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A LOWER BOUND ON HEAVY LEPTON PRODUCTION BY NEUTRINOS

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A B S T R A C T

A lower bound on $\sigma(\nu A \to M \ldots)$ is given, in terms of the structure functions which can most easily be measured in $\nu + A \to \mu + \ldots$ reactions, for heavy leptons (M) of the type required by renormalizable models cf weak interactions.

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INTRODUCTION

The old idea [1] that undiscovered "heavy leptons" may exist has recently attracted renewed interest because they are required in a large class of renormalizable models of weak interactions [2]. The most auspicious reaction for producing charged heavy leptons is $e^+e^- \to M^+M^-$ but if sufficiently energetic e^+e^- colliding beams are not available the reaction $\nu_\mu + A \to M^+ + \ldots$ (first considered by Gerstein and Folomeshkin [3]) may provide the best hope of detecting heavy leptons of the type required in renormalizable models (which are the only ones considered in this paper).

In Ref. 2) was estimated $\sigma(\nu + A \to M^+ \ldots)$ using a simple model. While this estimate may be useful for planning experiments, model independent calculations are obviously desirable for interpreting negative experimental results. In principle the hadronic vertex which describes the process $\nu_\mu \to M^+ (\bar{\nu}_\mu \to M^-)$ can be completely determined from $\bar{\nu}_\mu \to \mu^+ \;(\nu_\mu \to \mu^-)$ experiments [4]. In practice, however, the smallness of the muon's mass makes it almost impossible to measure all the relevant structure functions in "ordinary" neutrino experiments. In this note we record a lower bound for heavy lepton production in terms of the structure functions which can be measured (relatively) easily in conventional neutrino reactions.

Before stating the bound, we recall the well-known argument which shows why heavy leptons are needed in many models and why (at least in simple cases) it is the heavy M^+ which has the same lepton number as ν_μ and μ^-, as we assume below (it is this property which gives $M^\pm$ events such a good signature in neutrino reactions). In all models in which the weak interactions are mediated by vector bosons Fig. 1 must exist. On its own this diagram has a "bad" high energy behaviour which renders the corresponding fourth order amplitude for $\nu\bar{\nu} \to \nu\bar{\nu}$ unrenormalizably infinite; in renormalizable models this "bad" behaviour is cancelled by introducing new particle exchanges in the s channel (neutral current) or u channel (heavy lepton) - or both. Note that cancellation can only be achieved using a heavy lepton alone if it is in the u channel (the lepton mass can be neglected to leading order at high energy and two t channel contributions obviously make additive contributions); i.e., we need Fig. 2 and ν_μ, μ^- and M^+ therefore have the same lepton number. Furthermore, in simple models in which the complete amplitude for $\nu\bar{\nu} \to W^+W^-$ is given by μ^- and M^+ exchange alone in lowest order, cancellation requires [5]:

$$(g_{\nu\mu W})^2 = (g_{\nu M W})^2$$

THE BOUND

We are interested in the processes in Fig. 3. The hadronic vertex for a virtual W^+ (W^-) meson of four-momentum q incident on an unpolarized nucleon is described by the tensor:

$$W^{\pm}_{\mu\nu} = \frac{1}{2}\sum_F \overline{\sum} \langle p|J^{\mp}_\mu(0)|F\rangle\langle F|J^{\pm}_\nu(0)|p\rangle (2\pi)^3 \delta^4(q+p-p_F)$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1^{\pm} + \left(p_\mu - \frac{q_\mu \nu}{q^2}\right)\left(p_\nu - \frac{q_\nu \nu}{q^2}\right)\frac{W_2^{\pm}}{M^2} \qquad (1)$$

$$- \frac{i\epsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta}{2M^2} W_3^{\pm} + \frac{q_\mu q_\nu W_4^{\pm}}{M^2}$$

$$+ \frac{(q_\mu p_\nu + q_\nu p_\mu)}{2M^2} W_5^{\pm} + i\frac{(q_\mu p_\nu - q_\nu p_\mu)}{2M^2} W_6^{\pm},$$

$$W_i^{\pm} = W_i^{\pm}(q^2, \nu = q\cdot p),$$

where $\overline{\sum}$ denotes an average over nucleon spins [the metric is such that spacelike q^2 is negative; the normalization etc., is the same as in Ref. 6)]. The differential neutrino cross-section is given by contracting $W_{\mu\nu}$ with the spin averaged lepton tensor and is:

