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PION-NUCLEON SCATTERING AMPLITUDES AT ENERGIES BETWEEN 1.5 AND 14 GeV AND SMALL MOMENTUM TRANSFER

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ABSTRACT

From differential cross-section data of $\pi^{\pm}p$ elastic and charge exchange scattering as well as from the polarizations of the elastic processes we evaluate the πN scattering amplitudes for energies between 1.5 and 14 GeV and momentum transfers between -0.05 and -0.4 $(GeV/c)^2$. Our analysis starts from approximations of the amplitudes which are improved by iteration, using fixed-t dispersion relations and the experimental data. The resulting charge exchange amplitudes show a structure consistent with the dual absorptive model, whereas we found no indication that tensor exchange is present. Although not used in the analysis our amplitudes give the charge exchange polarization and spin rotation parameters in agreement with measurements.

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1. INTRODUCTION

It is well known that, assuming isospin invariance, πN scattering is described by four complex amplitudes which can be determined up to a common phase without any theoretical constraint, if all 7 independent measurements at a particular kinematical point are available. Such an analysis has been carried out by several authors[1-4] at 6 GeV/c and small momentum transfers where this complete data set exists.

At all other energies there are always some measurements missing, at the very least the A and R parameters. Therefore, in addition to the incomplete data some theoretical input is needed to extract the amplitudes.

For many years amplitude analysis of πN-scattering was done in terms of phase shifts[5,6]. This method is however limited to low energies up to 2 or 3 GeV/c, because at higher energies, too many partial waves have to be taken into account and moreover unitarity no longer constrains the solutions since all partial waves become highly inelastic. Consequently, the ambiguities of the phase shift solutions are growing with increasing energy[7]. Therefore, since at higher energies unitarity seems to be such a weak condition, the partial wave analysis should be replaced by an analysis in terms of the four complex amplitudes imposing the generally accepted properties like s-u crossing symmetry and fixed-t analyticity as theoretical constraints. An analysis along these lines has been carried out by Pietarinen[8].

He expanded the πN amplitudes in terms of analytic functions which have the correct s-u crossing symmetry and satisfy fixed-t dispersion relations. The stability and the convergence of the solutions are maintained by the use of a particular probability measure[9] for the expansion coefficients. For his analysis of the πN-data in the energy range up to 10 GeV/c and for five different values of the momentum transfer between -0.1 and -0.5 $(GeV/c)^2$ Pietarinen needs about 600 coefficients. It seems to us that a disadvantage of his method is the use of one and the same expansion in the resonance region and above. Probably this is the reason for some wiggles which his amplitudes show at high energies. (See for example Figs. 1 and 2.) Furthermore it may turn out to be difficult to find a suitable starting solution which is an essential condition that the minimization problem in this high dimensional space can be solved within a reasonable amount of computing time.

In this investigation we also perform an amplitude analysis for πN-scattering within the scheme described above, but we use a different method. We think that this work is necessary because of ambiguities resulting from statistical and systematical errors of the experimental data and from the incomplete set of existing data. Furthermore, the above mentioned behaviour of Pietarinen's amplitudes at higher energies should be checked.

The starting point of our analysis is a zero order approximation of the invariant amplitudes $C^{(\pm)}$ and $B^{(\pm)}$. These are improved successively by iteration using fixed-t dispersion relations (DR) and the experimenal data. The method will be described in detail in the next section. As a zero order approximation we use the model amplitudes of Barger and Phillips[10)] which should not be too far away from fixed-t analyticity because these authors have used continuous moment sum rules[*)], and also information from model independent studies of πN data[11-14)].

Furthermore, for the low energy region we use the phase shift results of Almehed and Lovelace[5)] and we make suitable assumptions concerning the high energy behaviour of the amplitudes. Our analysis will be performed for energies between 1.5 GeV/c and 14 GeV/c. The upper limit is imposed by the existing data. The region from 1.5 to 2.0 GeV/c is treated in the same way to study and,if possible, to overcome shortcomings in the phase shift results.

The momentum transfer region of this analysis is $0 > t > -0.4\ (\mathrm{GeV}/c)^2$. At $t = 0$ the amplitudes were determined in earlier works of the Karlsruhe group[11,14)]. The reason for the lower limit in t will be mentioned in the the next sections.

Since we are using DR reliable results can only be expected if the input, i.e. the experimental data is prepared carefully. Therefore we spent much effort on the interpolation of the available data[**)] in the

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*) We found that the amplitudes of Ref. 10 are not consistent with fixed-t dispersion relations and recent phase shift solutions[5,6)].

**) In our analysis we have taken as a basis an updated version of the compilation of πN scattering data collected by Almehed and Lovelace[5)]. Tables of our interpolated data and a list of references to the data which we have used are available on request.

relevant combinations to a definite set of k- and t-values as given in Tables 1.1 to 1.6. All interpolated data were checked for isospin conservation, i.e. we ensured that isospin inequalities hold. With experimental errors taken into account no violation of isospin bounds are found.

We wish to stress that our analysis contains as little theoretical input as possible, i.e. besides fixed-t dispersion relations and isospin invariance we only make some assumptions about the asymptotical behaviour of the amplitudes. Obviously, the influence of the assumed behaviour has to be discussed. However, we do not use analytic expressions for the amplitudes in the kinematical region studied by us. In this respect our method differs from the model dependent analyses of other authors, e.g. Barger and Phillips[10)] or from the study of the C^+-amplitudes of Höhler and Jakob[15)]. These models are generally high energy models, that means that the analytic expressions for the amplitudes provide smooth results and show no resonance structure. They can give at best mean values -- if duality works -- in the resonance region. Therefore, such models must necessarily fail at lower energies.

2. THE METHOD

In this section we describe the main features of the method we applied to determine the πN-amplitudes. Specific assumptions and approximations, which eventually become necessary and which differ for each amplitude, will be discussed in the Sections 3 to 5, where the determination of each amplitude will be described separately.

The starting point of our analysis is to construct a zero order approximation of the πN-amplitudes. From the imaginary parts of these amplitudes we compute by means of fixed-t dispersion relations (DR) the corresponding real parts. In the evaluation of the DR we use the CERN 1971 phase shifts for the low energy region and suitable assumptions for the high energy behaviour of the amplitudes. The new real parts obtained in this way will differ more or less from the old ones.

In the second step we calculate from the new real parts and the experimental data new imaginary parts. These two steps are repeated successively, until a stable solution is found. One might note that after the first step in the iteration the amplitudes obtained are analytic but do not necessarily reproduce the experimental data, while after the second step the situation is reversed. In principle the stable solution ultimately obtained will be analytic and describe the experimental data.

The iteration procedure outlined above does not necessarily lead to a stable solution, but it may. Positive but not definitive criteria for obtaining a solution are:

i) good zero order approximations of the amplitudes are available,

ii) the amplitudes for low energies as constructed from the phase shifts reproduce the experimental data and satisfy the DR,

iii) the relevant observable is dominated by either the real or imaginary part of a specific amplitude. This is, for example, the case for the amplitude $C^{(+)}$ *).

Certainly, it is hopeless to find a solution with such an iteration procedure unless the system of non-linear equations that determines the amplitudes is weakly coupled. This is, in fact, true for suitable combinations of the observables in πN scattering for $|t| < 0.4\ (\mathrm{GeV}/c)^2$:

Let us, at first, discuss the combination

$$\Sigma\sigma = \frac{1}{2}\left\{\frac{d\sigma_+}{dt} + \frac{d\sigma_-}{dt} - \frac{d\sigma_0}{dt}\right\} \tag{2.1}$$

of differential cross sections for $\pi^{\pm}p$-elastic and charge-exchange scattering, respectively. As can be seen from Eqs. (A.2) and (A.3) of the Appendix, only the crossing even amplitudes contribute to $\Sigma\sigma$:

$$\Sigma\sigma = \frac{1}{16\pi}\left\{\frac{1-t/4M^2}{k^2}|C^{(+)}|^2 - \frac{t}{4M^2}\,\frac{1+t/4q^2}{1-t/4M^2}|B^{(+)}|^2\right\} \tag{2.2}$$

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*) Since we are dealing with DR in this analysis we have to consider the invariant amplitudes $C^{(\pm)}$ and $B^{(\pm)}$. Their relations to the observables and other pertinent kinematical relations are given in the Appendix.

Because of the factor $t/4M^2$ in front of $|B^{(+)}|^2$, the contribution to $\Sigma\sigma$ due to $B^{(+)}$ is suppressed in the t-region under consideration. Moreover, we use the approximation $k|B^{(+)}| = |C^{(+)}|$ which is supported by the phase-shift results for energies above 1.5 GeV and also by the results of Höhler et al.[13] at higher energies. These authors have calculated the amplitude

$$A^{(+)} = C^{(+)} - \omega \frac{1 + t/4M\omega}{1 - t/4M^2} B^{(+)} \qquad (2.3)$$

at 6 GeV, using the R and A parameters of de Lesquen et al.[16]. From these two results and from the assumption that they also apply in the entire kinematic region under consideration it follows that the contributions of A^+, i.e. deviations from our assumption, are less than the errors of the experimental data. This assumption allows us to compute $C^{(+)}$ independently from all other amplitudes.

The next combination which we consider is

$$\sum P = \frac{4\pi\sqrt{s}}{\sin\vartheta} \left\{ P_+ \frac{d\sigma_+}{dt} + P_- \frac{d\sigma_-}{dt} \right\} \qquad (2.4)$$

where $P_\pm$ are the polarizations of the elastic processes. From the formulas of the Appendix one derives the relation of ΣP to the invariant amplitudes:

$$\sum P = Im\,(B^{(+)}C^{(+)*}) + Im\,(B^{(-)}C^{(-)*}). \qquad (2.5)$$

The second term is related to the charge exchange polarization. It can be neglected since its contribution is within the experimantal errors of ΣP. This can be seen from a bound following from the assumption $P_0 = 1$:

$$\left| Im\,(B^{(-)}C^{(-)*}) \right| \leq \frac{8\pi\sqrt{s}}{\sin\vartheta} \frac{d\sigma_0}{dt} \qquad (2.6)$$

Therefore, once $C^{(+)}$ is known, $B^{(+)}$ can in principle be evaluated from ΣP. The relevant combinations of observables which determine the isospin odd amplitudes are:

$$\Delta\sigma = \frac{1}{2}\left\{\frac{d\sigma_-}{dt} - \frac{d\sigma_+}{dt}\right\} =$$

$$= \frac{1 - t/4M^2}{8\pi k^2}\left\{Re\left(C^{(+)}C^{(-)*}\right) - \frac{t}{4M^2}\,\frac{1+t/4q^2}{1-t/4M^2}\,k^2\,Re\left(B^{(+)}B^{(-)*}\right)\right\} \tag{2.7}$$

$$\Delta P = \frac{4\pi\sqrt{s}}{\sin\vartheta}\left\{P_-\frac{d\sigma_-}{dt} - P_+\frac{d\sigma_+}{dt}\right\} =$$

$$= Im\left(B^{(-)}C^{(+)*}\right) + Im\left(B^{(+)}C^{(-)*}\right) \tag{2.8}$$

and the accurately measured charge-exchange differential cross section:

$$\frac{d\sigma_0}{dt} = \frac{1 - t/4M^2}{8\pi k^2}\left\{|C^{(-)}|^2 - \frac{t}{4M^2}\,\frac{1+t/4q^2}{(1-t/4M^2)^2}\,k^2\,|B^{(-)}|^2\right\} \tag{2.9}$$

These three equations are not decoupled for $C^{(-)}$ and $B^{(-)}$. Therefore, we have to iterate them simultaneously using a DR for $C^{(-)}$. Nevertheless, since $|C^{(-)}| \ll k|B^{(-)}|$ (flip dominance) the $C^{(-)}$ contribution to ΔP and $d\sigma_0/dt$ is small in comparison with the $B^{(-)}$ contribution in the kinematic region under consideration. This means that $B^{(-)}$ is almost determined from experimental data alone without any dispersion calculation. We note that the influence of $B^{(+)}$ on Eqs. (2.7) and (2.8) is rather small; therefore, the result for the charge exchange amplitudes depends essentially only on C^+ and the l.h.s of Eqs. (2.7), (2.8), and (2.9).

In the following sections we describe the details and results of the calculations for each amplitude separately.

3. THE CROSSING SYMMETRIC AMPLITUDE $C^{(+)}$

The determination of C^{+} was performed by using the once subtracted DR:

$$\mathrm{Re}\, C^{(+)}(\nu,t) = \frac{\nu}{1-t/4M^2} F_B^{(+)}(\nu,t) + C^{(+)}(0,t) + \frac{2\nu^2}{\pi}\int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'} \frac{\mathrm{Im}\, C^{(+)}(\nu',t)}{\nu'^2-\nu^2} \tag{3.1}$$

where $\nu_0 = \mu + t/4M$; the crossing symmetric variable ν is defined by

$$\nu = \frac{s-u}{4M} = \omega + \frac{t}{4M} \tag{3.2}$$

and $F_B^{(+)}(\nu,t)$, the nucleon Born term, is given in the Appendix, Eq. (A.5).

The low energy part of the integral and the subtraction function $C^{(+)}(0,t)$ were calculated by a consistency condition and the CERN 1971 phase-shifts[1]. Details of the determination of $C^{+}(0,t)$ are given in Ref. 15. For our purposes the subtraction function can be well approximated by

$$C^{(+)}(0,t) = 190 + 440\,t \quad [\mathrm{GeV}^{-1}] \tag{3.3}$$

in the region $|t| \leq 4(\mathrm{GeV}/c)^2$.

For the high energy behaviour of Im C^{+} the following parametrization was used:

$$\mathrm{Im}\, C^{(+)}(\nu,t) = \hat{\sigma}\, k \frac{s}{4q^2}\, e^{bt} \qquad k > 14\ \mathrm{GeV}/c \tag{3.4}$$

where $\hat{\sigma}$ = 23.25 mb is fixed by the total cross section data up to 60 GeV/c [17]. The value of b was adjusted at each t value separately in order to join smoothly the high energy part of Im C^{+} to the upper end of Im C^+ of our analysis. Since data on $d\sigma/dt$ for $\pi^- p$ elastic scattering at 25 and 40 GeV/c are available [Antipov et al.[18]] and charge exchange amplitudes are practically negligible with respect to C^+ at these energies, we determined the value of b by requiring consistency of our solution with these high energy data.

