Ref.TH.1600-CERN

DUALITY CONSTRAINTS ON QUARK PARTON MODEL DESCRIPTION OF DEEP INELASTIC SEMI-INCLUSIVE AND ANNIHILATION PROCESSES

M. Chaichian
University of Bielefeld

and

S. Kitakado *)
CERN - Geneva

ABSTRACT

By imposing the concept of duality on the quark parton model description of deep inelastic semi-inclusive and annihilation processes, relations as well as inequalities among the structure functions are obtained. In particular, the excess of electroproduction of π^+ over π^- off proton target comes out as a consequence and the differential cross-section for $e^+e^- \to \pi$ + anything is fully determined by the electroproduction of pions on nucleon. The discussion is limited to the current fragmentation region.

---

*) Alexander von Humboldt Fellow, on leave of absence from Institute of Physics, College of General Education, University of Tokyo.

Ref.TH.1600-CERN
20 December 1972

The quark parton ideas have been successfully applied to deep inelastic leptonic processes [1]. These ideas have been further used to describe the deep inelastic semi-inclusive as well as annihilation processes [2],[3]. On the other hand, the general constraints of duality, borrowed from strong interaction phenomena, were imposed on total inclusive deep inelastic structure functions, both within the framework of quark parton model [4] and without explicitly assuming it [5]. There, in analogy with hadronic processes, the existence of two components [6] was assumed - one being due to non-exotic contributions in two channels and the other one due to the Pomeron exchange.

In this letter [7] we would like to proceed further and apply similar duality considerations to deep inelastic semi-inclusive reactions in the current fragmentation region and to electron-positron annihilation processes within the framework of the quark parton model of Refs. 2) and 3) (call it QPM).

Consider the reaction

$$a + b \longrightarrow c + \text{anything} \ , \qquad (1)$$

where a is electromagnetic or weak current with a large space-like four momentum q, b is the nucleon target with momentum p and c is the detected particle like π^+, π^-, K^+, K^-, p, n, etc., with momentum h.

As proposed by Feynman [2], in the deep inelastic current fragmentation region [8] defined by

$$q^2 \longrightarrow -\infty \ , \ \nu = \frac{p \cdot q}{M} \longrightarrow \infty \ ; \ x = \frac{-q^2}{2M\nu} \ , \ z = \frac{h \cdot p}{p \cdot q} \ , h_T \text{ finite} \qquad (2)$$

the current, say photon, first interacts with a probability $d_i^b(x)$ with one of the partons of type i which carries a fraction x of the total target momentum p and then this parton, with a probability $D_i^h(z,h_T)$, fragments into a particular hadron h with a fraction z of the parton momentum in the longitudinal direction.

In this way, the structure function $L_{1,h}^{ep}$ denoted simply by L_h^{ep} for the process $\gamma_V + p \rightarrow h$ + anything can be written as

$$L_h^{ep}(x,z,h_T) = \frac{1}{2}\sum_i Q_i^2\, d_i^p(x) D_i^h(z,h_T) \quad , \tag{3}$$

where i runs over all the quarks and antiquarks u, d, s, $\bar{u}$, $\bar{d}$ and $\bar{s}$ and Q_i is the charge of i type quark.

The $d_i^p(x)$ are the parton distribution functions which enter the total deep inelastic structure function $F_1^{ep}(x)$ as

$$F_1^{ep}(x) = \frac{1}{2}\sum_i Q_i^2\, d_i^p(x) \quad . \tag{4}$$

The so-called parton fragmentation functions [2] D_i^h on the other hand describe the annihilation channel $e^+e^- \to h$ + anything as follows :

$$\frac{d\sigma(e^+e^- \to h + X)}{dz\, dh_T} = \frac{8\pi\alpha^2}{3Q^2}\tilde{F}^h(z,h_T) \tag{5}$$

with

$$\tilde{F}^h(z,h_T) = \frac{1}{2}\sum_i Q_i^2\, D_i^h(z,h_T) \quad , \tag{6}$$

where z is the fraction of the parton momentum carried by the observed hadron h and is given now by

$$z = \frac{2h \cdot Q}{Q^2} \quad , \tag{7}$$

where Q is the momentum of the timelike photon.

