Quantum corrections to the MSSM $h^0 b\bar{b}$ vertex: Decoupling limit

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We consider the leading one-loop Yukawa-coupling corrections to the $h^0 b\bar{b}$ coupling at $\mathcal{O}(m_t^2)$ in the MSSM in the decoupling limit. The decoupling behavior of the corrections from the various MSSM sectors is analyzed in the case of having some or all of the supersymmetric mass parameters and/or the CP-odd Higgs mass large as compared to the electroweak scale.

1. Introduction

It is well known that the tree-level couplings of the lightest Minimal Supersymmetric Standard Model (MSSM) Higgs boson (h^0) to fermion pairs and gauge bosons tend to their Standard Model (SM) values in the decoupling limit, $M_A \gg M_Z$ [1]. As a consequence of this decoupling, distinguishing the lightest MSSM Higgs boson in the large M_A limit from the Higgs boson of the SM will be very difficult. Our aim is to determine the nature of the decoupling limit at one-loop for the couplings of h^0 to SM particles. If some non-decoupling behavior of supersymmetric (SUSY) particles is found, it will provide a clear signal for some low energy observables, even if $M_{SUSY} \sim \mathcal{O}(\text{TeV})$.

In this paper, we focus on the h^0 coupling to $b\bar{b}$. This coupling determines the partial width of $h^0 \to b\bar{b}$, which is by far the dominant decay mode of h^0 in most of the MSSM parameter space. Therefore, accurate knowledge of the $h^0 b\bar{b}$ coupling is very important for Higgs boson searches. In particular, we study the $\mathcal{O}(m_t^2)$ Yukawa coupling MSSM radiative corrections to the $h^0 b\bar{b}$ vertex at one loop level, and we explore their behavior in the decoupling limit. A detailed discussion of the SUSY-QCD corrections that arise from gluino and bottom-squark (sbottom) exchange have been previously given in [2]. It has been shown that in the decoupling limit of both large SUSY mass parameters and large CP-odd Higgs mass, the $h^0 \to b\bar{b}$ decay width approaches its SM value at one loop, with the onset of decoupling delayed for large $\tan\beta$ values. However, this decoupling does not occur if just the SUSY mass parameters are taken large.

The full diagrammatic formula for the on-shell EW (electroweak)-Yukawa corrections to the $h^0 b\bar{b}$ coupling will be presented in [3]. Here we summarized the results obtained in this paper. In Section 2 we briefly review the decoupling limit in the Higgs sector and the SUSY sector of the MSSM. The $\mathcal{O}(m_t^2)$ Yukawa corrections to the $h^0 b\bar{b}$ coupling are presented in Section 3. Some details of the renormalization procedure and a discussion of the decoupling properties of these corrections are included in this section. Analytical and numerical results are collected in section 3.2. We conclude in Section 4.

2. Decoupling limit in the MSSM

The properties of the MSSM Higgs sector at the tree-level are determined by just two free parameters, conventionally chosen as the mass of

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the CP-odd neutral Higgs boson (A^0), M_A, and the ratio of the vacuum expectation values (*vevs*) of each doublet, $\tan\beta = v_2/v_1$ [4].

The decoupling limit in the MSSM is defined by considering the parameter regime where $M_A \gg M_Z$. In this limit, the expressions for the Higgs masses and mixing angle simplify [1] and two consequences are immediately apparent. First, $M_A \simeq M_{H^0} \simeq M_{H^\pm}$, up to corrections of $\mathcal{O}(M_Z^2/M_A)$, and $M_{h^0} \simeq M_Z|\cos 2\beta|$. Second, $\cos(\beta-\alpha) = 0$ up to corrections of $\mathcal{O}(M_Z^2/M_A^2)$. Consequently, the effective Higgs sector consists only of one light CP-even Higgs boson, h^0, whose couplings to SM particles are indistinguishable from those of the SM Higgs boson. When radiative corrections to the CP-even Higgs mass-squared matrix are taken into account, the upper bound on M_{h^0} increases substantially to $M_{h^0} \lesssim 135$ GeV [5].

