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S CHANNEL STRUCTURE OF THE TWO-REGGEON CUT

T. A. DeGrand *),+)

Department of Theoretical Physics
University of Geneva

and

C. E. DeTar **),+)

CERN - Geneva

ABSTRACT

The three contributions to the s channel discontinuity (called "diffractive", "absorbed multiperipheral" and "polyperipheral") are evaluated in the limit $s \to \infty$ in two contrasting models, a simple dual resonance model for the cut, and the Mandelstam graph. The models differ in the relative importance of the three contributions, contrary to prevailing belief. The analysis presented here provides a link between general investigations of the asymptotic singularity structure of Regge-behaved amplitudes and the s channel structure of Reggeon diagrams.

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*) National Science Foundation Graduate Fellow.

**) Alfred P. Sloan Foundation Fellow.

+) Permanent address : Department of Physics, Massachusetts Institute of Technology, Cambridge, Mass. 02139, U.S.A.

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It is in the spirit of the Reggeon calculus that there exists a perturbation expansion for scattering amplitudes in which the elementary propagators correspond to factorizable Regge poles (Reggeons) and the elementary vertices to scattering amplitudes for Reggeons and particles which are irreducible with respect to Reggeons. The motivation for the existence of such an expansion is most directly found in traditional perturbation theory [1),2)] in which the Reggeons are defined as the family of bound states and resonances in a particular channel composed of two elementary fields. Thus the simplest graph in $\lambda\phi^3$ perturbation theory generating the two-Reggeon cut is the Mandelstam graph, Fig. 1, in which the Reggeons are produced by the summation over all numbers of rungs in the ladders. It is reasonable to think of the Reggeon calculus as a technique for summing all possible graphs in perturbation theory in a particular order, in which the Reggeons are first identified [2)].

The validity of the Reggeon calculus should, however, transcend the particular field theory that is chosen as its basis. This belief is supported by the work of Gribov, Pomeranchuk, and Ter Martirosyan and White [3)], who, starting from S matrix principles, i.e., without reference to a perturbation theory, derived crossed-channel j plane unitarity formulae for discontinuities across Reggeon cuts. White found solutions to these analogous to the effective range expansion. His solutions can be thought of as a rudimentary Reggeon calculus.

Although the Reggeon calculus is now usually formulated as a field theory for the j plane representation of scattering amplitudes, it is particularly useful to keep in mind the energy plane representation of the Reggeon calculus, as, for example, provided by perturbation theory, since it then becomes possible to discuss the s channel structure of the various terms in the series. Thus there are three classes of contributions to the s channel discontinuity of the Mandelstam graph, corresponding to the Cutkosky slicings illustrated in Fig. 2 which slice two Reggeons, one, or none. These have been studied by Abramovskii, Gribov and Kancheli [4)] and Halliday and Sachrajda [5)]. The Mandelstam graph provides a good illustration of a general approach to the analysis of the simplest two-Reggeon cut diagram in the Reggeon calculus. In general the two-particle, two-Reggeon amplitude can be found from the helicity asymptotic limit [6)] of a six-particle scattering amplitude which is irreducible with respect to Reggeon cuts. The two-Reggeon cut diagram is in turn constructed by piecing together two such amplitudes. The three classes of Cutkosky slicings correspond to three discontinuities of the same six-particle scattering amplitude as illustrated for the Mandelstam graph in Fig. 2 and for general diagrams in Fig. 3. Thus a knowledge of the singularity structure of the six-particle amplitude and the behaviour of its discontinuities in the helicity asymptotic limit is required in order to find the asymptotic behaviour of the three classes of discontinuities of the two-Reggeon diagram.

The physical significance of the three classes of discontinuities is especially evident for the case that the Reggeons are Pomerons [4)]. It is popular to think of the intermediate states which collectively define the absorptive part of the Pomeron as having a multiperipheral distribution in momentum [7)]. Thus, the discontinuity slicing no Pomerons (a) would correspond to diffractive excitation, that slicing one Pomeron (b) would correspond to a multiperipheral event (with absorption) and that slicing both Pomerons (c) to a polyperipheral event, i.e., with twice the multiplicity of particles as in an ordinary multiperipheral event, such as might be expected to occur in the collision of two deuterons if the constituent nucleons should each scatter independently in a multiperipheral fashion. The various production amplitudes corresponding to the slicings of the Mandelstam graph are shown in Fig. 4. Thus, in principle, an understanding of the s channel structure of the two-Reggeon cut provides a second contact with phenomenology, the first being the usual prediction of the behaviour of the elastic two-body scattering amplitude.

In addition to the phenomenological interest in the study of discontinuities of Reggeon diagrams, it is useful theoretically to establish general techniques for evaluating such discontinuities [4)], since they are a prerequisite to understanding the s channel properties of the Reggeon calculus, in particular in understanding how it satisfies s channel unitarity. A general understanding of the procedure for evaluating discontinuities of Reggeon diagrams is useful also in the study of the rôle of Reggeon cuts in inclusive reactions [4),8)] and in the study of multiple production in nuclei [9)]. The analysis which we present here is also of interest, because it provides a link between the general investigation of the asymptotic singularities of Regge-behaved amplitudes [6)] and the study of the s channel structure of the Reggeon calculus.

The organization of the paper is as follows. In Section 2 we discuss the asymptotic behaviour of the two-Reggeon graph and its various discontinuities. In Section 3, we apply these results to two contrasting models for the two-Reggeon - two-particle vertex, one provided by the Mandelstam graph and the other by the dual resonance model in the tree approximation. We present results of an investigation of the effect of the various Cutkosky slicings upon the arrangement of singularities in the two-Reggeon - two-particle vertex, the detailed analysis of which is described elsewhere [10]. In particular we find that the various slicings have a profound effect upon the arrangement of poles and normal thresholds in the vertex, which leads to model-dependent results for the relative importance of their asymptotic contributions to the total cross-section. In Section 4 we speculate on the implications of these results for the general two-Reggeon - two-particle vertex.

