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THEORETICAL INTERPRETATION OF STORAGE RING
RESULTS AND VECTOR MESON DOMINANCE

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LECTURE I

ONE PHOTON EXCHANGE APPROXIMATION

1. TOTAL ANNIHILATION CROSS-SECTION

1.1 We consider the electron-positron annihilation into a final state F
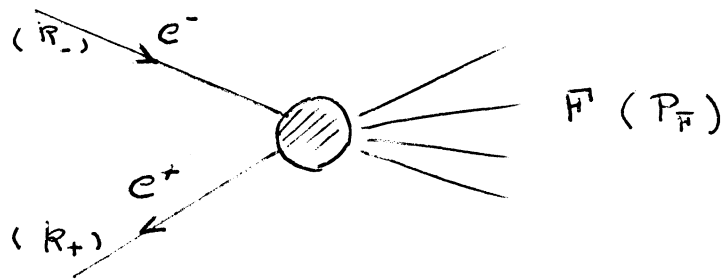
Fig. 1

The cross-section is related to the transition matrix element by

$$d\sigma(e^+e^- \to F) = (2\pi)^4 \delta_4(k_+ + k_- - P_F)\left(\frac{m^2}{4} \sum_{\substack{spins \\ e^+e^-}}\right) S_F \frac{1}{[(k_+ \cdot k_-)^2 - m^4]^{1/2}} \left|T(e^+e^- \to F)\right|^2$$

where m is the lepton mass and S_F a symbol including a summation over the polarizations and a phase space integration for the particles belonging to F.

The kinematics being indicated on Fig. 1, we define

$$q = k_+ + k_- \qquad\qquad s = -q^2 = 2m^2 - 2k_+ \cdot k_-$$

so that the total cross-section is written as

$$\sigma_{TOT}(e^+e^- \to F) = \frac{1}{[s(s - 4m^2)]^{1/2}} Q^F$$

with

$$\overline{\sigma}^{\,F} = (2\pi)^4 \delta_4 (q - p_F) \sum_F \left(\frac{m^2}{2} \sum_{\substack{spins \\ e^+e^-}} \right) | T(e^+e^- \Rightarrow F) |^2 \; .$$

1.2 In the one photon exchange approximation, the transition matrix element is written as the product of the two matrix elements of the electromagnetic current
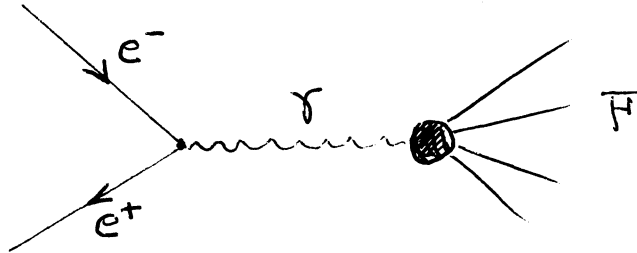
<u>Fig. 2</u>

$$T(e^+e^- \Rightarrow F) = -\frac{e^2}{s} \overline{v}(k_+ \sigma_+) \gamma^\mu u(k_- \sigma_-) \langle F | J^{em}_\mu (0) | 0 \rangle \qquad (\text{I.1})$$

where $u(k_- \sigma_-)$ is the Dirac spinor for the electron of momentum k_- and spin σ_-
  $v(k_+ \sigma_+)$ is the Dirac spinor for the positron of momentum k_+ and spin σ_+.
The electric charge is normalized so that

$$\frac{e^2}{4\pi} = \alpha = \frac{1}{137} \; .$$

We now define two tensors

$$m^{\mu\nu} = \frac{m^2}{2} \sum_{\sigma_+ \sigma_-} \left[\overline{v}(k_+ \sigma_+) \gamma^\mu u(k_- \sigma_-) \right] \left[\overline{v}(k_+ \sigma_+) \gamma^\nu u(k_- \sigma_-) \right]^* \qquad (\text{I.2})$$

$$M^F_{\mu\nu} = (2\pi)^6 \sum_F \delta_4 (q - p_F) \langle F | J^{em}_\mu (0) | 0 \rangle \langle F | J^{em}_\nu (0) | 0 \rangle^* \qquad (\text{I.3})$$

so that the Lorentz invariant quantity ζ^F is simply written as

$$\zeta^F = \frac{4\alpha^2}{s^2} \, m^{\mu\nu} \, M^F_{\mu\nu} \quad .$$

It is straightforward to compute the leptonic tensor $m^{\mu\nu}$ and the result is simply

$$m^{\mu\nu} = \frac{1}{2}\left[k^\mu_+ k^\nu_- + k^\mu_- k^\nu_+ + \frac{s}{2} g^{\mu\nu} \right] \quad . \tag{I.4}$$

As a consequence of the hermiticity and of the conservation of the electromagnetic current, we observe the two interesting properties

$$m^{\mu\nu} = m^{\nu\mu} \qquad\qquad q_\mu \, m^{\mu\nu} = 0 \quad .$$

For the hadronic part $M^F_{\mu\nu}$, because of the conservation of the electromagnetic current, we can construct only one Lorentz covariant

$$M^F_{\mu\nu} = \frac{1}{3}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \, Q^2 \, M^F(s) \quad .$$

Saturating the leptonic and hadronic tensors we obtain

$$m^{\mu\nu} M^F_{\mu\nu} = \frac{1}{6} \left(s + 2m^2 \right) Q^2 \, M^F(s)$$

$$\zeta^F = \frac{2\alpha^2}{3} \frac{s + 2m^2}{s^2} \, Q^2 \, M^F(s)$$

and the total cross-section takes the final form

$$\sigma_{TOT}\left(e^+ e^- \Rightarrow F \right) = \frac{2\alpha^2}{3} \frac{1}{s^2} \frac{1 + \frac{2m^2}{s}}{\left(1 - \frac{4m^2}{s} \right)^{1/2}} \, Q^2 \, M^F(s) \quad .$$

For relativistic electrons and positrons we always have $\frac{m^2}{s} \ll 1$ and the total cross-section is simply

$$\widetilde{\sigma}_{TOT}(e^+e^- \to F) = \frac{2\alpha^2}{3} \frac{1}{s^2} \sigma_2 M^F(s) . \qquad (I.5)$$

1.3 The width for the decay of a virtual time-like photon of mass $\sqrt{s}$ into a final state F is given by

$$\sqrt{s} \, \Gamma(\gamma \to F) = \frac{\alpha}{2\pi} \, \varepsilon^\mu(q) \varepsilon^{\nu*}(q) \, M^F_{\mu\nu}$$

where $\varepsilon(q)$ is the photon polarization four-vector. Using the property

$$\varepsilon^\mu \varepsilon^{\nu*} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) = \varepsilon^2 = 1$$

we obtain

$$\sqrt{s} \, \Gamma(\gamma \to F) = \frac{\alpha}{3\pi} \, \sigma_2 M^F$$

and the total cross-section can also be written in the form

$$\sigma_{TOT}(e^+e^- \to F) = \frac{4\pi\alpha}{s^{3/2}} \, \Gamma(\gamma \to F) . \qquad (I.6)$$

2. ONE PARTICLE DETECTED IN THE FINAL STATE

2.1 We study the case where only one particle of mass M and spin J is detected in the final state, and we call p its energy momentum four vector

$$|F\rangle \Rightarrow \sum_{F'} |p, F'\rangle .$$

The operator S_F is then written as

$$S_F \Rightarrow \frac{d_3p}{E} \frac{N_p}{(2\pi)^3} \sum_{p_p^\ell}^{'} \sum_{F'} S_{F'}$$

where N_p is a normalization factor ($N_p = \frac{1}{2}$ for bosons, $N_p = M$ for non-zero mass fermions).

By analogy with the case of inelastic lepton-nucleon scattering we introduce the hadronic tensor $\widetilde{T}_{\mu\nu}$

$$\widetilde{T}_{\mu\nu} = \frac{N_p}{2J+1} \sum_{pol} \,\backslash \sum_{\Gamma} S_{\eta} \, (2\pi)^3 \, \delta_4(q-p-P_\eta) \langle P, \Gamma | J_\mu^{em}(\omega)|c\rangle \langle P, \Gamma | J_\nu^{em}(\omega)|c\rangle^* \qquad (I.7)$$

and the original tensor $M_{\mu\nu}^F$ becomes

$$M_{\mu\nu}^F \Rightarrow (2J+1) \frac{d_3p}{E} \widetilde{T}_{\mu\nu} \;.$$

In the centre of mass system we have the following kinematics

$$k_- = (\vec{k}, k_0) \qquad k_+ = (-\vec{k}, k_0) \qquad p = (\vec{p}, E)$$

$$Z = \hat{k} \cdot \hat{p}$$

where $\hat{}$ indicates unit vector.

The invariant differential element d^3p/E is written, in the c.m. system, as

$$\frac{d_3p}{E} \Rightarrow 2\pi \, p \, dE \, dZ$$

and the differential cross-section takes the form

$$\frac{d^2\sigma}{dZ \, dE} = \frac{1}{[S(S-4m^2)]^{1/2}} \frac{8\pi\alpha^2 (2J+1)}{S^2} \, p \, m^{\mu\nu} \, \widetilde{T}_{\mu\nu} \;.$$

2.2 Because of the properties of the leptonic tensor $m^{\mu\nu}$, or equivalently as because of the hermiticity and of the conservation of the electromagnetic current at the hadronic vertex, the tensor $\tilde{T}_{\mu\nu}$ can be decomposed on the basis of two tensors, it is convenient to write

$$\tilde{T}_{\mu\nu} = \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right)\frac{1}{M^2} \tilde{V}_1 (s,u^2) + \frac{1}{2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)\tilde{V}_2 (s,u^2) \quad (\text{I.8})$$

The two structure functions $\tilde{V}_1$ and $\tilde{V}_2$ depend only on the two scalars one can construct

$$s = -q^2 \qquad u^2 = -(q-p)^2 = s + M^2 + 2 \, p \cdot q$$

We saturate the leptonic and hadronic tensors defining two invariants I_1 and I_2 by

$$I_1 = \frac{p_\mu p_\nu}{M^2} m^{\mu\nu} = \frac{1}{M^2}(p \cdot k_+)(p \cdot k_-) - \frac{1}{4} s$$

$$I_2 = \frac{1}{4}(s + 2m^2)$$

and the differential cross-section is written as

$$\frac{d^2\sigma}{dz \, dE} = \frac{8\pi\alpha^2(2J+1)}{[s(s-4m^2)]^{1/2}} \frac{p}{s^2} \left\{ I_1 \, \tilde{V}_1 (s,u^2) + I_2 \, \tilde{V}_2 (s,u^2) \right\}$$

Let us use the c.m. system variables

$$I_1 = \frac{1}{4} \frac{p^2}{M^2} \left\{ s(1 - z^2) + 4m^2 z^2 \right\}$$

$$I_2 = \frac{1}{4}(s + 2m^2)$$

For relativistic leptons $[(m^2/s) \ll 1]$ the differential cross-section

takes the simple form

$$\frac{d^2\sigma}{dz\,dE} = \frac{2\pi\alpha^2(2J+1)}{s^2} \, p \, \left\{ (1-z^2)\frac{p^2}{M^2}\widetilde{V}_1(s,u^2) + \widetilde{V}_2(s,u^2) \right\} \tag{I.9}$$

Instead of the c.m. energy E of the observed particle one can use the invariant variable u^2 which, in the c.m. system reduces to

$$u^2 = s + M^2 - 2E\sqrt{s} \qquad p = \frac{1}{2\sqrt{s}}\left\{ [s-(u+M)^2][s-(u-M)^2] \right\}^{1/2}$$

and we have

$$\frac{d^2\sigma}{dz\,du^2} = \frac{\pi\alpha^2(2J+1)}{s^2} \, \frac{p}{\sqrt{s}} \, \left\{ (1-z^2)\frac{p^2}{M^2}\widetilde{V}_1(s,u^2) + \widetilde{V}_2(s,u^2) \right\} \tag{I.10}$$

2.3 The diagonal elements of the tensor $\widetilde{T}_{\mu\nu}$ are obviously definite positive functions of s, u^2 in the physical region. We then obtain inequalities for the structure functions

$$\widetilde{V}_2(s,u^2) \geqslant 0 \tag{I.11}$$

$$\left[\frac{(p\cdot q)^2}{M^2 s} - 1 \right]\widetilde{V}_1(s,u^2) + \frac{1}{2}\widetilde{V}_2(s,u^2) \geqslant 0 \tag{I.12}$$

The differential cross-section is then equivalently written as

$$\frac{d^2\sigma}{dz\,dE} = \frac{2\pi\alpha^2(2J+1)}{s^2} \, p \, \left\{ (1-z^2)\left[\frac{p^2}{M^2}\widetilde{V}_1(s,u^2) + \frac{1}{2}\widetilde{V}_2(s,u^2) \right] + (1+z^2)\frac{1}{2}\widetilde{V}_2(s,u^2) \right\} \tag{I.13}$$

and the coefficients of $1 - z^2$ and $1 + z^2$ are both definite positive functions of s and u^2.

2.4 The structure functions $\tilde{V}_1$ and $\tilde{V}_2$ can be related to partial decay widths of polarized time-like photons, in the same way as the structure functions for inelastic lepton scattering can be interpreted in terms of total cross-sections with polarized space-like photons[1]

$$\sum_{\Gamma}{}' \sqrt{3}\, \frac{d}{dE}\, \Gamma\left(\gamma \Rightarrow p+\Gamma\right) = 4\alpha\,(2J+1)\, p\; \varepsilon^\mu \varepsilon^{\nu *} \tilde{T}_{\mu\nu}$$

The Lorentz invariant combination $2\Gamma_T + \Gamma_L$ corresponds to the projection operator

$$\sum_{p,\ell}{}' \varepsilon^\mu \varepsilon^{\nu *} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

and we find

$$\sqrt{3}\left[2\,\frac{d\Gamma_T}{dE} + \frac{d\Gamma_L}{dE}\right] = 2\alpha\,(2J+1)\, p\left[\frac{p^2}{M^2}\tilde{V}_1 + \frac{3}{2}\tilde{V}_2\right]$$

The transverse virtual photon polarizations are defined in the photon rest system with the projection operator

$$\sum_{p,\ell T}{}' \varepsilon^m \varepsilon^{n *} = \delta^{mn} - \frac{p^m p^n}{p^2}$$

and we find

$$\sqrt{3}\; 2\,\frac{d\Gamma_T}{dE} = 2\alpha\,(2J+1)\, p\,\tilde{V}_2$$

It follows the reciprocal relations

$$\tilde{V}_1 = \frac{1}{2\alpha\,(2J+1)}\; \frac{\sqrt{3}}{p}\; \frac{M^2}{p^2}\left[\frac{d\Gamma_L}{dE} - \frac{d\Gamma_T}{dE}\right] \tag{I.14}$$

$$\tilde{V}_2 = \frac{1}{\alpha\,(2J+1)}\; \frac{\sqrt{3}}{p}\; \frac{d\Gamma_T}{dE} \tag{I.15}$$

and

$$\frac{d\,\Gamma_T}{dE} = \alpha\,(2J+1)\,\frac{p}{\sqrt{s}}\,\widetilde{V}_2 \tag{I.16}$$

$$\frac{d\,\Gamma_L}{dE} = 2\alpha\,(2J+1)\,\frac{p}{\sqrt{s}}\left[\frac{p^2}{M^2}\,\widetilde{V}_1 + \frac{1}{2}\,\widetilde{V}_2\right] \tag{I.17}$$

The differential cross-section (I.13) takes the form

$$\frac{d^2\sigma}{dz\,dE} = \frac{\pi\alpha}{s^{3/2}}\left\{(1-z^2)\frac{d\,\Gamma_L}{dE} + (1+z^2)\frac{d\,\Gamma_T}{dE}\right\} \tag{I.18}$$

and after angular integration

$$\frac{d\sigma}{dE} = \frac{4\pi\alpha}{3\,s^{3/2}}\left\{\frac{d\,\Gamma_L}{dE} + 2\,\frac{d\,\Gamma_T}{dE}\right\}$$

The positivity conditions (I.11) and (I.12) become obvious from the relations (I.16) and (I.17).

3. THE ELASTIC CROSS-SECTIONS

    A final state F, consisting of a particle of mass M and spin J and its antiparticle will be called "elastic" by definition.

    The energy E takes the value $\sqrt{s}/2$ and the invariant variable u^2 is simply M^2. The structure functions $\widetilde{V}_1$ and $\widetilde{V}_2$ become the product of a delta distribution $\delta(u^2 - M^2)$ by a function of s only. From the relation between u^2 and E

$$\delta\,(u^2 - M^2) = \frac{1}{2\sqrt{s}}\,\delta\left(E - \frac{\sqrt{s}}{2}\right)$$

3.1 Spin zero case ($\pi^+\pi^-$, K^+K^-, $K^0\bar{K}^0$, etc.)

From the Lorentz covariance, and using the conservation of the electromagnetic current, we have

$$\langle p, \bar{p} \mid J_\mu^{em}(o) \mid o \rangle = (p - \bar{p}_\mu) \, \bar{F}(s) \qquad \text{(I.19)}$$

The electric factor of the spin zero particle is normalized at s = 0 to its electric charge Q. The structure functions $\tilde{V}_1$ and $\tilde{V}_2$ reduce to

$$\tilde{V}_1 \Rightarrow 2M^2 \delta(u^2 - M^2) \mid \bar{F}(s) \mid^2$$

$$\tilde{V}_2 \Rightarrow 0$$

The angular distribution is of the pure $1 - Z^2$ type[2]

$$\frac{d\sigma}{dZ} = \frac{\pi\alpha^2}{4s} \left(1 - \frac{4M^2}{s}\right)^{3/2} (1 - Z^2) \mid \bar{F}(s) \mid^2 \qquad \text{(I.20)}$$

After integration over Z we obtain the total cross-section

$$\sigma_{TOT} = \frac{\pi\alpha^2}{3s} \left(1 - \frac{4M^2}{s}\right)^{3/2} \mid \bar{F}(s) \mid^2 \qquad \text{(I.21)}$$

3.2 Spin ½ case ($N\bar{N}$, $\Lambda\bar{\Lambda}$, $\Sigma\bar{\Sigma}$, etc.)

From the Lorentz covariance, the space reflexion invariance and using the conservation of the electromagnetic current, we can introduce two electromagnetic form factors

$$\langle p, \bar{p} \mid J_\mu^{em}(o) \mid o \rangle = i \, \bar{u}(p) \left[\gamma_\mu \, \bar{F}_1(s) + \frac{i}{4M} [\gamma_\mu, \gamma_\nu] (p + \bar{p})^\nu \, \bar{F}_2(s) \right] v(\bar{p})$$

$$\text{(I.22)}$$

where $F_1(0) = Q$ the electric charge, and $F_2(0) = \kappa$ the anomalous magnetic moment of the spin $\frac{1}{2}$ particle.

The structure functions reduce to

$$\tilde{V}_1 \Rightarrow 2M^2 \, \delta(u^2 - M^2)\left[-|\overline{F}_1(s)|^2 + \frac{s}{4M^2} |\overline{F}_2(s)|^2 \right]$$

$$\tilde{V}_2 \Rightarrow 2M^2 \, \delta(u^2 - M^2) \, \frac{s}{2M^2} \, |\overline{F}_1(s) + \overline{F}_2(s)|^2$$

The angular distribution is written as[2]

$$\frac{d\sigma}{dz} = \frac{2\pi\alpha^2 M^2}{s^2} \left(1 - \frac{4M^2}{s}\right)^{1/2} \left\{ (1-z^2)|\overline{F}_1(s) + \frac{s}{4M^2}\overline{F}_2(s)|^2 + (1+z^2)\frac{s}{4M^2}|\overline{F}_1(s) + \overline{F}_2(s)|^2 \right\}$$

$$(I.23)$$

Integrating over Z we get the total cross-section

$$\sigma_{\text{Tot}} = \frac{8\pi\alpha^2 M^2}{3 s^2}\left(1 - \frac{4M^2}{s}\right)^{1/2}\left\{ |\overline{F}_1(s) + \frac{s}{4M^2}\overline{F}_2(s)|^2 + \frac{s}{2M^2}|\overline{F}_1(s) + \overline{F}_2(s)|^2 \right\} \quad (I.24)$$

For a final $\mu^+\mu^-$ system in pure quantum electrodynamics (Q.E.D.) $F_1 = 1$ and $F_2 = 0$

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{dz} = \frac{\pi\alpha^2}{2s} \frac{\left(1 - \frac{4m_\mu^2}{s}\right)^{1/2}}{\left(1 - \frac{4m_e^2}{s}\right)^{1/2}}\left[1 + \left(1 - \frac{4m_\mu^2}{s}\right)\left(1 - \frac{4m_e^2}{s}\right)z^2 + 4\frac{(m_e^2 + m_\mu^2)}{s}\right]$$

In the relativistic limit $\dfrac{m_e^2}{s} \ll 1$ $\dfrac{m_\mu^2}{s} \ll 1$

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{dz} = \frac{\pi\alpha^2}{2s}(1 + z^2) \qquad (I.25)$$

$$\sigma_{\text{Tot}}(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} \qquad (I.26)$$

3.3 Spin 1 case ($\rho^+ \rho^-$, $K^* \bar{K}^*$, $W^+ W^-$, etc.)

From the Lorentz covariance, the space and time reflexion invariances, we can define three independent form factors F_0, F_1, and F_2 normalized at $s = 0$ to the electric charge, the magnetic dipole moment, and the electric quadrupole moment[3,4].

The angular distribution is written as

$$\frac{d\sigma}{d\Omega} = \frac{3\pi\alpha^2}{4s}\left(1 - \frac{4M^2}{s}\right)^{3/2}\left\{(1-Z^2)\left[|F_0(s)|^2 + \frac{s^2}{18M^4}|F_2(s)|^2\right] + (1+Z^2)\frac{s}{6M^2}|F_1(s)|^2\right\}$$

(I.27)

and for the total cross-section we obtain

$$\sigma_{Tot} = \frac{\pi\alpha^2}{s}\left(1 - \frac{4M^2}{s}\right)^{3/2}\left\{|F_0(s)|^2 + \frac{s}{4M^2}|F_1(s)|^2 + \frac{s^2}{18M^4}|F_2(s)|^2\right\}$$

(I.28)

3.4 General case

From the Lorentz covariance, the space and time reflexion invariances, the number of linearly independent electromagnetic form factors for a spin J particle is $2J + 1$, and these form factors are normalized, at $s = 0$, to the static moments of the spin J particle[5].

The angular distribution measures only two positive functions $E(s)$ and $M(s)$, where $E(s)$ is the sum of the squares of the moduli of the electric form factors with $E(0) = Q^2$ the square of the electric charge, and $M(s)$ is the sum of the square of the modulus of magnetic form factors with $M(0) = [(J + 1)/3J]\mu^2$ the square of the dipole magnetic moment.

For integer spins, the angular distribution is given by

$$\frac{d\sigma}{d\Omega} = \frac{\pi\alpha^2(2J+1)}{4s}\left(1 - \frac{4M^2}{s}\right)^{3/2}\left\{(1-Z^2)E(s) + (1+Z^2)\frac{s}{4M^2}M(s)\right\}$$

and for half-integer spins we have

$$\frac{d\sigma}{dz} = \frac{\pi \alpha^2 (2J+1) M^2}{s^2} \left(1 - \frac{4M^2}{s}\right)^{1/2} \left\{ (1-z^2) E(s) + (1+z^2) \frac{s}{4M^2} M(s) \right\}$$

4. INELASTIC TWO-BODY FINAL STATES

We give two examples of two body final states which are not of the "elastic" type.

