PARTICLE AND NUCLEAR SCATTERING AT LARGE MOMENTUM TRANSFERS

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1. INTRODUCTION

The investigation of the reactions with large momentum transfers was found to be very fruitful in showing the quark structure of matter. These studies are now a wide area of activity, and the present lectures can by no means pretend to be any kind of a complete review*). I shall dwell mainly on those aspects of the reactions with large momentum transfers which were little discussed at the previous CERN-JINR School of Physics, as well as on some new results. Most of all, this concerns nuclear aspects, the so-called relativistic nuclear physics. As recent studies showed, the nuclear aspects of the quark theory of matter are very important for the construction of both the theory of strong interactions and modern nuclear theory. They were also found to be important for a number of experiments performed and being performed, in particular, for the joint JINR-CERN NA 4 experiment, which will be discussed below. I am sure that these aspects will noticeably affect further research programmes of a number of accelerator centres of the world. In particular, relativistic nuclear physics is of considerable importance in the JINR research programme.

The concept of the quark is nowadays the basic one in the theory of strong interactions. Initially we were led to this notion by composite models of hadrons, and from this point of view the hadrons are naturally thought of as systems that are analogous to the light nuclei. Subsequent extensive studies of hard collisions also led to the quark structure of hadrons. The parton model, which is an analogue of the impulse approximation of nuclear physics and postulates the existence of point-like constituents inside hadrons, enabled us to make sure that the quantum numbers of these constituents are the same as the quantum numbers of quarks considered in composite models of hadrons.

The quarks were found to be relatively weakly bound in hadrons. This is the most important fact for the following. Quantum chromodynamics (QCD) as an asymptotic free theory has succeeded in explaining this fundamental property of quarks and therefore being very promising has won general sympathy. The description of hard collisions in the framework of asymptotic free theories is close to the picture given by the parton structure of hadrons. The fact that the interaction between the QCD objects becomes stronger with increasing distance between these objects also gives hopes for obtaining confinement in this theory. The problem of confinement is still far from being solved. However, it is closely connected with interactions at large distances and quarks of low relative momenta. This makes it possible to factorize such effects and to consider the physics of large momentum transfers and small distances as a relatively independent subject.

The division of complicated motion into fast and slow parts -- the link between which is seen to be non-essential or weak -- is a fairly old recipe of classical physics. In

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*) A large background of information on the reactions with large p_T can be found in the lectures of Matveev at the 1979 JINR-CERN School of Physics and at the present School. This subject was also discussed at many other CERN Schools of Physics (see Ref. 1).

particular, it was proved to be very successful in the theory of accelerators. I imply the division of particle motion into radial-phase and betatron oscillations. The series expansion parameter which defines the applicability of this method is the ratio of characteristic frequences ω/Ω.

The applicability of the impulse approximation of nuclear physics is also defined by the ratio of the characteristic time of collision to the time of slow internuclear relative motion, or the ratio of the binding energy to the energy of a particle that interacts with the nucleus, ε/E.

In quantum field theory, especially in its applications to solid-state physics, elementary excitations are separated on the basis of the concept of quasi-particles. The range of applicability of this concept is also limited by the values of the energy-momentum transfers. As the energy-momentum transfers increase it is necessary to proceed to new quasi-particles.

As experiments on deep inelastic scattering of leptons show, for momentum transfers larger than 1 GeV the hadrons can on longer be assumed to be elementary particles, and it is necessary to take into account their quark structure. Hence, it follows that at such momentum transfers the nucleons as the quasi-particles of nuclear matter are no longer adequate to the problem. The nucleus should be viewed as a multiquark system, in which the interaction is mediated by the gluon field in just the same way as the electromagnetic field bonds electrons and nuclei into atoms and molecules. However, this does not mean that the new theory "cancels" the canonical nuclear physics constructed on the basis of the proton-neutron nuclear model and the non-relativistic quantum mechanics. In non-relativistic nuclear physics, the new fundamental theory must explain the phenomenological characteristics of nuclear forces and other parameters by expressing them in terms of the fundamental constants in just the same way as the Van der Waal forces are described by electrodynamics. However, at large momentum transfers p determined by the condition

$$\frac{m^2}{p^2} \ll 1 \ , \qquad (1)$$

where m is the characteristic mass, canonical nuclear physics becomes invalid and QCD must play a decisive part.

The criterion (1) makes it possible to divide the process into a slow motion part with characteristic time of the order of $\tau \sim 1/m$ and a fast motion one with $T \sim 1/p$. The independence of these two regimes is the fundamental property expressed as factorization. The inclusive cross-section is thus expressed as a product of the probability of finding a constituent (quark or gluon) in the hadron (or the nucleus) by the elementary interaction cross-section for the constituents. As yet there has been no success in describing slow motions on the basis of QCD. For the time being, QCD only pretends to the role of the theory of fast motions that can be described in the framework of perturbation theory.

The algorithm of such calculations (the Feynman rules) can be found, for example, in the lectures by Matveev[1d]. The main hope is placed on the small "running coupling constant" α_s, which decreases logarithmically with increasing squared four-momentum transfer

$$\alpha_s = \frac{12\pi}{(33 - 2n_f) \log Q^2/\Lambda^2} \ .$$

Here n_f if the number of quark flavours and Λ is the scale parameter of some hundreds of MeV which is important in all applications of QCD. According to Eq. (1), it defines the order of magnitude of the characteristic time of slow motion.

Unfortunately, the success achieved in quantitative calculations of hard processes on the basis of QCD is still very modest, even in the description of deep inelastic scattering of leptons. The higher order perturbative corrections in the whole Q^2 region under investigation are found to be large and cannot be reliably calculated. Nevertheless, there are serious grounds to hope that it is quite possible to make important qualitative QCD predictions for hard processes. This just explains the great efforts presently being made by experimenters in studying quantitative laws of hard processes.

What kind of new information is extracted from nuclear interactions? At large momentum transfers [criterion (1)] the interaction of particles and nuclei with nuclei should be considered at the quark level. When $p^2 \geq 1$ GeV^2 the interaction proceeds in a local manner, that is with one quark. This is commonly admitted in the literature. However, in the analysis of experimental data, the nucleons are often thought of as autonomous systems inside the nucleus and the interaction proceeds with the quark belonging to one of the nucleons of the nucleus. As will be seen from the foregoing, this assumption, that is widely used for example when studying deep inelastic scattering of leptons on nuclei, is, generally speaking, wrong.

The study of high-energy nuclear interactions makes it possible to obtain information about multiquark states and possibly about a new state of nuclear matter, the quark-gluon plasma.

As was mentioned above, the processes induced by the quark interactions at large distances are slow, $\tau \sim 1/m$. These times are expected to determine also the process of transformation of a knocked-out quark into hadrons. With due account of relativistic time dilation (Lorentz factor E/m), we have $\tau' \sim 1/m \cdot E/m$. The time τ' for fragmentation of a quark into hadrons is so large that for hard collisions the appropriate formation length exceeds the nucleus size. In other words, in nuclear collisions, particles with large momentum values are formed outside the nucleus. In this sense the nucleus can be regarded as a tool for the registration of free quarks. It is also necessary to take into account the fact that the quark-quark interaction cross-section at large momentum transfers is small, and double and triple collisions are therefore unlikely. Only one quark of the incident hadron is involved in the interaction. This means that the spectator quark (or quarks) of the high-energy incident hadron passes through the nucleus without interacting.

The above qualitative arguments show that nuclear reactions with large momentum transfers yield essentially new information about the quark nature of nuclear matter.

Section 2 of the present lectures is devoted to the introduction of the basic concepts and variables of relativistic nuclear physics. Section 3 contains the most essential experimental data. Section 4 is a brief discussion of perspectives.

2. THE BASIC CONCEPTS AND QUANTITIES

Back in 1970, when we started studying collisions of relativistic nuclei[2,3], many people thought that nothing would come of it since the structure of the collisions would be complicated and would give little information. However, these collisions were not found to be more complicated than those of elementary hadrons. Moreover, the properties of nuclear collisions were found to be similar to the properties of proton collisions but for higher energies. Figure 1 shows the event where a carbon nucleus of an energy 50 GeV collides with a carbon nucleus in propane in the JINR propane bubble chamber, with a subsequent collision of the fragment nucleus with a carbon nucleus. The angle between the momenta of the bombarding nucleus and the fragment nucleus is very small.

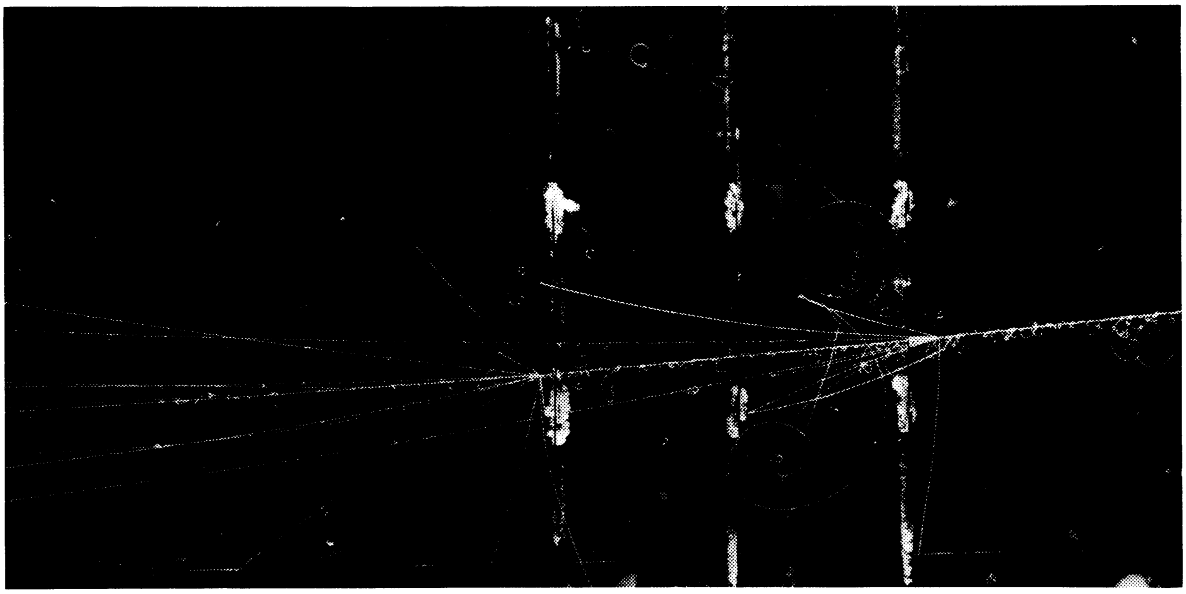

Fig. 1

In the collision of relativistic nuclei with small momentum transfers, we are dealing with ordinary nuclear physics considered in a fast-moving coordinate system. The basic parameter that defines this type of collision is the nucleon binding energy. The characteristic features of these reactions are given by *the pole approximation.*

The amplitude of the process

$$I + II \to 1 + X \tag{2}$$

has the form

$$T_{fi} = \frac{1}{2} \sum_j \frac{T_{fj} T_{ji}}{(p_I - p_1)^2 - m_2^2} , \tag{3}$$

where p_I and p_1 are the four-momenta of particles I and 1, and m_2 is the mass of a particle exchanged in the t-channel.