2. - THE ASYMPTOTIC BEHAVIOUR OF THE TWO-REGGEON DIAGRAM

The general asymptotic expression for the two-Reggeon diagram of the four-particle amplitude,

$$A_4(s,t) \underset{s\to\infty}{\sim} \frac{-i}{(2\pi)^4}\int d^4k\, B(M_1^2, t, t_u, t_l)\, s^{\alpha(t_l)+\alpha(t_u)} \times \xi_l\, \xi_u\, \Gamma[-\alpha(t_l)]\, \Gamma[-\alpha(t_u)]\, B(M_2^2, t, t_u, t_l) \tag{1}$$

is obtained from a Feynman loop momentum integration over the product of two Reggeon-particle elastic scattering amplitudes B and two Reggeon propagators $s^{\alpha}\xi\Gamma(-\alpha)$, where $\alpha(t)$ is the Reggeon trajectory function $2\xi(t) = [\exp[-i\pi\alpha(t)] + \tau]$ is the signature factor for signature $\tau = \pm 1$, and the Γ function gives the poles for the Reggeon recurrences. The Reggeon-particle elastic scattering amplitude is in turn found from the helicity asymptotic behaviour of the six-particle amplitude (Fig. 3)

$$s_u \to \infty,\ s_l \to \infty\ ;\ s_u/s_l\ ,\ M_1^2,\ t_u,\ t_l,\ t,\ u_1\ \ \text{fixed}$$

$$A_6 \sim B(M_1^2, t, t_u, t_\ell)\,(s_\ell)^{\alpha(t_\ell)}(s_u)^{\alpha(t_u)}\,\xi_\ell\,\xi_u \times \Gamma[-\alpha(t_u)]\,\Gamma[-\alpha(t_\ell)]\,\beta(t_\ell)\beta(t_u) + \text{other terms}, \tag{2}$$

where $\beta(t)$ is the two-particle-one-Reggeon vertex function and the "other terms" lack singularities in M_1^2 and so do not enter into the determination of the asymptotic behaviour of A_4 [2].

A convenient general technique for evaluating the asymptotic behaviour of the expression (2) and its discontinuities is due to Rothe [11], which we review here. It consists in making the change of variables over the internal Reggeon loop

$$d^4k \underset{s\to\infty}{\sim} \frac{dM_1^2\, dM_2^2\, dt_\ell\, dt_u\, \theta(-\lambda)}{2|s|\,[-\lambda(t, t_\ell, t_u)]^{1/2}}, \tag{3}$$

where

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac.$$

It is assumed that the limit $s \to \infty$ can be taken inside the integration ; i.e., that the M_1^2 and M_2^2 integrations are sufficiently convergent that the dominant contribution to the integral comes from the region $M_1^2, M_2^2 \ll s$. If this is not the case, the asymptotic behaviour is more readily found using a Mellin transform technique [2]. Since our major conclusions also follow from the Mellin analysis, we prefer to use the more transparent Rothe technique here, despite its limitations.

The limits of integration over M_1^2 and M_2^2 can be set at $\pm\infty$ due to the assumed convergence. Thus the integration can be carried out separately over M_1^2 and M_2^2. The integrand inherits the usual right- and left-hand cuts in M_1^2 from B_1, corresponding to singularities in the channels M_1^2 and u_1 of the six-point function. These are shown in Fig. 5. The integral can be carried out by closing the contour in the lower half plane, making use of the assumed convergence. Thus the asymptotic behaviour is given by

$$A_4 \sim \frac{i}{8\pi^2}\int \frac{dt_\ell\, dt_u\ \theta(-\lambda)}{(-\lambda)^{1/2}} N^2(t,t_u,t_\ell)\, s^{\alpha(t_u)+\alpha(t_\ell)-1} \times \xi_\ell\, \xi_u\ , \tag{4}$$

where

$$N(t,t_u,t_\ell) = \frac{1}{2\pi i}\int dM_1^2\ \mathrm{disc}_{M_1^2}\, B(M_1^2,t,t_u,t_\ell), \tag{5}$$

is often called the "residue of the fixed pole" in B. For the Pomeron $\alpha \approx 1$, $\xi \approx i$, the asymptotic behaviour is therefore approximately

$$A_4 \propto -is \quad \text{and} \quad \frac{1}{2i}\,\mathrm{disc}\, A_4 \propto -s, \tag{6}$$

thereby yielding a negative contribution to the total cross-section.

The foregoing is, of course, a well-known result. We would like to proceed to the question of how the total discontinuity (6) is divided asymptotically among the three separate contributions, designated by the corresponding labels in Fig. 3

$$\frac{1}{2i}\,\mathrm{disc}_{total}\, A_4 = \frac{1}{2i}\,\mathrm{disc}_a A_4 + \frac{1}{2i}\,\mathrm{disc}_b A_4 + \frac{1}{2i}\,\mathrm{disc}_c A_4 . \tag{7}$$

We consider them in order.

A) - Diffractive contribution

The Cutkosky rule together with the unitarity $i\epsilon$ prescription gives directly

$$\frac{1}{2i}\,\mathrm{disc}_a A_4(s,t) \underset{s\to\infty}{\sim} \frac{2}{2i}\,\frac{i}{8\pi^2}\int \frac{dt_\ell\, dt_u}{(-\lambda)^{1/2}}\,\theta(-\lambda)\int dM_1^2\, dM_2^2\ \frac{\mathrm{disc}_{M_1^2}}{2\pi i} B(M_1^2,t,t_\ell,t_u) \times (s)^{\alpha(t_u)+\alpha(t_\ell)}\xi_\ell\,\xi_u^*\,\Gamma[-\alpha(t_\ell)]\,\Gamma[-\alpha(t_u)]\,\frac{\mathrm{disc}_{M_2^2}}{2\pi i} B(M_2^2,t,t_\ell,t_u), \tag{8}$$

where the extra factor of 2 on the right comes from the fact that cuts in s are generated by a combination of right-hand cuts in M_1^2 and M_2^2 as well as left-hand cuts in M_1^2 and M_2^2, the integral over the latter discontinuity being exactly equal to the former

$$\int dM_1^2 \, \mathrm{disc}_{M_1^2} B(M_1^2, t, t_\ell, t_u) = \int du_1 \, \mathrm{disc}_{u_1} B(u_1, t, t_\ell, t_u), \tag{9}$$

because of the assumed convergence at large M_1^2. The only difference between the present integrand and that of Eq. (4) is that the signature factor ξ_u is now complex-conjugated, because of the unitarity $i\epsilon$ prescription. Thus in the notation of Eq. (6) for the Pomeron

$$\frac{1}{2i} \, \mathrm{disc}_a A_4 \propto +s, \tag{10}$$

corresponding to a positive contribution to the total cross-section.

