CERN-PH-TH/2006-102
MAN/HEP/2006/18
hep-ph/0605288
May 2006

Higgs Phenomenology with CPsuperH

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ABSTRACT

The MSSM contains CP-violating phases that may have important observable effects in Higgs physics. We review recent highlights in Higgs phenomenology obtained with the code `CPsuperH`, a useful tool for studies of the production, mixing and decay of a coupled system of the neutral Higgs bosons at future high energy colliders such as the LHC, ILC (γLC), and a muon collider (MC). `CPsuperH` implements the constraints from upper limits on electric dipole moments, and may be extended to include other related low-energy observables, such as $b \to s\gamma$ and $B \to K l\, l$, and to compute the relic abundance of the lightest neutralino.

1 Introduction

CP violation is one of the most sensitive probes for new physics beyond the Standard Model, and some additional source of CP violation is needed to account for cosmological baryogenesis. Ongoing experiments provide no indications of any flavour or CP-violating physics beyond the CKM mixing paradigm. Nevertheless, B-factories and the Tevatron collider are constantly refining their probes of the CKM mechanism, and the LHC and other TeV-scale colliders will open up new vistas in the study of CP violation. Supersymmetry is one of the more attractive extensions of the Standard Model, and there are many possible supplementary sources of CP violation even within the minimal supersymmetric extension of the Standard Model (MSSM). These are reduced to just two if one assumes universal soft supersymmetry-breaking parameters, which would be sufficient to facilitate baryogenesis at the electroweak scale. They would also have a wealth of implications for many different areas in particle-physics phenomenology, notably through radiative corrections in the Higgs sector [1–7]. CP-violating effects may play a prominent role in Higgs production at future high energy colliders, such as the LHC [8–25], ILC [26, 27], γLC [28–36] and MC [37–45]. In addition, CP-violating phenomena mediated by Higgs-boson exchanges may manifest themselves in a number of low-energy observables such as electric dipole moments [46–61]. The CP phases of the MSSM may also affect flavour-changing neutral-current processes and CP asymmetries involving K and B mesons [62–85]. Moreover, CP-violating Higgs effects may influence the annihilation rates of cosmic relics and hence the abundance of dark matter in the Universe [86–89]. Finally, we recall that an accurate determination of the Higgs spectrum in the presence of CP violation is crucial for testing the viability of electroweak baryogenesis in the MSSM [90–102]

The `Fortran` code `CPsuperH` [103] is a powerful and efficient computational tool for understanding quantitatively such phenomenological subjects within the framework of the MSSM with explicit CP violation. It calculates the mass spectrum and decay widths of the neutral and charged Higgs bosons in the most general MSSM including CP-violating phases. In addition, it computes all the couplings of the neutral Higgs bosons $H_{1,2,3}$ and the charged Higgs boson H^+. The program is based on the results obtained in Refs. [104–106] and the most recent renormalization-group-improved effective-potential approach, which includes dominant higher-order logarithmic and threshold corrections, b-quark Yukawa-coupling resummation effects, and Higgs-boson pole-mass shifts [6, 7] The masses and couplings of the charged and neutral Higgs bosons are computed at a similar high-precision level. Even in the CP-conserving case, `CPsuperH` is unique in computing the neutral and charged Higgs-boson couplings and masses with equally high levels of precision, and is therefore a useful tool for the study of MSSM Higgs phenomenology at present and future colliders.

2 CPsuperH

The tarred and gzipped program file `CPsuperH.tgz` can be downloaded from [1]:

`http://www.hep.man.ac.uk/u/jslee/CPsuperH.html`

When running `CPsuperH`, the main numerical output is stored in arrays. These include the masses of the three neutral Higgs bosons, labelled in order of increasing mass such that $M_{H_1} \leq M_{H_2} \leq M_{H_3}$. Since the neutral pseudoscalar Higgs bosons mixes with the neutral scalars in the presence of CP violation, the charged Higgs-boson pole mass $M_{H^\pm}$ is used as an input parameter. `CPsuperH` also yields the 3×3 Higgs mixing matrix, $O_{\alpha i}$: $(\phi_1, \phi_2, a)^T_\alpha = O_{\alpha i}(H_1, H_2, H_3)^T_i$ and all the couplings of the neutral and charged Higgs bosons. These include Higgs couplings to leptons, quarks, neutralinos, charginos, stops, sbottoms, staus, tau sneutrinos, gluons, photons, and massive vector bosons, as well as Higgs-boson self-couplings. We note that, in addition to quantities in the Higgs sector, including decay widths and branching fractions, `CPsuperH` also calculates and stores the masses and mixing matrices of the stops, sbottoms, staus, charginos, and neutralinos. For a full description, we refer to Ref. [103].

3 Collider Signatures

To analyze CP-violating phenomena in the production, mixing and decay of a coupled system [107] of multiple CP-violating MSSM neutral Higgs bosons at colliders, we need a "full" 3×3 propagator matrix $D(s)$, given by [108]

$$D(\hat{s}) = \hat{s} \begin{pmatrix} \hat{s} - M^2_{H_1} + i\Im\mathrm{m}\widehat{\Pi}_{11}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{12}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{13}(\hat{s}) \\ i\Im\mathrm{m}\widehat{\Pi}_{21}(\hat{s}) & \hat{s} - M^2_{H_2} + i\Im\mathrm{m}\widehat{\Pi}_{22}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{23}(\hat{s}) \\ i\Im\mathrm{m}\widehat{\Pi}_{31}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{32}(\hat{s}) & \hat{s} - M^2_{H_3} + i\Im\mathrm{m}\widehat{\Pi}_{33}(\hat{s}) \end{pmatrix}^{-1}, \tag{1}$$

where $\hat{s}$ is the center-of-mass energy squared, $M_{H_{1,2,3}}$ are the one-loop Higgs-boson pole masses, and the absorptive parts of the Higgs self-energies $\Im\mathrm{m}\widehat{\Pi}_{ij}(\hat{s})$ receive contributions from loops of fermions, vector bosons, associated pairs of Higgs and vector bosons, Higgs-boson pairs, and sfermions. The computed resummed Higgs-boson propagator matrix has been stored in the array `DH(3,3)`.

The so called tri-mixing scenario has been taken for studying the production, mixing and decay of a coupled system of the neutral Higgs bosons at colliders. This scenario is characterized by a large value of $\tan\beta = 50$ and the light charged Higgs boson $M_{H^\pm} = 155$

[1] Some new features appearing in this write-up, for example, the propagator matrix `DH(3,3)` and some low-energy observables, will be implemented in a forthcoming version of `CPsuperH`.

GeV. All the three-Higgs states mix significantly in this scenario in the presence of CP-violating mixing. Without CP violation, only two CP-even states mix. For details of the scenario, see Refs. [108–111].

