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CP asymmetries in chargino production and decay: the three-body decay case

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Abstract

We study CP violation in chargino production and decay in the Minimal Supersymmetric Standard Model (MSSM) with complex parameters at an e^+e^- linear collider with longitudinally polarized beams. We propose CP-sensitive asymmetries by means of triple product correlations and study their dependence on the complex parameters M_1 and μ. We give numerical predictions for the asymmetries and their measurability at the future International Linear Collider. Our results show that the CP asymmetries can be measured with very good statistical significances in a large region of the MSSM parameter space.

1 Introduction

In the Minimal Supersymmetric Standard Model (MSSM) [1] the supersymmetric partners of the gauge bosons and Higgs bosons with the same electric charge mix and form the neutralinos $\tilde{\chi}^0_i$ ($i = 1, \ldots, 4$) and the charginos $\tilde{\chi}^+_k$ ($k = 1, 2$), as the neutral and charged mass eigenstates, respectively. The charginos and the neutralinos are of particular interest, as they will presumably be among the lightest supersymmetric (SUSY) particles. One of the main goals of the International Linear Collider (ILC) will be the determination of the underlying SUSY parameters [2]. Those parameters that enter the neutralino/chargino system at tree level are the gaugino mass parameters M_1 and M_2, the higgsino mass parameter μ, and the ratio of the vacuum expectation values of the Higgs fields, $\tan\beta$. Among these parameters M_1 and μ can be complex, while M_2 and $\tan\beta$ can be chosen real.

The phases ϕ_μ and ϕ_{M_1} of μ and M_1 may be constrained or correlated by the experimental upper bounds on the electric dipole moments (EDMs). These constraints, however, are rather model-dependent [3]. While the restriction on the phase ϕ_μ, due to the electron EDM, is rather severe in a constrained MSSM with selectron masses of the order of 100 GeV [4], it may disappear if lepton-flavour-violating terms in the MSSM Lagrangian are included [5]. Recently it has been pointed out that for large trilinear scalar couplings A we can simultaneously fulfil the EDM constraints of electron, neutron, and of the atoms ^{199}Hg and ^{205}Tl where, at the same time, $\phi_\mu \sim O(1)$ [6]. The size of the phase ϕ_{M_1}, on the other hand, is less strongly restricted in the MSSM. Thus, the CP phases ϕ_{M_1} and ϕ_μ can have a big influence on the production and decay of charginos and neutralinos at the ILC. In particular, they give rise to CP-sensitive observables that may be accessible at future collider experiments. Measurements of CP-sensitive observables are necessary to prove that CP is violated. Furthermore, only the inclusion of CP-sensitive observables allows us to deduce the underlying model parameters in an unambiguous way. In neutralino production with subsequent decay, CP-sensitive observables based on triple product correlations have been investigated in [7, 8, 9]. Also for the case of chargino production and decay, various CP-sensitive observables have been studied. CP-sensitive asymmetries based on triple product correlations have been analysed for the subsequent two-body decays $\tilde{\chi}^-_j \to \tilde{\chi}^0_1 W^-$ [10] and $\tilde{\chi}^-_j \to \tilde{\nu}_\ell \ell^-$ [11]. For the case of transverse $e^\pm$ beam polarization azimuthal asymmetries have been studied for the same two-body decays, showing a pronounced dependence on ϕ_{M_1} and ϕ_μ [12]. In the present paper we extend previous investigations of CP violation in chargino production and decay to the case of chargino three-body decays.

We study the production processes

$$e^+e^- \to \tilde{\chi}^+_1 \tilde{\chi}^-_k, \qquad k = 1, 2 \; , \tag{1}$$

at a linear collider with longitudinal beam polarizations, and subsequent leptonic or hadronic three-body decays of the $\tilde{\chi}^+_1$,

$$\tilde{\chi}^+_1 \to \tilde{\chi}^0_1 \; \bar{\nu} \; \ell^+ \; , \quad \ell = e, \mu \; , \tag{2}$$

and

$$\tilde{\chi}^+_1 \to \tilde{\chi}^0_1 \; \bar{s} \; c \; , \tag{3}$$

where we assume that the momenta $\vec{p}_{\tilde{\chi}_1^+}$, $\vec{p}_\ell$, $\vec{p}_c$ and $\vec{p}_s$ of the associated particles can be measured or reconstructed. We study two T-odd observables based on triple product correlations of momentum vectors:

$$\mathcal{T}_\ell = \vec{p}_{\ell^+} \cdot (\vec{p}_{e^-} \times \vec{p}_{\tilde{\chi}_1^+}) \; , \tag{4}$$

$$\mathcal{T}_q = \vec{p}_{\bar{s}} \cdot (\vec{p}_c \times \vec{p}_{e^-}) \; . \tag{5}$$

The triple product $\mathcal{T}_\ell$, Eq. (4), relates momenta of initial, intermediate and final particles, whereas $\mathcal{T}_q$, Eq. (5), uses only momenta from the initial and final states. Therefore, both triple products show a different dependence from production and decay processes.

The triple product $\mathcal{T}_\ell$, Eq. (4), involves the momentum of the decay lepton that usually can be very accurately measured. However, the momentum of the chargino has to be reconstructed with information from the decay of the second chargino $\tilde{\chi}_k^-$ [9]. For the triple product $\mathcal{T}_q$, Eq. (5), it is necessary to identify the c-quark, which is expected to be possible with reasonable efficiency and purity [13]. To derive the CP-violating asymmetry also the charge of the c-quark has to be detected, which can be done with specific vertex detectors [14, 15]. The corresponding T-odd asymmetries are defined by

$$A_T(\mathcal{T}_{\ell,q}) = \frac{N[\mathcal{T}_{\ell,q} > 0] - N[\mathcal{T}_{\ell,q} < 0]}{N[\mathcal{T}_{\ell,q} > 0] + N[\mathcal{T}_{\ell,q} < 0]} \; , \tag{6}$$

where $N[\mathcal{T}_{\ell,q} > (<)\; 0]$ is the number of events for which $\mathcal{T}_{\ell,q} > (<)\; 0$.

Finally we recall that a non-zero value of the T-odd asymmetries does not immediately imply that the CP symmetry is violated since final-state interactions give rise (although only at the one-loop level) to the same asymmetries. However, a genuine signal of CP violation can be obtained when one combines $A_T(\mathcal{T}_{\ell,q})$ with the corresponding asymmetry $\bar{A}_T(\mathcal{T}_{\ell,q})$ for the charge-conjugated processes. Then in the CP asymmetries

$$A_{\mathrm{CP}}(\mathcal{T}_{\ell,q}) = \frac{A_T(\mathcal{T}_{\ell,q}) - \bar{A}_T(\mathcal{T}_{\ell,q})}{2} \; , \tag{7}$$

the effect of final-state interactions cancels out.

The paper is organized as follows. In Section 2 we present the relevant Lagrangians and couplings of the processes (1) and (2). Moreover, we briefly recall the formalism, which we use to calculate the cross sections and the CP asymmetries. We present our numerical results in Section 3. Finally, we summarize and conclude in Section 4.

2 Cross section and CP asymmetries

2.1 Lagrangian and couplings

The chargino production process (1) proceeds via γ and Z^0 exchange in the s-channel and via $\tilde{\nu}_e$ exchange in the t-channel (Fig. 1). The decay processes (2) and (3) contain contributions from W^+, $\tilde{\ell}_L$ ($\ell = e, \mu$) and $\tilde{\nu}_\ell$ exchange in the leptonic case and from W^+, $\tilde{c}_L$ and $\tilde{s}_L$ exchange in the hadronic case (Fig. 2).

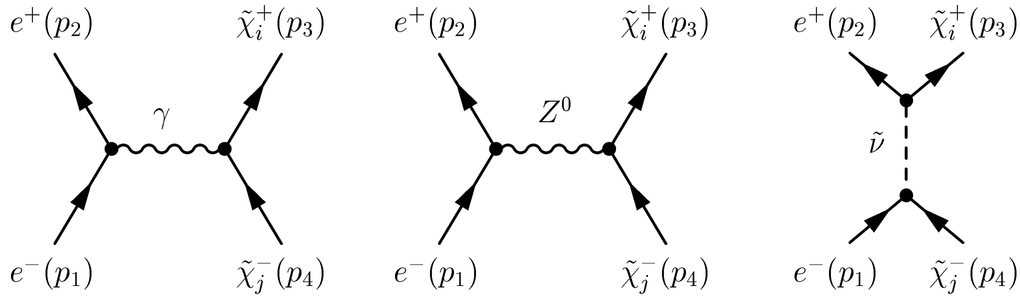

Figure 1: Feynman diagrams of the production process $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$.

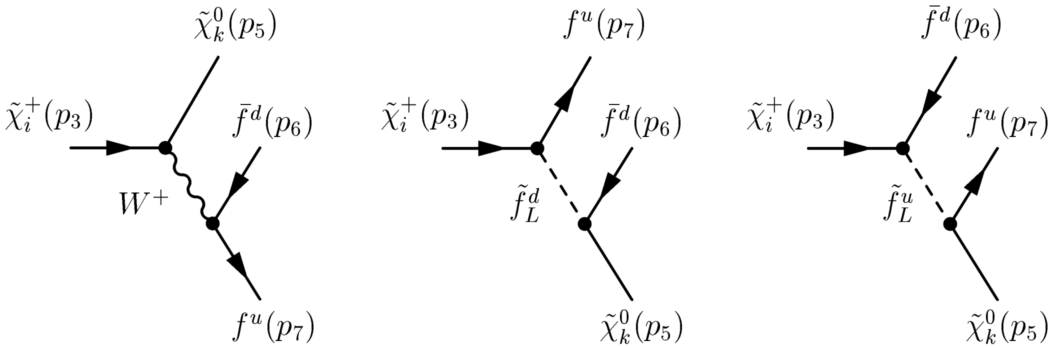

Figure 2: Feynman diagrams of the three-body decay $\tilde{\chi}_i^+ \to \tilde{\chi}_k^0\bar{f}^d f^u$.