B) - Absorbed multiperipheral contribution

For this contribution we must consider a contour rotation analogous to that of the undiscontinued amplitude. The Cutkosky prescription yields

$$A_4(s,t) \underset{s\to\infty}{\sim} \frac{-i}{(2\pi)^4} \int d^4k \, B_b(M_1^2, t, t_u, t_\ell)(s)^{\alpha(t_\ell)+\alpha(t_u)}$$
$$\times \, \xi_\ell \, [\mathrm{disc}\, \xi_u] \, \Gamma[-\alpha(t_\ell)] \Gamma[-\alpha(t_u)] B_b(M_2^2, t, t_u, t_\ell), \tag{11}$$

where the discontinued two-Reggeon, two-particle vertex B_b is obtained from the appropriate discontinuity of the six-point function in the helicity asymptotic limit

$$\mathrm{disc}_b A_6 \sim B_b(M_1^2, t, t_u, t_\ell)(s_\ell)^{\alpha(t_\ell)}(s_u)^{\alpha(t_u)} \xi_u \, [\mathrm{disc}\, \xi_\ell]$$
$$\times \, \Gamma[-\alpha(t_\ell)] \Gamma[-\alpha(t_u)] \beta(t_\ell)\beta(t_u) + \text{other terms}, \tag{12}$$

as illustrated in Fig. 3.

Our main observation is that this discontinuity profoundly alters the singularity structure of the two-Reggeon, two-particle amplitude. In particular, the amplitude B_b cannot have particle poles or normal threshold branch points in the channels overlapping *) the one in which the discontinuity is taken. In the present case (Fig. 6) the amplitude cannot have poles and normal threshold branch points in the u_1 channel, though they are permitted in the M_1^2 channel. One might well expect that such an effect would profoundly alter the asymptotic behaviour. Thus we would have (assuming the contours could be closed as before)

$$\frac{1}{2i}\,\mathrm{disc}_b\,A_4(s,t) \underset{s\to\infty}{\sim} \frac{1}{2i}\frac{4i}{8\pi^2}\int\frac{dt_\ell\,dt_u\,\theta(-\lambda)}{(-\lambda)^{1/2}}\,N_b^{\,2}(t,t_u,t_\ell) \times\left[\mathrm{disc}\,\xi_\ell\right]\mathrm{Im}\,\xi_u\; s^{\alpha(t_u)+\alpha(t_\ell)-1}, \tag{13}$$

where

$$N_b(t,t_u,t_\ell) = \frac{1}{2\pi i}\int dM_1^2\,\mathrm{disc}_{M_1^2}\,B_b(M_1^2,t,t_u,t_\ell),$$

may well differ from $N(t,t_u,t_\ell)$, so we keep the subscript b. In (13), the factors 4 and $\mathrm{Im}\,\xi_u$ are due to the four possible slicings of one Reggeon, shown in Fig. 7.

C) - Polyperipheral contribution

In the present case, both the M_1^2 and u_1 channels overlap the channel in which the discontinuity is taken (Fig. 3). In general, we have, in analogy with (13),

$$\frac{1}{2i}\,\mathrm{disc}_c\,A_4(s,t) \underset{s\to\infty}{\sim} -\frac{1}{2i}\,\frac{i}{8\pi^2}\int\frac{dt_\ell\,dt_u\,\theta(-\lambda)}{(-\lambda)^{1/2}}\,N_c^{\,2}(t,t_u,t_\ell) \times\left[\mathrm{disc}\,\xi_u\right]\left[\mathrm{disc}\,\xi_\ell\right] s^{\alpha(t_u)+\alpha(t_\ell)-1}. \tag{14}$$

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*) If one draws lines through a diagram in such a way that the incoming lines for each of two channels in question are separated from the outgoing lines, then the channels are said to overlap, if the lines must intersect (cf. Fig. 6b). Otherwise they are non-overlapping (cf. Fig. 6a).

In conclusion, we find that because the contribution A) is given directly in terms of discontinuities in M_1^2 and M_2^2, it is unaffected by the problems which plague contributions B) and C), and is related to the total discontinuity in a model-independent way. However, without resorting to specific models, we can say no more about the contributions B) and C) except that the arithmetic of Eq. (7) requires that the last two terms must make up the difference between the total and the contribution A).

3. - CALCULATIONS IN SPECIFIC MODELS

To investigate further the relative weights of the contributions to the total discontinuity, we consider two models for the six-point function, one taken from the dual resonance model in the tree approximation [12], and one taken from the Mandelstam graph in $\lambda\phi^3$ perturbation theory (Figs. 1 and 2).

A) - Dual resonance model

The only tree graph with a non-vanishing fixed pole residue is the one with poles in both M_1^2 and u_1 (Fig. 8c). Since the only allowed singularities are poles, it follows from the reasoning of Section 2C that the discontinuity associated with the polyperipheral contribution (Fig. 3) can have no singularities in M_1^2 or u_1 whatsoever. In other words, in the dual resonance model,

$$\begin{aligned} B_c\,(M_1^2, t, t_u, t_\ell) &= 0 \\ N_c\,(t, t_u, t_\ell) &= 0. \end{aligned} \tag{15}$$

[In the language of Eq. (2) the discontinuity in question is contained entirely in the "other terms", which are not associated with the two-Reggeon cut diagram.] Consequently, one expects that, in this model, the cross-section for the polyperipheral production process does not have the asymptotic behaviour associated with the two-Reggeon cut.

In calculating the absorbed multiperipheral contribution following the prescription of Section 2B, the discontinuity of Fig. 3 results in a function B_b with poles in the M_1^2 channel but none in the u_1 channel. Since the residues of the M_1^2 poles in B_b are exactly the same as in B, the full amplitude, it follows that

$$N_b(t, t_u, t_\ell) = N(t, t_u, t_\ell) = \frac{1}{2\pi i}\int dM_1^2 \, \mathrm{disc}_{M_1^2} B(M_1^2, t, t_u, t_\ell). \tag{16}$$