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{G^2}{8M^2\pi E}\Big[2W_1(m^2-q^2)\left(1+\frac{m^2}{2q^2}\right) + W_2\Big\{4EE' + q^2 - m^2 + \frac{E\nu m^2}{Mq^2} - \frac{m^2\nu^2(q^2-m^2)}{q^4M^4}\Big\} + \xi\frac{W_3}{M}\left(E(q^2+m^2) + E'(q^2-m^2)\right) + Y\Big]$$

where

$$Y = \frac{m^2(m^2-q^2)W_4}{M^2} - \frac{2Em^2W_5}{M} \qquad (2)$$

$E(E')$ = lab. energy of neutrino (outgoing lepton).

and for

$$\nu_\mu \rightarrow \mu^- : \quad W_i = W_i^+, \quad \xi = +1.$$

$$\bar{\nu}_\mu \rightarrow \mu^+ : \quad W_i = W_i^-, \quad \xi = -1.$$

$$\nu_\mu \rightarrow M^+ : \quad W_i = W_i^-, \quad \xi = +1.$$

$$\bar{\nu}_\mu \rightarrow M^- : \quad W_i = W_i^+, \quad \xi = -1.$$

for μ production:

$$\frac{G^2}{2} = \frac{g^2_{\nu\mu W}}{(q^2 - M_W^2)^2}$$

for M production:

$$\frac{G^2}{2} = \frac{g^2_{\nu M W}}{(q^2 - M_W^2)^2}$$

The physical region for the reaction is most easily stated in terms of the variables $W = \sqrt{2\nu + q^2 + M^2}$ (the "missing mass") and q^2:

$$M \leqslant W \leqslant \sqrt{s} - m$$

$$(s = (k_\nu + p)^2 = 2ME + M^2)$$

$$(q^2)^{\max}_{\min} = \frac{1}{2s}\Big[2M^2m^2 - (s - M^2)(s - m^2 - W^2) \pm (s - M^2)\sqrt{(s - (W+m)^2)(s - (W-m)^2)}\Big] \tag{3}$$

Note the factor m^2 multiplying $W_{4,5}$ in Eq. (2), because of which $W_{4,5}$ may make important contributions to M production but not to μ production ($W_{4,5}$ can in fact only be separated from $W_{1,2}$ by measuring the outgoing lepton's polarization as well as $d^2\sigma/dq^2 d\nu$). However, the W_i satisfy certain inequalities [7] from which it is straightforward to find a lower bound for the differential cross-section.

To see the origin of the bound it is more convenient to use a slightly different notation. We introduce four independent vectors which form a basis for the current (W meson) helicity states: three of the vectors having spin one, which satisfy $q^\lambda \epsilon_\lambda^{R,L,S} = 0$ and are "right handed" (R), "left handed" (L) and "longitudinal" (S) with respect to $\vec{q}$ in the lab. (and frames connected by Lorentz transforming along $\vec{q}$), and one spin zero ("divergence") vector $\epsilon_\lambda^D = q_\lambda / \sqrt{-q^2}$. We can then form the "W nucleon cross-sections":

$$\sigma_i^{\pm} = C(\nu, q^2)\, \epsilon_\mu^{i*} \epsilon_\nu^{i}\, W_{\mu\nu}^{\pm} \geqslant 0$$

$$\sigma_{DS} = \sigma_{SD}^{*} = C(\nu, q^2)\, \epsilon_\mu^{D*} \epsilon_\nu^{S}\, W_{\mu\nu}^{\pm}$$

where C is a "flux factor" which is rather arbitrary and need not be specified for our purposes. The interference terms satisfy the Schwartz inequality [8]:

$$\sigma_{DS}^2 + \sigma_{SD}^2 \leqslant 2\,\sigma_S\, \sigma_D \,.$$

Suppose the final lepton has a definite spin (denoted by a) with respect to some axis. The lepton current matrix elements can be calculated and written in the form:

$$j_\lambda^a = \sum_\alpha C_\alpha^a\, \epsilon_\lambda^\alpha \,.$$

Then

$$d\sigma^a \sim |C_R^a|^2\sigma_R + |C_L^a|^2\sigma_L + |C_S^a|^2\sigma_S + |C_D^a|^2\sigma_D$$
$$+ \overset{*}{C}{}_S^a C_D^a \sigma_{SD} + \overset{*}{C}{}_D^a C_S^a \sigma_{DS} .$$