The interaction Lagrangians for these processes are [1] (in our notation and conventions we follow [16]):

$$\mathcal{L}_{Z^0ff} = -\frac{g}{\cos\theta_W} Z_\mu \bar{f}\gamma^\mu [L_f P_{\rm L} + R_f P_{\rm R}] f, \tag{8}$$

$$\mathcal{L}_{\gamma ff} = -e_f A_\mu \bar{f}\gamma^\mu f, \tag{9}$$

$$\mathcal{L}_{\gamma\tilde{\chi}_i^+\tilde{\chi}_j^+} = -eA_\mu \bar{\tilde{\chi}}_i^+ \gamma^\mu \tilde{\chi}_j^+ \delta_{ij}, \quad e > 0, \tag{10}$$

$$\mathcal{L}_{Z^0\tilde{\chi}_i^+\tilde{\chi}_j^+} = \frac{g}{\cos\theta_W} Z_\mu \bar{\tilde{\chi}}_i^+ \gamma^\mu [O'^{\rm L}_{ij} P_{\rm L} + O'^{\rm R}_{ij} P_{\rm R}] \tilde{\chi}_j^+, \tag{11}$$

$$\mathcal{L}_{f^d\tilde{f}^u\tilde{\chi}_i^+} = -gU_{i1}^* \bar{\tilde{\chi}}_i^+ P_{\rm L} f^u \tilde{f}_{\rm L}^{d*} - gV_{i1}^* \bar{\tilde{\chi}}_i^{+C} P_{\rm L} f^d \tilde{f}_{\rm L}^{u*} + {\rm h.c.}, \tag{12}$$

$$\mathcal{L}_{W^+f^df^u} = -\frac{g}{\sqrt{2}} W_\mu^+ \bar{f}^u \gamma^\mu f^d + {\rm h.c.}, \tag{13}$$

$$\mathcal{L}_{W^-\tilde{\chi}_i^+\tilde{\chi}_k^0} = gW_\mu^- \bar{\tilde{\chi}}_k^0 \gamma^\mu [O^{\rm L}_{ki} P_{\rm L} + O^{\rm R}_{ki} P_{\rm R}] \tilde{\chi}_i^+ + {\rm h.c.}, \tag{14}$$

$$\mathcal{L}_{f\tilde{f}\tilde{\chi}_k^0} = g f^{\rm L}_{fk} \bar{f} P_{\rm R} \tilde{\chi}_k^0 \tilde{f}_{\rm L} + g f^{\rm R}_{fk} \bar{f} P_{\rm L} \tilde{\chi}_k^0 \tilde{f}_{\rm R} + {\rm h.c.}\ , \tag{15}$$

where $f^u = \nu, c$ and $f^d = \ell, s$ denote up-type and down-type fermions, respectively. The couplings are

$$L_f = T_{3f} - e_f \sin^2\theta_W, \quad R_f = -e_f \sin^2\theta_W, \tag{16}$$

$$f^{\rm L}_{fk} = -\sqrt{2}\Big[\frac{1}{\cos\theta_W}(T_{3f} - e_f\sin^2\theta_W)N_{k2} + e_f\sin\theta_W N_{k1}\Big], \tag{17}$$

$$f^{\rm R}_{fk} = -\sqrt{2}e_f\sin\theta_W\Big[\tan\theta_W N^*_{k2} - N^*_{k1}\Big], \tag{18}$$

$$O'^{\rm L}_{ij} = -V_{i1}V^*_{j1} - \frac{1}{2}V_{i2}V^*_{j2} + \delta_{ij}\sin^2\theta_W, \tag{19}$$

$$O'^{\rm R}_{ij} = -U^*_{i1}U_{j1} - \frac{1}{2}U^*_{i2}U_{j2} + \delta_{ij}\sin^2\theta_W, \tag{20}$$

$$O^{\rm L}_{ki} = -1/\sqrt{2}\Big(\cos\beta N_{k4} - \sin\beta N_{k3}\Big)V^*_{i2} + \Big(\sin\theta_W N_{k1} + \cos\theta_W N_{k2}\Big)V^*_{i1}, \tag{21}$$

$$O^{\rm R}_{ki} = +1/\sqrt{2}\Big(\sin\beta N^*_{k4} + \cos\beta N^*_{k3}\Big)U_{i2} + \Big(\sin\theta_W N^*_{k1} + \cos\theta_W N^*_{k2}\Big)U_{i1}, \tag{22}$$

where $P_{\rm L,R} = \frac{1}{2}(1 \mp \gamma_5)$, g is the weak coupling constant ($g = e/\sin\theta_W$, $e > 0$); e_f and T_{3f} are the charge (in units of e) and the third component of the weak isospin of the fermion f; θ_W is the weak mixing angle, and $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of the Higgs fields. The unitary (4×4) matrix N, which diagonalizes the complex symmetric neutralino mass matrix, is in the basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b)$ [17]. The unitary (2×2) matrices U and V diagonalize the complex chargino mass matrix.

2.2 Cross section

For the calculation of the squared amplitude of the whole process $e^+e^- \to \tilde{\chi}^+_i\tilde{\chi}^-_j \to \tilde{\chi}^0_1 \bar{f}^d f^u \tilde{\chi}^-_j$, we use the spin-density matrix formalism [16, 18]. The squared amplitude can then be written as

$$|T|^2 = 2|\Delta(\tilde{\chi}^+_i)|^2 \sum_{\lambda_i,\lambda'_i} {\rho_P}^{\lambda_i\lambda'_i}\; {\rho_D}_{\lambda'_i\lambda_i}\ , \tag{23}$$

with the propagator $\Delta(\tilde{\chi}^+_i) = 1/[p^2_{\tilde{\chi}^+_i} - m^2_i + i m_i\Gamma_i]$. Here, λ_i, λ'_i, m_i, Γ_i denote the helicities, masses and widths of the chargino $\tilde{\chi}^+_i$. The factor 2 in Eq. (23) is due to the summation over the helicities of chargino $\tilde{\chi}^-_j$, whose decay is not considered. The squared amplitude is composed of the unnormalized spin-density matrices ρ_P for the production and ρ_D for the decay, which carry the helicity indices λ_i, λ'_i of the chargino $\tilde{\chi}^+_i$. Introducing a set of polarization basis 4-vectors $s^a_{\chi_i}$ $(a = 1,2,3)$ for the charginos $\tilde{\chi}^+_i$, where $s^3_{\chi_i}$ describes the longitudinal polarization and $s^1_{\chi_i}$, $s^2_{\chi_i}$ the transverse polarization in and perpendicular to the production plane, respectively, and which fulfil the orthonormality relations $s^a_{\chi_i} \cdot s^b_{\chi_i} = -\delta^{ab}$ and $s^a_{\chi_i} \cdot p_{\chi_i} = 0$, the density matrices can be expanded in terms of the Pauli matrices:

$$\rho_P{}^{\lambda_i\lambda_i'} = \delta_{\lambda_i\lambda_i'} P + \sum_{a=1}^{3} \sigma^a_{\lambda_i\lambda_i'} \Sigma^a_P \ , \tag{24}$$

$$\rho_{D\,\lambda_i'\lambda_i} = \delta_{\lambda_i'\lambda_i} D + \sum_{a=1}^{3} \sigma^a_{\lambda_i'\lambda_i} \Sigma^a_D \ . \tag{25}$$

Then the squared amplitude is given by

$$|T|^2 = 4|\Delta(\tilde{\chi}^+_i)|^2 \left\{ P(\tilde{\chi}^+_i\tilde{\chi}^-_j)D(\tilde{\chi}^+_i) + \sum_{a=1}^{3} \Sigma^a_P(\tilde{\chi}^+_i)\Sigma^a_D(\tilde{\chi}^+_i) \right\} , \tag{26}$$

where $P(\tilde{\chi}^+_i\tilde{\chi}^-_j)$ and $D(\tilde{\chi}^+_i)$ are those parts of the spin density production and decay matrices, that are independent of the polarization of the charginos. The contributions $\Sigma^a_P(\tilde{\chi}^+_i)$ and $\Sigma^a_D(\tilde{\chi}^+_i)$ depend on the polarization vector s^a of the decaying chargino $\tilde{\chi}^+_i$. Finally, the differential cross section is given by

$$d\sigma = \frac{1}{8E_b^2}|T|^2(2\pi)^4\delta^4\left(p_1 + p_2 - \sum_{i=4}^{7} p_i\right) d\text{lips}(p_3 \cdots p_7) \ , \tag{27}$$

where E_b is the beam energy and $d$lips$(p_3 \cdots p_7)$ is the Lorentz-invariant phase-space element.