We started our iteration procedure by taking the Im C^+ from the model of Barger and Phillips[10] with modifications below 3 GeV/c in order to join it smoothly to the low energy amplitude from phase-shifts at 1.5 GeV/c. With the real part of C^+ from Eq. (3.1) we evaluated Eq. (2.2) and obtained a new Im C^+, taking into account the approximation $k|B^+| = |C^+|$. Now we went back to Eq. (3.1) and repeated the steps described above until we arrived at a stable solution.

Our results are presented in Fig. 1 and Table 1. We found that the C^+ amplitudes reconstructed from phase-shifts are not compatible with DR for energies between 1.5 and 2.0 GeV/c. The deviations from our solution are shown in Fig. 1 for t = -0.1 and -0.3 $(\mathrm{GeV/c})^2$. It seems to us that the phase-shift analysis allows only a good determination of $|C^+|$ whereas the phase has a rather large error*). By rotating the phase most of the discrepancies could be removed.

In order to study the influence of the high-energy assumption on the behaviour of Re $C^{(+)}$ one can change the energy dependence in Eq. (3.4) above 50 GeV [up to this energy Eq. (3.4) is in agreement with the experimental data] to a constant or to a logarithmic energy dependence like $\mathrm{Im}\, C^{(+)}_{(k,t)} = k[\hat{\sigma}_1 + \hat{\sigma}_2\, \ln^2(k/50)]\, e^{bt}$. The resulting correction to Re $C^{(+)}$ is less than 2% at 14 GeV and even less at lower energies**).

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*) The same problem appears if the Saclay phase-shifts[6] are used. This discrepancy was already noticed in Refs. 15 and 19. The results of Pietarinen[8] also differ from the phase-shifts results in this energy region (see also Fig. 1 and 2).

**) Following the ideas of Höhler et al.[15], $\hat{\sigma}_2$ is taken from a fit to the prediction of $\sigma^{(+)}$ by Cheng et al.[20].

Based on this evidence we conclude that the uncertainties in our values for Re C^+ arise mainly from the low energy contribution to the dispersion integral. Typical errors of our results are also shown in Figs. 1 and 2. These errors, however, are only an estimate, since a detailed calculation requires the full error matrix of the phase-shifts which is not available at present. Our estimation of the errors takes into account the experimental errors of $\Sigma\sigma$, uncertainties due to the unknown asymptotic behaviour of Im $C^{(+)}$ above 50 GeV, as well as the uncertainties in the low energy contribution to the DR.

Our results agree well with those of Höhler et al.[15] above 5 GeV and, within the errors, with those of Pietarinen[8], even though his amplitude shows some structures at higher energies.

At $t = -0.1$ $(\mathrm{GeV/c})^2$ we compared our results for $C^{(+)}$ with the analysis of McClure et al.[21]. These authors have used the modulus representation[22] instead of the usual DR [Eq. (3.1)]. Their results essentially agree with ours; however, their amplitude shows structures at high energies, which cannot arise from the data.

4. THE AMPLITUDE $B^{(+)}$

From Eq. (2.5) we can calculate the component of $B^{(+)}$ transverse to $C^{(+)}$:

$$B_{\perp}^{(+)} \simeq \frac{\Sigma P}{|C^{(+)}|} \tag{4.1}$$

The experimental information as well as the phase shift results for $B^{(+)}$ are very poor. Moreover, ΣP is the small difference between two large terms:

$$\Sigma P \simeq \mathrm{Im}\, B^{(+)} \mathrm{Re}\, C^{(+)} - \mathrm{Im}\, C^{(+)} \mathrm{Re}\, B^{(+)} \tag{4.2}$$

For these reasons we were unable to find a satisfactory solution for $B^{(+)}$ by iterating ΣP and the usual DR for this amplitude. However, information on the second component of $B^{(+)}$ could, in principle, be obtained from a modified DR for which the input is proportional to ΣP. A DR of this type was proposed by Pietarinen[8]. However, this DR is dominated by the high energy part of $\mathrm{Im}[B^{(+)}\, C^{(+)*}]$ which is completely unknown (because of the unknown behaviour of $A^{(+)}$) and it is not possible to get reliable results by using this relation. Therefore in this investigation we confine ourselves to an approximation of $B^{(+)}$ obtained from the assumption $k|B^{(+)}| = |C^{(+)}|$ and from ΣP which determines the phase of this amplitude. The results are presented in Table 1 and Fig. 2. This procedure has found, *a posteriori*, justification in the following:

i) At 6 GeV our amplitudes agree with the ones of Ref. 13, that means the R-parameters as well as all other observables are reproduced within the errors (see Fig. 6).

ii) Inserting the imaginary part of $B^{(+)}$ in the DR:

$$\mathrm{Re}\, B^{(+)}(\nu,t) = F_B^{(+)}(\nu,t) + \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im}\, B^{(+)}(\nu',t)}{\nu'^2 - \nu^2} \qquad (4.3)$$

with the low energy part of $\mathrm{Im}\, B^{(+)}$ constructed from the CERN 1971 phase-shifts and the asymptotic behaviour

$$\mathrm{Im}\, B^{(+)} = C \frac{s}{4q^2} \qquad q \geq 2.5 \text{ GeV/c} \qquad (4.4)$$

we found consistency of our solution within the errors.

The uncertainties of $B^{(+)}$ do not affect very much the determination of the isospin odd amplitudes because the term $\mathrm{Im}\,[B^{(+)}\, C^{(-)*}]$ in Eq. (2.8) is only of the order of 10% compared to $\mathrm{Im}\,[B^{(-)}\, C^{(+)*}]$ and because the influence of $B^{(+)}$ on $\Delta\sigma$ is suppressed by the factor $t/(4M^2)$ [see Eq. (2.7)] and thus would only change the results for $C^{(-)}$ within their errors.

5. THE CROSSING ANTISYMMETRIC AMPLITUDE $C^{(-)}$

Once $C^{(+)}$ and $B^{(+)}$ are known, we obtain $C^{(-)}$ from the iteration of $\Delta\sigma$, ΔP and $d\sigma_0/dt$, together with the DR for $C^{(-)}$:

$$\mathrm{Re}\, C^{(-)}(\nu,t) = \frac{\nu}{1-t/4M^2} F_B^{(-)}(\nu,t) + \frac{2\nu}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im}\, C^{(-)}(\nu',t)}{\nu'^2-\nu^2} \tag{5.1}$$

Again, the input for the DR at low energies was taken from the CERN 1971 phase-shifts. For the behaviour of Im $C^{(-)}$ above 14 GeV/c we assumed a simple power law:

$$\mathrm{Im}\, C^{(-)} = \mathrm{Im}\, C^{(-)}(14\ \mathrm{GeV/c}, t)\left(\frac{k}{14}\right)^{\alpha(t)} \tag{5.2}$$

The optimal α was determined for each t-value separately from Im $C^{(-)}$ above 10 GeV in order to take into account to some extent a possible non-Regge behaviour of this amplitude. More elaborate parametrizations for the asymptotic behaviour were not tried since the experimental errors of $\Delta\sigma$ are rather large.

The results for this amplitude are given in Table 1; for t = -0.1 and -0.3 $(\mathrm{GeV/c})^2$ curves are shown in Figs. 3a and 3b, respectively. The widths of the error channels given are estimated from the errors of $\Delta\sigma$ which should be dominant. Since DR are used the errors are correlated in energy. The errors of the real and imaginary parts of the amplitudes are also strongly correlated.

Our results for $C^{(-)}$ show the $\Delta(2420)$ resonance[23] and energy dependent zeros of the real and imaginary parts. The zero of Im $C^{(-)}$, which is strongly related to the cross-over point of the elastic differential cross-sections, moves from t = -0.08 $(\mathrm{GeV/c})^2$ at 3 GeV to t = -0.13 $(\mathrm{GeV/c})^2$ at 14 GeV. This energy dependence is much stronger than existing $\rho + \rho'$ and ρ + cut models predict. The zero of the real part is found at lower t-values.

Comparing this with the slope of $C^{(-)}$ at $t = 0$ [12] we conclude that the linear approximation is good for Im $C^{(-)}$ up to $t = -0.1$ (GeV/c)^2 and energies between 3 and 8 GeV. For higher energies as well as for the real part at all energies higher derivatives of the amplitude at $t = 0$ are important, since the zeros occur at smaller t-values. However, we are aware of the fact that the data in the two regions above 6 GeV and between 3 and 6 GeV are of different quality: whereas in the latter region the high statistics cross-section data of the Argonne group[24] are now available (this experiment was performed with the intention of determining $\Delta\sigma$), for higher energies only the older data of Harting et al.[25] and those of Foley et al.[26] exist. We may not exclude that new measurements in this energy region would change our results for the energy dependence of the zeros.

In Fig. 3 the results of Barger and Phillips as well as those of Pietarinen are also shown. The deviations from our results -- mainly close to the zeros of the real and imaginary parts -- are much larger than it was the case for the amplitude $C^{(+)}$. Again we remark that Pietarinen's amplitude shows considerable structures at higher energies which are not confirmed by our results.

6. THE AMPLITUDE $B^{(-)}$

The results for the amplitude $B^{(-)}$ are given in Table 1 as well as in Fig. 4. The errors of $B^{(-)}$ are estimated from the errors of ΔP and $d\sigma_0/dt$. However, the errors of the real and the imaginary part of $B^{(-)}$ are strongly correlated since the modulus of $B^{(-)}$ can be accurately determined from $d\sigma_0/dt$ while the phase determined from ΔP is much less accurate.

Our amplitude shows a resonance structure between 2 and 3 GeV, which is associated with the $\Delta(2420)$ resonance[23]. The real part of $B^{(-)}$ as well as the imaginary part follow a power law above 6 GeV, which can be well described by a pure ρ-Regge trajectory:

$$B^{(-)} = 166 \text{ GeV}^{-2} \alpha_\rho \left(\frac{k}{k_0}\right)^{\alpha_\rho(t)-1} \left(i + \tan\left(\frac{\pi\alpha_\rho}{2}\right)\right) e^{2.17t} \tag{6.1}$$

where $\alpha_s(t) = 0.55 + 0.98\ t$ and $k_0 = 1$ GeV/c. This agrees with an earlier result of Dronkers et al.[11] for $B^{(-)}$ at $t = 0$. Secondary contributions due to a ρ' or a cut seem to be small. Furthermore, we have checked our results for consistency with the DR

$$\mathrm{Re}\, B^{(-)}(\nu,t) = \mathcal{F}_B^{(-)}(\nu,t) + \frac{2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\nu'\, \mathrm{Im}\, B^{(-)}(\nu',t)}{\nu'^2 - \nu^2} \tag{6.2}$$

in the same way as it was done for $B^{(+)}$ using an asymptotic behaviour as given by Eq. (6.1). The dispersive real part agrees within the errors with the one calculated directly from the data. Nevertheless, below 2 GeV there seems to be a discrepancy in the phase shift results: the imaginary part of $B^{(-)}$ shows a dip -- which is also seen in the $d\sigma_0/dt$ data -- whereas no dip is seen in the real part which, however, is demanded by the DR. Recently $\pi^- p$ polarization measurements have been performed at 40 GeV [27]. Since $P\sigma_-$ at this energy is essentially given by the term $\mathrm{Im}\left[B^{(-)}\ C^{(+)*}\right]$ we have calculated the polarization at 40 GeV taking $B^{(-)}$ from Eq. (6.1) and $C^{(+)}$ from Eq. (3.4) and the DR. We found perfect agreement with the experiment which is certainly a strong support in favour of our high energy assumptions.

Apart from $C^{(+)}$ this amplitude turns out to be the best determined one. Therefore it is not astonishing that the results of Barger and Phillips (above 4 GeV) as well as those of Pietarinen agree very well with our results. However, in the resonance region we found some deviations between our $B^{(-)}$ and Pietarinen's.

7. CONSISTENCY WITH CHARGE EXCHANGE POLARIZATION AND SPIN ROTATION PARAMETERS

Since the data on charge exchange polarization and spin rotation parameters did not enter into our analysis they represent an independent test for our amplitudes. Figure 5 shows the existing measurements of the charge exchange polarization and the same quantity as calculated from our solution. We found good agreement with the data which confirms in particular our high energy assumption for C^-.

Unfortunately we are not able to discriminate clearly between the data of Bonamy et al.[30] at 4.9 GeV and the data of the Argonne Group[28] at 5 GeV which are inconsistent at small momentum transfers. However the data of Argonne seem to be favoured.

A further test can be performed by comparing our amplitudes with the spin rotation parameter R which in terms of the s-channel helicity amplitudes*) is given by

$$R \frac{d\sigma}{dt} = -\left\{ |F_{++}|^2 - |F_{+-}|^2 \right\} \cos \vartheta_R + 2 \, \mathrm{Re} \left(F_{++} F_{+-}^{*} \right) \sin \vartheta_R \qquad (7.1)$$

where ϕ_R is the angle of the recoil nucleon in the lab. system. ϕ_R is related to the c.m. scattering angle ϕ by:

$$\sin \vartheta_R = \frac{1}{\sqrt{1 - t/4M^2}} \cos\left(\frac{\vartheta}{2}\right) \qquad (7.2)$$

Figure 6 shows the data of Ref. 16 on $R(\pi^{\pm} p \rightarrow \pi^{\pm} p)$ at 6 GeV and the corresponding quantities following from our analysis. Within large errors the agreement with data is fairly good, however, we are aware of the fact that our result in this respect also suffers from large uncertainties which are mainly due to the large errors of ΣP and to the approximation $k|B^{+}| = |C^{+}|$.

