Maximum Dynamic Flow Finding Task with the Given Vitality Degree

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Abstract. This paper is devoted to the task of the maximum flow finding with nonzero lower flow bounds taking into account given vitality degree. Transportation network with the flow is considered in fuzzy conditions due to the fuzzy character of the network's parameters. Arcs of the network are assigned by the fuzzy arc capacities and nonzero lower flow bounds, vitality parameters and crisp transit times. All network's parameters can vary over time, therefore, it allows to consider network as dynamic one. The vitality parameter assigned to the arcs means ability of its objects to be resistant to weather conditions, traffic accidents and save and restore objects themselves, arc capacities of the network's sections in case of damage. The nonzero lower flow bounds are used to assess economic reliability of the transportation. Such methods can be applied in the real railways, roads and air roads solving the task of the optimal cargo transportation.

Keywords: Fuzzy dynamic graph, fuzzy nonzero lower flow bound, fuzzy vitality degree.

1 Introduction

The flow tasks [1] considered during the study of transportation networks are relevant due to their wide practical application, in particular, when finding the maximum amount of traffic between selected nodes on the road map, determining the routes of the optimal cost.

Important sphere of researches is dynamic networks [2-4], that take into account transit times along the arcs and don't assume instant flow distribution along the arcs. Another significant tool is considering dependence of arc capacities and lower flow bounds on flow departure time [5] and operating with fully dynamic networks instead of stationary-dynamic ones [6], using the notions of the time-expanded graphs [7-8].

Flow problems are connected with uncertainty of some kind, as changes in environment, measurement errors influence such network parameters, as arc flow bounds and vitality parameters. Therefore, we propose to consider these tasks in fuzzy conditions and we turn to the fuzzy graphs for solving such problems.

Vitality parameter [9-10] peculiar to arcs of the network usually isn't taken into account while studying networks. Its conventional definition was introduced by the authors H. Frank and I. Frisch in [11] as sensitivity of the network to damages.

However, vitality applied to the networks is ability of its objects and links among them to be resistant to weather conditions, traffic accidents and its combinations, and save and restore (fully or partially) objects themselves and their connections, arc capacities of the network's sections in case of damage. Nowadays, vitality of the network isn't taken into account, while railways and roads include the complex objects, such as stations, distillation ways, culverts, wagon, passenger and cargo managements. Sometimes network's parameters can be set qualitatively. Thus, one can set the notion "vitality degree" considering the roads and railways. In this case "vitality degree" is considered as probability of trouble-free operation of the road section and some subjective value, such as importance and reliability, etc.

Other words this paper presents method of the maximum flow finding with nonzero lower flow bounds in fuzzy dynamic network with given vitality degree.

The paper is structured as follows. In the Section 2 we give basic definitions and rules. Section 3 presents the proposed method. Section 4 provides numerical example illustrating the main steps of the proposed method. Section 5 is conclusion and future work.

2 Definitions and Rules

The proposed approach is based on the following notion of vitality.

Fuzzy directed path $\tilde{P}(x_i, x_m)$ of the graph $\tilde{G} = (X, \tilde{A})$ is a sequence of fuzzy directed arcs from the node x_i to the node x_m:

$$\tilde{P}(x_i, x_m) = < \mu_{\tilde{A}} < x_i, x_j > / < x_i, x_j >>,$$

$$\mu_{\tilde{A}} < x_j, x_k > / < x_j, x_k >>, ..., \mu_{\tilde{A}} < x_l, x_m > / < x_l, x_m >>.$$

Conjunctive durability of the path $\mu(\tilde{P}(x_i, x_m))$ is defined as

$$\mu(\tilde{P}(x_i, x_m)) = \underset{<x_\alpha, x_\beta> \in \tilde{P}(x_i, x_m)}{\&} \mu_{\tilde{A}} < x_\alpha, x_\beta >.$$

Fuzzy directed path $\tilde{P}(x_i, x_m)$ is called a simple path between vertices x_i and x_m if its part is not a path between the same vertices.

Vertex y is called a fuzzy accessible from the vertex x in the graph $\tilde{G} = (X, \tilde{A})$ if the fuzzy directed path from the node x to the node y exists.

The accessible degree of the node y from the node x, $(x \neq y)$ is defined by the following expression:

$$\gamma(x, y) = \max_\alpha (\mu(\tilde{P}_\alpha(x, y)), \alpha = 1, 2, ..., p,$$

where p is the number of various simple directed paths from vertex x to vertex y.