2.3 CP asymmetries

The T-odd asymmetries defined in Eq. (6) are calculated as

$$A_T(\mathcal{T}_{\ell,q}) = \frac{\int \text{sign}\{\mathcal{T}_{\ell,q}\}\ |T|^2\ d\text{lips}}{\int\ |T|^2\ d\text{lips}} \ , \tag{28}$$

where we weight the sign of the triple product correlations in Eqs. (4) and (5) with the associated squared amplitude. Since in the numerator $\int \text{sign}\{\mathcal{T}_{\ell,q}\}P(\tilde{\chi}^+_i\tilde{\chi}^-_j)D(\tilde{\chi}^+_i)d\text{lips} = 0$ and in the denominator $\int \Sigma^a_P(\tilde{\chi}^+_i)\Sigma^a_D(\tilde{\chi}^+_i)d\text{lips} = 0$, we obtain by inserting the squared amplitude, Eq. (26), into Eq. (28):

$$A_T(\mathcal{T}_{\ell,q}) = \frac{\int \text{sign}\{\mathcal{T}_{\ell,q}\}\Sigma^a_P(\tilde{\chi}^+_i)\Sigma^a_D(\tilde{\chi}^+_i)d\text{lips}}{\int P(\tilde{\chi}^+_i\tilde{\chi}^-_j)D(\tilde{\chi}^+_i)d\text{lips}} \ . \tag{29}$$

We split $\Sigma^a_P(\tilde{\chi}^+_i)$ and $\Sigma^a_D(\tilde{\chi}^+_i)$ into the T-odd terms $\Sigma^{a,\text{O}}_P(\tilde{\chi}^+_i)$ and $\Sigma^{a,\text{O}}_D(\tilde{\chi}^+_i)$, which contain the respective triple product, and T-even terms $\Sigma^{a,\text{E}}_P(\tilde{\chi}^+_i)$ and $\Sigma^{a,\text{E}}_D(\tilde{\chi}^+_i)$ without triple products:

$$\Sigma^a_P(\tilde{\chi}^+_i) = \Sigma^{a,\text{O}}_P(\tilde{\chi}^+_i) + \Sigma^{a,\text{E}}_P(\tilde{\chi}^+_i) \ , \qquad \Sigma^a_D(\tilde{\chi}^+_i) = \Sigma^{a,\text{O}}_D(\tilde{\chi}^+_i) + \Sigma^{a,\text{E}}_D(\tilde{\chi}^+_i) \ . \tag{30}$$

The terms of $|T|^2$, Eq. (26), which contribute to the numerator of A_T are

$$\Sigma^{a,\text{O}}_P(\tilde{\chi}^+_i)\Sigma^{a,\text{E}}_D(\tilde{\chi}^+_i) + \Sigma^{a,\text{E}}_P(\tilde{\chi}^+_i)\Sigma^{a,\text{O}}_D(\tilde{\chi}^+_i) \ , \tag{31}$$

Scenario	A	B	C
M_2	280	150	120
$\|\mu\|$	200	320	320
$\tan\beta$	5	5	5
$m_{\tilde{\nu}}$	250	250	250
$m_{\tilde{u}_L}$	500	500	500

Table 1: Input parameters M_2, $|\mu|$, $\tan\beta$, $m_{\tilde{\nu}}$ and $m_{\tilde{u}_L} = m_{\tilde{c}_L}$. $|M_1|$ is fixed by the GUT-inspired relation $|M_1| = 5/3 \tan^2\theta_W M_2$ and the masses of the down-type sfermions by the SU(2) relation. All masses are given in GeV.

where the first (second) term is sensitive to the CP phases in the production (decay) process of the chargino $\tilde{\chi}_i^+$. The explicit expressions for the T-odd and T-even contributions in Eq. (31) are given in Appendix B. (The analytical expressions of the quantities $P(\tilde{\chi}_i^+\tilde{\chi}_j^-)$ and $D(\tilde{\chi}_i^+)$ can be found in [16].) With $A_T(\mathcal{T}_{\ell,q})$ we calculate the corresponding CP asymmetries $A_{\rm CP}(\mathcal{T}_{\ell,q})$ according to Eq. (7).

3 Numerical results

In this section we give numerical results for the CP asymmetries $A_{\rm CP}(\mathcal{T}_{\ell,q})$, Eq. (7), for the reactions (1), (2), (3), at an e^+e^- linear collider with centre-of-mass energy $\sqrt{s} = 500$ GeV and longitudinally polarized beams. We analyse the hadronic decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \bar{s}c$ and the leptonic decays $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \ell^+\nu$, $\ell = e, \mu$. To this end, we consider three scenarios (see Table 1) for which $m_{\tilde{\chi}_1^+} < m_W + m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^+} < m_{\tilde{f}_L^{u,d}}$ to rule out two-body decays of $\tilde{\chi}_1^+$. The chargino decay widths and branching ratios have been calculated with the computer program SPheno [19].

The statistical significance to which $A_{\rm CP}$ can be determined to be non-zero can be estimated in the following way: The absolute error of $A_{\rm CP}$ is given by

$$\Delta A_{\rm CP} = \mathcal{N}_\sigma \frac{\sqrt{1 - A_{\rm CP}^2}}{\sqrt{\sigma \mathcal{L}_{\rm int}}} , \tag{32}$$

where $\mathcal{N}_\sigma$ denotes the respective number of standard deviations, $\sigma = \sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_j^-) \cdot B(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 f'\bar{f})$ being the corresponding cross section of the combined production and decay processes and $\mathcal{L}_{\rm int}$ is the integrated luminosity, where we assume $\mathcal{L}_{\rm int} = 500$ fb^{-1} in the theoretical estimates below. For $A_{\rm CP} \lesssim 10\%$, i.e. $A_{\rm CP}^2 \lesssim 0.01$, it is $\Delta A_{\rm CP} = \mathcal{N}_\sigma/\sqrt{\sigma\mathcal{L}_{\rm int}}$ in good approximation. If we require $A_{\rm CP} > \Delta A_{\rm CP}$ for $A_{\rm CP}$ to be measurable we obtain

$$\mathcal{N}_\sigma = \sqrt{A_{\rm CP}^2 \sigma \mathcal{L}_{\rm int}} . \tag{33}$$

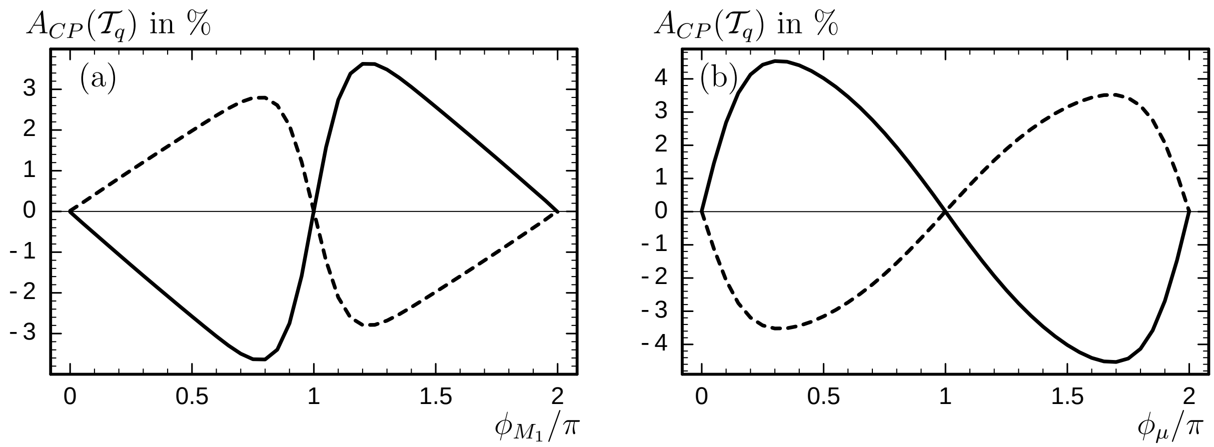

Figure 3: CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ with subsequent decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0\bar{s}c$ in scenario A of Table 1 (a) with $\phi_\mu = 0$ and (b) with $\phi_{M_1} = \pi$, for $\sqrt{s} = 500$ GeV and for the beam polarizations $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ (solid), $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$ (dashed).

3.1 CP asymmetry for $\tilde{\chi}_1^+\tilde{\chi}_1^-$ production and $\tilde{\chi}_1^+$ decay

In the case of pair production, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, only CP-violating couplings from the decay (second term in Eq. (31)) can give rise to a CP-violating effect, because in the production (first term in Eq. (31)) only the absolute squares of the couplings enter. Thus, the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$ is sensitive to the CP violation in the decay (because of ϕ_μ and ϕ_{M_1}). It should be noted that in this case the charge of the c-jet of the $\tilde{\chi}_1^+$ decay can also be derived via detecting the charge of the decay lepton of the $\tilde{\chi}_1^-$ in semileptonic decays. Fig. 3 (a) shows the asymmetry $A_{\rm CP}(\mathcal{T}_q)$ as a function of the phase ϕ_{M_1} for scenario A in Table 1 for $\phi_\mu = 0$. The masses of the squarks are chosen to be $m_{\tilde{c}} = 500$ GeV and $m_{\tilde{s}} = 505.9$ GeV. The centre-of-mass energy $\sqrt{s} = 500$ GeV and the two sets of longitudinal $e^\pm$ beam polarizations are fixed in our study at $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$ and $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$. The CP asymmetry reaches its largest value of about 3.7% for $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ at $\phi_{M_1} = 1.2\pi$. Note that the asymmetry changes its sign for the two different sets of beam polarization due to the prefactor (Eq. (48), Appendix B) which depends on the longitudinal beam polarization. Note further that the asymmetry does not have its largest absolute value for $\phi_{M_1} = 0.5\pi, 1.5\pi$. This behaviour is due to a complex interplay of the ϕ_{M_1} dependence of the numerator and denominator of the asymmetry in Eq. (29). In Fig. 3 (b) we show the dependence of the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$ on ϕ_μ for the same scenario taking $\phi_{M_1} = \pi$. The maximum value of about 4.6% of $A_{\rm CP}(\mathcal{T}_q)$ is reached at $\phi_\mu = 0.3\pi$ for $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$.