4.1 Pseudoscalar meson (M^0) -- Photon (γ) final state

From Lorentz covariance and space reflexion invariance we introduce only one form factor

Fig. 3

$$\langle M^0 \gamma | J_\mu^{em}(o) | o \rangle = e \frac{G_{\gamma M^0 \gamma}(s)}{m_o} \mathcal{E}_{\mu\nu\gamma\delta} q^\nu k_\gamma^\gamma \mathcal{E}_\gamma^\delta \tag{I.29}$$
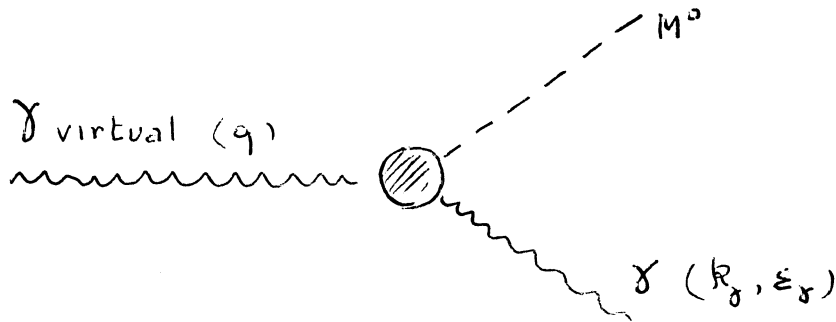
where m_o is the pseudoscalar meson mass introduced so that G(s) is dimensionless.

A straightforward calculation gives

$$\frac{d\sigma}{dz} = \frac{\pi \alpha^2}{4 m_o^2} \left(1 - \frac{m_o^2}{s}\right)^3 (1+z^2) \left| G_{M^0\gamma}(s) \right|^2 \tag{I.30}$$

$$\sigma_{TOT}\left(e^+e^- \Rightarrow M^0\gamma\right) = \frac{2\pi^2\alpha^3}{3\,m_c^2}\left(1 - \frac{m_c^2}{s}\right)^3 \left|G_{M^0\gamma}(s)\right|^2 \qquad (I.31)$$

The form factor $G(s)$ at $s = 0$, describes the 2γ decay mode of the pseudo-scalar meson M^0.

$$\gamma_1 \left(k_1, \varepsilon_1\right)$$

$$M^0$$

$$\gamma_2 \left(k_2, \varepsilon_2\right)$$
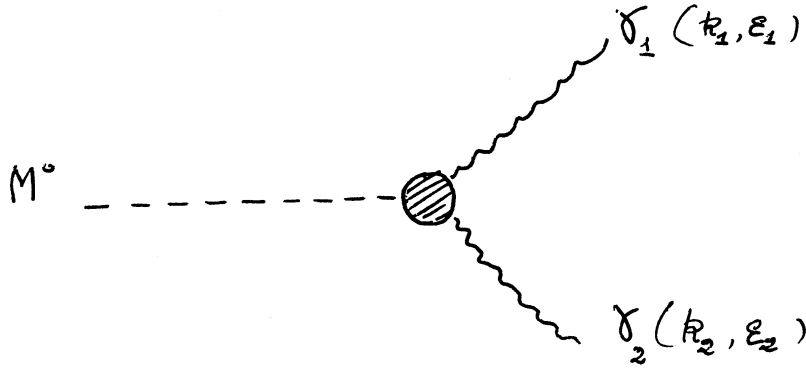
Fig. 4

$$\left\langle \gamma_1 \gamma_2 \mid T \mid M^0 \right\rangle = e^2 \frac{G_{M^0\gamma}(0)}{m_c} \varepsilon_{\mu\nu\rho\sigma} k_1^\mu \varepsilon_1^\nu k_2^\rho \varepsilon_2^\sigma$$

and the decay width is given, in terms of the coupling constant by

$$\Gamma\left(M^0 \Rightarrow 2\gamma\right) = \frac{\pi\alpha^2}{4} m_c \left|G_{\pi^0\gamma}(0)\right| \qquad (I.32)$$

Equation (I.32) can be used to rewrite the cross-sections (I.30) and (I.31) into the form

$$\frac{d\sigma}{dz} = \frac{\alpha}{2} \frac{\Gamma(M^0 \Rightarrow 2\gamma)}{m_c^3}\left(1 - \frac{m_c^2}{s}\right)^3\left(1 + z^2\right)\left|\frac{G_{M^0\gamma}(s)}{G_{M^0\gamma}(0)}\right|^2 \qquad (I.33)$$

$$\sigma_{TOT}\left(e^+e^- \Rightarrow M^0\gamma\right) = \frac{8\pi\alpha}{3} \frac{\Gamma(M^0 \Rightarrow 2\gamma)}{m_c^3}\left(1 - \frac{m_c^2}{s}\right)^3\left|\frac{G_{M^0\gamma}(s)}{G_{M^0\gamma}(0)}\right|^2 \qquad (I.34)$$

4.2 $\Lambda\bar{\Sigma}^0$ or $\Sigma^0\bar{\Lambda}$ final state

From Lorentz covariance, space reflexion invariance, and the conservation of the electromagnetic current, we have only two electromagnetic form factors

$$\Sigma^0 \ (\bar{p},\bar{\lambda})$$

$$\gamma_{virtual} \ (q)$$

$$\Lambda \ (p,\lambda)$$

Fig. 5

$$\langle p,\bar{p} \mid J_\mu^{em}(o) \mid o \rangle = i \ \bar{u}_\Lambda(p,\lambda)\left\{\left[\gamma_\mu - i\frac{M_\Sigma - M_\Lambda}{s}q_\mu\right]f_1^c(s) + \left[\gamma_\mu + i\frac{(P_\mu - \bar{P}_\mu)}{M_\Sigma + M_\Lambda}\right]f_2^c(s)\right\}V_{\bar{\Sigma}}(\bar{p},\bar{\lambda})$$

The structure functions $\tilde{V}_1$ and $\tilde{V}_2$ reduce to

$$\tilde{V}_1(s,u^2) \Rightarrow \delta(u^2 - M_{\bar{\Sigma}}^2)2M_\Lambda^2\left\{-\mid f_1^c(s)\mid^2 + \frac{s}{(M_\Sigma + M_\Lambda)^2}\mid f_2^c(s)\mid^2\right\}$$

$$\tilde{V}_2(s,u^2) \Rightarrow \delta(u^2 - M_{\bar{\Sigma}}^2) \ s\left[1 - \frac{(M_{\bar{\Sigma}} - M_\Lambda)^2}{s}\right]\mid f_1^c(s) + f_2^c(s)\mid^2$$

and for the angular distribution we obtain the expression[2)]

$$\frac{d\bar{\sigma}}{dz} = \frac{2\pi\alpha^2}{s^2}\left(\frac{M_{\bar{\Sigma}} + M_\Lambda}{2}\right)^2\left[1 - \frac{(M_{\bar{\Sigma}} + M_\Lambda)^2}{s}\right]^{1/2}\left[1 - \frac{(M_{\bar{\Sigma}} - M_\Lambda)^2}{s}\right]^{3/2} \times$$

$$\times \left\{(1 - z^2)\mid f_1^c(s) + \frac{s}{(M_\Sigma + M_\Lambda)^2}f_2^c(s)\mid^2 + (1 + z^2)\frac{s}{(M_\Sigma + M_\Lambda)^2}\mid f_1^c(s) + f_2^c(s)\mid^2\right\}$$

$$\text{(I.35)}$$

which in the limit $M_\Sigma = M_\Lambda = M$ reduces to equation (I.23) with $f_1 \rightarrow F_1$ and $f_2 \rightarrow F_2$.

5. MULTIPION FINAL STATE

5.1 Let us consider a final state with $n\pi$ mesons. The quantum numbers are

$$J^P = 1^- \qquad C = -1 \qquad Q = 0 \qquad G = (-1)^n$$

and we have the selection rules:

a) $n\pi^0$ are forbidden by C conservation (for $2\pi^0$, TCP is enough);

b) n   even G = +1 so that I = 1;

c) n   odd G = -1 so that I = 0.

5.2 In the particular case n = 2 we have a $\pi^+\pi^-$ system with total isotopic spin I = 1. We can construct only one Lorentz covariant $(p_+ - p_-)_\mu$, and we introduce one electromagnetic form factor $F_\pi(s)$.

5.3 For n = 3 we have a $\pi^+\pi^-\pi^0$ system with total isotopic spin I = 0. Again we can construct only one Lorentz covariant $\varepsilon_{\mu\nu\rho\sigma} \, p_+^\nu \, p_-^\rho \, p_0^\sigma$, and we define one electromagnetic form factor $F_{3\pi}$.

5.4 For n = 4 we have two possible states of total isotopic spin, I = 1 $\pi^+\pi^+\pi^-\pi^-$, and $\pi^+\pi^-\pi^0\pi^0$. Quasi two body states can be $\omega\pi^0$, $\phi\pi^0$, $\rho^0\varepsilon$, $\rho^+\rho^-$, but the $\rho^0\rho^0$ state is forbidden by TCP in the one photon exchange approximation. We can construct three linearly independent Lorentz covariants, an and therefore define three electromagnetic form factors.

5.5 We now consider the case n = 3 of a $\pi^+\pi^-\pi^0$ system but the generalization to a final state with three spin zero particles of total intrinsic parity -1 is straightforward ($K\bar{K}\pi$, etc.). From Lorentz covariance and space reflexion invariance we define one electromagnetic form factor

$$\langle 1,2,3 | \bar{J}_\mu^{em}(o) | o \rangle = \varepsilon_{\mu\nu\rho\sigma} \left(\frac{p_1}{m_1}\right)^\nu \left(\frac{p_2}{m_2}\right)^\rho \left(\frac{p_3}{m_3}\right)^\sigma \bar{F}(s_1,s_2,s_3)$$

where m_j is the particle j mass and s_j a scalar variable defined by

$$S_j = -P_j \cdot q \qquad\qquad q = P_1 + P_2 + P_3$$

In the centre of mass system, we use the energies E_1, E_2, and E_3 of the produced particles,

$$S_j = \sqrt{s}\, E_j \qquad\qquad \sqrt{s} = E_1 + E_2 + E_3$$

and the differential cross-section takes the form[2]

$$\frac{d^2\sigma}{dE_1\, dE_2} = \frac{\alpha^2}{12\pi} \frac{1}{s} \frac{|\vec{P_1} \times \vec{P_2}|^2}{(m_1 m_2 m_3)^2} \; \left| F(E_1, E_2, E_3) \right|^2 \qquad (I.36)$$

After integration over the Dalitz plot, we obtain the total cross-section

$$\sigma_{Tot}(e^+ e^- \to 1+2+3) = \frac{\alpha^2}{12\pi} \frac{1}{s} \frac{1}{(m_1 m_2 m_3)^2} \iint_{D(s)} |\vec{P_1} \times \vec{P_2}|^2 \, |F(E_1, E_2, E_3)|^2 dE_1\, dE_2 \qquad (I.37)$$

where the integration domain $D(s)$ is defined by the condition that p_1, p_2, and p_3 are sides of a triangle.

6. <u>POLARIZATION OF THE SPIN $\frac{1}{2}$ PARTICLE DETECTED IN THE FINAL STATE</u>

6.1 Let us go back to the hadronic tensor avoiding now the summation over the polarization of the observed particle p

$$\tilde{T}_{\mu\nu} + \tilde{S}_{\mu\nu} = N_p \sum_\Gamma \sum_\Gamma (2\pi)^3 \delta_4(q-p-p_\Gamma) \langle p, \Gamma | J_\mu^{em}(o)|o\rangle \langle p, \Gamma | J_\nu^{em}(o)|o\rangle^*$$

The spin-independent part $\tilde{T}_{\mu\nu}$ is decomposed as previously (I.8) with two structure functions $\tilde{V}_1$ and $\tilde{V}_2$.

The spin-dependent part $\tilde{S}_{\mu\nu}$ is linear in the spin ½ particle polarization. We introduce a space-like unit pseudovector S_μ orthogonal to the energy momentum p

$$S^2 = 1 \qquad\qquad S \cdot p = 0$$

and we construct a four vector n_μ linear in S

$$n_\mu = \frac{1}{M} \, \varepsilon_{\mu\nu\varsigma\sigma} \, p^\nu q^\varsigma S^\sigma \qquad\qquad (I.38)$$

Because of the symmetry character of the leptonic tensor $m^{\mu\nu}$, we have to consider only the symmetric part of $\tilde{S}_{\mu\nu}$. Using Lorentz covariance, space reflexion invariance, and the conservation of the electromagnetic current, we can write

$$\tilde{S}^{SYM}_{\mu\nu} = \frac{1}{2M^2} \left\{ n_\mu \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) n_\nu \right\} \tilde{Y}(s, u^2) \qquad (I.39)$$

Let us remark that in the case of inelastic lepton scattering, the corresponding structure function Y vanishes when time reversal invariance holds[6].

Now saturating the leptonic and hadronic tensors

$$m^{\mu\nu} \tilde{T}_{\mu\nu} = \bar{I}_1 \tilde{V}_1 (s, u^2) + \bar{I}_2 \tilde{V}_2 (s, u^2)$$

$$m^{\mu\nu} \tilde{S}_{\mu\nu} = L \, \tilde{Y}(s, u^2)$$

where the invariant L is given by

$$L = \frac{1}{M^2} \, n_\mu \, p_\nu \, m^{\mu\nu} = \frac{1}{2M^3} \, p \cdot (k_- - k_+) \, \varepsilon_{\lambda\varsigma\sigma z} \, k_-^\lambda k_+^\varsigma p^\sigma S^z$$

In the centre of mass system we introduce a unit vector, $\vec{N}$, orthogonal to the reaction plane

$$\vec{k} \times \vec{p} = k\,p\sqrt{1-z^2}\ \vec{N} \qquad\qquad \vec{N}^2 = 1$$

and the invariant L has the simple form

$$L = -\ \vec{s}\cdot\vec{N}\ 2z\sqrt{1-z^2}\ \frac{\sqrt{s}}{2M}\ \frac{p^2}{M^2}\ \frac{s}{4}\left(1 - \frac{4m^2}{s}\right)$$

For relativistic electrons $(m^2/s) \ll 1$, and the differential cross-section has the expression

$$\frac{d^2\sigma}{dz\,dE} = \frac{2\pi\alpha^2}{s^2}\,p\left\{(1-z^2)\frac{p^2}{M^2}\tilde{V}_1(s,u^2) + \tilde{V}_2(s,u^2) - (\vec{s}\cdot\vec{N})\frac{p^2}{M^2}\frac{\sqrt{s}}{2M}\,2z\sqrt{1-z^2}\ \tilde{Y}(s,u^2)\right\}$$

$$(I.40)$$

In addition to (I.11) and (I.12) we have the positivity condition[6]

$$\tilde{V}_1(s,u^2)\left\{\tilde{V}_1(s,u^2) + \frac{1}{2}\frac{M^2 s}{(p\cdot q)^2 - M^2 s}\ \tilde{V}_2(s,u^2)\right\} \geqslant \frac{(p\cdot q)^2 - M^2 s}{2M^4}\ \left|\tilde{Y}(s,u^2)\right|^2$$

$$(I.41)$$

6.2 For "elastic" reactions like nucleon-antinucleon, hyperon-antihyperon production, the structure function $\tilde{Y}$ has the form

$$\tilde{Y} \Rightarrow -\ 2M^2\delta(u^2-M^2)\ Im\left[F_1(s)\,F_2^*(s)\right]$$

and we obtain the angular distribution with a polarized fermion[2].

$$\frac{d\sigma}{dz} = \frac{\pi\alpha^2 M^2}{s}\left(1 - \frac{4M^2}{s}\right)^{1/2}\left\{(1-z^2)\,|F_E(s)|^2 + (1+z^2)\frac{s}{4M^2}\,|F_M(s)|^2\right.$$

$$\left. -\ \vec{s}\cdot\vec{N}\ 2z\sqrt{1-z^2}\ \frac{\sqrt{s}}{2M}\ Im\left[F_E(s)\,F_M^*(s)\right]\right\} \qquad (I.42)$$

where

$$\bar{F}_E(s) = \bar{F}_1(s) + \frac{s}{4M^2}\bar{F}_2(s) \qquad \bar{F}_M(s) = \bar{F}_1(s) + \bar{F}_2(s)$$

6.3 Analogous expressions can be derived for "inelastic" reactions like $\Lambda\bar{\Sigma}^0$ production

$$\tilde{\gamma} \Rightarrow -2M_\Lambda^2\delta(u^2 - M_\Sigma^2)\frac{2M_\Lambda}{M_\Sigma + M_\Lambda}\quad Im\left[f_1(s)\,f_2^*(s)\right]$$

and the angular distribution is written as

$$\frac{d\sigma}{dZ} = \frac{\pi\alpha^2}{s^2}\left(\frac{M_\Sigma + M_\Lambda}{2}\right)^2\left[1 - \frac{(M_\Sigma + M_\Lambda)^2}{s}\right]^{1/2}\left[1 - \frac{(M_\Sigma - M_\Lambda)^2}{s}\right]^{3/2} \times$$

$$\times \left\{(1-Z^2)|f_E(s)|^2 + (1+Z^2)\frac{s}{(M_\Sigma + M_\Lambda)^2}|f_M(s)|^2 - \vec{S}\cdot\vec{N}\,2Z\sqrt{1-Z^2}\frac{\sqrt{s}}{M_\Sigma + M_\Lambda}\,Im\left[f_E(s)f_M^*(s)\right]\right\}$$

where

$$f_E(s) = f_1(s) + \frac{s}{(M_\Sigma + M_\Lambda)^2}f_2(s) \qquad f_M(s) = f_1(s) + f_2(s)$$

7. DISCUSSION

7.1 The one photon exchange approximation has not yet been carefully tested experimentally for time-like photons, and our assumption is based on elastic and inelastic lepton scattering data, where there is no evidence for the presence of a multiphoton exchange.

7.2 When only one particle is detected in the final state the angular distribution for e^+e^- annihilation reactions is a linear function of Z^2, and we have given a physical interpretation of the $(1 - Z^2)$ and $(1 + Z^2)$ coefficients in terms of the virtual photon polarization. The same result obviously holds for any two body final state.

7.3 In the case of a multiphoton exchange the angular distribution is a polynomial in Z

$$\frac{d^2\sigma(e^+e^- \Rightarrow p + \Gamma)}{dZ \, du^2} \propto \sum_{n=0}^{n=2N} C_n(s, u^2) Z^n$$

where N is the maximum value of the spin exchanged between leptons and hadrons.

The coefficients of odd powers in Z (n = 2j + 1) correspond to interferences between C = -1 and C = +1 contributions, whose dominant part must be respectively 1γ and 2γ exchange on the basis of an order of α argument.

A detailed discussion of multiphoton exchange in lepton scattering has been given by A. Martin and the author[7], using in fact the annihilation channel.

7.4 Finally, the cleanest test for the 2γ exchange is certainly to search for final states forbidden by TCP in the 1γ exchange, like for instance, $\pi^0\pi^0$, η η, $\rho^0\rho^0$, ωω, φφ, etc.

If particle-antiparticle conjugation is a good symmetry of the electromagnetic interactions of hadrons, final states which have C = +1 like π^0η, ρ^0ω, ρ^0φ, and ωφ can proceed only via a 2γ exchange.

LECTURE II

VECTOR MESON DOMINANCE MODEL

1. VECTOR MESON DOMINANCE MODEL

1.1 We present the vector meson dominance (VMD) model in the form of the vector meson-photon analogy[8], using a direct coupling between the vector meson and the photon of strength $e \dfrac{m_V^2}{f_V}$
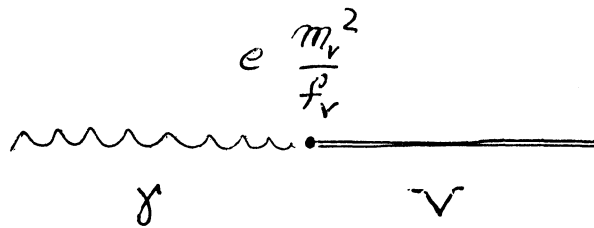
$$ e \; \frac{m_V^2}{f_V} $$

Fig. 6

The coupling constant f_V is dimensionless, and we do not discuss the problems of gauge invariance for such a coupling[9].

The matrix elements of the electromagnetic current between two arbitrary states, A and B, are assumed to be well represented by the sum of the vector meson contributions
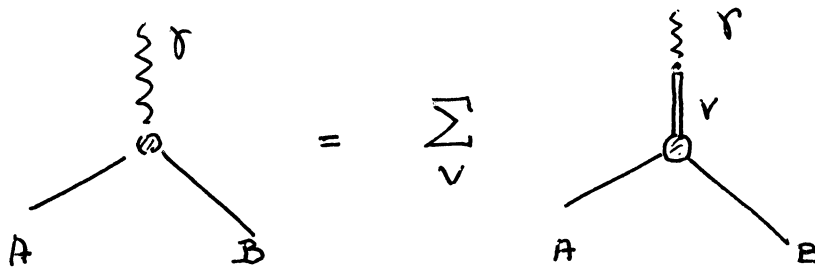
Fig. 7

$$\langle B | J^{em}_{\mu}(0) | A \rangle = \sum_{V}' \frac{1}{f_V} \frac{m_V^2}{W_V(s) - s} \langle B | J^{V}_{\mu}(0) | A \rangle \qquad \text{(II.1)}$$

where $W_V(s) - s$ is the vector meson inverse propagator.

The matrix element $\langle B | J^{V}_{\mu}(0) | A \rangle$ describes a vector meson reaction amplitude with an off mass shell vector meson of mass $\sqrt{s}$. We will discuss in the next section the extrapolation problem in the general case. For time-like photons, the vector mesons can be produced as physical unstable particles. The resonance region associated with ρ, ω, and ϕ is

$$500 \leq \sqrt{s} \leq 1100 \text{ MeV}$$

and in that domain the VMD model is really an isobaric model, the variation, with energy of the matrix elements of the electromagnetic current, being essentially due to the vector meson propagator as in a Breit-Wigner theory.

1.2 Assumption H_1 (coupling constant f_V)

The vector meson-photon coupling constants f_V are independent of the photon mass $\sqrt{s}$. They can be obtained from storage ring experiments, where they are measured for on mass shell vector mesons.

If this strong form of H_1 turns out to be wrong, a weaker form can be the following: f_V is a smooth function of s identical for all the phenomena induced by virtual photons having the same s.

1.3 Assumption H_2 (Mass extrapolation)

The extrapolation in the s variable of the $V \to F$ amplitude, from an on mass shell vector meson to an off mass shell vector meson, is smooth.

We immediately have a difficulty with the current conservation relation written as

$$q_c \langle B | J_c | A \rangle = |\vec{q}| \langle B | \hat{q} \cdot \vec{J} | A \rangle$$

where $\hat{q}$ is the unit vector in the photon momentum direction. Clearly, H_2 is incompatible with the current conservation relation, because $|\vec{q}|/q_0$ is not a slowly varying function of s.

We then have two types of ambiguities:

a) For the choice of the current components where the VMD relations can be used with the smoothness assumption H_2;

b) For the choice of the scalar variables to keep fixed in the mass extrapolation.

For e^+e^- annihilation reactions in the laboratory system the photon momentum $\vec{q}$ is zero, so that

$$< F \mid J_0 (o) \mid o > = O$$

It is then natural to apply the VMD relations with H_2 to the matrix elements of the space components of the current.

For inelastic lepton-scattering (space-like photon), or for lepton-antilepton pair production (time-like photon), the photon momentum $\vec{q}$ is not zero in the laboratory system, and the ambiguity $\underline{a}$ is not so easily overcome.

In the helicity frame of reference, the transverse photon amplitudes are associated to the transverse space components (orthogonal to $\hat{q}$) of the current, and we will use the VMD relations with the smoothness assumption H_2 for the matrix elements of these two components. The longitudinal photon amplitude can be related to the space component $\hat{q} \cdot \vec{J}$ or to the time component J_0, but we have, in addition, to introduce a kinematical factor depending on s because of Lorentz invariance. It makes a big difference to apply the VMD relations with H_2 to one of these matrix elements, including or not the kinematical factors.

There is no ambiguity of type $\underline{b}$ for a two-body final state, where the only possible scalar variable is s.

1.4 Assumption H₃

The virtual or real photon and the physical vector mesons have the same polarization in a given frame of reference.

On the basis of kinematical and dynamical arguments, the helicity frame seems to be the most convenient for the VMD relations with assumption H_2.