3.1 LHC

At the LHC, the matrix element for the process $g(\lambda_1)g(\lambda_2) \to H \to f(\sigma)\bar{f}(\bar{\sigma})$ can conveniently be represented by the helicity amplitude

$$\mathcal{M}^{gg}(\sigma\bar{\sigma};\lambda_1\lambda_2) \;=\; \frac{\alpha_s\, g_f\, \sqrt{\hat{s}}\,\delta^{ab}}{4\pi v}\langle\sigma;\lambda_1\rangle_g \delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2}\,, \tag{2}$$

where a and b are indices of the SU(3) generators in the adjoint representation and σ, $\bar{\sigma}$, and $\lambda_{1,2}$ denote the helicities of fermion, antifermion, and gluons, respectively. The amplitude $\langle\sigma;\lambda\rangle_g$ is defined as

$$\langle\sigma;\lambda\rangle_g \equiv \sum_{i,j=1,2,3} [S_i^g(\sqrt{\hat{s}}) + i\lambda P_i^g(\sqrt{\hat{s}})]\; D_{ij}(\hat{s})\; (\sigma\beta_f g^S_{H_j\bar{f}f} - i g^P_{H_j\bar{f}f})\,, \tag{3}$$

where

$$\begin{aligned} S_i^g(\sqrt{\hat{s}}) &= \sum_{f=b,t} g_f g^S_{H_i\bar{f}f}\,\frac{v}{m_f} F_{sf}(\tau_f) - \sum_{\tilde{f}_j=\tilde{t}_1,\tilde{t}_2,\tilde{b}_1,\tilde{b}_2} g_{H_i\tilde{f}_j^*\tilde{f}_j}\,\frac{v^2}{4m^2_{\tilde{f}_j}} F_0(\tau_{\tilde{f}_j})\,, \\ P_i^g(\sqrt{\hat{s}}) &= \sum_{f=b,t} g_f g^P_{H_i\bar{f}f}\,\frac{v}{m_f} F_{pf}(\tau_f)\,, \end{aligned} \tag{4}$$

where $\beta_f = \sqrt{1-4m_f^2/\hat{s}}$, $\tau_x = \hat{s}/4m_x^2$ and $\hat{s}$-dependent S_i^g and P_i^g are scalar and pseudoscalar form factors[2], respectively. The Higgs-boson couplings to quarks $g^{S,P}_{H_i\bar{f}f}$ and squarks $g_{H_i\tilde{f}_j^*\tilde{f}_j}$, and the explicit forms of the functions $F_{sf,pf,0}$ are coded in `CPsuperH` [103]. When $f = \tau\,, t\,, \chi^0\,, \chi^\pm$, etc, one can construct CP asymmetries in the longitudinal and/or transverse polarizations of the final fermions which can be observed at the LHC. The CP asymmetries can be defined similarly in the case of other production mechanisms such as b-quark and weak-boson fusions.

When $\tan\beta$ is large, b-quark fusion is an important mechanism for producing neutral Higgs bosons. In the centre-of-mass coordinate system for the $b\bar{b}$ pair, the helicity amplitudes for the process $b(\lambda)\,\bar{b}(\bar{\lambda}) \to H \to f(\sigma)\bar{f}(\bar{\sigma})$ are given by

$$\mathcal{M}^{b\bar{b}}(\sigma\bar{\sigma};\lambda\bar{\lambda}) \;=\; -\frac{g\, m_b\, g_f}{2M_W}\,\langle\sigma;\lambda\rangle_b \delta_{\sigma\bar{\sigma}}\delta_{\lambda\bar{\lambda}}\,, \tag{5}$$

where

$$\langle\sigma;\lambda\rangle_b \;\equiv\; \sum_{i,j=1,2,3} (\lambda\beta_b\, g^S_{H_i\bar{b}b} + i g^P_{H_i\bar{b}b})\; D_{ij}(\hat{s})\; (\sigma\beta_f\, g^S_{H_j\bar{f}f} - i g^P_{H_j\bar{f}f})\,. \tag{6}$$

[2]These $\hat{s}$-dependent gluon-gluon-Higgs couplings are stored in arrays `SGLUE(3)` and `PGLUE(3)`.

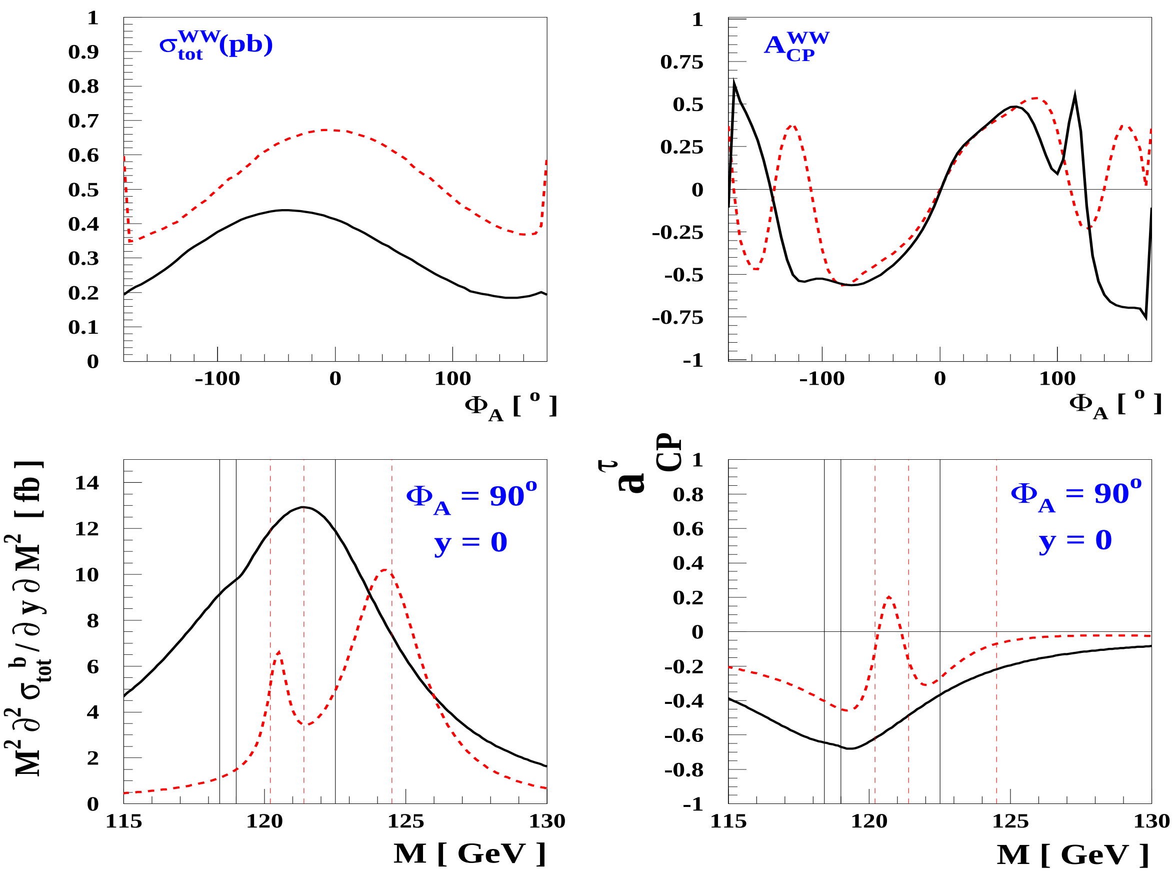

Figure 1: *The cross section $\sigma^{WW}_{\rm tot}[pp(WW)\to\tau^+\tau^-X]$ (upper-left panel) at the LHC and its associated total CP asymmetry $\mathcal{A}^{WW}_{\rm CP}\equiv[\sigma^{WW}_{RR}-\sigma^{WW}_{LL}]/[\sigma^{WW}_{RR}+\sigma^{WW}_{LL}]$ (upper-right panel) as functions of $\Phi_A=\Phi_{A_t}=\Phi_{A_b}=\Phi_{A_\tau}$, where $\sigma^{WW}_{RR,LL}\equiv\sigma(pp\,(WW)\to\tau^+_{R,L}\tau^-_{R,L}X)$. We have considered a tri-mixing scenario with $\Phi_3=-10^\circ$ (dotted lines) and -90° (solid lines). For details of the scenario and the CP asymmetry, see [108]. The lower frames are for Higgs bosons produced in diffractive collisions at the LHC. The lower-left frame shows the hadron-level cross sections when the Higgs bosons decay into b quarks, as functions of the invariant mass M with $\Phi_A=90^\circ$ and rapidity $y=0$. The vertical lines indicate the three Higgs-boson pole-mass positions. The lower-right frame shows the CP-violating asymmetry when the Higgs bosons decay into τ leptons. See [110] for details.*