To this amplitude there corresponds the cross-section

$$\frac{d\sigma}{db_{I1}} = \frac{F_1}{(\alpha + b_{I1})^2} . \tag{4}$$

Here we introduce the variable

$$b_{I1} = 2 \left[\frac{p_I \cdot p_1}{m_I m_1} - 1 \right]$$

and the parameter

$$\alpha = \frac{1}{m_I m_1} (m_1 + m_2 - m_I)(m_I + m_2 - m_1) \ .$$

The factor F_1 is weakly dependent on the invariant variable b_{I1}.

The simple one-pole formula (4) describes a great deal of experimental data on nuclear fragment production at relativistic energies. The quantity $\alpha \approx 2\varepsilon |m_I - m_1| / m_I \cdot m_1$ determines the width of the very narrow energy and angle distribution of the secondary particles.

In rapidity space $y_1 = \frac{1}{2} \ln (E_1 + p_{1z})/(E_1 - p_{1z})$ the cross-section (4) describes a narrow forward peak with half-width

$$\Delta y \approx \sqrt{2\varepsilon \frac{|m_I - m_1|}{m_I m_1}} \approx 0.1 \ .$$

Here ε is the binding energy. One has $\varepsilon = m_I - m_1 - m_2$ in the stripping reaction and $\varepsilon = m_1 - m_I - m_2$ in the pick-up reaction. According to Eq. (4), the longitudinal momentum distribution has sharp peaks for

$$p_{1z} = m_1 \frac{p_I}{m_1} = m_1 \cdot \text{const.} \ .$$

That is when the momentum per nucleon of the fragment is approximately equal to the momentum per nucleon of the incident nucleus.

The distribution width can be written as

$$\alpha m_1^2 = 2\varepsilon \frac{(B - F)F}{B} \ ,$$

where $B = m_I$ and $F = m_1$. This formula is known in the literature as the *"parabolic law"*.

Figure 2 shows the longitudinal momentum distribution for the isotopes of carbon obtained in the reaction $^{16}O + Be \rightarrow {}^{A}C + X$ with an oxygen nucleus energy of 2.1 GeV/nucleon. These data were obtained at the Bevatron (Berkeley, USA), where -- just as at Dubna -- studies with relativistic nuclei have begun. The range of small Δy and small momentum transfers yields no essentially new information even at the highest energies. Collisions of such a type are well described in the framework of ordinary nuclear physics, where the nucleon is a good quasi-particle.

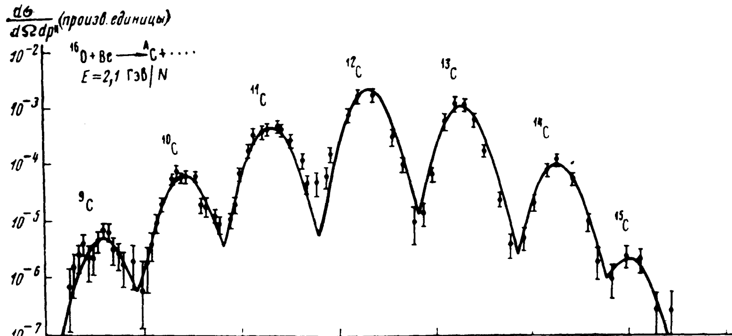

Fig. 2

Relativistic nucleus collision processes are described in the same way as usual multiple particle production processes in hadron physics, namely on the basis of average characteristics; for instance, average multiplicity, one- and two-particle distributions, correlation functions, etc.

The study of the multiple processes in nucleus-nucleus collisions has led our physicists to the conclusion that the basic characteristics of these processes can be described as resulting from a superposition of nucleon-nucleon collisions. As it is said, *the additive model works*. This is not difficult to understand on the basis of the above-mentioned ideas. In multiple production processes the average $\langle p_T \rangle$ value is about 300 MeV and is independent of the colliding energy. Thus, the validity of the additive model results from small momentum transfers in processes which give the decisive contribution to the total cross-section. The total inelastic interaction cross-sections for relativistic nuclei are, to a good approximation, described by the naive *geometric model* (see Fig. 3, where A_i and A_t are the atomic masses of the projectile and the target, respectively, and $A^{1/3}$ is proportional to the radius of the nucleus).

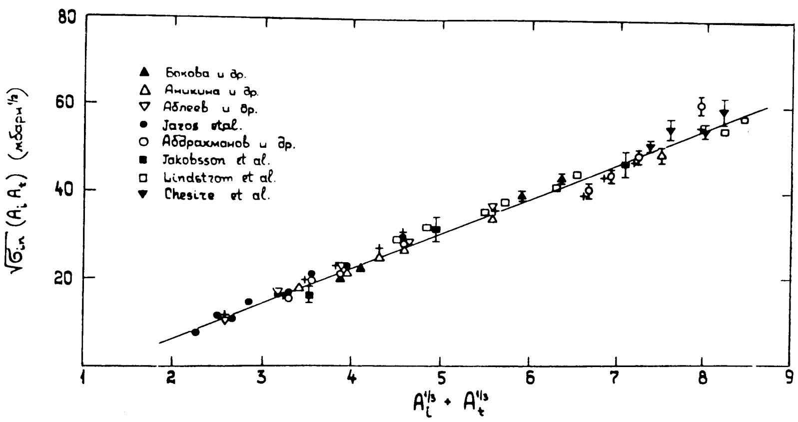

Fig. 3

Of great interest are the reactions with large momentum transfers. In spite of the fact that these reactions have small cross-sections, they definitely answer questions about the quark structure of nuclei. The study of these reactions at the Laboratory of High Energies of JINR has resulted in discovering one of the most interesting phenomenon in high-energy physics, the cumulative effect, which will be discussed below. Certain characteristics of the cumulative effect can also be obtained by means of track devices. However, their advantage of giving total information about nucleus-nucleus collisions turns, in this case, into a shortcoming. The cumulative effect can be studied well even with very little information extracted by means of track devices.

In the analysis of experimental data in hadron physics, invariant inclusive cross-sections

$$\rho_1 = \frac{E}{\sigma_{in}} \frac{d\sigma}{d\vec{p}} , \quad \rho_2 = \frac{E_1 E_2}{\sigma_{in}} \frac{d\sigma}{d\vec{p}_1 d\vec{p}_2} , \quad \dots , \tag{5}$$

which correspond to fixing the final state of one-, two-, ... particles

$$I + II \to 1 + X$$
$$I + II \to 1 + 2 + X\ , \tag{6}$$

are used as measurable quantities. Here ρ_1, ρ_2, ... depend on the relativistic invariants and, in particular, on $s = (p_I + p_{II})^2$; σ_{in} is the total inelastic cross-section for the collision of systems I and II.

The study of the limiting fragmentation suggested by Yang et al. was found to be very useful. This idea is very simply expressed in terms of the so-called short-range correlations in the rapidity space

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \text{arg sinh} \frac{p_z}{\mu}\ , \tag{7}$$

where

$$\mu_i = \sqrt{\vec{r}_i^{\,2} + m_i^2} \quad \text{and} \quad \vec{r} \equiv \vec{p}_T\ .$$

Owing to relativistic invariance, the invariant parameters ρ_1, ρ_2, ... depend on rapidity differences (the $\vec{r}^{\,2}$ dependence is, for the moment, omitted). This reads:

$$\rho_1(y_1 - y_I,\ y_I - y_{II})$$
$$\rho_2(y_1 - y_I,\ y_1 - y_2,\ y_2 - y_{II})\ . \tag{8}$$

For definiteness we assume rapidities to be ordered as

$$y_{II} < y_2 < y_I < y_1\ . \tag{9}$$

The short-range nature of correlations are determined from the following conditions:

i) If the rapidity difference is much larger than the characteristic correlation length $L \approx 2$, then ρ_m is independent of this variable. In particular, the independence of ρ of $(y_I - y_{II})$ can be considered as a definition of *limiting fragmentation.*

ii) If $y_1 - y_2 > L$, then ρ_2 is factorized as the product of one-particle distributions

$$\rho_2(y_1 - y_I,\ y_1 - y_2,\ y_2 - y_{II}) \simeq \rho_1(y_1 - y_I)\cdot\rho_1(y_2 - y_{II})\ . \tag{10}$$

These conditions are generalized, in a trivial manner, to any ρ_m, where $m > 3$.

A priori there is no special reason to believe that there exists a universal correlation length L which is valid for all types of high-energy reactions. Moreover, strictly speaking, the correlations should not be only of a short-range nature, let alone because of the restrictions imposed by energy-momentum conservation. Nevertheless, the short-range correlation model describes well many characteristics of multiple particle production and may be viewed as an approximate universal property of hadron interactions.

Nuclear collisions should obey the laws discovered in hadron physics and, in particular, the use of short-range correlations is found to be especially efficient in relativistic nuclear physics.

This model enables us to predict the region of approximate validity of limiting fragmentation. Recall

$$(p_I\cdot p_{II}) = m_I m_{II} \text{ch}(y_I - y_{II}) \approx m_I m_{II} \frac{1}{2} \exp |y_I - y_{II}|\ .$$

The case $|y_I - y_{II}| \gtrsim 2$ corresponds to this approximate boundary, or

$$(p_I \cdot p_{II}) = E_I m_{II} \approx m_I m_{II} \frac{1}{2} \exp |y_I - y_{II}| \gtrsim m_I m_{II} \frac{1}{2} \exp 2 ,$$

or $2(E_I/m_I) \gtrsim e^2 \approx 7.4$, i.e. at an energy E_I of about 4 GeV/nucleon.

If the rapidity difference $(y_I - y_{II})$ obeys the condition

$$(y_I - y_{II}) > L \approx 2 , \tag{11}$$

then the cross-section is factorized. What occurs near the left boundary (y_{II}) does not affect the vicinity of the right boundary (y_I), and vice versa. In particular, it then follows that for the study of the limiting fragmentation of heavy nuclei there is no necessity to accelerate them. It is enough to study the production of particles with large momentum transfers off heavy nuclei under the action of any particles of sufficiently high energies so that the condition (11) should be fulfilled. The cross-sections obtained in such a way can be transformed into a coordinate frame, where the heavy nucleus is moving, and secondary beams, which will be obtained, for example, from accelerated uranium, can be predicted.

It is just this approach that has been used to study the main regularities of the limiting fragmentation of nuclei at Dubna, starting in 1971.