We consider the degree of fuzzy graph vitality as a degree of strong connection [10, 11], so it will be defined by the following expression:

$$V(\tilde{G}) = \underset{x_i \in X}{\&} \underset{x_j \in X}{\&} \gamma(x_i, x_j).$$

It means that there is a route between each pair of the graph vertices with a conjunctive strength not less than value V.

Let us introduce basic rules and definitions underlying this method.

Turn to the fuzzy graph $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$ from $\tilde{G}_p = (X_p, \tilde{A}_p)$. Introduce the artificial source s^* and sink t^* and arcs connecting the node-time pair $(t, \forall \theta \in T)$ and $(s, \forall \theta \in T)$ with $\tilde{u}^*(t,s, \forall \theta \in T, \forall \theta \in T) = \infty$, $\tilde{l}^*(t, s, \forall \theta \in T, \forall \theta \in T) = \tilde{0}$, $\tilde{v}^*(t,s,\forall \theta \in T, \forall \theta \in T) = \tilde{1}$. in the graph $\tilde{G}_p$. For arcs with $\tilde{l}(x_i, x_j, \theta, \vartheta) \neq \tilde{0}$: 1) reduce $\tilde{u}(x_i, x_j, \theta, \vartheta)$ to $\tilde{u}^*(x_i, x_j, \theta, \vartheta) = \tilde{u}(x_i, x_j, \theta, \vartheta) - \tilde{l}(x_i, x_j, \theta, \vartheta)$, $\tilde{l}(x_i, x_j, \theta, \vartheta)$ to $\tilde{0}$, $v^*(x_i, x_j, \theta, \vartheta) = v(x_i, x_j, \theta, \vartheta)$. 2) Introduce the arcs connecting s^* with (x_j, ϑ), and the arcs connecting t^* with (x_i, θ) with $\tilde{u}^*(s^*, x_j, \forall, \vartheta) = \tilde{u}^*(x_i, t, \theta, \forall) = \tilde{l}(x_i, x_j, \theta, \vartheta)$ zero lower fuzzy flow bounds $\tilde{l}^*(s^*, x_j, \forall, \vartheta) = \tilde{l}^*(x_i, t, \theta, \forall) = \tilde{0}$, $\tilde{v}^*(x_i, x_j, \theta, \vartheta) = \tilde{v}(x_i, x_j, \theta, \vartheta)$.

Definition 1 of the fuzzy residual network of the time-expanded graph.

Fuzzy residual network $\tilde{G}_p^{*\mu} = (X_p^{*\mu}, \tilde{A}_p^{*\mu})$ is the network without lower flow bounds $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$, which is constructed according to the following rules: if

$$\begin{cases} \tilde{\xi}^*(x_i, x_j, \theta, \vartheta) < \tilde{u}^*(x_i, x_j, \theta, \vartheta), \\ \tilde{v}^*(x_i, x_j, \theta, \vartheta) \geq \tilde{v}_{req}, \end{cases}$$

,then include the corresponding arc from $(x_i^{*\mu}, \theta)$ to $(x_j^{*\mu}, \vartheta)$ in $\tilde{G}_p^{*\mu}$ with $\tilde{u}^{*\mu}(x_i, x_j, \theta, \vartheta) = \tilde{u}^*(x_i, x_j, \theta, \vartheta) - \tilde{\xi}^*(x_i, x_j, \theta, \vartheta)$ and $\tau^{*\mu}(x_i, x_j, \theta, \vartheta) = \tau^*(x_i, x_j, \theta, \vartheta)$.

If

$$\begin{cases} \tilde{\xi}^*(x_i, x_j, \vartheta, \theta) > \tilde{0}, \\ \tilde{v}(x_i, x_j, \theta, \vartheta) \geq \tilde{v}_{req}. \end{cases}$$

Then include the corresponding arc from $(x_j^{*\mu}, \vartheta)$ to $(x_i^{*\mu}, \theta)$ in $\tilde{G}_p^{*\mu}$ with $\tilde{u}^{*\mu}(x_j, x_i, \vartheta, \theta) = \tilde{\xi}^*(x_i, x_j, \theta, \vartheta)$ and $\tau^{*\mu}(x_j, x_i, \vartheta, \theta) = -\tau^*(x_i, x_j, \theta, \vartheta)$.

