Query Answering over Fact Bases for Fuzzy Interval Ontologies

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Abstract. Temporal ontologies contain events that are concepts and roles with references to temporal intervals. Therefore, a temporal ontology induces the interval ontology. We consider fuzzy interval ontologies written in a fuzzy Boolean extension of Allen's interval logic. Syntactically, the extended logic **ELA** is the set of all Boolean combinations of propositional variables and sentences of Allen's interval logic. Semantics of **ELA** is defined using fuzzy interpretations of propositional variables and atomic sentences of Allen's logic. An interval ontology in **ELA** is a finite set **ELA** sentences (formulas). A fact is an estimate of a formula i.e. an expression of the form $r \leq \varphi \leq s$ where φ is a **ELA** formula and $0 \leq r \leq s \leq 1$. A fact base for an interval ontology is a finite set of facts with formulas from the ontology. We present a method of finding answers to queries addressed to fact bases for fuzzy interval ontologies. The method uses analytical tableaux.

Keywords: knowledge representation, ontologies, fuzzy ontologies, temporal logics, Allen's interval logic, query answering

1 Introduction

Temporal ontologies contain events that are concepts and roles with references to temporal intervals. Therefore, a temporal ontology induces the interval ontology. Consider an example.

Example 1. Suppose, we should define the structure of the concept *Agent* in some ontology for a multi-agent system. Then we may write declarations such as

Agent[*Name*: *String*, *Carry_out*: *Action*(*)],

Action[*Name*: *String*, *Interval*: (*Integer,Integer*), *Procedure*: *Program*].

The terms *Agent*(*Name*=*rob*07) and *Agent*(*Name*=*rob*07)`.`*Carry_out*`.`*Interval* denote the robot *rob*07 and the temporal intervals of the actions carrying out by *rob*07. Let the robot *rob*07 is able to carry out the actions *a*, *b* and *c*, i.e. *Agent*(*Name*=*rob*07)`.`*Carry_out* = {*a*, *b*, *c*}. These actions spend certain time. Thus, temporal intervals *A, B* and *C* are associated with the actions.

Suppose, there is the following knowledge about the intervals:

(1) If *p* is true then there is no time point at which both actions *a* and *b* are carried out;

(2) If *q* is true then the action *b* is carried out only when the action *c* is carried out. Consider the question:

(3) What Allen’s relations are impossible between the *C* and *A* if both conditions *p* and *q* take place?

In Allen’s interval logic (see [1, 2]) with implication, the statements (1) and (2) can be written as the interval ontology ***O*** $=\{p \rightarrow A$ ***bb*****B**, $q \rightarrow B$ ***edfs*** $C\}$(see further). The query (3) is written as $?x - p \wedge q \rightarrow C - x\, A$.

(*End of Example 1.*)

In Allen’s interval logic **LA**, there are 7 basic relations between intervals: ***e*** (equals), ***b*** (before), ***m*** (meets), ***o*** (overlaps), ***f*** (finishes), ***s*** (starts), ***d*** (during). (See Table 1 for interpretation of these relations, where A^- and A^+ denote the left and the right ends of the interval *A*). Let tr($A\ \theta\ B$) be the set of inequalities characterized of the basic Allen’s relation θ (see the third column of Table 1). For example, tr($A\ \boldsymbol{f}\ B$) = $\{A^- > B^-, A^+ \geq B^+, B^+ \geq A^+\}$.

Table 1. Basic relations of Allen’s interval logic

<table>
<tr><th>Interval relation</th><th>Illustration</th><th>Inequalities for the ends of intervals</th></tr>
<tr><td>A b B</td><td>|===A===| |===B===|</td><td>$B^- > A^+$</td></tr>
<tr><td>A m B</td><td>|===A===|=====B=====|</td><td>$A^+ \geq B^-$, $B^- \geq A^+$</td></tr>
<tr><td>A o B</td><td>|===A===|
|=====B=====|</td><td>$B^- > A^-, A^+ > B^-, B^+ > A^+$</td></tr>
<tr><td>A d B</td><td>|===A===|
|=====B=====|</td><td>$A^- > B^-, B^+ > A^+$</td></tr>
<tr><td>A s B</td><td>|===A===|
|=====B=====|</td><td>$A^- \geq B^-, B^- \geq A^-, B^+ > A^+$</td></tr>
<tr><td>A f B</td><td>|===A===|
|=====B=====|</td><td>$A^- > B^-, A^+ \geq B^+, B^+ \geq A^+$</td></tr>
<tr><td>A e B</td><td>|=====A=====|
|=====B=====|</td><td>$A^- \geq B^-$, $B^- \geq A^-, A^+ \geq B^+, A^+ \geq B^+$</td></tr>
</table>