In Fig. 4 (a) and (b) the contours of the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), are shown in the M_2-$|\mu|$ plane. The other MSSM parameters are chosen to be $\tan\beta = 5$, $m_{\tilde{\nu}} = 250$ GeV, $m_{\tilde{c}} = 500$ GeV, $m_{\tilde{s}} = 505.9$ GeV, $|M_1| = 5/3\tan^2\theta_W M_2$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$. For both polarization configurations, $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ and $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$, the absolute value of $A_{\rm CP}(\mathcal{T}_q)$ is largest in the region $|\mu| \approx 260$ GeV and $M_2 \approx 360$ GeV with asymmetries of about -5% (4%) for $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ ($(+0.8, -0.6)$). The main contributions to the numerator of the asymmetry are due to the W^+-$\tilde{s}_L$ and W^+-$\tilde{c}_L$

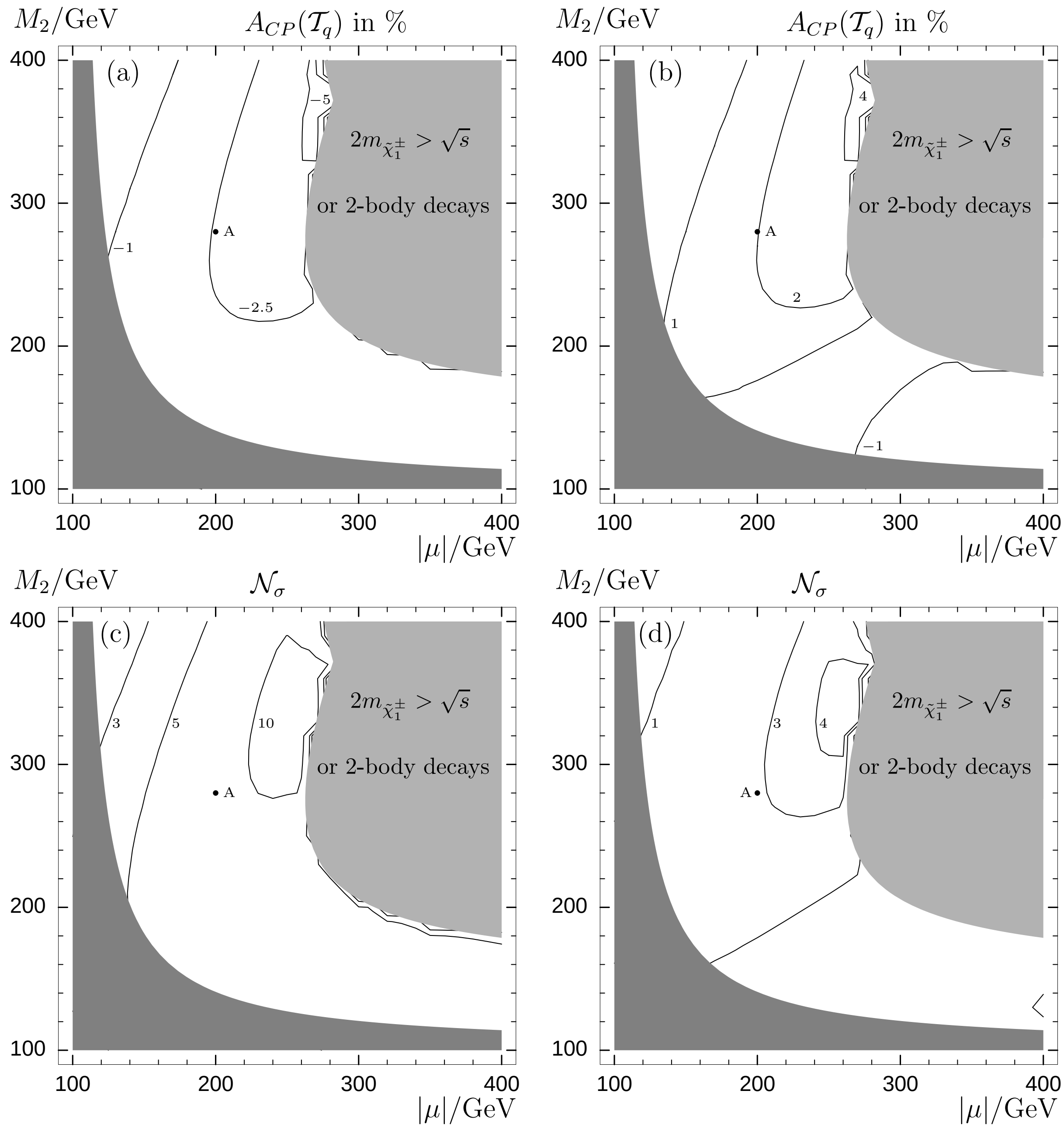

Figure 4: (a), (b) Contours of the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), in % for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ at $\sqrt{s} = 500$ GeV with subsequent decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \bar{s} c$ and (c), (d) contours of the number of standard deviations $\mathcal{N}_\sigma$, Eq. (33), for an integrated luminosity $\mathcal{L}_{\rm int} = 500$ fb^{-1}, respectively. The parameters are $\tan\beta = 5$, $m_{\tilde{\nu}} = 250$ GeV, $m_{\tilde{c}} = 500$ GeV, $m_{\tilde{s}} = 505.9$ GeV, $|M_1|/M_2 = 5/3 \tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$, $\phi_\mu = 0$. The beam polarizations are in (a), (c), $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ and in (b), (d), $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$. The point marks the scenario A of Table 1. In the dark-shaded area is $m_{\tilde{\chi}_1^\pm} < 103.5$ GeV, excluded by LEP [20]. The light-shaded area shows the region that either is kinematically not accessible or in which the three-body decay is strongly suppressed because $m_{\tilde{\chi}_1^+} > m_W + m_{\tilde{\chi}_1^0}$.

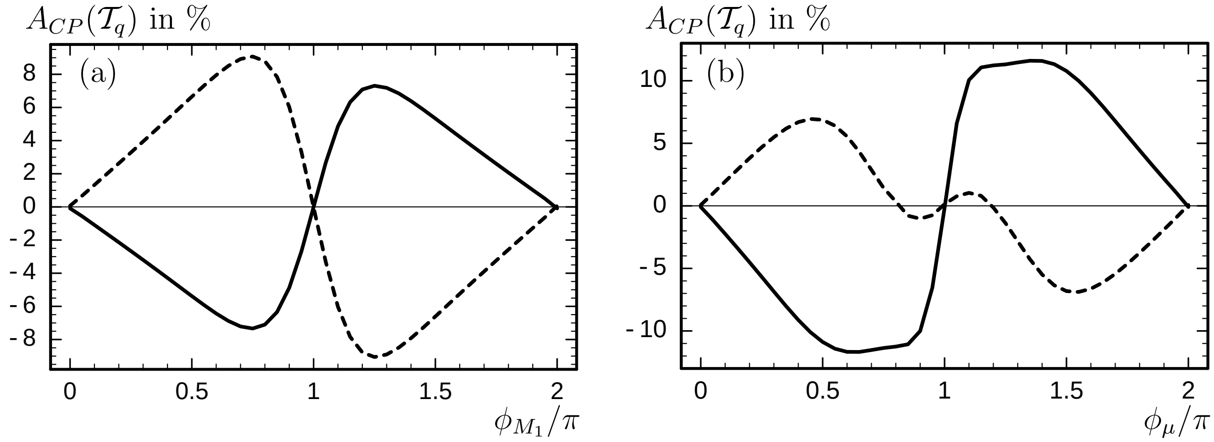

Figure 5: CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), for $e^+e^- \to \tilde\chi^+_1\tilde\chi^-_2$ with subsequent decay $\tilde\chi^+_1 \to \tilde\chi^0_1 \bar s c$ in scenario B of Table 1 (a) with $\phi_\mu = 0$ and (b) with $\phi_{M_1} = 0$, for $\sqrt{s} = 500$ GeV and for the beam polarizations $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ (solid), $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$ (dashed).

interference terms.

Figs. 4 (c) and (d) show the contours of the corresponding number of standard deviations $\mathcal{N}_\sigma$ for an integrated luminosity $\mathcal{L}_{\rm int} = 500$ fb^{-1} in the M_2-$|\mu|$ plane. Quite generally, the choice $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ for longitudinal beam polarizations yields better results than $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$, because it enhances the sneutrino-exchange contribution to the production cross section. It is interesting to note that the asymmetry $A_{\rm CP}(\mathcal{T}_q)$ is measurable with a 5σ significance in a large region of the parameter space.

