Published by Institute of Physics Publishing for SISSA
Received: *June 6, 2006*
Accepted: *July 31, 2006*
Published: *September 6, 2006*

Flavour matters in leptogenesis

Asmaa Abada,[a] **Sacha Davidson,**[b] **Alejandro Ibarra,**[c] **François-Xavier Josse-Michaux,**[a] **Marta Losada**[d] **and Antonio Riotto**[e]

[a] *Laboratoire de Physique Théorique, Université de Paris-Sud 11, Bâtiment 210, 91405 Orsay Cedex, France*
[b] *Institut de Physique Nucléaire de Lyon, Université C-B Lyon-1, 69622 Villeurbanne, cedex, France*
[c] *Instituto de Física Teórica, CSIC/UAM, C-XVI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*
[d] *Centro de Investigaciones, Universidad Antonio Nariño, Cll. 58A No. 37-94, Bogotá, Colombia*
[e] *CERN Theory Division, Geneve 23, CH-1211, Switzerland*
E-mail: abada@th.u-psud.fr, s.davidson@ipnl.in2p3.fr, alejandro.ibarra@cern.ch, francois-xavier.josse-michaux@th.u-psud.fr, martha.losada@cern.ch, antonio.riotto@pd.infn.it

Abstract: We give analytic approximations to the baryon asymmetry produced by thermal leptogenesis with hierarchical right-handed neutrinos. Our calculation includes flavour-dependent washout processes and CP violation in scattering, and neglects gauge interactions and finite temperature corrections. Our approximate formulae depend upon the three CP asymmetries in the individual lepton flavours as well as on three flavour-dependent efficiency factors. We show that the commonly used expressions for the lepton asymmetry, which depend on the total CP asymmetry and one single efficiency factor, may fail to reproduce the correct lepton asymmetry in a number of cases. We illustrate the importance of using the flavour-dependent formulae in the context of a two right-handed neutrino model.

Keywords: Beyond Standard Model, Neutrino Physics.

Contents

1. Introduction

There are robust observational evidences that a tiny excess of matter over antimatter was produced in our Universe [1], but its origin is still a mystery. Baryogenesis through Leptogenesis [2] is a simple mechanism to explain this baryon asymmetry of the Universe. A lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to $(B+L)$-violating sphaleron interactions [3] which exist in the Standard Model (SM).

A simple model in which this mechanism can be implemented is "Seesaw"(type I) [4], consisting of the Standard Model (SM) plus two or three right-handed (RH) Majorana neutrinos. In this simple extension of the SM, the usual scenario that is explored (referred to as "thermal leptogenesis") consists of a hierarchical spectrum for the RH neutrinos, such that the lightest of the RH neutrinos is produced by thermal scattering after inflation, and subsequently decays out-of-equilibrium in a lepton number and CP-violating way, thus satisfying Sakharov's constraints.

In recent years, a lot of work [5 − 7], has been devoted to a thorough analysis of this model, giving limited attention to the issue of lepton flavour [8]. The dynamics of leptogenesis is usually addressed within the 'one-flavour' approximation, where Boltzmann equations are written for the abundance of the lightest RH neutrino, responsible for the out of equilibrium and CP asymmetric decays, and for the total lepton asymmetry. However, this 'one-flavour' approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium.

In ref. [8], flavoured Boltzmann Equations were written down. Flavour effects in "resonant leptogenesis" were studied in [9], discussed for thermal leptogenesis in the two-right-handed neutrino model in [10], and used in [11] to protect an asymmetry made in the decay of the middle right-handed neutrino. In the four generation models of [12], flavour was used to enhance the asymmetry, foreshadowing the results we obtain here. The impact of flavour in thermal leptogenesis has been recently studied in some detail [13, 14], including the quantum oscillations/correlations of the asymmetries in lepton flavour space [13]. It was shown that the Boltzmann equations describing the asymmetries in flavour space have additional terms which can significantly affect the result for the final baryon asymmetry. In [13], we focused on how flavour effects can enlarge the area of parameter space where leptogenesis can work: the lower bound on the reheat temperature is mildly decreased, and the upper bound on the light neutrino mass scale no longer holds.[1]

Flavour effects have not usually been included in leptogenesis calculations. This is perhaps because perturbatively, they seem to be a small correction. For instance, if the asymmetry is a consequence of the very-out-of-equilibrium decay of an initial population of right-handed neutrinos, then the total lepton asymmetry is of order ϵ/g_*, where ϵ is the total CP asymmetry in the decay, and g_* counts for the entropy dilution factor. Clearly the small charged lepton Yukawa couplings have no effect on ϵ. However, realistic leptogenesis is a drawn-out dynamical process, involving the production and destruction of right-handed neutrinos, and of a lepton asymmetry that is distributed among *distinguishable* flavours. The processes which wash out lepton number are flavour dependent, *e.g* the inverse decays from electrons can destroy the lepton asymmetry carried by, and only by, the electrons. The asymmetries in each flavour are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of washout in the one-flavour approximation, where indistinguishable leptons propagate between decays and inverse decays, so inverse decays from all flavours are taken to wash out asymmetries in any flavour.[2]

[1] The bound is removed when flavour effects are relevant, which is the case for leptogenesis at temperatures $\lesssim 10^{12}$ GeV.

[2] The "one-flavour" formulae describe leptogenesis that takes place at temperatures larger than 10^{12} GeV, before the charged lepton Yukawas come into equilibrium. They are also appropriate for right-handed neutrinos who decay only to one flavour. (But note from eqns (4.16), (4.17), that flavour effects can be

In this paper we provide the necessary analytical expressions for the computation of the baryon asymmetry including flavour that the interested reader may apply to their preferred model. By comparing to the usually adopted 'one-flavour' approximation, we will show that the commonly used expressions for the lepton asymmetry, which depend on the total CP asymmetry and one single efficiency factor, fail to reproduce the correct lepton asymmetry in a large number of cases. As an application, we also present in this paper a detailed analysis of flavour effects on lepton asymmetries for a two right-handed neutrino model. Explicit examples in which sizeable enhancements can be obtained are also given.

The paper is organized as follows. In section 2, we review the conventional flavoured-blind computation of the baryon asymmetry, and present some analytic approximations that will be used later. In section 3, we introduce the Boltzmann equations we will solve, which differ by the inclusion of flavour and of CP violation in $\Delta L = 1$ processes. The following section provides a list of rules and expressions to apply in order to obtain an estimate of the baryon asymmetry which includes flavour effects. Section 5 contains the analysis in the context of two right-handed neutrino model which make manifest the difference between the results when flavours are and are not included. In section 6 different textures for the neutrino Yukawa coupling matrix and their implications are explored. In section 7 we discuss the special case in which there is no CP violation in the right-handed neutrino sector, and finally in section 8 we draw our conclusions.

2. The conventional computation of the baryon asymmetry

This section introduces notation and reviews the calculation of the lepton asymmetry when the charged lepton Yukawa couplings are neglected. As we shall see, the commonly used formulae for the final lepton asymmetry, which we report in this section, may not be appropriate once flavours are considered.

Our starting point is the Lagrangian of the Standard Model (SM) with the addition of three right-handed neutrinos N_i $(i = 1, 2, 3)$ with heavy Majorana masses $M_3 > M_2 > M_1$ and Yukawa couplings $\lambda_{i\alpha}$. Working in the basis in which the Yukawa couplings for the charged leptons are diagonal, the Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \left(\frac{M_i}{2} N_i^2 + \lambda_{i\alpha} N_i \ell_\alpha \, H + h_\alpha \, H^c \, \bar{e}_{R\alpha} \ell_\alpha + \text{h.c.} \right) . \tag{2.1}$$

Here ℓ_α and $e_{R\alpha}$ indicate the lepton doublet and singlet with flavour $(\alpha = e, \mu, \tau)$ respectively, and H is the Higgs doublet whose neutral component has a vacuum expectation value equal to $v = 246\,\text{GeV}$.

We assume that right-handed neutrinos are hierarchical, $M_{2,3} \gg M_1$ so that studying the evolution of the number density of N_1 suffices. The final amount of $(\mathcal{B} - \mathcal{L})$ asymmetry can be parametrized as $Y_{\mathcal{B}-\mathcal{L}} = n_{\mathcal{B}-\mathcal{L}}/s$, where $s = 2\pi^2 g_* T^3/45$ is the entropy density and g_* counts the effective number of spin-degrees of freedom in thermal equilibrium ($g_* =$

important when there are small branching ratios to other flavours.)

217/2 in the SM with a single generation of right-handed neutrinos). After reprocessing by sphaleron transitions, the baryon asymmetry is related to the $\mathcal{L}$ asymmetry by [15]

$$Y_{\mathcal{B}} = -\left(\frac{8n_G + 4n_H}{14n_G + 9n_H}\right) Y_{\mathcal{L}}, \tag{2.2}$$

where n_H is the number of Higgs doublets, and n_G the number of fermion generations (in equilibrium).

One defines the CP asymmetry generated by N_1 decays as

$$\epsilon_1 \equiv \frac{\sum_\alpha [\Gamma(N_1 \to H\ell_\alpha) - \Gamma(N_1 \to \overline{H\ell_\alpha})]}{\sum_\alpha [\Gamma(N_1 \to H\ell_\alpha) + \Gamma(N_1 \to \overline{H\ell_\alpha})]} = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\mathrm{Im}\left[(\lambda\lambda^\dagger)^2_{j1}\right]}{[\lambda\lambda^\dagger]_{11}} g\left(\frac{M_j^2}{M_1^2}\right), \tag{2.3}$$

where the wavefunction plus vertex contributions are included in [16]

$$g(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right] \xrightarrow{x \gg 1} -\frac{3}{2\sqrt{x}}. \tag{2.4}$$

Notice, in particular, that ϵ_1 denotes the CP asymmetry in the total number (the trace) of flavours.

Besides the CP parameter ϵ_1, the final baryon asymmetry depends on a single wash-out parameter,

$$K \equiv \frac{\sum_\alpha \Gamma(N_1 \to H\ell_\alpha)}{H(M_1)} \equiv \left(\frac{\widetilde{m}_1}{\widetilde{m}^*}\right), \tag{2.5}$$

where $H(M_1)$ denotes the value of the Hubble rate evaluated at a temperature $T = M_1$ ($\widetilde{m}^* \sim 3 \times 10^{-3}\,\mathrm{eV}$) and

$$\tilde{m}_1 \equiv \frac{(\lambda\lambda^\dagger)_{11} v^2}{M_1} \tag{2.6}$$

is proportional to the total decay rate of the right-handed neutrino N_1.

By defining the variable $z = M_1/T$, the Boltzmann equations for the lepton asymmetry $Y_{\mathcal{L}}$, and the right-handed neutrino number density Y_{N_1} (both normalised to the entropy s), may be written in a compact form as

$$\frac{d(Y_{N_1} - Y_{N_1}^{\mathrm{EQ}})}{dz} = -\frac{z}{sH(M_1)}(\gamma_D + \gamma_{\Delta L=1})\left(\frac{Y_{N_1}}{Y_{N_1}^{\mathrm{EQ}}} - 1\right) - \frac{dY_{N_1}^{\mathrm{EQ}}}{dz}, \tag{2.7}$$

$$\frac{dY_{\mathcal{L}}}{dz} = \frac{z}{sH(M_1)}\left[\left(\frac{Y_{N_1}}{Y_{N_1}^{\mathrm{EQ}}} - 1\right)\epsilon_1\gamma_D - \frac{Y_{\mathcal{L}}}{Y_L^{\mathrm{EQ}}}(\gamma_D + \gamma_{\Delta L=1} + \gamma_{\Delta L=2})\right]. \tag{2.8}$$

The processes taken into account in these equations are decays and inverse decays with rate γ_D, $\Delta L = 1$ scatterings such as ($qt^c \to N\ell$), and $\Delta L = 2$ processes mediated by heavy neutrinos. The first three modify the abundance of the lightest right-handed neutrinos. The $\Delta L = 2$ scatterings mediated by $N_{2,3}$ are neglected in our analysis for simplicity.[3] The

[3]See, e.g. the appendix of [13]. We discuss later the restrictions this implies.

various γ are thermally averaged rates, including all contributions summed over flavour (s, t channel interference etc); explicit expressions can be found in the literature (see for example [5, 6]). Notice that in this "usual" analysis, $\Delta L = 1$ scattering contributes to the creation of N_1's and not to the production of a lepton asymmetry, only to the washout.[4] This is a minor point in the single flavour analysis; it is more relevant when flavour is included, and will be discussed in the following two sections.

Approximate analytic solutions for $Y_{\mathcal{L}}$ and $\Delta_{N_1} \equiv Y_{N_1} - Y_{N_1}^{\rm EQ}$, which reproduce the numerical plots [5, 6], can be obtained from simplified equations [8, 6]. Calculating in zero temperature field theory for simplicity,[5] one obtains

$$\gamma_D \simeq s Y_{N_1}^{EQ} \frac{K_1(z)}{K_2(z)} \Gamma_D, \qquad Y_{N_1}^{\rm EQ} \simeq \frac{1}{4 g_*} z^2 \, K_2(z) \,. \tag{2.9}$$

The Boltzmann equations can be approximated

$$\Delta_{N_1}' = -zK \frac{K_1(z)}{K_2(z)} \, f_1(z) \, \Delta_{N_1} - Y_{N_1}^{\rm EQ \, \prime} \,, \tag{2.10}$$

$$Y_{\mathcal{L}}' = \epsilon_1 K z \, \frac{K_1(z)}{K_2(z)} \Delta_{N_1} - \frac{1}{2} z^3 \, K \, K_1(z) \, f_2(z) \; Y_{\mathcal{L}} \tag{2.11}$$

where K_1 and K_2 are modified Bessel functions of the second kind. The function $f_1(z)$ accounts for the presence of $\Delta L = 1$ scatterings [5, 6], and $f_2(z)$ accounts for scatterings in the washout term of the asymmetry. They can be approximated [6], in interesting limits, as

$$f_1(z) \simeq \begin{cases} 1 & \text{for } z \gg 1 \\ \frac{N_c^2 m_t^2}{4\pi^2 v^2 z^2} & \text{for } z \lesssim 1 \,, \end{cases} \tag{2.12}$$

and

$$f_2(z) \simeq \begin{cases} 1 & \text{for } z \gg 1 \\ \frac{a_K N_c^2 m_t^2}{8\pi^2 v^2 z^2} & \text{for } z \lesssim 1 \,, \end{cases} \tag{2.13}$$

where $\frac{N_c^2 m_t^2}{8\pi^2 v^2} \equiv K_s/K \; \sim 0.1$ parametrizes the strength of the $\Delta L = 1$ scatterings and $a_K = 4/3\,(2)$ for the weak (strong) wash out case. A good approximation to the rate $Kz(K_1(z)/K_2(z))f_1(z)$ is given by the function $(K_s + Kz)$ [5, 6] while the wash out term $-(1/2)z^3 K K_1(z) f_2(z) Y_{\mathcal{L}}$ is well approximated at small z by $-a_K K_s Y_{\mathcal{L}}$.

In the strong wash-out regime, the parameter $K \gg 1$ and the right handed neutrinos N_1's are nearly in thermal equilibrium. Under these circumstances, one can set $\Delta_{N_1}' \simeq 0$ and $\Delta_{N_1} \simeq (zK_2/4g_* K)$. Exploiting a saddle-point approximation in eq. (2.11) one easily reproduces the fit to the numerical results [5, 6]

$$Y_{\mathcal{L}} \simeq 0.3 \, \frac{\epsilon_1}{g_*} \left(\frac{0.55 \times 10^{-3} \, \rm eV}{\tilde{m}_1} \right)^{1.16} \,. \tag{2.14}$$

In the opposite weak wash-out regime, assuming that no right-handed neutrinos are initially present in the plasma, there could be a cancellation in the final lepton asymmetry

[4]We thank A. Strumia, A. Pilaftsis, G. Giudice and E. Nardi for useful conversations about this point.