2. LEPTON-ANTILEPTON ANNIHILATION CROSS-SECTIONS

2.1 The interesting quantity is the tensor $M_{\mu\nu}^F$ defined in equation (I.3),

$$M_{\mu\nu}^{\bar{F}} = S_F (2\pi)^6 \sum_4 (q - P_{\bar{F}}) \langle \bar{F} | J_\mu^{em}(o) | o \rangle \langle \bar{F} | J_\nu^{em}(o) | o \rangle^*$$

Using the fundamental VMD relation (II.1), we obtain

$$M_{\mu\nu}^{\bar{F}} = \sum_{V_1,V_2}' \left[\frac{1}{f_{V_1}} \frac{m_{V_1}^2}{W_{V_1}(s) - s} \right] \left[\frac{1}{f_{V_2}} \frac{m_{V_2}^2}{W_{V_2}(s) - s} \right]^* S_F (2\pi)^6 \sum_4 (q - P_{\bar{F}}) \langle \bar{F} | J_\mu^{V_1}(o) | o \rangle \langle \bar{F} | J_\nu^{V_2}(o) | o \rangle^*$$

2.2 Let us restrict ourselves to the case of total cross-sections, for which the relevant quantity is $g^2 M^F$, and define the overlap function of vector meson amplitudes by

$$\sum_{V_2 V_1}'^{\bar{F}}(s) = \frac{1}{6} S_F (2\pi)^4 \delta_4 (q - P_{\bar{F}}) \langle \bar{F} | J_\mu^{V_1}(o) | o \rangle \langle \bar{F} | J_\nu^{V_2}(o) | o \rangle^* g^{\mu\nu}$$

The diagonal terms of Σ^F on mass shell are related to the partial decay widths by

$$\sum_{VV}'^{\bar{F}} (m_V^2) = m_V \, \Gamma(V \Rightarrow \bar{F}) \tag{II.2}$$

if F is a physical decay state for the vector meson V. The set of functions $\Sigma^F_{V_2 V_1}$ (s) can be considered as matrix elements in the 3-dimensional space of the neutral vector mesons. In the same space we introduce an inverse propagator matrix W(s), whose diagonal elements in the physical basis are the functions W_V(s).

The unitarity property of the S matrix implies the following relations:

a) For the diagonal part of Σ

$$- \text{Im } W_V(s) = \sum_F \Sigma^F_{VV} (s)$$

where the summation is extended over all physical states. In particular, using (II.2), we obtain

$$- \text{Im } W_V (m_V^2) = m_V \Gamma_V \qquad\qquad (II.3)$$

where Γ_V is the total vector meson decay width.

b) For the non-diagonal part of Σ we obtain a Bell-Steinberger[9,10] type relation

$$\frac{1}{2i} \left[W^*_{V_2}(s) - W_{V_1}(s) \right] \langle V_2 | V_1 \rangle = \sum_F \Sigma^F_{V_2 V_1}(s) \qquad (II.4)$$

Let us notice that when strong and electromagnetic interactions are taken into account, the physical states $|V >$ are not orthogonal, so that relation (II.4) is not empty.

2.3 The total cross-section for the process $e^+ e^- \to F$ is then given in terms of the matrix Σ^F by

$$\sigma_{\text{TOT}} (e^+ e^- \to F)_{\text{VMD}} = \frac{16\pi^2 \alpha^2}{s^2} \sum_{V_2, V_1} \left[\frac{1}{f_{V_1}} \frac{m_{V_1}^2}{W_{V_1}(s) - s} \right] \left[\frac{1}{f_{V_2}} \frac{m_{V_2}^2}{W_{V_2}(s) - s} \right]^* \sum_F \Sigma^F_{V_2 V_1}(s)$$

In order to obtain a more symmetrical expression, it is convenient to introduce the matrix Σ associated with an electron-positron state. In the one-photon exchange approximation,

$$V\ (q,\lambda) \qquad\qquad \gamma \qquad \ell\ (p_-\sigma_-)$$
$$\bar{\ell}\ (p_+\sigma_+)$$
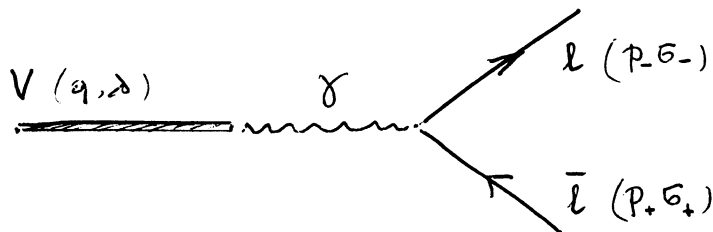
Fig. 8

the amplitude for the decay of an off mass shell vector meson of mass $\sqrt{s}$ into a lepton-antilepton pair is given by

$$\langle \ell\bar{\ell} \mid T \mid V \rangle = \frac{e^2}{s} \left[\bar{u}_{\sigma_-}(p_-)\ \gamma^\mu\ v_{\sigma_+}(p_+) \right] \frac{m_V^2}{f_V}\ e_\mu\ (q,\lambda) \qquad\qquad (II.5)$$

where $u_{\sigma_-}(p_-)$ is the lepton free Dirac spinor;

$v_{\sigma_+}(p_+)$ is the antilepton free Dirac spinor;

$e_\mu(q,\lambda)$ is the vector meson polarization vector.

The kinematics is indicated on Fig. 8.

The function $\Sigma^{\ell\bar{\ell}}_{V_2 V_1}(s)$ is computed from the expression (II.5)

$$\sum^{\ell\bar{\ell}}_{V_2 V_1}(s) = \frac{4\pi\alpha^2}{3s}\ \frac{m_{V_1}^2}{f_{V_1}}\ \frac{m_{V_2}^2}{f_{V_2}^*} \qquad\qquad (II.6)$$

and the final result, for the total cross-section $\sigma_{tot}(e^+e^- \to F)$, is simply

$$\sigma_{tot}(e^+e^- \to F)_{VMD} = \frac{12\pi}{s} \sum_{V_2 V_1} \frac{\sum^{e^+e^-}_{V_2 V_1}(s) \sum^{F}_{V_2 V_1}(s)}{\left[W_{V_1}(s) - s \right]\left[W_{V_2}(s) - s \right]^*}$$

As a consequence of equations (II.2) and (II.6), we obtain

$$\Gamma\left(V \Rightarrow \ell\bar{\ell}\right) = \frac{4\pi\alpha^2}{3} \frac{1}{|f_V|^2} \, m_V \tag{II.7}$$

up to terms of order $(m_\ell/m_V)^4$ that one can obviously neglect.

2.4 In the particular case where only one vector meson contributes, the total cross-section is simply written as

$$\sigma_{Tot}\left(e^+e^- \Rightarrow V \Rightarrow \bar{F}\right) = \frac{12\pi}{s} \frac{\sum_{VV}^{e^+e^-}(s) \, \sum_{VV}^{\prime\bar{F}}(s)}{|W_V(s) - s|^2} \tag{II.8}$$

At $s = m_V^2$, using (II.2), we have

$$\sum_{VV}^{e^+e^-}(m_V^2) = m_V \, \Gamma\left(V \Rightarrow e^+e^-\right) \qquad \sum_{VV}^{\prime\bar{F}}(m_V^2) = m_V \, \Gamma\left(V \Rightarrow \bar{F}\right)$$

For the inverse propagator, using (II.3) and the definition of the resonance mass

$$W_V(m_V^2) = m_V^2 - i \, m_V \, \Gamma_V$$

Formula (II.8) reduces to

$$\sigma_{Tot}\left(e^+e^- \Rightarrow V \Rightarrow \bar{F}\right)_{s=m_V^2} = \frac{12\pi}{m_V^2} \frac{\Gamma\left(V \Rightarrow e^+e^-\right)}{\Gamma_V} \frac{\Gamma\left(V \Rightarrow \bar{F}\right)}{\Gamma_V} \tag{II.9}$$

2.5 The total cross-section for production of a vector meson V is defined by a summation over the final states F

$$\sigma_{Tot}\left(e^+e^- \Rightarrow V\right) = \sum_{\bar{F}} \sigma_{Tot}\left(e^+e^- \Rightarrow V \Rightarrow F\right)$$

From equation (II.8) and the unitarity relation, we have

$$\sigma_{TOT}(e^+e^- \Rightarrow V) = \frac{12\pi}{s} \sum_{VV}^{e^+e^-}(s) \frac{-Im\, W_V(s)}{|W_V(s) - s|^2}$$

or explicitly substituting for $\Sigma_{VV}^{e^+e^-}(s)$

$$\sigma_{TOT}(e^+e^- \Rightarrow V) = \frac{12\pi}{s} \frac{m_V^2}{s} m_V \Gamma(V \Rightarrow e^+e^-) \frac{-Im\, W_V(s)}{|W_V(s) - s|^2} \qquad (II.10)$$

Let us now assume that the function $\left[W_V(s) - s\right]^{-1}$ satisfies an unsubtracted dispersion relation

$$\frac{1}{W_V(s) - s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{-Im\, W_V(t)}{|W_V(t) - t|^2} \frac{dt}{t - s - i\epsilon}$$

At s = 0 we deduce a sum rule

$$\frac{1}{W_V(0)} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{-Im\, W_V(t)}{|W_V(t) - t|^2} \frac{dt}{t}$$

which can be written, using equation (II.10), as

$$\frac{m_V^2}{W_V(0)} = \frac{1}{12\pi^2} \frac{\int_{4m_\pi^2}^{\infty} t\, \sigma_{TOT}(e^+e^- \Rightarrow V)\, dt}{m_V \Gamma(V \Rightarrow e^+e^-)} \qquad (II.11)$$

Allowing now one subtraction for the dispersion relation on $\left[W_V(s) - s\right]^{-1}$, it is straightforward to derive in a similar way a second sum rule

$$\left[\frac{m_V^2}{W_V(0)}\right]^2 \left[1 - W_V'(0)\right] = \frac{1}{12\pi^2} \frac{\int_{4m_\pi^2}^{\infty} \sigma_{TOT}(e^+e^- \Rightarrow V)\, dt}{\dfrac{\Gamma(V \Rightarrow e^+e^-)}{m_V}} \qquad (II.12)$$

3. ELECTROMAGNETIC FORM FACTORS

3.1 The matrix elements of the electromagnetic current

$$\langle \bar{F} \mid J_\mu^{em}(o) \mid o \rangle$$

can be decomposed on a basis of Lorentz covariants $I_\mu^{(j)}$, with scalar coefficients generally called electromagnetic form factors:

$$\langle \bar{F} \mid J_\mu^{em}(o) \mid o \rangle = \sum_j I_\mu^{(j)} \, G_{\gamma\bar{F}}^{(j)}(s;u)$$

Here u represents the set of scalar variables independent of s. The same basis can obviously be used for the matrix elements of the vector meson current

$$\langle \bar{F} \mid J_\mu^V(o) \mid o \rangle = \sum_j I_\mu^{(j)} \, G_{V\bar{F}}^{(j)}(s;u)$$

The vector meson dominance model for the electromagnetic form factors $G_{\gamma\bar{F}}^{(j)}$ is simply written as

$$G_{\gamma\bar{F}}^{(j)}(s;u) = \sum_V{}' \frac{1}{f_V} \frac{m_V^2}{W_V(s)-s} \, G_{V\bar{F}}^{(j)}(s;u) \qquad (II.13)$$

The quantity $G_{V\bar{F}}^{(j)}(s;u)$ represents a transition amplitude $V \to F$ with an off mass shell vector meson of mass $\sqrt{s}$.

3.2 In order to study the implications of the smoothness assumption H_2, we factorize the vector meson amplitudes as follows.

$$G_{V\bar{F}}^{(j)}(s;u) = f_{V\bar{F}}^{(j)} \, \Phi_{\bar{F}}^{(j)}(s;u) \qquad (II.14)$$

Let us first formulate a weak form of H_2: the coupling constants $f_{V\bar{F}}^{(j)}$ are independent of s. From a strong form of H_2, the phase space functions $\Phi_{\bar{F}}^{(j)}(s;u)$ are slowly varying with s.

In the particular case, where the final state F is a two body state, the variables u disappear and $\Phi_F^{(j)}$ must remain essentially constant during the extrapolation. The strong version of H_2 seems to agree with experiment for the π meson electromagnetic form factor, but certainly leads to serious difficulties for the nucleon electromagnetic form factors.

Finally, using equations (II.13) and (II.14) with the weak form of H_2, we obtain for the electromagnetic form factors

$$G_{\gamma F}^{(j)}(s, u) = \bar{\Phi}_F^{(j)}(s; u) \sum_V{}' \frac{f_{VF}^{(j)}}{f_V} \frac{m_V^2}{W_V(s) - s}$$

$$(II.15)$$

LECTURE III

THE RESONANCE REGION

1. THE π-MESON ELECTROMAGNETIC FORM FACTOR

1.1 The matrix element of the electromagnetic current between the vacuum and an $\pi^+\pi^-$ state is given by Lorentz covariance

$$\langle \pi^+\pi^- | \bar{J}_\mu^{em}(0) | o \rangle = (p_+ - p_-)_\mu \, \overline{F}_\pi(s)$$

where the kinematics is indicated on Fig. 9.
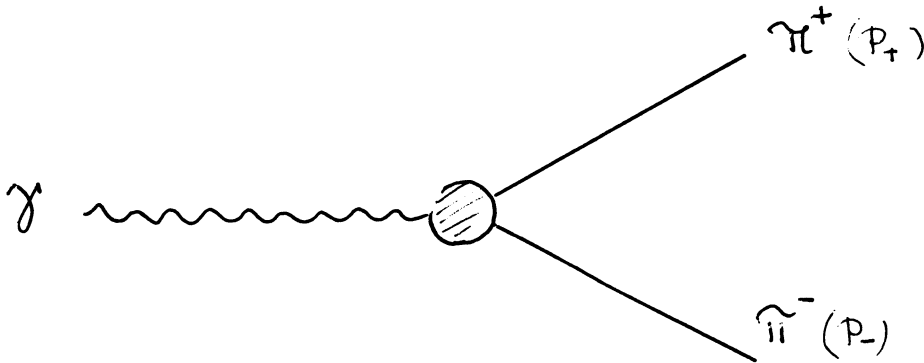
Fig. 9

The variable s is always defined as $s = -(p_+ + p_-)^2$.

From the TCP theorem, the π^0 meson electromagnetic form factor is identically zero. Using now the hermiticity property of the electromagnetic current, we easily check that $F_\pi(s)$ is real in the space like region $s < 0$.

The electric charge of the π^+ meson is unity and the normalization condition at $s = 0$ follows: $F_\pi(0) = 1$.

1.2 The π meson electromagnetic form factor $F_\pi(s)$ is the boundary value of an analytic function F(Z).

$$\bar{F}_\pi(s) = \lim_{\varepsilon \to 0^+} \bar{F}(s+i\varepsilon) \qquad\qquad s \geqslant 4m_\pi^2$$

$$\bar{F}_\pi(s) = \bar{F}(s) \qquad\qquad s < 4m_\pi^2$$

From now s is a real variable and Z a complex variable. The various physical regions are defined on Fig. 10.

$$\begin{array}{ccc} O & 4m_\pi^2 & \\ \end{array}$$

$$e^-\pi^\pm \to e^-\pi^\pm \qquad\qquad \bar{F}_\pi(\cdot)=1 \qquad\qquad e^+e^- \to \pi^+\pi^-$$

space-like photons   time-like photons
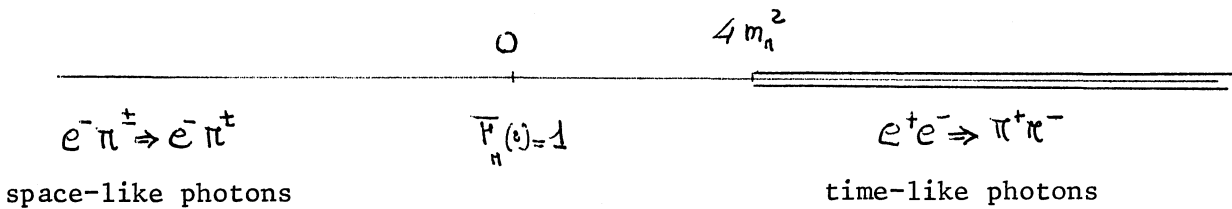
Fig. 10

The properties of F(Z) are

  i) analytic function of Z in the cut Z plane, the cut starting from from $4m_\pi^2$ to $+\infty$ on the real positive axis;

 ii) Schwartz reflection principle $F(Z^*) = F^*(Z)$;

iii) F(Z) is bounded by a power of $|Z|$ as $|Z| \to \infty$ in all directions.

A representation of F(Z) equivalent to these three properties is[12]

$$\bar{F}(Z) = \int_{-\infty}^{+\infty} S(\alpha)\, e^{i\alpha(Z-4m_\pi^2)^{1/2}}\, d\alpha \qquad\qquad (III.1)$$

where S(x) is a real valued tempered distribution whose support is contained in $\{ x \mid x \geq a \geq 0 \}$.

1.3 If a > 0, the form factor in the space-like region is exponentially decreasing[12]

$$F_\pi(s) \simeq \exp\left[-a\sqrt{-s}\right] \qquad \text{when} \quad s \Rightarrow -\infty \qquad \text{(III.2)}$$

and the high energy behaviour in the time-like region is restricted only by the condition iii).

1.4 When a = 0, we need more assumptions on F(Z) in order to obtain more information on the form factor $F_\pi(s)$.

If F(Z) has only a finite number of zeros in the complex Z plane, we define the polynomial P(Z) with real coefficients having the same zeros as F(Z) and normalized to unity at Z = 0. The function G(Z) defined by

$$G(Z) = P^{-1}(Z)\, F(Z) \qquad \text{(III.3)}$$

obviously satisfies the set of properties i), ii)   and iii) has no zero in the complex Z plane and is normalized to unity at Z = 0.

If, in addition, the phase of G(Z) is bounded, we can write for G(Z) a phase representation[13]

$$G(Z) = \exp\frac{Z}{\pi}\int_{4m_\pi^2}^{\sim}\frac{\varphi_G(t)}{t(t-Z)}\,dt \qquad \text{(III.4)}$$

or a modulus representation[14]

$$G(Z) = \exp\frac{Z(Z-4m_\pi^2)^{\frac12}}{i\pi}\int_{4m_\pi^2}^{\sim}\frac{\log|G_\pi(t)|}{t(t-4m_\pi^2)^{\frac12}(t-Z)}\,dt \qquad \text{(III.5)}$$

1.5 The interest of the modulus representation in the actual problem is that the integral (III.5) involves the modulus of the form factor in the time-like region as measured in $e^+e^- \to \pi^+\pi^-$ experiments. Unfortunately, the relation between $G_\pi(t)$ and $F_\pi(t)$ involves the unknown polynomial P(t), but it is possible to deduce from the modulus representation, sum rules measuring the importance of the zeros[11,14].

1.6 In the elastic unitary approximation the phase $\phi_G(t)$ is identified with the phase $\delta_{11}(t)$ of the $\pi\pi$-scattering amplitude $I = J = 1$. The corresponding form of G is called the Omnes function[15]

$$G_{11}(z) = \exp \frac{z}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\delta_{11}(t)}{t(t-z)}\, dt \qquad \text{(III.6)}$$

Experimentally the phase shift $\delta_{11}(t)$ is not known, and we must compute it in a model dependent way.

1.7 A somewhat general method is to write the $\pi\pi$-scattering amplitude $I = J = 1$,

$$h_{11}(s) = \frac{\sqrt{s}}{k^3(s)}\, e^{i\delta_{11}(s)} \sin\delta_{11}(s) \qquad \text{(III.7)}$$

where $k(s) = \frac{1}{2}(s - 4m_\pi^2)^{\frac{1}{2}}$ is the π meson centre of mass momentum, in the form

$$h_{11}(s) = \frac{N(s)}{D(s)} \qquad \text{(III.8)}$$

where N(s) has only left-hand cut singularities, and D(s) has only right-hand cut singularities. In particular, the discontinuity of D across the right-hand cut is computed from (III.7) and (III.8) to be

$$\operatorname{Im} D(s) = -\frac{k^3(s)}{\sqrt{s}} N(s)$$

We then write a dispersion relation for D(s).

Models built in that way correspond to particular assumptions on the numerator function N(s) or, equivalently, on the left-hand cut singularities of the scattering amplitude. The extra information injected in the calculation is the existence of the ρ meson resonance of mass m_ρ and width Γ_ρ with the constraints

$$\delta_{11} \left(m_s^2 \right) = \frac{\pi}{2} \qquad \frac{d}{ds} \delta_{11} \left(m_s^2 \right) = \frac{1}{m_s \Gamma_s} \qquad \text{(III.9)}$$

In the Frazer-Fulco approach[16] the left-hand cut is approximated by a pole at $s = -s_1$, and the dispersion relation for $D(s)$ is written with one subtraction.

In the Gounaris-Sakurai[17] model the $N(s)$ function is assumed to be a constant, and the dispersion relation for $D(s)$ is written with two subtractions.

The analytic form of $h(Z)$ turns out to be the same in both cases, and depends on two arbitrary constants that one determines with the conditions (III.9),

$$h^{-1}(Z) = (Z - 4 m_\pi^2) f(Z) + a + bZ \qquad \text{(III.10)}$$

where

$$f(Z) = \frac{1}{8\pi} \left(1 - \frac{4 m_\pi^2}{Z} \right)^{1/2} \text{Log} \frac{(Z - 4 m_\pi^2)^{1/2} + Z^{1/2}}{(Z - 4 m_\pi^2)^{1/2} - Z^{1/2}} \qquad \text{(III.11)}$$

The phase-shift $\delta_{11}(s)$ is given by a so-called generalized effective range formula

$$\frac{k^3(s)}{\sqrt{s}} \text{ctn} \, \delta_{11}(s) = k^2(s) h(s) + a + bs \qquad \text{(III.12)}$$

where $h(s)$ is deduced from (III.11) to be

$$h(s) = \frac{2}{\pi} \frac{k(s)}{\sqrt{s}} \text{Log} \frac{2 k(s) + \sqrt{s}}{2 m_\pi} \qquad \text{(III.13)}$$

The constants a and b are related to the ρ meson parameters by

$$\alpha = m_\pi^2 h_\rho + m_\rho^2 \left[\frac{k_\rho^3}{m_\rho^2 \Gamma_\rho} + k_\rho^2 h'_\rho \right]$$

$$b = -\frac{1}{4} h_\rho - \left[\frac{k_\rho^3}{m_\rho^2 \Gamma_\rho} + k_\rho^2 h'_\rho \right]$$

where

$$k_\rho = k(m_\rho^2) \qquad \text{and} \qquad h_\rho = h(m_\rho^2)$$

The value of s_1, for which $h^{-1}(-s_1) = 0$, is found to be very large

$$s_1 \simeq 9.6 \ 10^6 \ m_\rho^2$$

1.8 The Omnès function, associated to the Frazer-Fulco-Gounaris-Sakurai (FFGS) model, is simply given by

$$G_{11}(z) = \left(1 + \frac{z}{s_1}\right) \frac{a - \frac{m_\pi^2}{\pi}}{(z - 4m_\pi^2) f(z) + a + bz} \tag{III.14}$$

Finally, we identify the π meson form factor with the Omnes function, and the FFGS model gives

$$F_\pi(s) = \left(1 + \frac{s}{s_1}\right) \frac{m_\rho^2 \left(1 + d_s \frac{\Gamma_\rho}{m_\rho}\right)}{m_\rho^2 - s + \frac{m_\rho^2 \Gamma_\rho}{k_\rho^3} \left\{ k^2(s) \left[h(s) - h_s \right] + (m_\rho^2 - s) k_\rho^2 h'_s \right\} - i \ m_\rho \Gamma_\rho \frac{k^3(s)}{k_\rho^3} \frac{m_\rho}{\sqrt{s}}} \tag{III.15}$$

where the constant d_ρ, interpreted as measuring finite width corrections, is given by

$$d_\rho = \frac{3}{2} \frac{m_\pi^2 \, m_\rho}{\hbar_\rho^3} \, \hbar_\rho + \frac{m_\rho}{2\pi \, \hbar_\rho} - \frac{m_\pi^2 \, m_\rho}{\pi \, \hbar_\rho^3} \qquad (III.16)$$

The numerical value of d_ρ is computed to be $d_\rho \approx 0.48$.