However,

$$B_b(M_1^2, t, t_u, t_\ell) \underset{M_1^2\to\infty}{\sim} \frac{N(t, t_u, t_\ell)}{M_1^2}, \tag{17}$$

since it lacks poles in u_1. The asymptotic behaviour of $B_b(M_2^2, t, t_u, t_\ell)$ is the same. The integrand in (13) is therefore insufficiently damped at large M_1^2 and M_2^2 to permit the contour rotation or the approximation $M_1^2, M_2^2 \ll s$ which led to the Jacobian (3). A careful treatment [10)] shows that the effect of this slow convergence is simply to introduce an extra factor of $\frac{1}{2}$ in Eq. (13), yielding in this model

$$\frac{1}{2i}\,\mathrm{disc}_b\, A_4(s,t) \underset{s\to\infty}{\sim} \frac{1}{2i}\,\frac{2i}{8\pi^3}\int \frac{dt_\ell\, dt_u\, \theta(-\lambda)}{(-\lambda)^{1/2}} N^2(t, t_u, t_\ell) \times [\mathrm{disc}\,\xi_\ell]\,\mathrm{Im}\,\xi_u\, s^{\alpha(t_u)+\alpha(t_\ell)-1}, \tag{18}$$

so as to satisfy the arithmetic of Eq. (7). Thus, for the Pomeron in the approximation of Eqs. (6) and (10), we have

$$\frac{1}{2i}\,\mathrm{disc}_b\, A_4 \propto -2s \tag{19}$$

and the terms of Eq. (7) are proportional to

$$-1 = 1 - 2 + 0 \tag{20}$$

B) - Mandelstam graph

The model of Fig. 2b represents, in a sense, the opposite extreme, since the six-point function contains no poles in M_1^2, only branch points. If we first examine the discontinuity B_c associated with the polyperipheral contribution, we find, as expected from the reasoning of Section 2C, that the normal thresholds in M_1^2 and u_1 have been removed. In fact there remain no real axis singularities in M_1^2. This result is consistent with the Steinmann relations [13], which forbid real axis singularities from occurring simultaneously in overlapping channels in the physical region. However, there are complex M_1^2 branch points associated with various higher order Landau singularities [14] located symmetrically about the real axis [*]. One type is the anomalous triangle singularity in which the poles for the three propagators indicated in Fig. 9a trap the contour of integration over the internal loop of the six-point function. (There are four pairs of these singularities.) These singularities are ordinarily present in the full amplitude, but are on the second sheet of the M_1^2 normal threshold. However, they are exposed when the normal threshold is removed. Another type of triangle singularity, also in the complex plane, involves two of the same poles and a normal threshold in the absorptive part of the Reggeon (Fig. 9b). (There are two pairs for each normal threshold.) As a consequence of the presence of these singularities, the integral over $B_c(M_1^2, t, t_u, t_\ell)$ is convergent, yielding a non-vanishing fixed pole residue N_c. What is remarkable is that when it is evaluated explicitly [4),5),18)], it is found that

$$N_c(t, t_u, t_\ell) = N(t, t_u, t_\ell), \tag{21}$$

i.e., the slicing has no effect upon the fixed pole residue in this model, despite the drastically altered singularity structure of the two-Reggeon-two-particle vertex itself. Actually, the result can be seen to be less surprising if one exploits the special structure of the six-point function in this model, and does the integration over M_1^2 before completing the integration over the internal loop of the six-point function [4),5)].

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*) The appearance of these singularities is of interest in and of themselves, since they suggest limitations on generalizations of the Steinmann relations [6].

If we examine next the absorbed multiperipheral slicing, we find that half of the normal threshold singularities are removed, as expected from the reasoning of Section 2B, but complex branch points appear, which assure the normal convergence of the M_1^2 and M_2^2 integrals and, in fact, give

$$N_b(t, t_u, t_\ell) = N(t, t_u, t_\ell) \quad . \tag{22}$$

Thus, for the Pomeron in the approximation of Eqs. (6) and (10), we find for the relative weighting of the terms in Eq. (7)

$$-1 = 1 - 4 + 2 \; . \tag{23}$$

4. - CONCLUSION AND SPECULATION

We have calculated in two models the weighting of the various s channel diffractive, absorbed multiperipheral, and polyperipheral contributions, to the two-Reggeon cut diagram and found that although both models agree in the relative weighting of the total discontinuity and diffractive contribution, the relative weighting of the absorbed multiperipheral and polyperipheral contributions is model dependent. For the Pomeron, the weighting in the dual resonance model, is given approximately by Eq. (20) and in the Mandelstam graph, by Eq. (23).

Abramovskii, Gribov and Kancheli [4] (AGK) give general arguments in favour of a counting scheme for an arbitrary Reggeon calculus diagram similar to that found for the Mandelstam graph. What we find in our two examples does not contradict their claim, since they consider only perturbation theory models of the Reggeon-particle and Reggeon-Reggeon vertices. The dual resonance model does not conform to their assumptions. However, even in the context of a perturbation theory model one may still wonder whether a simple general rule for the weighting of the contributions exists. For example, in present analyses [4),5)] it is necessary to assume that scattering amplitudes involving elementary fields vanish rapidly off the mass shell. But even if such an assumption is granted, the AGK argument in its present formulation [4)] is unfortunately incomplete. A crucial step in the AGK proof involves using a contour rotation in a Sudakov variable to demonstrate the equivalence of a discontinuity and an integration over the real axis in that variable. The argument does not take account of the complication that several channels may in general produce normal threshold singularities in that same variable, which can impede the contour rotation. Nevertheless a general argument for perturbation theory vertices may still be possible.

Even if one is found, however, the existence of a counter-example compels us to conclude that the result is not generally correct in a larger class of models.

We have been concerned here only with the two-particle - two-Reggeon vertex. One might well imagine that similar difficulties may affect the counting scheme in more complicated vertices such as the three-Reggeon and four-Reggeon vertex [15)]. This is a natural subject for future investigation. The lack of a simple rule would be unfortunate for a number of applications. For instance, it would render more difficult the treatment of Reggeon cut modifications to inclusive cross-sections [4),8),16)]. Some recent attempts have been made to incorporate more complicated discontinuity rules into a general theory [17)].

It is conceivable that for the two-particle, two-Reggeon vertex, the correct rule may depend on the particle in question. It may well be that the Mandelstam graph could give a reasonable description of the two-deuteron - two-Reggeon coupling, since the anomalous singularities in this case are very close to the region of integration. On the other hand, our dual-resonance model result may be more appropriate for other processes in which the composite nature of the particle in question is not so obvious.

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REFERENCES

1) V.N. Gribov, I.Ya. Pomeranchuk and K.A. Ter Martirosyan - Yadernaya Fizika 2, 361 (1965) [Soviet J.Nuclear Phys. 2, 258 (1965)], Phys.Rev. 139, B184 (1965).

2) C. DeTar - MIT-CTP-421 (1974), Phys.Rev.D to be published.