[5]Significant finite temperature corrections were found in [5], which have $\mathcal{O}(1)$ effects on the final asymmetry.

between the (anti-) asymmetry generated in N_1 production, and the lepton asymmetry produced as the N_1 decay. However, this cancellation does not occur, in eqs. (2.7) and (2.8), because CP violation in the $\Delta L = 1$ scatterings is not included. $\Delta L = 1$ processes contribute significantly to the production of right-handed neutrinos, without making any associated (anti-) lepton asymmetry, and the N_1 later produce a lepton asymmetry in decay. The number of N_1 produced is $\propto K$, and the final lepton asymmetry can be approximated [5, 6]

$$Y_{\mathcal{L}} \simeq 0.3 \, \frac{\epsilon_1}{g_*} \left(\frac{\tilde{m}_1}{3.3 \times 10^{-3} \, \mathrm{eV}} \right) . \tag{2.15}$$

Notice that the final baryon asymmetry in the 'one flavour approximation' depends always upon the trace of the CP asymmetries over flavours, ϵ_1, times a function of the trace over flavours of the decay rate of the right-handed neutrinos, K. This is due to the fact that the inverse decay term in eq. (2.7) is proportional to the trace over the flavours of the lepton asymmetry times the trace over the flavours of the decay rate of N_1's. The reader is invited to remember this point in the following when we explain why the 'one flavour approximation' fails to predict the exact baryon asymmetry.

3. Including flavours and CP violation in scattering

In this section, we introduce the Boltzmann equations for individual flavour asymmetries, [8]. We define $Y_{\alpha\alpha}$ to be the lepton asymmetry in flavour α, where the α are the lepton mass eigenstates at the temperature of leptogenesis. As discussed in [13], the $Y_{\alpha\alpha}$ are the diagonal elements of a matrix $[Y]$ in flavour space, whose trace is the total lepton asymmetry. In this paper, the off-diagonal elements are neglected.[6]

The mass eigenstates for the particles in the Boltzmann equations(BE) are determined by the interactions which are fast compared to those processes included in the BE. The interaction rate for Yukawa coupling h_α can be estimated as [17]

$$\Gamma_\alpha \simeq 5 \times 10^{-3} h_\alpha^2 \, T, \tag{3.1}$$

so interactions involving the tau (mu) Yukawa coupling are out-of-equilibrium in the primeval plasma if $T \gtrsim 10^{12}\,\mathrm{GeV}$ ($T \gtrsim 10^9\,\mathrm{GeV}$).[7] Thermal leptogenesis takes place at temperatures on the order of M_1, and the asymmetry is generated when the rates $\lesssim H$, so we conclude that the τ (μ) lepton doublet is a distinguishable mass eigenstate, for the purposes of leptogenesis, at $T < 10^{12} (10^9)\,\mathrm{GeV}$.

The Boltzmann equations that we will use in this paper, for the flavour asymmetries $Y_{\alpha\alpha}$, are listed below. They differ from those of [13] in two respects.

[6] See [13] for a discussion. The equations of motion for the matrix $[Y]$ are more complicated than the Boltzmann equations, but at most temperatures are equivalent to Boltzmann equations written in the mass eigenstate basis of the leptons in the plasma. The off-diagonal elements of $[Y]$ could have some effect on the lepton asymmetry, if leptogenesis takes place just as a charged lepton Yukawa coupling is coming into equilibrium (so the mass eigenstate basis is changing).

[7] The electron Yukawa coupling mediates interactions relevant in the early Universe only for temperatures beneath $\sim 10^5$ GeV and can be safely disregarded.

First, we have neglected the off-diagonal terms of the matrix $[Y]$. The second and most significant difference is that we have included CP violation in the $\Delta L = 1$ scattering rate, which will give $Y_L \propto K^2$ for weak washout (instead of K in eq. (2.15)). That is, the function $\gamma_{\Delta L=1}$, which appears in the N_1 creation term, also now appears in the first term of equation (3.2), which describes the production of the lepton flavour asymmetry. Later in this section we will calculate the CP asymmetry in scattering, and show that it gives the same ϵ as decay and inverse decay, in the limit of hierarchical right-handed neutrinos. As in [13], we continue to neglect the non-resonant contribution to $\Delta L = 2$ scatterings, and its associated flavour effects [8]. At the end of section 4.1.2, we discuss the parameter range where this is acceptable.

Equation (2.7) for the N_1 number density remains unchanged, and the equation for the flavoured lepton asymmetry is

$$\frac{dY^{\alpha\alpha}}{dz} = \frac{z}{sH(M_1)}\left[\left(\frac{Y_{N_1}}{Y^{\rm EQ}_{N_1}} - 1\right)\epsilon^{\alpha\alpha}(\gamma_D + \gamma_{\Delta L=1}) - \frac{Y^{\alpha\alpha}}{Y^{\rm EQ}_L}\left(\gamma^{\alpha\alpha}_D + \gamma^{\alpha\alpha}_{\Delta L=1}\right)\right], \tag{3.2}$$

where there is no sum over α in the last term of equation (3.2) (or (3.3)).

The rates [6] and asymmetries are calculated in zero temperature field theory, and include processes mediated by the neutrino and top Yukawa couplings. This is simple, and parametrically consistent. However, finite temperature and gauge corrections can be significant [5], and should in principle be included.

To obtain analytic solutions we simplify this, with the approximations introduced in the previous section, to

$$Y'_{\alpha\alpha} = \epsilon_{\alpha\alpha} K z\,\frac{K_1(z)}{K_2(z)} f_1(z)\Delta_{N_1} - \frac{1}{2} z^3 K_1(z)\, f_2(z)\, K_{\alpha\alpha} Y_{\alpha\alpha} \tag{3.3}$$

$$\Delta'_{N_1} = -z\,K\,\frac{K_1(z)}{K_2(z)}\, f_1(z)\,\Delta_{N_1} - Y^{\rm EQ}_{N_1}{}', \tag{3.4}$$

where

$$K_{\alpha\alpha} = K\frac{\lambda_{1\alpha}\lambda^*_{1\alpha}}{\sum_\gamma |\lambda_{1\gamma}|^2} = \left(\frac{\tilde{m}_{\alpha\alpha}}{3\times 10^{-3}\,{\rm eV}}\right),\ K = \sum_\alpha K_{\alpha\alpha}\,. \tag{3.5}$$

$K_{\alpha\alpha}$ parametrizes the decay rate of N_1 to the α-th flavour, and the trace $\sum_\alpha K_{\alpha\alpha}$, coincides with the K parameter defined in the previous section, see eq. (2.5).

Notice in particular that the dynamics of the right-handed neutrinos is always set by the total K.

The CP asymmetry in the α-th flavour is $\epsilon_{\alpha\alpha}$ and is normalised by the total decay rate

$$\epsilon_{\alpha\alpha} = \frac{1}{(8\pi)}\frac{1}{[\lambda\lambda^\dagger]_{11}}\sum_j {\rm Im}\left\{(\lambda_{1\alpha})(\lambda\lambda^\dagger)_{1j}\lambda^*_{j\alpha}\right\} g\left(\frac{M_j^2}{M_1^2}\right) \tag{3.6}$$

$$\to \frac{3}{(8\pi)[\lambda\lambda^\dagger]_{11}}{\rm Im}\left\{\lambda_{1\beta}\frac{[m^*]_{\beta\alpha}}{v^2}\lambda_{1\alpha}\right\} \tag{3.7}$$

where the second line is in the limit of hierarchical N_J, and $m = U^* D_m U^\dagger = v^2\lambda^T M^{-1}\lambda$ is the light neutrino mass matrix. If $m_{\rm max}$ is the heaviest light neutrino mass ($= m_{\rm atm}$

for the non-degenerate case) and we define $\epsilon^{\max} = 3\Delta m^2_{\rm atm} M_1/(8\pi v^2 m_{\max})$ [18], then the flavour dependent CP asymmetries are bounded by

$$\epsilon_{\alpha\alpha} \leq \frac{3M_1 m_{\max}}{8\pi v^2}\sqrt{\frac{K_{\alpha\alpha}}{K}} = \epsilon^{\max}\frac{m^2_{\max}}{\Delta m^2_{\rm atm}}\sqrt{\frac{K_{\alpha\alpha}}{K}} \tag{3.8}$$

so the maximum CP asymmetry in a given flavour is unsuppressed for degenerate light neutrinos [13], but decreases as the square root of the branching ratio to that flavour $= K_{\alpha\alpha}/K$.

The CP asymmetry $\epsilon_{\alpha\alpha}$ can be written in terms of the diagonal matrix of the light neutrino mass eigenvalues $m = \mathrm{Diag}(m_1, m_2, m_3)$, the diagonal matrix of the the right handed neutrino masses $M = \mathrm{Diag}(M_1, M_2, M_3)$ and an orthogonal complex matrix $R = v\, M^{-1/2}\, \lambda\, U\, m^{-1/2}$ [19], where U is the leptonic mixing matrix, which ensures that the correct low-energy parameters are obtained

$$\epsilon_{\alpha\alpha} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U^*_{\alpha\beta} U_{\alpha\rho} R_{\beta 1} R_{\rho 1}\right)}{\sum_\beta m_\beta\,|R_{1\beta}|^2}. \tag{3.9}$$

As noted in [14], for a real R matrix, the individual CP asymmetries $\epsilon_{\alpha\alpha}$ may not vanish because of the presence of CP violation in the U matrix. On the contrary, the total CP asymmetry $\epsilon_1 = \sum_\alpha \epsilon_{\alpha\alpha}$ vanishes.

3.1 The CP asymmetry in $\Delta L = 1$ scattering

We now wish to show that the CP asymmetry in scattering processes such as $(qt^c \to N\ell_\alpha)$ is the same as in decays and inverse decays.[8] This result was found in [9, 20], for the case of resonant leptogenesis. CP violation in scattering is usually neglected in thermal leptogenesis [5, 6], because observed neutrino masses favour the strong washout regime $K > 1$. In strong washout, any contribution to the lepton asymmetry by scattering processes during N_1 production is rapidly washed out, so the CP violation in these processes can be neglected, for lack of observable consequences. However, we wish to include this CP violation, because washout in one flavour could be small, even though $K \gg 1$ and we wish to correctly include the contribution of weakly washed out lepton flavours to the final lepton asymmetry.

For simplicity, we work at zero temperature, in the limit of hierachical right-handed neutrinos. This means we calculate in an effective field theory with particle content of the SM $+N_1$, and the effects of the heavier N_2 and N_3 appear in a dimension five operator $(HL_\alpha)(HL_\beta)$. For computing one-loop CP violating effects involving N_1, we can take the coefficient of this operator $\propto [m_\nu]_{\alpha\beta}/v^2$.

We define the CP asymmetries in $\Delta L = 1$ scattering (mediated by s and t-channel Higgs boson exchange) as

$$\hat{\epsilon}_s^{\alpha\alpha} = \frac{\sigma(t^c q \to N L_\alpha) - \bar{\sigma}(\bar{q}\overline{t^c} \to N \bar{L}_\alpha)}{\sigma + \bar{\sigma}} \tag{3.10}$$

[8]We thank E. Nardi for discussions of his work in progress.

$$\hat{\epsilon}_t^{\alpha\alpha} = \frac{\sigma(qN \to \overline{t^c} L_\alpha) - \bar{\sigma}(\bar{q}N \to t^c \bar{L}_\alpha)}{\sigma + \bar{\sigma}} = \frac{\sigma(q\bar{L}_\alpha \to \overline{t^c} N) - \bar{\sigma}(\bar{q}L_\alpha \to t^c N)}{\sigma + \bar{\sigma}} \tag{3.11}$$

where barred fields are the antiparticles. The initial state density factors cancel in the ratio, so the cross-sections σ, $\bar{\sigma}$ can be replaced by the matrix elements squared $|\mathcal{M}|^2$, integrated over final state phase space $\int d\Pi$. If the tree + loop matrix element is separated into a coupling constant part c and an amplitude $\mathcal{A}$:

$$\mathcal{M} = c^t \mathcal{A}^t + c^\ell \mathcal{A}^\ell \,, \tag{3.12}$$

where the matrix element for the CP conjugate process is $\bar{\mathcal{M}} = c^{t*}\mathcal{A}^t + c^{\ell *}\mathcal{A}^\ell$, then the CP asymmetry can be written

$$\epsilon = \frac{4\mathrm{Im}\{c^t c^{l*}\}}{|c^t|^2} \frac{\int \mathrm{Im}\{\mathcal{A}^t \mathcal{A}^{l*}\} d\Pi}{\int |\mathcal{A}^t|^2 d\Pi} \,. \tag{3.13}$$

The loop amplitude has an imaginary part when there are branch cuts corresponding to intermediate on-shell particles, which can arise here in a bubble on the N line at the NHL_α vertex, e.g. for s-channel Higgs exchange:

$$\mathrm{Im}\{\mathcal{A}^t \mathcal{A}^{l*}\} = \mathcal{A}^t(t^c q \to NL_\alpha) \int \mathcal{A}^{t*}(t^c q \to L_\alpha L'_\beta H') d\Pi' \mathcal{A}^{t*}(L'_\beta H' \to N) \,. \tag{3.14}$$

H' and L'_β are the (assumed massless) intermediate on-shell particles, and $d\Pi'$ is the integration over their phase space.

In the scattering process $c^t = h_t \lambda^*_{1\alpha}$ and $c^\ell = 3 h_t \lambda_{1\beta} [m^*]_{\beta\alpha}/v^2$, where h_t is the top Yukawa coupling, so the complex coupling constant combination in $\hat{\epsilon}_{\alpha\alpha}(A\bar{H} \to NL_\alpha)$ is clearly the same as in $\epsilon_{\alpha\alpha}$ of eq. (3.7).[9] To obtain the amplitude ratio (the second fraction in eqn (3.13)), we take, for instance $\mathcal{A}^t(N \to \bar{H}\bar{L^\alpha}) = \bar{u}_\ell P_L u_N$, and after straightforward spin sums, one sees that it is the same for scattering and N decay, so $\hat{\epsilon}_s^{\alpha\alpha} = \hat{\epsilon}_t^{\alpha\alpha} = \epsilon^{\alpha\alpha}$.

The equality (3.11), and the result that the s and t-channel scattering asymmetries are equal to the decay asymmetry, may be an artifact of our effective field theory calculation, where there is no momentum exchanged on the $N_{2,3}$ line in the loop.

In the finite temperature calculation of [5], the Higgs boson has a large thermal mass due to its interactions with the top quarks. At $T \gg M_1$ it can therefore decay to NL, and ref. [5] found a CP asymmetry in this decay. Some part of the N production that is included in zero temperature scattering computations, is resummed in the finite temperature H decays, so it is consistent that in both processes, a lepton asymmetry can be generated.

Checking CPT and unitarity

We would like to verify that the CP violating matrix elements $\mathcal{M}$ for $\Delta L = 1$ scattering processes, satisfy the constraints following from CPT invariance and S-matrix unitarity.

[9] In both cases, there is an overall factor of three, from the weak SU(2) index contractions. This can be seen by reinstating the $N_{2,3}$ propagators: the charged and neutral component of the intermediate H' and L' contribute in the N_1 wave-function correction, giving a 2, but only the charged or the neutral H' and L' appear in the vertex correction.