3.2 CP-odd asymmetry for $\tilde\chi^+_1\tilde\chi^-_2$ production and $\tilde\chi^+_1$ decay

Now we consider the production process $e^+e^- \to \tilde\chi^+_1\tilde\chi^-_2$ at $\sqrt{s} = 500$ GeV with subsequent decays of the $\tilde\chi^+_1$. In this case $A_{\rm CP}(\mathcal{T}_q)$ is sensitive to the CP-violating couplings in the production and decay amplitudes (i.e. it is sensitive to both terms in (31)).

3.2.1 Hadronic decay $\tilde\chi^+_1 \to \tilde\chi^0_1 \bar s c$

In the case of hadronic decays, $\tilde\chi^+_1 \to \tilde\chi^0_1 \bar s c$, c-charge tagging is highly desirable because of the complicated cascade decays of the heavy chargino.

In Fig. 5 (a) we show the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), as a function of ϕ_{M_1} for scenario B in Table 1 with $\phi_\mu = 0$. The longitudinal beam polarization is $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ ($(+0.8, -0.6)$). The asymmetry reaches its largest value of about 9% (7%) for $\phi_{M_1} = 0.7\pi$ (1.2π). Fig. 5 (b) shows $A_{\rm CP}(\mathcal{T}_q)$ as a function of ϕ_μ for $\phi_{M_1} = 0$. The largest value of the CP asymmetry is reached at $\phi_\mu = 1.4\pi$ (0.5π). Note that the asymmetry can be large ($\sim 10\%$), even for values of ϕ_μ close to π. As can be seen in Figs. 5 (a) and (b), it changes the sign for the two choices of beam polarizations.

In Fig. 6 (a) and (b) the contours of $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), are shown in the M_2-$|\mu|$ plane for $\tan\beta = 5$, $m_{\tilde\nu} = 250$ GeV, $m_{\tilde c} = 500$ GeV, $m_{\tilde s} = 505.9$ GeV, $|M_1| = 5/3\tan^2\theta_W M_2$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$. Figs. 6 (c) and (d) show the corresponding contours for $\mathcal{N}_\sigma$,

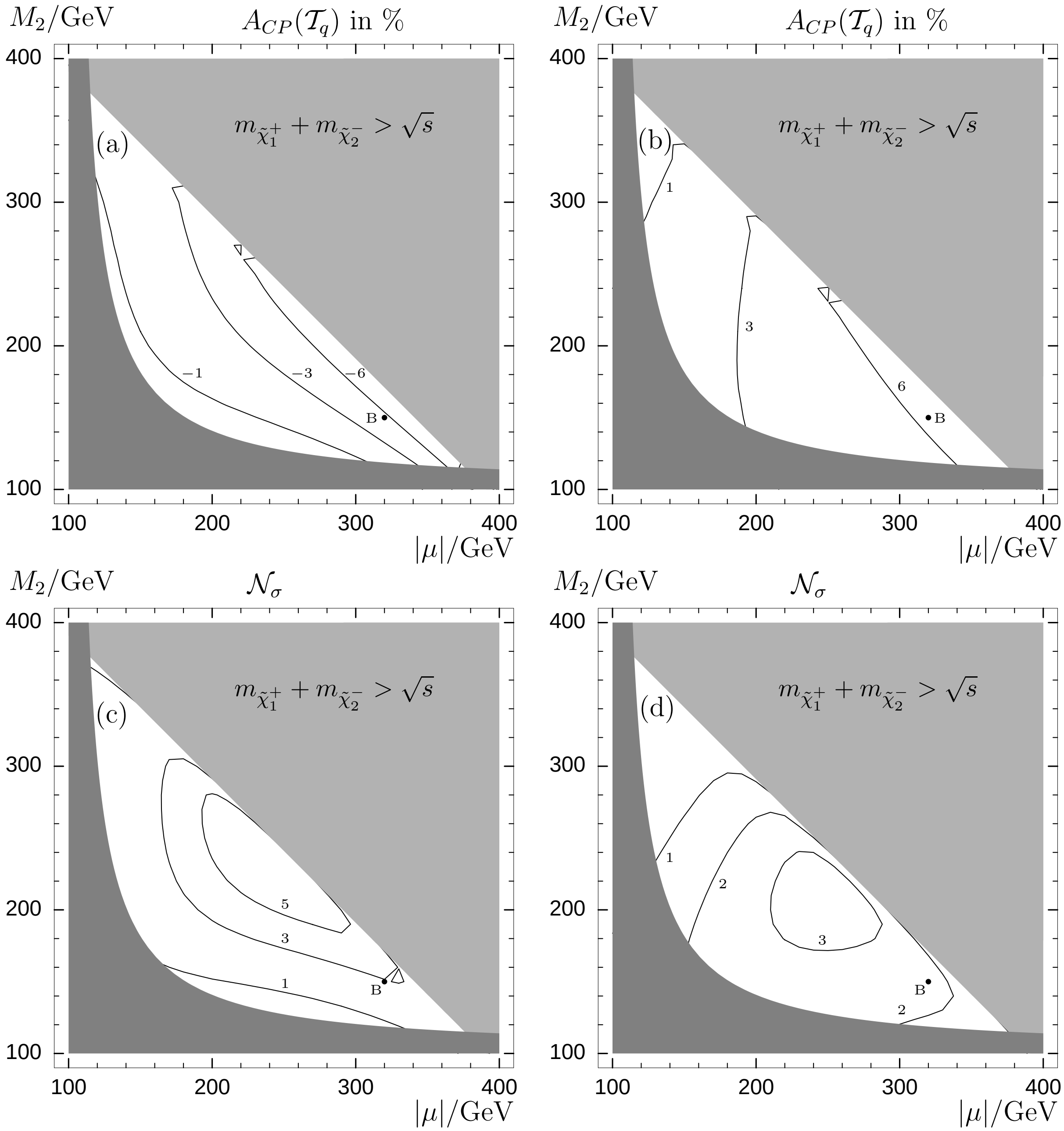

Figure 6: (a), (b) Contours of the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$, Eq. (7), in % for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ at $\sqrt{s} = 500$ GeV with subsequent decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \bar{s} c$ and (c), (d) contours of the number of standard deviations $\mathcal{N}_\sigma$, Eq. (33), for an integrated luminosity $\mathcal{L}_{\rm int} = 500\ {\rm fb}^{-1}$, respectively. The parameters are $\tan\beta = 5$, $m_{\tilde{\nu}} = 250$ GeV, $m_{\tilde{c}} = 500$ GeV, $m_{\tilde{s}} = 505.9$ GeV, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$. The beam polarizations are in (a), (c) $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ and in (b), (d) $(P_{e^-}, P_{e^+}) = (+0.8, -0.6)$. The point marks the scenario B of Table 1. In the dark-shaded area is $m_{\tilde{\chi}_1^\pm} < 103.5$ GeV, excluded by LEP [20]. The light-shaded area is kinematically not accessible.

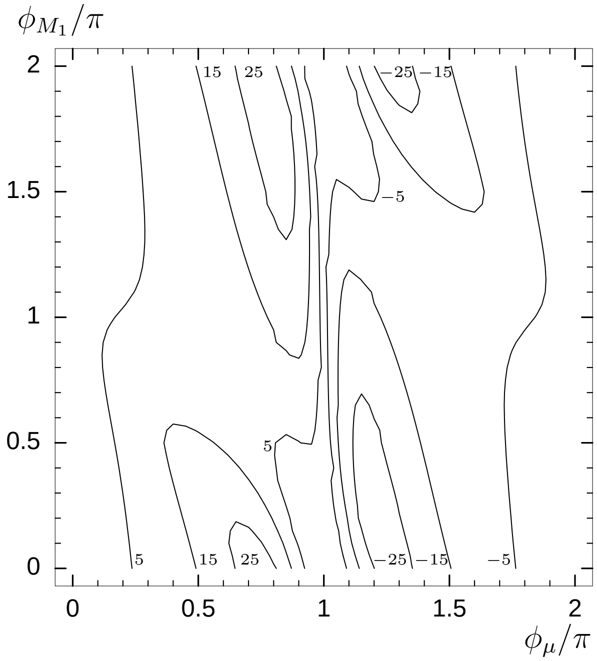

Figure 7: Contours of the CP asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$, Eq. (7), in % for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ at $\sqrt{s} = 500$ GeV with subsequent decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0\ell^+\nu$ in scenario C of Table 1 for $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$.

Eq. (33), for $\mathcal{L}_{\rm int} = 500$ fb^{-1} in the M_2-$|\mu|$ plane. Also in this case the choice $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$ enhances the statistical significance for a measurement of $A_T(\mathcal{T}_q)$.

3.2.2 Leptonic decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0\ell^+\nu$

In this section we analyse the CP asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$, Eq. (7), based on the triple product correlation $\mathcal{T}_\ell = \vec{p}_{\ell^+} \cdot (\vec{p}_{e^-} \times \vec{p}_{\tilde{\chi}_1^+})$, (4)[1]. For the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ the asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$ is only sensitive to CP-violating couplings in the production amplitude, which are involved in the first term of (31), because the CP sensitive couplings in the decay (c.f. Eqs. (80)–(82)) do not contain the triple product $\mathcal{T}_\ell$. This means $A_{\rm CP}(\mathcal{T}_\ell)$ is proportional to $\sin(\phi_\mu)$ and therefore $A_{\rm CP}(\mathcal{T}_\ell) \equiv 0$ for $\phi_\mu = 0, \pi, 2\pi, \ldots$, independently of ϕ_{M_1}. Hence, by measuring the CP asymmetries $A_{\rm CP}(\mathcal{T}_\ell)$ and $A_{\rm CP}(\mathcal{T}_q)$ one can separately study the influence of ϕ_μ and ϕ_{M_1}.