For the tri-mixing scenario, b-quark fusion is the main production mechanism, and dilutes the possibly large CP asymmetry in longitudinally-polarized gluon fusion when $f=\tau$. This dilution is basically due to the dominance of the b-quark loop contribution to the self energies and the experimental impossibility of distinguishing gluon and b-quark fusion events. In this case, the most promising channel for probing Higgs-sector CP violation may be the weak-boson fusion process, which can be separated from gg and $b\bar{b}$ collisions by applying a number of kinematic cuts [112], including the imposition of a veto on any hadronic activity between the jets [113–117]: $W^+W^-\to H_{1,2,3}\to\tau^+\tau^-$. The cross section $\sigma[pp(W^+W^-)\to H\to\tau^+\tau^-X]$ lies between 0.2 and 0.6 pb and the CP asymmetry is large for a wide range of CP phases, see the two upper frames of Fig. 1.

Higgs-boson production in an exclusive diffractive collision $p+p \to p+H_i+p$ offers unique possibilities for exploring Higgs physics in ways that would be difficult or even impossible in inclusive Higgs production [110]. In spite of the low and theoretically uncertain luminosity of the process, what makes diffraction so attractive compared to the inclusive processes are the clean environment due to the large rapidity gap and the good Higgs-mass resolution of the order of 1 GeV which may be achievable by precise measurements of the momenta of the outgoing protons in detectors a long way downstream from the interaction point. It may be possible to disentangle nearly-degenerate Higgs bosons by examining the production lineshape of the coupled system of neutral Higgs bosons, see lower-left frame of Fig. 1. Moreover, the CP-odd polarization asymmetry can be measured when the polarization information of Higgs decay products is available, see the lower-right frame of Fig. 1.

3.2 ILC

A future e^+e^- linear collider, such as the projected ILC, will have the potential to probe the Higgs sector with higher precision than the LHC. At the ILC, the Higgs-boson coupling to a pair of vector bosons, $g_{H_iVV} = c_\beta O_{\phi_1 i} + s_\beta O_{\phi_2 i}$, plays a crucial role. There are three main processes for producing the neutral Higgs bosons: Higgsstrahlung, WW fusion, and pair production, see Fig. 2.

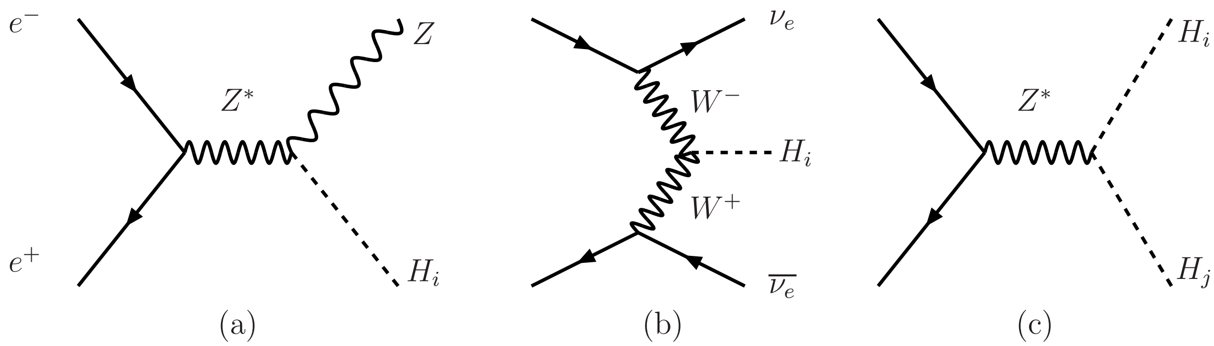

Figure 2: *Three main production mechanisms of the neutral Higgs bosons at the ILC: (a) Higgsstrahlung, (b) WW fusion, and (c) pair production.*

The cross section of each process is given by

$$\begin{aligned}
\sigma(e^+e^- \to ZH_i) &= g^2_{H_iVV}\, \sigma^{HZ}_{\rm SM}(M_{H_{\rm SM}} \to M_{H_i})\,, \\
\sigma(e^+e^- \to \nu\nu H_i) &= g^2_{H_iVV}\, \sigma^{WW}_{\rm SM}(M_{H_{\rm SM}} \to M_{H_i})\,, \\
\sigma(e^+e^- \to H_iH_j) &= g^2_{H_iH_jV}\, \frac{G_F^2 M_Z^4}{6\pi s}(v_e^2+a_e^2)\frac{\lambda^{3/2}(1, M^2_{H_i}/s, M^2_{H_j}/s)}{(1-M_Z^2/s)^2}\,, \qquad (7)
\end{aligned}$$

where $g_{H_iH_jV} = \text{sign}[\det(O)]\epsilon_{ijk}\, g_{H_kVV}$, $v_e = -1/4 + s_W^2$, $a_e = 1/4$ and $\sigma^{HZ,WW}_{\rm SM}$ denotes the corresponding production cross section of the SM Higgs boson. As is well known, the WW