With the usual duality assumptions and the Pomeron being an isospin singlet in t channel (call it SU_2 duality) the constraints on the parton distribution functions for proton target $d_i(x) \equiv d_i^p(x)$ in (4) have been previously obtained [4),5)]. They are

$$\begin{aligned} d_u(x) &= v_u(x) + S(x) \\ d_d(x) &= v_d(x) + S(x) \\ d_{\bar{u}}(x) &= d_{\bar{d}}(x) = S(x) \\ d_s(x) &= d_{\bar{s}}(x) = S'(x) \end{aligned} \tag{8}$$

where v_u and v_d are the distributions of the valence quarks in the proton corresponding to non-exotic contributions, while $S(x)$ and $S'(x)$ are those of a SU_2 singlet sea of quark-antiquark pairs which are due to Pomeron effects. If we assume Pomeron to be a t channel SU_3 singlet (call it SU_3 duality) we have

$$S'(x) = S(x) \quad . \tag{9}$$

Relations (8) have been used for the semi-inclusive electroproduction of pions in Ref. 9).

In order to impose duality on reaction (1) which, in QPM is given by Eq. (3) and displayed in Fig. 1a, we could :

- either start by applying the duality restrictions on D_i^h for the case of emission of bosons and baryons in the annihilation channel given by (6) and then combine them in Eq. (3) with the results of (8) which have already been derived in deep inelastic scattering,
- or, for the purely non-exotic part, for the case of only bosons as emitted particle, use the duality diagrams directly for the process (1) now given in QPM only by one diagram (Fig. 1a) and include the Pomeron contributions to lower and upper parts of Fig. 1 as a SU_2 or SU_3 singlet exchanges in $p\bar{p}$ and $h\bar{h}$ channels. Both these approaches, for the boson case, give identical results.

After duality is imposed we get for the parton fragmentation functions $D_i^{\pi^+}$ the following results :

$$D_u^{\pi^+} = R^{\pi} + P^{\pi} \, , \; D_d^{\pi^+} = P^{\pi} \;\; \text{and} \;\; D_s^{\pi^+} = P'^{\pi} \; . \tag{10}$$

SU_3 duality gives

$$P'^{\pi} = P^{\pi} \quad . \tag{11}$$

R^{π} and P^{π} correspond to the fragmentation of the quarks which finally end up in the valence quarks of the observed hadron and the sea and in turn correspond to resonance and Pomeron contributions respectively.

For the emission of nucleons we get

$$
\begin{aligned}
D_u^p &= R_u^p + P^p \\
D_d^p &= R_d^p + P^p \\
D_{\bar{u}}^p &= D_{\bar{d}}^p = P^p \\
D_s^p &= D_{\bar{s}}^p = P'^p
\end{aligned}
\tag{12}
$$

The notations and the complete analogy between (8) and (12) are obvious. SU_3 duality implies $P'^p = P^p$.

By utilizing the SU_2 duality relations (8) for the parton distribution functions in Eq. (3), we obtain the following sum rules :

$$
\begin{aligned}
5\left(L_{\pi^+}^{\nu n} - L_{\pi^+}^{\nu p}\right) &= 6\left[4\left(L_{\pi^+}^{ep} - L_{\pi^+}^{en}\right) + L_{\pi^-}^{ep} - L_{\pi^-}^{en}\right] \\
5\left(L_{\pi^-}^{\nu n} - L_{\pi^-}^{\nu p}\right) &= 6\left[4\left(L_{\pi^-}^{ep} - L_{\pi^-}^{en}\right) + L_{\pi^+}^{ep} - L_{\pi^+}^{en}\right]
\end{aligned}
\tag{13}
$$

The difference of these two sum rules does not depend on duality constraints and is generally valid in the QPM.