The last four terms can be combined in the form

$$\sum_F |C_S^a \epsilon_\lambda^S \langle F|J^\lambda|N\rangle + C_D^a \epsilon_\lambda^D \langle F|J^\lambda|N\rangle|^2 \geq 0.$$

A trivial lower bound can therefore be obtained by neglecting the longitudinal and divergence contributions altogether. Since the contributions of $\sigma_{R,L}$ do not vanish as $m_{\text{lepton}} \to 0$ they can be measured in ordinary ν reactions; this bound therefore has the desired form. Furthermore, it obviously remains true when we sum over lepton spins; however, we can do better in this case. The spin averaged cross-section is given by

$$\sum_a \left(\sum_\alpha |C_\alpha^a|^2 \sigma_\alpha + \frac{1}{2} (\overset{*}{C}{}_S^a C_D^a + \overset{*}{C}{}_D^a C_S^a)(\sigma_{SD} + \sigma_{DS}) \right)$$

There is no contribution from the T violating quantity $\sigma_{SD} - \sigma_{DS}$ because the lepton vertex conserves T and therefore $\sum_a (C_S^{*a} C_D^a - C_D^{*a} C_S^a) = 0$ automatically. We then 1) put $\sigma_D = \epsilon\sigma_S$, $\sigma_{SD} + \sigma_{DS} = 2\eta\sigma_S$, 2) use the minimum value of ϵ allowed by the Schwartz inequality $\epsilon \geq \eta^2$ (in the weaker and more obvious form $\sigma_{SD} + \sigma_{DS} \leq 2\sqrt{\sigma_S\sigma_D}$) and 3) minimize with respect to η. This gives the bound:

$$\sum_a \left(|C_S^a|^2 \sigma_S + |C_D^a|^2 \sigma_D + \frac{1}{2} (\overset{*}{C}{}_S^a C_D^a + \overset{*}{C}{}_D^a C_S^a)(\sigma_{SD} + \sigma_{DS}) \right)$$

$$\geq \sigma_S \left(\sum_a |C_S^a|^2 - \frac{\sum_a (\overset{*}{C}{}_S^a C_D^a + \overset{*}{C}{}_D^a C_S^a)}{4 \sum_a |C_D^a|^2} \right)$$

The bracket is positive semidefinite (this is the Schwartz inequality for the leptonic vertex); this bound is therefore better than that obtained by neglecting σ_S, σ_D and $\sigma_{SD}+\sigma_{DS}$ altogether (unless $\sigma_S = 0$).

We now restate the bounds explicitly in terms of the structure functions [9]:

- A "<u>trivial bound</u>" is given by neglecting longitudinal and divergence contributions i.e., by putting $W_{4,5} = 0$ and

$$W_2\left(1-\frac{\nu^2}{q^2M^2}\right) \longrightarrow W_1$$

in Eq. (2).

- A <u>stronger bound</u> is given by setting

$$Y \geq \frac{m^2\left[\nu(m^2-q^2)+2EMq^2\right]^2}{q^2(m^2-q^2)(\nu^2-M^2q^2)}\left[\left(1-\frac{\nu^2}{M^2q^2}\right)W_2 - W_1\right]$$

In both cases we obtain a lower bound on M production in terms of the functions $W_{1,2,3}$ which can be measured in μ production.

<u>REMARKS</u>

1) To bound $\sigma(\nu p \to M^+)$ $[\sigma(\bar{\nu}p \to M^-)]$ we need data for $d\sigma(\bar{\nu}p \to \mu^+)$ $[d\sigma(\nu p \to \mu^-)]$ in general. If we neglect strangeness changing processes and assume the charge symmetry condition;

$$W_i^+ \begin{pmatrix}\text{proton target}\\ \text{neutron target}\end{pmatrix} = W_i^- \begin{pmatrix}\text{neutron target}\\ \text{proton target}\end{pmatrix},$$

$\sigma(\nu p \to M^+)$ can be bounded in terms of neutrino data alone. However, renormalizable models generally involve "charmed" particles and it is likely that in many of them the charge symmetry condition may be violated. It is therefore dangerous to assume it without further tests.