1.9 A systematic analysis of the experiments performed in Novosibirsk and Orsay has been made by Roos and Pišut[18], using different parametrizations of the Omnès function, and disregarding the possibility of zeros of $F_\pi(s)$. Unfortunately, the experimental data are not accurate enough to allow a precise determination of the ρ meson shape, and it is not possible to choose clearly the best form for $F_\pi(s)$.

From the Novosibirsk[19] and Orsay[20] data we get

	Orsay FFGS	Novosibirsk	
		FFGS	Simple BW
m_ρ (MeV)	(773.6 ± 5.3)	(768 ± 10)	(754 ± 9)
Γ_ρ (MeV)	(110.7 5.3)	(140 ± 14)	(105 ± 20)

Table 1

Apparently, the Orsay and Novosibirsk data are not in good agreement, and, in particular, the FFGS model has some difficulty to fit the Novosibirsk data with a reasonable χ^2.

As a second remark, the small value of the ρ meson width $\Gamma_\rho \simeq 110$ MeV, obtained in Orsay with the FFGS fit, is certainly incompatible with the large values $\Gamma_\rho \simeq 130-140$ MeV, usually needed in the analysis of ρ meson photoproduction experiments. The storage ring experiments are, in principle, the cleanest way to measure the ρ meson parameters, but with the present accuracy of the experiments, the ρ meson shape cannot be very well determined, and the analysis of the data remains strongly model-dependent, so that the numerical values of the ρ meson parameters are also model-dependent.

1.10 It is clear that, in the ρ meson region, terms of order $(m_\rho^2 - s)^2$ can be neglected, and the formula (III.15) is very well approximated by

$$\overline{F}_\pi(s) \simeq \frac{m_\rho^2 \left(1 + d_s \frac{\Gamma_\rho}{m_\rho}\right)}{m_\rho^2 - s - i\, m_\rho \Gamma_\rho \frac{k^3(s)}{k_\rho^3} \frac{m_\rho}{\sqrt{s}}}$$

Relaxing now the constraint $d_\rho \simeq 0.48$ of the FFGS model, we can fit the Orsay data with three free parameters, and we get

$$m_\rho = (776 \pm 6)\ \text{MeV} \qquad \Gamma_\rho = (127.3 \pm 12.5)\ \text{MeV} \qquad d_\rho = 1.1 \pm 0.41 \ .$$

The most interesting feature of this fit is the relatively large value obtained for the ρ meson width making possible an agreement between storage ring and photoproduction experiments. More accurate data in the ρ meson region and also in the low-energy region are needed to test the FFGS model and to know what is the correct value of the ρ meson width.

1.11 The experimental data have also been fitted with phenomenological expressions of $F_\pi(s)$ different from the Omnès function. For instance

Curry and Moffat[21] have constructed a model where $F_\pi(s)$ is identified
with the Dirac nucleon isovector form factor $F_1{}^V(s)$ and has the explicit
form

$$\overline{F}_\pi(s) = \left(1 + \frac{s}{s_0}\right) \frac{m_s^2 \left[1 + \frac{m_\pi}{R_s} \frac{\Gamma_s}{m_s}\right]}{m_s^2 - s - i\, m_s \Gamma_s \frac{R(s)}{R_s}}$$

There is a zero on the negative real axis at $s = -s_0$ and the authors don't
worry about the wrong threshold behaviour at $s = 4\, m_\pi^2$. Agreement with
experiment can be achieved with

$$m_\rho \approx 765 \text{ MeV} \qquad \Gamma_\rho \approx 112 \text{ MeV} \qquad s_0 \approx 12.8\, m_\rho^2 \ .$$

1.12 For completeness we quote the Veneziano-type approach of the meson
electromagnetic form factor and we refer the reader to the specialized
literature.

2. DECAY OF VECTOR MESONS INTO A LEPTON-ANTILEPTON PAIR

2.1 In the time-like region around the vector meson masses where experi-
ments have been performed the VMD model reduces simply to an isobaric
model. We neglect, for the moment, the interferences between vector meson
contributions (see Lecture VI) and we analyze the data with the formula
(II.9)

$$\sigma_{\text{Tot}}\left(e^+e^- \Rightarrow V \Rightarrow F\right)_{s = m_V^2} = \frac{12\pi}{m_V^2} \frac{\Gamma(V \Rightarrow e^+e^-)}{\Gamma_V} \frac{\Gamma(V \Rightarrow F)}{\Gamma_V} \qquad \text{(III.17)}$$

which represents simply the diagram
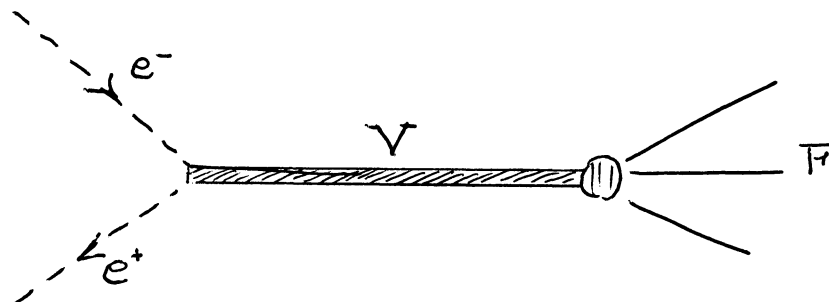
Fig. 11

Experiments have been performed in Orsay (O) and Novosibirsk (N)
looking at the following final states

$$F = \pi^+\pi^- \quad (O,N) \;, \qquad F = \pi^+\pi^-\pi^0 \quad (O) \;, \qquad F = K^0\bar{K}^0 \quad (O) \quad K^+K^- \quad (O,N) \;.$$

2.2 The $\pi^+\pi^-$ data in the ρ meson region are analyzed with a particular
model for the electromagnetic form factor $F_\pi(s)$. The Orsay results with
the FFGS model give[20]

$$\Gamma(\rho \to e^+e^-) = (7.4 \pm 0.7) \text{ keV} \;. \tag{III.18}$$

The Novosibirsk data are somewhat different and give a lower value for
the partial decay width

with FFGS $\qquad \Gamma(\rho \to e^+e^-) = (6.05 \pm 0.5)$ keV

with BW $\qquad \Gamma(\rho \to e^+e^-) = (5.25 \pm 0.9)$ keV .

2.3 The ω meson production is detected through the $\pi^+\pi^-\pi^0$ mode and the
data are fitted with a Breit-Wigner formula whose parameters are taken
from the world average

$$m_\omega = 783 \text{ MeV} \qquad \Gamma_\omega = 12.2 \text{ MeV} \;.$$

The value of the branching ratio $\Gamma(\omega \to 3\pi)/\Gamma_\omega$ has also been taken from the world average

$$\frac{\Gamma(\omega \to 3\pi)}{\Gamma_\omega} = 0.90 \pm 0.010 \ .$$

The result is[22]

$$\Gamma(\omega \to e^+e^-) = (1.0 \pm 0.18) \text{ keV} \ . \tag{III.19}$$

2.4 Three decay modes of the ϕ meson have been observed in the Orsay experiments: $K^0\bar{K}^0$, K^+K^-, $\pi^+\pi^-\pi^0$. Assuming the other modes to be negligible one obtains[23,24]

$$\Gamma(\phi \to e^+e^-) = (1.41 \pm 0.12) \text{ keV} \tag{III.20}$$

The total ϕ meson width has been measured:

$$\Gamma_\phi = (4.09 \pm 0.29) \text{ MeV} \tag{III.21}$$

and the branching ratios for the $K^0\bar{K}^0$, K^+K^- and $\pi^+\pi^-\pi^0$ decay modes

$$B(\phi \to K^0\bar{K}^0) = (30.1 \pm 4.1)\%$$

$$B(\phi \to K^+K^-) = (49.3 \pm 4.4)\%$$

$$B(\phi \to \pi^+\pi^-\pi^0) = (20.6 \pm 3.6)\%$$

In particular the ratio

$$\frac{\Gamma(\phi \to K^+K^-)}{\Gamma(\phi \to K^0\bar{K}^0)} = 1.64 \pm 0.23$$

is in good agreement with the SU(2) prediction corrected for the phase space (K^+-K^0 mass difference) and the Coulomb interaction[25]

$$\frac{\Gamma(\phi \to K^+K^-)}{\Gamma(\phi \to K^0\bar{K}^0)}_{th} = 1.60 \ .$$

The K^+K^- mode has also been detected in Novosibirsk and the results are in very good agreement with the Orsay data[26]

$$\Gamma(\phi \Rightarrow e^+e^-) = (1.42 \pm 0.15)\ keV$$

$$\Gamma_\phi = (4.1 \pm 0.5)\ MeV .$$

2.5 In the one photon exchange approximation the partial decay widths $\Gamma(v \rightarrow e^+e^-)$ are related to the coupling constants f_V by the formula (II.7).

$$\Gamma(V \Rightarrow e^+e^-) = \frac{\alpha^2}{3} m_V \frac{4\pi}{|f_V|^2}$$

From the Orsay data we obtain

$$\frac{|f_\phi|^2}{4\pi} = 1.85 \pm 0.17 \qquad\qquad (III.22)$$

$$\frac{|f_\omega|^2}{4\pi} = 13.9 \pm 2.5 \qquad\qquad (III.23)$$

$$\frac{|f_\rho|^2}{4\pi} = 12.85 \pm 1.10 . \qquad\qquad (III.24)$$

3. VECTOR MESON DOMINANCE MODEL FOR F_π, F_K, $G_{\pi\gamma}$

The vector meson states ρ, ω and ϕ are defined so that

a) assuming time reversal invariance

b) neglecting the electromagnetic effects

the three coupling constants f_V are real and positive.

3.1 π meson electromagnetic form factor $F_\pi(s)$

The ρ meson contribution to $F_\pi(s)$ is given, in the VMD model by

$$\overline{\Gamma}_\pi^{(s)}(s) = \frac{1}{f_\rho^0} \frac{m_\rho^2}{W_\rho(s) - s} f_{\rho\pi\pi}(s) \tag{III.25}$$

On mass shell the decay coupling constant $f_{\rho\pi\pi} \equiv f_{\rho\pi\pi}(m_\rho^2)$ is related to the $\rho \to \pi\pi$ decay width by

$$\Gamma(\rho \Rightarrow \pi^+\pi^-) = \frac{f_{\rho\pi\pi}^2}{4\pi} \frac{m_\rho}{12} \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{3/2} \tag{III.26}$$

Off mass shell the vertex function $f_{\rho\pi\pi}(s)$ is used to compute the decay function $\sum_{\rho\rho}^{(2\pi)}(s)$

$$\sum_{\rho\rho}^{(2\pi)}(s) = \frac{f_{\rho\pi\pi}^2(s)}{4\pi} \frac{s}{12} \left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} \tag{III.27}$$

On the other hand, the propagator function $W_\rho(s)$ is analytic in the complex s cut plane and it satisfies a dispersion relation. The spectral function is determined by the unitarity condition

$$- \operatorname{Im} W_\rho(s) = \sum_F' \sum_{\rho\rho}^F(s) \tag{III.28}$$

In practice, only the 2π contribution to the sum (III.28) is taken into account and we obtain a model-dependent expression for $W_\rho(s)$ corresponding to a particular assumption on $f_{\rho\pi\pi}(s)$. This situation is the reflect of the corresponding one with the N/D method as explained in the first section.

For instance, the Gounaris-Sakurai model assumes, in this language, the vertex function $f_{\rho\pi\pi}(s)$ to be a constant. Two subtractions are then necessary for writing a dispersion relation for $W_\rho(s)$ and these two unknown constants are determined by the ρ meson parameters m_ρ and Γ_ρ.

a) Let us now assume that $F_\pi(s)$ satisfies an unsubtracted dispersion relation. The normalization condition $F_\pi(0) = 1$ gives the sum rule

$$\frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\operatorname{Im} \bar{F}_\pi(t)}{t}\, dt = 1 \qquad (III.28)$$

If the sum rule can be saturated by the ρ meson contribution we deduce, from (III.25) the relation

$$\frac{f_{\rho\pi\pi}^\rho(0)}{f_\rho} \; \frac{m_\rho^2}{W_\rho(0)} = 1 \qquad (III.29)$$

In the zero width approximation $W_\rho(0) = m_\rho^2$ and we obtain from (III.29) the famous universality relation[27]

$$f_{\rho\pi\pi}^\rho(0) = f_\rho^\rho$$

Because of the large ρ meson width, finite width corrections must be taken into account, one writes as

$$W_\rho^\Gamma(0) = a_\rho\, m_\rho^2$$

and the normalization sum rule (III.29) becomes

$$\frac{f_{\rho\pi\pi}(o)}{f_\rho} = a_\rho \qquad\qquad (III.30)$$

Unfortunately the parameter a_ρ cannot be determined in a model independent way. For instance in the FFGS model where $f_{\rho\pi\pi}(0) = f_\rho$ we have $a_\rho \simeq 1.07$.

The Orsay data as given in Table 1 have been analysed with the FFGS model so that the relation (III.30) is automatically satisfied

$$\frac{f_{\rho\pi\pi}^2}{4\pi} = 2,11 \pm 0,13 \qquad\qquad \frac{f_\rho^2}{4\pi} = 1,85 \pm 0,17 \qquad\qquad (III.31)$$

and

$$\frac{f_{\rho\pi\pi}}{f_\rho} = 1,07$$

If now the three parameter fit is used we obtain different values for the coupling constants

$$\frac{f_{\rho\pi\pi}^2}{4\pi} = 2,42 \pm 0,25 \qquad\qquad \frac{f_\rho^2}{4\pi} = 1,74 \pm 0,17 \qquad\qquad (III.32)$$

but the relation (III.30) cannot be tested without further information about the energy variation of the vertex function $f_{\rho\pi\pi}(s)$. The only indication given by these last numbers is that $f_{\rho\pi\pi}(s)$ is certainly a slowly varying function of s between $s = 0$ and $s = m_\rho^2$ with a slight tendency to be increasing.

In all what follows we will assume all the vertex functions $f_{VF}(s)$ to be well represented by a constant in the range $s = 0$ $s = m_V^2$, keeping in mind the possibility of a small variation.

b) Even if one subtraction in the dispersion relation for $F_\pi(s)$ is needed we can obtain a second sum rule

$$\frac{1}{\pi} \int_{4m_\pi^2}^{\sim} \frac{Im\,\tilde{F}_\pi(t)}{t^2}\, dt = \bar{F}_\pi^{\,\prime}(o) = \frac{1}{6} \langle r_\pi^2 \rangle$$

Assuming again the sum rule to be saturated by the ρ meson contribution, we obtain a relation analogous to (III.29) and involving the root mean square radius of the π meson charge distribution.

$$\frac{f_{\rho\pi\pi}}{f_\rho} \left(\frac{m_\rho^2}{W_\rho(o)} \right)^2 \left(1 - W_\rho^{\,\prime}(o) \right) = \frac{1}{6} \langle r_\pi^2 \rangle\, m_\rho^2 \qquad (III.33)$$

In the zero width approximation we obtain the VMD prediction

$$\langle r_\pi^2 \rangle^{1/2} = \frac{\sqrt{6}}{m_\rho} = 0.64 \text{ fermi.}$$

The agreement with experiment is not too bad

$$\langle r_\pi^2 \rangle^{1/2} = (0.80 \pm 0.10) \text{ fermi } [28]$$

$$\langle r_\pi^2 \rangle^{1/2} = (0.86 \pm 0.14) \text{ fermi } [29]$$

Again the finite width corrections

$$W_\rho^{\,\prime}(o) = -\, e_\rho\, \frac{\Gamma_\rho}{m_\rho}$$

are model dependent. The FFGS model gives $e_\rho \simeq 0.58$ so that finite width corrections in (III.33) are small in that model.

3.2 K meson electromagnetic form factors $F_K(s)$

We have two K meson electromagnetic form factors $F_{K^+}(s)$ and $F_{K^0}(s)$ normalized, at $s = 0$, to the K meson charge

$$F_{K^+}(0) = 1 , \qquad F_{K^0}(0) = 0 .$$

From isospin invariance and neglecting electromagnetic corrections we can now introduce two new form factors

$$\bar{F}_K^{I=0}(s) = \frac{1}{2}\left[\bar{F}_{K^+}(s) + \bar{F}_{K^0}(s)\right]$$

$$\bar{F}_K^{I=1}(s) = \frac{1}{2}\left[\bar{F}_{K^+}(s) - \bar{F}_{K^0}(s)\right]$$

The ρ meson contribution to the K meson isovector form factor is given in the VMD model by

$$\bar{F}_K^{(\rho)}(s) = \frac{f_{\rho K\bar{K}}}{f_\rho} \frac{m_\rho^2}{W_\rho(s) - s} = \frac{f_{\rho K\bar{K}}}{f_{\rho\pi\pi}} \bar{F}_\pi^{(\rho)}(s) \tag{III.34}$$

The normalization condition implies the coupling constant relation $f_{\rho K\bar{K}} = \frac{1}{2} f_{\rho\pi\pi}$ which holds in the SU(3) symmetry even broken in a particular way (see Lecture V).

The ω and ϕ meson contributions to the K meson isoscalar form factor are given, in the VMD model by

$$\bar{F}_K^{I=0}(s) \simeq \frac{f_{\omega K\bar{K}}}{f_\omega} \frac{m_\omega^2}{W_\omega(s) - s} + \frac{f_{\phi K\bar{K}}}{f_\phi} \frac{m_\phi^2}{W_\phi(s) - s} \tag{III.35}$$

If the ω and ϕ mesons saturate the normalization sum rule, we obtain a relation analogous to (III.29)

$$\frac{f_{\omega K\bar{K}}^c}{f_\omega} \frac{m_\omega^2}{W_\omega^r(o)} + \frac{f_{\phi K\bar{K}}^c}{f_\phi} \frac{m_{\phi^2}}{W_\phi^r(o)} = \frac{1}{2} \qquad \text{(III.36)}$$

The zero width approximation is certainly valid for the ω meson and $W_\omega(0) = m_\omega^2$.

For the ϕ meson the situation is different because of the smallness of the phase space available in the $K\bar{K}$ decay modes. Finite width corrections have been computed by Renard[30] in a particular model of the FFGS type; he finds

$$W_\phi^r(o) \simeq 0.85\ m_\phi^2$$

Relation (III.31) becomes

$$\frac{f_{\omega K\bar{K}}^c}{f_\omega} + 1.17\ \frac{f_{\phi K\bar{K}}^c}{f_\phi} = \frac{1}{2} \qquad \text{(III.37)}$$

The ϕ meson contribution to Eq. (III.37) can be directly measured from the value of the cross-section $\sigma_{TOT}(e^+e^- \to K\bar{K})$ at the ϕ meson mass:

$$\left(\frac{f_{\phi K\bar{K}}^c}{f_\phi} \right)^2 = \frac{3}{\pi \alpha^2} \frac{\Gamma_\phi^2\ \sigma_{Tot}(e^+e^- \to K\bar{K})_{s=m_\phi^2}}{\left(1 - \frac{4 m_K^2}{m_\phi^2} \right)^{3/2}}$$

Using the Orsay data for the K^+K^- mode we obtain[24]

$$\left| \frac{f_{\phi K^+K^-}}{f_\phi} \right|_{exp} = 0,349 \pm 0,025 \qquad \text{(III.38)}$$

Because of the large experimental and theoretical uncertainties we can, at this stage, neglect the Coulomb corrections[25] and use the same coupling constant in both K^+K^- and $K^0\bar{K}^0$ modes.

Combining now the experimental results (III.23) and (III.38) we compute the $\omega \to K\bar{K}$ coupling constant from the normalization relation (III.37)

$$\frac{f^2_{\omega K\bar{K}}}{4\pi} = 0,11 \,^{+0,09}_{-0,06} \qquad \text{(III.39)}$$

The experimental value of the $\phi \to K\bar{K}$ coupling constant is deduced from Eqs. (III.38) and (III.24)

$$\frac{f^2_{\phi K\bar{K}}}{4\pi} = 1,55 \pm 0,18 \qquad \text{(III.40)}$$

In Lecture V we will compare these two values with the predictions of exact and broken SU(3) symmetry.

3.3 $\pi^0\gamma$ electromagnetic form factors $G_{\pi^0\gamma}(s)$[31]

The form factor $G_{\pi^0\gamma}(s)$ can be split into two terms depending on the isospin character of the photon

$$G_{\pi^0\gamma}(s) = G_{\pi^0\gamma}^{I=0}(s) + G_{\pi^0\gamma}^{I=1}(s)$$

In the $\pi^0 \to 2\gamma$ decay, because of the Bose symmetry for the two photon final state, one photon is isoscalar, one photon isovector. It follows the normalization condition

$$G_{\pi^0\gamma}^{I=0}(0) = G_{\pi^0\gamma}^{I=1}(0) = \frac{1}{2} G_{\pi^0\gamma}(0) \qquad \text{(III.41)}$$

As has been seen in Lecture I the constant $G_{\pi^0\gamma}(0)$ is related to the $\pi^0 \to 2\gamma$ decay width by

$$\Gamma(\pi^0 \to 2\gamma) = \frac{\pi\alpha^2}{4} m_{\pi^0} \left[G_{\pi^0\gamma}(0) \right]^2 \qquad \text{(III.42)}$$

The π^0 state is defined so that $G_{\pi^0\gamma}(0)$ is real and positive. Using the recent experimental value[32]

$$\Gamma(\pi^0 \to 2\gamma) = (11.7 \pm 1.2) \text{ eV} \qquad \text{(III.43)}$$

we obtain

$$G_{\pi^0\gamma}(0) = (4.56 \pm 0.23)10^{-2}. \qquad \text{(III.44)}$$

In the VMD model, the form factor $G_{\pi^0\gamma}(s)$ is assumed to be well described by the sum of the vector meson contributions
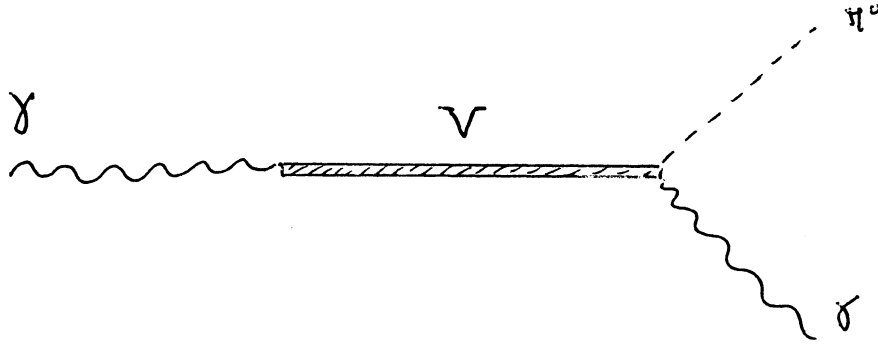
Fig. 12

The coupling constants $f_{V\pi^0\gamma}$ are defined by

$$\langle \pi^0\gamma \mid T \mid V \rangle = e \; \frac{f_{V\pi^0\gamma}}{m_{\pi^0}} \; \mathcal{E}_{\mu\nu\rho\sigma} \; P_V^\mu \, e_V^\nu \, P_\sigma^\rho \, \mathcal{E}_\gamma^\sigma \qquad (III.45)$$

where e_V and ε_γ are the polarization four vectors for the vector meson and the photon. The $V \to \pi^0\gamma$ widths are computed to be

$$\Gamma(V \Rightarrow \pi^0\gamma) = \frac{\alpha}{24} \, f_{V\pi^0\gamma}^2 \left(\frac{m_V}{m_{\pi^0}} \right)^2 \left(1 - \frac{m_{\pi^0}^2}{m_V^2} \right)^3 m_V \qquad (III.46)$$