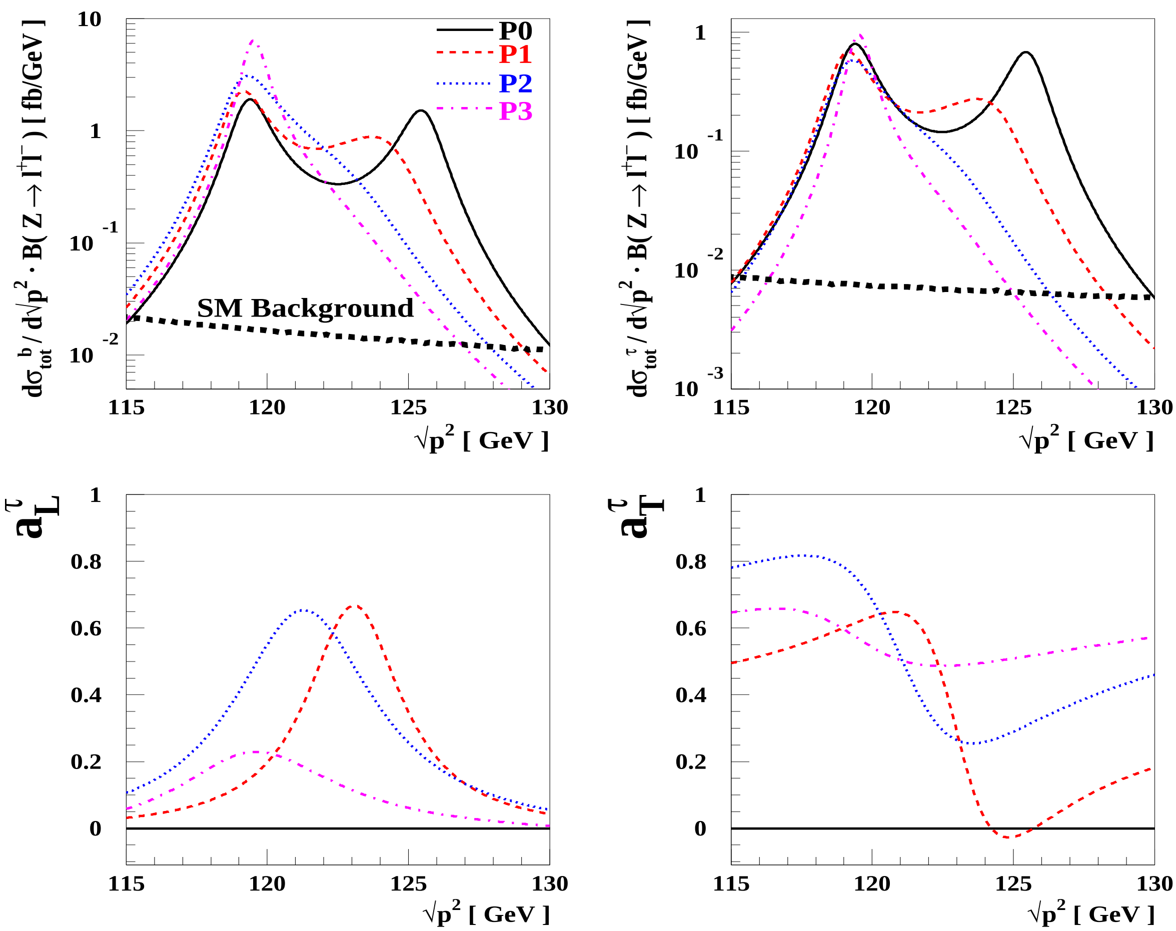

Figure 3: *The differential total cross section $d\hat{\sigma}^f_{\rm tot}(e^-e^+ \to ZH_i \to Z\bar{f}f)/d\sqrt{p^2}$ multiplied by $B(Z \to l^+l^-) = B(Z \to e^+e^-) + B(Z \to \mu^+\mu^-)$ as functions of the invariant mass of the Higgs decay products $\sqrt{p^2}$ in units of fb/GeV when $f = b$ (upper-left panel) and $f = \tau$ (upper-right panel). The CP-conserving two-way mixing (**P0**) and three CP-violating tri-mixing (**P1**-**P3**) scenarios have been taken. The lower two frames show the CP-violating asymmetries when Higgs bosons decay into tau leptons. The asymmetries are defined using the longitudinal (lower-left panel) and transverse (lower-right panel) polarizations of τ leptons. See [111] for details.*

fusion cross section grows as $\ln(s)$ compared to the Higgsstrahlung and becomes dominant for large center-of-mass energy $\sqrt{s}$. In the decoupling limit, $M_{H^\pm} \gtrsim 200$ GeV, the couplings for heavier Higgs bosons $g_{H_{2,3}VV}$ are suppressed. In this case, for the production of H_2 and H_3, the pair production mechanism is active since $|g_{H_2H_3V}| = |g_{H_1VV}| \sim 1$. When $M_{H^\pm} \lesssim 200$ GeV, the excellent energy and momentum resolution of electrons and muons coming from measurements of the Z boson in Higgsstrahlung may help to resolve a coupled system of neutral Higgs bosons by analyzing the production lineshape, see the two upper panels of Fig. 3.

In the CP-invariant MSSM framework, there is a selection rule that only two CP-even Higgs bosons can be produced *via* the Higgsstrahlung process and only two combinations

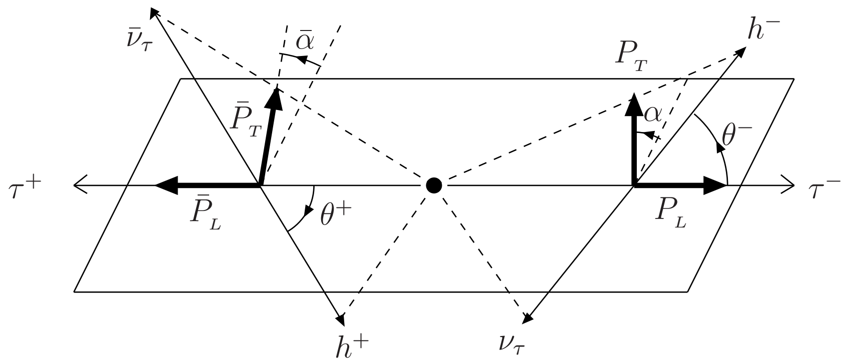

Figure 4: *The $\tau^+\tau^-$ production plane in the Higgs-boson rest frame, in the case when the τ leptons decay into hadrons $h^\pm$ and neutrinos. The longitudinal-polarization vector $P_L(\bar{P}_L)$ and the transverse-polarization vector $P_T(\bar{P}_T)$ with the azimuthal angle $\alpha(\bar{\alpha})$ of τ^- (τ^+) are shown.*

of CP-even and CP-odd Higgs bosons can be produced. In other words, the observation of three distinct Higgs bosons in Higgsstrahlung and WW fusion and/or of all three pairs of Higgs bosons in pair production could be interpreted as a signal of CP violation in the MSSM framework. However, such an interpretation relies on the hypothesis that there exist no additional singlet or doublet Higgs fields. To confirm the existence of genuine CP violation, one needs to measure other observables such as CP asymmetries. In this light, the final fermion spin-spin correlations in Higgs decays into tau leptons, neutralinos, charginos, and top quarks need to be investigated. For the Higgsstrahlung process in which the produced Higgs bosons decay into a fermion pair $f\bar{f}$, the differential cross-section for the Higgsstrahlung process is given by

$$\begin{aligned} p^2\frac{d\sigma}{dp^2} &= \sigma^{HZ}_{\rm SM}(p^2)\times\frac{N_f\, g_f^2\beta_f(p^2)}{16\pi^2}\Big\{(1+P_L\bar{P}_L)C_1^f(p^2)+(P_L+\bar{P}_L)C_2^f(p^2) \\ &\quad + P_T\bar{P}_T\Big[\cos(\alpha-\bar{\alpha})C_3^f(p^2)+\sin(\alpha-\bar{\alpha})C_4^f(p^2)\Big]\Big\}\,, \end{aligned} \tag{8}$$

where p^2 and N_f are the invariant mass squared and the color factor of the final-state fermions, respectively. The polarization coefficients C_i^f may conveniently be expressed as follows:

$$\begin{aligned} C_1^f(p^2) &= \frac{1}{4}\left(\left|\langle+\rangle^f\right|^2+\left|\langle-\rangle^f\right|^2\right), & C_2^f(p^2) &= \frac{1}{4}\left(\left|\langle+\rangle^f\right|^2-\left|\langle-\rangle^f\right|^2\right), \\ C_3^f(p^2) &= -\frac{1}{2}\Re\mathrm{e}\left(\langle+\rangle^f\langle-\rangle^{f*}\right), & C_4^f(p^2) &= \frac{1}{2}\Im\mathrm{m}\left(\langle+\rangle^f\langle-\rangle^{f*}\right), \end{aligned} \tag{9}$$

with

$$\langle\sigma\rangle^f = \sum_{i,j} g_{H_iVV}D_{ij}(p^2)\,(\sigma\beta g^S_{H_j\bar{f}f}-ig^P_{H_j\bar{f}f})\,. \tag{10}$$

