B}_{12}(\mathcal{T}_1)$. Suppose that $G \sqsubseteq H_1 \sqcup \cdots \sqcup H_n \in \mathfrak{C}_{12}$, then there is a mapping $1 : A \xrightarrow{\sqsupseteq} 2 : G$, and n mappings $1 : B_k \xrightarrow{\sqsubseteq} 2 : H_k$. If $x \in G^{\mathcal{I}_2}$, then by satisfiability of the onto-mapping there is $y \in A^{\mathcal{I}_1}$ with $(y, x) \in r_{12}$. The fact $\mathcal{T}_1 \models A \sqsubseteq B_1 \sqcup \cdots \sqcup B_n$, implies that $y \in B_k^{\mathcal{I}_1}$ for some k, and by the into-mapping, $x \in H_k$, and therefore in $H_1 \sqcup \cdots \sqcup H_n$. Analogous reasoning can be done for proving the soundness of $\mathfrak{R}_{12}(\mathcal{T}_1)$. Finally, suppose that $\exists S.\neg G \sqsubseteq H \in \mathfrak{CR}_{12}$ with $G = \sqcup_{h=1}^{p} G_k$ and $H = \sqcup_{k=1}^{m} H_k$. If $x \in (\exists S.\neg G)^{\mathcal{I}_2}$, then there is an x' such that $(x, x') \in S^{\mathcal{I}_2}$, and by the onto-mapping, $(y, y') \in R^{\mathcal{I}_1}$ for with $(y, x), (y', x') \in r_{12}$. By the into mappings we have that $y \notin A^{\mathcal{I}_1}$, and since $\mathcal{T}_1 \models \exists R.\neg A \sqsubseteq B$, we have that $y' \in B^{\mathcal{I}_1}$. This implies that $x' \in H^{\mathcal{I}_2}$.

To prove completeness, the contrapositive is proved. If $\mathcal{T}_2 \cup \mathfrak{B}_{12}(\mathcal{T}_1) \not\models X \sqsubseteq Y$ then there exists a distributed interpretation $\mathfrak{I}$ such that $\mathfrak{T}_{12} \not\models 2 : X \sqsubseteq Y$.

Let $\mathcal{I}_2$ be an interpretation of $\mathcal{T}_2 \cup \mathfrak{B}_{12}(\mathcal{T}_1)$ such that $\mathcal{I}_2 \not\models X \sqsubseteq Y$. The rest of the proof is devoted to the construction of an interpretation $\mathcal{I}_1$ and a domain relation r_{12} such that $\langle \mathcal{I}_1, \mathcal{I}_2, r_{12} \rangle$ is a distributed interpretation $\mathfrak{I}$ such that $\mathfrak{I} \models \mathfrak{T}_{12}$ and $\mathfrak{I} \not\models 2 : X \sqsubseteq Y$.

Let β be a concept-onto-concept bridge rule of the form $1 : A \xrightarrow{\sqsupseteq} 2 : G$. Starting from $\mathcal{I}_2$ and β, we define an interpretation $\mathcal{I}_\beta$, and a domain relation r_β as follows: for

[2] $\neg \bigsqcup_{h=1}^{0} G_h$ is equivalent to $\neg\bot$ which is equivalent to $\top$

[3] The concept of disjoint union of (distributed) interpretations used in the proofs is introduced and studied in [14].

every $d \in G^{\mathcal{I}_2}$, let us consider all the possible concept-into-concept bridge rules $1 : B_k \xrightarrow{\sqsubseteq} 2 : H_k$ in $\mathfrak{B}_{12}$ and indicate with $\mathbf{H} = \{H_1, \ldots, H_n\}$ the set of all subsequents H_k such that $d \notin H_k^{\mathcal{I}_2}$, and with $\mathbf{B} = \{B_1, \ldots, B_n\}$ the set of all antecedent B_k of the bridge rules $1 : B_k \xrightarrow{\sqsubseteq} 2 : H_k$ such that H_k belongs to $\mathbf{H}$. Intuitively the formulas $H_k \in \mathbf{H}$ are the ones such that $\mathcal{I}_2 \models G \sqsubseteq \bigsqcup_{H_k \in \mathbf{H}} H_k$ does not hold. This means that, $\mathcal{T}_1 \not\models A \sqsubseteq \bigsqcup_{B_k \in \mathbf{B}} B_k$. Let $\mathcal{I}_\beta^d$ as a model of $\mathcal{T}_1$ which satisfies the tree model property[4] such that $\mathcal{I}_\beta^d \not\models A \sqsubseteq \bigsqcup_{B_k \in \mathbf{B}} B_k$. Let d' be the object in $\Delta^{\mathcal{I}_\beta^d}$ such that $d' \in A^{\mathcal{I}_\beta^d}$ and $d' \notin (\bigsqcup_{B_k \in \mathbf{B}} B_k)^{\mathcal{I}_\beta^d}$. We define r_β^d as the pair $\langle d', d \rangle$.