Turn to the graph $\tilde{G}_p$ from the graph $\tilde{G}_p^*$ as following: reject artificial nodes and arcs, connecting them with other nodes. The feasible flow vector $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \theta, \vartheta))$ of the value $\tilde{\sigma}$ is defined as: $\tilde{\xi}(x_i, x_j, \theta, \vartheta) = \tilde{\xi}^*(x_i, x_j, \theta, \vartheta) + \tilde{l}(x_i, x_j, \theta, \vartheta)$, where $\tilde{\xi}^*(x_i, x_j, \theta, \vartheta)$ – the flows, going along the arcs of the graph $\tilde{G}_p^*$ after deleting all artificial nodes and connecting arcs.

for all arcs, if $\tilde{\xi}(x_i, x_j, \vartheta, \theta) < \tilde{u}(x_i, x_j, \vartheta, \theta)$, then include the corresponding arc

(x_i^μ, θ) from the node-time pair to the node-time pair (x_j^μ, ϑ) in $\tilde{G}_p^\mu(\tilde{\xi})$ with arc capacity $\tilde{u}^\mu(x_i, x_j, \theta, \vartheta) = \tilde{u}(x_i, x_j, \theta, \vartheta) - \tilde{\xi}(x_i, x_j, \theta, \vartheta)$ and transit time $\tau^\mu(x_i, x_j, \theta, \vartheta) = \tau(x_i, x_j, \theta, \vartheta)$. For all arcs, if $\tilde{\xi}(x_i, x_j, \vartheta, \theta) > \tilde{l}(x_i, x_j, \vartheta, \theta)$, then include the corresponding arc, going from the node-time pair (x_j^μ, ϑ) to the node-time pair (x_i^μ, θ) in $\tilde{G}_p^\mu(\tilde{\xi})$ with arc capacity $\tilde{u}^\mu(x_j, x_i, \vartheta, \theta) = \tilde{\xi}(x_i, x_j, \theta, \vartheta) - \tilde{l}(x_i, x_j, \vartheta, \theta)$ and transit time $\tau^\mu(x_j, x_i, \vartheta, \theta) = -\tau(x_i, x_j, \theta, \vartheta)$.

Therefore, the proposed method of the maximum flow finding with nonzero lower flow bounds in fuzzy dynamic network consists in the maximum flow finding in the network without lower flow bounds. We turn to the time-expanded fuzzy graph and consequently to the graph without lower flow bounds for it and try to find the maximum flow in the graph. Based on the formulated rules and definitions, turn to the maximum flow finding with nonzero lower flow bounds in dynamic network in terms of partial uncertainty.

3 Presented Method of the Maximum Flow Finding Task with Nonzero Lower Flow Bounds in the Fuzzy Dynamic Network

Let us introduce the task of the maximum flow finding with nonzero lower flow bounds in dynamic network in terms of partial uncertainty and given vitality degree, represented by the model (1)-(6).

$$Maximize \ \tilde{v}(p) \tag{1}$$

$$\sum_{\theta=0}^{p} \left(\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}(\theta) - \sum_{x_j \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta)) \right) = \tilde{v}(p), x_i = s, \tag{2}$$

$$\sum_{\theta=0}^{p} \left(\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}(\theta) - \sum_{x_j \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta)) \right) = \tilde{0}, x_i \neq s, t; \theta \in T, \tag{3}$$

$$\sum_{\theta=0}^{p} \left(\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}(\theta) - \sum_{x_j \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta)) \right) = -\tilde{v}(p), x_i = t, \tag{4}$$

$$\tilde{l}_{ij}(\theta) \leq \tilde{\xi}_{ij}(\theta) \leq \tilde{u}_{ij}(\theta), \theta + \tau_{ij}(\theta) \leq p, \theta \in T, \tag{5}$$

$$\tilde{v}_{ij}(\theta) \geq \tilde{v}_{req}, s(\theta) + \tau_{st}(\theta) \leq p, \theta \in T. \tag{6}$$

Step 1. Go to the time-expanded fuzzy static graph $\tilde{G}_p$ from the given fuzzy dynamic graph $\tilde{G}$.

Step 2. Turn to the graph $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$ according to the *rule 1*.