When $f=\tau$ and tau leptons subsequently decay into charged hadrons $h^\mp$ and neutrinos, see Fig. 4, we have

$$P_L = \cos\theta^- \,,\ P_T = \sin\theta^- \,,\ \alpha = \varphi^- \,;\quad \bar{P}_L = \cos\theta^+ \,,\ \bar{P}_T = \sin\theta^+ \,,\ \bar{\alpha} = \varphi^+ - \pi \,, \tag{11}$$

by identifying the polarization analyser for $\tau^\mp$ as $\hat{a}^\mp = \pm\hat{h}^\mp$ with $\hat{h}^\mp$ denoting unit vectors parallel to the $h^\mp$ momenta in the $\tau^\mp$ rest frame. The angles $\theta^\pm$ and $\varphi^\pm$ are the polar and azimuthal angles of $h^\pm$, respectively, in the $\tau^\pm$ rest frame. This implies that the polarization coefficients $C_i^\tau(p^2)$ can be determined by examining the angular distributions of the charged hadrons coming from the τ-lepton decays. See the lower panels of Fig. 3 for the CP asymmetries $a_L^\tau = C_2^\tau/C_1^\tau$ and $a_T^\tau = C_4^\tau/C_1^\tau$ [111].

3.3 γLC

The two-photon collider option of the ILC, the γLC, offers unique capabilities for probing CP violation in the MSSM Higgs sector, because one may vary the initial-state polarizations as well as measure the polarizations of some final states in Higgs decays [109]. The amplitude contributing to $\gamma(\lambda_1)\gamma(\lambda_2) \to H \to f(\sigma)\bar{f}(\bar{\sigma})$ is given by

$$\mathcal{M}_H \;=\; \frac{\alpha\, m_f\, \sqrt{\hat{s}}}{4\pi v^2} \langle\sigma;\lambda_1\rangle_H^f \delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2}, \tag{12}$$

where the reduced amplitude

$$\langle\sigma;\lambda\rangle_H^f \;=\; \sum_{i,j=1}^{3} [S_i^\gamma(\sqrt{\hat{s}}) + i\lambda P_i^\gamma(\sqrt{\hat{s}})]\; D_{ij}(\hat{s})\; (\sigma\beta_f g^S_{H_j\bar{f}f} - i g^P_{H_j\bar{f}f}), \tag{13}$$

is a quantity given by the Higgs-boson propagator matrix Eq. (1) combined with the production and decay vertices. The one-loop induced complex couplings of the $\gamma\gamma H_i$ vertex, $S_i^\gamma(\sqrt{\hat{s}})$ and $P_i^\gamma(\sqrt{\hat{s}})$, get dominant contributions from charged particles such as the bottom and top quarks, tau leptons, $W^\pm$ bosons, charginos, third-generation sfermions and charged Higgs bosons [3] :

$$\begin{aligned}
S_i^\gamma(\sqrt{\hat{s}}) \;&=\; 2\sum_{f=b,t,\tilde{\chi}^\pm_{1,2}} N_C\, Q_f^2\, g_f g^S_{H_i\bar{f}f}\, \frac{v}{m_f} F_{sf}(\tau_f) \;-\; \sum_{\tilde{f}_j=\tilde{t}_{1,2},\tilde{b}_{1,2},\tilde{\tau}_{1,2}} N_C\, Q_f^2 g_{H_i\tilde{f}_j^*\tilde{f}_j} \frac{v^2}{2m^2_{\tilde{f}_j}} F_0(\tau_{\tilde{f}_j}) \\
&\quad - g_{H_iVV}\, F_1(\tau_W) - g_{H_iH^+H^-} \frac{v^2}{2M^2_{H^\pm}} F_0(\tau_{H^\pm})\,, \\
P_i^\gamma(\sqrt{\hat{s}}) \;&=\; 2\sum_{f=b,t,\tilde{\chi}^\pm_{1,2}} N_C\, Q_f^2\, g_f g^P_{H_i\bar{f}f}\, \frac{v}{m_f} F_{pf}(\tau_f)\,,
\end{aligned} \tag{14}$$

where $\tau_x = \hat{s}/4m_x^2$, $N_C = 3$ for (s)quarks and $N_C = 1$ for staus and charginos, respectively. For the explicit forms of F_1 and couplings, see [103].

One advantage of the γLC option over e^+e^- collisions at the ILC is that one can construct CP asymmetries even when Higgs bosons decay into muons and b quarks, by