If we further integrate over x equations of the type (3) and exploit the relations

$$
\begin{aligned}
\int_0^1 \left[d_u(x) - d_{\bar{u}}(x)\right] dx &= 2 \\
\int_0^1 \left[d_d(x) - d_{\bar{d}}(x)\right] dx &= 1
\end{aligned}
\tag{14}
$$

we obtain $D_u^{\pi^+}$ and $D_d^{\pi^+}$ in terms of pion electroproduction structure functions integrated over x. By additional use of duality relations (10) and (11) for the functions $D_i^{\pi^+}$ the differential cross-section for $e^+e^- \to \pi$ + anything is then fully determined by the electroproduction of pions on nucleon :

$$
\frac{d\sigma(e^+e^- \to \pi^{\pm}_{0} + X)}{dz\, dk_T} = \frac{4\pi\alpha^2}{3Q^2} \cdot \frac{2}{\tau} \int_0^1 \left(13 L_{\pi^+}^{ep} + 15 L_{\pi^-}^{ep} - 14 L_{\pi^+}^{en} - 14 L_{\pi^-}^{en}\right) dx \tag{15}
$$

Within the framework of QPM one can obtain inequalities among the semi-inclusive structure functions analogous to Nachtmann [10] ones since all the probability functions d_i and D_i^h are positive quantities. In addition, they satisfy the isospin inequalities [10]:

$$2d_u - d_d \geq 0 \ , \qquad 2d_{\bar{d}} - d_{\bar{u}} \geq 0 \tag{16}$$

and similarly D_i^h satisfy

$$2D_u^p - D_d^p \geq 0 \ , \qquad 2D_{\bar{d}}^p - D_{\bar{u}}^p \geq 0 \ , \qquad 3D_u^{\pi^+} - D_d^{\pi^+} \geq 0 \ , \ \text{etc.} \tag{17}$$

The resulting inequalities are not so stringent, e.g., they cannot explain the excess of π^+ over π^- in electroproduction (see the Table).

However, if we apply the duality constraints (8), (10) and (12) together with the assumption of separate positivity for non-exotic and the Pomeron components in Eqs. (8), (10) and (12), we obtain much more stringent results which are displayed in the second rows of the Tables. The experimentally observed [11] excess of π^+ electroproduction off proton over π^- now comes about as a consequence of duality imposed on D_i^π and the similar excess of π^+ is predicted for neutrino reactions.

The inequalities among the integrated over x structure functions are still stronger. We present some of them in the third row of the Table.

Finally, we would like to make some remarks on relations which can be obtained near the boundary values of the kinematical variables $x \to 1$ or $z \to 1$ when we make use of the duality constraints on d_i and D_i^h [Eqs. (8) and (10)]. If the experimental ratio $(F^{en})/(F^{ep})$ actually approaches its boundary value $1/4$ as $x \to 1$, then one can conclude that

$$\frac{v_d(x)}{v_u(x)} \to 0 \ , \ \frac{S(x)}{v_u(x)} \to 0 \ , \ \frac{S'(x)}{v_u(x)} \to 0 \quad \text{as } x \to 1 \tag{18}$$

Assuming (18) to be an experimental fact, we expect the following relations to be valid as $x \to 1$:

$$\frac{L^{en}_{\pi^-}}{L^{ep}_{\pi^+}} \to \frac{1}{4}\ , \quad \frac{L^{en}_{\pi^+}}{L^{ep}_{\pi^-}} \to \frac{1}{4}\ , \quad \frac{L^{en}_{n}}{L^{ep}_{p}} \to \frac{1}{4} \quad \text{and} \quad \frac{L^{en}_{p}}{L^{ep}_{n}} \to \frac{1}{4}\ . \qquad (19)$$

$$(\text{for all } z)$$

As $z \to 1$, which is the approach to the threshold, one expects the Pomeron to become less important than the resonance component and then from the duality relation (10) one may conclude that [in this connection, see a similar remark in Ref. 2] :

$$\frac{D^{\pi^+}_{d}}{D^{\pi^+}_{u}} \to 0\ , \quad \frac{D^{\pi^+}_{s}}{D^{\pi^+}_{u}} \to 0 \quad \text{as } z \to 1\ . \qquad (20)$$

From (20) one expects the following relation to be valid as $z \to 1$:

$$\frac{L^{en}_{\pi^-} - L^{ep}_{\pi^-}}{L^{ep}_{\pi^+} - L^{en}_{\pi^+}} \to \frac{1}{4} \qquad (\text{for all } x)\ .$$

If both $x \to 1$ and $z \to 1$ we obtain relations like

$$\frac{L^{ep}_{\pi^-}}{L^{ep}_{\pi^+}} \to 0\ , \quad \frac{L^{en}_{\pi^+}}{L^{en}_{\pi^-}} \to 0 \quad \text{and} \quad \frac{L^{ed}_{\pi^-}}{L^{ed}_{\pi^+}} \to \frac{1}{4}\ .$$

ACKNOWLEDGEMENTS

It is our pleasure to thank A.J.G. Hey, S. Pallua, B. Renner and H. Satz for critical discussions on the significance of these results and B. Renner for reading the manuscript. We would like to thank the Theoretical Study Division of CERN for hospitality.