See e.g. [21] for useful related discussions. For i any state, and $\bar{i}$ its CP conjugate, CPT and unitarity imply [22]

$$\sum_{\bar{j}} |\mathcal{M}(\bar{i} \to \bar{j})|^2 = \sum_{\bar{j}} |\mathcal{M}(\bar{j} \to \bar{i})|^2 = \sum_{j} |\mathcal{M}(i \to j)|^2, \tag{3.15}$$

where $\{j\}$ is the set of accessible states. We want to check that this is consistent with having a CP asymmetry in scattering processes like $qt^c \to N\ell$.

First, as a warm-up, let us consider the case of inverse decays $H\ell \to N$ in the single flavour approximation. It is well-known that there is a CP asymmetry here, and this will show how to apply CPT and unitarity bounds with unstable particles (N) in the final state.

Suppose that

$$|\mathcal{M}(H\ell \to N)|^2 \equiv \frac{|\mathcal{M}_0|^2}{2}(1+\epsilon)\,, \qquad |\mathcal{M}(\bar{H}\bar{\ell} \to N)|^2 \equiv \frac{|\mathcal{M}_0|^2}{2}(1-\epsilon)\,, \tag{3.16}$$

where $|\mathcal{M}_0|^2$ is order λ^2 and $|\mathcal{M}_0|^2\epsilon$ is order λ^4.

At order λ^2, the CPT + unitarity constraint is verified, but at order λ^4, $|\mathcal{M}(H\ell \to N)|^2 \neq |\mathcal{M}(\bar{H}\bar{\ell} \to N)|^2$, because there can be additional final states: $\bar{H}\bar{\ell}$ and $H\ell$. As in the case of the Boltzman Equations, when we include these $2 \leftrightarrow 2$ processes, we must subtract out the real intermediate N_1, because we have already included this by treating the N_1 as a final state particle. So at order λ^4:

$$\begin{aligned}
|\mathcal{M}(H\ell \to X)|^2 &= |\mathcal{M}(H\ell \to N)|^2 + |\mathcal{M}(H\ell \to \bar{H}\bar{\ell})|^2 - |\mathcal{M}^{\rm RIS}(H\ell \to \bar{H}\bar{\ell})|^2 \\
&\quad + |\mathcal{M}(H\ell \to H\ell)|^2 - |\mathcal{M}^{\rm RIS}(H\ell \to H\ell)|^2 \\
&= \frac{|\mathcal{M}_0|^2}{2}(1+\epsilon) + |\mathcal{M}(H\ell \to \bar{H}\bar{\ell})|^2 - \frac{|\mathcal{M}_0|^2}{2}(1+\epsilon)\frac{(1+\epsilon)}{2} \\
&\quad + |\mathcal{M}(H\ell \to H\ell)|^2 - \frac{|\mathcal{M}_0|^2}{2}(1+\epsilon)\frac{(1-\epsilon)}{2} \\
&= |\mathcal{M}(H\ell \to \bar{H}\bar{\ell})|^2 + |\mathcal{M}(H\ell \to H\ell)|^2 + \cdots
\end{aligned} \tag{3.17}$$

where X is all possible final states, and the on-shell intermediate N_1 has been subtracted in the narrow width approximation [5]. Repeating for $(\bar{H}\bar{\ell} \to X)$ will give the same rate, because at $\mathcal{O}(\lambda^4)$, $|\mathcal{M}(H\ell \to \bar{H}\bar{\ell})|^2 = |\mathcal{M}(\bar{H}\bar{\ell} \to H\ell)|^2$, and $|\mathcal{M}(H\ell \to H\ell)|^2 = |\mathcal{M}(\bar{H}\bar{\ell} \to \bar{H}\bar{\ell})|^2$. So CPT and unitarity are satisfied for inverse decays, as they ought to be.

CPT and unitarity are realised in the scattering process ($qt^c \to N\ell_\alpha$), in a similar way to inverse decays. CPT and unitarity should hold order by order in perturbation theory, so we work at order $\lambda^2\lambda_\alpha^2 h_t^2$, and define

$$|\mathcal{M}(qt^c \to N\ell_\alpha)|^2 = |\mathcal{M}_s|^2(1+\epsilon^{\alpha\alpha})\,, \tag{3.18}$$

where $|\mathcal{M}_s|^2 \propto \lambda_\alpha^2 h_t^2$, and $|\mathcal{M}_s|^2\epsilon \propto \lambda^2\lambda_\alpha^2 h_t^2$. At order $\lambda^2\lambda_\alpha^2 h_t^2$, we should also include various tree diagrams without N in the final state. Following the inverse decay discussion, one can write

$$|\mathcal{M}(qt^c \to X\ell_\alpha)|^2 = |\mathcal{M}(qt^c \to N\ell_\alpha)|^2$$

$$
\begin{aligned}
&+\sum_\beta \left[|\mathcal{M}(qt^c \to \ell_\beta H \ell_\alpha)|^2 - |\mathcal{M}^{\rm RIS}(qt^c \to \ell_\beta H \ell_\alpha)|^2\right] \\
&+\sum_\beta \left[|\mathcal{M}(qt^c \to \overline{\ell}_\beta \overline{H} \ell_\alpha)|^2 - |\mathcal{M}^{\rm RIS}(qt^c \to \overline{\ell}_\beta \overline{H} \ell_\alpha)|^2\right] \\
=\ & |\mathcal{M}_s|^2(1+\epsilon^{\alpha\alpha}) + |\mathcal{M}(qt^c \to \ell H \ell_\alpha)|^2 - |\mathcal{M}_s|^2(1+\epsilon^{\alpha\alpha})\frac{(1+\epsilon)}{2} \\
&+|\mathcal{M}(qt^c \to \overline{\ell H}\ell_\alpha)|^2 - |\mathcal{M}_s|^2(1+\epsilon^{\alpha\alpha})\frac{(1-\epsilon)}{2} \\
=\ & \sum_\beta \left[|\mathcal{M}(qt^c \to \ell_\beta H \ell_\alpha)|^2 + |\mathcal{M}(qt^c \to \overline{\ell}_\beta \overline{H} \ell_\alpha)|^2\right] ,
\end{aligned} \tag{3.19}
$$

where, in the narrow width approximation,

$$
|\mathcal{M}^{\rm RIS}(qt^c \to \overline{\ell}_\beta \bar{H} \ell_\alpha)|^2 = |\mathcal{M}(qt^c \to N\ell_\alpha)|^2 \times {\rm BR}(N \to \bar{H}\bar{\ell}_\beta) \tag{3.20}
$$

In eq. (3.19), the CP asymmetry $\epsilon^{\alpha\alpha}$ has disappeared, so if we repeat the calculation for the CP conjugate initial state $\bar{q}\overline{t^c}$, we should obtain the same result, verifying that a CP asymmetry in $qt^c \to N\ell_\alpha$ is consistent with CPT and unitarity. Furthermore, eq. (3.19) is reassuring, because the unstable state N has disappeared. There is no CP violation in the total rate for $qt^c \to$ asymptotic (stable) final states, but CP violation in the partial rate to the unstable N is possible. This can be relevant to the final value of the baryon symmetry when some of the lepton flavours are weakly washed out (see section 4.1.2).

4. Approximate formulae for the baryon asymmetry

In this section we present analytical formulae for the final baryon asymmetry in the case in which flavours are taken into account. We can have different possible cases according to which interaction mediated by the charged Yukawa couplings is in equilibrium, [8, 14].

4.1 μ and τ Yukawa couplings in equilibrium: $M_1 \lesssim 10^9$ GeV

The μ and τ doublet leptons (and by default, the electron doublet) will be mass eigenstates at the temperature of leptogenesis when the mass of the lightest right-handed neutrino M_1 is smaller than about 10^9 GeV. The Boltzmann equation for the diagonal entry $Y_{\alpha\alpha}$ reads (no summation over the index α)

$$
Y'_{\alpha\alpha} = \epsilon_{\alpha\alpha} K z \frac{K_1(z)}{K_2(z)} f_1(z)\Delta_{N_1} - \frac{1}{2} z^3 K_1(z)\, f_2(z)\, K_{\alpha\alpha} Y_{\alpha\alpha} \,. \tag{4.1}
$$

Let us now solve analytically eq. (4.1) according to the magnitude of the various $K_{\alpha\alpha}$.

4.1.1 Strong wash-out regime for all flavours

In such a case all the $K_{\alpha\alpha} \gg 1$. The right-handed neutrinos N_1's are nearly in thermal equilibrium. Under these circumstances, one can set $\Delta'_{N_1} \simeq 0$ and $\Delta_{N_1} \simeq (zK_2/4g_* K)$. The lepton asymmetry for the flavour α is given by

$$Y_{\alpha\alpha} \simeq \epsilon_{\alpha\alpha} \int_0^\infty dz\, \frac{K_1}{4g_*}\, z^2\, e^{-\int_z^\infty dz'\,((z')^3/2) K_1(z') K_{\alpha\alpha}}\,. \tag{4.2}$$

Using the steepest descent method to evaluate the integral, one finds that it gets the major contribution at $\overline{z}$ such that $\overline{z} = \log\, K_{\alpha\alpha} + (5 \ln\, \overline{z}/2)$ when inverse decays become inefficient. The lepton asymmetry in the flavour α becomes

$$Y_{\alpha\alpha} \simeq 0.3\, \frac{\epsilon_{\alpha\alpha}}{g_*} \left(\frac{0.55 \times 10^{-3}\, \mathrm{eV}}{\tilde{m}_{\alpha\alpha}} \right)^{1.16}\,. \tag{4.3}$$

To get convinced that this result differs from the one usually considered in the literature, let us take the total lepton asymmetry $Y_{\mathcal{L}} = \sum_\alpha Y_{\alpha\alpha}$

$$Y_{\mathcal{L}} \simeq \sum_\alpha 0.3\, \frac{\epsilon_{\alpha\alpha}}{g_*} \left(\frac{0.55 \times 10^{-3}\, \mathrm{eV}}{\tilde{m}_{\alpha\alpha}} \right)^{1.16}\,. \tag{4.4}$$

It does not coincide with total lepton asymmetry result (2.14) in the strong wash-out regime. Indeed, the total lepton asymmetry (4.4) is the sum of the $\epsilon_{\alpha\alpha}$, each weighted by the wash-out factor $K_{\alpha\alpha}$ and not the sum of the $\epsilon_{\alpha\alpha}$ divided by the sum of the $K_{\alpha\alpha}$. Eqs. (2.14) and (4.4) coincide only if one family is dominating the contribution to the total CP asymmetry and the corresponding wash-out factor is the tiniest.

4.1.2 Weak wash-out regime for all flavours

In this case all the $K_{\alpha\alpha} \ll 1$. We assume that right-handed neutrinos are not initially present in the plasma, but they are generated by inverse decays and scatterings. As explained in section 2, the equation of motion for Y_{N_1} is well approximated by

$$Y'_{N_1} = -(K_s + Kz) \left(Y_{N_1} - Y_{N_1}^{\mathrm{EQ}} \right)\,, \tag{4.5}$$

We split the solution in two pieces. Let us define z_{EQ} the value of z at which $Y_{N_1}(z_{\mathrm{EQ}}) = Y_{N_1}^{\mathrm{EQ}}(z_{\mathrm{EQ}})$. This value has to be found a posteriori. For $z \ll z_{\mathrm{EQ}}$, we may suppose that $Y_{N_1} \ll Y_{N_1}^{\mathrm{EQ}}$ and eq. (4.5) is solved by

$$\begin{aligned} Y_{N_1}^-(z) \simeq \int_0^z dz'\,(K_s + Kz') Y_{N_1}^{\mathrm{EQ}} &= \frac{1}{4g_*} \int_0^z dz'\,(K_s + Kz')(z')^2\, K_2(z') \\ &= \frac{K}{4g_\star} \left(\frac{K_s}{K} I_1(z) + I_2(z) \right) \end{aligned} \tag{4.6}$$

With I_1 and I_2 integral involving the modified Bessel functions:

$$I_1(z) = \int_0^z x^2 K_2(x) dx \simeq f(z) + z^3 K_2(z) \tag{4.7}$$

where [6]

$$f(z) = \frac{3\pi z^3}{((9\pi)^c + (2z^3)^c)^{1/c}}, c = 0.7 \tag{4.8}$$

The integral I_2 is well known, and equals

$$I_2(z) = \int_0^z x^3 K_2(x) dx = 8 - z^3 K_3(z) \tag{4.9}$$

Therefore

$$Y_{N_1}^-(z) \simeq \frac{K}{4g_\star}\left(\frac{K_s}{K}(f(z) + z^3 K_2(z)) + 8 - z^3 K_3(z)\right) \tag{4.10}$$

As expected for weak washout, we find that the maximum number density of N_1 is proportional to K (recall $K_s \propto K$).

Let us now compute the value of z_{EQ}. We expect it to be $\gg 1$ and we therefore approximate, up to $\mathcal{O}(z^{-3/2})$: $K_2(z) \simeq K_3(z) \simeq \sqrt{\frac{\pi}{2}} z^{-1/2} e^{-z}$. Imposing $Y_{N_1}^-(z_{\mathrm{EQ}}) = Y_{N_1}^{\mathrm{EQ}}(z_{\mathrm{EQ}})$, we find

$$z_{\mathrm{EQ}} \simeq \frac{3}{2} \ln z_{\mathrm{EQ}} - \ln\left(\frac{8}{\sqrt{\pi/2}} K + 3\sqrt{\frac{\pi}{2}} K_s\right). \tag{4.11}$$

This solution is a good approximation to the real value for $K \ll 1$.

For $z > z_{\mathrm{EQ}}$, we have $(K_s + Kz) \simeq Kz$ and

$$Y_{N_1}(z) \simeq Y_{N_1}^{\mathrm{EQ}}(z_{\mathrm{EQ}}) e^{K/2\left(z_{\mathrm{EQ}}^2 - z^2\right)}. \tag{4.12}$$

We have included CP violation in $\Delta L = 1$ scattering, unlike the usual analysis, so we expect our solution for $Y_L^{\alpha\alpha}$ to have a different scaling with K than eqn (2.15). The reason is as discussed in [5, 6]: if $\not{C}\!P$ in scattering is neglected, then $Y_N \propto K$, and the N_1 decay out of equilibrium, so one expects $Y_L^{\alpha\alpha} \propto K\epsilon^{\alpha\alpha}$. However, if $\not{C}\!P$ in N_1 production ($\simeq$ scattering) is included, and washout is neglected, then the equations for Y_{N_1} and $Y_L^{\alpha\alpha}$ are identical, so $Y_L^{\alpha\alpha}(z \to \infty)$ vanishes. That is, for every $|1/\epsilon^{\alpha\alpha}|$ N_1's that are created, be it by inverse decay or scattering, an (anti)-lepton α is produced. This (anti-)asymmetry will approximately cancel against the lepton asymmetry generated later on, when the N_1 decay. However the cancellation will be imperfect, because the anti-asymmetry has more time to be washed out, so the final asymmetry should scale as $KK_{\alpha\alpha}$. After integrating by parts, this is what we find for the asymmetry in the flavour α, which is given by

$$\begin{aligned} Y_{\alpha\alpha} &\simeq \epsilon_{\alpha\alpha} \int_0^\infty dz' Y_{N_1}(z') g_{\alpha\alpha}(z') e^{-\int_{z'}^\infty dz'' g_{\alpha\alpha}(z'')}, \;\; g_{\alpha\alpha}(z) = \frac{1}{2} z^3 K_{\alpha\alpha} K_1(z) f_2(z), \\ &\simeq 1.5\, \frac{\epsilon_{\alpha\alpha}}{g_*}\left(\frac{\tilde{m}_1}{3.3\times 10^{-3}\,\mathrm{eV}}\right)\left(\frac{\tilde{m}_{\alpha\alpha}}{3.3\times 10^{-3}\,\mathrm{eV}}\right). \end{aligned} \tag{4.13}$$

We have checked numerically that this analytical formula fits the numerical results to a 30%.

