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Top-quark pair + jet production at next-to-leading order QCD*

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In this talk I report on the status of the calculation of the next-to-leading order (NLO) QCD corrections to top-quark pair production together with a jet in hadronic collisions. A precise understanding of this process is of great importance, because $pp \to t\bar{t} + 1$ jet is an important background for the Higgs search in the Higgs mass range 120–180 GeV. In addition the reaction is also an interesting process for precise measurements in the top sector.

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1. Introduction

The top quark is the heaviest known elementary particle so far. With a mass of 172.7 ± 2.9 GeV [1] it is almost as heavy as a gold atom. Because of its large mass, the top quark plays a special rôle in the Standard Model and its extensions. In particular it is the only fermion that couples with a strength of order 1 to the not yet discovered Higgs boson. In the Standard Model the top quark completes the third flavour family. As a consequence its quantum numbers are completely fixed by the structure of the Standard Model. As up-type partner of the bottom quark, its electric charge is $+2/3$. The left-handed component has weak isospin $+1/2$, while the right-handed component has weak isospin 0. So far our knowledge of these quantum numbers is only from indirect measurements. In particular, such a fundamental quantity as the electric charge of the top quark has not been measured yet [2]. It is thus an important goal to check experimentally that the top quark indeed behaves as predicted by the Standard Model.

Apart from the interest in top-quark physics as a signal process, it also plays an important rôle as background in many Higgs studies. For example, in the Higgs

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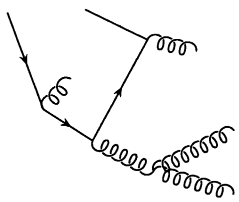
mass range 120–180 GeV, the most important discovery channel is the weak boson fusion process (WBF) [3, 4]. The dominant background for the WBF process comes from top-quark pair production together with a jet [5]. A very precise knowledge of this reaction is thus mandatory for the discovery of the Higgs boson. It is also obvious that, for precise measurements of the couplings, a detailed background determination is equally important. For example, it has been shown in Ref. [4] that even if one assumes an only 10% uncertainty of the $t\bar{t}+\text{jet}$ cross section it is still the dominant theoretical uncertainty in the measurement of $\sigma_{\rm H} = \sigma_{\rm WBF} \times B(H \to WW)$. As also pointed out in Ref. [4] this accuracy might be achievable only through a full next-to-leading-order calculation. In what follows, I will describe the status of the calculation of the next-to-leading-order corrections to $t\bar{t}+\text{jet}$ production in hadronic collisions.

2. Outline of the calculation

In this section I briefly summarize the calculation of the NLO corrections for the subprocess $gg \to t\bar{t}g$. This is an ongoing project in collaboration with A. Brandenburg, S. Dittmaier and S. Weinzierl. In view of the number of scales, it is clear that analytic results, even partial ones, are in general quite lengthy. Therefore I will restrict myself in what follows to a few technical aspects, which are important for the construction of a numerical stable program.

2.1. *Real corrections*

The calculation of the required matrix elements is straightforward. A sample



QGRAF-diagram 20

Fig. 1. Sample Feynman diagram contributing to the real corrections.

diagram for the reaction $gg \to t\bar{t}gg$ is shown in Fig. 1. We used two different methods to obtain the required colour-ordered helicity amplitudes:

1. A Feynman-diagram-based approach, where we evaluate all the diagrams contributing to one specific colour-ordered subamplitude.

2. The recurrence relations à la Berends and Giele [6].

We find complete agreement in the results of the two methods. Furthermore we also checked that our results agree with the ones obtained using Madgraph [7]. To extract the singularities from collinear or soft partons, and to render the phase integration over the real corrections finite, we use the dipole-subtraction method [8, 9, 10]. The idea of the subtraction method is to add and subtract a term that, on the one hand, cancels pointwise the singularities of the matrix elements in the singular regions of the phase space and, on the other hand, is easy enough to be integrated analytically. Schematically the NLO contribution is then obtained from the following formula:

$$\begin{aligned}\sigma_{\mathrm{NLO}} &= \underbrace{\int dR_{m+1}\left[\sigma_{\mathrm{real}}-\sigma_{\mathrm{sub}}\right]}_{\text{finite}} + \underbrace{\int dR_m \left[\sigma_{\mathrm{virt.}} + \int dR_1 \bar{\sigma}^1_{\mathrm{sub}}\right]}_{\text{finite}} \\ &+ \underbrace{\int dx \int dR_m \left[\sigma_{\mathrm{fact.}}(x) + \bar{\sigma}_{\mathrm{sub}}(x)\right]}_{\text{finite}}. \qquad (1)\end{aligned}$$