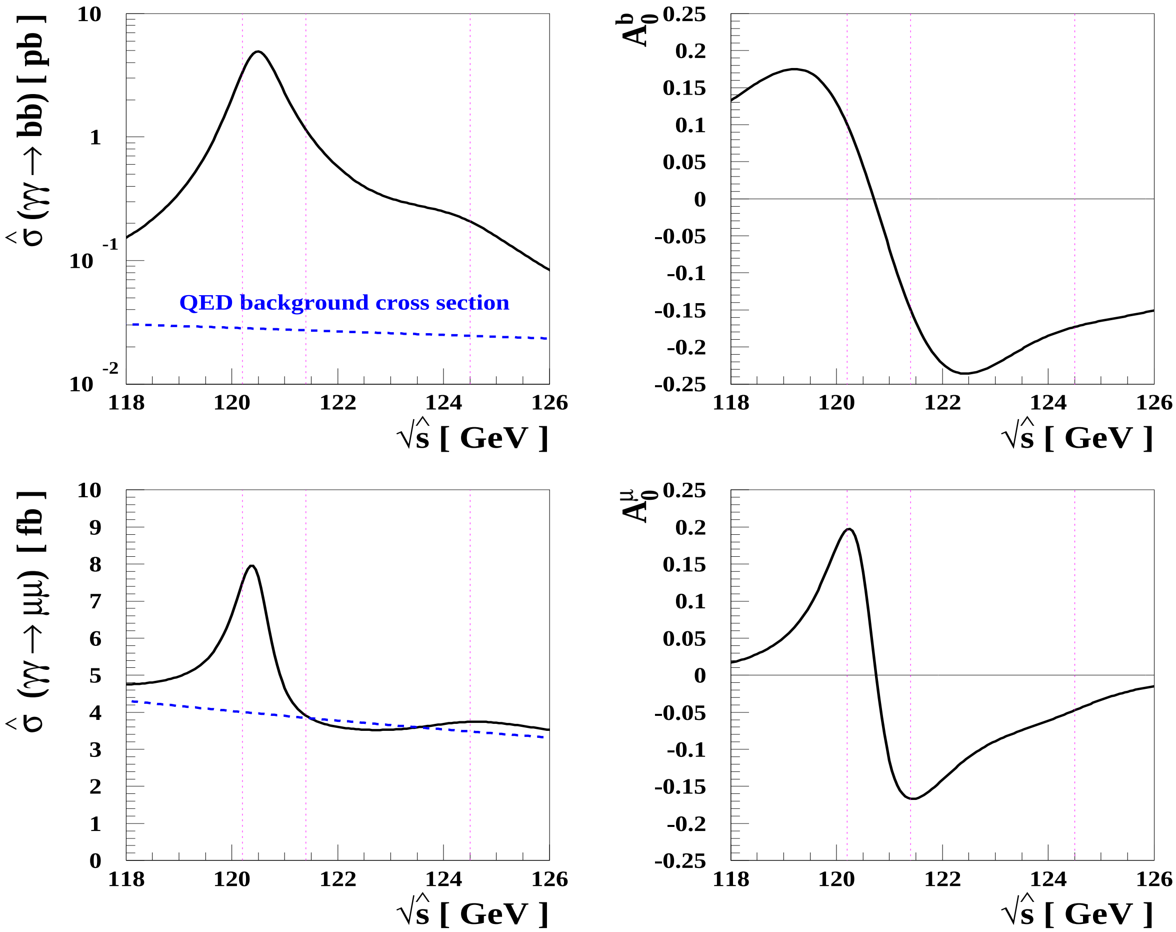

Figure 5: *The cross sections (left column) and the CP asymmetries $\mathcal{A}_0^f$ (right column) for the processes $\gamma\gamma \to \bar{b}b$ (upper) and $\gamma\gamma \to \mu^+\mu^-$ (lower). The QED continuum contributions to the cross sections are also shown. The tri-mixing scenario with $\Phi_3 = -10^\circ$ and $\Phi_A = 90^\circ$ has been considered, making the angle cuts $\theta^b_{\rm cut} = 280$ mrad and $\theta^\mu_{\rm cut} = 130$ mrad. The three Higgs masses are indicated by vertical lines. See [109] for details.*

exploiting the controllable beam polarizations of the colliding photons. In the case of identical photon helicities $\lambda_1 = \lambda_2 = \lambda$, corresponding to the different combinations of helicities of the initial-state photons λ and final-state fermions σ, we have four cross sections:

$$\hat{\sigma}^f_{\sigma\lambda}(\gamma\gamma \to f\bar{f}) = \frac{\beta_f N_C}{32\pi}\left(\frac{\alpha\, m_f}{4\pi v^2}\right)^2 \mathcal{Y}^f_{\sigma\lambda} \tag{15}$$

where $\sigma\,,\lambda = \pm$ and

$$\mathcal{Y}^f_{\sigma\lambda} \equiv 2\left|\langle\sigma;\lambda\rangle^f_H\right|^2 + 2R(\hat{s})^2\, F_1^{z_f}\left|\langle\sigma;\lambda\rangle^f_C\right|^2 + 2R(\hat{s})\, F_2^{z_f}\ \Re{\rm e}(\langle\sigma;\lambda\rangle^f_H\langle\sigma;\lambda\rangle^{f*}_C)\,. \tag{16}$$

We have included the QED continuum contribution and an experimental cut on the fermion polar angle θ has been introduced: $|\cos\theta| \leq z_f$ and $\cos\theta^f_{\rm cut} = z_f$. For the explicit forms

[3]The arrays `SPHO(3)` and `PPHO(3)` are used for the $\hat{s}$-dependent γ-γ-Higgs couplings.

of the functions $F_1^{z_f}$, $F_2^{z_f}$, $R(\hat{s})$, and the QED amplitude $\langle\sigma;\lambda\rangle_C^f$, we refer to Ref. [109]. In this case, one can construct CP asymmetries with no need to determine the helicities of the final states as

$$\mathcal{A}_0^f \equiv \frac{\sum_{\sigma=\pm}(\hat{\sigma}^f_{\sigma+} - \hat{\sigma}^f_{\sigma-})}{4\,\hat{\sigma}(\gamma\gamma \to f\bar{f})}\,, \tag{17}$$

where $\hat{\sigma}(\gamma\gamma \to f\bar{f}) = \left(\hat{\sigma}^f_{++} + \hat{\sigma}^f_{--} + \hat{\sigma}^f_{+-} + \hat{\sigma}^f_{-+}\right)/4$. Figure 5 shows the cross section $\hat{\sigma}(\gamma\gamma \to f\bar{f})$ and the CP asymmetry $\mathcal{A}_0^f$ when $f = b$ and τ.

By considering the interference of Higgs-mediated resonant and QED continuum amplitudes and the possibly measurable final-state polarizations, one can construct more than 20 independent observables for each decay mode and half of them are CP odd. This suggests that one can investigate all possible spin-spin correlations in the final states such as tau leptons, neutralinos, charginos, top quarks, vector bosons, etc., with the goal of complete determination of CP-violating Higgs-boson couplings to them. The cases of tau-lepton and top-quark final states are demonstrated in [109].

3.4 MC

For the complete determination of the CP-violating Higgs-boson couplings to SM as well as supersymmetric particles, a MC is even better than the γLC. At a muon collider, it is possible to control the energy resolution and polarizations of both the muon and the anti-muon. Compared to the γLC case, the center-of-mass frame is known very accurately, providing much better resolving power for a nearly-degenerate system of Higgs bosons.

For example, let us consider the process $\mu^-(\lambda)\mu^+(\bar{\lambda}) \to f(\sigma)\bar{f}(\bar{\sigma})$. When the helicities of the muons are equal: $\lambda = \bar{\lambda}$, the γ- and Z-mediated processes are suppressed by a kinematical factor of the muon mass, and the Higgs-mediated amplitude contributing to the process is given by

$$\mathcal{M}_H \;=\; -g_\mu\, g_f \langle\sigma;\lambda\rangle_\mu^f\, \delta_{\sigma\bar{\sigma}}\delta_{\lambda\bar{\lambda}}\,, \tag{18}$$

where $g_f = gm_f/2M_W$. The reduced amplitude is

$$\langle\sigma;\lambda\rangle_\mu^f \;=\; \sum_{i,j=1}^{3} (\lambda\beta_\mu\, g^S_{H_i\bar{\mu}\mu} + i g^P_{H_i\bar{\mu}\mu})\; D_{ij}(s)\; (\sigma\beta_f g^S_{H_j\bar{f}f} - i g^P_{H_j\bar{f}f})\,, \tag{19}$$

where s is the invariant muon-cillider energy squared. The observables obtained by controlling solely the muon and anti-muon polarizations are given by [41]

$$\sigma_{_{RR/LL}} = \frac{g_\mu^2\, g_f^2\, N_C\, \beta_f}{8\,\pi\, s}\; \left[\, C_3^f \pm C_4^f \,\right]\,, \qquad \sigma_{\,\|/\perp} = \frac{g_\mu^2\, g_f^2\, N_C\, \beta_f}{16\,\pi\, s}\,(\pm)\, C^f_{15/16}\,, \tag{20}$$

where the coefficients are

$$\begin{aligned} C_3^f \;&=\; \frac{1}{4}\sum_{\sigma=\pm}\left[|\langle\sigma;+\rangle_\mu^f|^2 + |\langle\sigma;-\rangle_\mu^f|^2\right]\,, \qquad C_4^f = \frac{1}{4}\sum_{\sigma=\pm}\left[|\langle\sigma;+\rangle_\mu^f|^2 - |\langle\sigma;-\rangle_\mu^f|^2\right]\,, \\ C_{15}^f \;&=\; -\frac{1}{2}\,\Re\mathrm{e}\sum_{\sigma=\pm}\left[\langle\sigma;-\rangle_\mu^f\langle\sigma;+\rangle_\mu^{f\,*}\right]\,, \qquad C_{16}^f = \frac{1}{2}\,\Im\mathrm{m}\sum_{\sigma=\pm}\left[\langle\sigma;-\rangle_\mu^f\langle\sigma;+\rangle_\mu^{f\,*}\right]\,. \end{aligned} \tag{21}$$