Let β be a role-onto-role bridge rule of the form $1 : R \xrightarrow{\sqsupseteq} 2 : T$, and let $\overline{d} = \langle d_1, d_2 \rangle$ be a pair in $T^{\mathcal{I}_2}$. Let $\mathbf{U} = \{U_1, \ldots U_k\}$ be the set of all U_i such that $1 : S_k \xrightarrow{\sqsubseteq} 2 : U_k$ is a role-into-role bridge rule and $\langle d_1, d_2 \rangle \notin U_k^{\mathcal{I}_2}$, and $\mathbf{S} = \{S_1, \ldots S_k\}$ the set of the corresponding antecedents. Let also $\mathbf{G} = \{G_1 \ldots G_l\}$ be the set of all G_i such that $1 : A_k \xrightarrow{\sqsubseteq} 2 : G_k$ is a concept-into bridge rule and $d_2 \notin G_k^{\mathcal{I}_2}$ (and thus $d_1 \in \exists T.\neg G_k^{\mathcal{I}_2}$) and let $\mathbf{A} = \{A_1 \ldots A_l\}$ be the set of the corresponding antecedent . Finally, let $\mathbf{H} = \{H_1, \ldots, H_t\}$ be the set of concepts H_k such that $1 : B_k \xrightarrow{\sqsubseteq} 2 : H_k$ is a concept-into-concept bridge rule and $d_1 \notin H_k^{\mathcal{I}_2}$, and let $\mathbf{B} = \{B_1, \ldots, B_t\}$ the set of corresponding antecedents.

This implies that there is a model $\mathcal{I}_\beta^{\overline{d}}$ of $\mathcal{T}_1$ such that: $\mathcal{I}_\beta^{\overline{d}} \not\models R \sqsubseteq S_i$, for all $S_i \in \mathbf{S}$, and $\mathcal{I}_\beta^{\overline{d}} \not\models \exists R.\neg \bigsqcup_{A_k \in \mathbf{A}} A_k \sqsubseteq \bigsqcup_{B_k \in \mathbf{B}} B_k$. Let us indicate with $\langle d_1', d_2' \rangle \in \Delta^{\mathcal{I}_\beta^{\overline{d}}} \times \Delta^{\mathcal{I}_\beta^{\overline{d}}}$ a pair such that: $\langle d_1', d_2' \rangle \in R^{\mathcal{I}_\beta^{\overline{d}}}$ and $\langle d_1', d_2' \rangle \notin S_i^{\mathcal{I}_\beta^{\overline{d}}}$ for all $S_i \in \mathbf{S}$; and $d_1' \in (\exists R.\neg \bigsqcup_{A_k \in \mathbf{A}} A_k)^{\mathcal{I}_\beta^{\overline{d}}}$ and $d_1' \notin (\bigsqcup_{B_k \in \mathbf{B}} B_k)^{\mathcal{I}_\beta^{\overline{d}}}$, We define $r_\beta^{\overline{d}}$ as the set $\{\langle d_1', d_1 \rangle, \langle d_2', d_2 \rangle\}$.

We define $\mathcal{I}_1$ as the disjoint union of the following two models

$$\biguplus_{\substack{1:A \xrightarrow{\sqsupseteq} 2:G \in \mathfrak{B}_{12} \\ d \in G^{\mathcal{I}_2}}} \mathcal{I}^d_{1:A \xrightarrow{\sqsupseteq} 2:G} \qquad \biguplus_{\substack{1:R \xrightarrow{\sqsupseteq} S:G \in \mathfrak{B}_{12} \\ (d,d') \in S^{\mathcal{I}_2}}} \mathcal{I}^{(d,d')}_{1:R \xrightarrow{\sqsupseteq} 2:S}$$

and r_{12} is the disjoint union of the following two relations

$$\biguplus_{\substack{1:A \xrightarrow{\sqsupseteq} 2:G \in \mathfrak{B}_{12} \\ d \in G^{\mathcal{I}_2}}} r^d_{1:A \xrightarrow{\sqsupseteq} 2:G} \qquad \biguplus_{\substack{1:R \xrightarrow{\sqsupseteq} S:G \in \mathfrak{B}_{12} \\ (d,d') \in S^{\mathcal{I}_2}}} r^{(d,d')}_{1:R \xrightarrow{\sqsupseteq} 2:S}$$