3) V.N. Gribov, I.Ya. Pomeranchuk and K.A. Ter Martirosyan - Yadernaya Fizika 2, 361 (1965) [Soviet J.Nuclear Phys. 2, 258 (1965)], Phys.Rev. 139, B184 (1965) ;

A.R. White - Nuclear Phys. B50, 93, 130 (1972) ; Phys.Rev. D10, 1236 (1974).

4) V.A. Abramovskii, O.V. Kancheli and V.N. Gribov - XVI International Conference on High Energy Physics, Vol. I, p. 389 (1972) ; Yadernaya Fizika 18, 595 (1973) [Soviet J.Nuclear Phys. 18, 308 (1974)].

5) I.G. Halliday and C.T. Sachrajda - Phys.Rev. D8, 3598 (1973).

6) R.C. Brower, C.E. DeTar and J.H. Weis - MIT-CTP-395 (1974), CERN Preprint TH. 1817 (1974) (Physics Reports, to be published).

7) K. Wilson - Cornell University Report, CNLS-131 (1970) ;

W. Frazer, R. Peccei, S. Pinsky and C.-I. Tan - Phys.Rev. D7, 2647 (1973) ; for other references, see Ref. 2).

8) M. Creutz, F.E. Paige and Pu Shen - Phys.Rev. D9, 684 (1974).

9) O.V. Kancheli - JETP Pis.Red. 18, 465 (1973) [JETP Letters 18, 274 (1973)] ;

E. Lehman and G.A. Winbow - Daresbury Report DL/P206 (1974), Phys.Rev.D, to be published.

10) T. DeGrand - "S Channel Discontinuities and the Two-Reggeon Cut", University of Geneva Preprint (1974).

11) H. Rothe - Phys.Rev. 159, 1471 (1967).

12) For a recent review of the dual resonance model, see :

J.H. Schwarz - Physics Reports 8C, 269 (1973).

13) H.P. Stapp - Phys.Rev. D3, 3177 (1971) ;

K.E. Cahill and H.P. Stapp - Phys.Rev. D6, 1007 (1972) ; Phys.Rev. D8, 2714 (1973).

14) R.J. Eden, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne - "The Analytic S Matrix", Cambridge University Press (1966).

15) E.S. Lehman and G.A. Winbow - Daresbury Report (December 1974).

16) L. Caneschi and R. Jengo - CERN Preprint TH. 1939 (1974).

17) M. Ciafaloni and G. Marchesini - CERN Preprint TH. 1932 (1974).

FIGURE CAPTIONS

Figure 1 — The Mandelstam graph in $\lambda\phi^3$ perturbation theory. The two-body elastic scattering amplitude, obtained by summing over all lengths of both ladders and including the possibility of a single twist in each ladder, contains the two-Reggeon cut.

Figure 2 — a) The three main classes of discontinuities of the Mandelstam graph : a. diffractive, b. absorbed multiperipheral, c. polyperipheral.
b) The corresponding discontinuities of the internal six-point function.

Figure 3 — The generalization of Fig. 2 to an arbitrary amplitude containing a two-Reggeon cut, and the corresponding six-point function that defines the component two-Reggeon, two-particle amplitude. The discontinuities of the six-point function are more readily visualized when the Reggeons are drawn on opposite sides.

Figure 4 — Three classes of production amplitudes which, when multiplied together, yield the corresponding slices of the Mandelstam graph.

Figure 5 — Location of normal threshold singularities of the full two-Reggeon, two-particle amplitude.

Figure 6 — a) Non-overlapping and b) overlapping channels in the discontinuity corresponding to the absorbed multiperipheral slicing.

Figure 7 — The four slicings of a single Reggeon after Abramovskii, Gribov, and Kancheli, and the corresponding discontinuities of the six-particle amplitude.

Figure 8 — Tree graphs in the dual resonance model. Only (c) has a non-vanishing fixed-pole residue.

Figure 9 — The slashed lines indicate propagators whose poles conspire to generate anomalous triangle singularities. These are encountered in the complex M_1^2 plane in vertices sliced through one or both Reggeons.