The inverted relations are marked by asterisks: ***b**** (after), ***m**** (met-by), ***o**** (overlapped-by), ***f**** (finished-by), ***s**** (started-by), ***d**** (contains); so, $A\ \alpha^* B \Leftrightarrow B\ \alpha\ A$.

Let Ω_0 = {***e***, ***b***, ***m***, ***o***, ***f***, ***s***, ***d***} and $\Omega = \Omega_0 \cup$ { ***b****, ***m****, ***o****, ***f****, ***s****, ***d****} = {***e***, ***b***, ***m***, ***o***, ***f***, ***s***, ***d***, ***b****, ***m****, ***o****, ***f****, ***s****, ***d****}.

A *sentence* (*formula*) of **LA** is an expression of the form $A\ \omega\ B$ where ω is any subset of the set Ω and *A, B* are interval variables. If $\omega = \{\alpha\}$, then instead of $A\{\alpha\}B$ we write simply $A\ \alpha\ B$. If $\omega = \{\alpha_1, \alpha_2,\ldots, \alpha_k\}$ then we write $A\ \alpha_1\alpha_2\ldots\alpha_k\ B$ instead of $A\{\alpha_1, \alpha_2,\ldots, \alpha_k\}B$. By definition, the formula $A\ \alpha_1\alpha_2\ldots\alpha_k\ B$ is true if it is true at least one formula $A\ \alpha_i\ B$ ($1 \le i \le k$). The sentences of the form $A\ \alpha\ B$ with $\alpha \in \Omega_0$ are called *atomic*.

The fuzzy Boolean extension **ELA** Allen's interval logic is defined as follows.

SYNTAX of **ELA**:

• propositional variables are **ELA** formulas;

• every **LA** formula belongs to **ELA**, i.e. **LA** ⊆ **ELA**;

• if φ and ψ are **ELA** formulas then ~φ, φ ∧ ψ and φ ∨ ψ are also **ELA** formulas, and

φ → ψ is **ELA** formula considered as shorthand for ~φ ∨ ψ.

An (*interval*) *ontology* is a finite set of **ELA** formulas. Let P(***O***) be the set of all propositional variables entering the formulas from ***O***, and A(***O***) be the set of all atomic sentences entering the formulas from ***O***. Let B(***O***) = ⋃{tr(β) | β ∈ A(***O***)}. For example, if ***O*** = {*A* ***o*** *B* → (*B* ***mf*** *C*) ∧ *p*, *q* → *A* ***s*** *C*, *C* ***o**** *A*} then P(***O***) = {*p, q*} and A(***O***) = {*B* ***m*** *C*, *B* ***f*** *C*, *A* ***s*** *C*, *A* ***o*** *C*}, and B(***O***) = $\{B^+ \ge C^-, C^- \ge B^+, B^- > C^-, B^+ \ge C^+, C^+ \ge B^+, A^- \ge C^-, C^- \ge A^-, C^+ > A^+, C^- > A^-, A^+ > C^-, C^+ > A^+\}$.

SEMANTICS of **ELA** is defined using fuzzy interpretations.

A *fuzzy interpretation* of an ontology ***O*** is any function "..." from P(***O***) ∪ B(***O***) to $[0,1] = \{x \mid 0 \le x \le 1\}$ with the following constraints:

(a) If $X < Y$ and $Y \le X$ belong to B(***O***) then "$X < Y$" + "$Y \le X$" = 1;

(b) If $X = Y$, $X < Y$ and $Y < X$ belong to B(***O***) then "$X = Y$"+ max{"$X < Y$", "$Y < X$"}=1;

(c) If $X < Y$, $Y < Z$ and $X < Z$ belong to B(***O***) then "$X < Z$" ≥ min{"$X < Y$" ,"$Y < Z$"}, and the similar constraints which are obtained by replacing signs "<" by signs "≤" or "=".