The last step is verify that the distributed interpretation $\langle \mathcal{I}_1, \mathcal{I}_2, r_{12} \rangle$ we have constructed satisfies $\mathfrak{T}$.

T-boxes $\mathcal{I}_2 \models \mathcal{T}_2$ by definition, $\mathcal{I}_1 \models \mathcal{T}_1$, as each $\mathcal{I}_\beta^d$ and $\mathcal{I}_\beta^{(d,d')}$ models $\mathcal{T}_1$.

Onto-bridge rules Let $\beta' = 1 : X \xrightarrow{\sqsupseteq} 2 : Y$ be some onto-rule. By construction, there is interpretation $\mathcal{I}_{\beta'}$ such that $r_{\beta'}(X^{\mathcal{I}_{\beta'}}) \supseteq Y^{\mathcal{I}_2}$. Since $A^{\mathcal{I}_1} = \biguplus_\beta A^{\mathcal{I}_\beta}$, and $r_{12}(A^{\mathcal{I}_1})$ contains $\biguplus_\beta (r_\beta(A^{\mathcal{I}_\beta}))$ by construction, we have the desired result.

[4] This exists since $\mathcal{SHIQ}$ satisfies the tree model property.

Into-bridge rules $B^{\mathcal{I}_1} = \biguplus_\beta B^{\mathcal{I}_\beta}$, By construction, for each β, $r_\beta(B^{\mathcal{I}_\beta}) \subseteq H^{\mathcal{I}_2}$ and therefore $\bigcup_\beta r_\beta(B^{\mathcal{I}_\beta}) \subseteq H^{\mathcal{I}_2}$. Which implies that $r_{12}(B^{\mathcal{I}_\beta}) \subseteq H^{\mathcal{I}_2}$.

5 A distributed tableaux algorithm for DDL

In this section we use the previous theoretical results to define a tableaux-based decision procedure for $\mathfrak{T} \models i : X \sqsubseteq Y$, for the case of simple distributed T-boxes of the form $\langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle$. The generalization to the case of general acyclic DTBs is similar to the one presented in [13], while to prevent the infinite looping due to the possible cycles in the mappings, we require that all the **DTab** procedures are named with a unique identifier. Intuitively, the identifiers are used for all sub-requests (subsumption or instantiation) and are necessary to check if a certain request, after traversing the distributed knowledge base via mappings, returns back to the initial tableau, and needs to be blocked. With no loss of generality we can also assume that the consequences of bridge rules are atomic [5]. In addition we use the usual notion of axiom internalization, as in [9]: given a T-box $\mathcal{T}_i$, the concept $C_{\mathcal{T}_i}$ is defined as $C_{\mathcal{T}_i} = \bigsqcap_{E \sqsubseteq D \in \mathcal{T}_i} \neg E \sqcup D$; also, the role hierarchy $R_{\mathcal{T}_i}$ contains the role axioms of $\mathcal{T}_i$, plus additional axioms $P \sqsubseteq U$, for each role P of $\mathcal{T}_i$, with U some fresh role.

We follow the approach proposed in [13]. The key idea is to build a decision procedure which is distributed among the different ontologies, and combines local decision procedures, that is, procedures for testing the satisfiability of concept expressions, in specific ontologies. More in detail, the algorithm for testing j-satisfiability of a concept expression X (i.e., checking $\mathfrak{T} \not\models_\epsilon j : X \sqsubseteq \bot$) builds, as usual, a finite representation of a distributed interpretation $\mathfrak{I}$, by running local *autonomous* $\mathcal{SHIQ}$ tableaux procedures to find each local interpretation $\mathcal{I}_i$ of $\mathfrak{I}$.