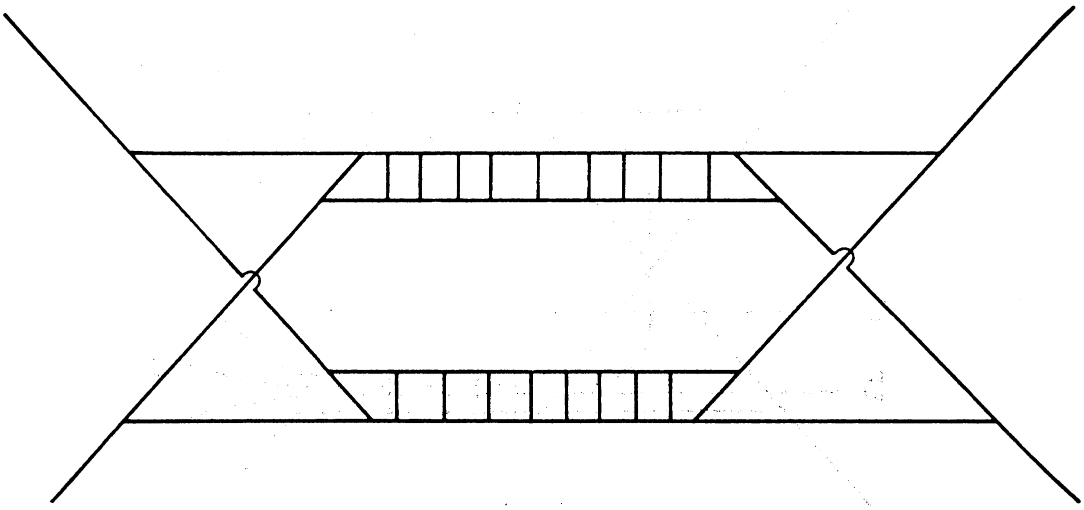


FIG. 1

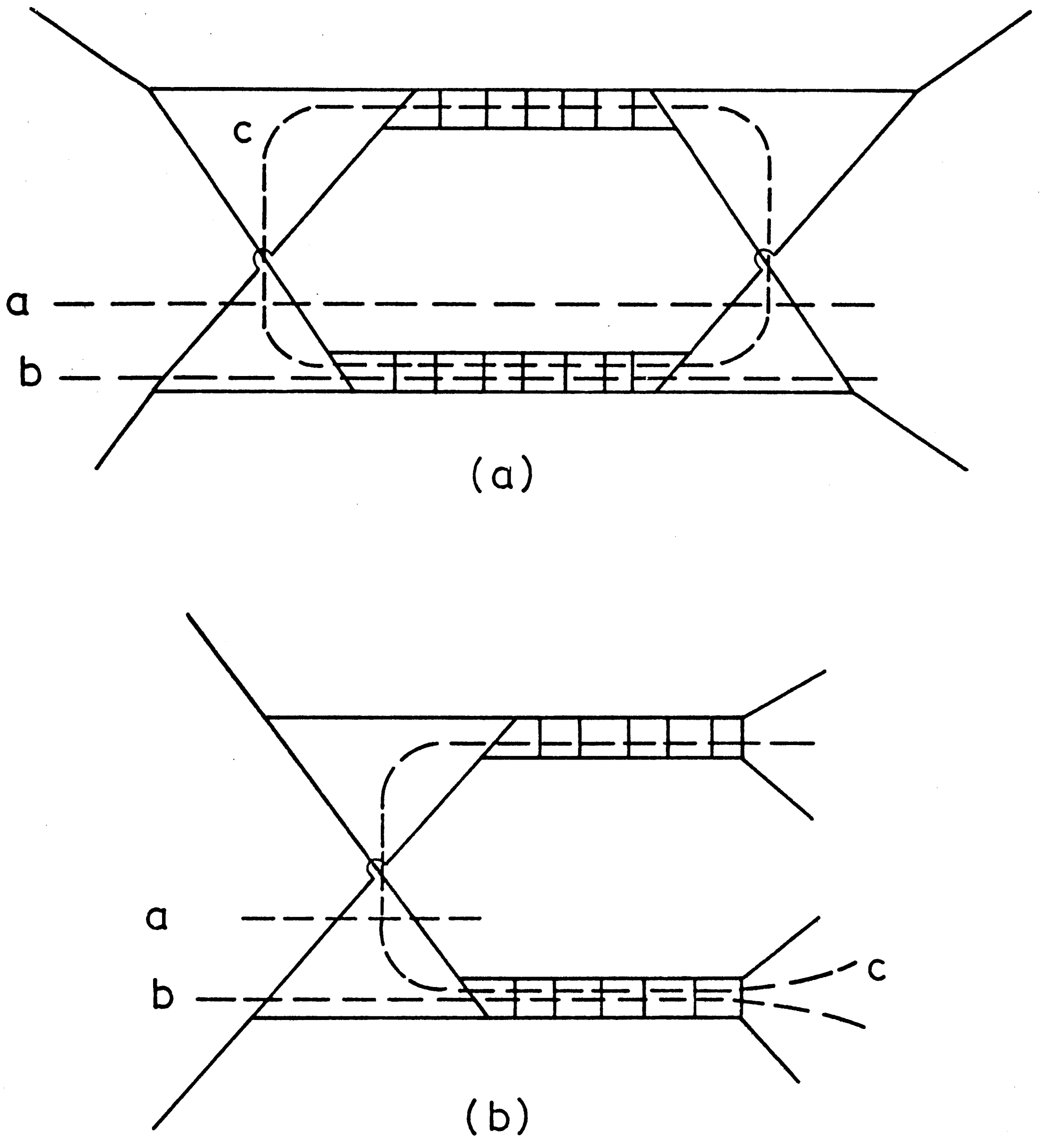


FIG. 2

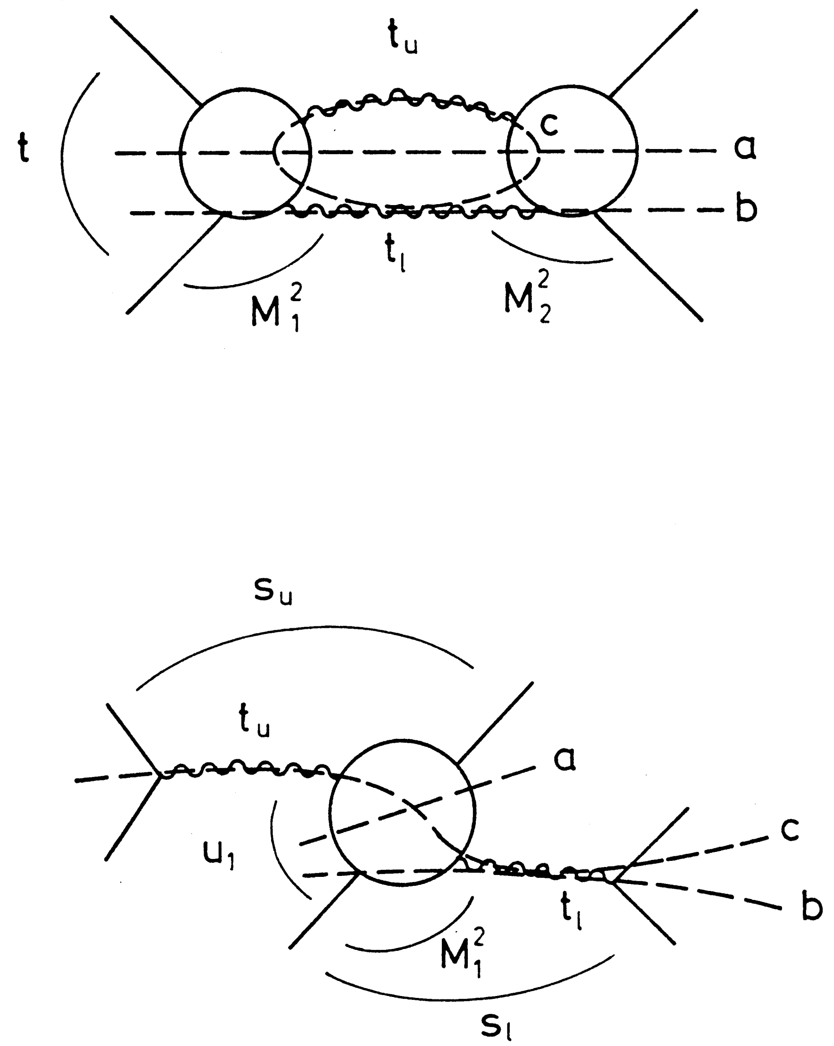


FIG.3

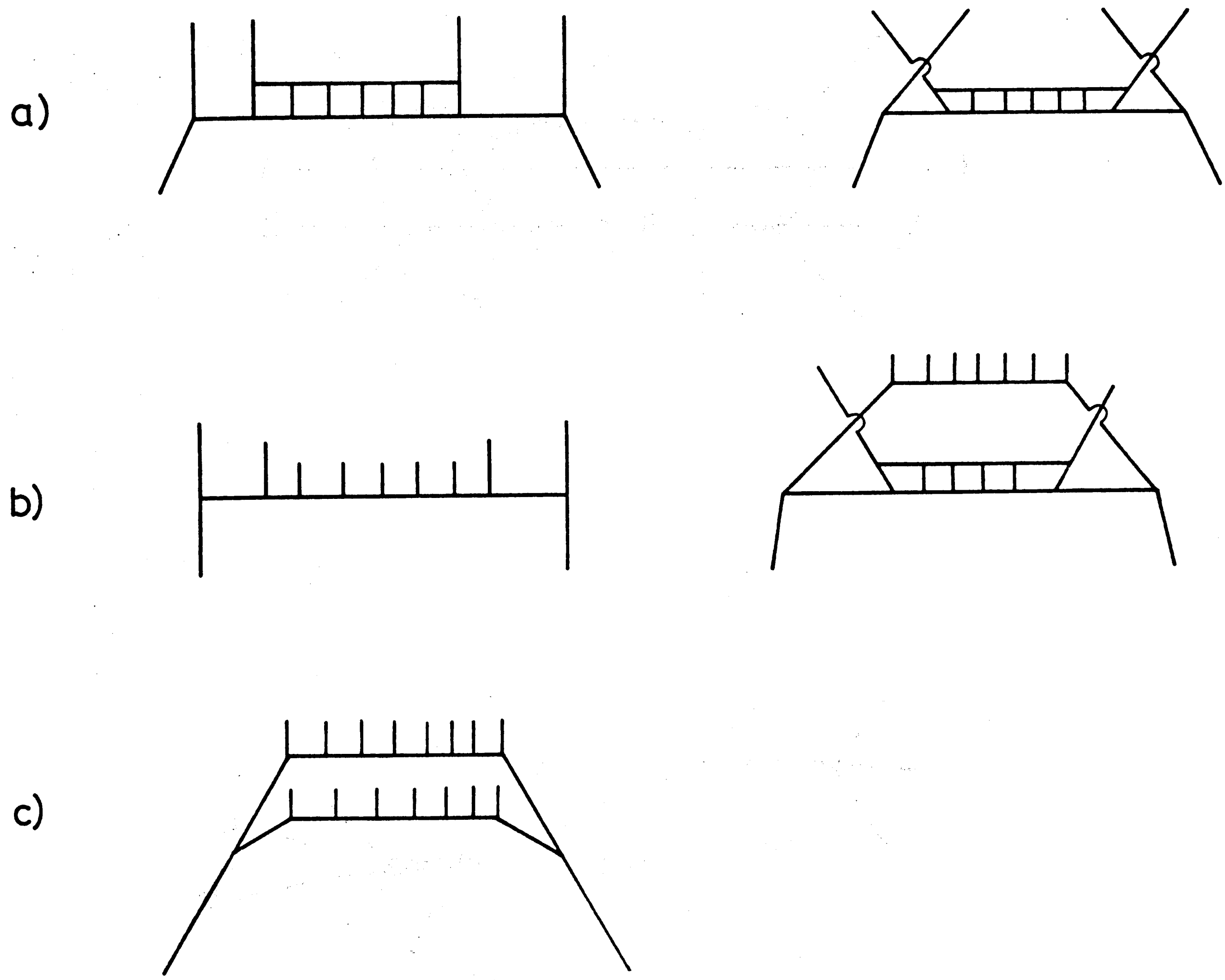