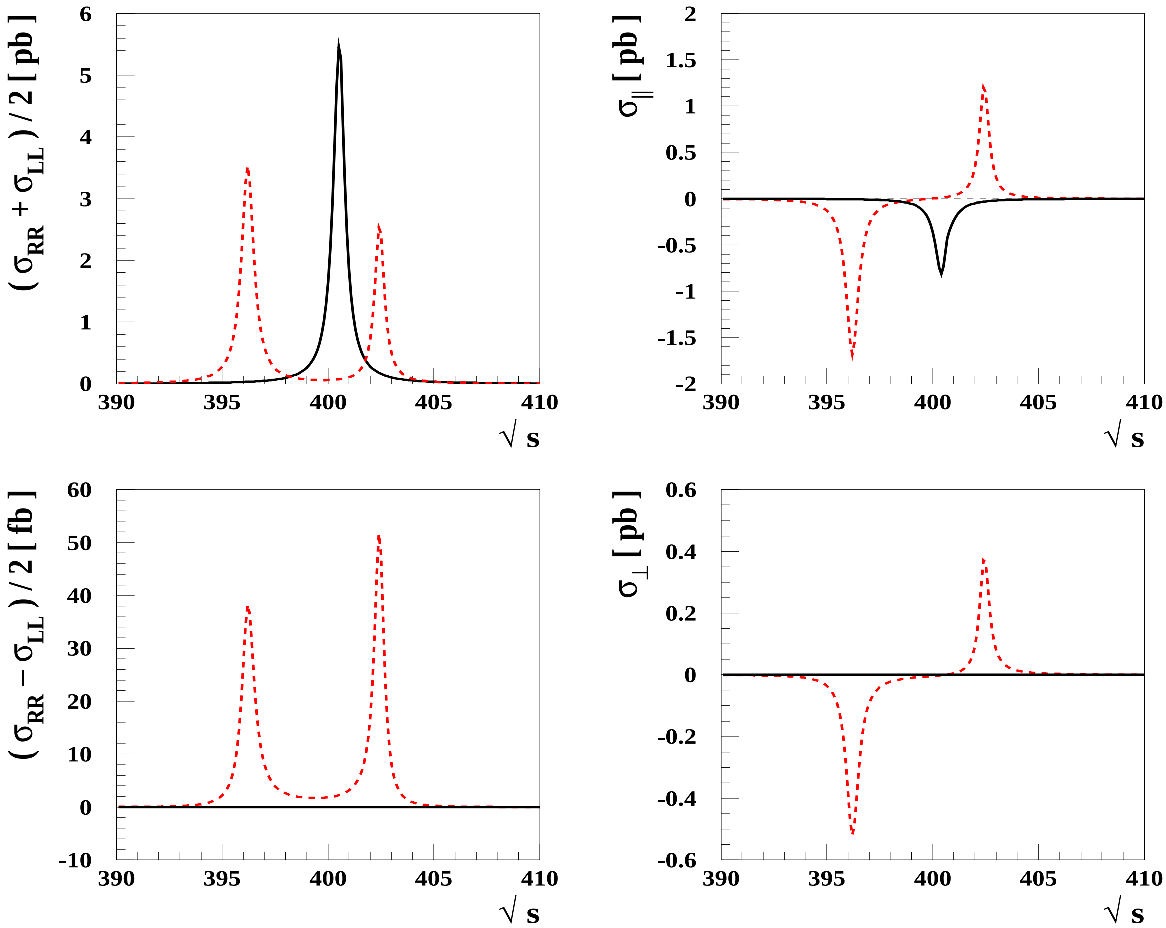

Figure 6: *The cross sections* $(\sigma_{RR}+\sigma_{LL})/2$ *(upper left),* $\sigma_{\parallel}$ *(upper right),* $(\sigma_{RR}-\sigma_{LL})/2$ *(lower left), and* $\sigma_{\perp}$ *(lower right) for the process* $\mu^-\mu^+ \to H \to t\bar{t}$ *as functions of* $\sqrt{s}$*. Here,* $(\sigma_{RR}\pm\sigma_{LL})/2$ *is given in units of fb but the other cross sections are in pb. The solid and dashed lines are for* $\Phi_A = 0^\circ$ *and* $\Phi_A = -90^\circ$*, respectively.*

We note that C_4^f and C_{16}^f are CP-odd observables. In Fig. 6, we show $(\sigma_{RR}\pm\sigma_{LL})/2$ and $\sigma_{\parallel/\perp}$ when Higgs bosons decay into top quarks $f=t$, as functions of $\sqrt{s}$. The parameters are chosen as in Ref. [41], except that $\tan\beta = 5$ and $M_{H^\pm} = 407$ GeV. The solid lines are for the CP-conserving case with $\Phi_A = 0^\circ$ and the dashed lines are for the CP-violating case with $\Phi_A = -90^\circ$. The masses are $M_{H_2} = 400.5$ GeV and $M_{H_3} = 400.6$ GeV in the CP-conserving case, but we have a larger mass splitting in the CP-violating case: $M_{H_2} = 396.2$ GeV and $M_{H_3} = 402.4$ GeV. The results have been updated, compared to those presented in Ref. [41], by including the off-diagonal absorptive parts in the Higgs-boson propagators, Higgs-boson pole mass shifts, b-quark Yukawa-coupling resummation effects, etc. More observables could be obtained by considering final-state spin correlations and interference effects with the processes mediated by γ and Z bosons.