Definition 8. *The function* $\mathbf{DTab}_2$ *takes as input a concept* X *and tries to build a representation of* $\mathcal{I}_2$ *with* $X^{\mathcal{I}_2} \neq \emptyset$ *(called a* completion tree *[9]) for the concept* $X \sqcap C_{\mathcal{T}_2} \sqcap \forall U.C_{\mathcal{T}_2}$*, using the* $\mathcal{SHIQ}$ *expansion rules, w.r.t. the role hierarchy* $R_{\mathcal{T}_2}$*, plus the "bridge" expansion rules described in Figure 1.*

The idea of these rules is inspired by the corresponding operators introduced in Section 4.4. Rule Unsat-$\mathfrak{C}_{12}$ corresponds to the operator $\mathfrak{C}_{12}(\cdot)$, and was first introduced in [13]. The idea behind this rule is that whenever $\mathbf{DTab}_2$ encounters a node x that contains a label G which is a consequence of a concept-onto-concept bridge rule, then if $G \sqsubseteq \sqcup\mathbf{H}$ is entailed by the bridge rules, the label $\bigsqcup \mathbf{H}$, is added to x. To determine if $G \sqsubseteq \sqcup\mathbf{H}$ is entailed by the bridge rules in $\mathfrak{B}_{12}$, $\mathbf{DTab}_2$ invokes $\mathbf{DTab}_1$ on the satisfiability of the concept $A \sqcap \neg(\sqcup\mathbf{B})$. $\mathbf{DTab}_1$ will build (independently from $\mathbf{DTab}_2$) an interpretation $\mathcal{I}_1$, in a manner similar to the one illustrated in Figure 2.

Rule Unsat-$\mathfrak{R}_{12}$ corresponds to the operator $\mathfrak{R}_{12}(\cdot)$. The idea behind this rule is that whenever $\mathbf{DTab}_2$ encounters a node x with a T-neighbour y, and T is a consequence of a role-onto-role bridge rule $1 : R \xrightarrow{\sqsupseteq} 2 : T$, then if $T \sqsubseteq U$ is entailed by the bridge rules, y is also a U-neighbour of x. To determine if $T \sqsubseteq U$ is entailed by the bridge rules $\mathfrak{R}_{12}$, we simply check if $S \sqsubseteq R$ belongs to the role hierarchy of $\mathcal{T}_1$ for all

[5] Non atomic mappings can easily be modeled by introducing names for the complex concepts.

Unsat-$\mathfrak{C}_{12}$-rule

if 1. $G \in \mathcal{L}(x)$, $1 : A \xrightarrow{\sqsupseteq} 2 : G \in \mathfrak{B}_{12}$, and
2. $\mathit{IsSat}_1(A \sqcap \neg \bigsqcup \mathbf{B}') = \mathit{False}$, for some $\mathbf{H}' \not\subseteq \mathcal{L}(x)$,
then $\mathcal{L}(x) \longrightarrow \mathcal{L}(x) \cup \{\bigsqcup \mathbf{H}'\}$

New-$\mathfrak{C}_{12}$-rule

if 1. $G \in \mathcal{L}(x)$, $1 : A \xrightarrow{\sqsupseteq} 2 : G \in \mathfrak{B}_{12}$, and
2. $\mathbf{B} \subseteq \{B | 1 : B \xrightarrow{\sqsubseteq} 2 : H \in \mathfrak{B}_{12}\}$, and
3. for no $\mathbf{B}' \subseteq \mathbf{B}$ is $\mathit{IsSat}_1(A \sqcap \neg \bigsqcup \mathbf{B}') = \mathit{False}$, and
4. for no $\mathbf{B}' \supseteq \mathbf{B}$ is $\mathit{IsSat}_1(A \sqcap \neg \bigsqcup \mathbf{B}') = \mathit{True}$,
then if $\mathbf{DTab}_1(A \sqcap \neg \bigsqcup \mathbf{B}) = \mathit{Satisfiable}$
then $\mathit{IsSat}_1(A \sqcap \neg \bigsqcup \mathbf{B}) = \mathit{True}$
else $\mathit{IsSat}_1(A \sqcap \neg \bigsqcup \mathbf{B}) = \mathit{False}$

Unsat-$\mathfrak{R}_{12}$-rule

if 1. y is a T-neighbour of x , $1 : R \xrightarrow{\sqsupseteq} 2 : T \in \mathfrak{B}_{12}$, and
2. $R \sqsubseteq S \in R_{\mathcal{T}_1}$, with $1 : S \xrightarrow{\sqsubseteq} 2 : U \in \mathfrak{B}_{12}$
then y is a U-neighbour of x