In Fig. 7 we show the contour lines of the CP-odd asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$, Eq. (7), for scenario C of Table 1 in the ϕ_{M_1}-ϕ_μ plane. Fig. 7 illustrates that the asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$ can be large for values of ϕ_μ close to π. For instance, for $\phi_{M_1} = 1.5\pi$ and $\phi_\mu = 0.9\pi$ one obtains an asymmetry of about 23%. However, the corresponding cross section is only about 0.16 fb.

In Fig. 8 (a) and (b), the CP asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$, Eq. (7), and the number of

[1]In order to be able to measure $A_{\rm CP}(\mathcal{T}_\ell)$ the production plane has to be reconstructed. Provided that the masses of the particles involved are known, this could be accomplished depending on the decay pattern of $\tilde{\chi}_2^-$. For instance, if $\tilde{\chi}_2^-$ decays according to $\tilde{\chi}_2^- \to \bar{\tilde{c}}s$ and $\bar{\tilde{c}}$ in turn decays to $\tilde{\chi}_1^0\bar{c}$, a reconstruction of the production plane can be performed up to a twofold ambiguity [9].

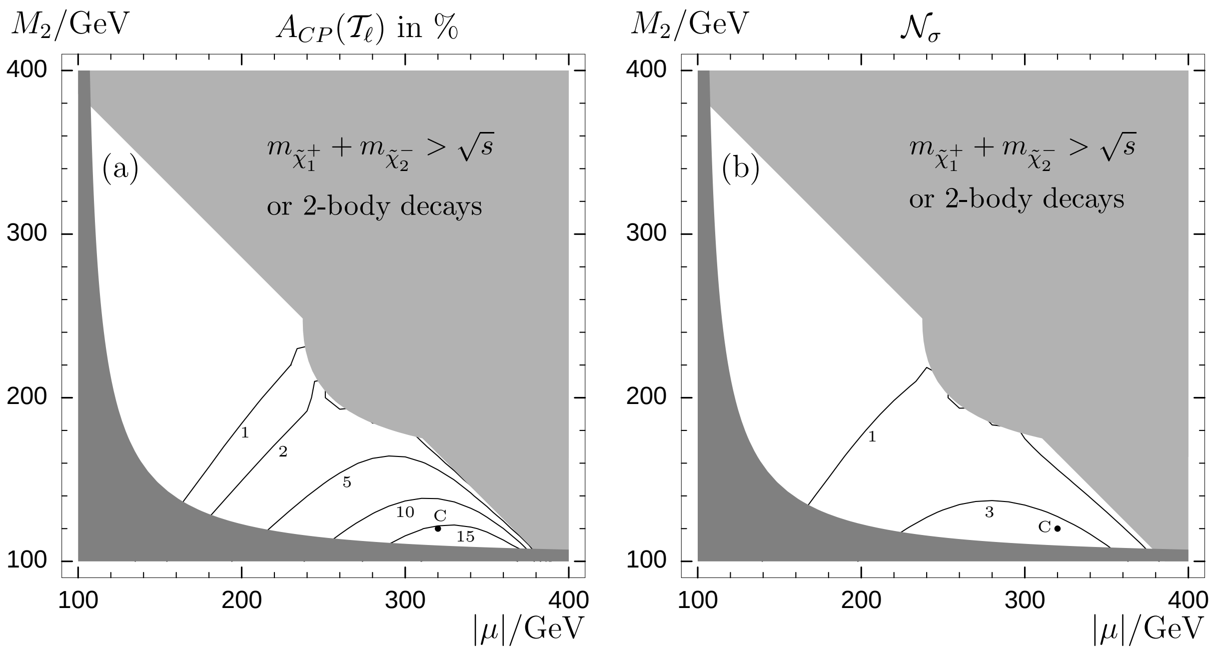

Figure 8: (a) Contours of the CP asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$, Eq. (7), in % for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ at $\sqrt{s} = 500$ GeV with subsequent decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \ell^+ \nu$ and (b) contours of the number of standard deviations $\mathcal{N}_\sigma$, Eq. (33), for an integrated luminosity $\mathcal{L}_{\rm int} = 500$ fb^{-1}, respectively. The parameters are $\tan\beta = 5$, $m_{\tilde{\nu}} = 250$ GeV, $m_{\tilde{\ell}} = 261.7$ GeV, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0$, $\phi_\mu = 0.5\pi$ and the beam polarizations are $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$. The point marks the scenario C of Table 1. In the dark-shaded area is $m_{\tilde{\chi}_1^\pm} < 103.5$ GeV, excluded by LEP [20]. The light-shaded area shows the region that either is not kinematically accessible or in which the three-body decay is strongly suppressed because $m_{\tilde{\chi}_1^+} > m_W + m_{\tilde{\chi}_1^0}$.

standard deviations $\mathcal{N}_\sigma$, Eq. (33), are shown for $\mathcal{L}_{\rm int} = 500$ fb^{-1}, respectively, in the M_2-$|\mu|$ plane. The MSSM parameters are $\tan\beta = 5$, $m_{\tilde{\nu}} = 250$ GeV, $m_{\tilde{\ell}} = 261.7$ GeV, $|M_1| = 5/3\tan^2\theta_W M_2$, $\phi_{M_1} = 0$ and $\phi_\mu = 0.5\pi$. The asymmetry reaches its largest values of about 15% in gaugino-like scenarios. For example, for scenario C, $A_{\rm CP}(\mathcal{T}_\ell)$ can be measured with a 3σ significance.

4 Summary and conclusions

We have proposed and analysed CP-sensitive observables in chargino production $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_{1,2}^-$ with subsequent hadronic and leptonic three-body decays $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \bar{f}^d f^u$ ($f^d = e, \mu, s$ and $f^u = \nu_e, \nu_\mu, c$) at an e^+e^- linear collider with centre-of-mass energy $\sqrt{s} = 500$ GeV, integrated luminosity $\mathcal{L}_{\rm int} = 500$ fb^{-1} and longitudinally polarized beams. Our framework has been the MSSM with complex parameters. We have constructed CP-odd asymmetries with the help of triple product correlations between the momenta of the incoming and outgoing particles.

Considering the production process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ followed by the hadronic three-body decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \bar{s} c$, we have defined the CP asymmetry $A_{\rm CP}(\mathcal{T}_q)$ that is based on the triple product $\mathcal{T}_q = \vec{p}_{\bar{s}} \cdot (\vec{p}_c \times \vec{p}_{e^-})$. The asymmetry $A_{\rm CP}(\mathcal{T}_q)$ is sensitive to CP violation in the decay and depends on the phases ϕ_μ and ϕ_{M_1} appearing in the chargino/neutralino system. We have shown that the measurability of the asymmetry $A_{\rm CP}(\mathcal{T}_q)$ can be significantly increased by a suitable choice of beam polarizations. Choosing $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$, $A_{\rm CP}(\mathcal{T}_q)$ can be probed at the 5σ level in a large region of the MSSM parameter space (in some regions even up to the 10σ level).

For the production process $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_2^-$ we have separately considered the hadronic three-body decay $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \bar{s} c$ and the leptonic three-body decays $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 \ell^+ \nu$, $\ell = e, \mu$. For the hadronic three-body decay we have studied again the CP asymmetry that is based on the triple product $\mathcal{T}_q$. In this case, the resulting CP asymmetry is sensitive to CP violation in production and decay. Also this asymmetry can be probed at the 5σ level in a large region of the MSSM parameter space. For the leptonic three-body decays, we have studied the asymmetry $A_{\rm CP}(\mathcal{T}_\ell)$ that is based on the triple product $\mathcal{T}_\ell = \vec{p}_{\ell^+} \cdot (\vec{p}_{e^-} \times \vec{p}_{\tilde{\chi}_i^+})$,which is sensitive to CP violation in the production only and hence to the phase ϕ_μ. We have found that the measurability of $A_{\rm CP}(\mathcal{T}_\ell)$ is somewhat decreased with respect to the previously considered asymmetries; however, it is accessible at the 3σ level. As the two types of CP-odd asymmetries are sensitive to various combinations of the phases ϕ_μ and ϕ_{M_1}, their measurement will allow CP violation to be tested in the chargino/neutralino sector. Moreover, we have demonstrated that the CP-odd asymmetries studied in this paper can be large even for small CP-violating phases ϕ_μ and ϕ_{M_1}, which are favoured by the EDM constraints.