We focus our attention on the one-particle distributions in the region of limiting fragmentation of nuclei which is kinematically forbidden for one-nucleon collisions. In the region $\exp |y_I - y_{II}| >> 1$, from energy-momentum conservation it follows that

$$\mu_1 \exp (y_1 - y_I) \lesssim m_I . \tag{12}$$

We determine the kinematic limits with the aid of the cumulative number N, i.e. the effective number of the nucleons of a fragmenting nucleus which are involved in the reaction. For the one-particle distributions the minimal N^{min} is determined by the kinematic limits imposed on the mass of the object participating in the collision

$$I + II \to 1 + X .$$

When $\exp |y_I - y_{II}| >> 1$ in the region of limiting fragmentation of nucleus I, the relativistic invariant quantity N^{min} assumes, according to Eq. (12), the following values:

$$N^{min} = \frac{\mu_1 \exp (y_1 - y_I)}{m_p} = \begin{cases} \dfrac{E_1 - p_{1z}}{m_p} & \text{in the rest system of nucleus I} \\ \dfrac{E_1 + p_{1z}}{E_I^0 + p_I^0} & \text{in the rest system of particle or nucleus II ,} \end{cases} \tag{13}$$

where p_I^0 is the momentum per nucleon, m_p the proton mass, y the rapidities, and p_{1z} the longitudinal momentum. *The cumulative effect* corresponds to the region defined as $N^{min} > 1$.

The cumulative effect was predicted[2)] on the basis of the following assumptions. In the spirit of the parton models, the one-particle distribution ρ_1 in the region of the limiting fragmentation of nucleus I is taken in the form of a superposition of the one-particle distributions which are due to the limiting fragmentation of the objects of mass $N \cdot m_p$ inside nucleus I

$$\rho_I^{II}(y_1 - y_I, \vec{r}_1) = \sum_N P_N \rho_N(y_1 - y_I, \vec{r}) . \tag{14}$$

Without further assumptions on the probability P_N of finding a constituent with mass Nm_p inside the nucleus and on an explicit form of ρ_N, the following properties of the cumulative effect can be obtained:

i) The dependences of ρ_I^{II} on the collision energy $(p_I \cdot p_{II})$ and the properties of the target (particle II) must be almost absent due to limiting fragmentation.

ii) We introduce in Eq. (14), instead of the rapidity difference, the quantity N^{min} according to Eq. (13), and then rewrite Eq. (14) in the form

$$\rho_I^{II}(N^{min}, \vec{r}) = \sum_N P_N \rho_N(N^{min}, \vec{r}) . \tag{15}$$

According to the definition of ρ_N and Eq. (13):

$$\rho_N(N^{min}, \vec{r}_1) = 0 \quad \text{for } N < N^{min} .$$

It is then clear that N^{min} defines the lower limit of summation (or integration if N is continuous). It is unlikely that many nucleons can get together in a cumulative effect. Consequently, P_N is a sharply decreasing function of N and it may be supposed that

$$\rho_I^{II}(N^{min}, \vec{r}_1) \approx P_{N^{min}} \cdot \rho_{N^{min}}(N^{min}, \vec{r}_1) .$$

Thus, according to our model, the main quantity which describes the cumulative effect ρ_I^{II} can be approximated by a fast decreasing function, e.g. the exponential

$$\rho = C \exp\left[-aN^{min}\right] , \tag{16}$$

where a and C are practically independent of the properties of particle II in the region of the limiting fragmentation of nucleus I. Equation (16) describes well the experimentally observed universal distributions of the cumulative particles over the variable N^{min}.

iii) Simple geometric considerations have led us (see, for example Refs. 2 and 3), to the following dependence of the coefficient C in Eq. (16) on the fragmenting nucleus atomic weight:

$$\sigma_{in}\rho \propto A^m \quad \text{where} \quad m = \frac{2}{3} + \frac{N^{min}}{3} . \tag{17}$$

Dependences of such a type were found to be rather non-trivial so that a number of experimental studies have been devoted to them. For large N^{min} the exponent m in the dependence A^m is larger than unity[3]. The so-called *anomalous A dependence* of the cross-sections for nuclear reactions with large momentum transfers is one of the most important signals which indicates that we are dealing with new and interesting physics (see below).

The study of particle production with large p_T on nuclei is directly related to the limiting fragmentation problem. In the experiments of the Cronin team[4], a strong A dependence, similar to those found in the cumulative effect studies, was discovered. Recently, a similar A dependence was detected[5] in hadron jet production off nuclei using $\pi^{\pm}$ and p beams of high energies. If it is parametrized as A^m, then m (with π incident) $\simeq 1.3$ and m (with p incident) $\simeq 1.45$ and increases linearly with increasing p_T. The large values of such exponents are a strong evidence for new dynamics.

The nuclear reactions with large momentum transfers (of the order of or larger than the nucleon mass) require a consistent relativistic approach and correspond to relative internucleon distances of the order of or smaller than the confinement radius where the quark degrees of freedom must be predominant. As a matter of fact, we are dealing here with the problems of hadron physics and quantum field theory. Correspondingly, the methods and approaches in these investigations, both theoretical and experimental, are essentially an adaptation and development of the high-energy physics methods. These investigations go beyond the framework of the canonical non-relativistic nuclear theory.

The relativistic description of multiparticle states encounters the following difficulties: i) we have to deal with a variable number of particles and, consequently, with an infinite number of degrees of freedom; ii) a formalism using different times is needed; iii) it is impossible to separate the contributions of particles and antiparticles in a relativistic invariant manner, in order to separate the internal motion from the motion of the composite system as a whole.

The description of states in *the Fock space* is the one most adequate to relativistic nuclear physics since it can be used to define states with a variable number of particles and, at the same time, allows an interpretation similar to that of the wave functions in non-relativistic theory.

The Fock column defined on the hyperplane t = 0 ("equal time") in the coordinate space is

$$\Phi = \left\{ \begin{array}{l} \Psi_1(x_1) \\ \Psi_2(x_1, x_2) \\ \dots \\ \Psi_n(x_1, \dots, x_n) \\ \dots \end{array} \right\} . \qquad (18)$$

The squared functions $\Psi_n(x_1, \dots, x_n)$ have the meaning of probability densities for n particles to be found in the system. It is not difficult to show (see, for example, Ref. 6) that in the non-relativistic case, when the Hamiltonian of the system commutes with the particle number operator, the Fock space disintegrates into subspaces. Each subspace has then its Schrödinger equation for an appropriate number of particles. While in the relativistic case, when particle production and annihilation can occur, neither the Hamiltonian and the momentum operators nor the Lorentz transformation operators commute with the particle number operator. This means that in a Lorentz transformation the lines of the Fock column get mixed up and, in different coordinate frames, the composition of a moving object, for example a nucleus, will be different.

The number of particles in a system depends on the momentum with which it moves. In this connection, of particular importance is the concept of a coordinate *frame moving with infinitely large momentum*[7,8] *(infinite momentum frame -- IMF)*. For a wide class of theories, a composite object in this frame becomes a set of almost non-interacting constituents and the consideration is completely analogous to that in the non-relativistic case. This idea underlies the parton models[9], which have successfully been applied to the collisions of "elementary" hadrons.

This approach is analogous to the impulse approximation in nuclear physics: owing to relativistic time dilation the characteristic times of the internal dynamics of the system are found to be much larger than the collision times. The collision cross-section for a composite system is expressed in terms of that for free constituents, the partons. Compared to the elementary particle, the nucleus in the relativistic energy region can successfully be thought of[2] as a parton gas, since the lifetime of the virtual nuclear state seen as an assembly of free nucleons is much longer than the lifetime of the nucleon seen as an assembly of partons. Thus, the methods developed in the quark-parton models provide us with a basis for considering relativistic collisions involving nuclei and enable us to overcome the above-mentioned troubles in the relativistic description of many-particle states.

The time development of the system is defined by the total energy which for a system of free particles is determined as

$$E = \sum_{i=1}^{n} \sqrt{\vec{p}_i^{\,2} + m_i^2} \ .$$

Let the motion along the axis z satisfy the basic criterion (1); then

$$E = \sum_{i=1}^{n} \sqrt{p_{iz}^2 + r_i^2 + m_i^2} \approx P_z + \sum_{i=1}^{n} \frac{r_i^2 + m_i^2}{2p_{iz}} \ , \tag{19}$$

and here and in what follows P is the total momentum of the system, and $\vec{r}_i^{\,2} = p_{iT}^2 = p_{ix}^2 + p_{iy}^2$. It is seen that in a coordinate system, where $P_z \to \infty$ (IMF)[8], it is possible to divide the motion of the system into the motion as a whole and the internal motion.

It should be stressed that our approach is based on the following hypothesis: there exists a P_z such that all the internal and transverse momenta are much smaller than this quantity. One often forgets this basic hypothesis when one tries to account for specific effects of relativistic nuclear physics on the basis of the Fermi motion of nucleons and the high-momentum components of nuclear wave functions. It is convenient to work in the IMF with the light cone coordinates which are linked with the ordinary coordinates in the following manner:

$$\tau = \frac{1}{\sqrt{2}} (t' + z'), \quad x = x', \quad y = y', \quad \zeta = \frac{1}{\sqrt{2}} (t' - z') \ .$$

The energy-momentum variables conjugate to the latter are obviously found from

$$p_\mu x^\mu = H\tau + \eta\zeta + p_x x + p_y y \ ,$$

and it follows that

$$H = \frac{1}{\sqrt{2}} (E - p_z), \quad p_x = p_x', \quad p_y = p_y', \quad \eta = \frac{1}{\sqrt{2}} (E + p_z) \ . \tag{20}$$

It is convenient to write the transformation from the lab. system to the IMF in terms of the hyperbolic angle ω between the time axes of these systems

$$\begin{aligned} p_z &= p'_z \cosh\omega + E' \sinh\omega \\ E &= p'_z \sinh\omega + E' \cosh\omega \\ \vec{r} &= \vec{r}' \ . \end{aligned}$$

The case we are considering corresponds to $\cosh\omega \to \sinh\omega \to \frac{1}{2}e^{\omega}$. In this case the transformation along the z axis assumes the form

$$\eta \to e^{\omega'}\eta, \quad \vec{r} \to \vec{r} \ , \tag{21}$$

and the rotations around the x and y axes the form

$$\begin{aligned} \vec{r} &\to \vec{r} + \vec{V}\eta \\ \eta &\to \eta \ . \end{aligned} \tag{22}$$