FIG. 4

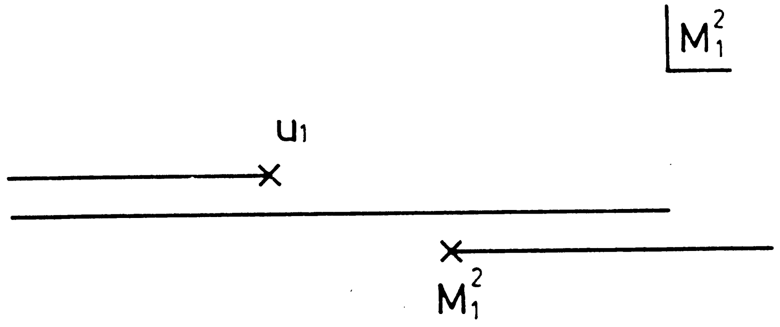


FIG.5

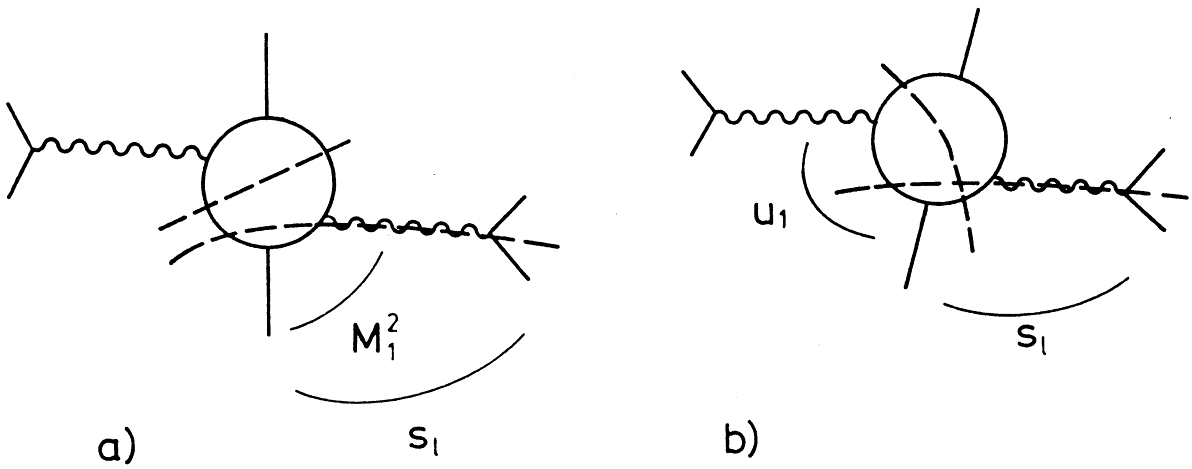
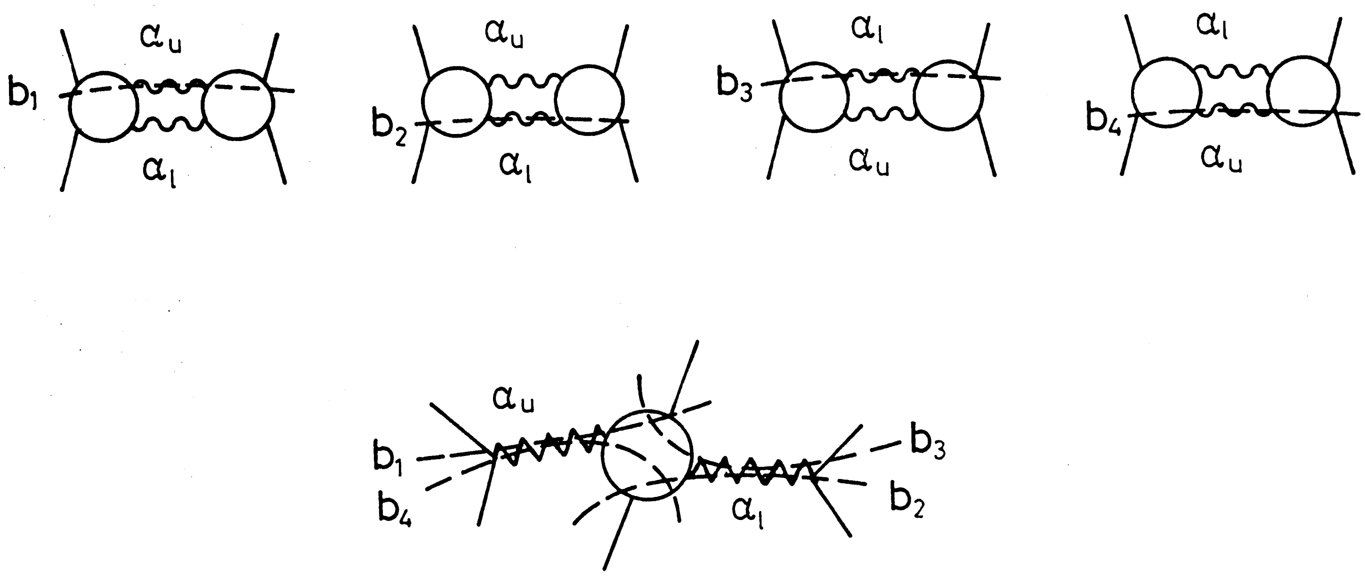
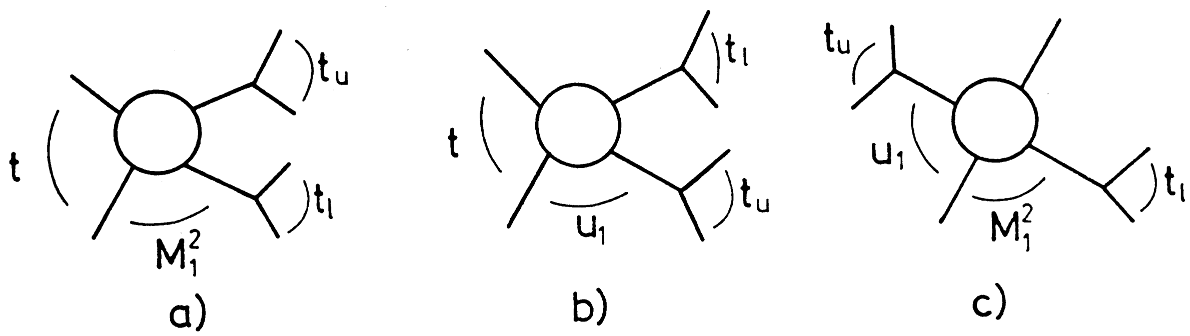


FIG.6