Here $\sigma_{\mathrm{fact.}}(x)$ denotes the contribution from the factorization of initial-state singularities due to the presence of coloured partons in the initial state; dR_m is the Lorentz-invariant phase-space measure for m particles in the final state. The contributions $\bar{\sigma}^1_{\mathrm{sub}}, \bar{\sigma}_{\mathrm{sub}}$ are obtained from σ_{sub} by integrating out the 'unresolved' parton. The result is split into the two terms $\bar{\sigma}^1_{\mathrm{sub}}, \bar{\sigma}_{\mathrm{sub}}$ to render the last two integrals individually finite. A remarkable feature of the subtraction method is that the analytic integration of the subtraction term has to be done only once and that no approximation is made in the whole procedure. This is made possible by the universality of soft and collinear factorization in QCD. The explicit expressions for $\sigma_{\mathrm{sub}}, \sigma^1_{\mathrm{sub}}$, and $\bar{\sigma}_{\mathrm{sub}}$ can be obtained from the colour-ordered subamplitudes for the subprocess $gg \longrightarrow t\bar{t}$, using the formulae given in Ref. [10]. In particular σ_{sub} is obtained from a sum over individual *dipole contributions*. In the case at hand we have to include the contribution from 36 individual dipoles. We do not consider the splitting $g \longrightarrow t\bar{t}$ because the divergence is regulated by the quark masses. (For light quarks with $m_q \neq 0$ one could consider the corresponding dipoles to render the integration numerically more stable.) We have checked that the combination of the 36 dipoles indeed reproduces all the singular limits arising from single unresolved configurations. Furthermore we have combined the subtraction terms together with the real corrections into a computer program, that allows the numerical integration over the phase space. The numerical result is shown in Fig. 2. To define the partonic cross section in Fig. 2 we made the idealized assumption that the top quarks are always observed. The remaining jets are then clustered according to the jet algorithm of Ellis and Soper [11]. In addition to the two top quarks we ask for one additional jet with a transverse momentum $k_\perp$ of at least 20 GeV. As can be seen from Fig. 2, even at very large (partonic) centre-of-mass energies we obtain stable

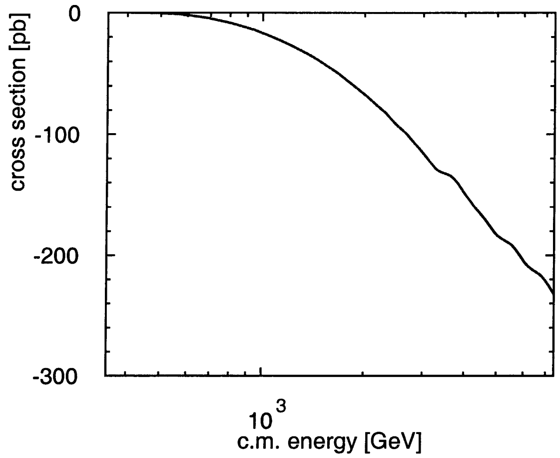


Fig. 2. Result for the 'real corrections' for the subprocess $gg \longrightarrow t\bar{t}g$ as defined by the first term in Eq. (1) ($k_\perp > 20$ GeV).

results. Note that to obtain a hadronic cross section the partonic contribution still needs to be folded with the parton distribution functions which put most of the weight to low centre-of-mass energies.

2.2. Virtual corrections

The calculation of the virtual corrections proceeds via the following steps:

1. Generation of the Feynman diagrams, using for example Feynarts [12] or QGRAF [13].
2. Reduction of the tensor integrals to scalar one-loop integrals.
3. Reduction of the amplitudes to standard matrix elements.
4. Numerical phase-space integration of the squared matrix elements, including appropriate phase-space cuts.

Technically the most complicated part is the evaluation of the pentagon diagrams. Two sample diagrams are shown in Fig. 3. Let us first address the evaluation of the scalar 5-point integrals. To calculate these, we use two different methods. One calculation is based on the method given in Refs. [14, 15]. The basic idea of this method is that finite 5-point integrals can be expressed in terms of 4-point integrals (see for example [16, 17, 18]). To apply this observation also to soft- and mass-singular integrals, they are rewritten according to Refs. [14, 15] in the following

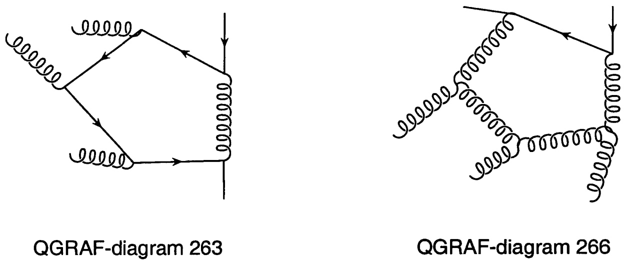


Fig. 3. Sample Feynman diagrams contributing to the virtual corrections.

way:

$$E^d = E^d_{\text{sing.}} + \left[E^{(\text{mass},d=4)} - E^{(\text{mass},d=4)}_{\text{sing.}} \right] . \tag{2}$$