Equation (22) is analogous to the Galilean transformation, provided that η is the analogue of the mass and $\vec{V}$ that of the relative motion velocity. This analogy becomes still more complete if we recall the expression for the energy in the IMF

$$H = \sum_{i=1}^{n} \frac{r_i^2 + m_i^2}{2\eta_i} \ . \tag{23}$$

The introduced notation and concepts make it possible to introduce *the wave function of the multiparticle state in the IMF:*

$$\Psi_{\eta p_T}(\eta_1,\vec{r}_1, \ \ldots, \ \eta_n,\vec{r}_n) \ .$$

Invariance under the transformations (21) and (22) requires that all dependences on η_i and $\vec{r}_i$ occur through the variables $\beta_i = \eta_i/\eta$ and the variables $\vec{R}_i = \vec{r}_i - (\eta_i/\eta)p_T$. The wave function assumes then the form

$$\Psi_n = \Psi_n(\beta_1, \ \ldots, \ \beta_n, \ \vec{R}_i\vec{R}_j, \ \ldots) \ . \tag{24}$$

$\beta_i = (E_i + p_{iz})/(E + P_z)$ is the fraction of the momentum which is carried by a subsystem (parton, nucleon, quark). The normalization of these functions is usually taken as

$$\langle \eta,\vec{r}|\eta',\vec{r}'\rangle = \eta\delta(\eta - \eta')\cdot\delta^2(\vec{r} - \vec{r}') \ . \tag{25}$$

The integration is performed over the invariant measure $(d\eta/\eta)d^2r$. In the introduced notation the normalization has the form

$$\sum_n \int \frac{d\beta_1 \ \ldots \ d\beta_n}{\beta_1 \ \ldots \ \beta_n} d^2r_1 \ \ldots \ d^2r_n \Psi_n^*(\beta_1, \ \ldots, \ \beta_n, \ \vec{R}_1, \ \ldots, \ \vec{R}_n)\Psi_n(\beta_1, \ \ldots, \ \beta_n, \ \vec{R}_1, \ \ldots, \ \vec{R}_n) = 1 \ . \tag{26}$$

The above-formulated hypothesis about the finiteness of $r_i = p_{iT}$ and, in general, of the momenta of the internal motion has led us to the fact that the wave function depends on the ratio of the momenta β alone. Thus, this implies the scale invariance of the matrix elements.

The Fock column which is composed of the functions (24) is a wave function of the parton model[9]. Owing to the fact that the interaction Hamiltonian vanishes at $P_z \to \infty$, as we have postulated, this function describes a mixture of practically non-interacting particle-partons. The parton model is a natural relativistic generalization of the impulse approximation.

In particular, it is not difficult to show (see Ref. 8) that the matrix element of the bilinear scalar density between two states of a hadron composed of two constituents is proportional to the integral

$$\int \Psi^*(\beta_1, R_1^2)\Psi[\beta_1, (\vec{R}_1 + \beta_1\vec{Q})^2] \frac{d\beta_1}{\beta_1(1 - \beta_1)^2} d^2R \ , \tag{27}$$

where $\vec{Q} = \vec{r} - \vec{r}'$ is the transverse momentum of an external effect. This formula has a simple non-relativistic analogue in the impulse approximation. In the initial state we have a hadron in a frame with zero transverse momentum; $\Psi(\beta_1, R_1^2)$ is the amplitude for a two-parton state with parton 1 having momentum $(\beta_1, \vec{R}_1)$, and parton 2 having momentum $[(1 - \beta_1), -\vec{R}_1]$. Then a transverse momentum Q is deposited on parton 2 to bring its momentum to $[(1 - \beta_1), -\vec{R}_1 + \vec{Q}]$. The hadron has a centre-of-mass velocity in the transverse plane given by Q. Thus, to project the state onto the final hadronic state we must transform the wave function of the final hadron by a transformation of types (21) and (22) to a frame in which it moves with velocity Q. This takes each transverse momentum and translates it by an amount βQ, so that the argument of the final-state function is $(\vec{R}_1 + \beta_1\vec{Q})^2$.

This method gives only a recipe for overcoming the troubles of reaching a relativistic description but does not answer the question as to how to construct wave functions of type (24).

Let us consider the pole-approximation amplitude (3) in terms of the light-cone variables. According to (20),

$$\eta H = \frac{1}{2}(E + p_z)(E - p_z) = \frac{1}{2}(m^2 + r^2)$$

and

$$H = \frac{m^2 + r^2}{2\eta} \ .$$

We find

$$p_I \cdot p_1 = E_I E_1 - p_I p_{1z} = \eta_I \frac{m_1^2 + r_1^2}{2\eta_1} + \eta_1 \frac{m_I^2}{2\eta_I} = \frac{m_1^2 + r_1^2}{2\beta} + \beta \frac{m_I^2}{2} \ ,$$

and for the denominator of the amplitude (3) we get

$$(p_I - p_1)^2 - m_2^2 = m_I^2(1 - \beta) + m_1^2 - m_2^2 - \frac{m_1^2 + r_1^2}{\beta} \ .$$

For the case when particle I is a deuteron, $m_I = m_d$, and particle 1 a nucleon, $m_1 = m_2 = m_p$, the denominator assumes the form

$$(m_d^2 - M^2)(1 - \beta) \ ,$$

where the following notation is introduced

$$M^2 = \frac{m_1^2 + r_1^2}{\beta(1 - \beta)} \ . \tag{28}$$

The mentioned transition to the non-relativistic impulse approximation on the basis of Eq. (7) enables us to use information about the non-relativistic wave functions in considering the interactions of high-energy particles with nuclei. The construction of the wave function comes in this case to a relativistic generalization of the known wave functions of relativistic nuclear physics. For example, in Ref. 10 use is made of the following relativistic generalization of the Hulthèn wave function for the analysis of the reaction $d + p \to p + p + n$:

$$\Phi_d = N(M^2 - C_1)^{-1}(M^2 - C_2)^{-1} ,$$

where M^2 is the same as in Eq. (28), and C_1 and C_2 are constants.

The authors of Ref. 11 consider that the applicability of a relativistic quantum mechanical description to, for example, the deuteron seen as a system of only two bodies, is based on the absence of noticeable inelasticities in the NN scattering phase shifts up to relative motion momenta $\lesssim$ 1 GeV/c. In particular, in the framework of this hypothesis, the one-particle distribution of fast protons in the process $h + A \to p + X$ is given by the formula[11])

$$\frac{1}{\sigma_{tot}} \cdot E \frac{d\sigma}{d\vec{p}} = A \cdot k \frac{\rho(M^2)}{1 - \beta} .$$

Here $\rho(M^2)$ is the probability density for a correlated nucleon pair. In the case of the deuteron it is normalized by the condition

$$\int \rho(M^2) \frac{d^2 r d\beta}{\beta(1 - \beta)} = 1 \quad \text{and} \quad \rho \propto |\Psi_d|^2 .$$

The quantity k takes a possible screening into account.

From the very beginning of the studies with relativistic nuclei it became evident that there must exist a type of nuclear collision for which it is necessary to switch from the quasi-particle-nucleons to the quasi-particle-quarks descriptions. In the case when the nucleons are taken to be partons we know the wave function for large relative distances and try to extrapolate it to small distances; while in the case of parton-quarks large relative distances cause great difficulties for the theory. Then it is necessary to choose the basis that leads to a clear-cut separation of the long- and short-distance physics. In particular, each observed (initial or final) hadron or nucleus can explicitly be expressed by a set of functions that describe the longitudinal (light-cone) momentum distributions of its constituents taken one at a time, two at a time, three at a time, etc.; the constituents are treated as on-shell and, therefore, necessarily moving collinearly with the relevant hadron. These multiconstituent longitudinal distribution functions are process independent. If we are interested in the interaction with one constituent then the functions (18), each line of which has the form (24) in IMF, can be used to construct the quantity

$$S\Phi^*\Phi = G(\beta_1, R_1^2) , \tag{29}$$

where S means summation and integration over all the variables, except over the variables belonging to particle 1. The quantity (29) means the probability (the expectation value) of finding in a hadron in the Φ state a constituent 1 (parton 1) with a momentum fraction β. This apparatus can be used to express the cross-section of any process in terms of the cross-section σ_b^f of interaction with parton b and the expectation value of the number of partons $G_{B/b}(\beta,R^2)$ by means of the following formula

$$\sigma_B^f(\beta,R^2) = \sum_b \int \frac{d\beta'}{\beta'} \sigma_b^f \left(\frac{\beta}{\beta'}\right) G_{B/b}(\beta',R^2) \ . \tag{30}$$

Here $G_{B/b}$ is the probability of finding a parton (quark) of type b in a hadron, e.g. nucleus, B with a momentum fraction β and any transverse momentum R. The summation is performed over all the quantum numbers of the parton (spin, colour, flavour).

For the lepton-hadron scattering

$$\ell + h \to \ell' + X$$

the cross-section of scattering of a lepton on a point-like parton can be written as $\sigma_b^f = \sigma_0^b \delta[1 - (\beta/\beta')]$. Then we find

$$\sigma_h(\beta,R^2) = \sum_b \sigma_0^b G_{h/b}(\beta,R^2) \ . \tag{31}$$

Thus the cross-section is expressed as a sum of the cross-sections for non-coherent scattering on all partons b which are, with a noticeable probability, present in hadron B. The electromagnetic interaction is susceptible only to the constituent charges. The weak interaction reacts upon quarks and antiquarks in a different manner owing to the fact that we have both neutrinos and antineutrinos. In experiments on deep inelastic lepton scattering a set of various projectiles can be used[1)] to obtain a detailed knowledge of the structure functions $G_{B/b}(\beta,R^2)$. Of much importance is the fact that the structure functions were found to be almost independent of R^2. This is just the famous scaling. Quantum chromodynamics not only contains the parton picture as a reasonable first approximation, but also allows the prediction of corrections violating Bjorken scaling. However the existing theory does not enable us to calculate explicitly the $G_{B/b}(\beta,R^2)$ functions. They are determined from experiment and can be used to express the cross-section for different processes on the basis of definite assumptions and information on the cross-sections for primary quark interactions.

We generalize these important results to nuclei, paying attention to the fact that the definition of the structure functions (29) contains summation and integration over an infinite number of variables. According to present concepts, hadrons consist of a small number of valence quarks (the second and third lines of the Fock column) and an infinite number of virtual "sea" quarks. In this sense the proton or meson, like the nuclei, are infinitely complicated objects. In this connection, it is natural to introduce *for the atomic nuclei the quark-parton structure functions* $G_{A/q}(\beta,R^2)$ as one of the basic concepts of relativistic nuclear physics.