We expand the function "..." to A(***O***) by "$A\ \theta\ B$"= min{"V" | $V \in$ tr($A\ \theta\ B$)}. Further, we expand "..." to formulas by the usual rules of Zadeh's fuzzy logic: "~ φ" = 1 –"φ", "φ ∧ ψ" = min{"φ","ψ"}, "φ ∨ ψ" = max{"φ","ψ"} [3].

Let r and s be numbers from [0,1] and φ be a **ELA** formula. Expressions of the forms $\varphi > r$, $\varphi \ge r$, $\varphi < r$ and $\varphi \le r$ are called *estimates* of the formula φ, and expressions of the form $r \le \varphi \le s$ (where $0 \le r \le s \le 1$) are called *bilateral estimates* of φ. The estimates are interpreted naturally. Let "..." be any interpretation of the ontology {φ}. Then "$\varphi > r$" $\Leftrightarrow_{df}$ "φ" > r, "$\varphi \ge r$" $\Leftrightarrow_{df}$ "φ" ≥ r, "$\varphi < r$" $\Leftrightarrow_{df}$ "φ" < r, "$\varphi \le r$" $\Leftrightarrow_{df}$ "φ" ≤ r, "$r \le \varphi \le s$" $\Leftrightarrow_{df}$ $r \le$ "φ" $\le s$.

The set **EST** of all estimates for **ELA** formulas can be considered as a crisp logic with fuzzy interpretations. As every logic, **EST** has the relation '|=" of *logical consequence*. Let ***E*** ⊆ **EST** and σ ∈ **EST.** We state ***E*** |= σ when there is no fuzzy interpretation "...": ***E*** → [0,1] such that all estimates from ***E*** are true but the estimate σ is false.

We consider estimates with the relation "≤" as *facts.* For any interval ontology ***O*** = $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ ($\varphi_i \in$ **ELA**), any set ***Fb*** = $\{r_1 \le \varphi_1 \le s_1, r_2 \le \varphi_2 \le s_2, \ldots, r_n \le \varphi_n \le s_n\}$ ($0 \le r_i, s_i \le 1$) of bilateral estimates is called a *fact base* for the ontology ***O***.

We can query a fact base and get the appropriate answers. Let $\psi = \psi[x_1, x_2, \ldots, x_n]$ be an ELA formula in which some of its Allen's connectives are replaced with variables $x_1, x_2, \ldots, x_n$ whose values are in Ω. A *query* is an expression of the form

$$?\ (x_1, x_2, \ldots, x_m) - \psi[x_1, x_2, \ldots, x_m], \quad (1.1)$$

where $\psi = \psi[x_1, x_2,\ldots, x_n]$ is an **ELA** formula in which some of its Allen's connectives are replaced with variables $x_1, x_2,\ldots, x_n$ whose values are in Ω. (For example, the expression $?(x_1, x_2) – (p \vee A$ ***bs*** $B) \rightarrow B\ x_1$***od*** $C \wedge {\sim}A\ x_2\ D$ is a query.)

The *answer* to query (1.1), addressed to the fact base ***Kb,*** is the set of all tuples $(g, h; \alpha_1, \alpha_2,\ldots, \alpha_m)$ with $\alpha_i \in \Omega$ and $g, h \in [0,1]$ such that ***Kb*** $\models g \leq \psi[\alpha_1, \alpha_2,\ldots, \alpha_m] \leq h$ with maximal g and minimal h. So, we have $g = \max\{r \mid$ ***Kb*** $\models r \leq \psi[\alpha_1, \alpha_2,\ldots, \alpha_m]\}$and $g = \min\{s \mid$ ***Kb*** $\models \psi[\alpha_1, \alpha_2,\ldots, \alpha_m] \leq s\}$.

Remarks. 1) It is easy to prove that the maximum and the minimum exist. 2) Since "$\varphi \leq r$" ⇔ "φ" $\leq r$ ⇔ 1–"φ" $\geq 1 - r$ ⇔ "~ φ" $\geq r$ ⇔ "~ $\varphi \geq 1- r$" and "$r \leq \varphi \leq s$" ⇔ $r \leq$ "φ" $\leq s$ ⇔ $r \leq$ "φ", "φ" $\leq s$ ⇔ "$\varphi \geq r$", "~ $\varphi \geq 1- s$", then any fact base with bilateral estimates is equivalent to a fact base with lower estimates i.e. of the form $\varphi_i \geq r_i$. We will consider further only fact bases with lower estimates.