The isovector and isoscalar form factors $G^I_{\pi^0\gamma}(s)$ are given, in the VMD model by

$$G^{I=1}_{\pi^0\gamma}(s) = \frac{f_{\rho\pi^0\gamma}}{f_\rho} \; \frac{m_\rho^2}{W_\rho(s) - s} \qquad (III.47)$$

$$G_{\pi^0\gamma}^{T^{I=0}}(s) = \frac{f_{\omega\pi^0\gamma}^{\rho}}{f_\omega} \frac{m_\omega^2}{W_\omega(s)-s} + \frac{f_{\phi\pi^0\gamma}^{\rho}}{f_\phi} \frac{m_\phi^2}{W_\phi(s)-s} \qquad \text{(III.48)}$$

In fact the isovector part $G_{\pi^0\gamma}^{I=1}(s)$ is proportional to the π meson electromagnetic form factor $F_\pi(s)$

$$G_{\pi^0\gamma}^{I=1}(s) = \frac{f_{\rho\pi^0\gamma}^{\rho}}{f_{\rho\pi\pi}^{\rho}} \overline{F}_\pi(s)$$

and the normalization condition at $s = 0$ gives the sum rule

$$\frac{f_{\rho\pi^0\gamma}^{\rho}}{f_{\rho\pi\pi}^{\rho}} = \frac{1}{2} G_{\pi^0\gamma}(0) \qquad \text{(III.49)}$$

Squaring this relation and introducing the convenient kinematical factors we immediately deduce the relation between partial decay widths

$$\frac{\Gamma(\rho \to \pi^0\gamma)}{\Gamma(\rho \to \pi\pi)} = \frac{2}{\alpha} \left(\frac{m_\rho}{m_{\pi^0}}\right)^2 \frac{\left(1 - \frac{m_{\pi^0}^2}{m_\rho^2}\right)^3}{\left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{3/2}} \frac{\Gamma(\pi^0 \to 2\gamma)}{m_{\pi^0}}$$

With the experimental value (III.43) for the π^0 life-time we predict

$$\frac{\Gamma(\rho \to \pi^0\gamma)}{\Gamma(\rho \to \pi\pi)} = (8.7 \pm 0.9)\, 10^{-4} \qquad \text{(III.50)}$$

to compare with the experimental upper limit[33]

$$\frac{\Gamma(\rho \Rightarrow \pi^o \gamma)}{\Gamma(\rho \Rightarrow \pi \pi)} \exp < 40 \; 10^{-4}$$

The coupling constant $f_{\rho\pi^0\gamma}$ is computed from (III.44) and (III.49) to be

$$\left|
\begin{array}{ll}
f^\rho_{\rho\pi^o\gamma} = 0,117 \pm 0,007 & \text{with} \quad \dfrac{f^2_{\rho\pi\pi}}{4\pi} = 2,11 \\[4mm]
f^\rho_{\rho\pi^o\gamma} = 0,126 \pm 0,009 & \text{with} \quad \dfrac{f^2_{\rho\pi\pi}}{4\pi} = 2,42
\end{array}
\right. \tag{III.51}$$

Let us now study the isoscalar part. If the ω and ϕ contributions saturate the normalization sum rule we obtain the relation

$$\frac{f^\rho_{\omega\pi^o\gamma}}{f_\omega} \frac{m^2_\omega}{W_\omega(o)} + \frac{f^\rho_{\phi\pi^o\gamma}}{f_\phi} \frac{m^2_\phi}{W_\phi(o)} = \frac{1}{2} G_{\pi^o\gamma}(o) \tag{III.52}$$

Again we introduce finite width corrections for the ϕ meson contribution and the relation (III.52), using the experimental result (III.44) becomes

$$\frac{f^\rho_{\omega\pi^o\gamma}}{f_\omega} + 1,17 \frac{f^\rho_{\phi\pi^o\gamma}}{f_\phi} = (2,28 \pm 0,12) \, 10^{-2} \tag{III.53}$$

The radiative decay width of the ω meson has been experimentally measured[34]

$$\Gamma\left(\omega \to \pi^0\gamma\right) = (1.1 \pm 0.2) \text{ MeV}$$

from which we determine up to a sign the $f_{\omega\pi^0\gamma}$ coupling constant

$$\left| f_{\omega\pi^0\gamma} \right| = 0.39 \pm 0.04 \quad .$$

Therefore the ω contribution to Eq. (III.48) can be computed from the measured $\omega \to \pi^0\gamma$ and $\omega \to e^+e^-$ partial decay widths. The result is

$$\left| \frac{f_{\omega\pi^0\gamma}}{f_\omega} \right|_{exp} = (2.95 \pm 0.41)10^{-2}$$

Comparing this value with the right-hand side of Eq. (III.53) we choose a positive sign for the coupling constant $f_{\omega\pi^0\gamma}$ and the ϕ meson contribution turns out to be of opposite sign and very small compared to the ω one as expected from the quark model (see Lecture V)

$$\frac{f_{\phi\pi^0\gamma}}{f_\phi} = (-0.55 \pm 0.38)10^{-2}$$

Using for f_ϕ the Orsay data (III.24) we can estimate a range of values for the radiative decay width of the ϕ meson

$$f_{\phi\pi^0\gamma} = -0.070 \pm 0.0048 \qquad\qquad \text{(III.54)}$$

$$\Gamma\left(\phi \to \pi^0\gamma\right) = (10-200) \text{ keV} \qquad\qquad \text{(III.55)}$$

The only experimental information we have is an upper limit[35]

$$\Gamma\left(\phi \Rightarrow \eta^\circ \gamma\right)_{exp} < 15 \text{ keV}$$

compatible with the prediction (III.55).

LECTURE IV

SUM RULES AND HIGH-ENERGY BEHAVIOUR

1. SPECTRAL REPRESENTATIONS

1.1 The trace of the tensor $M_{\mu\nu}$ defined in Lecture I (I.3) and the total cross-section for electron-positron annihilation into hadrons are related to the spectral function $\rho^{e.m.}(s)$ defined in the Källen-Lehman representation value of products of components of the electromagnetic current.

More generally, for a conserved current we have[36,37]

$$\langle 0| J_\mu^\nu(x)\, J_\nu^\beta(o)|0\rangle = \int_0^{\widetilde{\;}} \widetilde{\rho}^{\alpha\beta}(t)\left[g_{\mu\nu} - \frac{1}{t}\partial_\mu\partial_\nu\right] i\, \Delta^+(x;t)\, dt$$

$$\langle 0|[J_\mu^\eta(x),\, J_\nu^\beta(o)]|0\rangle = \int_b^{\widetilde{\;}} \widetilde{\rho}^{\alpha\beta}(t)\left[g_{\mu\nu} - \frac{1}{t}\partial_\mu\partial_\nu\right] i\, \Delta(x;t)\, dt$$

$$\langle 0|T(J_\mu^\alpha(x)\, J_\nu^\beta(o))|0\rangle = \int_0^{\overline{\;}} \overline{\rho}^{\alpha\beta}(t)\left[g_{\mu\nu} - \frac{1}{t}\partial_\mu\partial_\nu\right] i\, \Delta_F(x;t)\, dt$$

$$+ \frac{1}{i}\, g_{\mu 0} g_{\nu 0}\, \delta_4(x) \int_0^{\widetilde{\;}} \frac{\widetilde{\rho}^{\alpha\beta}(t)}{t}\, dt$$

where the invariant distributions $\Delta^+(x;t)$, $\Delta(x;t)$ are defined by their four-dimensional Fourier transform

$$\Delta^+(k;t) = -2i\pi\, \theta(k_0)\delta(k^2 + t)$$

$$\Delta(k;t) = -2i\pi\, \varepsilon(k_0)\, \delta(k^2 + t)$$

$$\Delta_F(k;t) = \lim_{\varepsilon \Rightarrow 0^+} \frac{1}{k^2 + t - i\varepsilon}$$

The precise relation between $\rho^{e.m.}$ and the total cross-section $\sigma(e^+e^- \to$ hadrons$)$ is[2]

$$\sigma_{TOT}(e^+e^- \to hadrons) = \frac{16\pi^3\alpha^2}{s^2} \rho^{em}(s)$$ (IV.1)

1.2 Schwinger terms[38]

From the properties of the invariant distribution $\Delta(x,t)$ it is straightforward to prove the existence of Schwinger terms in the equal time commutator of a time component of the current with a space component (k = 1,2,3)

$$\langle 0|[J_o^{em}(x), J_k^{em}(o)]|0\rangle = \frac{1}{i} \frac{\partial}{\partial x^k} \delta_4(x) \int_t \frac{\rho^{em}(t)}{t} dt$$

The last integral is called the vacuum expectation value of the Schwinger term – or the Schwinger term itself if it is a c-number. Using the relation (IV.1) we have

$$C^{em} \equiv \int_0 \frac{\rho^{em}(t)}{t} dt = \frac{1}{16\pi^3\alpha^2} \int_0 t \, \sigma_{TOT}(e^+e^- \to hadrons) dt$$ (IV.2)

1.3 Vacuum polarization[2]

The photon propagator $D_{\mu\nu}(q)$ is the vacuum expectation value of the time-ordered product of two components of the electromagnetic field $A_\mu(x)$. It can be written as the sum of the free field propagator and of a gauge invariant correction

$$D_{\mu\nu}(q) = \frac{g_{\mu\nu}}{s+i\varepsilon} - \left[g_{\mu\nu} + \frac{q_\mu q_\nu}{s+i\varepsilon} \right] \Pi(s)$$

where $q^2 + s = 0$.

1.4 Muon anomalous magnetic moment[39]

The hadronic contributions to the muon anomalous magnetic moment $a_\mu = \frac{1}{2}(g_\mu - 2)$ due to vacuum polarization corrections are also given as an integral over the e^+e^- annihilation total cross-section

$$a_\mu \text{ (hadrons)} = \frac{1}{4\pi^2\alpha} \int_0^\infty \sigma_{Tot} (e^+e^- \to \text{hadrons}) \, K_\mu^{(2)}(t) \, dt$$

(IV.4)

where the weight function $K_\mu^{(2)}(t)$ is the second order vertex function explicitly known[40]

$$K_\mu^{(2)}(t) = \frac{\alpha}{\pi} \left\{ \frac{y^2}{2}(2-y) + (1+y)^2(1+y^2) \frac{\log(1+y) - y + \frac{y^2}{2}}{y^2} + \frac{1+y}{1-y} y^2 \log y \right\}$$

where

$$y = \frac{1 - \left(1 - \frac{4m_\mu^2}{t}\right)^{1/2}}{1 + \left(1 - \frac{4m_\mu^2}{t}\right)^{1/2}}$$

We retain the two interesting properties of $K_\mu^{(2)}(t)$

$$K_\mu^{(2)}(0) = \frac{\alpha}{\pi}$$

$$K_\mu^{(2)}(t) \simeq \frac{1}{t} \qquad \text{for large } t .$$

1.5 Charge renormalization[9]

The bare, e_0, and renormalized, e, electric charges are related by

$$e^2 = e_0^2 - \delta e_0^2$$

and the correction δe_0^2 can be computed in terms of the spectral function $\rho^{e.m.}$.

$$\frac{\delta e_0^2}{e^2} = e^2 \int_0^\infty \frac{\rho^{em}(t)}{t^2} \, dt$$

The hadronic contributions to the charge renormalization take the form

$$\frac{\delta e_0^2}{e^2} \text{(hadrons)} = \frac{1}{4\pi^2\alpha} \int_\tau^\infty \sigma_{TOT} \, (e^+e^- \to \text{hadrons}) \, dt \qquad \text{(IV.5)}$$

1.6 Convergence of the integrals

We have written a set of integrals involving the total cross-section for electron-positron annihilation into hadrons $\sigma_{TOT}(s)$ and we try now to discuss the question of convergence of these integrals. We can distinguish three classes of integrals

Class	Example	Type
A	Schwinger term	$\int_0^\infty t \, \sigma_{TOT}(t) \, dt$
B	Charge renormalization	$\int_0^\infty \sigma_{TOT}(t) \, dt$
C	Vacuum polarization	$\int_0^\infty \frac{1}{t} \sigma_{TOT}(t) \, dt$

Table 2

For instance, in pure quantum electrodynamics the total cross-sections behave like $1/s$ for large s and therefore we have divergences of the linear type for A, of the logarithmic type for B, and convergence for C.

The same situation occurs in composite models of elementary particles as for instance the quark model[41]. But we can construct models where the three classes of integrals A, B and C are convergent. A detailed discussion of the various field algebra theories has been given by Gatto in his Daresbury talk[42] where the reader can refer for more details.

2. VECTOR MESON DOMINANCE MODEL FOR SPECTRAL REPRESENTATIONS

We replace the total cross-section $\sigma_{TOT}(e^+e^- \to$ hadrons$)$ by the sum of the vector meson contributions, neglecting the interferences

$$\sigma_{TOT}(e^+e^- \Rightarrow \text{hadrons}) \underset{VMD}{\simeq} \sum_V {}' \sigma_{TOT}(e^+e^- \Rightarrow V) \qquad (IV.6)$$

2.1 Schwinger term

If the function $\left[W_V(s) - s\right]^{-1}$ satisfies an unsubtracted dispersion relation, the Schwinger term is finite and using equation (II.11) we define a finite width correction parameter x_V by

$$x_V^2 = \frac{m_V^2}{W_V(0)} = \frac{1}{12\pi^2} \; \frac{\int_0^{\sim} t \, \sigma_{TOT}(e^+e^- \Rightarrow V) \, dt}{m_V \, \Gamma(V \Rightarrow e^+e^-)}$$

The Schwinger term $C^{e.m.}$ is then written as

$$C^{em} = \sum_V {}' C^V \qquad (IV.7)$$

and each vector meson contribution is given by

$$C^V = \frac{3}{4\pi\alpha^2}\, \alpha_V\, m_V\, \Gamma(V \to e^+ e^-) = \alpha_V\, \frac{m_V^2}{f_V^2} \qquad (IV.8)$$

Using

$$x_\rho = 0.93\ , \qquad x_\omega = 1\ , \qquad x_\phi = 1.15$$

and the Orsay data (III.22), (III.23) and (III.24), we obtain

$$c^\rho = (23.96 \pm 2.16)10^{-3}\ \text{GeV}^2 \qquad (IV.9)$$

$$c^\omega = (3.50 \pm 0.63)10^{-3}\ \text{GeV}^2 \qquad (IV.10)$$

$$c^\phi = (7.40 \pm 0.53)10^{-3}\ \text{GeV}^2 \qquad (IV.11)$$

so that

$$c^{e.m.} = (34.9 \pm 2.3)\ 10^{-3}\ \text{GeV}^2\ .$$

2.2 Charge renormalization

If the function $[W_V(s) - s]^{-1}$ satisfies a dispersion relation, with at most one subtraction, the hadronic contributions to the charge renormalization are finite in the VMD model and, using Eq. (II.12), we define a finite width correction parameter y_V by

$$y_V = \left(\frac{m_V^2}{W_V(0)}\right)^2 \left(1 - W_V'(0)\right) = \frac{1}{12\pi^2}\ \frac{\int_0^\infty \sigma_{TOT}(e^+ e^- \to V)\, dt}{\dfrac{\Gamma(V \to e^+ e^-)}{m_V}}$$

The charge renormalization is then written as

$$\frac{\delta e_0^2 \text{(hadrons)}}{e^2} = \sum_V{}' \frac{\delta e_0^2 (V)}{e^2} \tag{IV.12}$$

and each vector meson contribution is given by

$$\frac{\delta e_0^2 (V)}{e^2} = \frac{3}{\alpha} y_V \frac{\Gamma(V \Rightarrow e^+ e^-)}{m_V} = \alpha \, y_V \frac{4\pi}{f_V^2} \tag{IV.13}$$

Using

$$y_\rho = 0.95 \, , \quad y_\omega = 1 \, , \quad y_\phi = 1.075$$

and the Orsay data we get

$$\frac{\delta e_0^2}{e^2}(\rho) = (3.75 \pm 0.34)10^{-3} \tag{IV.14}$$

$$\frac{\delta e_0^2}{e^2}(\omega) = (0.52 \pm 0.09)10^{-3} \tag{IV.15}$$

$$\frac{\delta e_0^2}{e^2}(\phi) = (0.61 \pm 0.05)10^{-3} \tag{IV.16}$$

so that

$$\frac{\delta e_0^2 \text{(hadrons)}}{e^2} = (4.88 \pm 0.35)10^{-3} \, .$$

2.3 Muon anomalous magnetic moment[43]

The VMD model computation of a_μ (hadrons) defined in Eq. (IV.4) gives the result

$$a_\mu \text{(hadrons)} = (6.5 \pm 0.5)10^{-8} \, .$$

The theoretical prediction including 2nd, 4th, 6th terms is given by[44]

$$\partial_{\mu \ th} = (116587 \pm 2)10^{-8}$$

in very good agreement with the most recent experimental value[45]

$$\partial_{\mu \ exp} = (116616 \pm 31)10^{-8} \ .$$

3. ELECTROMAGNETIC SUM RULES

3.1 The electromagnetic current can be decomposed as a sum of its isotopic spin components

$$J_\mu^{e.m.} = J_\mu^3 + \frac{1}{\sqrt{3}} J_\mu^8 \quad .$$

In the SU(3) language

J_μ^3 corresponds to an isovector particle of the octuplet

J_μ^8 corresponds to an isoscalar particle of the octuplet.

For the special function $\rho^{e.m.}$ we have the decomposition

$$\rho^{e.m.} = \rho^{33} + \frac{1}{3}\rho^{88} + \frac{1}{\sqrt{3}} (\rho^{38} + \rho^{83}) \ . \qquad (IV.17)$$

The last term corresponds to an electromagnetic contribution and it will be neglected in this section (see Lecture VI). The spectral functions ρ^{33} and ρ^{88} are associated with total cross-sections for e^+e^- annihilation into hadronic states of definite isotopic spin, respectively I = 1 and I = 0.

3.2 First Weinberg sum rule[46]

From the Gell-Mann commutation relations for the current components of the SU(3) vector components and some additional technical assumptions Weinberg has derived a spectral function sum rule which it is convenient to write in the form

$$\int_0^\infty t \, \overline{\sigma}_{Tot} (e^+e^- \Rightarrow I=1) dt = 3 \int_0^\infty t \, \overline{\sigma}_{Tot} (e^+e^- \Rightarrow I=0) dt \qquad (IV.18)$$

In the Schwinger term language the first Weinberg sum rule is simply written as

$$c^{33} = c^{88} \qquad \text{(IV.19)}$$

where $c^{\alpha\beta}$ is defined from $\rho^{\alpha\beta}$ by an equation analogous to (IV.2). The VMD model for the Schwinger terms has been computed in the previous section

$$c^{33} = c^{\rho} \qquad c^{88} = 3(c^{\omega} + c^{\phi})$$

and we obtain a sum rule for the partial decay widths $\Gamma(V \to e^+e^-)$

$$x_\rho \, m_\rho \, \Gamma(\rho \to e^+e^-) = 3 \left[x_\omega \, m_\omega \, \Gamma(\omega \to e^+e^-) + x_\phi \, m_\phi \, \Gamma(\phi \to e^+e^-) \right] \qquad \text{(IV.20)}$$

which, in the zero width approximation ($x_V = 1$) is known as the Das-Mathur-Okubo sum rule[47].

A good way to compare the sum rules with experiment is to introduce the dimensionless parameter δ

$$\delta = \frac{[I = 1] - [I = 0]}{[I = 1] + [I = 0]}$$

where $[I]$ is the isotopic spin I contribution to the sum rule. From the Orsay measurements we obtain with and without finite width corrections

$$\delta_W = (-15.5 \pm 6)10^{-2} \qquad \delta_W^0 = (-7.3 \pm 6.1)10^{-2} \qquad \text{(IV.21)}$$

3.3 Charge renormalization sum rule[48,49]

From the Sugawara model[50] one can derive a spectral function sum rule

$$\int_0^\infty \sigma_{TOT}(e^+e^- \to I=1)\, dt = 3\int_0^\infty \sigma_{TOT}(e^+e^- \to I=0)\, dt \qquad (IV.22)$$

which implies the equality of the isovector and isoscalar contributions to the charge renormalization.

Using the VMD model (IV.13) we obtain a second sum rule for the partial decay widths $\Gamma(V \to e^+e^-)$

$$y_f \frac{\Gamma(\rho \to e^+e^-)}{m_\rho} = 3\left[y_\omega \frac{\Gamma(\omega \to e^+e^-)}{m_\omega} + y_\phi \frac{\Gamma(\phi \to e^+e^-)}{m_\phi} \right] \qquad (IV.23)$$

From the Orsay measurements we obtain with and without finite width corrections

$$\delta_G = (+4.7 \pm 6.3)10^{-2} \qquad \delta_G^0 = (+9.2 \pm 6.4)10^{-2} . \qquad (IV.24)$$

3.4 Sugawara sum rule[50]

The sum rules (IV.20) and (IV.23) cannot both be exact sum rules and in the framework of the Sugawara model the first Weinberg sum rule must be corrected with a specific factor corresponding to a particular breaking of the SU(3) symmetry. Relation (IV.13) becomes

$$\frac{C^{33}}{m_3^2} = \frac{C^{88}}{m_8^2} . \qquad (IV.25)$$

Using a Gell-Mann-Okubo formula for the vector meson octuplet we have

$$m_{\eta_3}^2 = m_\rho^2 \qquad\qquad m_\gamma^2 = \frac{1}{3}\left(4\, m_{K^*}^2 - m_\rho^2\right)$$

and instead of the Das-Mathur-Okubo sum rule we obtain the Sugawara sum rule

$$m_\rho\, \Gamma(\rho \to e^+ e^-) = 3\left[m_\omega\, \Gamma(\omega \to e^+ e^-) + m_\phi\, \Gamma(\phi \to e^+ e^-)\right] \frac{3\, m_\rho^2}{4\, m_{K^*}^2 - m_\rho^2} \qquad \text{(IV.26)}$$

The comparison with the Orsay data gives

$$\delta_S = (2.9 \pm 6.6)10^{-2} \qquad \delta_S^0 = (+11.1 \pm 6.7)10^{-2} \; . \qquad \text{(IV.27)}$$

3.5 Discussion

In the framework of the VMD model and assuming to be correct

    a)   the Orsay experimental data

    b)   the finite width corrections

we have obtained a $2\frac{1}{2}$ standard deviation effect for the first Weinberg sum rule and a good agreement with experiment for the charge renormalization and the Sugawara sum rules.

The first Weinberg sum rule is associated with a current type mixing between ω and ϕ whereas the charge renormalization and the Sugawara sum rules are both associated with a mass type mixing[48]. On the basis of the present calculations the mass mixing model is favoured.

Nevertheless we must keep in mind that the VMD has not been tested in the time-like region and high energy contributions to the sum rule integrals can change our conclusions.

4.   STRUCTURE FUNCTIONS $\tilde{V}_1$ AND $\tilde{V}_2$

4.1 We consider the three related processes

$$e^- + p \to e^- + \Gamma \qquad\qquad\qquad\qquad I$$

$$e^+ + e^- \to p + \Gamma \qquad\qquad\qquad\qquad II$$

$$\Gamma \to e^+ + e^- + p \qquad\qquad\qquad\qquad III$$

where p is a particle of mass M and Γ an arbitrary multi-particle state of effective mass W.

   We use the one photon exchange approximation and we call q the photon energy momentum four vector.

   In the q^2, W^2 plane the physical regions for reactions I, II, III are shown in Fig. 13.

   It will be useful later to introduce the dimensionless parameter ω corresponding to the slope of a straight line passing by the point 0

$$\omega = 1 + \frac{W^2 - M^2}{q^2} \quad .$$

For the reaction   I $1 < \omega$

                      II $0 < \omega < 1$

                     III $\omega < 0$.