Experiments on deep inelastic lepton-nucleus scattering at large β are very difficult and the most complete information on $G_{A/q}(\beta,R^2)$ was extracted from the studies of limiting fragmentation of nuclei. However, the limiting fragmentation of nuclei involves the quark hadronization process according to the quark-parton model. No-one has yet been able to calculate the probability of this process just as $G(\beta,R^2)$. To overcome this difficulty a hypothesis about soft hadronization is used, that is the β distribution of fast particles is supposed to differ little from that of the quarks from which they are formed. Qualitatively this hypothesis is formulated as an assumption that a quark whose virtuality is small $\sim \Lambda^2$ forms a hadron with probability $D(\gamma)$ which depends only on the hadron fraction γ of the energy of the parent quark. The short-range correlations in rapidity space for particles produced at large momentum transfers correspond to those in rapidity space for partons. Hence, it follows that a cumulative hadron can be produced only from a cumulative parton. This fundamental property was given grounds in quantum chromodynamics and is a reflection of the asymptotic freedom of quarks. Hence, it follows that the spectra (16) reflect the distribution of partons (quarks) in nuclei, in other words, $G_{B/b}(\beta)$.

As was often mentioned at the CERN Schools[1], the idea about factorization of slow and fast motion makes it possible to write the general expression for the cross-section of inclusive particle production in the collision of hadrons A and B (in our case, nuclei) in the form

$$E \frac{d\sigma}{d\vec{p}_C} = \sum_{a,b} \int d\alpha \int d\beta \; G_{A/a}(\alpha) G_{B/b}(\beta) D_{c/C}(\gamma) \frac{1}{\gamma} \frac{1}{\pi} \frac{d\sigma}{d\hat{t}} , \tag{32}$$

where $d\sigma/d\hat{t}$ is the differential cross-section for the parton interaction subprocess. This model must, in principle, describe the main laws of the cumulative processes. Specific features of nuclear interactions, for example the dependence of the cross-section on the atomic masses of colliding particles, are contained in the quark-parton functions of nuclei and the $D_{c/C}(\gamma)$ functions of fragmentation of parton c to cumulative particle C. We write down the general formula for the spin density matrix of the cumulative particle in this model[12]:

$$\rho_{\mu\mu'} E \frac{d\sigma}{d\vec{p}_C} = \int d\gamma \, d\Delta \; Q_{A/a}(\alpha) Q_{B/b}(\beta) \frac{1}{\pi} \rho_{\lambda\lambda'} \frac{d\sigma}{dt'} (s',t') D_{c/C}^{\lambda\mu\lambda'\mu'}(\gamma) , \tag{33}$$

where $Q(x) = xG(x)$; $G_{A/a}(x)$ and $G_{B/b}(x)$ are the number of partons of kinds a and b in hadrons A and B, respectively; the quantity $\rho_{\lambda\lambda'}(d\sigma/dt')(s',t')$ describes the process of scattering of point-like partons, $a + b \to c + d$; $D_{c/C}^{\lambda\mu\lambda'\mu'}$ is the matrix of fragmentation of parton c to hadron C with a momentum fraction γ:

$$\alpha = -\frac{x_1}{\gamma}(1 + e^{\Delta}), \quad \beta = -\frac{x_2}{\gamma}(1 + e^{-\Delta}), \quad x_1 = \frac{-u}{s}, \quad x_2 = -\frac{t}{s}$$

$$s' = \alpha\beta s, \quad t' = \frac{\alpha}{\gamma} t, \quad u' = \frac{\beta}{\gamma} u, \quad s = 2(P_A \cdot P_B)$$

$$t = -2(P_A \cdot P_C), \quad u = -2(P_B \cdot P_C) .$$

The limits of integration are as follows:

$$-\ln \frac{\gamma - x_1}{x_1} \le \Delta \le \frac{\gamma - x_2}{x_2} .$$

It is obvious that the polarization will not be zero only in the case when the amplitude of the process $a + b \to c + d$ has both an imaginary and a real part. The usual assumption about the possibility of describing the process of parton scattering in the Born approximation results in a purely real amplitude and zero polarization. In the quark-gluon model polarization arises from the interference of the one- and two-gluon exchange terms and is very sensitive to the properties of the model. The calculations for the case when A is a nucleus differ from those when A is a particle by the fact that $Q_{A/a}$ contains the A-dependent probability P_N of the type (17).

We draw the following qualitative conclusions[12] about the polarization properties:

a) polarization strongly depends on θ with a peak around 90°;

b) the product $k \cdot P_C$ weakly depends on the k value

$$x \approx \frac{E - p_z}{m_p} \approx k$$

(in our notation, $k = N^{min}$ is the cumulative number); P_C is the cumulative particle polarization;

c) for $E = s/2m_p \gtrsim$ 5-10 GeV polarization is energy independent;

d) polarization weakly depends on the beam and target.

The comparison of the measured polarization of Λ particles produced in a process with large p_T [12],

$$P + Be \to \Lambda + X \quad \text{at} \quad p_p = 300 \text{ GeV/c} ,$$

with that of cumulative Λ particles in processes $\pi + A \to \Lambda + X$ [13,14] and $n + A \to \Lambda + X$ [13] is in agreement with these conclusions. In the case of unpolarized particles (or particles with spin 0) Eq. (32) reduces to Eq. (15)

$$d\sigma = \sum_k P_k^A d\sigma_p \left(x_2, \frac{x_1}{k}\right)\left(1 - \frac{x_1}{k}\right)^{\sigma(k-1)} , \tag{34}$$

where $x_1 \approx N^{min}$; $d\sigma_p$ is the cross-section of a one-nucleon process, and the last factor is due to an excessive number of passive quarks (see also Ref. 15). We estimated P_N in Eq. (15) starting from the assumption of a uniform distribution of quarks over the nucleus. Essential deviations from the result of this estimation were obtained in P_N calculations[16] on the basis of the quark bag model.

The study of multiquark fluctuations in nuclei, the so-called fluctons, is of much importance from the point of view of the study of quark dynamics. One of the interesting predictions of Ref. 16 is that for $N^{min} > 4$ the cumulative effect probability must be negligible.

From Eq. (32) we know that the invariant cross-section for the C hadron production in the region of limiting fragmentation of hadron B is proportional to

$$E_C \frac{d\sigma}{d\vec{p}_C} \propto G_B(\beta^C, R_C^2) . \tag{35}$$

Thus, the measurement of the particle distributions in the region of limiting fragmentation of hadron B is the measurement of its quark-parton structure function. As applied to nuclei, this means that by measuring the yield of secondary particles in the region of limiting fragmentation of nuclei we obtain information about the quark-parton structure functions of nuclei.

According to Eq. (13), it is useful to introduce instead of β *the cumulative number* $N^{min} = \beta A = \beta^0$, where A is the fragmenting nucleus atomic mass. In the parton picture, the parton must be near its mass shell both before and after scattering. Neglecting parton virtualities and assuming that the nucleons in the nucleus are isolated systems, we would obtain N^{min} to be always smaller than unity. The detection of partons with $N^{min} > 1$ means that we have succeeded in knocking out a quark belonging to a nucleon group. Thus, studying the quark-parton structure functions in the cumulative region $\beta > 1/A$ we thereby study specific nuclear multiquark interactions which are due to quark collectivization (irreducibility of the quark-parton functions to one-nucleon ones). The cumulative effect is thus a non-trivial QCD phenomenon, which occurs in a domain where this theory can be formulated in a consistent way and verified.

3. SOME EXPERIMENTAL RESULTS

By now there is a large amount of information about relativistic nuclear physics (see Refs. 3, 11, 16-27). Below we give some experimental results which serve as an illustration and do not pretend to be complete.

Relativistic nuclei, just as many other objects of high-energy physics, were first observed in 1948 in cosmic rays. Bradt and Peters with their coworkers exposed nuclear emulsions at a height of 30 km. Detailed studies of this remarkable component of cosmic rays were performed by the Bristol group. The relativistic nuclei of cosmic radiation give very important information about its origin, isotope composition, and about the intergalactic medium which the nuclei passed through. But this information is still to be interpreted. It is quite obvious that passing through a medium nuclei break up and the part of the spectrum which is due to light nuclei is enriched at the expense of that for heavy nuclei. The relative nuclear fluxes of cosmic radiation (abundances) were found to be strongly (by orders of magnitude) different from the abundance of elements in the solar system. There is, as yet, no explanation for this fact.

It is very hard to study the interactions of relativistic nuclei in cosmic rays with all details mentioned in the previous sections because of low flux intensities and great difficulties in constructing effective detectors at a high altitude, e.g. on sputniks. The mechanisms of relativistic nucleus collisions will obviously be studied in detail by means of accelerators. This will then make it possible to understand many problems of astrophysics.

The general picture of nuclear collisions was obtained with the aid of track chambers, and bubble and streamer chambers. The possibility of observing and studying quantitatively individual nuclear collisions, measuring the energies and momenta of produced particles, is very important for the solution of many problems of relativistic nuclear physics.

Pictures such as the one shown in Fig. 1 make it possible to study short-lived states, i.e. the interaction of secondary particles and nuclei with matter. It is possible also to define the lifetime or the interaction cross-section by measuring the distance between the events. It is a kind of time development of events; a nuclear oscillograph is thus realized. The traditional methods of nuclear physics cannot give similar possibilities.

More than a million such pictures have been obtained by means of various track chambers at the Laboratory of High-Energy Physics of JINR. Hundreds of physicists from dozens of institutes of the JINR-member countries are engaged in treating them with the aim of obtaining information of the kind mentioned above. The experimental data show that the average number of the nucleons of a bombarding nucleus interacting with a target-nucleus $\langle \nu \rangle$ is rather large. For example, it reaches $\langle \nu \rangle = 6.00 \pm 0.60$ for the collision of a carbon nucleus with a tantalum nucleus. Tantalum plates were placed inside the working volume of the propane chamber. By the present time the general picture of nucleus-nucleus collisions at relativistic energies is to a large extent clarified [see review by Bartke[26]].

At the 19th International Conference on High Energy Physics (Tokyo), in my talk[19] devoted to multibaryon interactions at relativistic energies, arguments were given in favour of the following conclusions:

- The limiting fragmentation of nuclei is reached in nucleus-nucleus collisions at an energy above 3.5-4 GeV/nucleon.
- There exist universal regularities which describe the one-particle distributions in the cumulative region [Eq. (16)].
- Experiments show a large value of the cumulative particle polarization, strong A dependences of the cross-sections, and unusual dependences on the quantum numbers (flavours) and angles.
- The cumulative particle production is in an interesting accordance with the production of particles and hadron jets with large p_T on nuclei.

These results show that we deal with new phenomena, for the interpretation of which the quark degrees of freedom of the nucleus are important. At the same time they show the invalidity of the attempts to describe nuclear reactions with large momentum transfers by taking into account the nucleon degrees of freedom alone.

The experimental data summarized in the talk mentioned[19] allowed the following parametrization of the formula (16)

$$E \frac{d\sigma}{d\vec{p}} = \text{const.}\ A_{II}^{1/3} A_{I}^{m} \exp\left[-\alpha\beta^{0}\right] , \tag{36}$$

where A_I and A_{II} are the atomic weight of the fragmenting nucleus and the target-nucleus, respectively.