Step 3. Build a fuzzy residual network $\tilde{G}_p^{*\mu}$ due to the **_definition 1._**

Step 4. Search the augmenting shortest path (in terms of the number of arcs) $\tilde{P}_p^{*\mu}$ from the artificial source s^* to the artificial sink t^* in the constructed fuzzy residual network according to the breadth-first-search.

4.1 Go to the step 5 if the augmenting path $\tilde{P}_p^{*\mu}$ is found.

4.2 The flow value $\tilde{\phi}^* < \displaystyle\sum_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$ is obtained, which is the maximum flow in $\tilde{G}_p^*$, if the path is failed to find. Exit.

Step 5. Pass the minimum from the arc capacities $\tilde{\delta}_p^{*\mu} = \min[\tilde{u}(\tilde{P}_p^{*\mu})]$, $\tilde{u}(\tilde{P}_p^{*\mu}) = \min[\tilde{u}^{*\mu}(x_i,x_j,\theta,\vartheta)$, $(x_i,\theta),(x_j,\vartheta) \in \tilde{P}_p^{*\mu}$ along this path $\tilde{P}_p^{*\mu}$.

Step 6. Update the fuzzy flow values in the graph $\tilde{G}_p^*$: replace the fuzzy flow $\tilde{\xi}^*(x_j,x_i,\theta,\vartheta)$ along the corresponding arcs going from (x_j^*,θ) to (x_i^*,ϑ) from $\tilde{G}_p^*$ by $\tilde{\xi}^*(x_j,x_i,\theta,\vartheta) - \tilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair $(x_i^{*\mu},\vartheta)$ with $(x_j^{*\mu},\theta)$ in $\tilde{G}_p^{*\mu}$, such as $((x_i^{*\mu},\vartheta),(x_j^{*\mu},\theta)) \notin \tilde{A}_p^*$, $((x_i^{*\mu},\vartheta),(x_j^{*\mu},\theta)) \in \tilde{A}_p^{*\mu}$ and replace the fuzzy flow $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta)$ along the arcs going from (x_i^*,θ) to (x_j^*,ϑ) from $\tilde{G}_p^*$ by $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair $(x_i^{*\mu},\theta)$ with $(x_j^{*\mu},\vartheta)$ in $\tilde{G}_p^{*\mu}$, such as $((x_i^{*\mu},\theta),(x_j^{*\mu},\vartheta)) \in \tilde{A}_p^*$, $((x_i^{*\mu},\theta),(x_j^{*\mu},\vartheta)) \in \tilde{A}_p^{*\mu}$. Replace $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta)$ by $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$.

Step 7. Compare flow value $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$ and $\displaystyle\sum_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$:

7.1. If the flow value $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$ is less than $\displaystyle\sum_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$, go to the **step 3**.

7.2. If the flow value $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$ is equal to $\displaystyle\sum_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$, turn to the graph $\tilde{G}_p$ from the graph $\tilde{G}_p^*$ according to the *rule 2*. Go to the **step 8**.

Step 8. Construct the residual network $\tilde{G}_p^\mu(\tilde{\xi})$ according to the rule 3.

Step 9. Define the shortest path $\tilde{P}_p^\mu$ $\tilde{G}_p^\mu(\tilde{\xi})$.

 (I) Go to the **step 10** if the augmenting path $\tilde{P}_p^\mu$ is found.

(II) The maximum flow $\tilde{\xi}(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^\mu\tilde{P}_p^\mu = \tilde{v}(p)$ in $\tilde{G}_p(\tilde{\xi})$ is found if the path is failed to find, then the maximum flow in "time-expanded" static fuzzy graph can be found at the **step 12**.

Step 10. Pass the flow value $\tilde{\delta}_p^{\mu} = \min[\tilde{u}(\tilde{P}_p^{\mu})]$, $\tilde{u}(\tilde{P}_p^{\mu}) = \min[\tilde{u}^{\mu}(x_i, x_j, \theta, \vartheta)]$, $(x_i, \theta), (x_j, \vartheta) \in \tilde{P}_p^{\mu}$ along the found path.

Step 11. Update the flow values in the graph $\tilde{G}_p(\tilde{\xi})$.

Step 12. Turn to the initial dynamic graph $\tilde{G}$ as follows: reject the artificial nodes s', t' and arcs, connecting them with other nods.

4 Numerical Example

Let us describe the proposed algorithm. For example, assume that the original fuzzy dynamic network is shown in Fig. 1. It is necessary to find the maximum flow in the initial dynamic graph with the given vitality degree no less than $0,\tilde{7}$ and represent the result in the form of the triangular number.