Example 3. Consider the ontology ***O*** from Example 1 as a fuzzy ontology with the fact base ***Fb*** = {$p \rightarrow A$ ***bb*****B** ≥ 0.6, $q \rightarrow B$ ***edfs*** $C \geq 0.9$}. In the next section we show that the set {(0.6, ***d***), (0.6, ***e***), (0.6, ***f***), {(0.6, ***s***)} is the answer to the query $?x – p \wedge q \rightarrow {\sim}\ C\ x\ A$.

(*End of Example 3*.)

Generally, we can associate with any fuzzy logic the crisp logic of estimates whose sentences are expressions of the form $r \leq \varphi \leq s$ where φ are formulas of the fuzzy logic and $0 \leq r \leq s \leq 1$. Umberto Straccia have studied a fuzzy description logic which are the logics of estimates for description logics [4, 5]. The logic of estimates for propositional logic was considered in [6] where the method of query answering over fact bases was described.

In the paper, we present the method (based on analytical tableaux [6]) for finding the answers to queries addressed a fact base for an interval ontology.

2 Finding Answers to Queries Addressed to a Fact Base

The method of analytical tableaux can be applied to the problem of finding answers to queries addressed to fact bases for fuzzy interval ontologies. We show, by example, how to do this.

Example 3. Consider again the interval ontology ***O*** and the its fact base from Example 2: ***Fb*** = {$p \rightarrow A$ ***bb****$B \geq 0.6$, $q \rightarrow B$ ***edfs*** $C \geq 0.9$}. In Fig.1, it is shown the deduction tree constructed step by step from ***Fb*** and the estimate $p \wedge q \rightarrow {\sim}\ C\ x\ A < g$ which is corresponded to the body of the query $?x – p \wedge q \rightarrow {\sim}\ C\ x\ A$.

Constructing the deduction tree, we start with the initial branch containing the formulas $p \rightarrow A$ ***bb****$B \geq 0.6$, $q \rightarrow B$ ***edfs*** $C \geq 0.9$. At the first step we apply the rule from Table 2 in the fourth row and second column (denote by T2(4,2) this rule) to the formula $p \rightarrow A$ ***bb****$B \geq 0.6$ and we put the label "[1]" on the right of the formula. As a result of the application of the rule T2(4,2), the "fork" with the estimates $p \leq 0.4$ and A ***bb****$B \geq 0.6$ are added to the initial branch and the label "1:" is put on the left of each of the estimates. At the step 2, the rule T2(4,2) is applied to $q \rightarrow B$ ***edfs*** $C \geq 0.9$. As a result, the "fork" with the estimates $q \leq 0.1$ and B ***edfs*** $C \geq 0.9$ are added to each of two current branches. At the step 8, the rule T8(1,2) is applied to the estimates $q \leq$

0.1 and $q > 1 - g$. As a result, we get the inequality $g \leq 0.9$ that means the estimates $q \leq 0.1$ and $q \geq 1 - g$ are inconsistent (and therefore, the first branch is inconsistent) if and if $g \leq 0.9$. At step 9, the rule T8(1,2) is applied to the estimates $p \leq 0.4$ and $p > 1 - g$. As a result, we obtain that the second branch is inconsistent if and only if $g \leq 0.6$. At step 10, the rule T8(1,2) is applied to the estimates $q \leq 0.1$ and $q \geq 1 - g$. As a result, we obtain that the third branch is inconsistent if and only if $g \leq 0.9$. Thus, the first, second and third branch are inconsistent if and only if $g \leq \min\{0.9, 0.6, 0.9\} = 0.6$.

At step 12, the rule T4(1,1) is applied to the estimates B ***edfs*** $C \geq 0.9$ and B ***edfs*** $C \geq 0.9$, and as result, the estimate A ***bb*dfmm*oo*s*** $C \geq 0.6$ is obtained. Indeed, using Table 4 which is a fragment of the Allen's table of compositions (see [2]), we have

bb* ∘ ***edfs*** = ***b*** ∘ ***e*** ∪ ***b*** ∘ ***d*** ∪ ***b*** ∘ ***f*** ∪ ***b*** ∘ ***s*** ∪ ***b**** ∘ ***e*** ∪ ***b**** ∘ ***d*** ∪ ***b**** ∘ ***f*** ∪ ***b**** ∘ ***s*** =
b ∪ ***bdmos*** ∪ ***bdmos*** ∪ ***b*** ∪ ***b**** ∪ ***b*dfm*o**** ∪ ***b**** ∪ ***b*dfm*o**** =
bb*dfmm*oo*s.