A large amount of experimental data is also described by the formula

$$E \frac{d\sigma}{d\vec{p}} = \text{const.}\ A_{II}^{1/3} A_{I}^{m} \exp\left[-\frac{T}{T_0}\right] , \tag{37}$$

where T is the kinetic energy of a cumulative particle. Parameters a and T_0 reach their limiting values at an energy higher than 3-4 GeV/n.

The exponent m was predicted to be β^0 dependent: $m \approx 2/3 + 1/3\ \beta^0$. For $\beta^0 > 1$, $m > 1$. Equation (36) describes practically all the set of experimental data on the cumulative effect.

The assertion that the cumulative effect is the main source of information on the quark distribution in nuclei was given further support over the past years (see Ref. 28)*).

*) The quark-parton structure function of nuclei has recently been discussed by Dar and his co-workers[18] on the basis of a modified collective tube model. They have succeeded in describing a large amount of various experimental material relative to high-energy particle-nucleus collisions by means of this simple model.

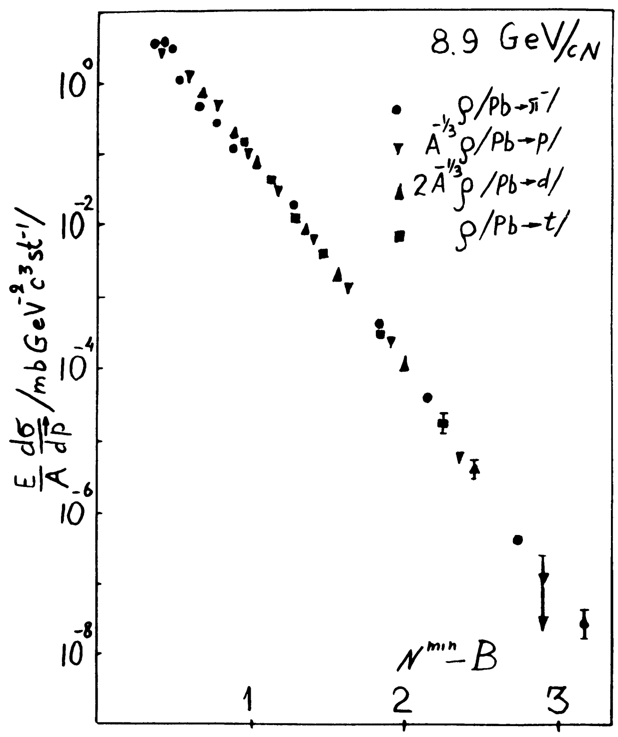

Fig. 4

The universal dependence of the type of Eq. (36) has been confirmed by subsequent experiments, among which the results of the Stavinsky group[29] are worth noting. In these experiments it was shown that the production cross-section for the cumulative particles, π, K, p, d, and t, can be described by the unique parameter a, that is the latter is independent of the quantum numbers of cumulative particles, and this appears to be the most important characteristic of quark distribution in nuclei (the functions G). These experimental results are given in Fig. 4. The processes of hadronization of quarks into "elementary" hadrons and nucleus fragments are shown to be similar.

The dependence of $E\, d\sigma/d\vec{p}$ on β^0 was retraced when $E\, d\sigma/d\vec{p}$ was changed by nine orders of magnitude.

The limiting fragmentation of the nuclei d, ^{4}He, ^{6}Li, ^{7}Li, ^{12}C, Al, Cu, ^{112}Sn, ^{124}Sn, ^{144}Sm, ^{154}Sm, ^{182}W, ^{186}W, Pb, and U was investigated. Figures 5 to 7 present the data on the cumulative meson production on nuclei in reactions $p + A \to \pi(180°)$ and $p + A \to K^+(\theta)$ for different proton momenta. The $E(d\sigma/d\vec{p})$ values, normalized to the atomic weight of the fragmenting nucleus, are given as a function of the pion kinetic energy T_π and N^{min}. The cumulative number $N^{min} = 1$ corresponds to $T_\pi = 270$ MeV and $N^{min} = 2$ to $T_\pi = 629$ MeV. An approximate equality of the yields $(E/A)(d\sigma/d\vec{p})$ for identical N^{min}, for π and K^+ mesons, is found to be a surprising and important result (Figs. 6 and 7). At the same time, $(E/A)(d\sigma/d\vec{p})$ for cumulative K^- mesons for identical N^{max} is smaller by about a factor of 30 than for K^+ mesons. This appears to be due to the fact that among valence quarks there is none that enters into the quark constitution $K^-(s\bar{u})$. Figures 5-8 illustrate well the universal character of the dependence of the invariant cross-section on N^{min}.

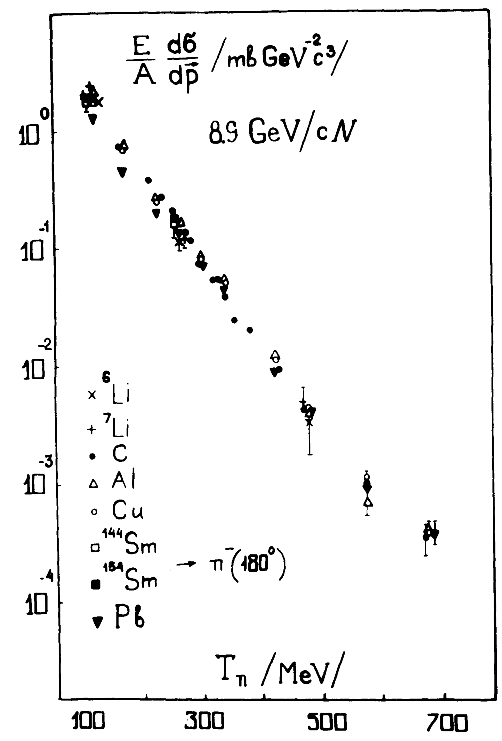

Fig. 5

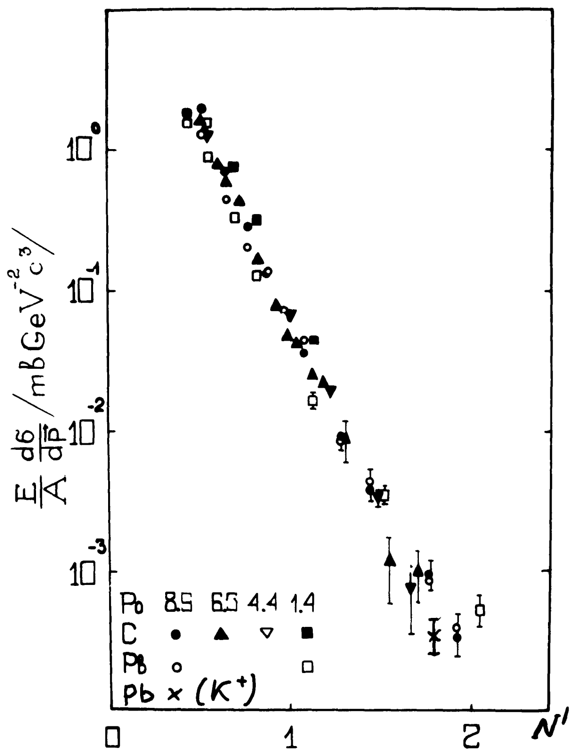

Fig. 6

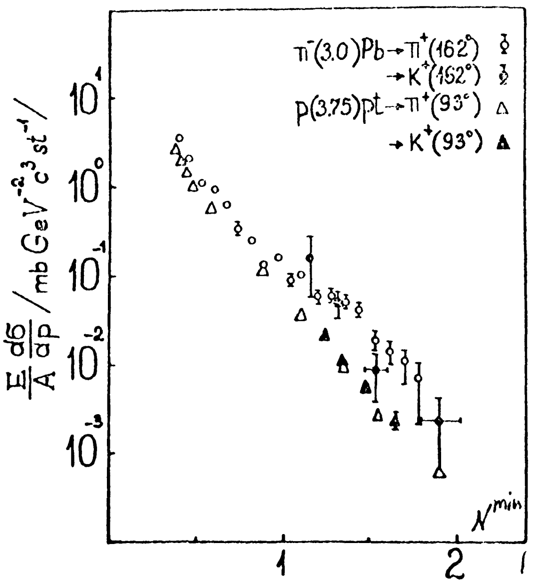

Fig. 7

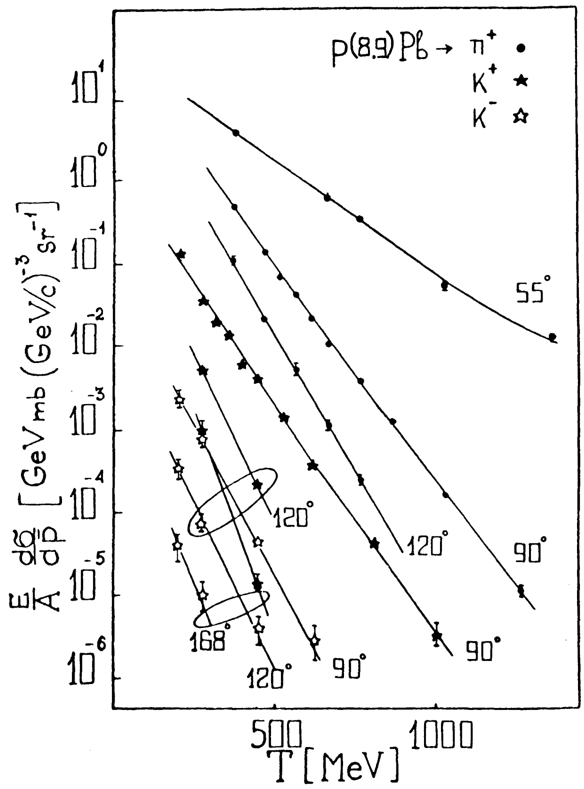

Fig. 8

An important confirmation that limiting fragmentation has been reached at an energy of 3.5 GeV/nucleon is the obtaining of data on the cumulative effect in the reactions $p + A \to 1 + X$ at an energy of 400 GeV [30]. These data yield the same universal parameter a that was obtained by us at an energy of 8.9 GeV.