Fuzzy upper flow bounds $\tilde{u}_{ij}$, depending on the flow departure time θ are shown in the Table I. Fuzzy lower flow bounds $\tilde{l}_{ij}$, depending on the flow departure time θ are shown in the Table II. Time parameters τ_{ij} depending on the flow departure time θ are shown in the Table III. Fuzzy vitality parameters $\tilde{v}_{ij}$, depending on the flow departure time θ are shown in the Table IV.
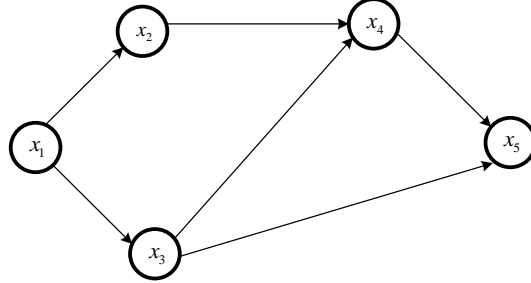

Fig. 1. Initial dynamic graph $\tilde{G}$

Construct time-expanded graph, as shown in Fig. 2.

Turn to the graph without lower flow bounds and find the augmenting paths for the graph in Fig. 3: $\tilde{P}_1^{*\mu} = s^*, (x_5, 2), (x_1, 0), t^*$ with $\tilde{7}$ flow units, $\tilde{P}_2^{*\mu} = s^*, (x_2, 1), (x_3, 2), (x_5, 3), (x_1, 0), t^*$ with $\tilde{3}$ flow units, $\tilde{P}_3^{*\mu} = s^*, (x_2, 1), (x_3, 2), (x_5, 3), (x_1, 0), (x_4, 1), t^*$ with $\tilde{7}$ flow units.

We obtain graph with the maximum flow in Fig. 4. Therefore, the task has a solution and we turn to the initial time-expanded graph with the feasible flow in Fig.

5. Finding the augmenting paths and pushing the flows among them, we obtain graph with the maximum flow in Fig. 6.

TABLE I. FUZZY UPPER FLOW BOUNDS $\tilde{u}_{ij}$, DEPENDING ON THE FLOW DEPARTURE TIME θ

Arcs of the graph	Fuzzy upper flow bounds $\tilde{u}_{ij}$ at the time periods θ, time units.			
	0	1	2	3
(x_1,x_2)	$2\tilde{5}$	$2\tilde{0}$	$2\tilde{5}$	$4\tilde{0}$
(x_1,x_4)	$1\tilde{0}$	$2\tilde{0}$	$2\tilde{5}$	$2\tilde{5}$
(x_1,x_5)	$1\tilde{8}$	$1\tilde{8}$	$3\tilde{0}$	$3\tilde{5}$
(x_2,x_3)	$3\tilde{5}$	$3\tilde{0}$	$3\tilde{5}$	$1\tilde{8}$
(x_3,x_4)	$1\tilde{5}$	$2\tilde{7}$	$3\tilde{3}$	$2\tilde{5}$
(x_3,x_5)	$5\tilde{5}$	$4\tilde{5}$	$4\tilde{0}$	$5\tilde{5}$
(x_4,x_5)	$2\tilde{0}$	$2\tilde{0}$	$1\tilde{8}$	$2\tilde{8}$

TABLE II. FUZZY LOWER FLOW BOUNDS $\tilde{l}_{ij}$, DEPENDING ON THE FLOW DEPARTURE TIME θ.

Arcs of the graph	Fuzzy lower flow bounds $\tilde{l}_{ij}$ at the time periods θ, time units.			
	0	1	2	3
(x_1,x_2)	$1\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
(x_1,x_4)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
(x_1,x_5)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$2\tilde{0}$
(x_2,x_3)	$\tilde{6}$	$\tilde{0}$	$1\tilde{5}$	$\tilde{0}$
(x_3,x_4)	$\tilde{0}$	$\tilde{8}$	$\tilde{0}$	$\tilde{0}$
(x_3,x_5)	$2\tilde{5}$	$1\tilde{5}$	$\tilde{0}$	$\tilde{0}$
(x_4,x_5)	$\tilde{0}$	$\tilde{5}$	$\tilde{0}$	$1\tilde{0}$