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*) The relation between the s-channel helicity amplitudes and the (invariant) t-channel helicity amplitudes is given by Eq. (A.4) in the Appendix.

8. REMARKS AND CONCLUSIONS

We have determined the πN amplitudes from differential cross-sections and elastic polarization data, using fixed-t DR as a theoretical constraint. $C^{(-)}$ and in particular $B^{(+)}$ have large errors. New measurements with high statistics of elastic differential cross-sections -- such as the experiment of the Argonne group[24] -- with accurate values of $\Delta\sigma$ would help to reduce the uncertainties of $C^{(-)}$ to a large extent. Furthermore, better data on the elastic polarizations between 2 and 3 GeV would be very helpful in working out resonance structures at intermediate energies. Measurements of any observable above 14 GeV would also improve an analysis along these lines since they help to establish realistic assumptions on the asymptotic behaviour.

A detailed error analysis is as yet impossible for reasons we have mentioned above. Therefore, we have confined ourselves to estimating errors -- which are highly correlated in energy and also between the different amplitudes and their real and imaginary part -- to get a rough idea of the uncertainties. These errors are shown in the figures.

We have compared our amplitudes with those of Pietarinen's analysis. The over-all agreement is good at least for the amplitudes $C^{(+)}$ and $B^{(-)}$. Nevertheless, his amplitudes show structures even at high energies which are responsible for some wiggles in the polarizations calculated from these amplitudes. This is demonstrated by the charge exchange polarization in Fig. 5 and as a typical example for the elastic polarization we show in Fig. 7 those at $t = -0.3\ (\text{GeV}/c)^2$.

We summarize the main features of the resulting amplitudes as follows:

i) The ratio Re $C^{(+)}$/Im $C^{(+)}$ is appreciable within the diffraction peak. The influence of the real part on the absorption corrections is therefore not negligible and should be taken into account in realistic models[40].

ii) The agreement of our results for $C^{(+)}$ with those of Ref. 15 suggests that the simple formula (3.4) provides a very good parametrization for Im C^{+} above 4 GeV up to 40 GeV/c. There seems to be no indication for an energy dependence characteristic for f_0-exchange, since the secondary term of Eq. (3.4) behaves like a constant.

iii) The imaginary part of the isospin odd s-channel helicity amplitudes above 4 GeV show a behaviour consistent with the dual absorption model[41], as far as this can be concluded from our limited t-region (compare Fig. 8 and 9 where we have shown the helicity amplitudes at 6 GeV). In particular we confirm the zero of the Bessel function J_0 around $t \sim -0.15$ $(GeV/c)^2$ in the non-flip amplitude.

iv) Above 6 GeV the flip amplitude $F_{+-}^{(-)}$ is well represented by a pure ρ-Regge formula:

$$F_{+-}^{(-)} = -\sin\left(\frac{\vartheta}{2}\right) 12.2 \; e^{.17t} \alpha \left(\frac{k}{k_0}\right)^{\alpha - 1} \left(i + \tan\left(\frac{\pi\alpha}{2}\right)\right) \tag{8.1}$$

where the trajectory is given by $\alpha = 0.56 + 1.04\ t$ and $k_0 = 1$ GeV/c. This is in agreement with the dual absorption model and confirmes the assumptions made in several studies of πN-scattering [for example: Elvekjaer et al. [42], Argyres et al. [43]]. Since $B^{(-)}$ dominates in $F_{+-}^{(-)}$ this result depends only on the assumption concerning the high energy behaviour of the imaginary part of $C^{(+)}$ which is however well known up to 40 GeV by experimental data.

v) No simple relation for the phase of $F_{++}^{(-)}$ can be established from the present analysis. We note that the large uncertainties of this amplitude can be reduced only if more accurate data on the elastic differential cross-section are available.

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APPENDIX

The usual Mandelstam variables are s, t, u, and the nucleon and pion masses are M and μ, respectively. The lab. energy and momentum of the incident pion are

$$\omega = (s - M^2 - \mu^2)/2M \tag{A.1a}$$

and

$$k = \sqrt{\omega^2 - \mu^2}, \tag{A.1b}$$

respectively.

The isospin decomposition of the invariant amplitudes are:

$$C_{\pm} = C^{(+)} \mp C^{(-)} \qquad \text{for } \pi^{\pm}p \rightarrow \pi^{\pm}p \tag{A.2a}$$

$$C_0 = -\sqrt{2}\, C^{(-)} \qquad \text{for } \pi^- p \rightarrow \pi^0 n \tag{A.2b}$$

(similar formulas hold for B).

The measurable quantities in terms of the invariant functions are well-known:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\left\{\frac{1 - t/4M^2}{k^2}|C|^2 - \frac{t}{4M^2}\,\frac{1 + t/4q^2}{1 - t/4M^2}|B|^2\right\} \tag{A.3a}$$

$$P\frac{d\sigma}{dt} = \frac{\sin\vartheta}{16\pi\sqrt{s}}\, \mathrm{Im}\,(BC^*) \tag{A.3b}$$

where θ is the c.m. scattering angle and

$$q = \sqrt{[s-(M+\mu)^2][s-(M-\mu)^2]/(4s)}\,. \tag{A.3c}$$

Relations between s-channel helicity amplitudes and the invariant amplitudes (c.m. frame) are:

$$F_{++} = \frac{\cos(\frac{\vartheta}{2})}{4\sqrt{\pi}\,k}\left\{C - \frac{t}{4M^2}\,\frac{M+\omega}{1-t/4M^2}\,B\right\}$$

$$F_{+-} = q\frac{\sin(\frac{\vartheta}{2})}{4\sqrt{\pi}\,M}\left\{\frac{M+\omega}{k^2}\,C - \frac{1+t/4q^2}{1-t/4M^2}\,B\right\} \tag{A.4}$$

The nucleon pole term is given by:

$$F_B^{(\pm)} = \frac{g^2}{M} \frac{1}{\nu_B^2 - \nu^2} \begin{pmatrix} \nu \\ \nu_B \end{pmatrix} , \qquad \text{(A.5)}$$

where $\nu_B = -\mu^2/(2M) + t/(4M)$ and $g^2/(4\pi) = 14.6$ is the πN coupling constant.

REFERENCES

1) F. Halzen and C. Michael, Phys. Letters 36 B, 367 (1971).

2) P. Johnson, K.E. Lassila, P. Koehler, R. Miller and A. Yokosawa, Phys. Rev. Letters 30, 343 (1973).

3) R.L. Kelly, Phys. Letters 39 B, 635 (1972).

4) G. Cozzika, Y. Ducros, A. Gaidot, A. de Lesquen, J.P. Merlo and L. van Rossum, Phys. Letters 40 B, 281 (1972).

5) S. Almehed and C. Lovelace, Nuclear Phys. B40, 157 (1971).

6) R. Ayed, P. Bareyre and Y. Lemoigne, preprint, CEN-Saclay (1972).

7) F. Wagner, preprint, München (1972).

8) E. Pietarinen, Nuclear Phys. B49, 315 (1972).

9) R. Cutcosky, Ann. Phys. 54, 350 (1969).
E. Pietarinen, Nuovo Cimento 12 A, 522 (1972).

10) V. Barger and R.J.N. Phillips, Phys. Rev. 187, 2210 (1969).

11) J. Dronkers and P. Kroll, Nuclear Phys. B47, 291 (1972).

12) H.P. Jakob and P. Kroll, to be published in Nuclear Phys. B.

13) G. Höhler, H.P. Jakob and F. Kaiser, preprint TKP 11/72, Karlsruhe (1972).

14) G. Höhler and H.P. Jakob, preprint TKP 23/72, Karlsruhe (1972).

15) G. Höhler and H.P. Jakob, preprint TKP 13/73, Karlsruhe (1973).

16) A. De Lesquen, B. Amblard, R. Beurtey, G. Cozzika, J. Bystricky, J. Deregel, Y. Ducros, J.M. Fontaine, A. Gaidot, M. Hansroul, F. Lehar, J.P. Merlo, S. Miyashita, J. Movchet and L. van Rossum, Phys. Letters 40 B, 277 (1972).

17) G. Höhler and H.P. Jakob, Nuovo Cimento Letters 6, 389 (1973).

18) Yu.M. Antipov, G. Ascoli, R. Busnello, G. Damgaard, M.N. Kienzle-Focacci, W. Kienzle, R. Klanner, L.G. Landsberg, A.A. Lebedev, C. Lechanoine, P. Lecomte, M. Martin, V. Roinishvili, R.D. Sard, A. Weitsch and F.A. Yoteh, Nuclear Phys. B57, 333 (1973).

19) G. Höhler and H.P. Jakob, Nuovo Cimento Letters 2, 485 (1971).

20) H. Cheng, J.K. Walker and T.T. Wu, Phys. Letters 44 B, 97 (1973).

21) J.A. McClure and L.E. Pitts, Phys. Rev. D5, 109 (1972).

22) R. Odorico, Nuovo Cimento 54 A, 96 (1968).

23) Particle Data Group, Phys. Letters 39 B, 1 (1972).

24) I. Ambats et al., Phys. Rev. Letters 29, 1415 (1972).

25) O. Harting, P. Blackall, B. Elsner, A.C. Helmholz, W.C. Middelkoop, B. Powell, B. Zacharov, P. Zanella, P. Dalpiaz, M.N. Focacci, S. Focardi, G. Giacomelli, L. Monari, J.A. Beaney, R.A. Donald, P. Mason, L.W. Jones and D.C. Caldwell, Nuovo Cimento 38, 60 (1965).

26) K.J. Foley, R.S. Jones, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.D. Platner, C.A. Quarles and E.H. Willen, Phys. Rev. 181, 1775 (1969).

27) C. Bruneton et al., Phys. Letters 44 B, 471 (1973).

28) D. Hill, P.F.M. Koehler, T.B. Novey, B. Sandler, H. Spinka, A. Yokosawa, D. Eartly, K. Pietzl, G. Burleson and H. Davis, Phys. Rev. Letters 30, 239 (1973).

29) P. Bonamy, P. Borgeaud, S. Brehin, C. Bruneton, P. Falk-Variant, O. Guisan, P. Sonderegger, C. Caverzasio, J.P. Guillaud, J. Schneider, M. Yvert, I. Mannelli, F. Sergiampietri and M.L. Vincelli, Nuclear Phys. B16, 335 (1970).

30) P. Bonamy, P. Borgeaud, M. Crozon, J.P. Guillaud, O. Guisan, P. Le Du, P. Sonderegger, J.K. Bienlein, S. Mango, L. Paul and H. Dinter, Amsterdam Int. Conf. on Elementary Particles (1971).

31) D.D. Drobnis et al., Phys. Rev. Letters 20, 274 (1968).

32) M. Borghini et al., to be published in Phys. Letters.

33) M. Borghini et al., Phys. Letters 24 B, 77 (1967) and Phys. Letters 31 B, 405 (1970).

34) R.J. Esterling et al., Phys. Rev. Letters 21, 1410 (1968).

35) N.E. Booth, Chicago preprint (1969).

36) A. Yokosawa, preprint ANL/HEP 7177 (1971),
G. Burleson et al., Phys. Rev. Letters 26, 338 (1971),
R.E. Hill et al., Phys. Rev. D1, 729 (1970).

37) P. Johnson, preprint UCRL-17683 (1967).

38) M. Albrow et al., Nuclear Phys. B25, 9 (1971).

39) M. Hansroul, preprint UCRL-17263 (1967).

40) G.L. Kane, Lecture given at the International Summer Institute on Theoretical Physics in Kaiserslautern (1972).

41) H. Harari, Phys. Rev. Letters 26, 1400 (1971).
H. Harari and A. Schwimmer, Phys. Rev. D5, 2780 (1972).

42) F. Elvekjaer, T. Inami and R.J.N. Phillips, Rutherford preprint RL 73-031 T56 (1973).

43) E.N. Argyres, A.P. Contogouris and M. Svec, to be published in Phys. Rev. D.

Table 1.1 πN AMPLITUDES AT T = -.05(GEV/C)**2

K GEV	RE C(+) 1/GEV	IM C(+) 1/GEV	RE C(-) 1/GEV	IM C(-) 1/GEV	RE B(+) 1/GEV**2	IM B(+) 1/GEV**2	RE B(-) 1/GEV**2	IM B(-) 1/GEV**2
1.50	-36.9	119.2	5.03	-7.14	-63.22	23.00	57.23	64.47
1.55	-42.7	122.5	9.16	-6.54	-69.00	11.10	46.36	59.25
1.60	-46.1	120.0	12.00	-2.94	-54.20	45.05	43.58	54.18
1.65	-46.1	120.6	13.13	-0.58	-41.95	56.35	43.94	49.81
1.70	-46.6	121.0	13.83	2.54	-32.73	61.51	43.55	43.32
1.80	-45.5	123.7	12.11	7.33	-21.89	64.81	44.64	39.39
1.90	-44.2	128.8	9.27	9.32	-18.17	63.67	45.57	40.75
2.00	-43.3	134.2	7.17	9.77	-17.93	61.03	39.52	52.51
2.20	-43.3	149.9	3.60	10.43	-34.04	50.96	39.35	64.22
2.40	-48.6	163.5	0.29	8.18	-38.97	47.69	20.60	69.03
2.60	-52.6	173.3	-0.06	4.46	-39.24	45.57	19.83	68.53
2.80	-55.0	183.9	1.65	2.62	-37.78	43.79	19.40	67.92
3.00	-58.0	195.4	3.48	2.04	-31.52	47.71	18.30	65.06
3.20	-61.6	205.4	4.82	2.53	-28.20	50.01	18.72	60.65
3.40	-64.5	215.1	5.32	3.17	-26.29	51.10	20.49	56.18
3.60	-66.8	224.1	5.50	3.38	-24.38	51.15	22.58	51.95
3.80	-68.4	233.9	5.78	3.44	-23.61	50.39	28.27	46.36
4.00	-70.5	243.9	6.13	3.50	-22.57	49.94	32.75	41.14
5.00	-77.9	290.4	7.74	4.34	-19.88	47.55	29.99	35.72
6.00	-82.6	340.3	8.88	5.10	-17.64	46.57	28.08	31.14
7.00	-86.6	388.2	10.01	5.85	-15.55	45.82	25.78	27.62
8.00	-89.5	438.3	11.10	6.75	-13.04	45.79	24.09	25.84
9.00	-92.6	487.8	12.02	7.83	-11.77	45.49	22.96	24.25
10.00	-95.5	538.4	12.57	9.02	-10.93	45.34	21.72	23.02
11.00	-99.2	588.5	12.72	9.99	-9.98	45.26	20.88	21.76
12.00	-102.8	637.6	12.86	10.48	-9.55	45.04	19.77	21.02
13.00	-105.9	686.8	13.22	10.89	-9.65	44.74	19.08	20.03
14.00	-108.8	735.3	13.63	11.35	-9.70	44.48	18.35	19.39