$W^2 = M^2 - q^2 + 2M\sqrt{-q^2}$

$W^2 = M^2 - q^2 - 2M\sqrt{-q^2}$

$\dfrac{W^2}{M^2} = \dfrac{U^2}{M^2}$

III

I

II

$e^+ e^- \Rightarrow p\bar{p}$

$+1$

$e^- p \Rightarrow e^- p$

-4

O

$\dfrac{q^2}{M^2}$
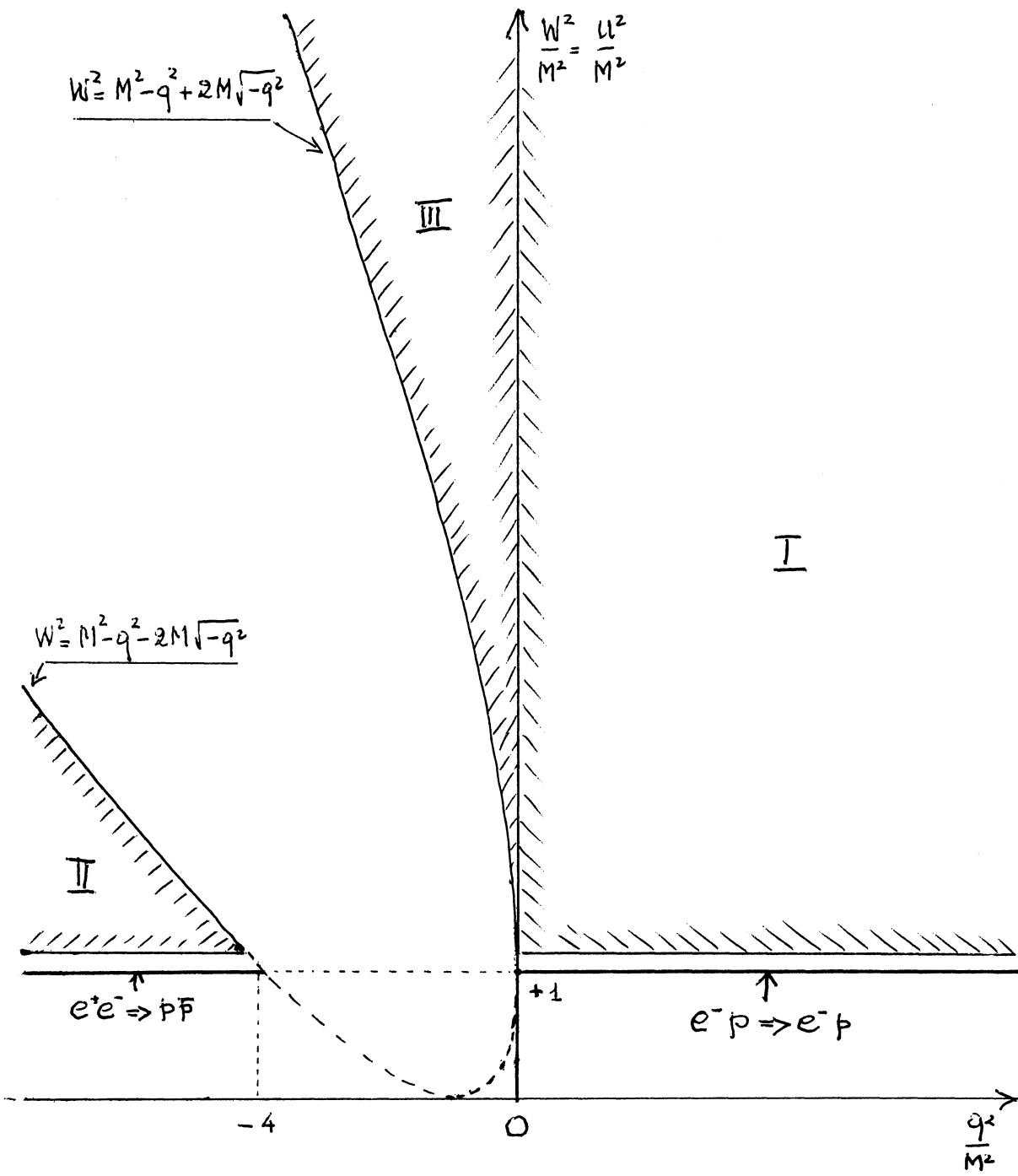
Fig. 13

4.2 The structure functions defined for the processes I, II, III are, in general, independent. Going from space-like photons to time-like photons we have to cross the q^2 cut and the possible branch points in the q^2 plane forbid the analytic continuation of the structure functions from one region to another region[51,52].

4.3 Nevertheless in some specific models such singularities are not important at high energy and the analytic continuation can be performed. To make precise the relations one obtains, in that case it is convenient to work with the variables W^2 and ω. We first define new structure functions

a) space-like photons $1 < \omega$, $q^2 > 0$

$$\frac{q^2}{4M^2} \, V_1(q^2, W^2) \equiv \mathcal{J}_1(W^2, \omega)$$

$$-V_2(q^2, W^2) \equiv \mathcal{J}_2(W^2, \omega)$$

b) time-like photons $0 < 1 < \omega$, $q^2 = -s < 0$

$$\frac{s}{4M^2} \, \tilde{V}_1(s, W^2) \equiv \tilde{\mathcal{J}}_1(W^2, \omega)$$

$$\tilde{V}_2(s, W^2) \equiv \tilde{\mathcal{J}}_2(W^2, \omega)$$

When the analytic continuation in q^2 or ω at fixed W^2 above its threshold is legitimate we have

$$\tilde{\mathcal{J}}_1(W^2, \omega) = \mathcal{J}_1(W^2, \omega) \tag{IV.28}$$

$$\tilde{\mathcal{J}}_2(W^2, \omega) = -\mathcal{J}_2(W^2, \omega) \tag{IV.29}$$

In particular from the positivity conditions

$$F_2(W^2,\omega) \geq 0 \quad \text{for} \quad \omega > 1 \;, \quad \tilde{F}_2(W^2,\omega) \geq 0 \quad \text{for} \quad 0 < \omega < 1$$

and the condition (IV.29) implies that $F_2(W^2,\omega)$ must change sign at $\omega = 1$.

4.4 In the high-energy limit the angular distribution at fixed ω for the annihilation process

$$e^+ + e^- \rightarrow p + \Gamma$$

is given by

$$\frac{d^2\sigma}{dZ\,d\omega} = \frac{\pi\alpha^2(2J+1)}{2s}\,\omega\left\{\omega^2\tilde{\mathcal{F}}_1(W^2,\omega)(1-Z^2) + \tilde{\mathcal{F}}_2(W^2,\omega)\right\} \qquad \text{(IV.30)}$$

If the structure functions $\tilde{F}_{1,2}(W^2,\omega)$ admit a non-trivial Bjorken limit (see for instance Bjorken's lectures)

$$\lim_{\substack{W^2 \Rightarrow \infty \\ \omega\ \text{fixed}}} \tilde{\mathcal{F}}_{1,2}(W^2,\omega) = \tilde{\mathcal{F}}_{1,2}(\omega) \qquad \text{(IV.31)}$$

the annihilation cross-section for the process $e^+e^- \rightarrow p + \Gamma$ behaves like a point structure cross-section

$$\lim_{\substack{s \Rightarrow \infty \\ \omega\ \text{fixed}}} \frac{d^2\sigma}{dZ\,d\omega} = \frac{\pi\alpha^2(2J+1)}{2s}\,\omega\left\{\omega^2\tilde{\mathcal{F}}_1(\omega)(1-Z^2) + \tilde{\mathcal{F}}_2(\omega)\right\} \qquad \text{(IV.32)}$$

4.5 In the Drell-Levy-Yan model of partons[56] we have the two properties

a) analytic continuation from the space-like region to the time-like region (IV.28) and (IV.29).

b) scaling of the structure functions at high energy (IV.31). Combining both properties

$$\tilde{F}_1(\omega) = F_2(\omega) \qquad \tilde{F}_2(\omega) = -F_2(\omega) \ .$$

Another interesting property is the relation between the angular distribution (IV.32) and the parton spin.

a) if the parton spin is zero, the photon has a longitudinal polarization in the high-energy limit and the angular distribution has the form $1 - z^2$ (I.18).

b) if the parton spin is $\frac{1}{2}$, the photon has a transverse polarization in the high-energy limit and the angular distribution has the form $1 + z^2$ (I.18).

5. HIGH-ENERGY MODELS

5.1 Preliminary results from ADONE in Frascati indicate a relatively copious production of four charged particles and perhaps of two charged particles in the region 1.5 GeV $\leq \sqrt{s} \leq$ 2 GeV. An order of magnitude for the cross-section is about 10^{-32} cm^2 [54]. Obviously the angular and energy distributions of the emitted particles have not been measured and we must remain very cautious about the interpretation of the ADONE results before being sure, for instance, that the radiative corrections have been correctly computed.

5.2 The VMD model discussed in Lecture II makes definite predictions about the high-energy behaviour of the annihilation cross-sections in terms of the vector meson propagator if the smoothness assumption H$_2$ remains correct at high energy.

For instance the π meson electromagnetic form factor has been measured in Orsay at the ϕ meson mass

$$\left| F_\pi(m_\phi^2) \right|^2 = 1.84 \pm 0.46 \ .$$

This value agrees with the extrapolated FFGS model.

Recently efforts have been made to look for high mass vector mesons as predicted by theoretical models. No experimental evidence has been found yet for these new particles which, if they exist, will modify the predictions of the simplest VMD model with only ρ, ω and ϕ mesons.

5.3 We now pay some attention to the two-jet production mechanism which receives serious support from strong interaction data. Let us begin with some kinematics for the process

$$e^+ + e^- \rightarrow \Gamma_1 + \Gamma_2$$
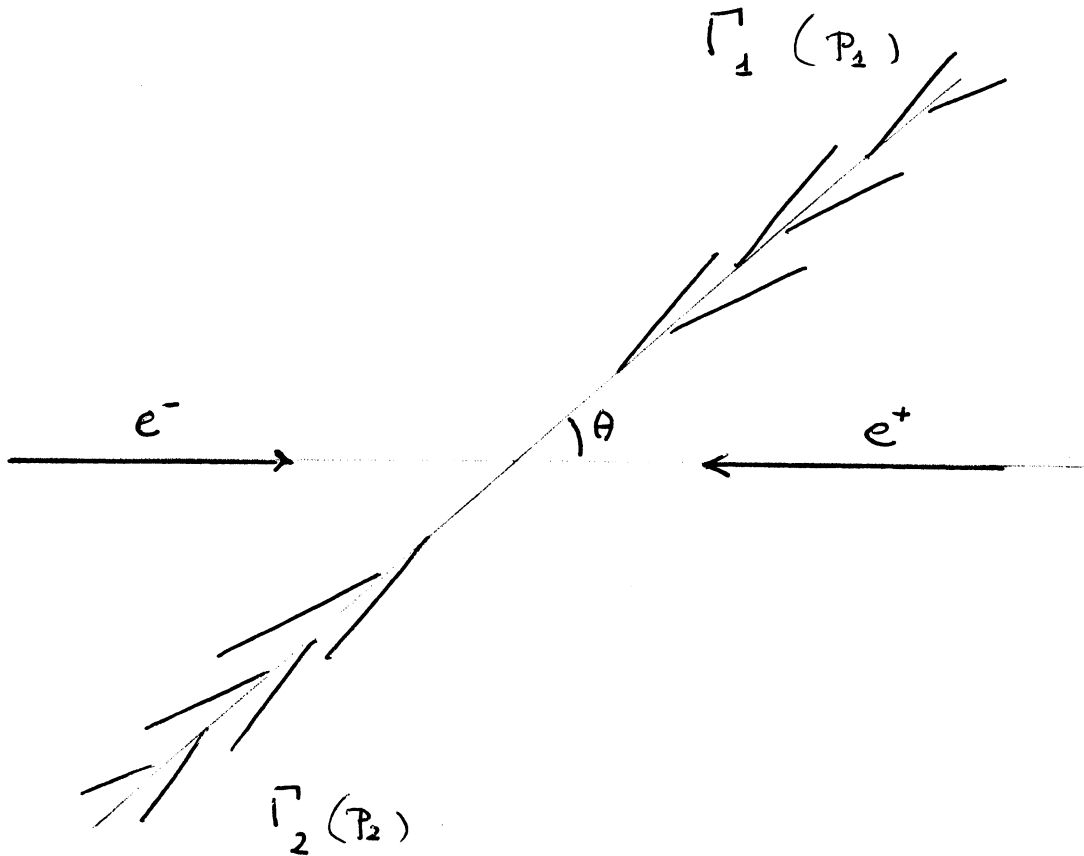
Fig. 14

The total energy momentum four vectors of the multiparticle systems Γ_1 and Γ_2 are called p_1 and p_2. We define the two effective masses

$$u_1^2 + p_1^2 = 0 \qquad u_2^2 + p_2^2 = 0 .$$

Only the effective masses and the jet direction are observed. In the one photon exchange approximation the differential cross-section has the structure

$$\frac{d_3\sigma}{dz \, du_1^2 \, du_2^2} = \frac{2\pi\alpha^2}{s^2} \frac{p}{\sqrt{s}} \left\{ (1-z^2) \, p^2 A(s, u_1^2, u_2^2) + B(s, u_1^2, u_2^2) \right\} \qquad \text{(IV.33)}$$

where the centre of mass momentum p is a function of s, u_1, u_2, given by

$$p = \frac{1}{2\sqrt{s}} \left\{ [s - (u_1 + u_2)^2][s - (u_1 - u_2)^2] \right\}^{1/2}$$

In the particular case where Γ_1 is a single particle state with mass M and spin J the structure functions A and B reduce to the functions $\tilde{V}_1$ and $\tilde{V}_2$ defined in Lecture I

$$A(s, u_1^2, u_2^2) \implies \delta(u_1^2 - M^2)(2J+1) \frac{1}{2M^2} \tilde{V}_1(s, u_2^2)$$

$$B(s, u_1^2, u_2^2) \implies \delta(u_1^2 - M^2)(2J+1) \frac{1}{2} \tilde{V}_2(s, u_2^2)$$

If now Γ_1 is a particle of mass M_1 and spin J, and Γ_2 an antiparticle of mass M_2 and same spin, the structure functions are completely factorized in the three variables with Dirac delta distribution for u_1 and u_2

$$A(s,u_1^2,u_2^2) \Rightarrow \delta(u_1^2 - M_1^2)\,\delta(u_2^2 - M_2^2)\,(2J+1)\,a(s)$$

$$B(s,u_1^2,u_2^2) \Rightarrow \delta(u_1^2 - M_1^2)\,\delta(u_2^2 - M_2^2)\,(2J+1)\,b(s)$$

where $a(s)$ and $b(s)$ described the electromagnetic vertex 1-2-γ. In the elastic case, for instance, we simply have (see Lecture I):

Spin 0 : $a(s) = |F(s)|^2$, $b(s) = 0$

Spin $\frac{1}{2}$: $a(s) = -|F_1(s)|^2 + \dfrac{s}{4M^2}\,|F_2(s)|^2$, $b(s) = \dfrac{s}{2}|F_1(s) + F_2(s)|^2$.

5.4 In the Cabibbo-Parisi-Testa model[55] the two jets are assumed to be induced by partons of spin 0 and/or $\frac{1}{2}$. The structure functions are factorized following

$$A(s,u_1^2,u_2^2) = \rho(u_1^2)\,\rho(u_2^2)\,(2J+1)\,a(s)$$

$$B(s,u_1^2,u_2^2) = \rho(u_1^2)\,\rho(u_2^2)\,(2J+1)\,b(s)$$

where $\rho(u^2)$ is the parton propagator spectral function. At high energy the parton electromagnetic structure is assumed to be point-like.

Spin 0 parton $a(s) = Q^2$ $b(s) = 0$

Spin $\frac{1}{2}$ parton $a(s) = -Q^2$ $b(s) = \frac{s}{2}Q^2$

where Q is the parton electric charge.

Retaining only spin 0 and $\frac{1}{2}$ partons the differential cross-section at high energy becomes

$$\frac{d_3\sigma}{dZ\,du_1^2\,du_2^2} = \frac{\pi\alpha^2}{4s}\left\{(1-Z^2)\sum_{j\in 0}Q_j^2 + 2(1+Z^2)\sum_{j\in\frac{1}{2}}Q_j^2\right\}\,\rho(u_1^2)\rho(u_2^2) \qquad \text{(IV.34)}$$

where $j \in 0$ and $j \in \frac{1}{2}$ mean partons of spin 0 and $\frac{1}{2}$ respectively. Integrating over Z, u_1^2 and u_2^2 with the normalization condition

$$\int_0^{\sim}\rho(u^2)\,du^2 = 1$$

we obtain the total cross section

$$\sigma_{\text{TOT}}(e^+e^- \to \text{hadrons}) = \frac{\pi\alpha^2}{3s}\left\{\sum_{j\in 0}Q_j^2 + 4\sum_{j\in\frac{1}{2}}Q_j^2\right\} \qquad \text{(IV.35)}$$

which behaves like a point structure cross-section.

Starting from the recent ADONE results the authors speculate about the possibility that π^+, K^+ and other spin zero charged mesons are partons. The direct consequence of this last assumption is a strong depression of the production of bosonic states with an odd number of π mesons as compared with those with an even number of π mesons. Experiment will quickly test the model.

LECTURE V

SU(3) SYMMETRY AND THE $\omega-\phi$ MIXING

1. GENERALITIES

1.1 Notation

The vector meson states $|\rho>$, $|\omega>$, $|\phi>$ are defined so that

    a) in the absence of electromagnetic mixing

    b) with time reversal invariance,

the three vector meson-photon coupling constants f_ρ, f_ω, f_ϕ are real and positive by convention.

1.2 In an exact SU(3) symmetry the particles are associated to the weights of irreducible representations. But we know that the symmetry is broken and this implies a non-degenerate mass spectrum for particles belonging to a given representation and also the possibility of configuration mixing.

We call as ϕ_8 and ω_1 the $I = Y = 0$ weights of the vector meson octuplet and singulet representations

$$\phi_8 \in 8 \qquad \omega_1 \in 1 \ .$$

We use the vector meson propagator language in order to define the physical states ϕ and ω as the eigenstates of the propagator matrix W. In this particular case, the widths of the vector mesons are small compared to the masses and the matrix W reduces essentially to the hermitian squared mass matrix R. It follows that the linear transformation in the two-dimensional space ϕ_8, ω_1 which diagonalize R is orthogonal, the mixing parameter real and the eigenstates ϕ and ω are orthogonal.

Assuming time reversal invariance, the condition of reality of the mixing parameter is simply

$$\left| \frac{m_\omega \Gamma_\omega - m_\phi \Gamma_\phi}{m_\omega^2 - m_\phi^2} \right| = 0.0186 \ll 1$$

We then introduce a mixing angle θ[56]

$$\phi = \cos \theta \, \phi_8 - \sin \theta \, \omega_1$$

$$\omega = \sin \theta \, \phi_8 + \cos \theta \, \omega_1$$

with the reciprocal formulae

$$\phi_8 = \cos \theta \, \phi + \sin \theta \, \omega$$

$$\omega_1 = -\sin \theta \, \phi + \cos \theta \, \omega$$

The physical states ϕ and ω are defined as the eigenstates of the squared mass matrix and the mixing angle θ is given in terms of the physical vector meson masses by a Gell-Mann - Okubo formula

$$4 \, m_{K^*}^2 - m_\rho^2 = 3 \, m_8^2 = 3 \left[\cos^2 \theta \, m_\phi^2 + \sin^2 \theta \, m_\omega^2 \right] \qquad \text{(V.1)}$$

The solution of (V.1) located in the first quadrant is

$$\theta = 40.2° \quad .$$

If the mass formula is written for the inverse squared masses[57] we obtain a different mixing angle θ_{inv}

$$\theta_{inv} = 28.6° \quad .$$

Both values will be used later in the comparison with experiment.

1.3 Quark model

The neutral non-strange vector mesons are defined in the quark model by:

$$\rho^0 = \frac{\bar{q}_1 q_1 - \bar{q}_2 q_2}{\sqrt{2}} \qquad \phi_8 = \frac{\bar{q}_1 q_1 + \bar{q}_2 q_2 - 2\bar{q}_3 q_3}{\sqrt{6}} \qquad \omega_1 = \frac{\bar{q}_1 q_1 + \bar{q}_2 q_2 + \bar{q}_3 q_3}{\sqrt{3}}$$

With the "ideal" mixing angle θ_q

$$\sin \theta_q = \sqrt{\frac{1}{3}} \qquad \cos \theta_q = \sqrt{\frac{2}{3}}$$

the physical particles are given by

$$\phi_q = -\bar{q}_3 q_3 \qquad \omega_1 = \frac{\bar{q}_1 q_1 + \bar{q}_2 q_2}{\sqrt{2}}$$

$$\theta_q \simeq 35.2^\circ$$

1.4 The relation between SU(3) symmetry and electromagnetic interactions is part of the lectures of Professor Joos and we discuss here only a few applications strongly connected with storage ring experiments. For a more systematic study we refer to specialized books[58].

2. DECAY OF VECTOR MESONS INTO A LEPTON-ANTILEPTON PAIR

2.1 SU(3) symmetry

In the SU(3) theory of electromagnetic interactions the electromagnetic current is assumed to be the U spin scalar of an adjoint representation.

Therefore the unitary singlet ω_1 is not coupled to the photon

$$(\gamma - \omega_1) = 0 \qquad\qquad\qquad (V.2)$$

and we have a relation between the two octuplet members ϕ_8 and ρ

$$(\gamma - \phi_8) = \frac{1}{\sqrt{3}} (\gamma - \rho) . \qquad (V.3)$$

Unfortunately because of the large mass differences between vector mesons (especially ρ and ϕ) and because of their instability some ambiguities remain concerning the precise meaning of these SU(3) relations.

To be more general let us introduce a finite width correction factor z_V and a mass term m_V^α where α is an arbitrary integer. We then apply the SU(3) symmetry to the quantities

$$F_V = z_V \frac{m_V^\alpha}{f_V}$$

and we obtain from (V.2) and (V.3)

$$-\sin\theta \; \overline{F}_\phi + \cos\theta \; \overline{F}_\omega = 0$$

$$\cos\theta \; \overline{F}_\phi + \sin\theta \; \overline{F}_\omega = \frac{1}{\sqrt{3}} \; \overline{F}_\rho$$

one can solve for F_ϕ and F_ω in terms of F_ρ

$$F_\phi = \frac{1}{\sqrt{3}} \cos\theta \; F_\rho$$

$$F_\omega = \frac{1}{\sqrt{3}} \sin\theta \; F_\rho .$$

Now eliminating the mixing angle θ we have a sum rule

$$\overline{F}_\rho^{\,2} = 3 \left[\overline{F}_\phi^{\,2} + \overline{F}_\omega^{\,2} \right] \qquad (V.4)$$

The VMD model for the first Weinberg sum rule corresponds to the particular choice (IV.20)

$$z_V = x_V \; , \qquad \alpha = 1$$

and we have a current mixing model[9],[48].

The VMD model for the charge renormalization sum rule corresponds to a different choice (IV.23)

$$z_V = y_V \; , \qquad \alpha = 0$$

and we have a mass mixing model[9],[48].

2.2 It is usual to introduce an auxiliary mixing angle θ_Y defined by

$$\tan \theta_Y = f_\phi / f_\omega$$

and directly related to experiments by

$$\tan^2 \theta_Y = \frac{m_\phi}{m_\omega} \frac{\Gamma(\omega \to e^+ e^-)}{\Gamma(\phi \to e^+ e^-)}$$

The relation between θ_Y and θ is simply

$$\tan \theta_Y = \tan \theta \left(\frac{m_\phi}{m_\omega} \right)^\alpha \frac{z_\phi}{z_\omega}$$

so that, for instance, in the mass mixing model we have $\theta_Y = \theta$ in the zero width approximation.

2.3 Comparison with experiment

We compute the partial decay widths $\Gamma(\omega \to e^+ e^-)$ and $\Gamma(\phi \to e^+ e^-)$ with, as an input, the experimental value of $\Gamma(e \to e^+ e^-) = (7.4 \pm 0.7)$ keV and we also predict the angle θ_Y.