Our findings hold provided that the non-resonant $\Delta L = 2$ scattering rates, in particular those mediated by the N_2 and N_3 heavy neutrinos, are slower than decays and $\Delta L = 1$ scatterings when most of the asymmetry is generated. We estimate that this applies when

$$\left(\frac{M_1}{10^{14}\,\mathrm{GeV}}\right) \ll 10^{-1}\,K_{\alpha\alpha}\,. \tag{4.14}$$

This means that $K_{\alpha\alpha}$ should be larger than 10^{-4}.

4.1.3 Strong wash-out for some flavours and either weak or mild wash-out for others

In the case in which $K_{\alpha\alpha} \gg 1$ for some flavour α, but $K_{\beta\beta} \ll 1$ for some other flavour β, it is impossible to match this case with any of the cases discussed in the section for the 'one-flavour' approximation. Because some flavours α strongly interact with the right-handed neutrinos, we may set $K = \sum_\alpha K_{\alpha\alpha} \gg 1$. Thus, right-handed neutrinos are brought to thermal equilibrium by inverse decays and by $\Delta L = 1$ scatterings to an abundance approximately given by

$$Y_{N_1}(z) \simeq Y_{N_1}^{\mathrm{EQ}}\left(1 - e^{-K_s z - K z^2/2}\right)\,. \tag{4.15}$$

The lepton asymmetry in flavour β is then given by

$$\begin{aligned} Y_{\beta\beta} &\simeq \epsilon_{\beta\beta}\int_0^\infty dz\, Y_{N_1}^{\mathrm{EQ}}(z')\left(1 - e^{-K_s z' - K z'^2/2}\right) g_{\beta\beta}(z') \\ &\simeq 0.4\,\frac{\epsilon_{\beta\beta}}{g_*}\left(\frac{\tilde{m}_{\beta\beta}}{3.3\times 10^{-3}\,\mathrm{eV}}\right)\,. \end{aligned} \tag{4.16}$$

Again, we have checked that this formula fits the numerical results to about 30 %. It has the dependence on $K_{\beta\beta}$ expected from our inclusion of $\not{C}\!P$ in scattering. The anti-asymmetry $\sim -\epsilon_{\beta\beta} s/g_*$ created during N_1 production has the leisure to be partially destroyed by processes violating β lepton number. The amount of this reduction is controlled by $K_{\beta\beta}$, so one expects the final β asymmetry $\propto \epsilon_{\beta\beta}K_{\beta\beta}$, as in eq. (4.16). Again, this result applies for $K_{\alpha\alpha} \gg 10^{-4}$.

In the case in which $K_{\alpha\alpha} \gg 1$ for some flavour α and the flavour β suffers a mild ($K_{\beta\beta} \sim 1$) wash-out, the final asymmetry in the flavour β is fitted within 30 % by the formula

$$Y_{\beta\beta} \simeq \frac{\epsilon_{\beta\beta}}{g_*}\left(\left(\frac{\tilde{m}_{\beta\beta}}{8.25\times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2\times 10^{-3}\,\mathrm{eV}}{\tilde{m}_{\beta\beta}}\right)^{-1.16}\right)^{-1}\,. \tag{4.17}$$

This formula reduces to eqs. (4.3) and (4.16) for the weak and strong wash out for the flavour β, respectively.

4.1.4 Recipe to go from the flavour asymmetries to the baryon asymmetry

To provide the complete analytical formulae for the baryon asymmetry in the case in which $M_1 \lesssim 10^9\,\mathrm{GeV}$, it suffices to provide the relationship between the lepton asymmetry for each flavour $Y_{\alpha\alpha}$ and the asymmetries Y_{Δ_α}, where $\Delta_\alpha = ((1/3)\mathcal{B} - Y_{\alpha\alpha})$ which are effectively conserved by the sphaleron interactions. The final baryon asymmetry is given by

$$Y_{\mathcal{B}} \simeq \frac{12}{37} \sum_\alpha Y_{\Delta_\alpha} \,, \tag{4.18}$$

where

$$\begin{pmatrix} Y_{\Delta_e} \\ Y_{\Delta_\mu} \\ Y_{\Delta_\tau} \end{pmatrix} = \begin{pmatrix} -22/9 & -4/9 & -4/9 \\ -4/9+5/39 & -4/9-3+8/39 & -4/9+8/39 \\ -4/9+5/39 & -4/9+8/39 & -4/9-3+8/39 \end{pmatrix} \begin{pmatrix} Y_{ee} \\ Y_{\mu\mu} \\ Y_{\tau\tau} \end{pmatrix} . \tag{4.19}$$

Thus the recipe to compute the baryon asymmetry in the case in which $M_1 \lesssim 10^9$ GeV is the following:

1) Compute the $K_{\alpha\alpha}$ parameters for the three flavours to see which of the three different previously discussed cases applies;

2) compute the lepton asymmetry in each flavour using the relevant formualae among equations (4.4), (4.13), (4.16);

3) compute the baryon asymmetry

$$Y_{\mathcal{B}} = -\frac{12}{37} \left(\frac{40}{13} Y_{ee} + \frac{51}{13} Y_{\mu\mu} + \frac{51}{13} Y_{\tau\tau} \right) . \tag{4.20}$$

Figures 1, 2 and 3 illustrate the differences between the final baryon asymmetry with flavours accounted for and the result obtained within the one-flavour approximation for some values of the wash-out parameters and CP asymmetries.

4.2 Only the tau Yukawa coupling in equilibrium: $(10^9 \lesssim M_1 \lesssim 10^{12})$ GeV

This case is realized when the mass of the lightest right-handed neutrino M_1 is larger than about 10^9 GeV, but smaller than about 10^{12} GeV.[10] Interactions mediated by h_τ are in equilibrium, but not those mediated by h_μ. The off-diagonal entries of the matrix $Y_{\tau\beta}$ all vanish and the muon and electron asymmetries are indistinguishable. The problem of finding the total baryon asymmetry reduces to a case of two flavours, the lepton ℓ_τ, and $\hat{\ell}_2$, the non-τ components of the lepton into which N_1 decays.[11] At tree level, $\ell_2 = \sum_{\alpha=e,\mu} \lambda_{1\alpha} \ell_\alpha / \left(\sum_{\alpha=e,\mu} |\lambda_{1\alpha}|^2 \right)^{1/2}$. One can therefore define two CP asymmetries, $\epsilon_{\tau\tau}$ and $\epsilon_2 = \epsilon_{ee} + \epsilon_{\mu\mu}$, and the corresponding wash-out parameters $K_{\tau\tau}$ and $K_2 = K_{ee} + K_{\mu\mu}$ for the two asymmetries $Y_{\tau\tau}$ and $Y_2 = Y_{ee} + Y_{\mu\mu}$. One then solves this set of equations as we described in the previous subsection depending upon the magnitude of the wash-out parameters $K_{\tau\tau}$ and K_2.

The recipe to compute the baryon asymmetry in the case in which $(10^9 \lesssim M_1 \lesssim 10^{12})$ GeV is the following:

1) Compute the $K_{\tau\tau}$ and K_2 parameters to see whether they are either both larger than unity, or both smaller than unity, or one larger and the other smaller than unity;

[10] In the case in which M_1 is around 10^9 GeV off-diagonal terms may be relevant. However, they are quickly damped away as soon as M_1 becomes larger than 10^9 GeV [13].

[11] Notice that at one-loop, $\overline{\ell}_2 \equiv$ the non-tau components of the *antilepton* into which N_1 decays, is not exactly ℓ_2^* [8, 14].

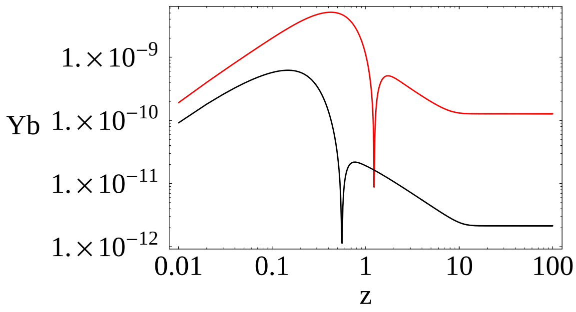

Figure 1: The total baryon asymmetry including flavours (upper) and within the one-flavour approximation (lower) as a function of z. The chosen parameters are $K_{ee} = 10$, $K_{\mu\mu} = 30$, $K_{\tau\tau} = 30$, $\epsilon_{ee} = 2.5 \times 10^{-6}$, $\epsilon_{\mu\mu} = -2 \times 10^{-6}$, $\epsilon_{\tau\tau} = 10^{-7}$ and $M_1 = 10^{10}\,\text{GeV}$.

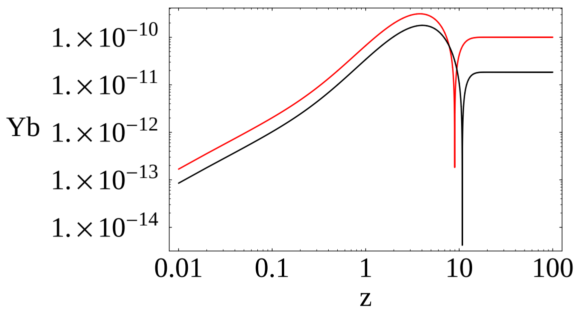

Figure 2: The total baryon asymmetry including flavours (upper) and within the one-flavour approximation (lower) as a function of z. The chosen parameters are $K_{ee} = 5 \times 10^{-2}$, $K_{\mu\mu} = 10^{-2}$, $K_{\tau\tau} = 10^{-3}$, $\epsilon_{ee} = 2.5 \times 10^{-6}$, $\epsilon_{\mu\mu} = -2 \times 10^{-6}$, $\epsilon_{\tau\tau} = 10^{-7}$ and $M_1 = 10^{10}\,\text{GeV}$.

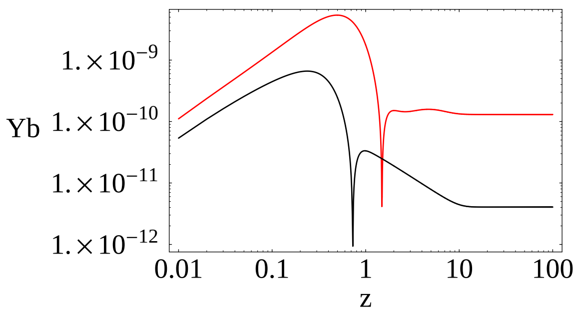

Figure 3: The total baryon asymmetry including flavours (upper) and within the one-flavour approximation (lower) as a function of z. The chosen parameters are $K_{ee} = 10$, $K_{\mu\mu} = 30$, $K_{\tau\tau} = 10^{-2}$, $\epsilon_{ee} = 2.5 \times 10^{-6}$, $\epsilon_{\mu\mu} = -2 \times 10^{-6}$, $\epsilon_{\tau\tau} = 10^{-7}$ and $M_1 = 10^{10}\,\text{GeV}$.

2) compute the CP asymmetry in each of the two flavours, and the lepton asymmetry using the relevant formulae among equations (4.4), (4.13), (4.16);

3) compute the baryon asymmetry

$$Y_{\mathcal{B}} = -\frac{12}{37}\left(\frac{115}{36}\,Y_2 + \frac{37}{9}\,Y_{\tau\tau}\right). \tag{4.21}$$

Again, our results apply if $\Delta L = 2$ scatterings are negligible, see eq. (4.14).

4.3 No charged Yukawa couplings in equilibrum: $M_1 \gtrsim 10^{12}$ GeV

This case is realized when the mass of the lightest right-handed neutrino M_1 is larger than about 10^{12} GeV. Interactions mediated by all charged lepton Yukawa couplings are out of equilibrium and all flavours are indistinguishable. The problem of finding the total baryon asymmetry reduces to a case of one flavour, the lepton

$$\hat{\ell}_1 = \sum_{\alpha=e,\mu,\tau} \lambda_{1\alpha}\ell_\alpha / \left(\sum_{\alpha=e,\mu,\tau} |\lambda_{1\alpha}|^2\right)^{1/2}.$$

One can therefore define a single CP asymmetry, $\epsilon_1 = \sum_\alpha \epsilon_{\alpha\alpha}$, and the corresponding wash out parameter $K = \sum_\alpha K_{\alpha\alpha}$. The corresponding Boltzmann equations read as those in eq. (2.11). Only in this case the commonly used formulae reported in section 2 hold with the eventual inclusion of the effects from the $\Delta L = 2$ scatterings.

5. General analysis of leptogenesis in the two right-handed neutrino model

In this section we will concentrate on the see-saw model with two right-handed neutrinos (2RHN). Oscillation experiments indicate that two new mass scales have to be introduced, in order to account for the solar and atmospheric mass splittings. These two mass scales could be associated to the masses of two right-handed neutrinos, therefore a see-saw model with just two right-handed neutrinos can already accommodate all the observations. An additional motivation to consider the two right-handed neutrino model is that it corresponds to some interesting limits of the complete see-saw model with three right-handed neutrinos, namely when the mass of the heaviest right-handed neutrino is much larger than the masses of the other two, or when the Yukawa couplings for the first generation of right-handed neutrinos are much smaller than for the other two generations.

The two right-handed neutrino model depends on many less parameters than the complete see-saw model and the theoretical analysis of leptogenesis becomes much more manageable, while preserving the key features of the model with three right-handed neutrinos. In the basis where the charged lepton Yukawa coupling and the right-handed mass matrices are diagonal, the model is defined at high energies by a 2×3 Yukawa matrix and two right-handed neutrino masses, M_1 and M_2. This amounts to eight moduli and three phases. On the other hand, at low energies the neutrino mass matrix is defined by five moduli (two masses and three mixing angles) plus two phases (the Dirac phase and the Majorana phase). In particular, since the mass matrix is rank 2 in this model, the lightest neutrino mass eigenvalue automatically vanishes and only two possible spectra may arise:

- Normal hierarchy: $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{\rm sol}}$, $m_3 = \sqrt{\Delta m^2_{\rm atm}}$
- Inverted hierarchy: $m_3 = 0$, $m_1 = \sqrt{\Delta m^2_{\rm atm} - \Delta m^2_{\rm sol}}$, $m_2 = \sqrt{\Delta m^2_{\rm atm}}$

The only Majorana phase corresponds to the phase difference between the two non-vanishing mass eigenvalues. Therefore, the number of unmeasurable parameters in the 2RHN model is reduced to three moduli and one phase.