At step 13, the rule T4(1,3) is applied to the estimate A ***bb*dfmm*oo*s*** $C \geq 0.6$, and we have C ***b*bd*f*m*mo*os**** $A \geq 0.6$. At step 14, the substitution $\{x :=$ ***defs***, $g := 0.6\}$ is applied to the estimate $C\ {-x}\ A < g$, and we have C ***b*bd*f*m*mo*os**** $A < 0.6$. Finally, at step15 we obtain the contradiction: C ***b*bd*f*m*mo*os**** $A \geq 0.6$ and C ***b*bd*f*m*mo*os**** $A < 0.6$. From the substitution we obtain the following answer to the query $?x - p \wedge q \rightarrow \sim C\ x\ A$: {(0.6, ***d***), (0.6, ***e***), (0.6, ***f***), {(0.6, ***s***)}.

(*End of Example 3.*)

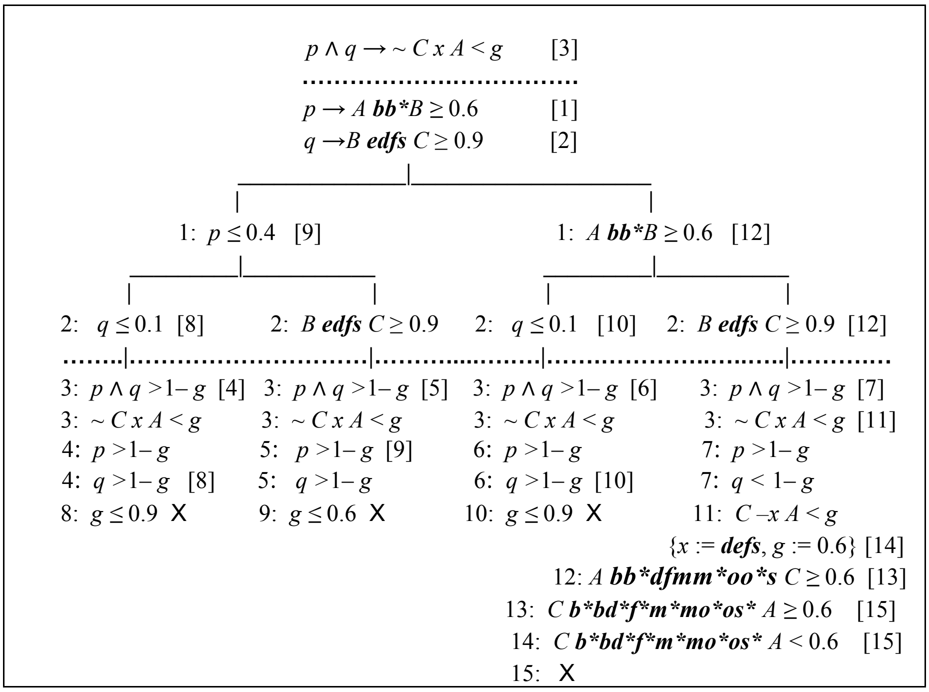

Fig. 1. Deduction tree for Example 2

Remark. In Example 2, the Tables 2, 3 and 4 were used to construct the deduction tree in Fig. 1. Generally, the Tables 5, 6, 7 may be needed. The inference rules entering all these tables are formed a complete system for query answering over fact bases for ontologies written in the language **ELA**.