Unsat-$\mathfrak{CR}_{12}$-rule

if 1. y is a S-neighbour of x , $1 : R \xrightarrow{\sqsupseteq} 2 : S \in \mathfrak{B}_{12}$, $\neg \bigcup \mathbf{G} \subseteq \mathcal{L}(y)$ and
2. $\mathit{IsSat}_1(\exists R.\neg \bigcup \mathbf{A} \sqcap \neg \bigsqcup \mathbf{B}') = \mathit{False}$, for some $\mathbf{H}' \not\subseteq \mathcal{L}(x)$,
then $\mathcal{L}(x) \longrightarrow \mathcal{L}(x) \cup \{\bigsqcup \mathbf{H}'\}$

New-$\mathfrak{CR}_{12}$-rule

if 1. y is a S-neighbour of x , $1 : R \xrightarrow{\sqsupseteq} 2 : S \in \mathfrak{B}_{12}$, and
2. $\mathbf{A} \subseteq \{A | 1 : A \xrightarrow{\sqsubseteq} 2 : G \in \mathfrak{B}_{12}$ and $\neg G \in \mathcal{L}(y)\}$, and
3. $\mathbf{B} \subseteq \{B | 1 : B \xrightarrow{\sqsubseteq} 2 : H \in \mathfrak{B}_{12}\}$, and
3. for no $\mathbf{B}' \subseteq \mathbf{B}$ is $\mathit{IsSat}_1(\exists R.\neg \bigcup \mathbf{A} \sqcap \neg \bigsqcup \mathbf{B}') = \mathit{False}$, and
4. for no $\mathbf{B}' \supseteq \mathbf{B}$ is $\mathit{IsSat}_1(\exists R.\neg \bigcup \mathbf{A} \sqcap \neg \bigsqcup \mathbf{B}') = \mathit{True}$,
then if $\mathbf{DTab}_1(\exists R.\neg \bigcup \mathbf{A} \sqcap \neg \bigsqcup \mathbf{B}) = \mathit{Satisfiable}$
then $\mathit{IsSat}_1(\exists R.\neg \bigcup \mathbf{A} \sqcap \neg \bigsqcup \mathbf{B}) = \mathit{True}$
else $\mathit{IsSat}_1(\exists R.\neg \bigcup \mathbf{A} \sqcap \neg \bigsqcup \mathbf{B}) = \mathit{False}$

Fig. 1. Additional expansion rules for $\mathbf{DTab}_2$

role-into-role bridge rules $1 : S \xrightarrow{\sqsubseteq} 2 : U \in \mathfrak{B}_{12}$. This because $\mathcal{SHIQ}$ does not allow any reasoning on roles.

Rule Unsat-$\mathfrak{CR}_{12}$ corresponds to the operator $\mathfrak{CR}_{12}(\cdot)$. The idea behind this rule is that whenever $\mathbf{DTab}_2$ encounters a node x with a S-neighbour y, and S is a consequence of a role-onto-role bridge rule $1 : R \xrightarrow{\sqsupseteq} 2 : S$, then if y contains a label $\neg G_i$ which is a consequence of a concept-into-concept bridge rule, then if $\exists S.\neg G \sqsubseteq H$ is entailed by the bridge rules, the label H, is added to x. To determine if $\exists S.G \sqsubseteq H$ is entailed by the bridge rules $\mathfrak{B}_{12}$, $\mathbf{DTab}_2$ invokes $\mathbf{DTab}_1$ on the satisfiability of the concept $\exists R.\neg A \sqcap \neg B$. $\mathbf{DTab}_1$ will build (independently from $\mathbf{DTab}_2$) an interpretation $\mathcal{I}_1$, in a manner similar to the construction illustrated in Figure 2.

To avoid redundant calls, $\mathbf{DTab}_2$ caches the calls to $\mathbf{DTab}_1$ in a data structure IsSat_1, which caches the subsumption propagations that have been computed so far.