Summarizing the parameters of the squark sector, the tree-level squark squared-mass matrix is:

$$\hat{M}_{\tilde{q}}^2 \equiv \begin{pmatrix} M_{\tilde{L}_q}^2 & m_q X_q \\ m_q X_q & M_{\tilde{R}_q}^2 \end{pmatrix} , \quad q \equiv t, b \qquad (1)$$

with,

$$\begin{aligned} M_{\tilde{L}_q}^2 &= M_{\tilde{Q}}^2 + m_q^2 + \cos 2\beta\, {M_Z}^2 (T_3^q - Q_q s_W^2) \\ M_{\tilde{R}_q}^2 &= M_{\tilde{U},\tilde{D}}^2 + m_q^2 + \cos 2\beta\, {M_Z}^2 Q_q s_W^2 \\ X_t &= A_t - \mu\cot\beta\,, \quad X_b = A_b - \mu\tan\beta\,, \end{aligned} \qquad (2)$$

and $s_W \equiv \sin\theta_W$. The parameters $M_{\tilde{Q}}$ and $M_{\tilde{U},\tilde{D}}$ are the soft-SUSY-breaking masses, A_t is a soft-SUSY-breaking trilinear coupling and μ is the bilinear coupling of the two Higgs doublet superfields.

In order to get heavy squarks, we need to choose large values for the appropriate soft SUSY breaking parameters and the μ-parameter. Since we are interested here in the limiting situation where the whole SUSY spectrum is heavier than the electroweak scale, we have made the following assumptions (see ref. [3] for more details),

$$M_{\tilde{Q},\tilde{U},\tilde{D}} \sim M_{\tilde{g}} \sim \mu \sim A_{t,b} \sim M_{SUSY} \gg M_Z, \qquad (3)$$

where M_{SUSY} represents generically a common large SUSY mass scale. In addition, we have considered two extreme cases, maximal and minimal mixing, which imply certain constraints on the squark mass differences: A. Close to maximal mixing ($\theta_{\tilde{q}} \sim \pm 45°$): $|M_L^2 - M_R^2| \ll m_q X_q \Rightarrow |M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2| \ll |M_{\tilde{q}_1}^2 + M_{\tilde{q}_2}^2|$, and B. Close to minimal mixing ($\theta_{\tilde{q}} \sim 0°$): $|M_L^2 - M_R^2| \gg m_q X_q \Rightarrow |M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2| \sim \mathcal{O}|M_{\tilde{q}_1}^2 + M_{\tilde{q}_2}^2|$. Eq. (3) also implies that the gluino is heavy.

Finally, the chargino mass matrix is given by,

$$\hat{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix} , \qquad (4)$$

and in order to get heavy charginos we consider the limit: $M_{SUSY} \sim \mu \sim M_2 \gg M_Z$.

3. $\mathcal{O}(m_t^2)$ Yukawa corrections to $h^0 \to b\,\bar{b}$

Here we present the one-loop corrections to the partial decay width $\Gamma(h^0 \to b\bar{b})$. We will then explore the decoupling behavior of these corrections for large SUSY masses, M_{SUSY}, and/or large M_A. Both numerical and analytical results will be presented elsewhere [3].

The tree-level $h^0 b\bar{b}$ coupling is given by

$$g_{hbb} = \frac{g m_b \sin\alpha}{2 M_W \cos\beta} \,. \qquad (5)$$

In lowest order this Higgs-fermion vertex represents the Yukawa coupling proportional to the fermion mass $m_f = m_b$. Note that in the limit of large M_A, $\sin\alpha \to -\cos\beta$ and g_{hbb} tends to the SM coupling, $g_{hbb}^{SM} = -g m_b/(2M_W)$.

The vertex functions obtained from the set of one-loop diagrams are in general UV-divergent. For finite one-particle irreducible (1PI) Green functions and physical observables, renormalization has to be performed by adding appropriate counterterms. We follow the conventions given in [6] for the renormalization procedure.

3.1. The $\alpha_{\rm eff}$-approximation

The radiatively-corrected $h^0 b\bar{b}$ coupling depends on the mixing angle α. At tree-level, α is determined by fixing $\tan\beta$ and M_A. At one-loop order, there are no $\mathcal{O}(\alpha_s)$ corrections to this mixing angle [2]. However, once one-loop SUSY-EW effects are included, the one-loop radiative corrections to α must be taken into account [7]. It turns out that the dominant contributions to the Higgs boson self-energies can be obtained by setting $q^2 = 0$. Consequently, these corrections can be absorbed into a redefinition of the CP-even neutral Higgs mixing angle α. This is the so-called α_{eff}-approximation.