Acknowledgements

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A Kinematics

We choose a coordinate frame in the laboratory system, where the momenta are

$$p_1 = E_b(1, -\sin\theta, 0, \cos\theta), \tag{34}$$

$$p_2 = E_b(1, \sin\theta, 0, -\cos\theta), \tag{35}$$

$$p_3 = (E_i, 0, 0, -q), \tag{36}$$

$$p_4 = (E_j, 0, 0, q). \tag{37}$$

E_b is the beam energy, θ is the scattering angle between the incoming $e^-(p_1)$ beam and the outgoing chargino $\tilde{\chi}_j^-(p_4)$; the azimuth Φ can be chosen equal to zero. The energies and the momenta of the charginos $\tilde{\chi}_j^-(p_4)$ and $\tilde{\chi}_i^+(p_3)$ are

$$E_i = \frac{s + m_i^2 - m_j^2}{2\sqrt{s}}, \quad E_j = \frac{s + m_j^2 - m_i^2}{2\sqrt{s}}, \quad q = \frac{\sqrt{\lambda(s, m_i^2, m_j^2)}}{2\sqrt{s}}, \tag{38}$$

where m_i, m_j are the masses of the charginos and λ is the kinematical triangle function $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

The polarization vectors $s^{a\mu}(\tilde{\chi}_i^+)$ and $s^{b\mu}(\tilde{\chi}_j^-)$ $(a,b = 1,2,3)$ of the charginos in the laboratory system are

$$s^{1\mu}(\tilde{\chi}_i^+) = (0,-1,0,0)\ , \qquad s^{1\mu}(\tilde{\chi}_j^-) = (0,1,0,0)\ , \tag{39}$$

$$s^{2\mu}(\tilde{\chi}_i^+) = (0,0,1,0)\ , \qquad s^{2\mu}(\tilde{\chi}_j^-) = (0,0,1,0)\ , \tag{40}$$

$$s^{3\mu}(\tilde{\chi}_i^+) = \frac{1}{m_i}(q,0,0,-E_i)\ , \qquad s^{3\mu}(\tilde{\chi}_j^-) = \frac{1}{m_j}(q,0,0,E_j)\ , \tag{41}$$

where $s^3(\tilde{\chi}_{i,j}^\pm)$ describes the longitudinal polarization, $s^1(\tilde{\chi}_{i,j}^\pm)$ the transverse polarization in the scattering plane, and $s^2(\tilde{\chi}_{i,j}^\pm)$ the transverse polarization perpendicular to the scattering plane.

B Formalism

B.1 T-odd terms of production and T-even terms of decay

It can be shown [8, 16] that all contributions to the T-odd terms $\Sigma_P^{a,\mathrm{O}}(\tilde{\chi}_i^+)$ in Eq. (31) contain a factor

$$f_4^a = i \cdot m_j \epsilon_{\mu\nu\rho\sigma} p_2^\mu p_1^\nu s^{a,\rho} p_3^\sigma \tag{42}$$

which vanishes for longitudinal polarization $(a = 3)$ and transverse polarization in the production plane $(a = 1)$ so that we have only to include the spin terms for transverse polarization of the chargino $\tilde{\chi}_i^+$ perpendicular to the production plane $(a = 2)$:

$$\Sigma_P^{2,\mathrm{O}}(\tilde{\chi}_i^+) = \Sigma_P^{2,\mathrm{O}}(\gamma Z) + \Sigma_P^{2,\mathrm{O}}(\gamma\tilde{\nu}) + \Sigma_P^{2,\mathrm{O}}(ZZ) + \Sigma_P^{2,\mathrm{O}}(Z\tilde{\nu}) \tag{43}$$

with

$$\Sigma_P^{2,\mathrm{O}}(\gamma Z) = 2\mathrm{Re}\Big\{\Delta(\gamma)\Delta^*(Z)\frac{1}{2}\delta_{ij}c_-^P(\gamma Z)(O_{ij}^{\prime L*} - O_{ij}^{\prime R*})f_4^{a=2}\Big\}, \tag{44}$$

$$\Sigma_P^{2,\mathrm{O}}(\gamma\tilde{\nu}) = -2\mathrm{Re}\Big\{\Delta(\gamma)\Delta_{ij}^*(\tilde{\nu})\frac{1}{4}\delta_{ij}c_+^P(\gamma\tilde{\nu})f_4^{a=2}\Big\}, \tag{45}$$

$$\Sigma_P^{2,\mathrm{O}}(ZZ) = |\Delta(Z)|^2\frac{1}{2}\Big[c_-^P(ZZ)(O_{ij}^{\prime R}O_{ij}^{\prime L*} - O_{ij}^{\prime L}O_{ij}^{\prime R*})f_4^{a=2}\Big], \tag{46}$$

$$\Sigma_P^{2,\mathrm{O}}(Z\tilde{\nu}) = -2\mathrm{Re}\Big\{\Delta(Z)\Delta_{ij}^*(\tilde{\nu})\frac{1}{4}c_+^P(Z\tilde{\nu})O_{ij}^{\prime R}f_4^{a=2}\Big\}. \tag{47}$$

Here

$$c_\pm^P(\alpha\beta) = \pm c^L(\alpha)c^L(\beta)(1 - P_{e^-})(1 + P_{e^+}) + c^R(\alpha)c^R(\beta)(1 + P_{e^-})(1 - P_{e^+}) \tag{48}$$

with

$$c^L(\gamma) = 1, \quad c^L(Z) = L_e, \quad c^L(\tilde{\nu}) = 1, \tag{49}$$

$$c^R(\gamma) = 1, \quad c^R(Z) = R_e, \quad c^R(\tilde{\nu}) = 0, \tag{50}$$

and P_{e^-} and P_{e^+} is the degree of longitudinal polarization of the electron beam and positron beam, respectively.

Note that f_4^a is purely imaginary, so that, for example, $\Sigma_P^{2,\mathrm{O}}(\gamma Z)$, Eq. (44), is non-vanishing only if the couplings $O_{ij}^{'L,R}$ are complex and gives a CP-sensitive contribution to the asymmetry $A_{\mathrm{CP}}(\mathcal{T}_{q,\ell})$. Analogous contributions come from the other terms in $\Sigma_P^{2,\mathrm{O}}$, Eqs. (45)–(47). We have to multiply $\Sigma_P^{2,\mathrm{O}}$ in Eq. (31) by

$$\begin{aligned}\Sigma_D^{2,\mathrm{E}}(\tilde{\chi}_i^+) \;=\; & \Sigma_D^{2,\mathrm{E}}(W^+W^+) + \Sigma_D^{2,\mathrm{E}}(W^+\tilde{f}_L^d) + \Sigma_D^{2,\mathrm{E}}(W^+\tilde{f}_L^u) \\ & + \Sigma_D^{2,\mathrm{E}}(\tilde{f}_L^d\tilde{f}_L^d) + \Sigma_D^{2,\mathrm{E}}(\tilde{f}_L^d\tilde{f}_L^u) + \Sigma_D^{2,\mathrm{E}}(\tilde{f}_L^u\tilde{f}_L^u),\end{aligned} \tag{51}$$

with

$$\begin{aligned}\Sigma_D^{2,\mathrm{E}}(W^+W^+) \;=\; & |\Delta(W)|^2 4\Big[(|O_{ki}^R|^2 - |O_{ki}^L|^2)(g_1^{a=2} + g_2^{a=2}) \\ & - (O_{ki}^{L*}O_{ki}^R + O_{ki}^L O_{ki}^{R*})g_3^{a=2} - (|O_{ki}^R|^2 + |O_{ki}^L|^2)(g_1^{a=2} - g_2^{a=2})\Big],\end{aligned} \tag{52}$$

$$\Sigma_D^{2,\mathrm{E}}(W^+\tilde{f}_L^d) \;=\; 2\mathrm{Re}\Big\{\Delta(W)\Delta_i^*(\tilde{f}_L^d)2\Big[2O_{ki}^R g_2^{a=2} - O_{ki}^L g_3^{a=2}\Big]\Big\}, \tag{53}$$

$$\Sigma_D^{2,\mathrm{E}}(W^+\tilde{f}_L^u) \;=\; 2\mathrm{Re}\Big\{\Delta(W)\Delta_i^*(\tilde{f}_L^u)2\Big[2O_{ki}^L g_1^{a=2} + O_{ki}^R g_3^{a=2}\Big]\Big\}, \tag{54}$$

$$\Sigma_D^{2,\mathrm{E}}(\tilde{f}_L^d\tilde{f}_L^d) \;=\; |\Delta_i(\tilde{f}_L^d)|^2 2g_2^{a=2}, \tag{55}$$

$$\Sigma_D^{2,\mathrm{E}}(\tilde{f}_L^d\tilde{f}_L^u) \;=\; 2\mathrm{Re}\{\Delta_i(\tilde{f}_L^d)\Delta_i^*(\tilde{f}_L^u)g_3^{a=2}\}, \tag{56}$$

$$\Sigma_D^{2,\mathrm{E}}(\tilde{f}_L^u\tilde{f}_L^u) \;=\; |\Delta_i(\tilde{f}_L^u)|^2 2(-g_1^{a=2}), \tag{57}$$

where

$$g_1^{a=2} \;=\; m_i(p_5p_7)(p_6s^{a=2}), \tag{58}$$

$$g_2^{a=2} \;=\; m_i(p_5p_6)(p_7s^{a=2}), \tag{59}$$

$$g_3^{a=2} \;=\; m_k[(p_3p_6)(p_7s^{a=2}) - (p_3p_7)(p_6s^{a=2})]. \tag{60}$$

The kinematic functions g_1^a, g_2^a, g_3^a, $a = 2$ are real. When multiplied by the purely imaginary $f_4^{a=2}$, Eq. (42), this leads to triple products sensitive to the CP phases of the couplings $O_{ij}^{'L,R}$ in the production process, which in the laboratory system read:

$$g_1^{a=2}\cdot f_4^{a=2} \;=\; i2E_b m_i m_j(p_5p_7)\vec{p}_6(\vec{p}_1\times\vec{p}_3), \tag{61}$$

$$g_2^{a=2}\cdot f_4^{a=2} \;=\; i2E_b m_i m_j(p_5p_6)\vec{p}_7(\vec{p}_1\times\vec{p}_3), \tag{62}$$

$$g_3^{a=2}\cdot f_4^{a=2} \;=\; i2E_b m_j m_k\{(p_3p_6)\vec{p}_7(\vec{p}_1\times\vec{p}_3) - (p_3p_7)\vec{p}_6(\vec{p}_1\times\vec{p}_3)\}. \tag{63}$$

As outlined above, these expressions will be multiplied in Eqs. (44)–(47) by the factors $i\cdot\mathrm{Im}\{(O_{ij}^{'L*} - O_{ij}^{'R*})\}$ etc., and contribute to the first term of Eq. (31) and, hence, to the numerator of the asymmetry A_T, Eq. (28).