The most general Yukawa coupling compatible with the low energy data is given by:

$$\lambda = M^{1/2} R m^{1/2} U^\dagger / v, \tag{5.1}$$

where $m = {\rm Diag}(m_1, m_2, m_3)$ is the diagonal matrix of the light neutrino mass eigenvalues (which has $m_1 = 0$ for the normal hierarchy and $m_3 = 0$ for the inverted hierarchy), $M = {\rm Diag}(M_1, M_2)$ is the diagonal matrix of the right handed neutrino masses, U is the leptonic mixing matrix, and R is an orthogonal matrix that in the 2RHN model has the following structure [24]

$$R = \begin{pmatrix} 0 & \cos\hat\theta & \xi\sin\hat\theta \\ 0 & -\sin\hat\theta & \xi\cos\hat\theta \end{pmatrix} \text{(normal hierarchy)}, \tag{5.2}$$

$$R = \begin{pmatrix} \cos\hat\theta & \xi\sin\hat\theta & 0 \\ -\sin\hat\theta & \xi\cos\hat\theta & 0 \end{pmatrix} \text{(inverted hierarchy)}, \tag{5.3}$$

with $\hat\theta$ a complex parameter and $\xi = \pm 1$ a discrete parameter that accounts for a discrete indeterminacy in R. In consequence, the elements of the neutrino Yukawa matrix read:

$$\begin{aligned} \lambda_{1\alpha} &= \sqrt{M_1}(\sqrt{m_2}\cos\hat\theta\, U^*_{\alpha 2} + \xi\sqrt{m_3}\sin\hat\theta\, U^*_{\alpha 3})/v, \\ \lambda_{2\alpha} &= \sqrt{M_2}(-\sqrt{m_2}\sin\hat\theta\, U^*_{\alpha 2} + \xi\sqrt{m_3}\cos\hat\theta\, U^*_{\alpha 3})/v, \end{aligned} \tag{5.4}$$

for the case with normal hierarchy and

$$\begin{aligned} \lambda_{1\alpha} &= \sqrt{M_1}(\sqrt{m_1}\cos\hat\theta\, U^*_{\alpha 1} + \xi\sqrt{m_2}\sin\hat\theta\, U^*_{\alpha 2})/v, \\ \lambda_{2\alpha} &= \sqrt{M_2}(-\sqrt{m_1}\sin\hat\theta\, U^*_{\alpha 1} + \xi\sqrt{m_2}\cos\hat\theta\, U^*_{\alpha 2})/v, \end{aligned} \tag{5.5}$$

for the case with inverted hierarchy. The three moduli and the phase that are not determined by low energy experiments are identified in this parametrization with the two right-handed masses M_1 and M_2, and the complex parameter $\hat\theta$.

Notice that we have included all the low energy phases in the definition of the matrix U, i.e. we have written the leptonic mixing matrix in the form $U = V\ {\rm Diag}(1, e^{-i\phi/2}, 1)$, where ϕ is the Majorana phase and V has the form of the CKM matrix:

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{5.6}$$

so that the neutrino mass matrix is $\mathcal{M} = U^* {\rm Diag}(m_1, m_2, m_3) U^\dagger$. It is straightforward to check that the Yukawa coupling eq. (5.1) indeed satisfies the see-saw formula $\mathcal{M} = \lambda^T {\rm Diag}(M_1^{-1}, M_2^{-1}) \lambda v^2$.

The flavour CP asymmetries can be readily computed in terms of low energy data and the unmeasurable parameters M_1, M_2 and $\hat{\theta}$ substituting the expression for the Yukawa coupling in eq. (3.6). In the limit $M_1 \ll M_2$ we obtain for the case with normal hierarchy the following result:

$$\begin{aligned}\epsilon_{\alpha\alpha} \simeq \frac{3}{8\pi v^2}\frac{M_1}{m_2|\cos^2\hat{\theta}|+m_3|\sin^2\hat{\theta}|}&[(m_3^2|U_{\alpha 3}|^2 - m_2^2|U_{\alpha 2}|^2)\,\mathrm{Im}\sin^2\hat{\theta}+\\&+\xi\sqrt{m_2 m_3}(m_3+m_2)\,\mathrm{Re}U^*_{\alpha 2}U_{\alpha 3}\,\mathrm{Im}\sin\hat{\theta}\cos\hat{\theta}+\\&+\xi\sqrt{m_2 m_3}(m_3-m_2)\,\mathrm{Im}U^*_{\alpha 2}U_{\alpha 3}\,\mathrm{Re}\sin\hat{\theta}\cos\hat{\theta}],\end{aligned} \tag{5.7}$$

and analogously for the case with inverted hierarchy, with the changes in the labels $3 \to 2$ and $2 \to 1$.

We will analyze numerically the predictions for the baryon asymmetry taking flavour properly into account, as described in section 3, and also for comparison following the conventional computation ignoring flavour, as described in section 2. We will perform this analysis both for the case of normal hierarchy and inverted hierarchy, fixing the atmospheric and solar mass splittings and mixing angles to the values suggested by oscillation experiments, $\Delta m^2_{\rm atm} \simeq 2.2\times 10^{-3}\,\mathrm{eV}^2$, $\Delta m^2_{\rm sol} \simeq 8.1\times 10^{-5}\,\mathrm{eV}^2$, $\theta_{23}\simeq \pi/4$ and $\theta_{12}\simeq\pi/6$, respectively [23]. The remaining parameters in the leptonic mixing matrix are fixed to $\theta_{13}=0.1$, $\delta=\pi/4$, $\phi=\pi/3$. In general, the results are not very sensitive to the value of θ_{13}. On the other hand, the aspect of the plots does depend on the precise value of the phases δ and ϕ, although our main conclusions remain valid.

We will show our results for different values of M_1, to cover the possibility that only the tau Yukawa interaction is in equilibrium ($M_1 \gtrsim 10^9\,\mathrm{GeV}$) or that the tau and the muon Yukawa interactions are in equilibrium ($M_1 \lesssim 10^9\,\mathrm{GeV}$), and for different values of the complex parameter $\hat{\theta}$, restricting ourselves to the region $|\hat{\theta}|<1$. We will also fix $\xi=1$, although the results for the case $\xi=-1$ can be read from our results by changing $\hat{\theta}\to-\hat{\theta}$, as can be checked from eq. (5.7).

In figure 4 we show the result of the calculation of the baryon asymmetry in the complex plane of $\hat{\theta}$ when the mass spectrum presents a normal hierarchy, following the calculation that takes flavour properly into account (left plots) and following the conventional calculation ignoring flavour (right plots). In the upper plots we show the results when $M_1 = 10^8\,\mathrm{GeV}$, so that the tau and muon Yukawa interactions are in equilibrium, whereas in the lower plots we take $M_1 = 10^{10}\,\mathrm{GeV}$, so that only the tau Yukawa interaction is in equilibrium. It is apparent from these plots that the proper treatment of flavour in the Boltzmann equations is necessary in order to calculate correctly the baryon asymmetry.[12]

The differences between the correct analysis of leptogenesis, taking flavour into account, and the conventional analysis are more acute along the axes $\mathrm{Im}\hat{\theta}=0$ and $\mathrm{Re}\hat{\theta}=0$, and around the values of $\hat{\theta}$ that correspond to texture zeros in the Yukawa coupling. The

[12] In particular, the prediction for the baryon asymmetry in the conventional computation is symmetric under $\mathrm{Re}\hat{\theta}\to-\mathrm{Re}\hat{\theta}$ and $\mathrm{Im}\hat{\theta}\to-\mathrm{Im}\hat{\theta}$, and consequently independent of the discrete parameter ξ. On the other hand, when flavour is taken into account, the parameter ξ indeed plays a role in the computation of the baryon asymmetry.

JHEP09(2006)010

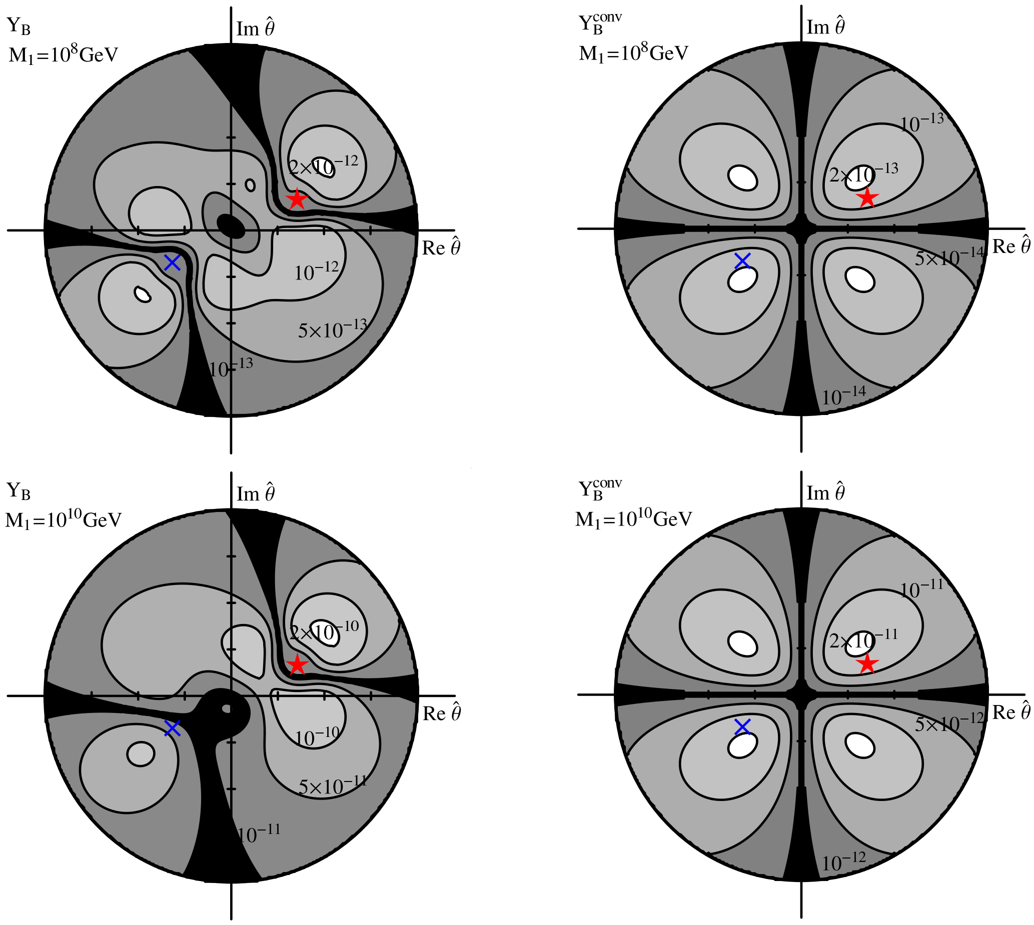

Figure 4: Baryon asymmetry in the 2RHN model when the neutrinos present a spectrum with normal hierarchy. In the left plots we show the result of the calculation that takes flavour into account, whereas in the right plots, the result of the conventional calculation that ignores flavour. On the top plots we take $M_1 = 10^8$GeV, so that the tau and muon Yukawa couplings are in equilibrium, while in the lower plots, $M_1 = 10^{10}$GeV, so that only the tau Yukawa coupling is in equilibrium. In these plots we have fixed $\theta_{13} = 0.1$, $\delta = \pi/4$, $\phi = \pi/3$ and the remaining neutrino parameters to their favoured experimental values.

corresponding values of $\hat{\theta}$ are indicated in the plots with a red star, when $|\lambda_{13}| \simeq 0$, and with a blue cross, when $|\lambda_{12}| \simeq 0$ (a texture zero in the (1,1) position appears at $|\hat{\theta}| > 1$). The point in the $\hat{\theta}$ complex plane where $|\lambda_{1\alpha}| \simeq 0$ can be derived from eq. (5.4), being the result:

$$\tan \hat{\theta}_0^{(\alpha)} \simeq -\xi \sqrt{\frac{m_2}{m_3}} \frac{U^*_{\alpha 2}}{U^*_{\alpha 3}}. \tag{5.8}$$

The reason why in the conventional analysis the baryon asymmetry vanishes along the axes can be easily understood from the expression of the total CP asymmetry in the 2RHN model. For instance, for the case with normal hierarchy:

$$\epsilon_1 \simeq \frac{3M_1}{8\pi v^2} \frac{(m_3^2 - m_2^2)\, \mathrm{Im} \sin^2 \hat{\theta}}{m_2 |\cos^2 \hat{\theta}| + m_3 |\sin^2 \hat{\theta}|}. \tag{5.9}$$

Hence, the total CP asymmetry vanishes when $\mathrm{Im}\hat{\theta} = 0$ (i.e. R real) and when $\mathrm{Re}\hat{\theta} = 0$ (since in this case $\sin^2\hat{\theta} = -\sinh^2|\hat{\theta}|$, and R is again real). This is not necessarily the case when flavour is properly taken into account, as can be realized from the expressions of the flavour CP asymmetries, eq. (3.7).

On the other hand, when the Yukawa coupling presents an approximate texture zero, the difference stems mainly from the washing-out of the asymmetry. When the spectrum has a normal hierarchy, the CP asymmetry in the α-th flavour is comparable to the total CP asymmetry. However, the smallness of the interaction of the lightest right-handed neutrino with ℓ_α translates into a weak wash-out of the asymmetry, in stark contrast with the result of the same analysis following the conventional computation, where the total CP asymmetry is necessarily strongly washed-out (recall that in the 2RHN model with normal hierarchy $\widetilde{m}_1 \geq \sqrt{\Delta m^2_{\mathrm{sol}}}$, so $K > 1$). As a consequence, when $\lambda_{1\alpha} \simeq 0$, the actual prediction for the baryon asymmetry can be around one order of magnitude larger than previously believed. Interestingly enough, in realistic models strict texture zeros rarely appear; there are usually subleading effects that produce small entries instead of strict texture zeros. To be precise, texture zeros normally appear in a basis where the charged lepton Yukawa coupling and/or the right-handed neutrino mass matrix are slightly non diagonal. Therefore, the diagonalization of these matricial couplings to express the Lagrangian as in eq. (2.1) will lift the texture zero in the neutrino Yukawa matrix, yielding a small entry instead. Finally, even if the texture zero was strict at the high energy scale, radiative corrections could lift them.

The differences between the computation of the baryon asymmetry taking into account flavour or not are even more acute in the case of a spectrum with inverted hierarchy, as can be realized from figure 5. As in the case of the spectrum with normal hierarchy, the maximum differences arise along the axes $\mathrm{Im}\hat{\theta} = 0$ and $\mathrm{Re}\hat{\theta} = 0$, and around the values of $\hat{\theta}$ that correspond to texture zeros in the first row of the Yukawa coupling, also indicated in these plots with a red star and a blue cross.[13] The precise value of $\hat{\theta}$ where $|\lambda_{1\alpha}| \simeq 0$ is:

$$\tan\hat{\theta}_0^{(\alpha)} \simeq -\xi\sqrt{\frac{m_1}{m_2}\frac{U^*_{\alpha 1}}{U^*_{\alpha 2}}}. \tag{5.10}$$

Around this value for $\hat{\theta}$, the difference between the conventional calculation (right plots) and the calculation taking into account flavour can be as large as three orders of magnitude when $M_1 = 10^8$ GeV (see upper left plot) or two orders of magnitude when $M_1 = 10^{10}$ GeV (see lower left plot).

The reason for this huge enhancement is double. First, in the conventional calculation ignoring flavour, when the spectrum has an inverted hierarchy the total CP asymmetry goes as $\Delta m^2_{\mathrm{sol}}/\sqrt{\Delta m^2_{\mathrm{atm}}}$. However, the flavour CP asymmetries go as $\sqrt{\Delta m^2_{\mathrm{atm}}}$, therefore, the individual flavour CP asymmetries can be a factor of 20 bigger than the total CP asymmetry. Secondly, the total lepton asymmetry computed ignoring flavour is strongly

[13]Notice the proximity of both points, which is due to the maximal atmospheric mixing. Furthermore, as θ_{13} approaches zero, the two points collapse into one, which reflects the fact that in the limit with $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, there is an exact $\mu \leftrightarrow \tau$ symmetry, and imposing $\lambda_{12} = 0$ automatically implies $\lambda_{13} = 0$.