2) If we assume scale invariance

$$\begin{array}{l} \lim.\ \nu \to \infty \\ x = -\frac{q^2}{2\nu} \text{ fixed} \end{array} \left\{ \begin{array}{l} W_1 \longrightarrow F_1(x) \\ \frac{\nu W_{2,3}}{M^2} \longrightarrow F_{2,3}(x) \end{array} \right.$$

then the bound for $d\sigma$ takes a simple form in the region where the limit is reached:

$$\frac{d^2\sigma}{dx\,dy} \geq \frac{g^2 s}{\pi(yxs+M_W^2)^2} \left\{ F_1(x)\left(2x + x(y^2-2y) + \frac{m^2}{s}(y-2)\right) + \xi \frac{F_3(x)}{2}\left[x(y^2-2y) + \frac{m^2 y}{s}\right] + \left[F_2(x) - 2xF_1(x)\right]\left(1 - y - \frac{m^2}{m^2+yxs}\right)\right\}$$

where

$$0 \leq y = \frac{\nu}{ME} \simeq \frac{2\nu}{s} \leq 1 - \frac{m^2}{sx}$$

$$\frac{m^2}{s} \leq x \leq 1$$

and terms of order M/E have been dropped. The "simple bound" is obtained by neglecting the last term, i.e., by putting $F_2 \to 2xF_1$ (this term is positive semidefinite in the physical region since $F_L = F_2 - 2xF_1 \geq 0$). Putting $M_W \to \infty$ and integrating over y we obtain:

$$\sigma \geq \frac{G^2 ME}{\pi} \int_{m^2/s}^{1} dx \left\{ 2x F_1(x) \left(1 - \frac{m^2}{sx}\right)^2 \left(1 + \frac{1}{12}\left[\frac{m^2}{sx} - 4\right]\right) \right.$$

$$+ \frac{\xi\, x F_3(x)}{12} \left(1 - \frac{m^2}{sx}\right)^2 \left(\frac{m^2}{sx} - 4\right)$$

$$\left. + \left[F_2(x) - 2x F_1(x)\right]\left[\left(1 - \frac{m^2}{sx}\right)\left(\frac{1}{2} + \frac{m^2}{2sx}\right) - \frac{m^2}{xs} \log\left(\frac{xs}{m^2}\right)\right]\right\}$$

A simpler bound may again be obtained by neglecting the last term. If we do this and also assume the charge symmetry condition, we obtain a bound which coincides with the result of the model calculation in Ref. 2) [10] [Eqs. (3.8) and (3.9)], when we make the identification

$$F_1(x) = f(x) + \bar{f}(x)$$

$$F_3(x) = 2\left(\bar{f}(x) - f(x)\right)$$

with the notation used there i.e., the model of Ref. 2) gives a lower bound in all models in which scaling and the charge symmetry condition hold, provided it is a good approximation to put $M_W \to \infty$. However, whenever this bound is needed it should be recalculated using the best data currently available and the complete formula if there is then evidence against the charge symmetry condition or if it turns out that F_L does not vanish .

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FOOTNOTES AND REFERENCES

1) A review of various possible types of heavy leptons and their experimental status has recently been given by M. Perl SLAC-PUB-1062 (1972), [see also Ref. 2)]. The first explicit discussion of heavy leptons in the literature of which I am aware is due to E.M. Lipmanov, JETP 16, 634 (1963) [Russian original: 43, 893 (1962)].

2) A discussion of the properties of the heavy leptons required in renormalizable models, has been given by J.D. Bjorken and C.H. Llewellyn Smith, SLAC-PUB-1107 (1972), to be published in Phys.Rev. Note that the factor $1+(m^2/xs)$ in Eqs. (3.9) and (3.13) should be $1+(m^2/2xs)$ (this does not change Fig. 12 in which the term multiplying this factor was neglected).