Table 1.2 πN AMPLITUDES AT T = -.10(GEV/C)**2

K GEV	RE C(+) 1/GEV	IM C(+) 1/GEV	RE C(-) 1/GEV	IM C(-) 1/GEV	RE B(+) 1/GEV**2	IM B(+) 1/GEV**2	RE B(-) 1/GEV**2	IM B(-) 1/GEV**2
1.50	-41.9	94.1	2.89	-4.99	-47.27	7.51	45.44	55.19
1.55	-44.2	98.1	5.73	-5.92	-62.86	11.17	41.88	50.78
1.60	-47.0	95.2	6.56	-2.95	-51.67	32.87	41.54	50.02
1.65	-46.3	96.2	6.97	-1.81	-40.97	42.88	41.17	46.38
1.70	-46.8	97.3	7.03	-0.28	-33.97	47.70	41.55	42.59
1.80	-47.1	99.4	6.37	1.81	-23.76	50.64	42.55	34.15
1.90	-45.6	102.0	5.17	3.33	-18.71	48.43	39.90	32.23
2.00	-43.7	106.9	3.90	3.98	-22.67	44.70	28.80	41.47
2.20	-43.6	120.9	1.29	4.37	-31.95	40.38	20.88	47.56
2.40	-48.9	132.3	-0.83	2.88	-35.83	38.03	19.43	47.66
2.60	-52.6	139.5	-0.90	0.36	-35.61	36.15	18.71	46.10
2.80	-54.5	147.8	0.32	-0.83	-33.70	36.47	20.71	43.11
3.00	-56.2	156.4	1.29	-0.90	-29.17	39.52	20.82	40.43
3.20	-58.5	165.8	1.79	-0.72	-26.30	41.29	22.21	36.46
3.40	-61.3	174.4	2.07	-0.55	-24.55	41.86	23.22	33.72
3.60	-64.4	182.5	2.24	-0.42	-23.07	42.04	25.31	30.40
3.80	-66.9	189.6	2.35	-0.29	-22.53	41.31	26.39	26.67
4.00	-68.9	197.3	2.43	-0.22	-21.62	40.95	24.45	25.75
5.00	-76.4	233.4	2.79	-0.11	-18.82	38.84	21.99	22.78
6.00	-81.5	271.5	3.28	-0.12	-16.52	37.91	19.98	20.44
7.00	-85.8	308.4	4.16	-0.13	-14.58	37.17	18.09	18.97
8.00	-88.7	345.9	5.02	0.55	-12.42	36.90	17.05	17.78
9.00	-90.9	383.7	5.51	1.29	-11.07	36.56	16.07	16.76
10.00	-92.9	422.1	5.80	2.12	-10.07	36.31	15.08	15.73
11.00	-94.7	460.6	5.77	2.95	-9.07	36.16	14.35	14.99
12.00	-96.5	499.2	5.42	3.54	-8.51	35.96	13.49	14.29
13.00	-98.4	538.1	5.15	3.68	-8.29	35.74	13.07	13.66
14.00	-100.1	576.2	5.07	3.82	-8.13	35.51	12.76	13.06

Table 1.3 πN AMPLITUDES AT T = -.15(GEV/C)**2

K GEV	RE C(+) 1/GEV	IM C(+) 1/GEV	RE C(-) 1/GEV	IM C(-) 1/GEV	RE B(+) 1/GEV**2	IM B(+) 1/GEV**2	RE B(-) 1/GEV**2	IM B(-) 1/GEV**2
1.50	-42.0	75.5	2.73	-4.02	-51.82	-9.24	39.81	39.77
1.55	-45.0	76.6	2.72	-3.51	-51.90	12.69	37.13	39.59
1.60	-46.5	75.2	2.76	-2.93	-45.12	24.77	34.98	38.78
1.65	-46.1	75.2	2.90	-2.58	-38.08	30.93	32.24	37.38
1.70	-46.0	75.6	2.99	-2.24	-32.64	34.19	29.70	36.03
1.80	-44.8	76.8	2.79	-1.43	-25.03	35.48	26.50	33.12
1.90	-42.8	80.1	2.37	-0.86	-19.99	35.88	26.39	28.01
2.00	-41.2	84.1	1.83	-0.48	-23.09	33.34	24.19	29.99
2.20	-39.8	96.0	0.75	-0.33	-30.51	29.67	22.41	32.55
2.40	-43.4	107.2	-0.44	-0.62	-32.84	29.84	21.19	32.42
2.60	-47.6	114.9	-0.68	-1.96	-31.57	31.23	20.34	30.43
2.80	-50.7	121.7	0.12	-2.87	-28.77	33.08	19.69	29.75
3.00	-53.1	128.5	0.57	-2.61	-26.01	34.56	18.91	26.47
3.20	-55.3	135.2	0.50	-2.33	-24.08	35.21	18.86	24.51
3.40	-57.4	141.9	0.33	-2.27	-22.53	35.45	18.57	23.22
3.60	-59.6	148.5	0.22	-2.31	-21.60	35.26	18.35	21.41
3.80	-61.3	154.9	0.14	-2.35	-20.78	35.00	19.11	17.50
4.00	-63.2	162.0	0.04	-2.39	-20.10	34.91	18.87	17.33
5.00	-74.1	192.6	-0.28	-2.95	-17.64	33.80	14.78	17.18
6.00	-82.8	220.5	-0.03	-3.59	-15.56	32.25	11.96	16.13
7.00	-88.6	245.9	0.58	-3.89	-14.04	30.45	11.78	14.92
8.00	-91.0	271.9	1.21	-3.74	-11.95	29.50	9.99	13.93
9.00	-91.9	299.7	1.55	-3.27	-10.58	28.93	9.08	13.22
10.00	-92.9	328.6	1.58	-2.82	-9.52	28.65	8.63	12.03
11.00	-94.0	357.8	1.44	-2.51	-8.63	28.46	8.15	11.21
12.00	-95.0	387.0	1.29	-2.39	-8.01	28.27	7.71	10.89
13.00	-95.6	416.5	1.23	-2.32	-7.62	28.08	7.43	10.13
14.00	-96.3	446.7	1.18	-2.24	-7.42	27.94	7.09	9.87

Table 1.4 πN AMPLITUDES AT T = -.20(GEV/C)**2

K GEV	RE C(+) 1/GEV	IM C(+) 1/GEV	RE C(-) 1/GEV	IM C(-) 1/GEV	RE B(+) 1/GEV**2	IM B(+) 1/GEV**2	RE B(-) 1/GEV**2	IM B(-) 1/GEV**2
1.50	-39.9	58.3	2.91	-2.60	-44.29	-13.84	27.24	34.65
1.55	-40.7	58.3	2.89	-3.22	-46.45	2.84	24.49	34.29
1.60	-40.2	58.0	3.08	-3.11	-41.90	15.82	21.23	33.66
1.65	-39.5	59.7	3.06	-2.93	-36.32	22.67	18.69	32.81
1.70	-40.0	60.9	2.98	-2.56	-31.67	26.13	17.40	32.19
1.80	-40.0	62.4	2.67	-1.99	-22.53	31.27	14.08	30.33
1.90	-39.0	64.9	2.13	-1.49	-19.07	32.24	12.10	27.97
2.00	-37.8	67.8	1.56	-1.23	-20.78	29.92	12.92	25.50
2.20	-37.3	78.1	0.26	-1.12	-24.28	27.34	13.31	26.17
2.40	-40.0	85.0	-0.68	-1.84	-26.03	25.09	12.25	25.68
2.60	-42.0	92.3	-0.13	-3.33	-25.35	25.40	11.56	24.76
2.80	-45.2	98.7	0.34	-3.57	-24.05	26.16	11.66	23.65
3.00	-47.2	103.2	0.44	-3.43	-22.08	26.61	11.94	21.11
3.20	-48.5	109.4	0.41	-3.31	-19.86	27.77	12.45	18.62
3.40	-50.5	115.0	0.30	-3.20	-18.89	27.93	13.01	16.96
3.60	-52.1	120.3	0.13	-3.12	-17.98	27.92	13.38	15.02
3.80	-53.9	126.2	-0.05	-3.09	-17.39	27.94	13.95	13.37
4.00	-55.9	131.4	-0.30	-3.06	-16.85	27.81	13.71	12.63
5.00	-64.3	156.6	-1.05	-3.97	-14.89	26.95	10.78	11.81
6.00	-72.6	179.4	-0.91	-4.80	-13.65	25.99	8.38	11.43
7.00	-77.1	198.4	-0.40	-5.38	-12.24	24.86	6.94	10.55
8.00	-78.4	221.1	0.23	-5.50	-10.69	24.50	6.29	10.32
9.00	-80.9	244.3	0.65	-5.26	-9.55	24.25	5.73	9.75
10.00	-82.9	266.7	0.81	-4.98	-8.56	23.95	5.32	9.30
11.00	-85.2	290.3	0.83	-4.77	-7.73	23.82	5.05	8.63
12.00	-86.8	311.4	0.84	-4.68	-7.15	23.45	4.84	8.42
13.00	-86.9	334.4	0.89	-4.60	-6.57	23.27	4.58	7.96
14.00	-87.2	357.7	0.97	-4.53	-6.13	23.11	4.46	7.56

Table 1.5 πN AMPLITUDES AT T = -.30(GEV/C)**2

K GEV	RE C(+) 1/GEV	IM C(+) 1/GEV	RE C(-) 1/GEV	IM C(-) 1/GEV	RE B(+) 1/GEV**2	IM B(+) 1/GEV**2	RE B(-) 1/GEV**2	IM B(-) 1/GEV**2
1.50	-29.0	33.4	2.02	-0.26	-36.31	2.33	13.25	22.75
1.55	-27.2	32.8	2.04	-0.64	-32.07	9.90	11.52	22.34
1.60	-24.9	34.9	1.99	-1.87	-27.06	16.47	9.12	21.62
1.65	-24.6	38.6	1.85	-1.99	-26.96	17.73	8.01	21.22
1.70	-26.1	41.4	1.64	-2.14	-25.94	20.97	7.12	20.64
1.80	-29.2	43.7	1.32	-2.27	-22.63	25.11	5.30	18.87
1.90	-30.6	44.9	0.88	-2.52	-20.56	25.95	4.02	16.58
2.00	-31.4	46.7	0.60	-2.69	-20.68	25.08	3.45	14.01
2.20	-31.9	49.5	0.67	-3.59	-21.13	22.27	4.42	16.50
2.40	-32.8	55.4	0.57	-3.83	-21.75	21.51	4.36	15.07
2.60	-35.1	59.3	0.35	-3.92	-22.16	20.33	3.64	13.53
2.80	-36.8	62.9	0.11	-4.04	-21.04	20.63	3.61	12.77
3.00	-38.3	66.1	-0.07	-4.19	-19.84	20.79	3.68	11.53
3.20	-39.3	69.4	0.12	-4.56	-18.62	21.02	4.07	10.31
3.40	-40.2	72.7	0.15	-4.54	-17.58	21.10	4.29	9.46
3.60	-41.1	76.3	-0.12	-4.29	-16.68	21.24	4.48	8.80
3.80	-42.1	79.6	-0.43	-4.20	-16.00	21.19	4.81	8.28
4.00	-42.9	82.8	-0.72	-4.23	-15.18	21.23	4.88	7.72
5.00	-46.4	100.5	-1.38	-5.21	-12.75	21.12	3.53	6.58
6.00	-50.9	117.1	-1.40	-5.82	-11.23	20.82	2.84	5.89
7.00	-55.2	132.7	-1.29	-6.26	-10.14	20.38	2.33	5.41
8.00	-58.5	146.8	-1.12	-6.53	-9.14	19.87	2.09	5.04
9.00	-60.2	161.1	-0.93	-6.67	-8.15	19.49	1.94	4.69
10.00	-61.3	175.9	-0.82	-6.68	-7.34	19.24	1.82	4.39
11.00	-62.6	191.3	-0.76	-6.69	-6.69	19.06	1.71	4.08
12.00	-63.7	206.3	-0.72	-6.71	-6.19	18.87	1.61	3.86
13.00	-64.8	221.8	-0.69	-6.72	-5.81	18.72	1.55	3.71
14.00	-66.1	236.7	-0.52	-6.78	-5.55	18.54	1.48	3.49