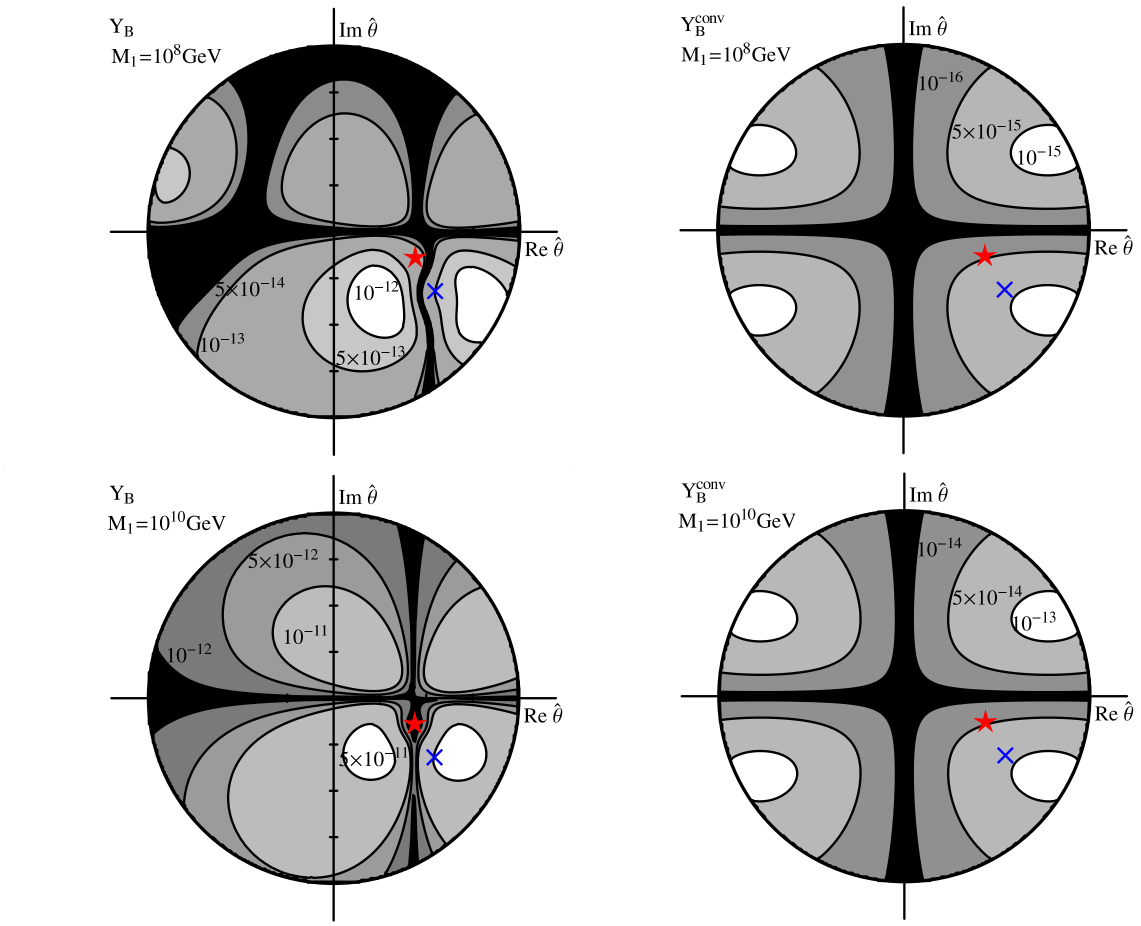

Figure 5: The same as figure (4), but for a spectrum with inverted hierarchy.

washed-out, since $\widetilde{m}_1 \geq \sqrt{\Delta m^2_{\rm atm}}$. yielding a suppressed baryon asymmetry. On the contrary, when there is an approximate texture zero, $\lambda_{1\alpha} \simeq 0$, the lepton asymmetry in the α-th flavour is only weakly washed-out. These two effects combined are the responsible of the huge enhancement of the baryon asymmetry when the spectrum has an inverted hierarchy and there is an approximate texture zero in the first row of the neutrino Yukawa coupling.

6. Texture zeros in the two right-handed neutrino model

In this section we would like to study carefully the predictions for the baryon asymmetry in the case that there are approximate texture zeros in the neutrino Yukawa coupling. Texture zeros commonly arise in model constructions based on the Froggatt-Nielsen mechanism [25]. The assignment of different charges under an extra symmetry to particles of different generations, translates into a Yukawa coupling with a non-trivial structure in the effective theory, once the extra symmetry is spontaneously broken. The assignment of charges could be such that the resulting Yukawa coupling could have one or several entries which are very small compared to the others. In some instances, the entry for the Yukawa coupling could exactly vanish at the high energy scale, although these vanishing entries are usually filled when the fields are brought to the basis where the charged lepton Yukawa coupling and the right-handed mass matrix are both diagonal. In addition to this, radiative effects can also fill these entries.

In the 2RHN model an exact texture zero in the Yukawa coupling fixes the value of $\tan\hat{\theta}$ in terms of low energy data. Namely, if $|\lambda_{1\beta}| = 0$,

$$\tan\hat{\theta}_0^{(\beta)} \simeq -\xi\sqrt{\frac{m_2}{m_3}}\frac{U^*_{\beta 2}}{U^*_{\beta 3}} \quad \text{(normal hierarchy)}, \tag{6.1}$$

$$\tan\hat{\theta}_0^{(\beta)} \simeq -\xi\sqrt{\frac{m_1}{m_2}}\frac{U^*_{\beta 1}}{U^*_{\beta 2}} \quad \text{(inverted hierarchy)}. \tag{6.2}$$

By perturbing the Yukawa coupling around this value of $\hat{\theta}_0^{(\beta)}$ it is possible to lift the texture zero, while still reproducing the observed masses and mixing angles. The R-matrices that yield a viable Yukawa coupling with an approximate texture zero are:

$$R = \begin{pmatrix} 0 & \cos\hat{\theta}_0^{(\beta)} & \xi\sin\hat{\theta}_0^{(\beta)} \\ 0 & -\sin\hat{\theta}_0^{(\beta)} & \xi\cos\hat{\theta}_0^{(\beta)} \end{pmatrix} - \rho e^{i\omega}\begin{pmatrix} 0 & \sin\hat{\theta}_0^{(\beta)} & -\xi\cos\hat{\theta}_0^{(\beta)} \\ 0 & \cos\hat{\theta}_0^{(\beta)} & \xi\sin\hat{\theta}_0^{(\beta)} \end{pmatrix} \quad \text{(normal hierarchy)}, \tag{6.3}$$

$$R = \begin{pmatrix} \cos\hat{\theta}_0^{(\beta)} & \xi\sin\hat{\theta}_0^{(\beta)} & 0 \\ -\sin\hat{\theta}_0^{(\beta)} & \xi\cos\hat{\theta}_0^{(\beta)} & 0 \end{pmatrix} - \rho e^{i\omega}\begin{pmatrix} \sin\hat{\theta}_0^{(\beta)} & -\xi\cos\hat{\theta}_0^{(\beta)} & 0 \\ \cos\hat{\theta}_0^{(\beta)} & \xi\sin\hat{\theta}_0^{(\beta)} & 0 \end{pmatrix} \quad \text{(inverted hierarchy)}, \tag{6.4}$$

where $\rho e^{i\omega}$ parametrizes the departure from the strict texture zero in the $\hat{\theta}$-parameter space, i.e. $\hat{\theta} = \hat{\theta}_0^{(\beta)} + \rho e^{i\omega}$. In the case with normal hierarchy the Yukawa couplings explicitly read,

$$\begin{aligned} \lambda_{1\alpha} &\simeq \sqrt{\tfrac{M_1}{\mathcal{M}_{\beta\beta}}}\left[\sqrt{m_2 m_3}\left(U^*_{\alpha 2}U^*_{\beta 3} - U^*_{\beta 2}U^*_{\alpha 3}\right) + \xi\rho e^{i\omega}\mathcal{M}_{\alpha\beta}\right]/v, \\ \lambda_{2\alpha} &\simeq \sqrt{\tfrac{M_2}{\mathcal{M}_{\beta\beta}}}\left[\xi\mathcal{M}_{\alpha\beta} - \sqrt{m_2 m_3}\rho e^{i\omega}\left(U^*_{\alpha 2}U^*_{\beta 3} - U^*_{\beta 2}U^*_{\alpha 3}\right)\right]/v, \end{aligned} \tag{6.5}$$

while in the case with inverted hierarchy,

$$\begin{aligned} \lambda_{1\alpha} &\simeq \sqrt{\tfrac{M_1}{\mathcal{M}_{\beta\beta}}}\left[\sqrt{m_1 m_2}\left(U^*_{\alpha 1}U^*_{\beta 2} - U^*_{\beta 1}U^*_{\alpha 2}\right) + \xi\rho e^{i\omega}\mathcal{M}_{\alpha\beta}\right]/v, \\ \lambda_{2\alpha} &\simeq \sqrt{\tfrac{M_2}{\mathcal{M}_{\beta\beta}}}\left[\xi\mathcal{M}_{\alpha\beta} - \sqrt{m_1 m_2}\rho e^{i\omega}\left(U^*_{\alpha 1}U^*_{\beta 2} - U^*_{\beta 1}U^*_{\alpha 2}\right)\right]/v. \end{aligned} \tag{6.6}$$

These expressions can be substituted in eqs. (3.7) and eq. (3.5) to derive the CP asymmetries and the washout factors in the α-th flavour, in the case that $|\lambda_{1\beta}| \simeq 0$. Expanding for small values of ρ and keeping the lowest order terms we obtain for the case with normal hierarchy the following CP flavour asymmetries:

$$\begin{aligned} \epsilon_{\alpha\alpha} &\simeq -\frac{3M_1 m_3}{8\pi v^2}\frac{1}{|U_{\beta 2}|^2 + |U_{\beta 3}|^2}\mathrm{Im}\left[e^{i\phi/2}\frac{\mathcal{M}^*_{\beta\beta}}{|\mathcal{M}_{\beta\beta}|}\sum_\gamma \epsilon_{\alpha\beta\gamma}\left(U_{\alpha_3}U^*_{\beta 2} - \frac{m_2}{m_3}U_{\alpha_2}U^*_{\beta 3}\right)U_{\gamma 1}\right] \\ &\qquad\qquad \text{if } \alpha \neq \beta, \\ \epsilon_{\beta\beta} &\simeq -\frac{3M_1 m_3}{8\pi v^2}\frac{|\mathcal{M}_{\beta\beta}|}{\sqrt{m_2 m_3}}\frac{\xi\rho}{|U_{\beta 2}|^2 + |U_{\beta 3}|^2}\mathrm{Im}\left[U^*_{\beta 2}U_{\beta_3}\left(e^{i\omega} + \frac{m_2}{m_3}e^{-i\omega}\right)\right], \end{aligned} \tag{6.7}$$

and the following washout factors:

$$K_{\alpha\alpha} \simeq \frac{m_2 m_3}{\tilde{m}^* |\mathcal{M}_{\beta\beta}|} \left| \sum_\gamma \epsilon_{\alpha\beta\gamma} U_{\gamma 1} \right|^2 \text{ if } \alpha \neq \beta,$$
$$K_{\beta\beta} \simeq \frac{|\mathcal{M}_{\beta\beta}|}{\tilde{m}^*} \rho^2. \tag{6.8}$$

The corresponding formulas for the case with inverted hierarchy are:

$$\epsilon_{\alpha\alpha} \simeq -\frac{3M_1 m_2}{8\pi v^2} \frac{1}{|U_{\beta 1}|^2 + |U_{\beta 2}|^2} \text{Im} \left[e^{i\phi/2} \frac{\mathcal{M}^*_{\beta\beta}}{|\mathcal{M}_{\beta\beta}|} \sum_\gamma \epsilon_{\alpha\beta\gamma} \left(U_{\alpha_2} U^*_{\beta 1} - \frac{m_1}{m_2} U_{\alpha_1} U^*_{\beta 2} \right) U_{\gamma 3} \right] \quad \text{if } \alpha \neq \beta,$$
$$\epsilon_{\beta\beta} \simeq -\frac{3M_1 m_2}{8\pi v^2} \frac{|\mathcal{M}_{\beta\beta}|}{\sqrt{m_1 m_2}} \frac{\xi\rho}{|U_{\beta 1}|^2 + |U_{\beta 2}|^2} \text{Im} \left[U^*_{\beta 1} U_{\beta_2} \left(e^{i\omega} + \frac{m_1}{m_2} e^{-i\omega} \right) \right], \tag{6.9}$$

and

$$K_{\alpha\alpha} \simeq \frac{m_1 m_2}{\tilde{m}^* |\mathcal{M}_{\beta\beta}|} \left| \sum_\gamma \epsilon_{\alpha\beta\gamma} U_{\gamma 3} \right|^2, \text{if } \alpha \neq \beta,$$
$$K_{\beta\beta} \simeq \frac{|\mathcal{M}_{\beta\beta}|}{\tilde{m}^*} \rho^2. \tag{6.10}$$

The predictions for the flavour asymmetries and the washout factors, and accordingly for the final baryon asymmetry, will depend on the low energy observables and the "texture zero uplifting" parameters ρ and ω. In some cases, the prediction of the baryon asymmetry will not depend on ρ and ω, and therefore it could be possible to establish a connection between leptogenesis and low energy observables [24, 26]. Let us analyze separately the different possibilites:

6.1 Texture zero in the (1,1) position

The possibility of a texture zero in the (1,1) position is perhaps the most interesting one from the phenomenological point of view. The postulates that the up and down quark matrices are symmetric in the first two generations and that they present a simultaneous zero in the (1,1) position, lead to the renown prediction for the Cabibbo angle $\lambda_C \simeq \sqrt{m_d/m_s}$ [27]. The success and robustness of this prediction may suggest we apply the same rationale to the leptonic sector, and impose a texture zero in the (1,1) position of the neutrino Yukawa matrix, and perhaps also in the charged lepton Yukawa coupling and the right-handed neutrino mass matrices.

The flavour CP asymmetries for the case with normal hierarchy can be straightforwardly obtained from eq. (6.7), being the result:

$$\epsilon_{ee} \simeq \frac{3M_1 m_3}{8\pi v^2} \frac{|\mathcal{M}_{ee}|}{\sqrt{m_2 m_3}} 2\xi\rho \sin\theta_{13} \sin(\delta - \phi/2 - \omega),$$
$$\epsilon_{\mu\mu} \simeq -\frac{3M_1 m_3}{8\pi v^2} \frac{\sqrt{3}}{8} \frac{m_3}{|\mathcal{M}_{ee}|} \sin\theta_{13} \left[\frac{m_2}{m_3} \sin\delta + \left(\frac{m_2}{m_3}\right)^2 \sin(\delta - \phi) + \frac{4}{\sqrt{3}} \sin\theta_{13} \sin(2\delta - \phi) \right],$$

$$\epsilon_{\tau\tau} \simeq \frac{3M_1 m_3}{8\pi v^2}\frac{\sqrt{3}}{8}\frac{m_3}{|\mathcal{M}_{ee}|}\sin\theta_{13}\left[\frac{m_2}{m_3}\sin\delta + \left(\frac{m_2}{m_3}\right)^2\sin(\delta-\phi) - \frac{4}{\sqrt{3}}\sin\theta_{13}\sin(2\delta-\phi)\right], \tag{6.11}$$

where $\mathcal{M}_{ee} \simeq m_3 e^{2i\delta}\sin^2\theta_{13} + e^{i\phi}m_2/4$ is the (1,1) element of the effective neutrino mass matrix (recall that this matrix element is precisely the relevant one for analyses of neutrinoless double beta decay).