3) S.S. Gerstein and V.N. Folomeshkin, Soviet J.Nuclear Phys. 8, 447 (1969) [Russian original: Yadernaya Fisika 8, 768 (1968)].

4) Assuming (as we do in this paper) that there is only one charged vector boson which couples to $\nu_\mu - \mu^-$ ($\bar\nu_\mu - \mu^+$) and $\bar\nu_\mu - M^-$ ($\nu_\mu - M^+$) or that if there are several they couple to $\nu - \mu$ and $\nu - M$ in the same combination.

5) All the models considered in Ref. 2) have this property.

6) C.H. Llewellyn Smith, Phys.Reports 3C, No 6 (1972). [Note that the definition of $W_{4,5}$ in Ref. 6) is different from that used here, which was chosen to simplify the inequalities.]

7) T.D. Lee and C.N. Yang, Phys.Rev. 126, 2239 (1962); M.G. Doncel and E. de Rafael, Nuovo Cimento 4A, 363 (1971). Explicitly the inequalities are

$$\frac{1}{2M^2}\sqrt{\nu^2-M^2q^2}\,|W_3| \leq W_1 \leq \left(1-\frac{\nu^2}{M^2q^2}\right)W_2$$

$$W_5^2+W_6^2 \leq \frac{4M^2}{\nu^2-M^2q^2}\left[\left(1-\frac{\nu^2}{M^2q^2}\right)W_2-W_1\right]\left(-\nu W_5-q^2W_4\right)$$

8) Together with the conditions $\sigma_\alpha \geq 0$ this is equivalent to the inequalities in Footnote 7).

9) Given Eq. (2), it is easier in practice to obtain the stronger bound by working directly with the W_i and the inequalities which they satisfy (we did it with both notations as a check).

10) Apart from the misprint in Ref. 2) mentioned above.

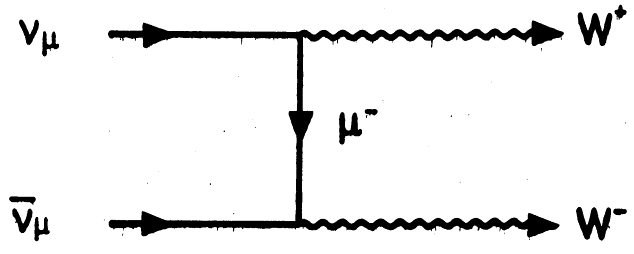

FIG. 1

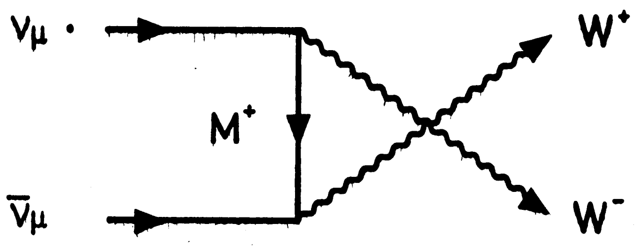

FIG. 2

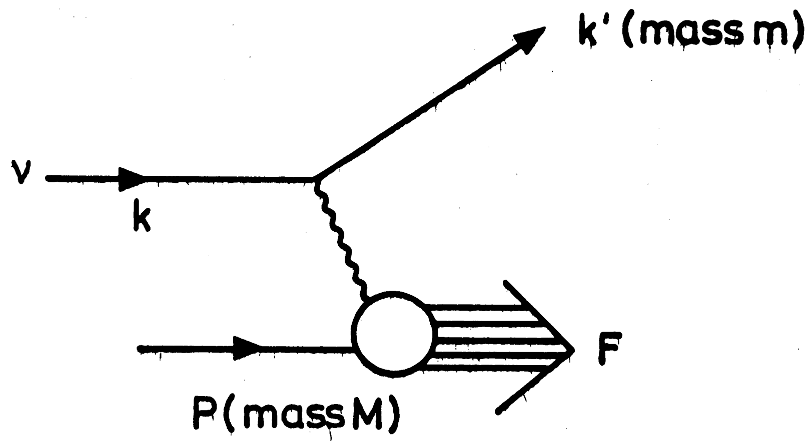

FIG. 3