Figure 9 from Ref. 30 shows the data on particle production in the cumulative region at an energy of 400 GeV; the Dubna data at 8.9 GeV energy are also plotted there. Figures 10 and 11 from Ref. 24 are the LBL (USA) data on cumulative π^- and π^+ meson production in proton-nucleus collisions. The authors used the parametrization (37). The data

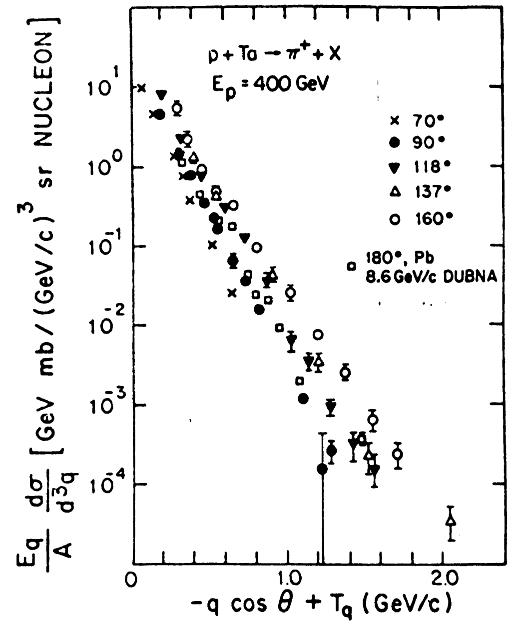

Fig. 9

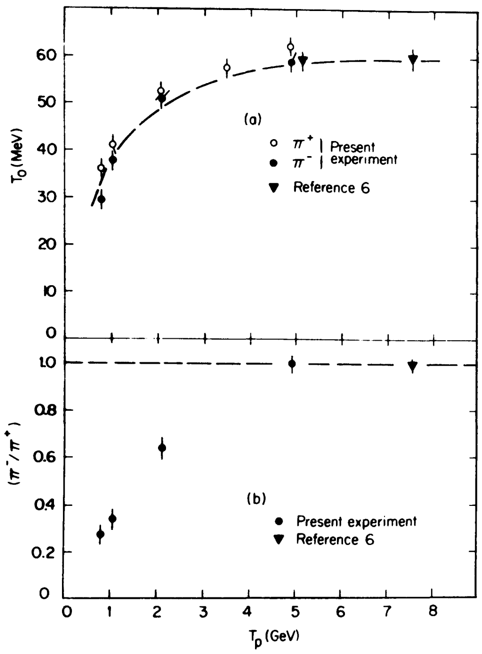

Fig. 10

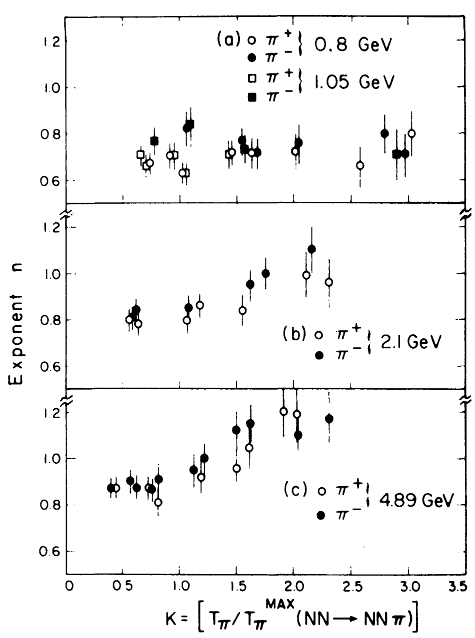

Fig. 11

marked as Ref. 6 are taken from the papers by the Dubna group. Figure 10 shows the energy dependence of the slope parameter T_0 and the ratio $R = \pi^-/\pi^+$ for a Cu target. Both are observed to rise with energy until about 3-4 GeV, after which a levelling off occurs. The trends observed in Fig. 10 are common to all targets. Above about 3-4 GeV, a limiting value is reached. Figure 11 shows the A dependence of the cumulative pion production cross-section according to parametrization A^n [Eq. (37)]. The K value is, to a good accuracy, equal to our cumulative number N^{min}. The variation observed in the A dependence between 0.8 and 4.89 GeV suggests that different mechanisms are responsible for pion production over this energy region with a smooth evolution from one to the other as the energy is increased.

The complicated and unusual A dependences of the cumulative production cross-sections, on the one hand, were confirmed and, on the other hand, exhibited new peculiarities. The most essential of them is the departure of the A dependence from the assumed asymptotic regime. Deviations from a simple dependence of the type A^m, where $m = 2/3 + 1/3\ \beta^0$, should be expected starting from the models suggested earlier. In fact, according to Ref. 19 for P_N we take the binomial distribution which, to a good accuracy, can be presented in the form

$$\frac{A!}{N!(A-N)!} q^N (1-q)^{A-N} \approx C_1 \frac{1}{N!} A^{N/3} \exp\left[-\left(\frac{r}{r_0}\right)^2 A^{1/3}\right] .$$

The exponential provides here a noticeable deflection from the simple exponential dependence A^m. However, using this formula we fail to describe the asymptotic regime discovered by the Stavinsky group (see Fig. 12).

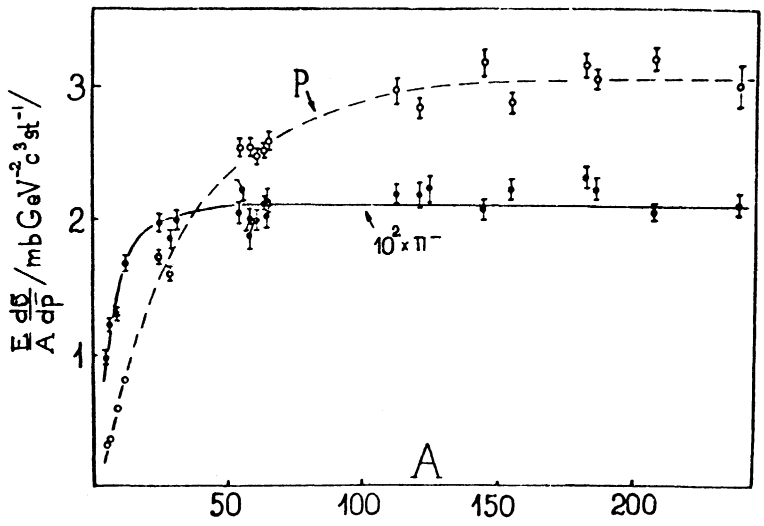

Fig. 12

The A dependence of the quark-parton function of nuclear fragmentation is still an unsolved problem for the theory. As is shown by the data on the cumulative effect and the production of particles with large p_T, the A and β dependences of G are not factorized. This is likely to indicate that the role of the effects of quark recombination into hadrons is essential. The important role of quark recombination follows also from the mentioned "reciprocity relation" $D(\gamma) = G(\beta)$, according to which the experimentally discovered A dependence of the cumulative production cross-section should be tracked back to both G and D.

This idea may explain an essential difference of the A dependence for the cumulative particles with different quark composition; namely, the increase of the A dependence of the cross-section by an additional factor $A^{B/3}$, where B is the baryon number of the cumulative particle.

It is interesting to note that a similar increase of the A dependence was detected for the formation of nuclear fragments with $p_T \sim 1$ GeV/c [31]. Figures 13 and 14 from Ref. 32 show the data on production of nuclear fragments with large transverse momentum. The figures illustrate well the change of the A dependence of the cross-section from the usual geometric $A^{2/3}$ to $A^{m(p_T)}$; in this case m reaches a value much larger than unity (up to m = 3). Data obtained at incident proton energies of 6.6 GeV (black circles) and of 400 GeV (open circles) are plotted in Fig. 13. The fact that the structure functions are the same in such a wide energy range is good evidence for limiting fragmentation of nuclei.

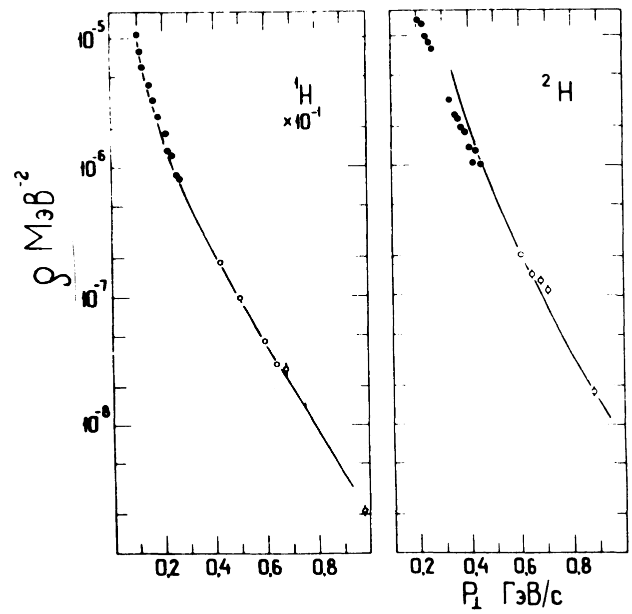

Fig. 13

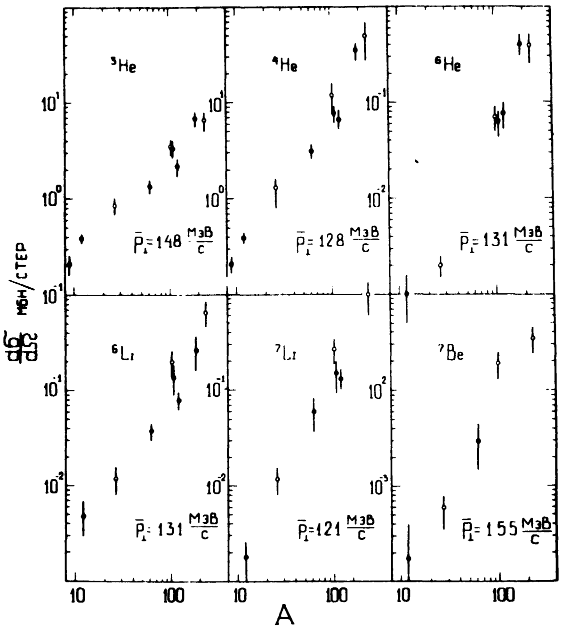

Fig. 14

The important result is a new measurement of m in the A dependence of the cumulative Λ particle production cross-section. From the experiment of the Shakhbasian group m was found to be 1.45 instead of the value derived from the earlier existing data of the Leksin group, m = 0.7. The new data are in good agreement with the mentioned dependence of m on the baryon number of the cumulative particle.

The interpretation of the experimental data on the cumulative effect in terms of the quark-parton structure functions of nuclei strongly depends on the validity of the hypotheses on quantum-number conservation in the process of the quark recombination into hadrons, the check of which is very important.

In this connection polarization experiments are of special interest. In Ref. 33 it was proved that in the region of large momentum transfers we are interested in the quark helicity conservation in hadronization. Measurements of the cumulative baryon and vector meson polarization are therefore measurements of the dependence of G and D on one more variable.

For the time being, we have only rough estimates for the cumulative proton and Λ hyperon polarization[13,14], which show that it is large enough and reaches 100%. The quantitative study of polarization phenomena in cumulative particle production is one of the most urgent experiments in the domain of multibaryon phenomena. In particular, most attention should be paid to the study of the cumulative effect caused by polarized particles.