On the other hand, the washout parameters are, from eq. (6.8):

$$\begin{aligned}
K_{ee} &\simeq \frac{|\mathcal{M}_{ee}|}{\tilde{m}^*}\rho^2,\\
K_{\mu\mu} &\simeq \frac{1}{8}\frac{m_2 m_3}{\tilde{m}^*|\mathcal{M}_{ee}|}(1-2\sqrt{3}\sin\theta_{13}\cos\delta),\\
K_{\tau\tau} &\simeq \frac{1}{8}\frac{m_2 m_3}{\tilde{m}^*|\mathcal{M}_{ee}|}(1+2\sqrt{3}\sin\theta_{13}\cos\delta),
\end{aligned} \tag{6.12}$$

with $\tilde{m}^* \simeq 3\times 10^{-3}\,\mathrm{eV}$. In view of the present experimental bound $\sin\theta_{13} \lesssim 0.2$, it follows that $|\mathcal{M}_{ee}| \lesssim m_2/4$ and accordingly $K_{\mu\mu,\tau\tau} \gtrsim m_3/(2\tilde{m}^*) \simeq 20$. Hence, when there is an approximate (1,1) texture zero and ρ is sufficiently small, the muon and the tau CP asymmetries are strongly washed-out, while the electron CP asymmetry is only weakly washed-out.

For the case of an inverted hierarchy of neutrinos, the flavour CP asymmetries read:

$$\begin{aligned}
\epsilon_{ee} &\simeq \frac{3M_1 m_2}{8\pi v^2}\frac{|\mathcal{M}_{ee}|}{\sqrt{m_1 m_2}}\xi\rho\frac{\sqrt{3}}{2}\cos\omega\sin(\phi/2),\\
\epsilon_{\mu\mu} &\simeq -\frac{3M_1 m_2}{8\pi v^2}\frac{3}{8}\left[\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}\frac{\sin\phi}{\sqrt{10+6\cos\phi}} + \frac{\sin\theta_{13}}{\sqrt{3}}\left(\frac{2\sin\delta+\sin(\delta-\phi)-3\sin(\delta+\phi)}{\sqrt{10+6\cos\phi}}\right)\right],\\
\epsilon_{\tau\tau} &\simeq -\frac{3M_1 m_2}{8\pi v^2}\frac{3}{8}\left[\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}\frac{\sin\phi}{\sqrt{10+6\cos\phi}} - \frac{\sin\theta_{13}}{\sqrt{3}}\left(\frac{2\sin\delta+\sin(\delta-\phi)-3\sin(\delta+\phi)}{\sqrt{10+6\cos\phi}}\right)\right],
\end{aligned} \tag{6.13}$$

and the washout-parameters,

$$\begin{aligned}
K_{ee} &\simeq \frac{|\mathcal{M}_{ee}|}{\tilde{m}^*}\rho^2,\\
K_{\mu\mu} &\simeq \frac{1}{2}\frac{m_1 m_2}{\tilde{m}^*|\mathcal{M}_{ee}|},\\
K_{\tau\tau} &\simeq \frac{1}{2}\frac{m_1 m_2}{\tilde{m}^*|\mathcal{M}_{ee}|},
\end{aligned} \tag{6.14}$$

where in this case $\mathcal{M}_{ee} \simeq (3m_1 + e^{i\phi}m_2)/4$. Therefore, it follows that

$$K_{\mu\mu,\tau\tau} \gtrsim \sqrt{\Delta m^2_{\mathrm{atm}}}/(2\tilde{m}^*),$$

so the muon and tau asymmetries are strongly washed-out, whereas the electron asymmetry is weakly washed-out.

These formulas can be straightforwardly applied to the recipes presented in subsection 4.1.3, to go from the flavour asymetries to the baryon asymmetry in the different regimes for M_1. We find numerically that when $M_1 \lesssim 10^9\,\mathrm{GeV}$ it is not possible to reproduce the observed baryon asymmetry; in order to reproduce the data it is necessary $M_1 \gtrsim 10^{11}\,\mathrm{GeV}$ so that only the tau Yukawa interactions are in thermal equilibrium. If this is the case, leptogenesis only depends on observables that are in principle measurable at low energies, since the electron asymmetry is always negligible compared to the muon asymmetry (recall that when $M_1 \gtrsim 10^{10}\,\mathrm{GeV}$ the relevant quantities to compute the baryon asymmetry are $Y_2 = Y_{ee} + Y_{\mu\mu}$ and $K_2 = K_{ee} + K_{\mu\mu}$). Using eqs. (4.16), (4.21) and taking into account that $K_{\mu\mu} \simeq K_{\tau\tau}$, it follows that when $\sin\theta_{13}$ is large, in the case with normal hierarchy $Y_{\mathcal{B}} \propto \sin^2\theta_{13}\sin(2\delta-\phi)$, while in the case with inverted hierarchy the relation is more complicated (it goes roughly as the term propotional to $\sin\theta_{13}$ in eq. (6.13)). On the other hand, when $\sin\theta_{13}$ is small, in the case with normal hierarchy the baryon asymmetry is very suppressed (it goes as $\sin^2\theta_{13}$), while in the case with inverted hierarchy $Y_{\mathcal{B}} \propto \sin\phi/\sqrt{10+6\cos\phi}$, being in this case the asymmetry suppressed by $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$.

6.2 Texture zero in the (1,2) position

In the case that there is an approximate texture zero in the (1,2) position of the neutrino Yukawa matrix, the flavour CP asymmetries read:

$$
\begin{aligned}
\epsilon_{ee} &\simeq \frac{3M_1 m_3}{8\pi v^2}\frac{\sqrt{3}}{7}\left[\sin\theta_{13}\sin(\delta-\phi) - \frac{\sqrt{3}}{4}\frac{m_2^2}{m_3^2}\sin\phi\right], \\
\epsilon_{\mu\mu} &\simeq -\frac{3M_1 m_3}{8\pi v^2}\sqrt{\frac{m_3}{m_2}}\frac{\sqrt{3}}{7}\xi\rho\sin(\phi/2+\omega), \\
\epsilon_{\tau\tau} &\simeq \frac{3M_1 m_3}{8\pi v^2}\frac{3}{7}\left[\sin\phi + \frac{\sin\theta_{13}}{\sqrt{3}}\sin(\delta-\phi)\right].
\end{aligned} \tag{6.15}
$$

On the other hand, the washout parameters read:

$$
\begin{aligned}
K_{ee} &\simeq \frac{1}{4}\frac{m_2}{\tilde{m}^*}(1-2\sqrt{3}\sin\theta_{13}\cos\delta), \\
K_{\mu\mu} &\simeq \frac{1}{2}\frac{m_3}{\tilde{m}^*}\rho^2, \\
K_{\tau\tau} &\simeq \frac{3}{2}\frac{m_2}{\tilde{m}^*}.
\end{aligned} \tag{6.16}
$$

Therefore, the electron asymmetry and the muon asymmetries are only weakly washed-out, while the tau asymmetry is strongly washed-out (when $M_1 \gtrsim 10^{10}\mathrm{GeV}$, Y_2 would be weakly washed-out and $Y_{\tau\tau}$, strongly washed-out).

We find that when there is an approximate texture zero in the (1,2) position, $M_1 \gtrsim 10^{10}\,\mathrm{GeV}$ is necessary in order to reproduce the observed baryon asymmetry. In the strict texture zero limit, the baryon asymmetry is dominated by the tau lepton asymmetry and therefore there is a well defined connection between leptogenesis and low energy observables, $Y_{\mathcal{B}} \propto \sin\phi$. Despite this connection it becomes more diffuse as we depart from the texture zero limit, the connection still holds in the region in the vicinity of the (1,2) texture zero where the baryon asymmetry is enhanced (see figure 4 lower left plot).

JHEP09(2006)010

We find a similar behaviour when the spectrum presents an inverted hierarchy. In this case the flavour asymmetries are:

$$\epsilon_{ee} \simeq -\frac{3M_1 m_2}{8\pi v^2}\frac{3}{4}\left[\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}\frac{\sin\phi}{\sqrt{10+6\cos\phi}}+\frac{\sin\theta_{13}}{\sqrt{3}}\left(\frac{2\sin\delta+\sin(\delta-\phi)-3\sin(\delta+\phi)}{\sqrt{10+6\cos\phi}}\right)\right],$$
$$\epsilon_{\mu\mu} \simeq -\frac{3M_1 m_2}{8\pi v^2}\sqrt{\frac{3}{2}}\frac{\xi\rho}{8}\sqrt{5+3\cos\phi}\sin(\phi/2)\cos\omega,$$
$$\epsilon_{\tau\tau} \simeq \frac{3M_1 m_2}{8\pi v^2}\frac{\sqrt{3}}{4}\sin\theta_{13}\left[\frac{2\sin\delta+\sin(\delta-\phi)-3\sin(\delta+\phi)}{\sqrt{10+6\cos\phi}}\right], \tag{6.17}$$

and the washout factors,

$$K_{ee} \simeq \frac{1}{2}\frac{m_1 m_2}{\tilde{m}^*|\mathcal{M}_{\mu\mu}|},$$
$$K_{\mu\mu} \simeq \frac{|\mathcal{M}_{\mu\mu}|}{\tilde{m}^*}\rho^2,$$
$$K_{\tau\tau} \simeq \frac{m_1 m_2}{\tilde{m}^*|\mathcal{M}_{\mu\mu}|}\sin^2\theta_{13}, \tag{6.18}$$

with $\mathcal{M}_{\mu\mu} \simeq m_1/8 + 3/8 m_2 e^{i\phi}$. Using that $|\mathcal{M}_{\mu\mu}| \lesssim m_2/2$, we find that the electron asymmetry is necessarily strongly washed-out, whereas the muon and the tau asymmetries are only weakly washed-out (when $M_1 \gtrsim 10^{10}$GeV, Y_2 would be strongly washed-out and $Y_{\tau\tau}$, weakly washed-out).

For the case with inverted hierarchy, we require again $M_1 \gtrsim 10^{10}$GeV to reproduce the observed asymmetry (or even larger, when $\sin\theta_{13}$ is small). The baryon asymmetry is in this case also dominated by the tau asymmetry, except when $\sin\theta_{13}$ is very small. In the case that the baryon asymmetry is dominated by the tau asymmetry, although there exists a connection between leptogenesis and low energy observables, this connection is too complicated to be of any practical use. On the other hand, when $\sin\theta_{13}$ is very small, there is no relation whatsoever, since leptogenesis would depend on the unobservable parameters ρ and ω.

6.3 Texture zero in the (1,3) position

Finally, in the case that the texture zero appears in the (1,3) position, the flavour CP asymmetries are:

$$\epsilon_{ee} \simeq -\frac{3M_1 m_3}{8\pi v^2}\frac{\sqrt{3}}{7}\left[\sin\theta_{13}\sin(\delta-\phi)+\frac{\sqrt{3}}{4}\frac{m_2^2}{m_3^2}\sin\phi\right],$$
$$\epsilon_{\mu\mu} \simeq \frac{3M_1 m_3}{8\pi v^2}\frac{3}{7}\left[\sin\phi-\frac{\sin\theta_{13}}{\sqrt{3}}\sin(\delta-\phi)\right],$$
$$\epsilon_{\tau\tau} \simeq \frac{3M_1 m_3}{8\pi v^2}\sqrt{\frac{m_3}{m_2}}\frac{\sqrt{3}}{7}\xi\rho\sin(\phi/2+\omega). \tag{6.19}$$

On the other hand, the washout parameters read:

$$K_{ee} \simeq \frac{1}{4}\frac{m_2}{\tilde{m}^*}(1+2\sqrt{3}\sin\theta_{13}\cos\delta),$$

$$\begin{aligned} K_{\mu\mu} &\simeq \frac{3}{2}\frac{m_2}{\tilde{m}^*}, \\ K_{\tau\tau} &\simeq \frac{1}{2}\frac{m_3}{\tilde{m}^*}\rho^2. \end{aligned} \tag{6.20}$$

Therefore, in this case, the electron and the tau asymmetries are weakly washed-out, and the muon asymmetry, strongly washed-out. On the other hand, for $M_1 \gtrsim 10^9$GeV the relevant quantity to estimate the washout is $K_2 = K_{ee} + K_{\mu\mu} > 1$, so in this regime Y_2 is strongly washed out and $Y_{\tau\tau}$ is weakly washed out.

Similarly to the case of the (1,2) texture zero, this case requires $M_1 \gtrsim 10^{10}$ GeV to reproduce the observations. Furthermore, the baryon asymmetry in the vicinity of the texture zero is dominated by the muon asymmetry and hence depends mainly on $\sin\phi$. This behaviour occurs in particular, in the region where the baryon asymmetry is enhanced in figure 4, lower left plot.

The case with inverted hierarchy presents some qualitative differences with respect to the case with normal hierarchy. When neutrinos have an inverted hierarchy, the flavour CP asymmetries read:

$$\begin{aligned} \epsilon_{ee} &\simeq -\frac{3M_1 m_2}{8\pi v^2}\frac{3}{4}\left[\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}\frac{\sin\phi}{\sqrt{10+6\cos\phi}} - \frac{\sin\theta_{13}}{\sqrt{3}}\left(\frac{2\sin\delta + \sin(\delta-\phi) - 3\sin(\delta+\phi)}{\sqrt{10+6\cos\phi}}\right)\right], \\ \epsilon_{\mu\mu} &\simeq -\frac{3M_1 m_2}{8\pi v^2}\frac{\sqrt{3}}{4}\sin\theta_{13}\left[\frac{2\sin\delta + \sin(\delta-\phi) - 3\sin(\delta+\phi)}{\sqrt{10+6\cos\phi}}\right], \\ \epsilon_{\tau\tau} &\simeq -\frac{3M_1 m_2}{8\pi v^2}\sqrt{\frac{3}{2}}\frac{\xi\rho}{8}\sqrt{5+3\cos\phi}\sin(\phi/2)\cos\omega, \end{aligned} \tag{6.21}$$

and the washout parameters,

$$\begin{aligned} K_{ee} &\simeq \frac{1}{2}\frac{m_1 m_2}{\tilde{m}^*|\mathcal{M}_{\tau\tau}|}, \\ K_{\mu\mu} &\simeq \frac{m_1 m_2}{\tilde{m}^*|\mathcal{M}_{\tau\tau}|}\sin^2\theta_{13}, \\ K_{\tau\tau} &\simeq \frac{|\mathcal{M}_{\tau\tau}|}{\tilde{m}^*}\rho^2, \end{aligned} \tag{6.22}$$

with $\mathcal{M}_{\tau\tau} \simeq m_1/8 + 3/8 m_2 e^{i\phi}$. As for the case with the (1,2) texture zero, we find that the electron asymmetry is necessarily strongly washed-out, whereas the muon and the tau asymmetries are only weakly washed-out (also, in the regime where only the tau Yukawa interactions are in equilibrium, Y_2 would be strongly washed-out and $Y_{\tau\tau}$, weakly washed-out).

It is important to note that all the flavour asymmetries in eq. (6.21) have a suppression factor, with different origins. As a consequence, in the limit of the strict texture zero, the resulting baryon asymmetry is very small and $M_1 \gtrsim 5\times 10^{12}$GeV would be necessary to accommodate observationl data. However, as we depart from the texture zero limit, we find a huge enhancement of the baryon asymmetry, that can allow right-handed neutrino masses as low as $M_1 \sim 10^{10}$GeV, independently of the value of $\sin\theta_{13}$. The reason is that the tau CP asymmetry can become less suppressed, and at the same time the resulting tau lepton asymmetry can be sizable since the tau asymmetry is only weakly washed-out (on the contrary, $\epsilon_2 = \epsilon_{ee} + \epsilon_{\mu\mu}$ could be comparable to $\epsilon_{\tau\tau}$, but is strongly washed-out). As a result, the baryon asymmetry is dominated by the tau asymmetry and hence any connection between leptogenesis and low energy observables is lost in this region of enhanced baryon asymmetry.