One of us (S.K.) wishes to thank H. Satz for the hospitality extended to him at Bielefeld. He would like to acknowledge the Alexander von Humboldt-Stiftung for financial support.

Model \ Ratios of L's	$\frac{ep \to \pi^-}{ep \to \pi^+}$	$\frac{en \to \pi^-}{ep \to \pi^+}$	$\frac{ed \to \pi^-}{ed \to \pi^+}$	$\frac{en \to \pi^+}{ep \to \pi^+}$	$\frac{en \to \pi^-}{ep \to \pi^-}$	$\frac{(en \to \pi^+) + (en \to \pi^-)}{(ep \to \pi^+) + (ep \to \pi^-)}$	$\frac{\nu p(n) \to \pi^-}{\nu p(n) \to \pi^+}$	$\frac{\nu d \to \pi^+}{ed \to \pi^+}$	$\frac{\nu d \to \pi^-}{ed \to \pi^-}$	$\frac{(\nu d \to \pi^+) + (\nu d \to \pi^-)}{(ed \to \pi^+) + (ed \to \pi^-)}$
Quark Parton Model	$0 \leqq r \leqq 8$	$\frac{1}{4} \leq r \leq 4$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r < 12$	$\frac{1}{12} \leq r < \infty$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 3$	$0 \leq r \leq 18$	$0 \leq r \leq \frac{54}{7}$	$0 \leq r \leq \frac{18}{5}$
QPM constrained by duality	$0 \leq r \leq 1$	$\frac{1}{4} \leq r \leq \frac{3}{2}$	$\frac{1}{4} \leq r \leq 1$	$0 \leq r \leq 2$	$\frac{1}{4} \leq r < \infty$	$\frac{1}{4} \leq r \leq \frac{3}{2}$	$0 \leq r \leq 1$	$3^* \leq r \leq \frac{36}{5}$	$0 \leq r \leq \frac{18}{5}$	$3^* \leq r \leq \frac{18}{5}$
QPM + duality for L integrated over x	$\frac{1}{8} \leq r \leq 1$	$\frac{1}{4} \leq r \leq 1$	$\frac{1}{4} \leq r \leq 1$	$\frac{1}{2} \leq r \leq 1$	$\frac{2}{3} \leq r \leq 2$	$\frac{2}{3} \leq r \leq 1$	$0 \leq r \leq 1$	$3^* \leq r \leq \frac{36}{5}$	$0 \leq r \leq \frac{18}{5}$	$3^* \leq r \leq \frac{18}{5}$

Model \ Ratios of L's or $\tilde{F}$'s	$\frac{ep \to n}{ep \to p}$	$\frac{ed \to n}{ed \to p}$	$\frac{en \to p}{ep \to p}$	$\frac{ep \to K^-}{ep \to K^+}$	$\frac{ed \to K^-}{ed \to K^+}$	$\frac{e^+e^- \to K^0}{e^+e^- \to K^+}$	$\frac{\nu d \to p}{ed \to p}$	$\frac{(\nu d \to p) + (\nu d \to n)}{(ed \to p) + (ed \to n)}$
Quark Parton Model	$0 \leq r \leq 8$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 8$	$0 \leq r \leq \infty$	$0 \leq r \leq \infty$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 18$	$0 \leq r \leq \frac{18}{5}$
QPM constrained by duality	$0 \leq r \leq 2$	$\frac{1}{4} \leq r \leq \frac{3}{2}$	$0 \leq r \leq 2$	$0 \leq r \leq 1$	$0 \leq r \leq 1$	$\frac{2}{5} \leq r \leq 1$	$3^* \leq r \leq \frac{18}{5}$	$3^* \leq r \leq \frac{18}{5}$

Table 1: Bounds on semi-inclusive electroproduction of pions, nucleons and kaons in current fragmentation region and on e^+e^- - annihilation processes. Letter d stands for the sum of proton and neutron e.g. $(ed \to \pi^-) \equiv (ep \to \pi^-) + (en \to \pi^-)$, etc. By duality in this table we imply SU_2 duality on both d_i and D_i^h. However a number with star indicates that it has been derived from SU_3 duality.