Here E^d denotes the original 5-point integral in d dimensions while $E^{(\text{mass},d=4)}$ is obtained from the original integral by dressing the massless propagators with a small mass λ. The subtraction term $E^{(\text{mass},d=4)}_{\text{sing.}}$, which has the same singular structure as the 5-point integral $E^{(\text{mass},d=4)}$ in the limit $\lambda \to 0$, is obtained by studying the soft and collinear behaviour of $E^{(\text{mass},d=4)}$; it can be expressed in terms of 3-point integrals [19]. Note that the term in square brackets in Eq. (2) is finite and regularization-scheme-independent. Rewriting now the finite integral $E^{(\text{mass},d=4)}$ in terms of 4-point integrals, we thus succeeded in expressing the original 5-point integral in terms of 3- and 4-point functions. A more detailed discussion can be found in Ref. [15]. The second method we used to calculate the five-point integrals is based on the fact that, even for divergent integrals, it is possible to obtain a representation as linear combination of 4-point integrals (see for example Ref. [18]). Expressing the 4-point function for $d = 4 - 2\varepsilon$ in terms of the finite 4-point function in 6 dimensions, plus a combination of 3-point integrals, allows us also to shift all the divergences to the 3-point integrals. Defining the 5-point functions through

$$\begin{aligned} & E^d(p_0, p_1, p_2, p_3, p_4, m_0, m_1, m_2, m_3, m_4) \\ = & \frac{1}{i\pi^2} \int d^d\ell \prod_{j=0}^{4} \frac{1}{(\ell + p_j)^2 - m_j^2 + i\varepsilon}, \end{aligned} \tag{3}$$

we obtain for example

$$\begin{aligned} & E_0(0, p_1, p_1 - p_3, p_4 - p_2, -p_2, m_t, m_t, 0, 0, m_t)|_{\text{sing.}} \\ = & \; P(t_{13})P(s_{45})C_0(p_1 - p_3, p_4 - p_2, p_1, 0, 0, m_t) \\ + & \; P(t_{24})P(s_{35})C_0(p_4 - p_2, -p_2, p_1 - p_3, 0, m_t, 0) \\ - & \; (t_{13} - t_{24})^2 P(t_{13})P(t_{24})P(s_{35})P(s_{45}) \\ & \times C_0(0, p_1 - p_3, p_4 - p_2, m_t, 0, 0), \end{aligned} \tag{4}$$

with $P(x) = 1/(x - m_t^2)$ and $s_{ij} = (p_i + p_j)^2$, $t_{ij} = (p_i - p_j)^2$. The parton momenta are assigned according to $g(p_1)g(p_2) \to t(p_3)\bar{t}(p_4)g(p_5)$. For the cases at hand it is possible to solve all the required box-integrals in 6 dimensions. We checked that the two methods yield the same results for the 5-point integrals E^d.

Having solved the scalar integrals, the next step is the reduction of the 5-point tensor integrals to scalar one-loop integrals. In principle one could attack this problem using the standard Passarino–Veltman approach [20]. This method leads to spurious singularities in individual terms at the phase-space boundary, which are due to vanishing Gram determinants in the denominator. These spurious singularities create numerical instabilities when doing the phase-space integration. Note that the spurious singularities will cancel if one combines the individual terms analytically before doing the numerical integration. One solution of this problem is a time-consuming extrapolation technique, as was used for example in Ref. [15]. As an alternative to the extrapolation technique a different reduction procedure [21] was also used in Ref. [15]. In this work we follow the method developed in Ref. [21]. Essentially the same technique to reduce scalar 5-point integrals to scalar 4-point integrals is also applied to the tensor integrals. In this way the 5-point tensor integrals are directly reduced to 4-point ones. The explicit calculation shows that the spurious singularities in individual terms, due to vanishing Gram determinants depending on 4 external momenta, are avoided [21]. Recently a refined version of the method presented in Ref. [21] as described in Ref. [22]. In addition to the aforementioned reduction procedure, we use a completely independent second technique [23, 24, 25] to cross-check our results. This second method is based on a two-step procedure. In the first step the tensor integrals are reduced to scalar integrals with raised powers of the propagators and in shifted dimensions [26]. In a second step these integrals are reduced to a small set of one-loop master integrals using identities derived from integration by parts [27]. Defining the scalar integrals by

$$I_n(d, \{q_l\}, \{\nu_i\}) = \int \frac{d^d\ell}{i\pi^{d/2}} \frac{1}{((\ell+q_1) - m_1^2)^{\nu_1}} \cdots \frac{1}{((\ell+q_n) - m_n^2)^{\nu_n}}, \tag{5}$$