Table 1.6 πN AMPLITUDES AT T = -.40(GEV/C)**2

K GEV	RE C(+) 1/GEV	IM C(+) 1/GEV	RE C(-) 1/GEV	IM C(-) 1/GEV	RE B(+) 1/GEV**2	IM B(+) 1/GEV**2	RE B(-) 1/GEV**2	IM B(-) 1/GEV**2
1.50	-21.5	15.9	-1.21	2.73	-23.08	-4.80	8.20	11.32
1.55	-19.5	16.6	-0.53	2.02	-28.48	6.07	7.73	11.02
1.60	-18.1	18.6	0.06	1.47	-28.86	13.38	6.36	10.43
1.65	-17.6	20.9	0.45	0.18	-27.03	15.70	5.77	10.15
1.70	-17.8	23.4	0.56	-1.15	-25.21	17.21	4.25	9.25
1.80	-20.3	26.6	0.54	-3.13	-23.83	19.16	2.35	6.55
1.90	-22.0	26.9	0.61	-3.62	-22.11	19.57	1.41	4.51
2.00	-22.1	27.6	1.05	-4.03	-21.14	18.52	1.09	2.95
2.20	-21.4	30.5	2.02	-5.20	-19.35	17.46	1.50	6.79
2.40	-22.6	35.8	2.11	-5.17	-19.55	18.10	1.15	6.53
2.60	-25.0	37.8	1.31	-4.21	-19.20	17.66	0.62	6.18
2.80	-25.2	39.4	0.66	-3.63	-17.16	17.85	0.43	5.57
3.00	-25.3	43.1	0.43	-3.91	-15.93	18.58	0.61	5.23
3.20	-26.5	46.3	0.41	-4.18	-15.33	18.94	0.73	4.85
3.40	-27.9	49.0	0.40	-4.37	-14.88	18.98	1.14	4.23
3.60	-29.0	51.3	0.11	-4.26	-14.42	18.87	1.35	3.79
3.80	-30.0	53.7	-0.30	-4.03	-13.85	18.88	1.58	3.40
4.00	-31.0	56.1	-0.69	-4.02	-13.37	18.87	1.61	3.12
5.00	-35.5	66.0	-1.31	-5.32	-11.57	18.03	0.90	2.31
6.00	-38.4	75.6	-1.23	-5.92	-10.12	17.32	0.63	1.88
7.00	-39.7	84.2	-1.08	-6.34	-8.81	16.60	0.52	1.76
8.00	-40.5	94.9	-0.82	-6.59	-7.70	16.46	0.45	1.69
9.00	-42.0	104.8	-0.70	-6.56	-6.97	16.21	0.40	1.52
10.00	-43.1	114.7	-0.67	-6.56	-6.30	16.02	0.38	1.41
11.00	-44.0	124.6	-0.66	-6.58	-5.78	15.83	0.34	1.30
12.00	-44.6	134.3	-0.63	-6.60	-5.32	15.64	0.31	1.20
13.00	-45.1	144.6	-0.60	-6.63	-4.92	15.56	0.30	1.12
14.00	-46.0	154.6	-0.34	-6.70	-4.67	15.43	0.28	1.08

Figure captions

Fig. 1a : $C^{(+)}/k$ as a function of the lab. momentum k at $t = -0.1$ $(\mathrm{GeV/c})^2$. The solid lines represent our results, the dashed regions indicate the error bounds of our results.
Dashed line from Barger and Phillips[10].
Dashed dotted line from Pietarinen[8].
▼ from Ref. 21.
o from Ref. 15.
x CERN 1971 phase-shifts.

Fig. 1b : $C^{(+)}/k$ at $t = -0.3$ $\mathrm{GeV/c}^2$. Symbols as in Fig. 1a.

Fig. 2a : $B^{(+)}$ at $t = -0.1$ $\mathrm{GeV/c}^2$. Symbols as in Fig. 1a.

Fig. 2b : $B^{(+)}$ at $t = -0.3$ $\mathrm{GeV/c}^2$. Symbols as in Fig. 1a.

Fig. 3a : $C^{(-)}$ at $t = -0.1$ $\mathrm{GeV/c}^2$. Symbols as in Fig. 1a.

Fig. 3b : $C^{(-)}$ at $t = -0.3$ $\mathrm{GeV/c}^2$. Symbols as in Fig. 1a.

Fig. 4a : $B^{(-)}$ at $t = -0.1$ $\mathrm{GeV/c}^2$. ⧳ Re $B^{(-)}$ following from the data of Ref. 27 and the assumption of a pure imaginary C^+ and $\Sigma P = 0$.

Fig. 4b : $B^{(-)}$ at $t = -0.3$ $\mathrm{GeV/c}^2$. ⧳ Re $B^{(-)}$ following from the data of Ref. 27 and the assumption of a pure imaginary C^+ and $\Sigma P = 0$.

Fig. 5 : The charge exchange polarization as calculated from our amplitudes. The data points are taken from Ref. 28 (⧳), Ref. 29 (⧮), Ref. 30 (⧯) and Ref. 31 (⧲). The data are interpolated in t if necessary.

Fig. 6 : The R parameters for $\pi^{\pm}p$ elastic and charge exchange scattering as calculated from our amplitudes at 6 GeV/c (⧳). Data points are taken from Ref. 16 (⧲). To indicate the energy dependence following from our result we have included the values at 3 and 12 GeV/c (solid and dashed lines respectively). These curves are smooth interpolations of our results in order to keep the diagram readable.

Fig. 7 : The elastic polarization at $t = -0.3\ \mathrm{GeV/c^2}$ compared to what follows from our amplitudes (solid line) and from Ref. 8 (dashed-dotted line). Data are taken from Ref. 32 (●), Ref. 33 (○), Ref. 34 (▲), Ref. 35 (△), Ref. 36 (■), Ref. 37 (⊕), Ref. 38 (✱), Ref. 39 (◆), and Ref. 27 (+).

Fig. 8 : The s-channel helicity non-flip amplitude $F_{++}^{(-)}$ at 6 GeV/c. The components of $F_{++}^{(-)}$ are given with respect to $F_{++}^{(+)}$. Our results (●) are compared to those of Ref. 1 (○) -- as an example for an analysis where the polarization of Bonamy et al.[29] was used -- and those of Ref. 2 (□) based on the polarization data of Ref. 28. The broken line indicates the errors given by Ref. 2. The value t = 0 (▼) was taken from the table of πN forward amplitudes[14].

Fig. 9 : The s-channel helicity flip amplitude $F_{+-}^{(-)}$ at 6 GeV/c. Symbols are the same as in Fig. 8.