Table 2. Inference rules for propositional connectives

$\sim\varphi > t$ / $\varphi < 1-t$	$\sim\varphi \geq t$ / $\varphi \leq 1-t$	$\sim\varphi < t$ / $\varphi > 1-t$	$\sim\varphi \leq t$ / $\varphi \geq 1-t$
$\varphi \wedge \psi > t$ / $\varphi > t$, $\psi > t$	$\varphi \wedge \psi \geq t$ / $\varphi > t$, $\psi > t$	$\varphi \wedge \psi < t$ / $\varphi < t \mid \psi < t$	$\varphi \wedge \psi \leq t$ / $\varphi \leq t \mid \psi \leq t$
$\varphi \vee \psi > t$ / $\varphi > t \mid \psi > t$	$\varphi \vee \psi \geq t$ / $\varphi \geq t \mid \psi \geq t$	$\varphi \vee \psi < t$ / $\varphi < t$, $\psi < t$	$\varphi \vee \psi \leq t$ / $\varphi \leq t$, $\psi \leq t$
$\varphi \to \psi > t$ / $\varphi < 1-t \mid \psi > t$	$\varphi \to \psi \geq t$ / $\varphi \leq 1-t \mid \psi \geq t$	$\varphi \to \psi < t$ / $\varphi > 1-t$, $\psi < t$	$\varphi \to \psi \leq t$ / $\varphi \geq 1-t$, $\psi \leq t$

Table 3. Fragment of Allen's table of compositions

	b	***d***	***f***	***s***
b	***b***	***bdmos***	***b***	***b***
b*	Ω	***b*dfm*o****	***b****	***b*dfm*o****

Table 4 . Inference rules with the composite relations ω and ρ

$A\ \omega\ B \geq r \quad B\ \rho\ C \geq s$ / $A\ \omega \circ \rho\ C \geq \min\{r, s\}$	$A\ \omega\ B \leq r \quad B\ \rho\ C \leq s$ / $A\ \omega \circ \rho\ C \leq \max\{r, s\}$	$A\ \omega\ B \geq r$ / $B\ \omega^* A \geq r$	$A\ \omega\ B \leq r$ / $B\ \omega^* A \leq r$
$A\ \omega\ B \geq r \quad A\ \rho\ B \geq s$ / $A\ \omega \cap B \geq \min\{r, s\}$	$A\ \omega\ B \leq r \quad A\ \rho\ B \leq s$ / $A\ \omega \cap B \leq \max\{r, s\}$	$A\ \alpha\omega\ B \geq r$ / $A\ \alpha\ B \geq r \mid A\ \omega\ B \geq r$	$A\ \alpha\omega\ B \leq r$ / $A\ \alpha\ B \leq r$, $A\ \omega\ B \leq r$

Table 5. Inference rules for modification of estimates

$(X \geq A^+) \geq t$ / $(X > A^-) \geq t$	$(X \geq A^+) > t$ / $(X > A^-) > t$	$(X \geq A^+) \leq t$ / $(X > A^-) \leq t$	$(X \geq A^+) < t$ / $(X > A^-) < t$

$(A^- \geq X) \geq t$ -------------- $(A^- \geq X) \geq t$	$(A^- \geq X) < t$ -------------- $(A^- \geq X) < t$	$(A^- \geq X) \leq t$ -------------- $(A^- \geq X) \leq t$	$(A^- \geq X) < t$ -------------- $(A^- \geq X) < t$
$X \in \{B^+, B^-\}$			