Experiments on the measurement of quark distribution in nuclei, which are free of uncertainties caused by the quark recombination into hadrons, are actually experiments on deep inelastic scattering of leptons off nuclei in the cumulative region. However, owing to the smallness of the cross-sections, the only present experiment in which such effects are to be measured is NA 4 at CERN. I suggested in Ref. 28 extracting the relevant information from the existing experimental data on the reaction $\mu + {}^{12}C \to \mu + X$. Using the data on G given above it is not difficult to estimate the cross-sections. The main dependence of the cross-section on X will be determined by a sharp exponential [Eq. (36)] with an exponent $\langle N \rangle = 0.16$. The fraction $\Delta\sigma/\sigma$ of the cross-section for the Bjorken variable $x > \ell$ is estimated as

$$\frac{\Delta\sigma}{\sigma} \approx e^{-\ell/0.16} = \begin{cases} 2 \times 10^{-3} & \text{for} \quad \ell = 1 \\ 1.2 \times 10^{-4} & \text{for} \quad \ell = 1.5 \\ 0.6 \times 10^{-5} & \text{for} \quad \ell = 2 \; . \end{cases}$$

From this it is seen that, for the existing and expected statistics, it is quite possible to measure the region of x up to 2 by means of the NA 4 equipment, which is of much interest not only from the point of view of direct measurements of quark-parton distributions in nuclei, but also from the point of view of the study of various effects at large Q^2.

Lately success has been arrived at extracting the quark-parton structure functions of the ${}^{12}C$ nucleus in the cumulative region from NA 4 experimental data (I take the opportunity to thank Drs. I.A. Savin and G.I. Smirnov for providing the relevant information and for discussions). Figure 15 shows the results of processing various experimental data on the basis of

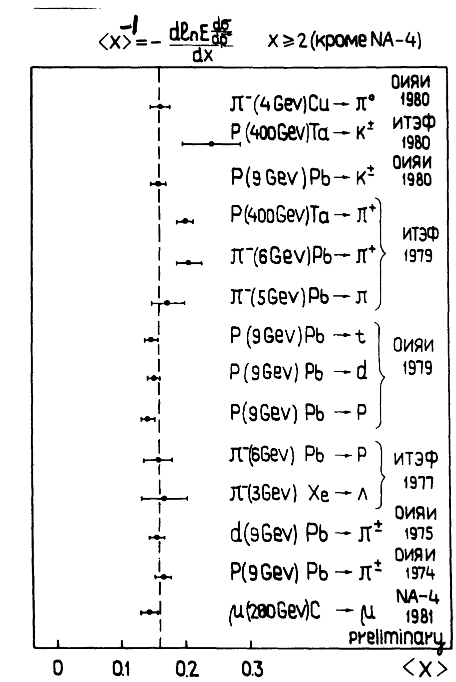

Fig. 15

parametrization of type (36). Also given are preliminary data obtained from the NA 4 experiment for $x > 0.7$. The quantity $1/\langle x \rangle = -[d \ln E(d\sigma/d\vec{p})]/dx$ that characterizes the longitudinal quark distribution in nuclei is seen to possess a remarkable universality. The dotted line corresponds to the average value $\langle x \rangle = 0.16$.

Now I would like to make some concluding remarks to this section.

i) The quark-parton structure functions of nuclei, $G(\beta^0, p_T^2)$, in the region $\beta^0 > 1$ as independent (irreducible to one-nucleon) objects of hadron physics have become of much importance and are being studied both for their theoretical and experimental aspects.

ii) The universal character of the structure functions is defined not only by the earlier established limiting nuclear fragmentation beginning at an energy 3.5-4 GeV/A in relativistic nucleus collisions. The parameter $a = - d \ln \rho/d\beta^0 \approx \langle \beta^0 \rangle^{-1}$, which characterizes the longitudinal quark distribution, was found to be universal, independent of the quantum numbers of cumulative particles.

iii) The parametrization of the A dependence of the D functions in the form A^m used earlier (where m was as large as 2) was found to be insufficient. There were observed asymptotic regimes in the A dependence:

$$\sigma \propto A^1 \ .$$

The strong dependence of m (in the old parametrization) on the cumulative number β^0 and the baryon number of a cumulative particle shows an interdependence (non-factorizability) of these parameters.

iv) An ever-growing amount of experimental information on cumulative particle production goes essentially beyond the framework of the results based on the quark-parton models of hard collisions and QCD.

v) Of special value is the direct measurement of the $D(\beta)$ functions in deep inelastic lepton-nuclear interactions and the study of polarization phenomena in the cumulative effect as sensitive methods for checking QCD.

4. FACILITIES AND PERSPECTIVES

For the next few years the main tools of relativistic nuclear physics will be proton synchrotrons and detectors commonly used in elementary particle physics. The intensity of the extracted nuclear beams is already now much higher than the intensity of the beams of secondary particles (pions, kaons, etc.) which the existing detectors can handle with appreciable rates.

The relativistic nuclear beams -- the parameters of which will be improved undoubtedly in the near future -- and the available detectors will make it possible to resolve many of the problems discussed.

Five electronic installations and three track detectors (a liquid-hydrogen bubble chamber, a two-metre propane bubble chamber, and a two-metre streamer chamber at the Dubna synchrophasotron) are used for studies in the field of relativistic nuclear physics. With the exception of the Stavinsky group's installation, which was used to obtain the main results on the cumulative effect, all these facilities were constructed for performing studies in the field of particle physics, and it is only lately that they have been adjusted for investigations with relativistic nuclei.

In the first installation of Stavinsky's group specially created for studying processes of the type $p + A \to \pi(180°)$, pions were detected by a DISC-type Čerenkov differential counter with a velocity resolution $\Delta\beta = \pm 3 \times 10^{-2}$ in a velocity range 0.7-1.0. The second version of this installation is a rotating magnetic spectrometer which has allowed the performance of detailed angular measurements of cumulative particle distributions. There events were extracted by an independent measurement of the time of flight on two bases (4 m and 1 m) with an accuracy of 150-200 ps and measurement of ionization losses and intensity of the Čerenkov burst in a solid radiator. The description of these facilities is given in Refs. 34 and 35.

Recently the method of thin internal targets[36] has been applied on the synchrophasotron for measuring the cross-sections of nuclear collisions with large transverse momenta[31].

The one-arm magnetic spectrometers with proportional chambers are used for measuring the inclusive cross-sections for relativistic nuclear collisions[37,38].

An installation called "Photon" is oriented to studying relativistic nuclear collisions with emission of neutral particles (π^0, η^0, ω^0). It is a 90 channel Čerenkov hodoscope of lead-glass in which the gamma-quantum energy is measured. The direction of gamma quanta is measured by 32 spark chambers with a magnetostrictive read-out. The accuracy of measurement of the gamma-quanta direction depends on the thickness of the converters and amounts to 3.4 mrad. A large complex of electronic apparatus and an on-line computer of the installation "Photon" make it possible to study effectively multiple photon emission in relativistic nuclear collisions, in particular the problem formulated in Ref. 39 on cumulative production of vector mesons.

Among track sensitive devices the 2 m propane chamber has been developed mainly with relativistic nuclear physics in mind.

Of particular interest are the search for and study of multibaryon resonances, the existence of which is predicted by the quark bag theory. The (Λp) and, possibly, ($\Lambda\Lambda$) and ($\Lambda\Lambda$p) resonances discovered by Shakhbasian on the basis of the study of photographs from the propane bubble chamber were lately interpreted[40,41] as multiquark formations in a single "bag". In the same experiments it is found to be possible to study the cumulative production of Λ particles, including the study of their polarization (see above). The relationship between the cumulative effect and the manifestation of quark plasmons is one of the most interesting and important objects of the investigations in relativistic nuclear physics.

For the study of exclusive reactions in relativistic nuclear physics, use is made of a 1 m liquid-hydrogen chamber. It is bombarded by ^{3}He and ^{4}He nuclei (see, for example, Ref. 42).

The first experiments of deuteron acceleration in the Dubna synchrophasotron in 1970 showed that in order to proceed to the acceleration of nuclei with the aid of ordinary proton accelerators the accelerating system needs not be strongly modified. Thus, any high-energy accelerator can be adapted to accelerate deuterons and α particles.

In order to pass to the acceleration of nuclei with large atomic masses, a number of technological problems should be resolved. The main one is that of obtaining stripped nuclei. The acceleration of partially ionized atoms imposes very strict requirements on the vacuum inside the accelerator chamber. To obtain stripped nuclei, it is suggested to create pre-accelerators with an intermediate stripping.

Our Laboratory has engaged in developing essentially new sources of heavy ions: electron-beam ion and laser sources. The electron-beam ion source invented by Donetz is a rather compact device which has been running reliably at our synchrophasotron for a long time under operating conditions. This source is sometimes called "CREBIS" (cryogenic electron beam ion source). The present status of the relative investigations enables us to hope to obtain stripped nuclei with an intensity of about $10^{11}/Z$.

Relativistic acceleration of heavy nuclei and even intermediate mass nuclei requires creation of special injection complexes, pre-accelerators at an energy of about 500 MeV/nucleon, which are of great value by themselves. The high-voltage injection also resolves vacuum problems in the main ring, since for ions of an energy higher than 500 MeV/nucleon the electron pick-up is non-essential even for rather moderate requirements on the vacuum.

Thus, the following research programme has been worked out at the Laboratory of High Energies of the JINR. During the next 4-5 years it is planned to use extensively the beams of relativistic nuclei of the synchrophasotron up to 5 GeV/nucleon energies. As we have already stressed, the limiting fragmentation of nuclei begins at an ion energy higher than 3 GeV/nucleon. This ion energy range has, as yet, been obtained in no other accelerator centres. The available detectors will make it possible to realize a rather wide programme of investigations. At the present time the construction of a large experimental hall of an area of about 10^4 m^2, in which a large number of simultaneously operating installations can be arranged on the extracted beams of the synchrophasotron, has been completed.

Further perspectives at our Laboratory are connected with the construction of a superconducting accelerator specialized for nuclei. It will replace the synchrophasotron. Some progress has also been made in the construction of superconducting magnets for accelerators (see, for example, Ref. 43). At the Laboratory there are some advances in the design of superconducting magnets with an iron-shaped magnetic field (see, for example, Ref. 43). Though the magnetic field is restricted by a value of 2.5 T, the construction of the magnet becomes, in turn, essentially simpler; the winding volume and the weight of the magnetic circuit decreases, which essentially facilitates their manufacture in the lab. conditions. The small superconductor volume and the low value of the energy stored in the winding enable us to hope to reach high frequencies of repetition of acceleration cycles (0.1-0.5 Hz) and to obtain a growth of the average beam intensity. A group of engineers and physicists of the Laboratory of High Energies, under the leadership of Shelaev, has got good dipole and quadrupole parameters.

Some preliminary suggestions concerning the design of a superconducting accelerator of relativistic nuclei, which was given the name "Nuclotron", are presented in Refs. 44 and 45.

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