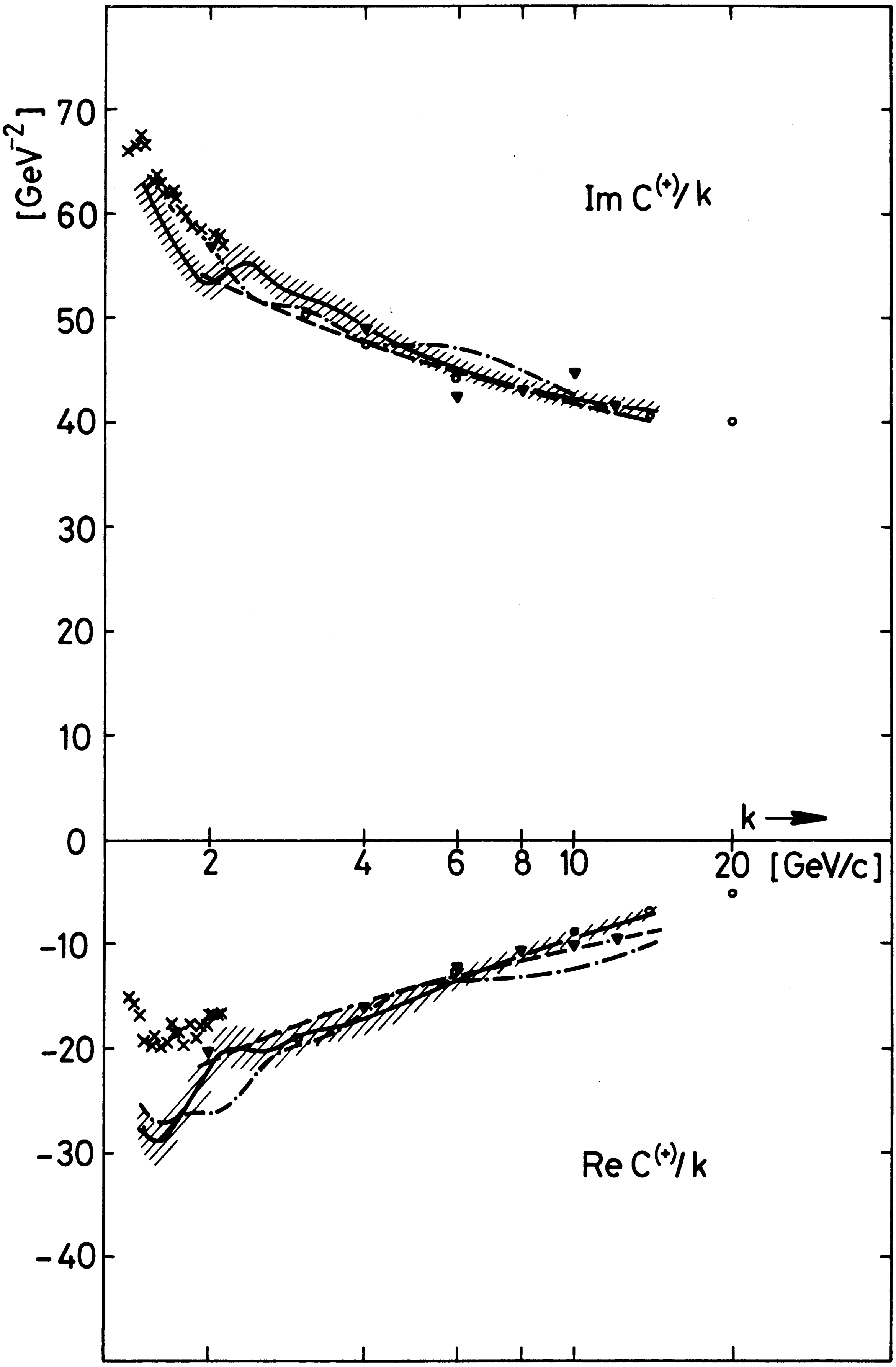


Fig. 1a

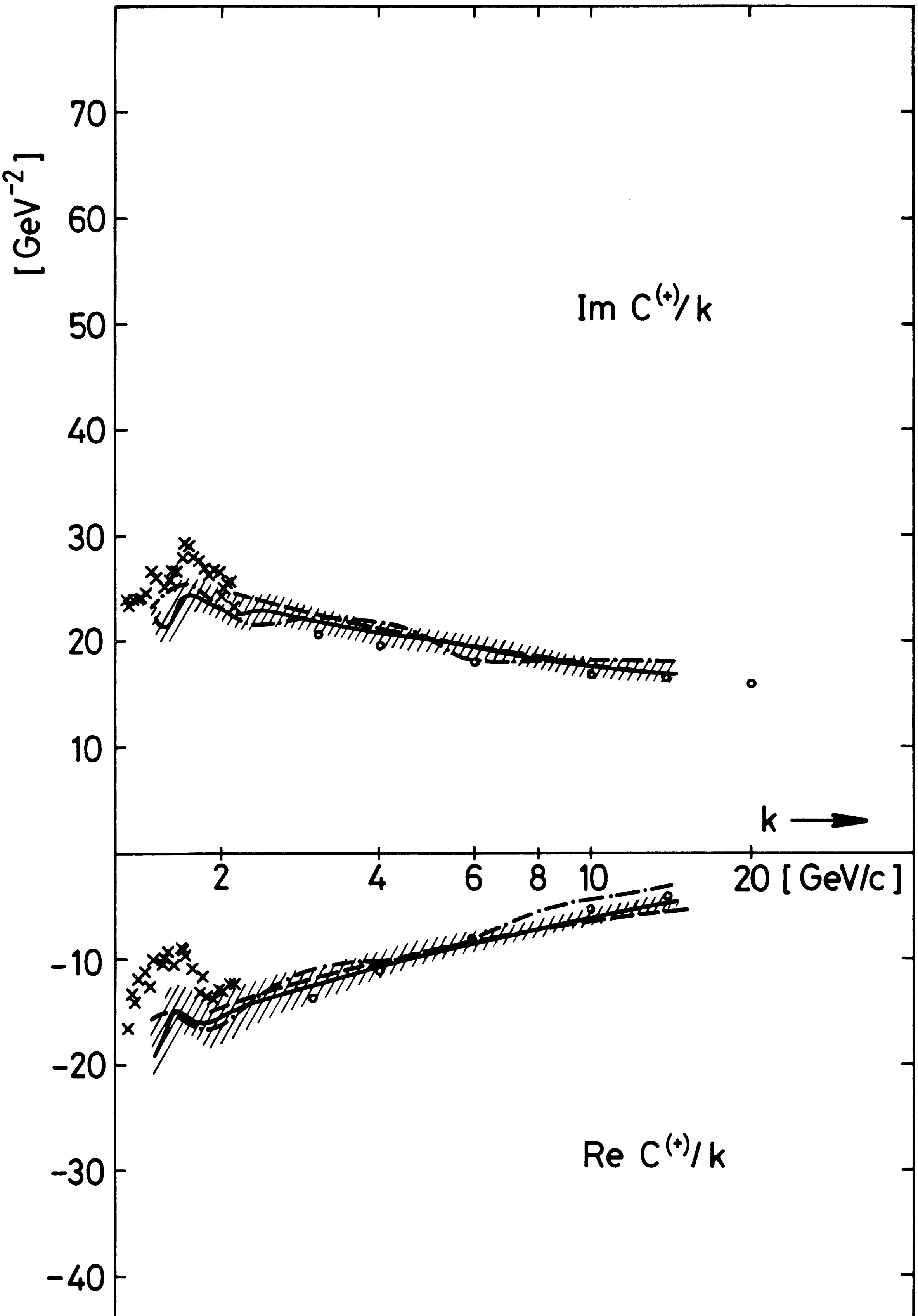


Fig. 1b

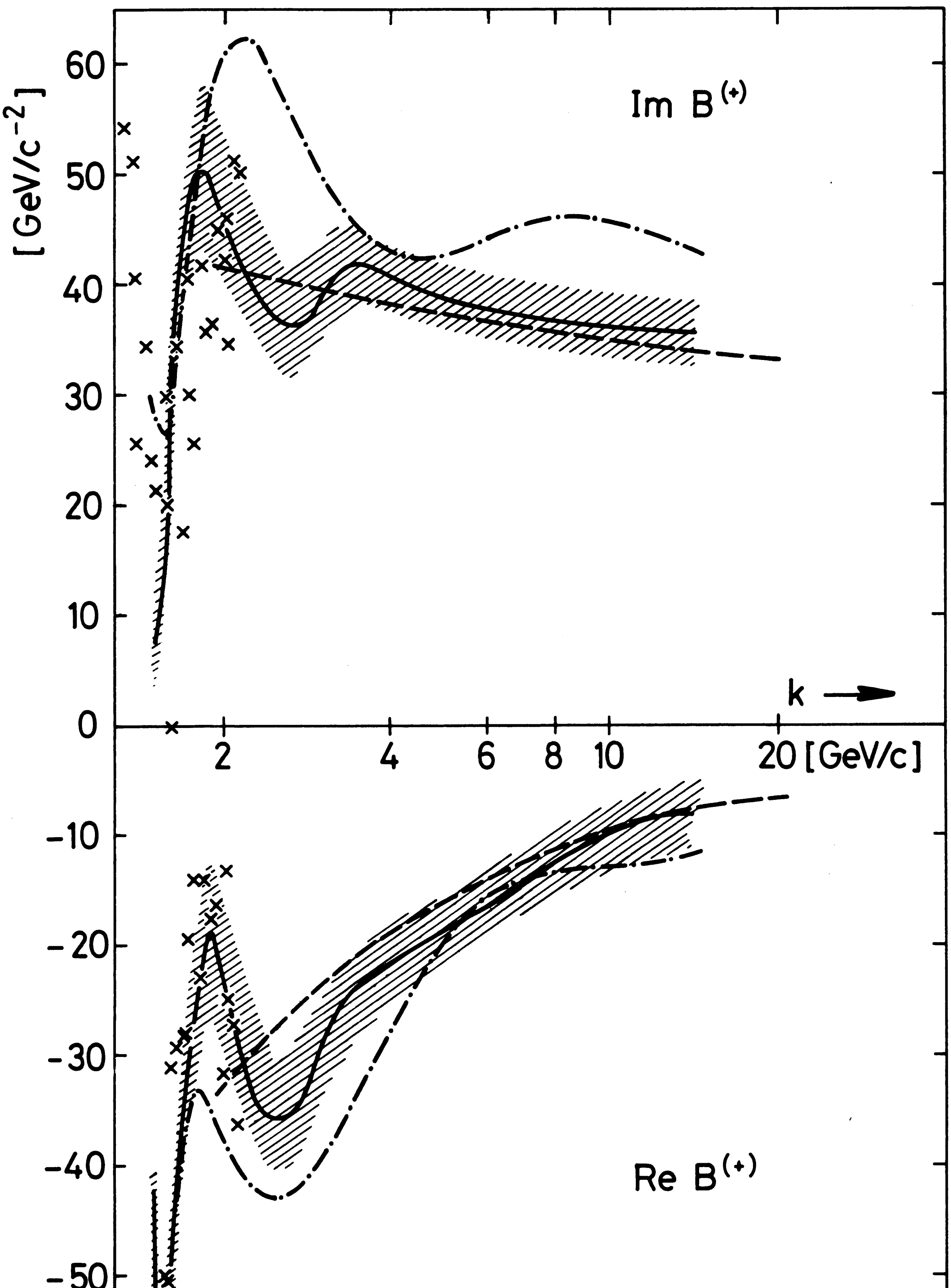


Fig. 2a

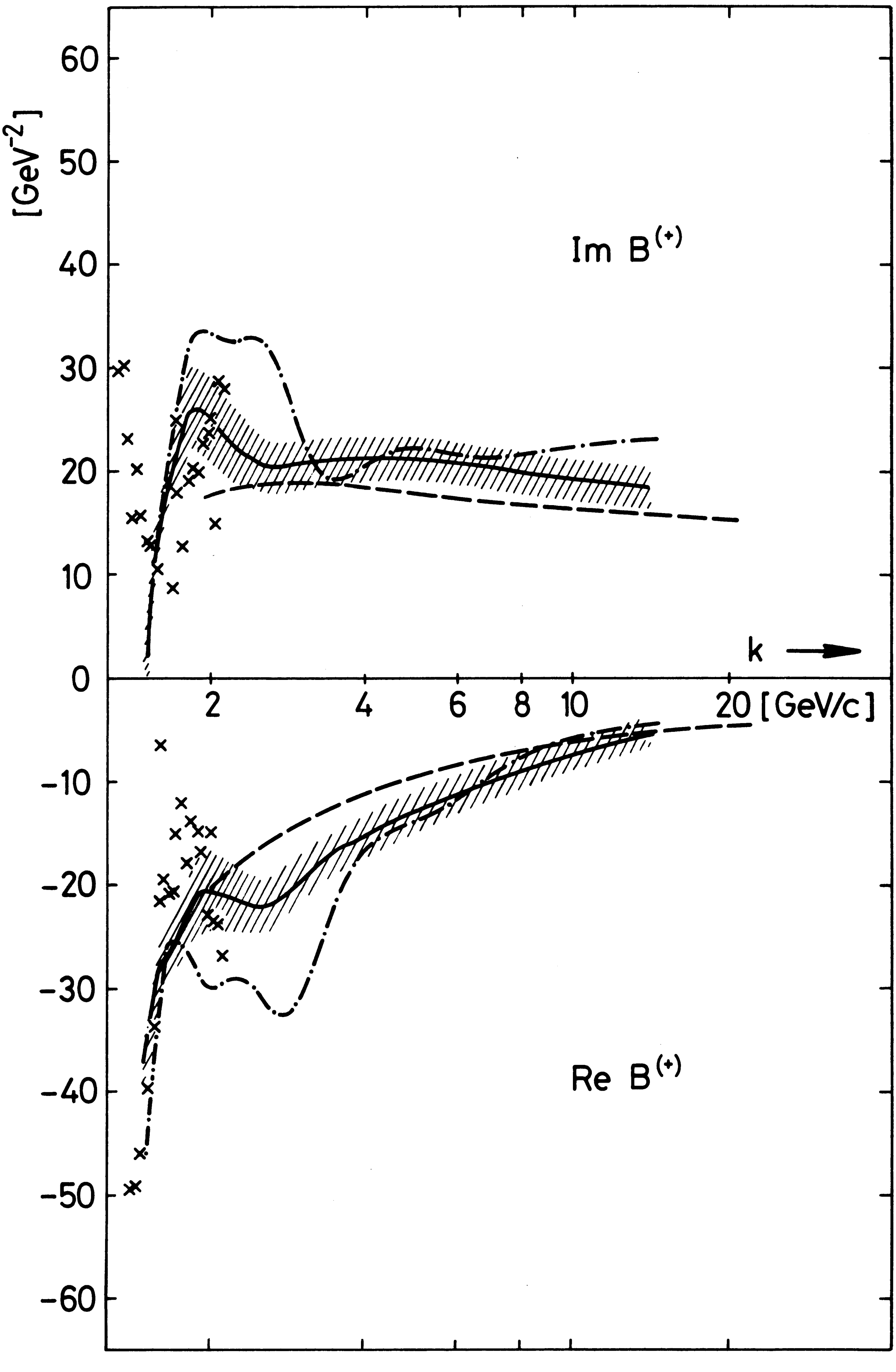


Fig. 2b

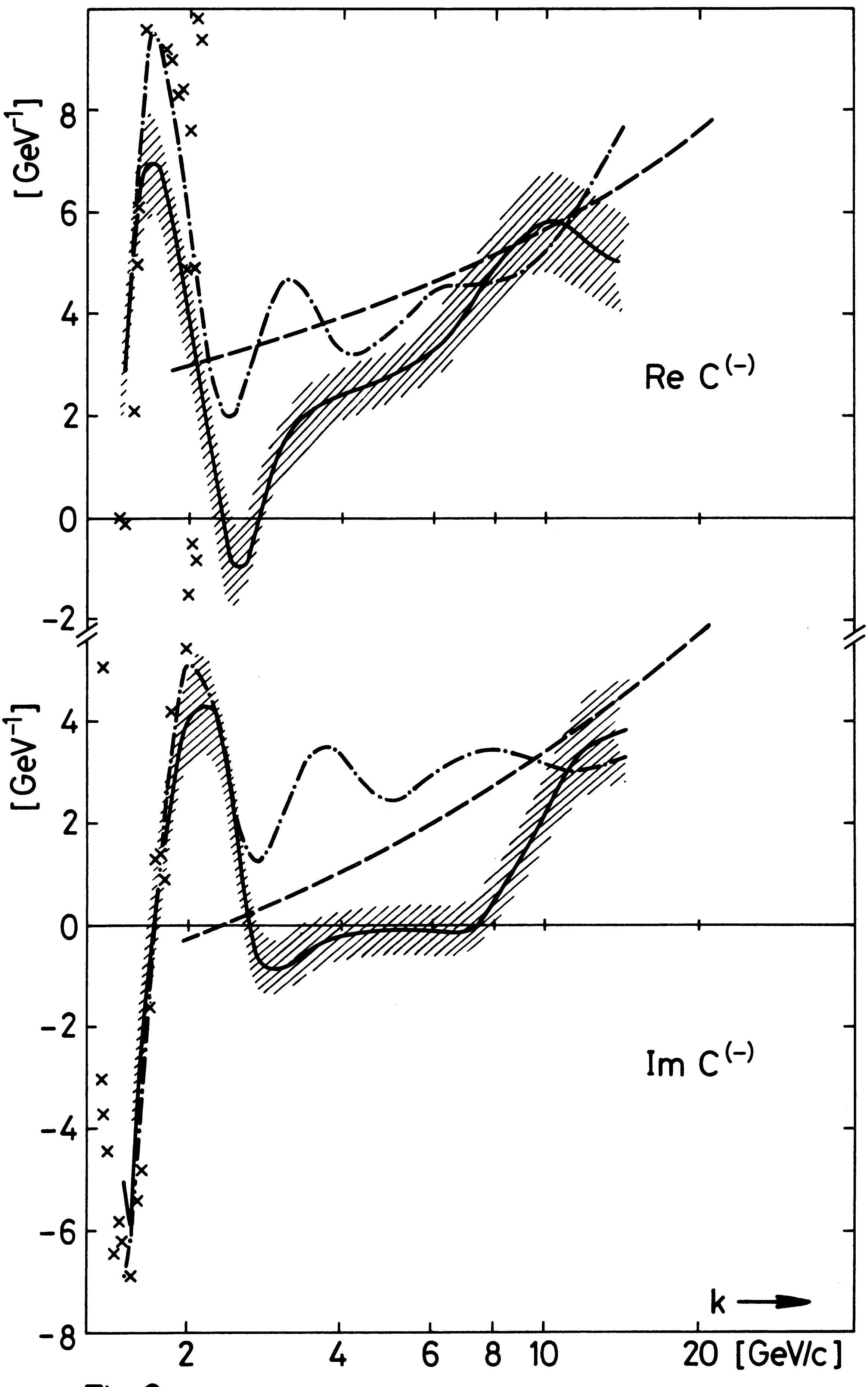


Fig. 3a

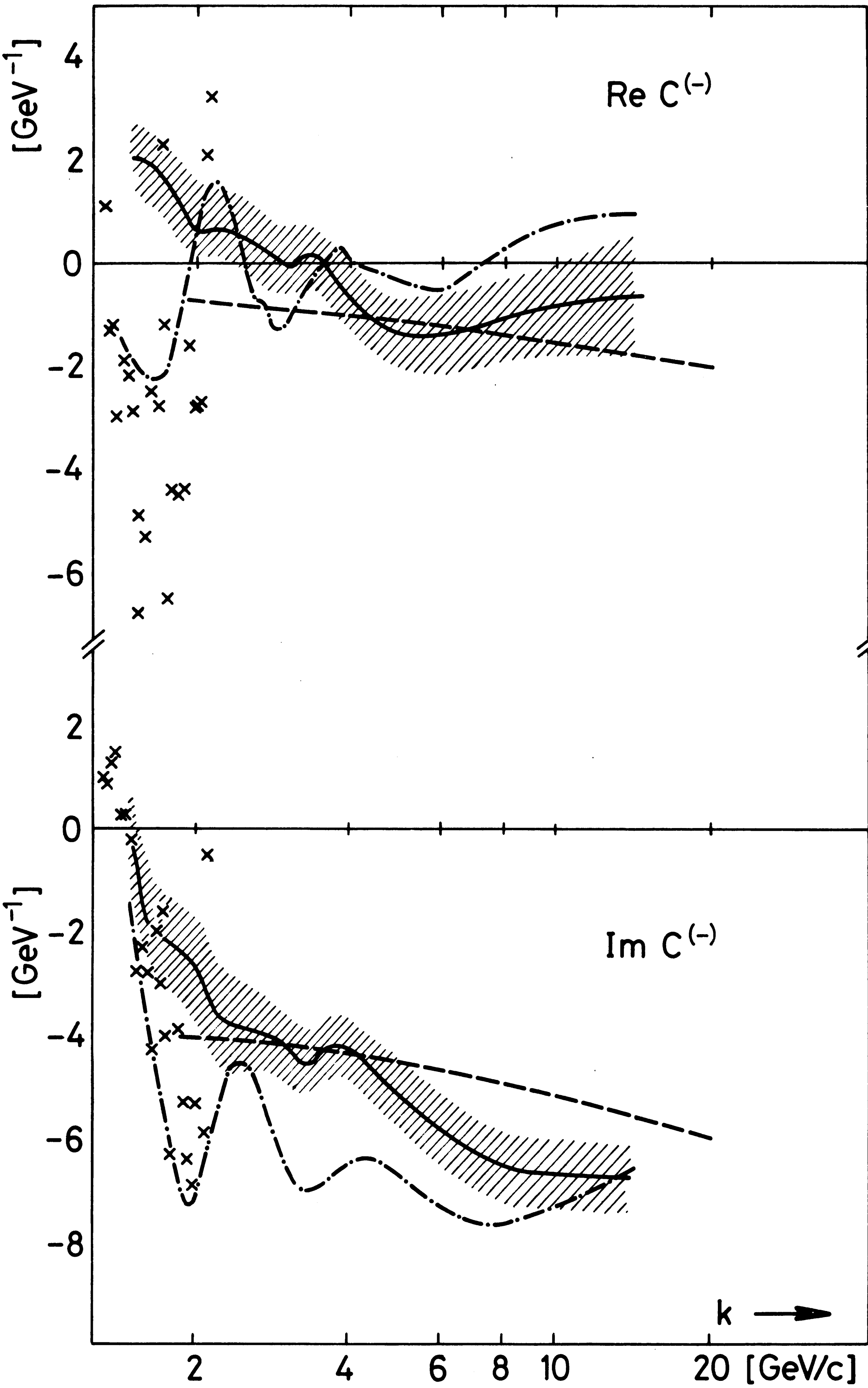
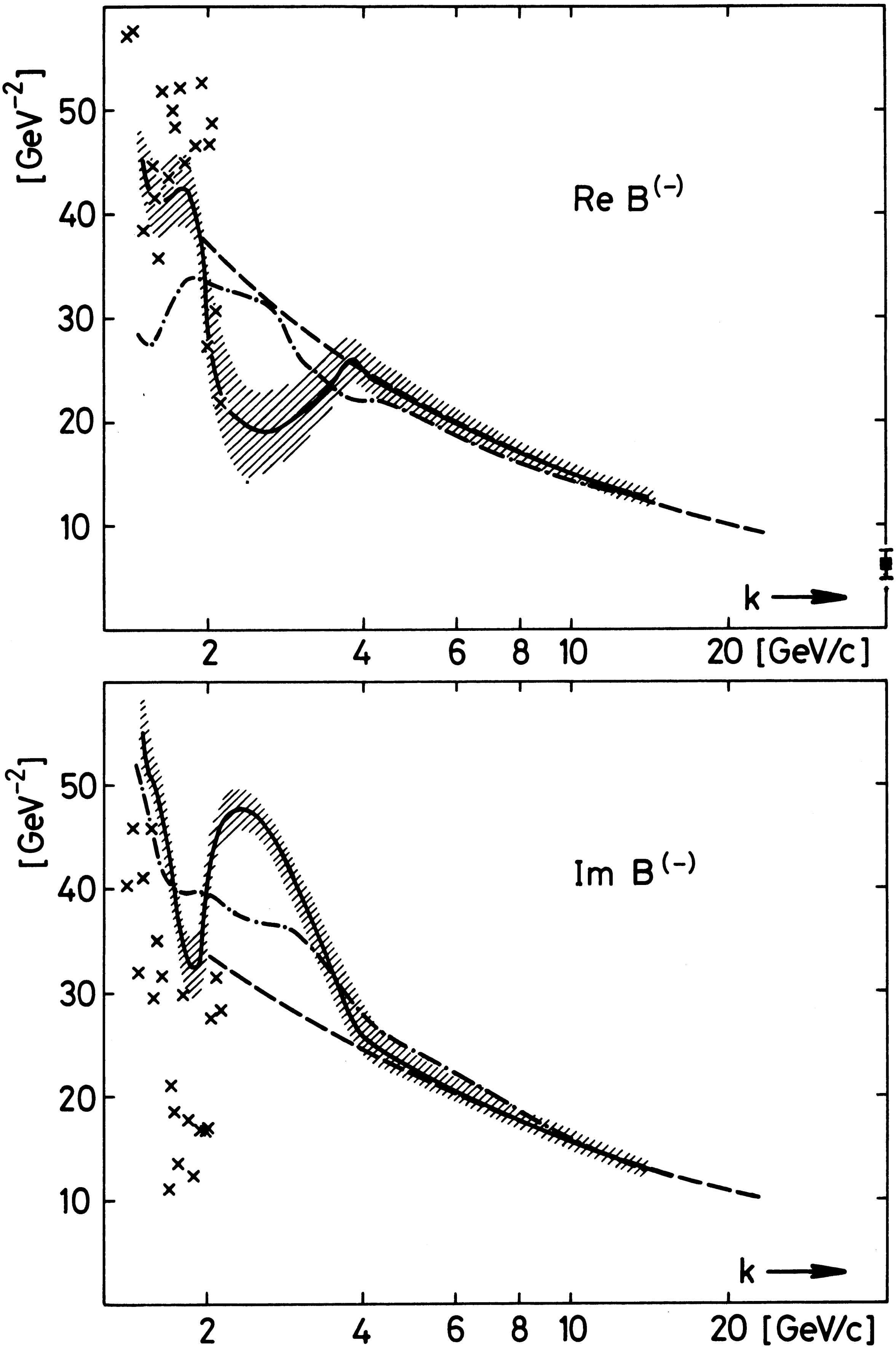
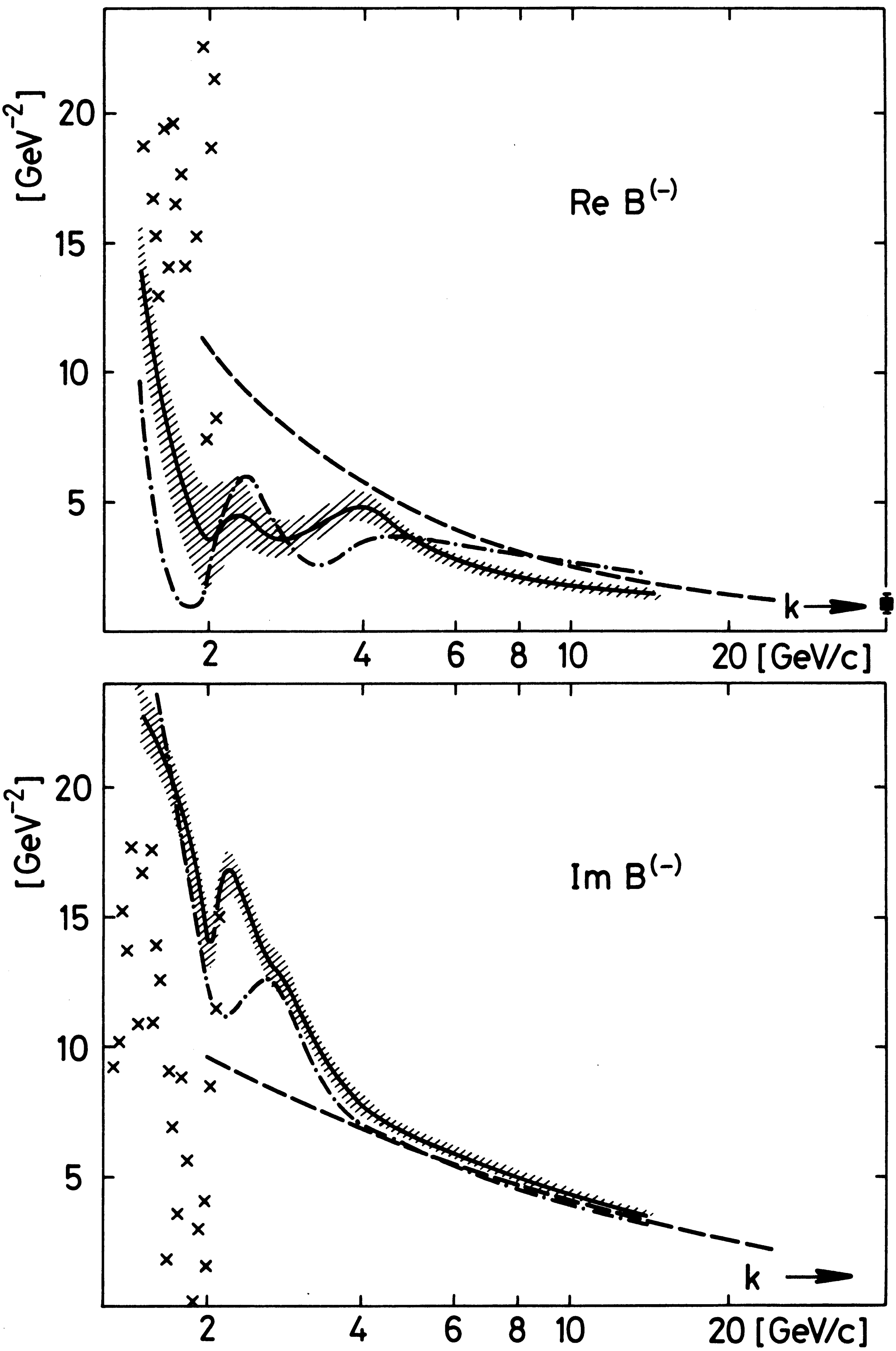


Fig. 3b



[GeV^{-2}]
50
40
30
20
10