TABLE III. TIME PARAMETERS τ_{ij} DEPENDING ON THE FLOW DEPARTURE TIME θ

Arcs of the graph	Time parameters τ_{ij} at time periods θ, time units.			
	0	1	2	3
(x_1,x_2)	1	1	1	2
(x_1,x_4)	1	3	2	2
(x_1,x_5)	4	4	1	1
(x_2,x_3)	4	1	1	1
(x_3,x_4)	1	1	2	2
(x_3,x_5)	2	2	1	1
(x_4,x_5)	5	4	1	3

TABLE IV. FUZZY VITALITY PARAMETERS v_{ij} , DEPENDING ON THE FLOW DEPARTURE TIME θ

Arcs of the graph	Fuzzy vitality parameters $\tilde{v}_{ij}$ at time periods θ, vitality units			
	0	*1*	*2*	*3*
(x_1, x_2)	$0,\tilde{8}$	$0,\tilde{4}$	$0,\tilde{6}$	$0,\tilde{5}$
(x_1, x_4)	$0,\tilde{7}$	$0,\tilde{2}$	$0,\tilde{8}$	$0,\tilde{9}$
(x_1, x_5)	$0,\tilde{4}$	$0,\tilde{8}$	$0,\tilde{6}$	$0,\tilde{3}$
(x_2, x_3)	$0,\tilde{7}$	$0,\tilde{8}$	$0,\tilde{7}$	$0,\tilde{4}$
(x_3, x_4)	$0,\tilde{7}$	$0,\tilde{9}$	$0,\tilde{7}$	$0,\tilde{6}$
(x_3, x_5)	$0,\tilde{3}$	$0,\tilde{4}$	$0,\tilde{7}$	$0,\tilde{4}$
(x_4, x_5)	$0,\tilde{8}$	$0,\tilde{3}$	$0,\tilde{3}$	$0,\tilde{4}$

The maximum flow in the initial graph with the vitality degree no less than $0,\tilde{7}$ is $2\tilde{5} + 1\tilde{0} = 3\tilde{5}$ flow units.

Let us define deviation borders of the obtained fuzzy number "near $3\tilde{5}$".

Since the calculations with fuzzy numbers are cumbersome and result in strong blurring of the resulting number's borders, we suggest to operate fuzzy numbers according to the method, described in [8]. In this case we will operate the central values of fuzzy numbers, blurring the result at the final step and presenting it as a triangular the number.

Therefore, deviation borders of the obtained fuzzy number "near $3\tilde{5}$" corresponded to the maximum flow in the graph $\tilde{G}$ are calculated according to the basic values of arc capacities in Fig. 7.

The detected result is between two adjacent basic values of the arc capacities: $3\tilde{1}$ with the left deviation $l_1^L = 8$, right deviation $- l_1^R = 7$ and $4\tilde{4}$ with the left deviation $l_2^L = 9$, right deviation $- l_2^R = 10$. We obtain deviations: $l_1^L \approx 8$, $l_1^R \approx 7$.

Therefore, the maximum flow in the fuzzy dynamic graph with the given vitality degree no less than $0,\tilde{7}$ can be represented by fuzzy triangular number (27, 35,42) units.
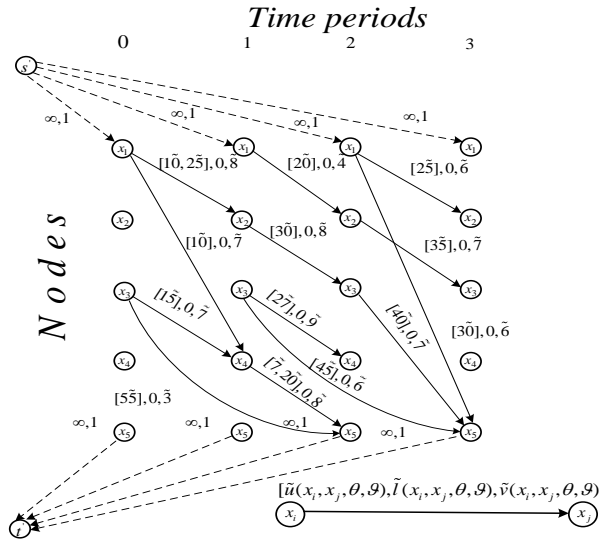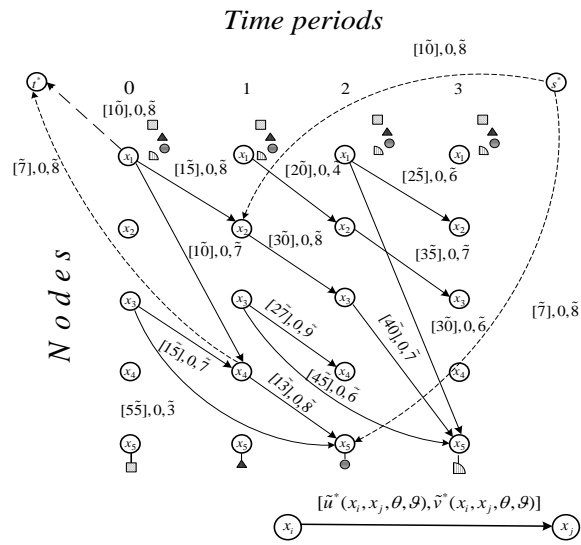
Fig. 2. Time-expanded graph $\tilde{G}_p$