B.2 T-odd terms of decay and T-even terms of production

The factor

$$\Sigma_D^{a,\mathrm{O}}(\tilde{\chi}_i^+) = \Sigma_D^{a,\mathrm{O}}(W^+W^+) + \Sigma_D^{a,\mathrm{O}}(W^+\tilde{f}_L^d) + \Sigma_D^{a,\mathrm{O}}(W^+\tilde{f}_L^u) + \Sigma_D^{a,\mathrm{O}}(\tilde{f}_L^d\tilde{f}_L^u) \tag{64}$$

in the second term in Eq. (31) with

$$\Sigma_D^{a,\mathrm{O}}(W^+W^+) \;=\; |\Delta(W)|^2 4\mathrm{Re}\{(O_{ki}^{L*}O_{ki}^{R} - O_{ki}^{L}O_{ki}^{R*})g_4^a\}, \tag{65}$$
$$\Sigma_D^{a,\mathrm{O}}(W^+\tilde{f}_L^d) \;=\; -2\mathrm{Re}\{\Delta(W)\Delta_i^*(\tilde{f}_L^d)2O_{ki}^{L}g_4^a\}, \tag{66}$$
$$\Sigma_D^{a,\mathrm{O}}(W^+\tilde{f}_L^u) \;=\; -2\mathrm{Re}\{\Delta(W)\Delta_i^*(\tilde{f}_L^u)2O_{ki}^{R}g_4^a\}, \tag{67}$$
$$\Sigma_D^{a,\mathrm{O}}(\tilde{f}_L^d\tilde{f}_L^u) \;=\; -2\mathrm{Re}\{\Delta_i(\tilde{f}_L^d)\Delta_i^*(\tilde{f}_L^u)g_4^a\} \tag{68}$$

is sensitive to CP violation in the decay of the chargino $\tilde{\chi}_i^+$ [8, 16] due to the purely imaginary kinematic factor

$$g_4^a = i\cdot m_k\epsilon_{\mu\nu\rho\sigma}s^{a\mu}p_3^{\nu}p_7^{\rho}p_6^{\sigma}\ . \tag{69}$$

For example in Eq. (65) it is multiplied by the factor $i\cdot\mathrm{Im}\{(O_{ki}^{L*}O_{ki}^{R} - O_{ki}^{L}O_{ki}^{R*})\}$, which depends on the phases ϕ_μ and ϕ_{M_1} and contributes to the CP asymmetry A_{CP}, Eq. (7). Analogous contributions follow from Eqs. (66)–(68).

The T-even contributions from the production process in Eq. (31) are

$$\begin{aligned}\Sigma_P^{a,\mathrm{E}}(\tilde{\chi}_i^+) \;=\;& \Sigma_P^{a,\mathrm{E}}(\gamma\gamma) + \Sigma_P^{a,\mathrm{E}}(\gamma Z) + \Sigma_P^{a,\mathrm{E}}(\gamma\tilde{\nu})\\ &+ \Sigma_P^{a,\mathrm{E}}(ZZ) + \Sigma_P^{a,\mathrm{E}}(Z\tilde{\nu}) + \Sigma_P^{a,\mathrm{E}}(\tilde{\nu}\tilde{\nu}),\end{aligned} \tag{70}$$

with

$$\Sigma_P^{a,\mathrm{E}}(\gamma\gamma) \;=\; |\Delta(\gamma)|^2 c_-^P(\gamma\gamma)\delta_{ij}(-f_1^a + f_2^a + f_3^a), \tag{71}$$
$$\begin{aligned}\Sigma_P^{a,\mathrm{E}}(\gamma Z) \;=\;& 2\mathrm{Re}\Big\{\Delta(\gamma)\Delta^*(Z)\frac{1}{2}\delta_{ij}\Big[c_+^P(\gamma Z)(O_{ij}^{\prime R*} - O_{ij}^{\prime L*})(f_1^a + f_2^a)\\ &+ c_-^P(\gamma Z)(O_{ij}^{\prime R*} + O_{ij}^{\prime L*})(-f_1^a + f_2^a + f_3^a)\Big]\Big\},\end{aligned} \tag{72}$$
$$\Sigma_P^{a,\mathrm{E}}(\gamma\tilde{\nu}) \;=\; -2\mathrm{Re}\{\Delta(\gamma)\Delta_{ij}^*(\tilde{\nu})\frac{1}{4}\delta_{ij}c_+^P(\gamma\tilde{\nu})(2f_2^a + f_3^a)\}, \tag{73}$$
$$\begin{aligned}\Sigma_P^{a,\mathrm{E}}(ZZ) \;=\;& |\Delta(Z)|^2\frac{1}{2}\Big[c_+^P(ZZ)(|O_{ij}^{\prime R}|^2 - |O_{ij}^{\prime L}|^2)(f_1^a + f_2^a)\\ &+c_-^P(ZZ)\Big((O_{ij}^{\prime L}O_{ij}^{\prime R*} + O_{ij}^{\prime R}O_{ij}^{\prime L*})f_3^a\\ &+(|O_{ij}^{\prime R}|^2 + |O_{ij}^{\prime L}|^2)(-f_1^a + f_2^a)\Big)\Big],\end{aligned} \tag{74}$$
$$\Sigma_P^{a,\mathrm{E}}(Z\tilde{\nu}) \;=\; -2\mathrm{Re}\{\Delta(Z)\Delta_{ij}^*(\tilde{\nu})\frac{1}{4}c_+^P(Z\tilde{\nu})(2O_{ij}^{\prime L}f_2^a + O_{ij}^{\prime R}f_3^a)\}, \tag{75}$$
$$\Sigma_P^{a,\mathrm{E}}(\tilde{\nu}\tilde{\nu}) \;=\; |\Delta_{ij}(\tilde{\nu})|^2\frac{1}{4}c_+^P(\tilde{\nu}\tilde{\nu})(-f_2^a), \tag{76}$$

where

$$f_1^a \;=\; m_i(p_2p_4)(p_1s^a), \tag{77}$$
$$f_2^a \;=\; m_i(p_1p_4)(p_2s^a), \tag{78}$$
$$f_3^a \;=\; m_j[(p_1p_3)(p_2s^a) - (p_2p_3)(p_1s^a)]. \tag{79}$$

Since $s^a(\tilde{\chi}_i^+)$ for $a = 2$ is perpendicular to the production plane, $\Sigma_P^{2,\mathrm{E}}(\tilde{\chi}_i^+)$ vanishes, so that in A_{CP} only the contributions of the longitudinal polarization ($a = 3$) and of the transverse polarization in the production plane ($a = 1$) have to be taken into account.

Finally, the triple products sensitive to the CP phases in the chargino decay in the laboratory system read

$$\begin{aligned}\sum_{a=1,3} f_1^a \cdot g_4^a \;=\; & i m_i m_k (p_2 p_4)\Big\{ - E_b \vec{p}_5(\vec{p}_7 \times \vec{p}_6) - E_7 \vec{p}_5(\vec{p}_6 \times \vec{p}_1) \\ & + E_6 \vec{p}_5(\vec{p}_7 \times \vec{p}_1) + E_5 \vec{p}_1(\vec{p}_7 \times \vec{p}_6)\Big\},\end{aligned} \tag{80}$$

$$\begin{aligned}\sum_{a=1,3} f_2^a \cdot g_4^a \;=\; & i m_i m_k (p_1 p_4)\Big\{ - E_b \vec{p}_5(\vec{p}_7 \times \vec{p}_6) + E_7 \vec{p}_5(\vec{p}_6 \times \vec{p}_1) \\ & - E_6 \vec{p}_5(\vec{p}_7 \times \vec{p}_1) - E_5 \vec{p}_1(\vec{p}_7 \times \vec{p}_6)\Big\},\end{aligned} \tag{81}$$

$$\begin{aligned}\sum_{a=1,3} f_3^a \cdot g_4^a \;=\; & i m_j m_k \Big\{ [(p_2 p_3) - (p_1 p_3)] E_b \vec{p}_5(\vec{p}_7 \times \vec{p}_6) \\ & + [(p_2 p_3) + (p_1 p_3)] \\ & \Big[E_7 \vec{p}_5(\vec{p}_6 \times \vec{p}_1) - E_6 \vec{p}_5(\vec{p}_7 \times \vec{p}_1) - E_5 \vec{p}_1(\vec{p}_7 \times \vec{p}_6)\Big]\Big\}.\end{aligned} \tag{82}$$

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