α_u
b_1
α_l
α_u
b_2
α_l
α_l
b_3
α_u
α_l
b_4
α_u
α_u
b_1
b_4
b_3
α_l
b_2

FIG.7



t_u
t
t_l
M_1^2
a)
t_l
t
t_u
u_1
b)
t_u
u_1
t_l
M_1^2
c)

FIG.8

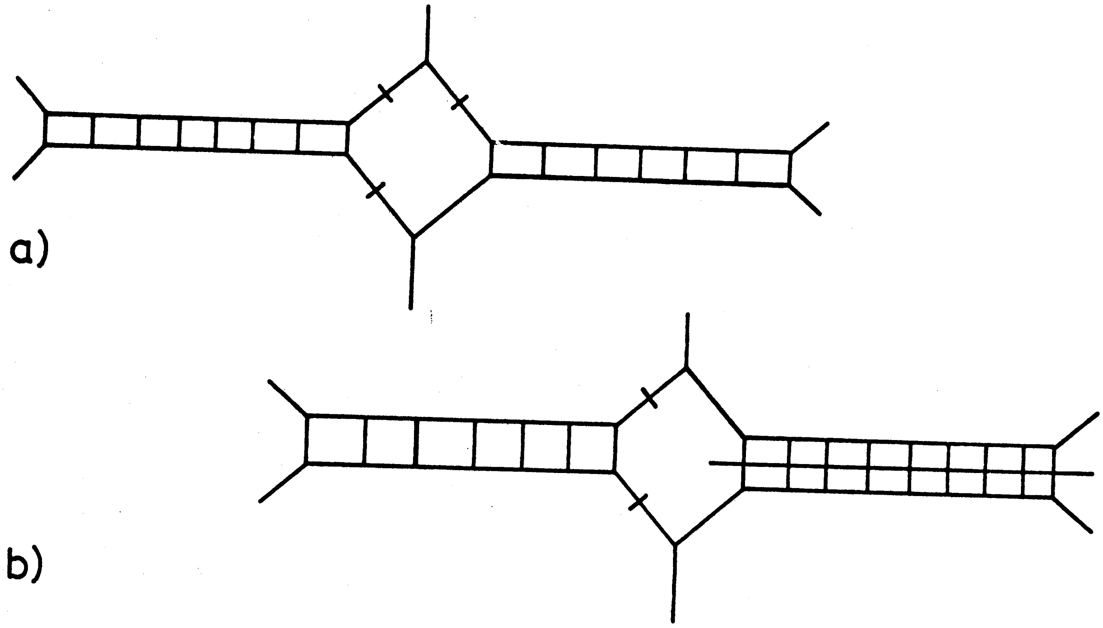


FIG. 9