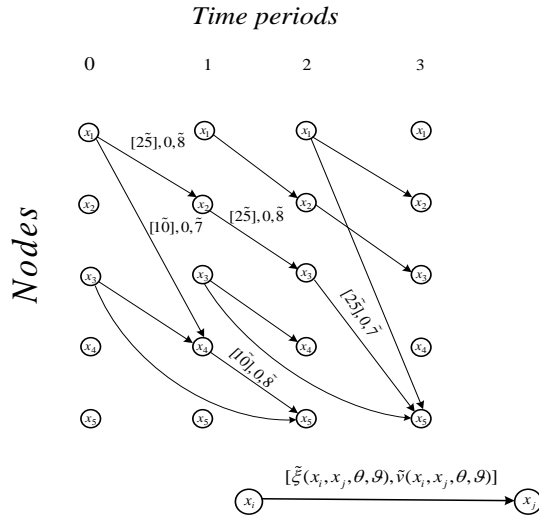Fig. 3. $\tilde{G}_p^*$ – Time-expanded graph without lower flow bounds $\tilde{G}$

Time periods

[1õ], 0, 8̃

0
[1õ], 0, 8̃

1   2   3

[7̃], 0, 8̃

N o d e s

[7̃], 0, 7̃

[1õ], 0, 8̃

[7̃], 1̃

[1õ], 1̃

[1õ], 0, 7̃

[7̃], 0, 8̃

$[\tilde{\bar{\xi}}^*(x_i, x_j, \theta, \vartheta), \tilde{v}^*(x_i, x_j, \theta, \vartheta)]$

Fig. 4. Graph $\tilde{G}_p^*$ with the maximum flow

Time periods

0   1   2   3

Nodes

[1õ], 0, 8̃

[7̃], 0, 7̃

[1õ], 0, 8̃

[1õ], 0, 7̃

[7̃], 0, 8̃

$[\tilde{\bar{\xi}}(x_i, x_j, \theta, \vartheta), \tilde{v}(x_i, x_j, \theta, \vartheta)]$

Fig. 5. Graph $\tilde{G}_p$ with the feasible flow

Time periods

0   1   2   3

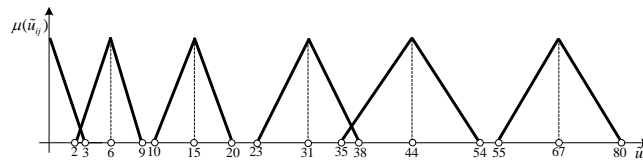Fig. 6. Graph $\tilde{G}_p$ with the maximum flow

Fig. 7. Membership functions of the basic values of arc capacities of the network $\tilde{G}$

5 Conclusion and Future Work

Paper presents proposed algorithm of the maximum flow finding with nonzero lower flow bounds and vitality degrees in the fuzzy dynamic network with the required vitality degree based on the formulated definitions and rules. The considered network is represented as fuzzy graph with parameters, depending on the flow departure time and varying over time. Given lower flow bounds are used for assessing economic reliability of transportation. Given vitality degree reflects ability of its objects to be resistant to weather conditions, traffic accidents and save and restore objects themselves, arc capacities of the network's sections in case of damage. The proposed method has important practical value in transportation implementing on the real types of roads. In the future works we will propose methods of increasing the vitality degree in fuzzy dynamic networks.

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