		$\Gamma(\omega \to e^+e^-)$ keV	$\Gamma(\phi \to e^+e^-)$ keV	$\tan^2 \theta_\gamma$
Current mixing	$\theta = 40.2°$	0.94 ± 0.09	0.88 ± 0.08	1.39
	$\theta_{inv} = 29.6°$	0.52 ± 0.05	1.16 ± 0.11	0.58
Mass mixing	$\theta = 40.2°$	0.99 ± 0.10	1.68 ± 0.16	0.77
	$\theta_{inv} = 29.6°$	0.55 ± 0.05	2.21 ± 0.21	0.32
Experiment		1.0 ± 0.18	1.41 ± 0.12	0.92 ± 0.17

Table 3

The models with a mixing angle $\theta_{inv} = 29.6°$ [59] seem to be excluded on the basis of the ω meson data, independently of the type of mixing because of the small $\omega - \rho$ mass difference. The mass mixing model seems to fit more nicely to the Orsay data and this conclusion is obviously identical to that obtained from the study of the electromagnetic sum rules.

3. STRONG DECAY OF VECTOR MESONS

3.1 SU(3) symmetry [58]

We consider the strong decay of the vector mesons into two pseudo-scalar mesons. From the point of view of SU(3) symmetry we have the transition

$$1 \oplus 8 \to 8 \otimes 8 \ .$$

In the exact symmetry there is only one reduced matrix element because of the generalized Pauli principle

$$8 \xrightarrow{} 8_a \qquad (1)$$

and in particular the transition $1 \overset{(1)}{\nrightarrow} 1$ is forbidden because of particle-antiparticle conjugation invariance. In the broken symmetry there exists, to first order, three independent ways of breaking

$$\underline{\alpha} \qquad 8 \overset{(8)}{\to} 8_a$$

$$\underline{\beta} \qquad 1 \overset{(1)}{\to} 8_a$$

$$\gamma \qquad 8 \overset{(8)}{\to} (10 - \overline{10}) \ .$$

The universality relation $f_{\rho K \bar{K}} = \frac{1}{2} f_{\rho \pi \pi}$ is preserved by the α and β type of breaking and in order to save this equality, also given by the VMD model, from now on we take $\gamma = 0$. The coupling constants of interest are then given by

$$\frac{f_{\rho \pi \pi}^2}{4\pi} = \frac{f_\rho^2}{4\pi} (1 + \alpha)^2$$

$$\frac{f_{K^* K \pi}^2}{4\pi} = \frac{3}{4} \frac{f_\rho^2}{4\pi} \left(1 - \frac{\alpha}{2}\right)^2$$

$$\frac{f_{\phi K \bar{K}}^2}{4\pi} = \frac{3}{4} \frac{f_\rho^2}{4\pi} \cos^2\theta \, (1-\alpha)^2 \left(1 - \beta \tan\theta\right)^2$$

$$\frac{f_{\omega K \bar{K}}^2}{4\pi} = \frac{3}{4} \frac{f_\rho^2}{4\pi} \sin^2\theta \, (1-\alpha)^2 \left(1 + \beta \cot\theta\right)^2$$

The comparison between SU(3) prediction and experiment is made using the physical masses in the phase space factors and assuming the ρ meson width to be given by

$$\Gamma(\rho \to \pi^+\pi^-) = (127.3 \pm 12.5) \text{ MeV} .$$

3.2 $K^* \to K + \pi$ decay

The exact SU(3) symmetry predicts a K^* width of 36 MeV whereas the experimental value is[34]

$$\Gamma(K^* \to K + \pi)_{exp} = (50 \pm 1) \text{ MeV} .$$

We introduce a first order breaking of the symmetry and the breaking parameter α is computed to be

$$\alpha = -0.101 \pm 0.034 \qquad\qquad (V.5)$$

3.3 $\phi \to K + \bar{K}$ decay

Again the exact SU(3) symmetry prediction is in disagreement with experiment. We first use the value (V.5) for α to compute the coupling constant $f_{\phi K\bar{K}}$ assuming $\beta = 0$. The result is

$$\frac{f^2_{\phi K\bar{K}}}{4\pi} = 1.58 \pm 0.12$$

in very good agreement with the Orsay result

$$\frac{f^2_{\phi K\bar{K}}}{4\pi}_{exp} = 1.55 \pm 0.18 .$$

If now we allow a non zero value for β, e.g. a second type of SU(3) breaking, we observe that β must be small and compatible with zero

$$\beta \tan\theta = 0.009 \pm 0.067 .$$

3.4 $\omega \to K\bar{K}$ transition

The decay $\omega \to K\bar{K}$ is not energetically possible and we cannot have a direct comparison between theory and experiment. Nevertheless, it is useful to compute the coupling constant $f_{\omega K\bar{K}}$ in order to compare with other predictions. Using only the α type breaking we obtain

$$\frac{f^2_{\omega K\bar{K}}}{4\pi} = 1.13 \pm 0.09$$

in very bad disagreement with the VMD model prediction (III.34)

$$\frac{f^2_{\omega K\bar{K}}}{4\pi} \text{ VMD} = 0.11 \begin{array}{c} + 0.09 \\ - 0.06 \end{array} .$$

The inclusion of a second type of breaking cannot reduce the discrepancy and we are faced with a serious difficulty which one can express in the following equivalent way. Consider the quantity Q defined by

$$Q = \frac{f_{\omega K\bar{K}}}{f_\omega} + 1.17 \frac{f_{\phi K\bar{K}}}{f_\phi} - 0.5$$

which in the VMD model must be zero if the ω and ϕ mesons saturate the isoscalar K meson electric charge. As explained in Lecture III the ϕ contribution and the coupling constant f_ω are deduced from the storage ring experiments. We now use for $f_{\omega K\bar{K}}$ the α-type broken SU(3) prediction $f_{\omega K\bar{K}} = \tan \theta \, f_{\phi K\bar{K}}$ and the result is

$$Q = 0.191 \pm 0.053 ,$$

showing a 3.5 standard deviation effect.

It is not obviously possible to determine, on this simple case, the origin of the discrepancy and we have the choice

a) the SU(3) symmetry even broken is not reliable;

b) ω and ϕ mesons do not saturate the K meson isoscalar charge;

c) f_ω and f_ϕ are not constants in the extrapolation.

4. RADIATIVE DECAY OF VECTOR MESONS

4.1 SU(3) symmetry[58]

We are now interested in the electromagnetic decay of the vector mesons into a pseudoscalar meson and a photon. As previously the photon is assumed to be the U spin scalar of an octuplet and from the point of view of SU(3) symmetry we have again the transition

$$1 \oplus 8 \rightarrow 8 \otimes 8 .$$

In the exact symmetry there are two reduced matrix elements using particle-antiparticle conjugation invariance

$$\begin{cases} 8 \overset{(1)}{\rightarrow} 8_s & g_{88} \\ \\ 1 \overset{(1)}{\rightarrow} 1 & g_{18} \end{cases}$$

4.2 $V \rightarrow M^0 + \gamma$ decays

In the particular case where the pseudoscalar meson is a π^0 we have three possible reactions which can be expressed in terms of g_{88} and g_{18} and of the ω-ϕ mixing angle

$$\left|
\begin{aligned}
f_{\rho\pi^0\gamma} &= g_{88} \\[2mm]
f_{\omega\pi^0\gamma} &= \sqrt{3}\left[\sin\theta\, g_{88} + \cos\theta\, g_{18} \right] \\[2mm]
f_{\phi\pi^0\gamma} &= \sqrt{3}\left[\cos\theta\, g_{88} - \sin\theta\, g_{18} \right]
\end{aligned}
\right.$$

We then have one relation

$$D = \sqrt{3}\, f_{\rho\pi^0\gamma} - \left[\cos\theta\, f_{\phi\pi^0\gamma} + \sin\theta\, f_{\omega\pi^0\gamma} \right] = 0 \qquad (V.6)$$

In the VMD model, the two coupling constants $f_{\rho\pi^0\gamma}$ and $f_{\phi\pi^0\gamma}$ can be computed from the π^0 life term and the measured decay width $\Gamma(\omega \to \pi^0\gamma)$ (see Lecture III). Using

$$\Gamma(\pi^0 \to 2\gamma) = (11.7 \pm 1.2)\,eV \qquad \Gamma(\omega \to \pi^0\gamma) = (1.1 \pm 0.2)\,MeV$$

we obtain

$$f_{\rho\pi^0\gamma} = 0.126 \pm 0.012 \qquad\qquad (V.7)$$

$$f_{\omega\pi^0\gamma} = 0.39 \ \pm 0.04 \qquad\qquad (V.8)$$

$$f_{\phi\pi^0\gamma} = -0.070 \pm 0.048 \qquad\qquad (V.9)$$

and the relation (V.6) is very well satisfied

$$\mathcal{D} \quad = 0.01 \pm 0.05 \ .$$

In the quark model the reduced matrix elements g_{88} and g_{18} are no longer independent; the strange quark-antiquark pair $\phi_q = -\bar{q}_3 q_3$ cannot decay into a $\pi^0\gamma$ system so that

$$g_{18} = \sqrt{2}\ g_{88} \ .$$

We now obtain two predictions instead of the single relation (V.6)

$$f_{\rho\pi^0\gamma} = \frac{1}{3\cos(\theta-\theta_q)}\ f_{\omega\pi^0\gamma} \qquad\qquad f_{\phi\pi^0\gamma} = -\tan(\theta-\theta_q)\ f_{\omega\pi^0\gamma}$$

Using for $f_{\omega\pi^0\gamma}$ the experimental value (V.8) we obtain

$$f_{\rho\pi^0\gamma} = \ 0.130 \pm 0.013 \qquad\qquad (V.10)$$

$$f_{\phi\pi^0\gamma} = -0.031 \pm 0.003 \qquad\qquad (V.11)$$

in good agreement with the VMD model predictions (V.7) and (V.9).

In particular, the radiative decay width $\Gamma(\phi \to \pi^0\gamma)$ deduced from (V.11) is compatible with the experimental upper limit of 15 keV.

LECTURE VI

THE ω-ρ MIXING

1. ## THE ω → 2π DECAY

1.1 The decay of the ω meson into 2π is forbidden by G parity invariance and it can proceed only via electromagnetic interactions.

Experimentally a considerable effort has been made in order to measure the partial decay width $\Gamma(\omega \to 2\pi)$ via an interference between the two amplitudes ρ → 2π and ω → 2π and therefore to detect a possible ω-ρ electromagnetic mixing. We classify the experiments into three categories.

a) strong production of ρ and ω mesons followed by a decay into the $\pi^+\pi^-$ mode;

b) electron-positron annihilation into 2π;

c) coherent photoproduction on nuclei of $\ell\bar{\ell}$ and $\pi^+\pi^-$ pairs.

1.2 An excellent review of the experimental situation in case a) has recently been given by Roos[60]. The experimental observation of a ω-ρ interference will determine two quantities, the branching ratio

$$\tau = \left| \frac{f_{\omega 2\pi}}{f_{\rho 2\pi}} \right| \approx \frac{\Gamma(\omega \to 2\pi)^{1/2}}{\Gamma(\rho \to 2\pi)^{1/2}}$$

and the relative phase for the ρ and ω amplitudes.

The value given by Roos for τ is

$$\tau = \left(4.5 \, {}^{+\,1.35}_{-\,0.75} \right) 10^{-2} . \tag{VI.1}$$

In the case b) the Orsay storage ring experiment obtains[61]

$$\tau = (6 \pm 2)10^{-2} \qquad (VI.2)$$

and finally in case c) the Daresbury measurements[62] agree also with these values

$$\tau = (2.7 \pm 0.6)10^{-2} \; . \qquad (VI.3)$$

In the following we will use the value (VI.1) for τ.

1.3 It is now well-known that the one-photon exchange model for the $\omega \rightarrow 2\pi$ transition gives for τ a very small value

$$\tau_{1\gamma} \simeq 1 \times 10^{-2}$$

which cannot explain the observed value (VI.1).

Therefore it is natural to look at the theoretical problem of a possible mixing of the ρ and ω mesons due to electromagnetic interactions, and which can be important because of the quasi degeneracy of the two vector meson states[63].

1.4 Because of the lack of time we cannot review all the theoretical investigations[64-73] and we only present a mixing formalism based on quantum mechanics[74] and using the propagator method proposed some years ago by Jacob and Sachs[75].

2. FORMALISM

2.1 We start with an orthonormal basis $|a >$

$$|a > = |\rho_0 > , \quad |\omega_0 >$$

defined by the strong interactions with isotopic spin invariance in the two dimensional space E_2 . The characteristic relations are

$$< a|b > = \delta_{ab} \qquad I = \sum_a |a > < a|$$

where I is the projector on E_2, e.g. the unit operator in E_2.

In the absence of electromagnetic interactions, the matrix W has a diagonal form in this basis.

Now introducing the electromagnetic interactions, we first have a slight modification of the diagonal elements of W and secondly the appearance of non-diagonal matrix elements connecting the ρ and ω^0 states.

2.2 We assume that, in the neighbourhood of the physical region W can be diagonalized by a linear transformation in E_2 represented by a complex regular matrix C

$$C = \begin{vmatrix} p & -q \\ \\ r & s \end{vmatrix} .$$

The right eigenvectors $|V>$ are defined by the homogeneous equation[*)]

$$\{W - W_V I\}|V> = 0$$

where W_V is the corresponding eigenvalue function. The linear transformation is then written

$$\left| \; |\rho> = p\left[\; |\rho_o> - \mathcal{E}_1 \; |\omega_o> \right] \right. \tag{VI.4}$$

$$\left| \; |\omega> = s\left[\mathcal{E}_2 \; |\rho_o> + \; |\omega_o> \right] \right. \tag{VI.5}$$

where we have introduced two complex mixing parameters

$$\varepsilon_1 = \frac{q}{p} \qquad \varepsilon_2 = \frac{r}{s} \; .$$

[*)] In order to simplify the notations we omit systematically the energy dependence for all the functions.

The hermitian conjugate vectors $< V|$ are not, in general, left eigen-vectors of W because W is not a hermitian matrix the vector mesons being unstable particles. The left eigenvectors defined by the homogeneous equation

$$\langle \tilde{V} | \left\{ W - W_v I \right\} = O$$

are related to the vector $< a|$ by the inverse linear transformation C^{-1}

$$\langle \tilde{\rho} | = \frac{s}{p_{s} + q_{r}} \left[\langle \rho_o | - \bar{\mathcal{E}}_2 \langle \omega_c | \right] \qquad \text{(VI.6)}$$

$$\langle \tilde{\omega} | = \frac{p}{p_{s} + q_{r}} \left[\bar{\mathcal{E}}_1 \langle \rho_o | + \langle \omega_c | \right]$$

$$\text{(VI.7)}$$

whereas the vectors $< V|$ are deduced, from the same vectors $< a|$ by the transformations C^*.

2.3 The matrix C is not unitary and the bases $|V >$ and $|\tilde{V} >$ are not an orthogonal basis

$$\langle \omega | \rho \rangle = p_{\bar{s}} \left[\bar{\mathcal{E}}_2 - \mathcal{E}_1 \right]$$

$$\langle \tilde{\omega} | \tilde{\rho} \rangle = \frac{p_{\bar{s}}}{|p_{s} + q_{r}|^2} \left[\mathcal{E}_1 - \bar{\mathcal{E}}_2 \right]$$

But using both bases we have very nice properties

$$\langle \tilde{\rho} | \rho \rangle = 1 = \langle \tilde{\omega} | \omega \rangle$$

$$\langle \tilde{\omega} | \rho \rangle = O = \langle \tilde{\rho} | \omega \rangle$$

The closure relation in the space E_2 is simply written using again both bases

$$\underline{I} = |\wp\rangle\langle\tilde{\wp}| + |\omega\rangle\langle\tilde{\omega}|$$

2.4 Let us assume that the matrix W is known in the bases $|a\rangle$. We can then compute the two mixing parameters ε_1, ε_2 and the two eigenvalues W_ρ, W_ω in terms of the W_{ab}'s.

$$\varepsilon_1 = \frac{2\eta_1}{1 + \sqrt{1 + 4\eta_1\eta_2}} \qquad\qquad \varepsilon_2 = \frac{2\eta_2}{1 + \sqrt{1 + 4\eta_1\eta_2}}$$

$$W_\wp = \frac{1}{2}\left(W_{\wp_0\wp_0} + W_{\omega_0\omega_0}\right) + \frac{1}{2}\left(W_{\wp_0\wp_0} - W_{\omega_0\omega_0}\right)\sqrt{1 + 4\eta_1\eta_2}$$ (VI.8)

$$W_\omega = \frac{1}{2}\left(W_{\wp_0\wp_0} + W_{\omega_0\omega_0}\right) - \frac{1}{2}\left(W_{\wp_0\wp_0} - W_{\omega_0\omega_0}\right)\sqrt{1 + 4\eta_1\eta_2}$$

where

$$\eta_1 = -\frac{W_{\omega_0\wp_0}}{W_{\wp_0\wp_0} - W_{\omega_0\omega_0}} \qquad\qquad \eta_2 = -\frac{W_{\wp_0\omega_0}}{W_{\wp_0\wp_0} - W_{\omega_0\omega_0}}$$ (VI.9)

2.5 If time reversal invariance holds the matrix W is symmetric in the basis $|a\rangle$ and we obtain interesting constraints on the linear transformation C: $r = q$ $s = p$.

The two quantities η_1 and η_2 are equal and we have only one independent mixing parameter

$$\varepsilon_1 = \varepsilon_2 = \varepsilon .$$

In particular the hermitian product $< \omega | \rho >$ becomes purely imaginary

$$< \omega | \rho > = -2i \ \text{Im}\varepsilon \ |p|^2 \ .$$

3. MIXING PARAMETERS

3.1 The $\omega \rightarrow 2\pi$ decay in this formalism can come either from the direct transition $\omega_0 \rightarrow 2\pi$ or from the strong decay $\rho_0 \rightarrow 2\pi$ the ρ_0 being mixed into the physical ω by the mixing parameter ε_2 or both. From Eq. (VI.5).

$$f_{\omega \ 2\pi} = S \left[f_{\omega_c \ 2\pi} + \varepsilon_2 \ f_{\rho_c \ 2\pi} \right] \qquad (VI.10)$$

It seems that the dominant effect is located in the mixing parameter ε_2 which can be larger than α because of the quasi degeneracy in mass of the ρ and ω mesons[63].

Nevertheless, with a mixing parameter ε_2 larger than α we still think that the direct transition $\omega_0 \rightarrow 2\pi$ should be of order α as compared with $\rho_0 = 2\pi$.

Similar consideration can be made for the $\rho \rightarrow 3\pi$ decay

$$f_{\rho \ 3\pi} = P \left[f_{\rho_c \ 3\pi} - \varepsilon_1 \ f_{\omega_c \ 3\pi} \right]$$

and again the direct transition $\rho_0 \rightarrow 3\pi$ is assumed to remain of order α as compared with $\omega_0 \rightarrow 3\pi$ the dominant effect being due to the mixing parameter ε_1.

3.2 We split the matrix W written in the $| a >$ basis into its hermitian R and skew hermitian Σ parts

$$(VI.11)$$

$$W = R - i \Sigma$$

The constraint on the vector meson propagator due to the unitarity of the S matrix is simply (see Lecture II, Section II)

$$\sum{}'_{ab}(s) = \frac{1}{6} \sum_{F} \int_{F} (2\pi)^4 \int_{\mathcal{L}} (q - p_F) \langle F|J^a_\mu(o)|o\rangle \langle F|J^b_\nu(o)|o\rangle^* g^{\mu\nu} \qquad (VI.12)$$

Let us recall that the diagonal elements, on the mass shell, are related to total widths by

$$\sum{}'_{aa}(m_a^2) = m_a \Gamma_a \qquad (VI.13)$$

If time reversal invariance holds all the matrix elements R_{ab} and Σ_{ab} are real functions of energy.

3.3 Let us study more carefully the parameter η_2 defined in Eq. (VI.9). Using the decomposition (VI.11) we can write

$$\eta_2 = \eta_2^R + i \, \eta_2^\Sigma$$

with

$$\eta_2^R = - \frac{R_{\rho_0 \omega_0}}{W_{\rho_0 \rho_0} - W_{\omega_0 \omega_0}} \qquad\qquad \eta_2^\Sigma = \frac{\Sigma_{\rho_0 \omega_0}}{W_{\rho_0 \rho_0} - W_{\omega_0 \omega_0}} \qquad (VI.14)$$

The term $R_{\rho_0 \omega_0}$ comes from transitions to a virtual intermediate state containing at least one photon. It is a self-energy type contribution of electromagnetic origin as those responsible for the mass differences

between charged and neutral particles in the same isotopic spin multiplet. $R_{\rho_0\omega_0}$ can be estimated using symmetry argument. For instance in the framework of the SU(3) theory of electromagnetic interactions with the usual assumptions we obtain[58]

$$\sqrt{3}\left[\cos\theta \; Re \left(R_{\rho_0\phi_0}\right) + \sin\theta \; Re\left(R_{\rho_0\omega_0}\right)\right] \qquad \text{(VI.15)}$$

$$= \left(m^2_{\rho_0} - m^2_{\rho^+}\right) + \left(m^2_{K^{*+}} - m^2_{K^{*0}}\right)$$

Unfortunately the electromagnetic mass differences for vector mesons are not experimentally known with a reliable accuracy[34]

$$m_{\rho_0} - m_{\rho^+} = (2.4 \pm 2.1)\text{MeV} \qquad m_{K^{*0}} - m_{K^{*+}} = (7.6 \pm 3.0)\text{MeV}$$

We then need more assumptions to estimate $R_{\omega_0\rho_0}$.

The quark model with ideal mixing angle predicts $R_{\rho_0\phi_q} = 0$ so that in a first approximation one can neglect the ϕ contribution to Eq. (VI.15) and we replace its left-hand side simply by Re $\left(R_{\rho_0\omega_0}\right)$. Moreover, the quark model - or the SU(6) symmetry - relates the pseudoscalar meson and vector meson mass differences[75].