Theorem 2 (Termination,Soundness, and completeness). $\mathbf{DTab}_2(X)$ *terminates, and* $2{:}X$ *is satisfiable in* $\mathfrak{T}_{12}$ *if and only if* $\mathbf{DTab}_2(X)$ *can generate a complete and clash-free completion tree.*

Example 1. To clarify how the distributed tableaux works let us consider the DTBox $\mathfrak{T}_{12} = \langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle$ where $\mathcal{T}_1$ contains the axioms $\exists R.\neg A_1 \sqsubseteq B_1$ and $\exists R.\neg A_2 \sqsubseteq B_2$, $\mathcal{T}_2$ does not contain any axiom, and $\mathfrak{B}_{12}$ contains the following bridge rules:

$$1 : R \xrightarrow{\sqsupseteq} 2 : S \quad (9)$$

$$1 : A_1 \xrightarrow{\sqsubseteq} 2 : G_1 \quad (10) \qquad 1 : B_1 \xrightarrow{\sqsubseteq} 2 : H_1 \quad (12)$$

$$1 : A_2 \xrightarrow{\sqsubseteq} 2 : G_2 \quad (11) \qquad 1 : B_2 \xrightarrow{\sqsubseteq} 2 : H_2 \quad (13)$$

Let us show that $\mathfrak{T}_{12} \models_d 2 : \exists S.\neg(G_1 \sqcup G_2) \sqsubseteq H_1 \sqcap H_2$, i.e. that for any distributed interpretation $\mathfrak{I} = \langle \mathcal{I}_1, \mathcal{I}_2, r_{12} \rangle$, $(\exists S.\neg(G_1 \sqcup G_2))^{\mathcal{I}_2} \subseteq (H_1 \sqcap H_2)^{\mathcal{I}_2}$.

1. Suppose that by contradiction there is an $x \in \Delta_2$ such that $x \in (\exists S.\neg(G_1 \sqcup G_2))^{\mathcal{I}_2}$ and $x \notin (H_1 \sqcap H_2)^{\mathcal{I}_2}$.
2. Then there exists a y such that $\langle x, y \rangle \in S^{\mathcal{I}_2}$ and $y \notin G_1^{\mathcal{I}_2}$, $y \notin G_2^{\mathcal{I}_2}$, and either $x \notin H_1^{\mathcal{I}_2}$ or $x \notin H_2^{\mathcal{I}_2}$.
3. Because of the bridge rule (9) there is a pair $\langle x', y' \rangle \in \Delta_1$ such that $\langle r_{12}(x'), r_{12}(y') \rangle = \langle x, y \rangle$ and $\langle x', y' \rangle \in R^{\mathcal{I}_1}$.
4. Let us consider the case where $x \notin H_1^{\mathcal{I}_2}$. From the fact that $y \notin G_1^{\mathcal{I}_2}$, then by the bridge rule (10), $y' \notin A_1^{\mathcal{I}_1}$.
5. Since $\exists R.\neg A_1 \sqsubseteq B_1$ is an axiom of $\mathcal{T}_1$, then $y' \in B_1^{\mathcal{I}_1}$, and by bridge rule (12) $y \in H_1^{\mathcal{I}_1}$. But this is a contradiction.
6. The case where $x \notin H_2^{\mathcal{I}_2}$ is analogous and we can conclude that $\mathfrak{T}_{12} \models_d 2 : \exists S.\neg(G_1 \sqcup G_2) \sqsubseteq H_1 \sqcap H_2$.

The above reasoning can be seen as a combination of a tableau in $\mathcal{T}_2$ with a tableau in $\mathcal{T}_1$. In Figure 2 we depict the construction of a tableau for $\mathcal{T}_2$ where dotted arrow lines represent the evolution of the tree while solid lines represent connections between neighbouring nodes. For the sake of space we depict only the tableau $\mathbf{Tab}_1(\neg B_1 \sqcap \exists R.\neg A_1)$. The tableau $\mathbf{Tab}_1(\neg B_2 \sqcap \exists R.\neg A_2)$ is analogous.

6 Related Work

The semantics of mappings proposed in this paper is in line with the semantic of mappings between relations proposed by Calvanese et al. in [4]. The only difference between our approach is that DDL admits multiple domains connected via domain relations rather than a single domain.

Role mappings can be encoded also in a unique top global ontology. To encode the semantics of role-mappings with heterogeneous domain, one needs to index each concept and role with the T-box it comes from, add concepts for local domains, and introduce a role R_{12} representing the domain relation from T-box 1 to T-box 2. The into-mapping $1 : R \xrightarrow{\sqsubseteq} 2 : S$ can be encoded with the role-axioms $R_{12}^{-} \circ R_1 \circ R_{12} \sqsubseteq S_2$.