More explicitly, we approximate the renormalized Higgs boson self-energies by $\hat{\Sigma}(q^2) \simeq \hat{\Sigma}(0) \equiv \hat{\Sigma}$. Consequently, the correction to the mixing angle, $\Delta\alpha$, is related to the renormalized self-energies and masses. Neglecting terms beyond one-loop level, one obtains

$$\tan\Delta\alpha = \frac{\hat{\Sigma}_{h^0 H^0}}{M^2_{h_o} - M^2_{H_o}}\,, \tag{6}$$

where $\hat{\Sigma}_{h^0 H^0}$ is the renormalized h^0–H^0 mixing propagator given in [6]. We can then absorb the contributions to the h^0–H^0 mixing, due to the squarks/chargino loops, in the redefinition of the effective mixing angle α. In this approximation one deduces that $\mathcal{Z}_R^{h^0} \approx 1$.

3.2. Analytic and numerical results

In this section we present explicit results for the $\mathcal{O}(m_t^2)$ EW-Yukawa correction to the $h^0 b\bar{b}$ vertex. The leading Yukawa contribution to the $h^0 b\bar{b}$ coupling arises from diagrams involving the exchange of virtual top-squarks, as shown in Fig. 1.

At the one-loop level and $\mathcal{O}(m_t^2)$, the $h^0 b\bar{b}$ coupling can be written as,

$$\bar{\Gamma}(h^0 \to b\bar{b}) = \Gamma(h^0 \to b\bar{b})(1 + 2\Delta_{\text{SUSYEW}})\,, \tag{7}$$

where $\bar{\Gamma}$ is the one-loop partial width and Γ is the tree-level partial width as in (5). Δ_{SUSYEW} denotes the $\mathcal{O}(m_t^2)$ radiative corrections to this vertex as given in Fig. 1,

$$\Delta_{\text{SUSYEW}} = \Delta^{\text{loops}}_{\text{SUSYEW}} + \Delta^{\text{CT}}_{\text{SUSYEW}}\,. \tag{8}$$

The triangle diagram, with the exchange of stops and charginos, contributes to $\Delta^{\text{loops}}_{\text{SUSYEW}}$, whereas the bottom self-energy diagram contributes to the contribution of the counterterms, $\Delta^{\text{CT}}_{\text{SUSYEW}}$. We have checked that other triangle contributions (with one top-squark and two charginos) and neutralinos contributions are subleading diagrams; hence, these diagrams are not included here. Note that our results are in agreement with the corresponding results of [8].

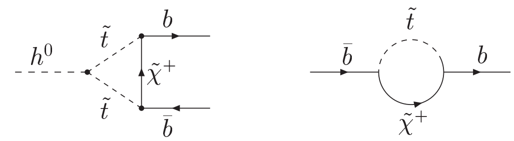

Figure 1. Feynman diagrams for the $\mathcal{O}(m_t^2)$ corrections to the $h^0 b\bar{b}$ coupling.

Expansions in inverse powers of SUSY masses are performed in order to examine the decoupling behavior when the SUSY masses are large compared to M_Z. Concretely, we perform expansions of the loop integrals and mixing angles for $M_{SUSY} \gg m_{EW}$ and isolate terms of $\mathcal{O}\left(m^2_{EW}/M^2_{SUSY})^n\right)$. Thus, by defining

$$\tilde{M}_S^2 \equiv \frac{1}{2}(M^2_{\tilde{t}_1} + M^2_{\tilde{t}_2})\,, \qquad R \equiv M_{\tilde{\chi}^+}/\tilde{M}_S\,, \tag{9}$$

and including the leading $\mathcal{O}(1)$ terms (*i.e.*, $n = 0$) in the expansion,, we obtain the following result for the maximal mixing case, $\theta_{\tilde{t}} \sim \pm 45°$:

$$\Delta_{\text{SUSYEW}} = \frac{g^2}{64\pi^2 m_W^2}\frac{1}{\sin^2\beta}\, m_t^2 \;\times \left\{\frac{-\mu A_t}{\tilde{M}_S^2}\,(\tan\beta + \cot\alpha)\, f_1(R) + \mathcal{O}\left(\frac{m^2_{EW}}{M^2_{SUSY}}\right)\right\}, \tag{10}$$

where the functions $f_i(R)$ are defined in ref. [2] and have been normalized as $f_i(1) = 1$.