In fact, models must be constructed to evaluate $R_{\rho_0\omega_0}$ and all the estimates give results with a negative sign and in the range[68,77,78],

$$- \frac{1}{m_\rho} Re \left(R_{\rho_0\omega_0}\right) = (5 - 10) \text{ MeV} .$$

3.4 The term $\sum_{\rho_0\omega_0}$ comes from the existence of real physical intermediate states like $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^0\gamma$, $\eta\gamma$, etc. and it can be represented as the sum of its various contributions

$$\sum{}'_{\rho_0\omega_0} = \sum{}'^{(2\pi)}_{\rho_0\omega_0} + \sum{}'^{(3\pi)}_{\rho_0\omega_0} + \sum{}'^{(\pi^0\gamma)}_{\rho_0\omega_0} + \cdots$$
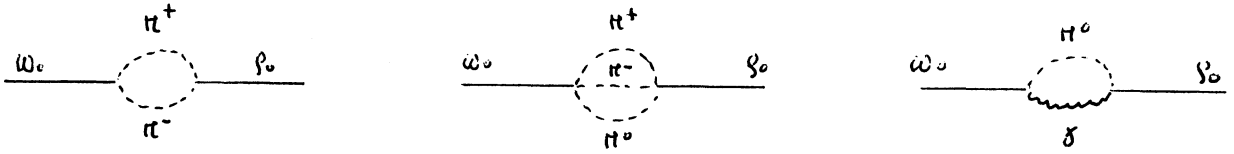
Fig. 13

From Eqs. (VI.12) and (VI.13) and neglecting the small ω-ρ mass difference

$$\frac{1}{m_\rho} \sum_{\rho_0\omega_0}'^{(2\pi)} = \frac{f_{\omega_0 2\pi}^\rho}{f_{\rho_0 2\pi}} \Gamma\left(\rho_0 \Rightarrow 2\pi\right)$$

$$\frac{1}{m_\rho} \sum_{\rho_0\omega_0}'^{(3\pi)} = \frac{f_{\rho_0 3\pi}^{\rho *}}{f_{\omega_0 3\pi}^{\rho *}} \Gamma\left(\omega_0 \Rightarrow 3\pi\right)$$

Accepting the order of α argument we obtain

$$\frac{1}{m_\rho} \left| \sum_{\rho_0\omega_0}'^{(2\pi)} \right| \simeq 0.8 \text{ MeV}$$

$$\frac{1}{m_\omega} \left| \sum_{\rho_0\omega_0}'^{(3\pi)} \right| \simeq 0.08 \text{ MeV}$$

For the $\pi^0\gamma$ intermediate state the experimental data are[34]

$$\Gamma(\omega^0 \to \pi^0\gamma) = (1.1 \pm 0.2)\text{MeV} \qquad \Gamma(\rho^0 \to \pi^0\gamma) < 0.4 \text{ MeV}$$

and hence there is an upper limit for the $\pi^0\gamma$ contribution

$$\frac{1}{m_\rho} \left| \Sigma_{\rho_0 \omega_0}^{(\pi^0 \gamma)} \right| < 0.7 \text{ MeV} .$$

A rough limit for $\left| \Sigma_{\rho_0 \omega_0} \right|$ seems to be

$$\frac{1}{m_\rho} \left| \Sigma_{\rho_0 \omega_0} \right| < 1.6 \text{ MeV} .$$

3.5   All these results can be easily translated in the language of the parameters η^R and η^Σ using the hermitian character of the matrices R and Σ.

$$R_{\omega_0 \rho_0} = R_{\rho_0 \omega_0}^* \qquad\qquad \Sigma'_{\omega_0 \rho_0} = \Sigma'_{\omega_0 \rho_0}^*$$

and for the diagonal terms $W_{\rho_0 \rho_0}$ and $W_{\omega_0 \omega_0}$ the simple forms

$$W_{\rho_0 \rho_0} \simeq m_{\rho_0}^2 - i\, m_{\rho_0} \Gamma_{\rho_0} \qquad\qquad W_{\omega_0 \omega_0} \simeq m_{\omega_0}^2 - i\, m_{\omega_0} \Gamma_{\omega_0}$$

Masses and widths are modified by electromagnetic corrections only at second order. Using the observed values we can summarize our estimates

$$\left| \eta_1^R \right| = \left| \eta_2^R \right| \simeq (4.5 \text{ to } 10) 10^{-2}$$

$$\left| \eta_1^\Sigma \right| = \left| \eta_2^\Sigma \right| < 1.4 \times 10^{-2} \quad .$$

3.6   The mixing parameters ε_1, ε_2 are given in terms of η_1 and η_2 by the Eqs. (VI.8). Expanding in powers of η_1 and η_2

$$\varepsilon_1 \simeq \eta_1 \left(1 - \eta_1 \eta_2 \right) \qquad\qquad \varepsilon_2 \simeq \eta_2 \left(1 - \eta_1 \eta_2 \right)$$

For the modulus of the mixing parameters, a first order calculation in η is certainly reliable

$$|\varepsilon_1| = |\eta_1| \; , \qquad |\varepsilon_2| = |\eta_2|$$

and from the previous discussion, both moduli must be dominated by the contributions coming from the self-energy terms

$$|\varepsilon_1| \simeq |\varepsilon_2| \simeq |\eta^R| \; .$$

3.7 Let us work again to lowest order with respect to electromagnetic interactions so that $\varepsilon_{1,2} \simeq \eta_{1,2}$. The physical states $|\rho>$ and $|\omega>$ are normalized to unity: $p = s = 1 + O(\varepsilon^2)$

$$f_{\omega\, 2\pi}^{\rho} \simeq f_{\omega_0\, 2\pi}^{\rho} + \eta_2\, f_{\rho_0\, 2\pi}^{\rho} \qquad\qquad (VI.16)$$

The direct transition $\omega_0 \to 2\pi$ affects the physical amplitude $\omega \to 2\pi$ in two ways

    a)   through the first term in Eq. (VI.16)

    b)   through the 2π contribution in η_2^{Σ}.

It is remarkable, and one can easily check by direct calculation, that these two contributions partially cancel and the effective importance of $f_{\omega_0\, 2\pi}$ in Eq. (VI.16) comes out damped by a factor of order $\Gamma_\omega/\Gamma_\rho$. The physical origin of this cancellation is the dominance of the 2π channel in our problem.

Therefore $f_{\omega_0\, 2\pi}$ has only a small effect on the physical transition $f_{\omega 2\pi}$ and it cannot explain the magnitude of the observed $\omega \to 2\pi$ decay width.

It is then convenient to introduce a parameter $\bar{\eta}_2^{\Sigma}$ defined as η_2^{Σ} but without the 2π contribution. Equation (VI.16) is then rewritten as

$$f_{\omega\, 2\pi}^{\rho} \simeq \left(\eta_2^R + i\, \bar{\eta}_2^{\Sigma} \right) f_{\rho_0\, 2\pi}^{\rho} + f_{\omega_0\, 2\pi}^{\rho}\, O\left(\frac{\Gamma_\omega}{\Gamma_\rho} \right) \qquad (VI.17)$$

Of course the estimate for $\bar{\eta}_2^{\Sigma}$ is reduced by a factor two as compared to that of η_2^{Σ}.

The only possible explanation for the large observed value of $f_{\omega 2\pi}$ is then the self-energy term η_2^R

$$\frac{f_{\omega 2\pi}^{\rho}}{f_{\rho 2\pi}^{\rho}} \backsim \eta_2^R \simeq \varepsilon_2^R$$

From the mean value of experimental measurements (VI.1) we get

$$|\eta_2^R| \tilde{=} \tau = \left(4.5 \begin{array}{c} + 0.75 \\ - 1.35 \end{array}\right)10^{-2} \qquad (VI.18)$$

and the corresponding self-energy needed

$$\frac{1}{m_\rho} \left| R_{\rho_0 \omega_0} \right| = \left(4.9 \begin{array}{c} + 0.8 \\ - 1.4 \end{array}\right) MeV$$

fits the models very well.

3.8 Because of the smallness of $|\eta|$, in all the applications we will use the lowest order approximation $\varepsilon = \eta$ neglecting, for the modulus and the phases, third order corrections.

4. THEORETICAL PREDICTIONS

4.1 We assume time reversal invariance

$$\eta_1 = \eta_2 = \eta \qquad \varepsilon_1 = \varepsilon_2 = \varepsilon \quad .$$

The function $R_{\rho_0 \omega_0}$ is real and all theoretical arguments give it a negative sign.

The phase of η^R is known, and using

$$m_\rho = (773.6 \pm 5.3)MeV \qquad m_\omega = 783 \ MeV$$

$$\Gamma_\rho = (110.7 \pm 5.3)MeV \quad and \quad \Gamma_\omega = 12.2 \ MeV$$

we obtain

$$\phi_{\eta R} = (101 \pm 6)°$$

where the error is essentially due to the uncertainty on the $\omega-\rho$ mass difference.

4.2 Let us call $\phi_{2\pi}$ the relative phase of the decay amplitudes

$$\phi_{2\pi} = \text{phase } \frac{f_{\omega 2\pi}}{f_{\rho 2\pi}} \quad .$$

The main contribution to $\phi_{2\pi}$ is given by the η^R term and we can also take into account the small correction due to the term η^Σ

$$\phi_{2\pi} = \phi_{\eta R} + \bar{\delta}$$

with

$$\bar{\delta} = - \frac{\Sigma_{\rho_0 \omega_0}}{R_{\rho_0 \omega_0}} \quad .$$

The only possible significant contribution to $\bar{\Sigma}_{\rho_0\omega_0}$ can be due to the $\pi^0\gamma$ intermediate state. As explained in Lectures II and V, in the framework of the vector meson dominance model, $f_{\omega^0\pi^0\gamma}$ and $f_{\rho^0\pi^0\gamma}$ have the same sign so that $\Sigma_{\rho_0\omega_0}^{(\pi^0\gamma)}$ is positive. We then obtain

$$\delta_{\pi^0\gamma} = (4 \pm 2)° \qquad \text{from the VMD model}$$

$$\delta_{\pi^0\gamma} = 8° \qquad \text{using the experimental limit for } \Gamma(\rho \to \pi^0\gamma).$$

4.3 The phase of η can be slightly different because of the 2π contributions entering in η^Σ. We write

$$\phi_\eta = \phi_{\eta R} + \delta_{2\pi} + \delta_{\pi^0\gamma}$$

and $\delta_{2\pi}$ is given by

$$\widetilde{\delta}_{2\pi} = - \frac{\overset{(2\pi)}{\sum} \rho_o \omega_o}{\mathcal{R}_{\rho_o \omega_o}}$$

Unfortunately, we don't know the sign of $\delta_{2\pi}$ and using the order of α argument we find $\left| \delta_{2\pi} \right| \simeq 8°$.

Therefore

$$\phi_\eta \simeq (105 \pm 16)° . \tag{VI.20}$$

4.4 The photon-vector meson coupling constants f_V^{-1} have a small phase ϕ_V because of the mixing. Equations (VI.6) and (VI.7) are used to compute these parameters

$$\phi_\omega - \phi_\rho = |\eta| \sin \phi_2 \left(\frac{f_{\omega_o}}{f_{\rho_o}} + \frac{f_{\rho_o}}{f_{\omega_o}} \right) - \frac{|\eta|^2}{2} \sin \phi_2 \cos \phi_2 \left(\frac{f_{\omega_o}^2}{f_{\rho_o}^2} - \frac{f_{\rho_o}^2}{f_{\omega_o}^2} \right) + O(\eta^3)$$

With the experimental values of f_{ω^0} and f_{ρ^0} as deduced from the Orsay storage ring experiments and the previous estimates of $|\eta|$ and ϕ_η, (VI.18) and (VI.19), we obtain

$$\phi_\omega - \phi_\rho = \left(8 \begin{array}{c} + 2 \\ - 3 \end{array} \right)° . \tag{VI.21}$$

4.5 The phase measured in the Orsay experiment is

$$\alpha_{2\pi} = \phi_\omega - \phi_\rho + \phi_{2\pi} .$$

The theoretical prediction is

$$\alpha_{2\pi \, th} = \left(113 \begin{array}{c} + 10 \\ - 11 \end{array} \right)° . \tag{VI.22}$$

The Orsay measurement is[61]

$$\alpha_{2\pi \, exp} = (164 \pm 28)^\circ \ .$$

The agreement between theory and experiment is not very good but a definite conclusion cannot be given for the moment, more accurate experiments being needed.

5. COHERENT PHOTOPRODUCTION ON NUCLEI

5.1 Let us disregard here all the complicated problems of background and shape in the vector meson photoproduction on nuclei. We also assume the production amplitude

$$\gamma + A \to V + A$$

to be spin independent and the total cross-section is simply written as

$$\sigma(\gamma A \Rightarrow VA \Rightarrow \bar{F} A) \sim \left| \sum_V \frac{C_V}{W_V(m^2) - m^2} \right|^2$$

where m^2 is the invariant squared mass of the final state F resulting from the vector meson decay $V \to F$.

The complex parameter C_V is the product of the production amplitude and the decay coupling constant

$$C_V = f^\circ_{VF} < \tilde{V}, A \mid T \mid \gamma, A >$$

5.2 The vector mesons V are the physical ones after electromagnetic mixing, and in the ω region we take into account only the ω and ρ contributions. The ω-ρ interference is described by the complex ratio

$$\frac{C_\omega}{C_\rho} = \xi \, e^{i\alpha} \qquad\qquad (VI.28)$$

and we have to study the two real parameters ξ and α. In the vector meson dominance model, the production amplitude is given by

$$\langle \hat{V}, A \mid T \mid \gamma, A \rangle = \sum_{V'}{}' \frac{e}{f_{V'}} \langle \tilde{V}, A \mid T \mid V', A \rangle$$

It is more convenient to work on the $|\rho_0 > |\omega_0 >$ basis where we assume the non-diagonal transitions $\omega_0-\rho_0$ to be negligible as compared to the diagonal $\omega_0-\omega_0$ and $\rho_0-\rho_0$ transitions. Defining

$$A_{\rho_0} = \langle \rho^0 A \mid T \mid \rho^0 A \rangle \qquad A_{\omega^c} = \langle \omega^c A \mid T \mid \omega^c A \rangle$$

we obtain, to first order in ε

$$\langle \tilde{\rho}, A \mid T \mid \rho, A \rangle = A_{\rho^c} \qquad \langle \tilde{\omega}, A \mid T \mid \omega, A \rangle = A_{\omega^c}$$

$$\langle \tilde{\rho}, A \mid T \mid \omega, A \rangle = \langle \tilde{\omega}, A \mid T \mid \rho, A \rangle = \varepsilon (A_{\rho^0} - A_{\omega^c})$$

The two elastic amplitudes A_{ρ_0} and A_{ω_0} are expected to be of the same order of magnitude in the quark model.

5.3 The ratio C_ω/C_ρ is given by

$$\frac{C_\omega}{C_\rho} = \frac{f^0_{\omega\tilde{\rho}}}{f_{\rho\tilde{\rho}}} \frac{A_{\omega^c}}{A_{\rho^0}} \frac{f^c_{\rho^0}}{f_{\omega^c}} \left[1 + \varepsilon \left(\frac{f_{\omega^c}}{f_{\rho^0}} \frac{A_{\rho^0}}{A_{\omega^c}} + \frac{f_{\rho^0}}{f_{\omega^0}} \frac{A_{\omega^0}}{A_{\rho^c}} \right) \right] \qquad \text{(VI.24)}$$

To lowest order in ε, the bracket in Eq. (VI.24) is a phase and we define the production phase Φ by

$$\underline{\Phi} = \underline{\Phi}_0 + |\varepsilon| \left\{ R_0 \frac{f_{\rho_0}}{f_{\omega_0}} \sin (\phi + \underline{\Phi}_0) + R_0^{-1} \frac{f_{\omega_0}}{f_{\rho_0}} \sin (\phi - \underline{\Phi}_0) \right\} \quad \text{(VI.25)}$$

where

$$\frac{A_{\omega_0}}{A_{\rho_0}} = R_0 \, e^{i \underline{\Phi}_0} \qquad\qquad \varepsilon = |\varepsilon| \, e^{i\phi}$$

Equation (VI.24) is then rewritten to lowest order in ε as

$$\frac{C_\omega}{C_\rho} = \frac{f_{\omega F}}{f_{\rho F}} \, R_0 \, \frac{f_{\rho_0}}{f_{\omega_0}} \, e^{i \underline{\Phi}}$$

5.4 In the particular final state $F \equiv \pi^+\pi^-$, we have previously obtained

$$\frac{f_{\omega 2\pi}}{f_{\rho 2\pi}} \simeq |\eta| \, e^{i \phi_{2\pi}}$$

and the ω–ρ interference parameters for the reaction $\gamma A \to \pi^+\pi^- A$ are given by

$$\xi_{2\pi} = |\eta| \, R_0 \, \frac{f_{\rho_0}}{f_{\omega_0}} \qquad\qquad \alpha_{2\pi} = \phi_{2\pi} + \underline{\Phi} \qquad \text{(VI.26)}$$

For the final state $F = e^+e^-$ or $\mu^+\mu^-$ a straightforward calculation gives

$$\frac{f^\rho_{\omega \bar{\ell \ell}}}{f^\rho_{\rho \bar{\ell \ell}}} = \left(\frac{m_\omega}{m_\rho}\right)^2 \frac{f^\rho_\rho}{f_\omega} \simeq \left(\frac{m_\omega}{m_\rho}\right)^2 \frac{f^\rho_{\rho_o}}{f_{\omega_o}} e^{i(\phi_\omega - \phi_\rho)}$$

and the ω-ρ interference parameters for the lepton pair photoproduction are given by

$$\xi_{\bar{\ell \ell}} = \left(\frac{m_\omega}{m_\rho}\right)^2 \left(\frac{f^\rho_{\rho_o}}{f_{\omega_o}}\right)^2 \mathcal{R}_o \qquad \alpha_{\bar{\ell \ell}} = \phi_\omega - \phi_\rho + \underline{\Phi} \qquad (VI.27)$$

5.5 The experimental data on lepton pair production are consistent with $R_0 = 1$. If, on the other hand, we make the assumption $\Phi_0 = 0$ as suggested by the quark model, we can make predictions because the production phase Φ is now essentially due to the ω-ρ mixing

$$\underline{\Phi} \simeq \phi_\omega - \phi_\rho \qquad (VI.28)$$

a) $\pi^+\pi^-$ photoproduction: the theoretical prediction turns out to be the same as for $\pi^+\pi^-$ production in e^+e^- annihilation

$$\alpha_{2\pi \ th} = (113 \ {}^{+ \ 10}_{- \ 11})^\circ$$

and a Daresbury measurement gives[62]

$$\alpha_{2\pi \ exp} = (104 \pm 5.1)^\circ$$

in excellent agreement with the theoretical value.

Let us remark that under the assumption $R_0 = 1$, $\Phi_0 = 0$, the relative phase α for the two processes $e^+e^- \to \pi^+\pi^-$ and $\gamma A \to \pi^+\pi^- A$ must be the same, independently of the details of the ω-ρ mixing theory which predicts its precise value.

b) $\ell\bar{\ell}$ photoproduction: the theoretical prediction for the phase $\alpha_{\ell\bar{\ell}}$ is entirely due to the ω-ρ mixing, $\alpha_{\ell\bar{\ell}} = 2(\phi_\omega - \phi_\rho)$

$$\alpha_{\ell\bar{\ell}\,\text{th}} = (16 \; {}^{+\;4}_{-\;6})^\circ$$

The experimental situation is not absolutely clear. The Daresbury experiment gives a very large result[62]

$$\alpha_{\ell\bar{\ell}\,\text{exp}} = (100 \; {}^{+\;38}_{-\;30})^\circ$$

but a preliminary analysis of a DESY experiment gives[78]

$$\alpha_{\ell\bar{\ell}\,\text{exp}} = (41 \pm 20)^\circ$$

in better agreement with the theoretical prediction

6. ϕ-ρ ELECTROMAGNETIC MIXING

6.1 The general problem of electromagnetic mixing between the neutral vector mesons is a three-dimensional one. Nevertheless in two particular situations it can be approximated by a two-dimensional one, because of the smallness of the ω and ϕ meson widths as compared with the ρ meson width.

a)   In the ω region, only the ω-ρ mixing is important.

b)   In the ϕ region only the ϕ-ρ mixing has to be considered.

We now discuss some aspects of the ϕ-ρ mixing in the ϕ meson region.

6.2 We assume, from the beginning, time-reversal invariance and we work in lowest order with respect to the mixing parameter λ

$$| \rho \rangle = | \rho_0 \rangle - \lambda | \phi_0 \rangle$$

$$| \phi \rangle = | \phi_0 \rangle + \lambda | \rho_0 \rangle$$

Again, λ is given in terms of the matrix elements of the matrix propagators W in the isotopic spin basis $| \rho_0 \rangle$, $| \phi_0 \rangle$

$$\lambda = - \frac{W_{\rho_0 \phi_0}}{W_{\rho_0 \rho_0} - W_{\phi_0 \phi_0}}$$

All the quantities are assumed to be taken at the ϕ meson mass. As previously, $W_{\rho_0 \phi_0}$ is split into two parts

$$W_{\rho_0 \phi_0} = R_{\rho_0 \phi_0} - i \, \Sigma_{\rho_0 \phi_0} \, .$$

The first term is a self-energy contribution and the second one can be computed using the unitarity relation.

6.3 Estimates based on the quark model predict $R_{\rho_0 \phi_0}$ smaller than $R_{\rho_0 \omega_0}$, by perhaps an order of magnitude, and positive. In that case the vector meson mass difference is large and it seems that the dominant part of $R_{\rho_0 \phi_0}$ is due to the one photon exchange

Fig. 14

The corresponding value of $R_{\rho_0 \phi_0}$ is then

$$\frac{\lambda}{m_\phi} \left(R_{\rho_0 \phi_0} \right)_{1\gamma} \simeq 0.8 \text{ MeV} \, .$$

6.4 We now decompose $\Sigma_{\rho_0\phi_0}$ as a sum over the physical intermediate states

$$\sideset{}{'}\sum_{\rho_0\phi_0} = \sideset{}{'}\sum_{\rho_0\phi_0}^{(K^+K^-)} + \sideset{}{'}\sum_{\rho_0\phi_0}^{(\pi^0\bar{K}^0)} + \sum_{\rho_0\phi_0}^{(\eta\gamma)} + \sum_{\rho_0\phi_0}^{(\pi^0\gamma)} + \sum_{\rho_0\phi_0}^{(3\pi)} + \sideset{}{'}\sum_{\rho_0\phi_0}^{(2\pi)} + \ldots$$

The two $K\bar{K}$ contributions have opposite sign but they do not cancel exactly because of the K^+-K^0 mass difference. In fact, we have

$$\left|\begin{array}{l} \dfrac{1}{m_\phi} \sideset{}{'}\sum_{\rho_0\phi_0}^{(K^+K^-)} = \dfrac{f^0_{\rho K^+K^-}}{f_{\phi K^+K^-}} \; \Gamma\left(\phi \to K^+K^-\right) \\[3em] \dfrac{1}{m_\phi} \sideset{}{'}\sum_{\rho_0\phi_0}^{(K^0\bar{K}^0)} = \dfrac{f_{\rho K^0\bar{K}^0}}{f^0_{\phi K^0\bar{K}^0}} \; \Gamma\left(\phi \to K^0\bar{K}^0\right) \end{array}\right.$$

Now neglecting the Coulomb corrections in the coupling constants, we obtain

$$\dfrac{1}{m_\phi}\left[\sideset{}{'}\sum_{\rho^0\phi_0}^{(K^+K^-)} + \sideset{}{'}\sum_{\rho^0\phi_0}^{(K^0\bar{K}^0)}\right] = \dfrac{f^0_{\rho K\bar{K}}}{f_{\phi K\bar{K}}}\left[\Gamma\left(\phi \to K^+K^-\right) - \Gamma\left(\phi \to K^0\bar{K}^0\right)\right]$$

Using the relation $f_{\rho K\bar{K}} = \tfrac{1}{2}f_{\rho\pi\pi}$ and the Orsay data we obtain

$$\dfrac{1}{m_\phi}\sideset{}{'}\sum_{\rho^0\phi_0}^{(K\bar{K})} \simeq (0.46 \pm 0.04)\text{MeV}.$$

For the $\eta\gamma$ contribution, we can only make speculations

$$\dfrac{1}{m_\phi}\sideset{}{'}\sum_{\rho^0\phi_0}^{(\eta\gamma)} = \dfrac{f^0_{\rho\eta\gamma}}{f^0_{\phi\eta\gamma}} \; \Gamma\left(\phi \to \eta\gamma\right)$$

One can estimate the coupling constants $f_{\rho 2\gamma}$ and $f_{\phi 2\gamma}$ using the SU(3) symmetry with the η-X^0 mixing and the VMD model[80]. The result turns out to give a negative contribution to $\Sigma_{\rho_0 \phi_0}$

$$\frac{1}{m_\phi} \Sigma'^{(2\pi)}_{\rho^0 \phi^0} \simeq -0.28 \text{ MeV}$$

The $\pi^0\gamma$ and 3π contributions are obviously smaller by an order of magnitude and the 2π contribution, which can be large, fortunately don't play any role for the particular 2π decay mode as previously.

Summarizing, a crude evaluation of $\bar{\Sigma}_{\rho_0 \phi_0}$ - where the 2π contribution is absent - gives

$$\frac{1}{m\phi} \bar{\Sigma}_{\rho_0 \phi_0} \simeq (0.2 \pm 0.1)\text{MeV} .$$

6.5   The $\phi \rightarrow 2\pi$ decay amplitude has the form

$$f_{\phi 2\pi} = (\lambda^R + i\lambda^{\bar{2}\bar{3}}) f_{\rho_0 2\pi} + f_{\phi_0 2\pi} O\left(\frac{\Gamma_\phi}{\Gamma_\rho}\right)$$

Neglecting the direct transition $f_{\phi_0 2\pi}$ reduced by a damping factor Γ_ϕ/Γ_ρ we obtain

$$\left| \frac{f_{\phi 2\pi}}{f_{\rho 2\pi}} \right| \simeq (2.3 \pm 0.1)10^{-3}$$

$$\text{Phase} \left(\frac{f_{\phi 2\pi}}{f_{\rho 2\pi}} \right) \simeq (-30 \pm 10)^\circ$$

where a large part of the uncertainty on the phase prediction is due to the unknown value of the ρ meson width at the ϕ meson mass.

Again we emphasize that these numbers are only crude estimates and the errors given cannot be taken very seriously. Nevertheless, they can perhaps be useful for planning experiments.

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