the relations obtained from integration-by-parts can be written as follows [23, 24]:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ji} y_i \nu_j I_n(d; \{q_l\}, \{\nu_l + \delta_{lj}\}) = -\sum_{i=1}^{n} y_i I_n(d-2; \{q_l\}, \{\nu_l - \delta_{li}\})$$
$$-(d - 1 - \sum_{j=1}^{n} \nu_j) \sum_{i=1}^{n} y_i I_n(d; \{q_l\}, \{\nu_l\}), \tag{6}$$

with

$$S_{ji} = (q_i - q_j)^2 - m_i{}^2 - m_j{}^2. \tag{7}$$

For $\det(S) \neq 0$ the equation $S_{ik}y_k = \delta_{ij}$ can be solved and the integrals with raised powers of the propagators can be systematically reduced to basic integrals with unit powers of the propagators. For $\det(S) = 0$ one can choose y_i out of the null space of S to obtain useful reduction formulae. Furthermore, for $\det(S) \neq 0$ but small, one can use recurrence relations that do not terminate but yield in every order higher powers of $\det(S)$ that can be neglected when the desired precision is reached [28, 25]. In particular this last feature, namely a systematic algorithm to handle exceptional momentum configurations, makes this second reduction method very promising. So far the method has been tested only in very few examples [25, 29]. Let us just mention that there are also other methods to solve the scalar 5-point integrals and perform the reduction of the tensor integrals. For example one could also use the methods developed in Refs. [30, 31, 32, 33].

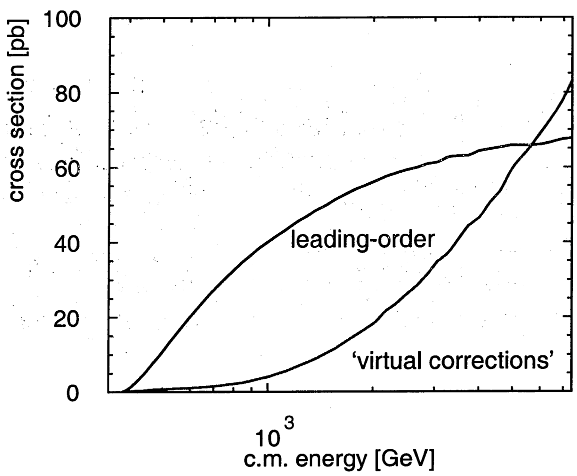


Fig. 4. Result for the virtual corrections for the subprocess $gg \to t\bar{t}g$ as defined by the second term in Eq. (1) ($k_\perp > 20$ GeV).

In Fig. 4 we show, as an illustration, the result at the parton level for the virtual corrections (defined as the second term in Eq. (1)) for different centre-of-mass energies, using the reduction procedure described in Ref. [21]. The observable is defined in the same way as in Fig. 2. As can be seen from Fig. 4 the method we used for the treatment of the tensor integrals indeed gives numerically stable results. Furthermore we note that the inclusion of $d\bar{\sigma}^1_{\rm sub}$ together with the renormalization of the coupling and the quark mass renders the second term in Eq. (1) finite, as it must be. This is an important cross-check.

3. Status

The current status of the project is as follows: Most of the separate contributions are already implemented in the form of computer programs that allows the numerical evaluation of the cross sections. As mentioned earlier we checked in particular that the integrand for the first contribution in Eq. (1) is finite for all single unresolved phase-space configurations. As can be seen from Fig. 2, the numerical phase-space integration yields stable results. Furthermore the real corrections are evaluated using two completely independent programs. Note that the subprocesses $q\bar{q} \to t\bar{t}gg$, $qg \to t\bar{t}qg$, which are also necessary for the full process $pp \to t\bar{t} + \text{jet}$, are much easier to compute and can be calculated using the same technology as discussed above. As far as the virtual corrections are concerned, many contributions have already been checked partially although a complete check of the pentagon diagrams is still missing. The numerical implementation of improved Passarino–Veltman-type reduction is finished and yields numerically stable results. The implementation of the reduction scheme presented in Refs. [23, 24] is finished and is now in the debugging phase.

4. Conclusions

Top-quark pair production together with an additional jet is an important reaction. The interest in this reaction is twofold. First of all it is an interesting signal process for top-quark physics. In particular, $t\bar{t} + \text{jet}$ events can be used to search for anomalous top–gluon couplings. Furthermore $t\bar{t} + \text{jet}$ can also appear as an important background for the Higgs searches. For example, in the case of Higgs production through weak boson fusion — which is the most important discovery channel for a Higgs mass in the range 120–180 GeV — the $t\bar{t} + \text{jet}$ process is the dominant background. A precise knowledge of this reaction is thus mandatory for the Higgs physics program at the LHC. The calculation of the NLO corrections for this process, although often considered as conceptually solved, is still a highly non-trivial task. In the present article I have reported on the current status of this calculation. Complete results will be published elsewhere.

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