Re B$^{(-)}$
k
2
4
6
8
10
20 [GeV/c]
[GeV^{-2}]
50
40
30
20
10
Im B$^{(-)}$
k
2
4
6
8
10
20 [GeV/c]

Fig. 4a



Re B^(-)
[GeV^-2]
20
15
10
5
2
4
6
8
10
20 [GeV/c]
k
Im B^(-)
[GeV^-2]
20
15
10
5
2
4
6
8
10
20 [GeV/c]
k

Fig. 4b

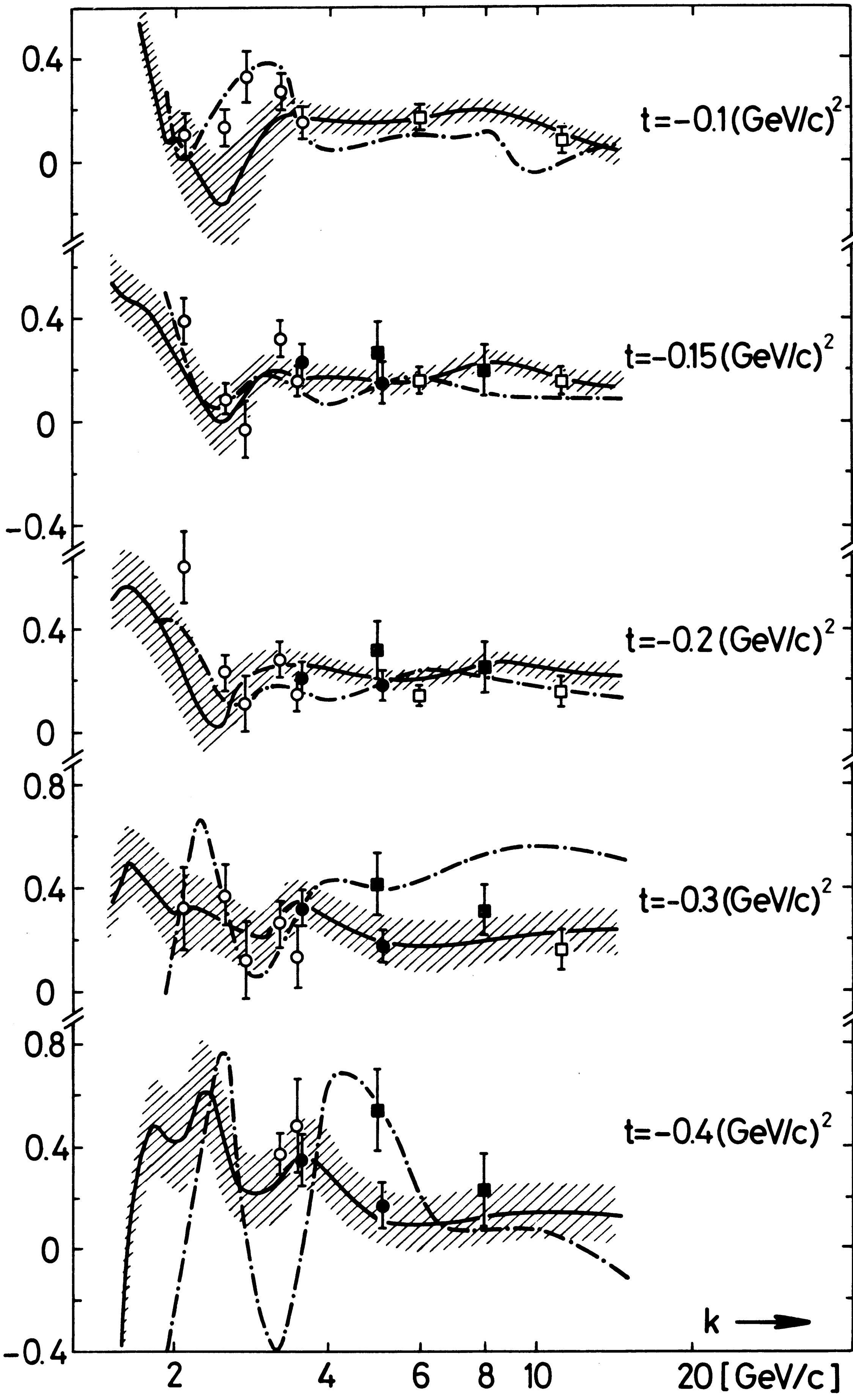

0.4
0
t=-0.1(GeV/c)^2
0.4
0
-0.4
t=-0.15(GeV/c)^2
0.4
0
t=-0.2(GeV/c)^2
0.8
0.4
0
t=-0.3(GeV/c)^2
0.8
0.4
0
-0.4
t=-0.4(GeV/c)^2
k
2
4
6
8
10
20 [GeV/c]