Notice that the first term in (10) is the dominant one in the limit of large M_{SUSY} [eq. (3)] and does not vanish in the asymptotic limit of infinitely large $\tilde{M}_S$, μ and A_t. Therefore this term gives a non-decoupling SUSY contribution to the $\Gamma(h^o \to b\bar{b})$ partial width which can be of phenomenological interest. Moreover, since this term is enhanced at large $\tan\beta$ it can provide important corrections to the $h^o \to b\bar{b}$ total width, even for a very heavy SUSY spectrum. The sign of these corrections are fixed by the sign of μA_t. We find similar results for the minimal mixing case, $\theta_{\tilde{b}} \sim \pm 0°$ [3].

From this result, we conclude that there is no decoupling of stops and charginos in the limit of large SUSY mass parameters for fixed M_A. Similar results have been found for the SUSY-QCD corrections [2]. In contrast, most numerical studies done so far on this subject indicate decoupling of heavy SUSY particles from SM physics. How do we then recover decoupling of the heavy MSSM spectra from the SM low energy physics? The answer to this question relies in the fact that in order to converge to SM predictions we need to consider not just a heavy SUSY spectra but also a heavy Higgs sector. That is, besides large M_{SUSY}, the condition of large M_A is also needed. Thus, if $M_A \gg M_Z$, one can easily derive:

$$\cot\alpha = -\tan\beta - \frac{2M_Z^2}{M_A^2}\tan\beta\cos 2\beta + \mathcal{O}\left(\frac{M_Z^4}{M_A^4}\right). \quad (11)$$

By substituting this into (10) we see that the non-decoupling terms cancel out and we end up with

$$\Delta_{\rm SUSYEW} = \frac{g^2}{32\pi^2 m_W^2}\frac{1}{\sin^2\beta}\; m_t^2 \;\times \left\{\frac{-\mu A_t}{\tilde{M}_S^2} f_1(R)\,\tan\beta\cos 2\beta\,\frac{m_Z^2}{m_A^2} + \mathcal{O}\left(\frac{m_{EW}^2}{M_{SUSY}^2}\right)\right\} \quad (12)$$

which clearly vanishes in the asymptotic limit of M_{SUSY} and $M_A \to \infty$. Therefore, we get decoupling if and only if both M_{SUSY} and M_A are large.

The above non-decoupling behavior is shown numerically in Figs. 2 and 3 for tan$\beta = 30$ and SUSY parameters as defined in (3). We show the dependence of $\Delta_{\rm SUSYEW}$ on M_{SUSY} (Fig. 2) and M_A (Fig. 3). Clearly, in the limit of large M_{SUSY}, $\Delta_{\rm SUSYEW}$ tends to a non-vanishing constant, and this constant tends to zero in the large M_A limit. Similarly, in the limit of large M_A, $\Delta_{\rm SUSYEW}$ tends to a non-vanishing constant, and this constant tends to zero in the large M_{SUSY} limit. The solid lines in these figures correspond to the exact computation of the squarks/chargino loops, and the dashed lines correspond to the results of the expansion as in (10). Note that we have just considered the leading $\mathcal{O}(1)$ term in the expansion. The agreement between the exact results and the approximation derived from the expansion is recovered when the second term in the expansion is included.

The non-decoupling behavior emerging when the SUSY mass scale is much larger than M_A, can be derived from the low-energy effective theory that is obtained by integrating out the SUSY particles. This low-energy effective theory contains two Higgs doublets, whose couplings to fermions are unrestricted (*i.e.*, each Higgs doublet couples to *both* up-type and down-type quarks), characteristic of the so-called general type-III two Higgs doublet model instead of the type-II model that is assumed in the MSSM with no radiative corrections included [9].

The fact that decoupling is recovered when all SUSY mass parameters and M_A are equal is shown in Fig. 4. The $\Delta_{\rm SUSYEW}$ corrections are plotted as a function of a common scale M_S for different values of tanβ. Clearly, $\Delta_{\rm SUSYEW}$ de-

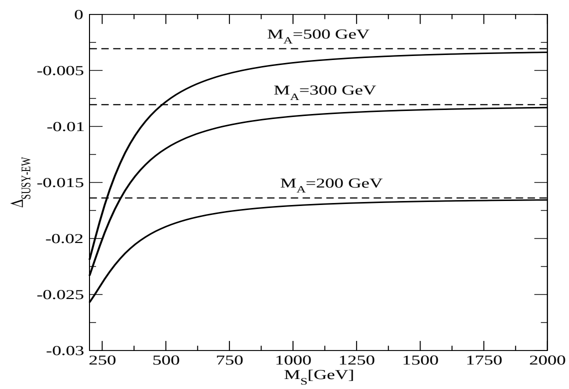

Figure 2. $\Delta_{\rm SUSYEW}$ as a function of M_{SUSY} for $M_A = 200$, 300, and 500 GeV and tan$\beta = 30$.