7. The case of R real

In section 5 it was discussed that there are two situations where the differences between the computation of the baryon asymmetry taking into account flavour or not are maximal, namely when there is an approximate texture zero in the neutrino Yukawa matrix, and when the matrix R is real. The case with R real physically corresponds to the class of models where CP is an exact symmetry in the right-handed neutrino sector. The reason for this can be more easily understood working in the basis where the charged lepton Yukawa coupling and the right-handed mass matrix are diagonal, so that the neutrino Yukawa matrix is the only coupling in the leptonic Lagrangian that violates CP. More specifically, the neutrino Yukawa coupling can be written in its singular value decomposition, $\lambda = V_R^\dagger \mathrm{Diag}(\lambda_1, \lambda_2, \lambda_3) V_L$. Hence, the CP violation in the right-handed neutrino sector is encoded in the phases in V_R, that can be extracted from diagonalizing the combination $\lambda\lambda^\dagger = V_R^\dagger \mathrm{Diag}(\lambda_1^2, \lambda_2^2, \lambda_3^2) V_R$. On the other hand, using the parametrization of the Yukawa coupling in eq. (5.1), this same combination of matrices can be written as $\lambda\lambda^\dagger = M^{1/2} R m R^\dagger M^{1/2}/v^2$. Comparing the two expressions it is apparent that R is real if and only if V_R is real, i.e. when there is no CP violation in the right-handed sector.[14]

In this limit the flavour CP asymmetries and the baryon asymmetry depend exclusively on the phases of the left-handed sector, that are in turn uniquely determined by the low energy phases. Consequently, in this limit the leptogenesis mechanism is tightly connected to the low energy phases. This connection is more apparent from the expression of the flavour CP asymmetries in the parametrization eq. (5.1):

$$\begin{aligned}\epsilon_{\alpha\alpha} &\simeq \frac{3M_1}{8\pi v^2} \frac{\mathrm{Im}\left(\sum_\beta \sqrt{m_\beta} R_{1\beta} U^*_{\alpha\beta}\right)\left(\sum_\beta \sqrt{m_\beta^3} R_{1\beta} U_{\alpha\beta}\right)}{\sum_\beta m_\beta R_{1\beta}^2} \\ &= \frac{3M_1}{8\pi v^2} \frac{\sum_\beta \sum_{\gamma>\beta} \sqrt{m_\beta m_\gamma}(m_\gamma - m_\beta) R_{1\beta} R_{1\gamma} \mathrm{Im} U^*_{\alpha\beta} U_{\alpha\gamma}}{\sum_\beta m_\beta R_{1\beta}^2}. \end{aligned} \tag{7.1}$$

This discussion suggests that the observation of low energy CP violation would constitute an important hint to the leptogenesis mechanism. In a general case with R complex, the low energy phases in the leptonic mixing matrix could stem from the phases in the left-handed sector, in the right-handed sector, or in both sectors. In *any* of the cases, and barring unnatural cancellations, a baryon asymmetry is necessarily generated through the mechanism of leptogenesis, as long as at least one of the lepton Yukawa interactions is in equilibrium (corresponding roughly to $M_1 \lesssim 10^{12}$ GeV). This result only follows when

[14] Furthermore, it can be checked that there is mixing in V_R if and only if there is mixing in R, and that mixing in any 2×2 block in R translates into mixing in the same block of the matrix V_R.

flavour is correctly taken into account in the Boltzmann equations. In previous analyses of leptogenesis ignoring flavour, the observation of low energy CP violation did not automatically imply the existence of a baryon asymmetry, since the possibility existed that the low energy phases could stem exclusively from the left-handed sector and hence be irrelevant for leptogenesis.

8. Conclusions

Thermal leptogenesis is an attractive and minimal mechanism to make the baryon asymmetry of the Universe. The asymmetry is commonly calculated by solving a Boltzmann equation for the total lepton asymmetry (one-flavour approximation). In a previous paper [13] we studied the impact of lepton flavours (charged lepton Yukawa couplings) on the Boltzmann equations (one for each lepton flavour) and discussed the phenomenological implications for leptogenesis.

It may be counter-intuitive that flavour matters in leptogenesis, since Yukawa couplings are a small perturbative correction. We have shown that flavour effects are relevant when the interaction rates mediated by the the charged Yukawa couplings are faster than the typical timescale for leptogenesis. The charged Yukawa rates may be dropped from the Boltzmann equations *provided* the latter are written in the flavour basis, where the charged Yukawa couplings cannot change the flavour of the asymmetries. This implies that one should solve Boltzmann equations for each flavour. In the region where our approximations are valid the equations are not coupled.

The final value of the baryon asymmetry depends on the CP asymmetry in each flavour α and on the washing out by the lepton number α violating processes. Taking into account these flavour dependent washing out factors generically enhances the baryon asymmetry with respect to the usual one-flavour approximation, in the limit of strong washout.

In this paper we have provided analytical approximations for the final baryon asymmetry with flavours accounted for. These depend on the temperature of leptogenesis, and can be obtained following the procedure of section 4.1.4, or of the end of section 4.2. We also included CP violation in the $\Delta L = 1$ scatterings relevant for N_1 production.

In the two right-handed neutrino (2RHN) model, we have compared our results obtained with flavoured Boltzmann equations against the usual one-flavour approximation, to illustrate the big impact that flavour has on leptogenesis. We have found that there are two situations where the differences between the treatment of leptogenesis taking flavour properly into account and previous analyses, that ignored flavour, are maximal. The first one arises when the neutrino Yukawa coupling present approximate texture zeros in the first row, so that the CP asymmetry in that flavour is only weakly washed-out. As a consequence, we have found that thermal leptogenesis in the 2RHN model can produce the observed baryon asymmetry for masses of the lightest right-handed neutrino smaller than previously believed, namely 10^{10} GeV for the case with normal hierarchy and 5×10^{10} GeV for the case with inverted hierarchy (to be compared with 10^{11} GeV and 10^{13} GeV, respectively, from the conventional computation ignoring flavour). The second situation corresponds to the limit in which CP is an exact symmetry in the right-handed neutrino sector. In this case, the conventional computation would yield an exactly vanishing baryon asymmetry, whereas the computation that takes flavour into account could predict a sizable baryon asymmetry.

Acknowledgments

It is a pleasure to thank E. Nardi for sharing with us his results and for many useful discussions. We also thank L. Covi, G.F. Giudice, A. Pilaftsis and A. Strumia for discussions. S.D. thanks in particular A. Strumia for many thought-provoking and enlightening emails, and A. Notari for related results. A.R. thanks M. Passera for usefull discussions. A.A. and F.X.J.M. acknowledge the support of the Agence Nationale de la Recherche ANR through the project JC05-43009, NEUPAC.

References

[1] A.D. Sakharov, *Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe*, *JETP Lett.* **5** (1967) 24;
For a review, see A. Riotto and M. Trodden, *Recent progress in baryogenesis*, *Ann. Rev. Nucl. Part. Sci.* **49** (1999) 35 [hep-ph/9901362].

[2] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett.* **B 174** (1986) 45.

[3] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, *On the anomalous electroweak baryon number nonconservation in the early universe*, *Phys. Lett.* **B 155** (1985) 36.

[4] P. Minkowski, *$\mu \to e\gamma$ at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the supergravity Stony Brook Workshop*, New York 1979, P. Van Nieuwenhuizen and D. Freedman eds.;
T. Yanagida, *Proceedinds of the workshop on unified theories and baryon number in the universe*, Tsukuba, Japan 1979, A. Sawada and A. Sugamoto eds.;
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[5] See, e.g. G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM*, *Nucl. Phys.* **B 685** (2004) 89 [hep-ph/0310123].

[6] W. Buchmuller, P. Di Bari and M. Plumacher, *Leptogenesis for pedestrians*, *Ann. Phys. (NY)* **315** (2005) 305 [hep-ph/0401240].

[7] A partial list: W. Buchmuller, P. Di Bari and M. Plumacher, *Cosmic microwave background, matter-antimatter asymmetry and neutrino masses*, *Nucl. Phys.* **B 643** (2002) 367 [hep-ph/0205349];
J.R. Ellis, M. Raidal and T. Yanagida, *Observable consequences of partially degenerate leptogenesis*, *Phys. Lett.* **B 546** (2002) 228 [hep-ph/0206300];
G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and M.N. Rebelo, *Leptogenesis, CP-violation and neutrino data: what can we learn?*, *Nucl. Phys.* **B 640** (2002) 202 [hep-ph/0202030];
R.N. Mohapatra, S. Nasri and H.-B. Yu, *Leptogenesis, μ-τ symmetry and θ_{13}*, *Phys. Lett.* **B 615** (2005) 231 [hep-ph/0502026];

A. Broncano, M.B. Gavela and E. Jenkins, *Neutrino physics in the seesaw model*, *Nucl. Phys.* **B 672** (2003) 163 [hep-ph/0307058];
A. Pilaftsis, *CP-violation and baryogenesis due to heavy majorana neutrinos*, *Phys. Rev.* **D 56** (1997) 5431 [hep-ph/9707235];
E. Nezri and J. Orloff, *Neutrino oscillations vs. leptogenesis in* SO(10) *models*, *JHEP* **04** (2003) 020 [hep-ph/0004227];
S. Davidson and A. Ibarra, *Leptogenesis and low-energy phases*, *Nucl. Phys.* **B 648** (2003) 345 [hep-ph/0206304];
S. Davidson, *From weak-scale observables to leptogenesis*, *JHEP* **03** (2003) 037 [hep-ph/0302075];
S.T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, *The see-saw mechanism, neutrino Yukawa couplings, LFV decays $l_i \to l_j + \gamma$ and leptogenesis*, *Nucl. Phys.* **B 739** (2006) 208 [hep-ph/0510404];
S. Lavignac, I. Masina and C. A. Savoy, *Large solar angle and seesaw mechanism: a bottom-up perspective*, *Nucl. Phys.* **B 633** (2002) 139 [hep-ph/0202086];
E.K. Akhmedov, M. Frigerio and A.Y. Smirnov, *Probing the seesaw mechanism with neutrino data and leptogenesis*, *JHEP* **09** (2003) 021 [hep-ph/0305322];
F. Deppisch, H. Pas, A. Redelbach and R. Ruckl, *Constraints on SUSY seesaw parameters from leptogenesis and lepton flavor violation*, *Phys. Rev.* **D 73** (2006) 033004 [hep-ph/0511062].

[8] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, *Baryogenesis through leptogenesis*, *Nucl. Phys.* **B 575** (2000) 61 [hep-ph/9911315].

[9] A. Pilaftsis and T.E.J. Underwood, *Electroweak-scale resonant leptogenesis*, *Phys. Rev.* **D 72** (2005) 113001 [hep-ph/0506107];
A. Anisimov, A. Broncano and M. Plumacher, *The CP-asymmetry in resonant leptogenesis*, *Nucl. Phys.* **B 737** (2006) 176 [hep-ph/0511248].

[10] T. Endoh, T. Morozumi and Z.-h. Xiong, *Primordial lepton family asymmetries in seesaw model*, *Prog. Theor. Phys.* **111** (2004) 123 [hep-ph/0308276];
T. Fujihara et al., *Cosmological family asymmetry and CP-violation*, *Phys. Rev.* **D 72** (2005) 016006 [hep-ph/0505076].

[11] O. Vives, *Flavoured leptogenesis: a successful thermal leptogenesis with N_1 mass below 10^8 GeV*, *Phys. Rev.* **D 73** (2006) 073006 [hep-ph/0512160].

[12] A. Abada and M. Losada, *Leptogenesis with four gauge singlets*, *Nucl. Phys.* **B 673** (2003) 319 [hep-ph/0306180];
A. Abada, H. Aissaoui and M. Losada, *A model for leptogenesis at the TeV scale*, *Nucl. Phys.* **B 728** (2005) 55 [hep-ph/0409343].

[13] A. Abada, S. Davidson, F.-X. Josse-Michaux, M. Losada and A. Riotto, *Flavour issues in leptogenesis*, *JCAP* **04** (2006) 004 [hep-ph/0601083].

[14] E. Nardi, Y. Nir, E. Roulet and J. Racker, *The importance of flavor in leptogenesis*, *JHEP* **01** (2006) 164 [hep-ph/0601084].

[15] J.A. Harvey and M.S. Turner, *Cosmological baryon and lepton number in the presence of electroweak fermion number violation*, *Phys. Rev.* **D 42** (1990) 3344.

[16] L. Covi, E. Roulet and F. Vissani, *CP-violating decays in leptogenesis scenarios*, *Phys. Lett.* **B 384** (1996) 169 [hep-ph/9605319].

[17] J.M. Cline, K. Kainulainen and K.A. Olive, *Protecting the primordial baryon asymmetry from erasure by sphalerons*, *Phys. Rev.* **D 49** (1994) 6394 [hep-ph/9401208].

[18] S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett.* **B 535** (2002) 25 [hep-ph/0202239];
K. Hamaguchi, H. Murayama and T. Yanagida, *Leptogenesis from sneutrino-dominated early universe*, *Phys. Rev.* **D 65** (2002) 043512 [hep-ph/0109030].

[19] J.A. Casas and A. Ibarra, *Oscillating neutrinos and $\mu \to e, \gamma$*, *Nucl. Phys.* **B 618** (2001) 171 [hep-ph/0103065].

[20] A. Pilaftsis and T.E.J. Underwood, *Resonant leptogenesis*, *Nucl. Phys.* **B 692** (2004) 303 [hep-ph/0309342].

[21] E. Roulet, L. Covi and F. Vissani, *On the CP asymmetries in Majorana neutrino decays*, *Phys. Lett.* **B 424** (1998) 101 [hep-ph/9712468];
W. Buchmuller and M. Plumacher, *CP asymmetry in Majorana neutrino decays*, *Phys. Lett.* **B 431** (1998) 354 [hep-ph/9710460].

[22] E.W. Kolb and S. Wolfram, *Baryon number generation in the early universe*, *Nucl. Phys.* **B 172** (1980) 224, erratum *ibid.* **B 195** (1982) 542.

[23] M. Maltoni, T. Schwetz, M.A. Tortola and J.W.F. Valle, *Status of global fits to neutrino oscillations*, *New J. Phys.* **6** (2004) 122 [hep-ph/0405172].

[24] A. Ibarra and G.G. Ross, *Neutrino properties from Yukawa structure*, *Phys. Lett.* **B 575** (2003) 279 [hep-ph/0307051]; *Neutrino phenomenology: the case of two right handed neutrinos*, *Phys. Lett.* **B 591** (2004) 285 [hep-ph/0312138]. See also Petcov et al., in ref. [7].

[25] C.D. Froggatt and H.B. Nielsen, *Hierarchy of quark masses, Cabibbo angles and CP-violation*, *Nucl. Phys.* **B 147** (1979) 277.

[26] P.H. Frampton, S.L. Glashow and T. Yanagida, *Cosmological sign of neutrino CP-violation*, *Phys. Lett.* **B 548** (2002) 119 [hep-ph/0208157];
M. Raidal and A. Strumia, *Predictions of the most minimal see-saw model*, *Phys. Lett.* **B 553** (2003) 72 [hep-ph/0210021];
S.F. King, *Leptogenesis-MNS link in unified models with natural neutrino mass hierarchy*, *Phys. Rev.* **D 67** (2003) 113010 [hep-ph/0211228].

[27] R. Gatto, G. Sartori and M. Tonin, *Weak selfmasses, Cabibbo angle and broken* $SU(2) \times SU(2)$, *Phys. Lett.* **B 28** (1968) 128.