Table 6. Inference rules for Allen's connectives

$A\ \boldsymbol{b}\ B > t$ ——— $(B^- \geq A^+) > t$	$A\ \boldsymbol{b}\ B \geq t$ ——— $(B^- \geq A^+) \geq t$	$A\ \boldsymbol{b}\ B < t$ ——— $(B^- \geq A^+) < t$	$A\ \boldsymbol{b}\ B < t$ --------------- $(B^- \geq A^+) < t$
$A\ \boldsymbol{m}\ B > t$ ——— $(A^+ \geq B^-) > t$ $(B^- \geq A^+) > t$	$A\ \boldsymbol{m}\ B \geq t$ ——— $(A^+ \geq B^-) \geq t$ $(B^- \geq A^+) > t$	$A\ \boldsymbol{m}\ B < t$ ——— $(A^+ \geq B^-) < t$ | $(B^- \geq A^+) < t$	$A\ \boldsymbol{m}\ B \leq t$ ——— $(A^+ \geq B^-)\ \ t$ | $(B^- \geq A^+) < t$
$A\ \boldsymbol{o}\ B > t$ ——— $(B^- > A^-) > t$ $(A^+ > B^-) > t$ $(A^+ < B^+) > t$	$A\ \boldsymbol{o}\ B \geq t$ ——— $(B^- > A^-) \geq t$ $(A^+ > B^-) \geq t$ $(A^+ < B^+) \geq t$	$A\ \boldsymbol{o}\ B < t$ ——— $(B^- > A^-) < t$ | $(A^+ > B^-) < t$ | $(A^+ < B^+) < t$	$A\ \boldsymbol{o}\ B \leq t$ ——— $(B^- > A^-) \leq t$ | $(A^+ > B^-) \leq t$ | $(A^+ < B^+) \leq t$
$A\ \boldsymbol{f}\ B > t$ ——— $(A^- > B^-) > t$ $(A^+ \geq B^+) > t$ $(A^+ \geq B^+) > t$	$A\ \boldsymbol{f}\ B \geq t$ ——— $(A^- > B^-) \geq t$ $(A^+ \geq B^+) \geq t$ $(B^+ \geq A^+) > t$	$A\ \boldsymbol{f}\ B < t$ ——— $(A^- > B^-) < t$ | $(A^+ \geq B^+) < t$ | $(A^+ \geq B^+) < t$	$A\ \boldsymbol{f}\ B \leq t$ ——— $(A^- > B^-) \leq t$ | $(A^+ \geq B^+) \leq t$ | $(A^+ \geq B^+) < t$
$A\ \boldsymbol{s}\ B > t$ ——— $(A^- > B^-) > t$ $(B^+ \geq A^+) > t$ $(B^+ > A^+) > t$	$A\ \boldsymbol{s}\ B \geq t$ ——— $(A^- > B^-) \geq t$ $(B^+ \geq A^+) \geq t$ $(B^+ > A^+) > t$	$A\ \boldsymbol{s}\ B < t$ ——— $(A^- \geq B^-) < t$ | $(B^+ \geq A^+) < t$ | $(B^+ > A^+) < t$	$A\ \boldsymbol{s}\ B \leq t$ ——— $(A^- \geq B^-) \leq t$ | $(B^+ \geq A^+) \leq t$ | $(B^+ > A^+) \leq t$
$A\ \boldsymbol{d}\ B > t$ ——— $(A^- > B^-) > t$ $(B^+ < A^+) > t$	$A\ \boldsymbol{d}\ B \geq t$ ——— $(A^- > B^-) \geq t$ $(B^+ < A^+) \geq t$	$A\ \boldsymbol{d}\ B < t$ ——— $(A^- > B^-) < t$ | $(B^+ < A^+) < t$	$A\ \boldsymbol{d}\ B \leq t$ ——— $(A^- > B^-) \leq t$ | $(B^+ < A^+) \leq t$
$A\ \boldsymbol{e}\ B > t$ ——— $(B^- \geq A^-) > t$ $(A^- \geq B^-) > t$ $(B^+ \geq A^+) > t$ $(A^+ \geq B^+) > t$	$A\ \boldsymbol{e}\ B > t$ ——— $(B^- \geq A^-) \geq t$ $(A^- \geq B^-) \geq t$ $(B^+ \geq A^+) \geq t$ $(A^+ \geq B^+) \geq t$	$A\ \boldsymbol{e}\ B < t$ ——— $(B^- \geq A^-) < t$ | $(A^- \geq B^-) < t$ | $(B^+ \geq A^+) < t$ | $(A^+ \geq B^+) < t$	$A\ \boldsymbol{e}\ B \leq t$ ——— $(B^- \geq A^-) \leq t$ | $(A^- \geq B^-) \leq t$ | $(B^+ \geq A^+) \leq t$ | $(A^+ \geq B^+) \leq t$

Table 7. Inference rules for contrary pairs (where $V \in \{X \geq Y, X > Y\}$)

$\dfrac{p < x \quad p \geq t}{x \leq t}$	$\dfrac{p > x \quad p \leq t}{x \geq t}$	$\dfrac{(V) < x \quad (V) \geq t}{x \leq t}$	$\dfrac{(V) > x \quad (V) \leq t}{x \geq t}$
$V \in \{X \geq Y, X > Y\})$			

3 Conclusion

We have defined the fuzzy Boolean extension of Allen's interval logic and considered ontologies written in the extension. Fact bases for such ontologies consist of bilateral estimates for formulas from the ontologies. We have considered the problem of query answering over fact bases. For decision of this problem the analytical tableaux method was applied.

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