Fig. 5

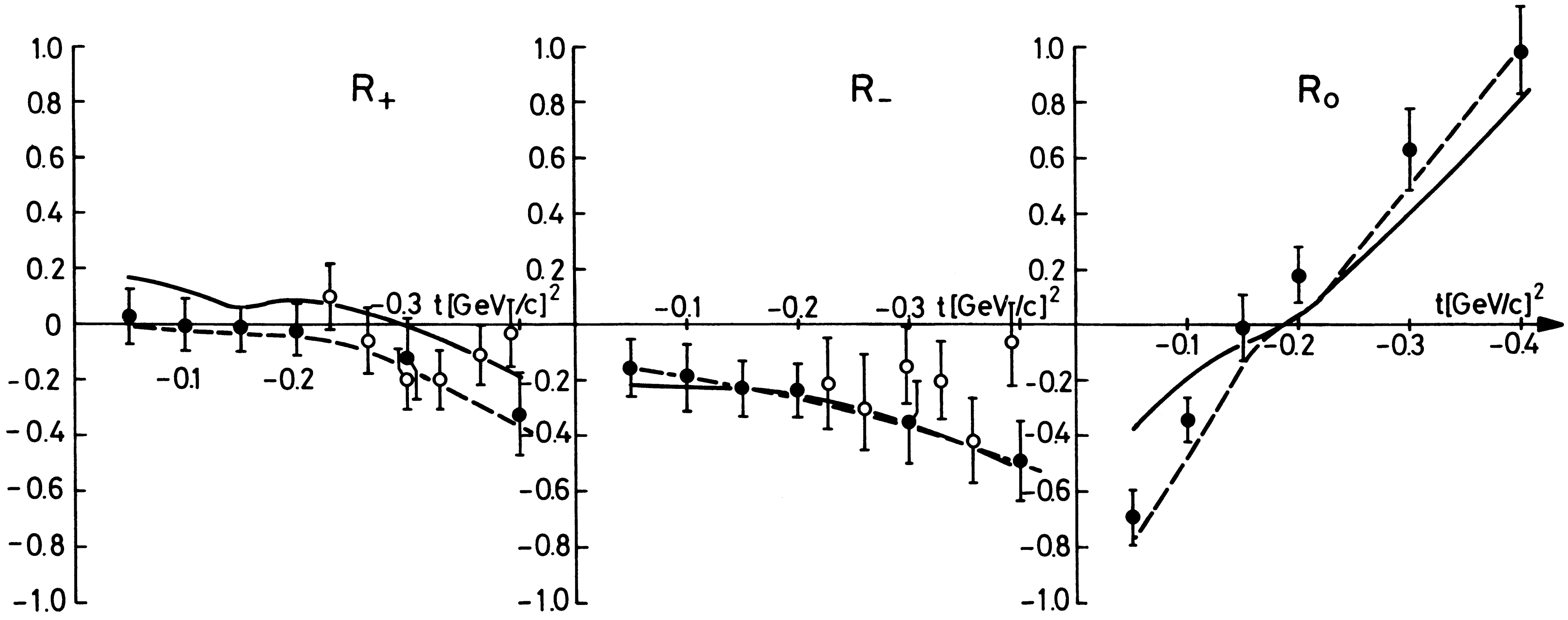


Fig. 6

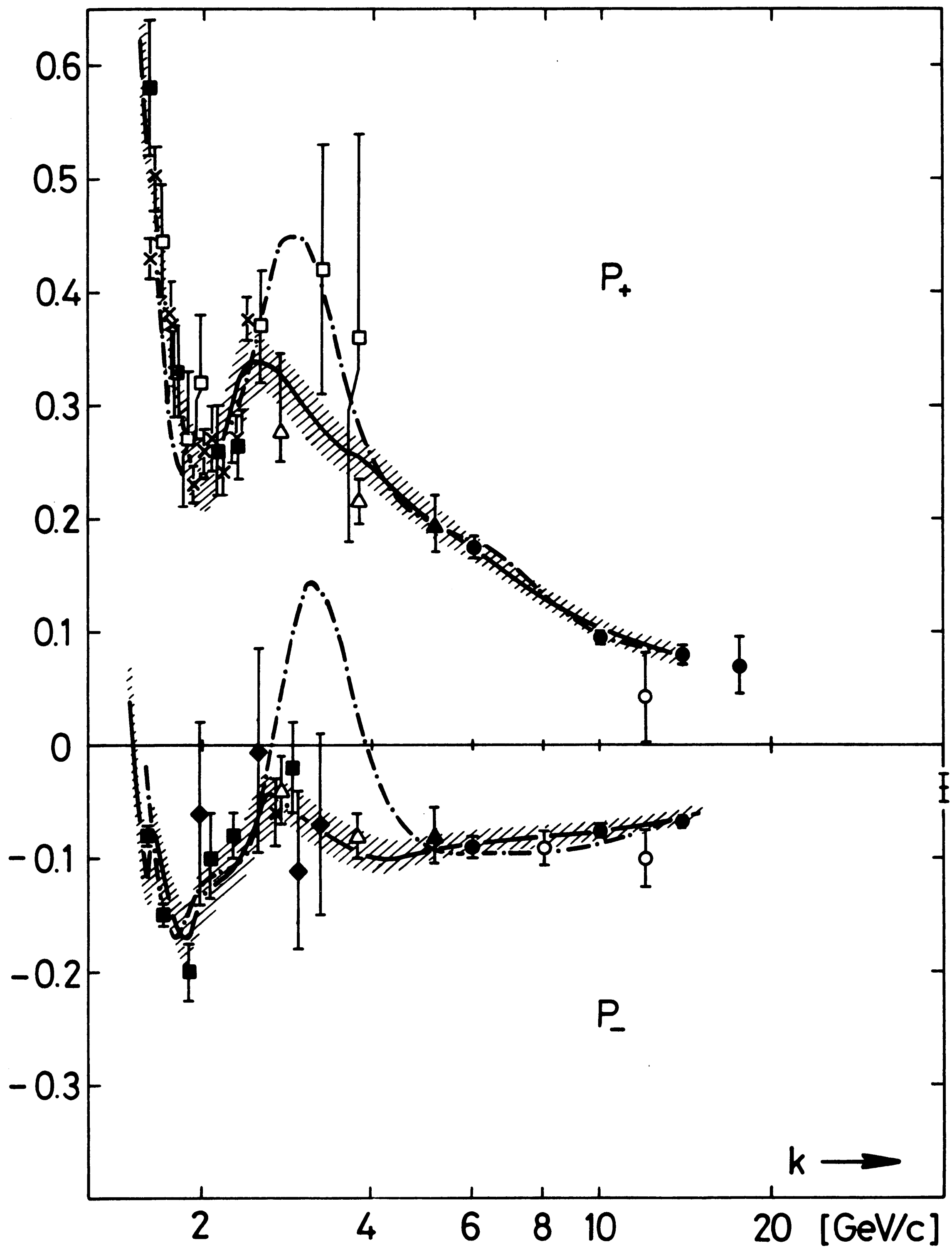


Fig. 7

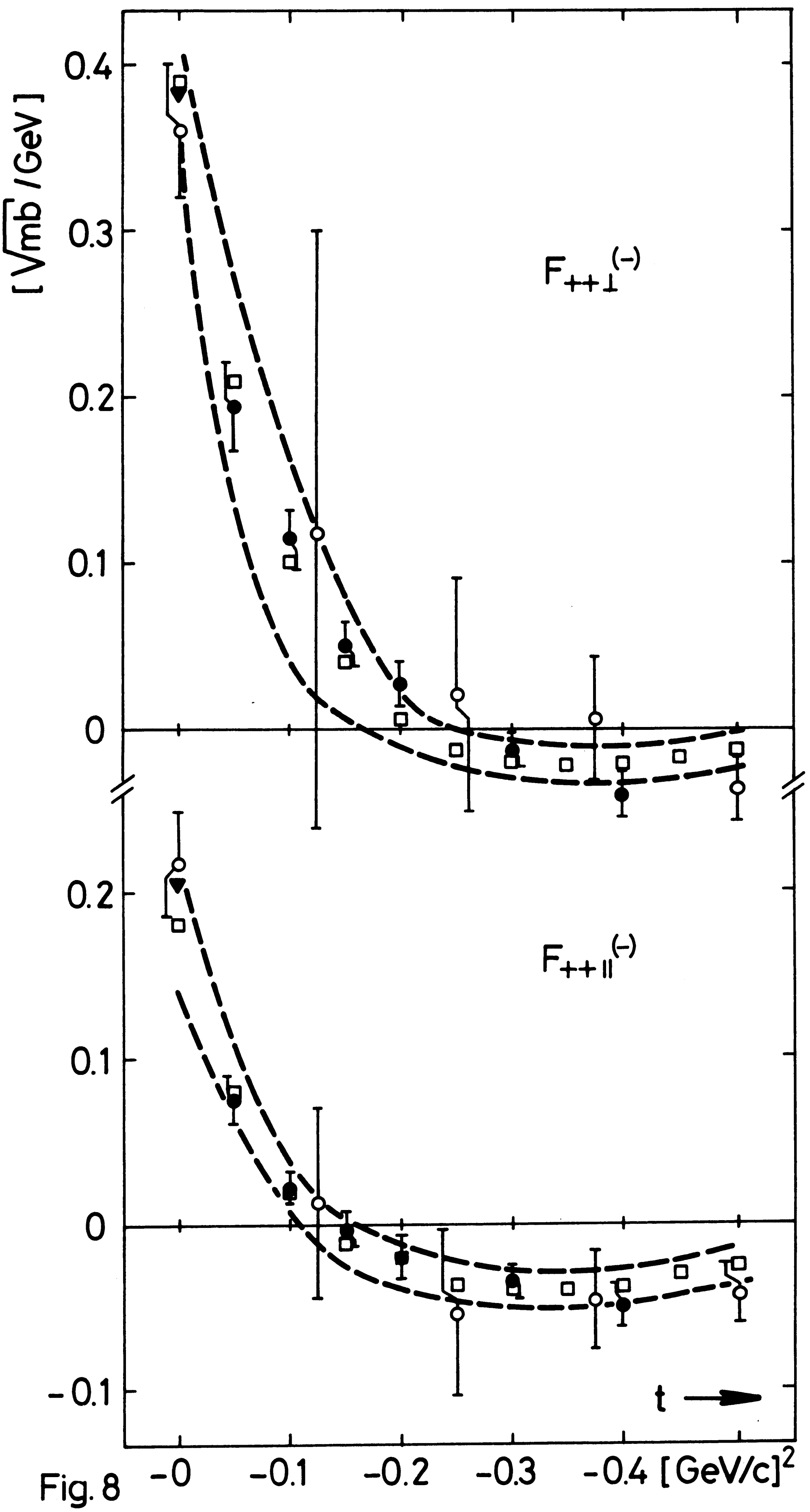

[√mb / GeV]
0.4
0.3
0.2
0.1
0
0.2
0.1
0
- 0.1
$F_{++\perp}^{(-)}$
$F_{++\parallel}^{(-)}$
t
-0
-0.1
-0.2
-0.3
-0.4 [GeV/c]²

Fig. 8

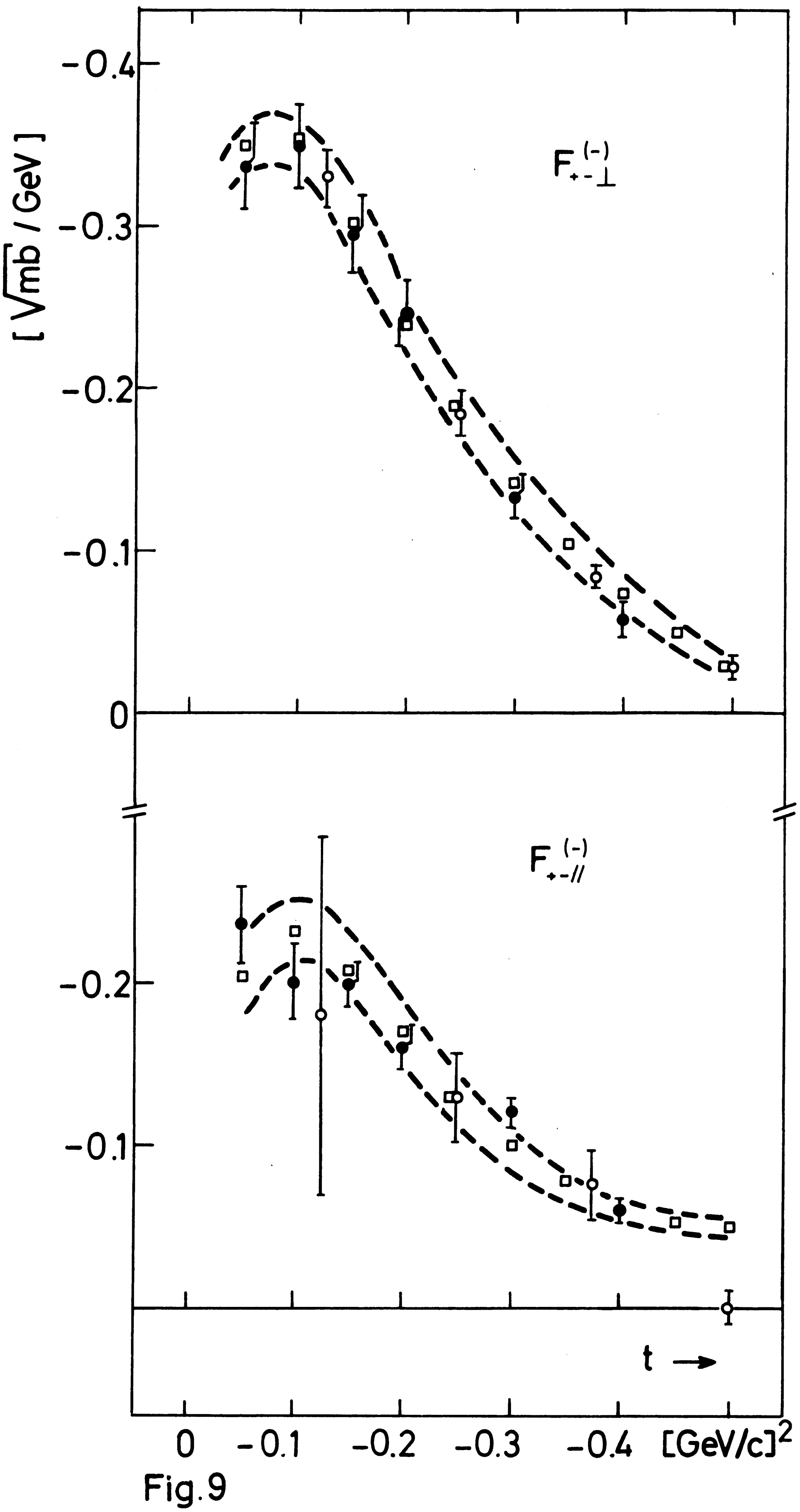


Fig.9