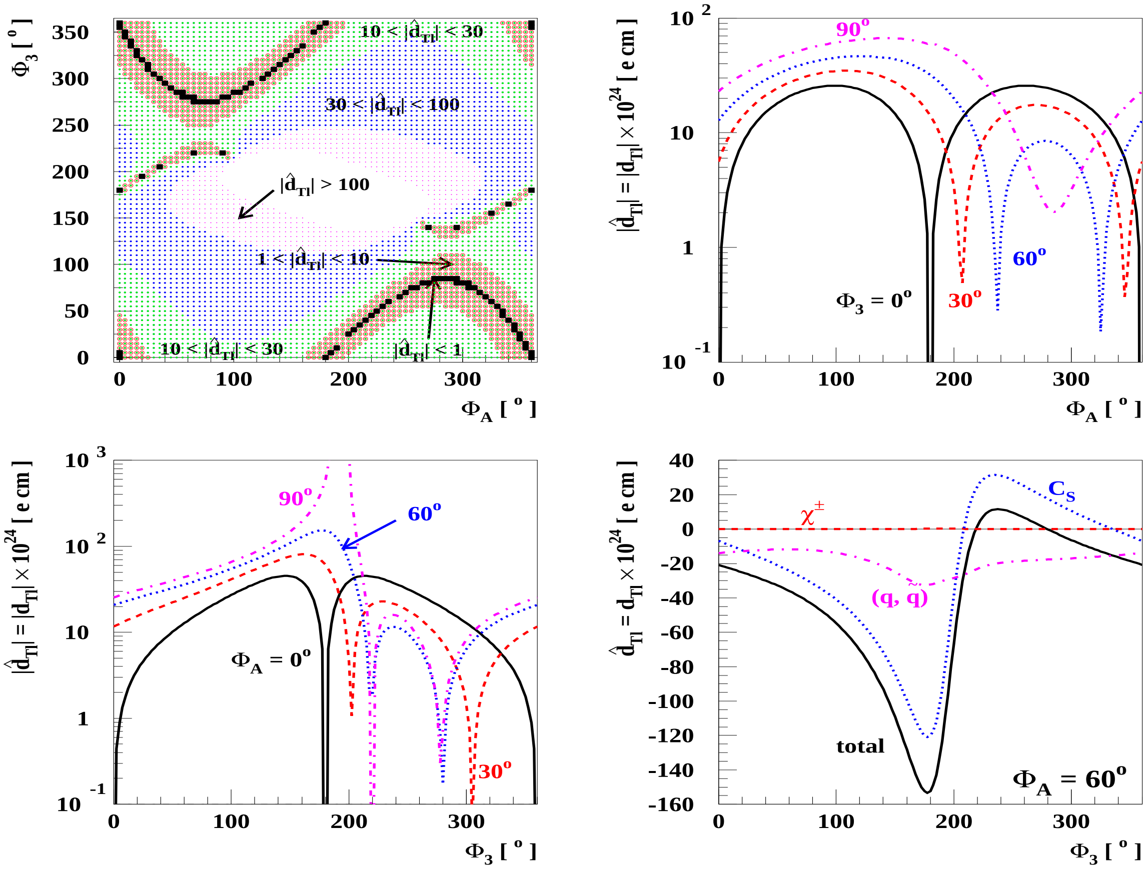

Figure 7: *The Thallium EDM* $\hat{d}_{\rm Tl} \equiv d_{\rm Tl} \times 10^{24}$ *e cm in the tri-mixing scenario. The upper-left frame displays* $|\hat{d}_{\rm Tl}|$ *in the* (Φ_A, Φ_3) *plane. The unshaded region around the point* $\Phi_3 = \Phi_A = 180^o$ *is not theoretically allowed. The different shaded regions correspond to different ranges of* $|\hat{d}_{\rm Tl}|$*, as shown: specifically,* $|\hat{d}_{\rm Tl}| < 1$ *in the narrow region denoted by filled black squares. In the upper-right frame, we show* $|\hat{d}_{\rm Tl}|$ *as a function of* Φ_A *for several values of* Φ_3*. In the lower-left frame, we show* $|\hat{d}_{\rm Tl}|$ *as a function of* Φ_3 *for four values of* Φ_A*. In the lower-right frame, we show the* C_S *(dotted line) and* d_e *(dash-dotted line) contributions to* $\hat{d}_{\rm Tl}$ *separately as functions of* Φ_3 *when* $\Phi_A = 60^o$*. As shown by the dashed line, the chargino contribution is negligible. See [111] for details.*

4 Low-energy Observables

Low-energy observables provide indirect constraints on the soft SUSY breaking parameters. The observables are particularly useful for identifying the favoured range of parameter space when the SM predictions for them are strongly suppressed and/or precise experimental measurements of them have been performed. Such observables include EDMs, $(g-2)_\mu$, $\mathrm{BR}(b \to s\gamma)$, $\mathcal{A}_{\rm CP}(b \to s\gamma)$, $\mathrm{BR}(B \to K\,l\,l)$ and $\mathrm{BR}(B_{s,d} \to l^+ l^-)$.

Currently, the EDM of the Thallium atom provides one of the best constraints on the CP-violating phases, depending on the SUSY scale. The main contributions to the atomic EDM of ^{235}Tl come from two terms. One of them is the electron EDM d_e and the other is the coefficient C_S of a CP-odd electron-nucleon interaction. The coefficient C_S is essentially given by the gluon-gluon-Higgs couplings and the two-loop Higgs-mediated electron EDM [58,59] is given by the sum of contributions from third-generation quarks and squarks and charginos [4]. We show in [111] that it is straightforward to obtain C_S and the Higgs-mediated d_e by use of couplings calculated by `CPsuperH`. As can be seen for example in Fig. 7, one can then implement the thallium EDM constraint on the CP-violating phases and demonstrate that they leave open the possibility of large CP-violating effects in Higgs production at the ILC.

The code `CPsuperH` will be extended in near future to include CP-violating effective FCNC Higgs-boson interactions to up- and down-type quarks [85, 118–120]. The determination of these effective interactions may be further improved in the framework of an effective potential approach, where the most significant subleading contributions to the couplings can be consistently incorporated. At large values of $\tan\beta$, Higgs-mediated interactions contribute significantly to the B-meson observables mentioned above and so may offer novel constraints on the parameter space of constrained versions of the MSSM, such as the scenario of minimal flavour violation.

5 Cosmological Connections and Future Directions

Imposing R parity, the Lightest Supersymmetric Particle (LSP) is stable and becomes one of the strongest candidates for the cold dark matter which provides some 23 % of the energy density of the Universe. When the lightest neutralino is the LSP, the annihilation of neutralino pairs and scattering off ordinary matter become relevant in Cosmology. Neutralino-pair annihilation cross sections are needed in calculations of the cosmological neutralino relic density and of the fluxes of cosmic-ray antiprotons, positrons and gamma-rays due to relic annihilations in the Galactic halo. The fluxes of high-energy neutrinos from the cores of the Sun and Earth depend on the scattering processes leading to neutralino capture, as well as on the rates for different annihilation channels. Scattering cross sections are also important for the direct detection of the neutralino via its interaction with matter.

In the presence of CP-violating mixing in Higgs sector, the couplings of Higgs bosons to the SM and SUSY particles are significantly modified and the contributions to the pair-annihilation and scattering cross sections from Higgs mediated processes could be largely affected. For example, for non-relativistic neutralinos, the amplitude of the process $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to H_i \to WW\,, H_1H_1$ is highly suppressed in CP-invariant theories, but may be enhanced when the intermediate Higgs bosons are CP-mixed states [89]. We note that

[4] Here we have not considered one-loop contributions, since they are independent in the absence of any assumptions relating the soft SUSY-breaking terms of the first two generations to those of the third generation.

CPsuperH has been used for the calculation of the relic abundance of the lightest neutralino in Ref. [121].

As mentioned above, CP-violating phases in the MSSM may also make possible baryogenesis at the electroweak scale [100–102, 122]. A key ingredient in verifying this and other cosmological connections will be laboratory measurements of CP-violating phenomena, and `CPsuperH` will be a key tool for evaluating their implications for the underlying model parameters by extending its scope to low-energy and cosmological observables.

Acknowledgements

The work of JSL was supported in part by The Korea Research Foundation and The Korean Federation of Science and Technology Societies Grant funded by Korea Government (MOEHRD, Basic Research Promotion Fund). The work of AP is supported in part by the PPARC research grant: PP/D000157/1.

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