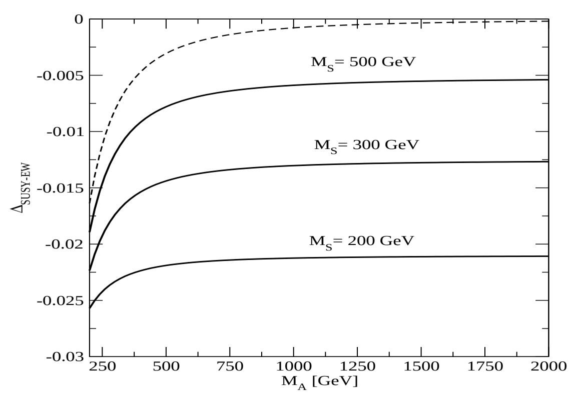

Figure 3. $\Delta_{\rm SUSYEW}$ as a function of M_A for $M_{SUSY} = 200$, 300, and 500 GeV and tan$\beta = 30$.

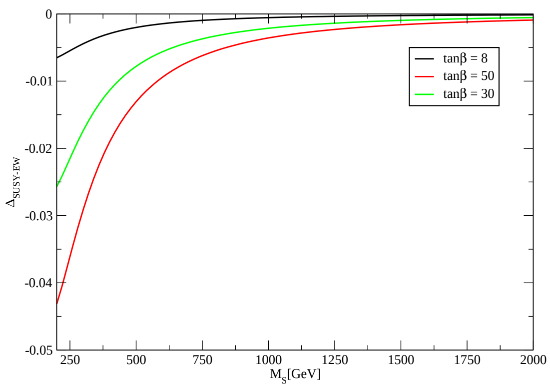

Figure 4. $\Delta_{\rm SUSYEW}$ as a function of M_{SUSY} for $M_{SUSY} = M_L = M_R = M_S = M_{\tilde{g}} = |\mu| = A_b = M_A$ and $\tan\beta = 8$, 30, 50.

couples, but this decoupling is delayed at large $\tan\beta$. For example, even at $M_{SUSY} = 1$ TeV, $|\Delta_{\rm SUSYEW}| \simeq 0.5\%$ for $\tan\beta \sim 30$. The corrections can be $|\Delta_{\rm SUSYEW}| \simeq 2\%$ for $\tan\beta \sim 30$ and $M_{SUSY} = 250$ GeV.

4. Conclusions

In this paper we have studied the $\mathcal{O}(m_t^2)$ one loop Yukawa corrections to the $h^0b\bar{b}$ coupling, coming from diagrams involving the exchange of virtual stops and charginos, in the limit of large SUSY masses. We have performed expansions for the SUSY mass parameters large compared to the electroweak scale. We demonstrate that in the limit of large M_A and large SUSY mass parameters, the corrections decouple like the inverse square of these mass parameters, and the SM expression for the $h^0b\bar{b}$ one-loop coupling is recovered. That is, the EW-Yukawa corrections to the $h^0b\bar{b}$ coupling decouple in the limit of large SUSY masses and large M_A. However, if the mass parameters are not all of the same size, then the decoupling behavior can be modified. If M_A is light, then the corrections to the $h^0b\bar{b}$ coupling generically do not decouple in the limit of large SUSY mass parameters.

The decoupling behavior of the radiative corrections to the $h^0b\bar{b}$ coupling implies that distinguishing h^0 from the SM Higgs boson will be very difficult if A^0 and the SUSY spectrum are heavy, even after one-loop SUSY corrections are taken into account. However, because of the enhancement at large $\tan\beta$, the onset of decoupling is delayed, and the corrections can still be at the percent level for $\tan\beta \sim 50$ and all SUSY mass parameters and M_A of order 1 TeV. Such effects could be detected in precision Higgs studies and provide a critical window to the TeV-scale.

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