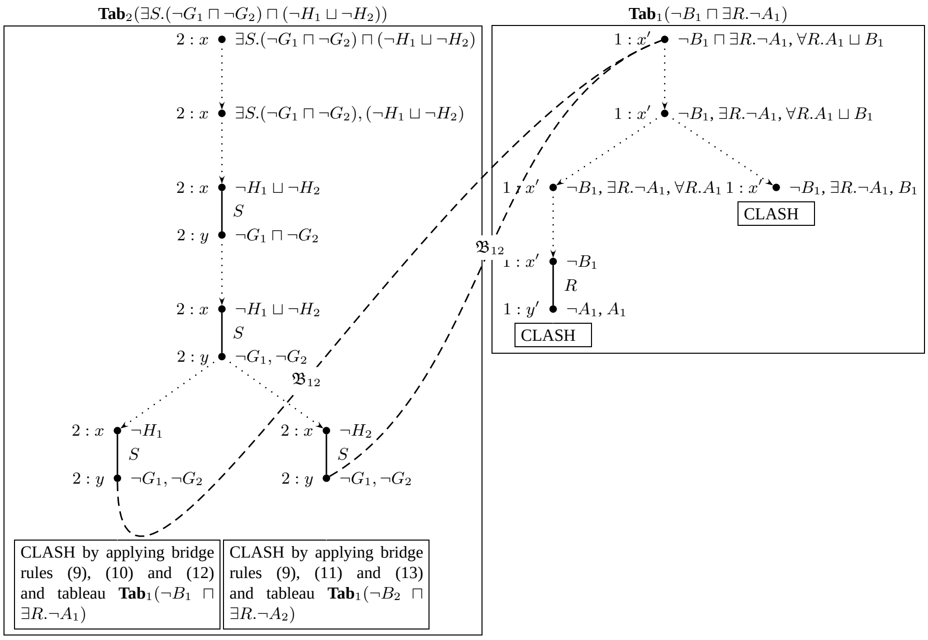

Fig. 2. An illustration of distributed tableaux for computation of DDL subsumption.

Similarly the onto mapping $1:R \xrightarrow{\sqsupseteq} 2:S$ is encoded with $S_2 \sqsubseteq R_{12}^{-} \circ R_1 \circ R_{12}$. The main drawback of this translation is that it is based on role composition, which in the general case is undecidable.

The formalism of $\mathcal{E}$-connections, proposed by Wolter et. al in, e.g., [10] allows to express mappings between concepts in different DL-based ontologies. To express mappings between roles in the formalism of $\mathcal{E}$-connections we need to consider the operator of role composition. Indeed, if we consider the translation of DDL into $\mathcal{E}$-connections proposed by [8], the translation of bridge rules makes explicit the domain relation r_{ij} in the syntax of $\mathcal{E}$-connections via a corresponding role, say R_{ij}. Using this role, they propose to translate onto and into bridge rules as $A \sqsubseteq \forall R_{ij}.B$ respectively $B \sqsubseteq \exists R_{ij}^{-}.A$. The semantic of mappings between roles cannot directly reproduced in $\mathcal{E}$-connection since there are no construct to state isa-hierarchy of $\mathcal{E}$-roles.

In comparing the formal setting described in this paper with ontology matching system such as Ontology Mapping Tool OMEN [12], we can say that the inferences supported by the bridge operators $\mathfrak{C}_{ij}, \mathfrak{R}_{ij}$ and $\mathfrak{C}\mathfrak{R}_{ij}$ allow to justify and explain, from a theoretical perspective, the heuristics inference rules implemented in such system. As an example, consider the following heuristic implemented in OMEN: If OMEN find that the concepts C_1 and D_1 are the domain and range of R in O_1 and C_2 and D_2 are the domain and range of S in O_2, then the fact that OMEN have maps C_1 in C_2 and D_1 in D_2 is an evidence that R is mapped into S. According to the formal semantics provided in this paper, this inference is justified since domain and ranges of roles are propagated by mappings between roles as explained in Section 4.

7 Concluding Remarks

The language, the semantics and the decision procedure presented in this paper constitute a genuine contribution in the direction of the integration of heterogeneous ontologies. The language proposed in this paper makes it possible to directly bind relations in different ontologies as well as concepts. vice-versa. The complete characterization of the logic together with the decision procedure presented in the paper make the logic ready to be implemented in a reasoning system for distributed ontologies, implementation that we leave for future work.

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