REFERENCES

1) R.P. Feynman - Proceedings of the Third High Energy Conference at Stony Brook, Gordon and Breach (1970) ;

J.D. Bjorken and E.A. Paschos - Phys.Rev. 185, 1975 (1969).

2) R.P. Feynman - Talk presented at the Neutrino Conference, Balatonfüred, Hungary (June 1972).

3) S.M. Berman, J.D. Bjorken and J.B. Kogut - Phys.Rev. D4, 3388 (1971).

4) P.V. Landshoff and J.C. Polkinghorne - Nuclear Phys. B28, 225 (1971).

5) M. Chaichian, S. Kitakado, S. Pallua, B. Renner and J. De Azcarraga - DESY Preprint 72/50 (to be published in Nuclear Phys.).

6) H. Harari - Phys.Rev.Letters 22, 1395 (1968) ;

P.G.O. Freund - Phys.Rev.Letters 22, 235 (1968).

7) A more general approach is being studied by H.J.G. Hey, S. Pallua, B. Renner and us. We hope to be able to report the results in a future publication.

8) We use the notations of :

M. Gronau, F. Ravndal and Y. Zarmi - Caltech Preprint CALT-68-367.

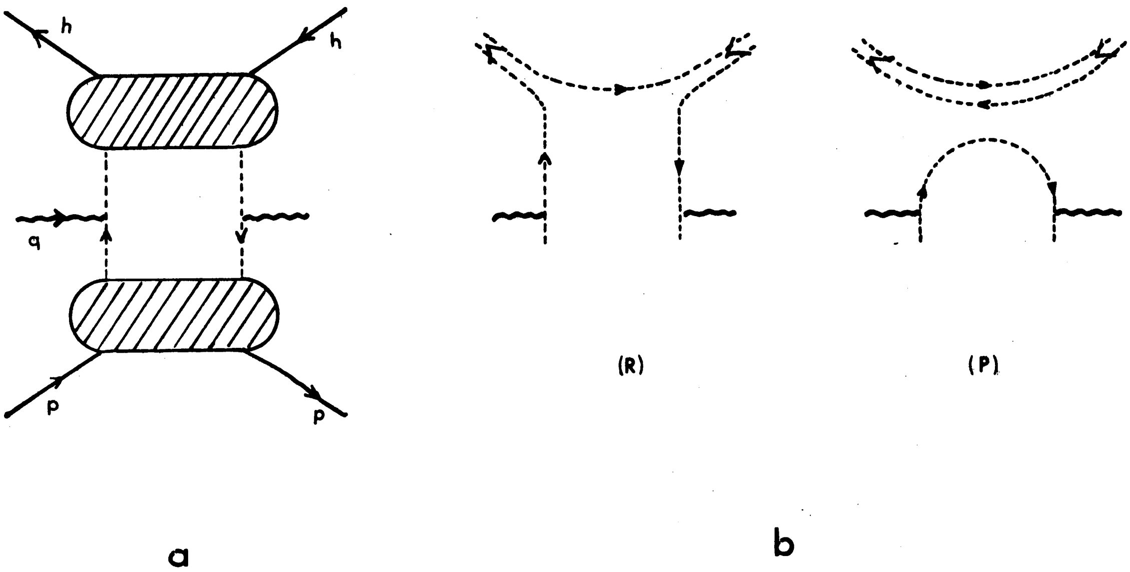
9) S. Pallua and Y. Zarmi - CERN Preprint Ref. TH. 1587 (1972).

10) O. Nachtmann - Nuclear Phys. B38, 397 (1972).

11) I. Dammann et al. - DESY Report, Contribution to XVI International Conference on High Energy Physics, NAL, Batavia (1972).

FIGURE CAPTION

The QPM diagram for deep inelastic semi-inclusive reactions in the current fragmentation region and the interpretation of the upper part in terms of duality diagrams [for the lower part, see Ref. 5].


h
h
q
p
p
(R)
(P)
a
b