Archives

COLLECTIVE ASPECTS OF HADRONS AND QUARKS IN NUCLEUS - NUCLEUS COLLISIONS

Joseph Kapusta

CERN -- Geneva

ABSTRACT

The primary goal of colliding heavy nuclei at high energy is to obtain information on the hadronic matter equation of state at high density and temperature. Hydrodynamical models are well suited theoretically for such studies. However, the degree of applicability of such models to realistic size nuclei is still an open question. Comparisons are made between numerical calculations and experiment for Ar + KCl at 800 MeV per nucleon. The prospects for, and consequences of, heavier nuclei and much higher energy are discussed.

1. - INTRODUCTION

Since 1973 there has been much theoretical and experimental interest in colliding heavy nuclei at high energy. Heavy nuclei means mass numbers greater than 40, and high energy means a beam energy greater than 200 MeV per nucleon, usually considerably greater. The hope is that we will be able to learn something about the properties of hadronic matter at high energy density occupying a large volume. The simplest scenario is that thermalization is achieved during central collisions of heavy nuclei, and therefore we ought to be able to extract some information on the equation of state of the produced matter. If thermalization is not achieved then one might still learn something about the properties of bulk matter at high energy density, but it will not be the equation of state. The aim of these studies then is orthogonal to e^+e^- annihilation physics where one likes to concentrate a large amount of energy in a small volume.

There has been considerable speculation on the types of exotic matter which may be formed in central heavy ion collisions. At high baryon density conjectures have centered on pion condensation[1)], Lee-Wick nuclear matter[2)], delta isomers[3)] and quark matter[4)]. At high temperature one might encounter a limiting temperature[5)] or a transition to quark-gluon matter[6)]. (Since the field is so large these and other references are meant to be illustrative but not exhaustive.)

Hydrodynamical models are well suited theoretically to the study of heavy ion collisions at high energy. This is because the only variable input for solving the hydrodynamic equations of motion for a given nucleus-nucleus collision is the hadronic matter equation of state. If heavy nuclei were collided at the highest energies attainable in current proton accelerators, the hydrodynamical model would predict energy densities so great that the resulting matter would be in the deconfined quark-gluon phase. However, caution must be used when comparing the results of hydrodynamic calculations with experiment, since real nuclei are not macroscopic objects in the classical sense of being composed of 10^{23} particles. It remains an open and intriguing question as to whether even uranium is large enough. If we were able to collide neutron stars, and some day mankind or his descendants may have that capability, there would be no controversy.

We can make some semi-quantitative estimates. The mean free path for a particle which belongs to an ensemble of particles which is in or near thermal equilibrium is $\lambda = 1/n\sigma\sqrt{2}$, where n is the particle number density and σ is the scattering cross-section. (If the particle in question does not belong to the ensemble but is shot into the gas at high velocity the $\sqrt{2}$ is taken away.) At normal nuclear density and with a cross-section of 40 mb we find that λ = 1.2 fm. Let L be a typical dimension of the nuclei which are colliding against each other. The radii and diameters of some typical nuclei are listed in the Table.

nucleus	radius (fm)	diameter (fm)
p	0.8	1.6
^{12}C	2.7	5.4
^{40}Ar	4.1	8.2
^{238}U	7.4	14.8

(Recall, however, that real nuclei have a diffuse surface of width 1 fm.) If $\lambda >> L$ then we would expect a single nucleon knock-out model to be valid[7]. That is, one nucleon from each of the nuclei would scatter together once and then leave the collision zone and fly off toward the detectors. If $\lambda \gtrsim L$, then the individual nucleons would undergo several binary collisions with other nucleons and we would expect the validity of the more complicated intranuclear cascade models[8],[9]. If λ was to become too small compared with L, then we might begin to worry about the effects of the dense packing of nucleons, many-body forces, off-mass-shell propagation, etc., which are not contained in the intranuclear cascade models. If $\lambda << L$ then these and other effects might be incorporated more conveniently or correctly by using a realistic equation of state and by solving the equations of motion of hydrodynamics. This is equivalent to saying that local thermal equilibrium is achieved during the collision. If $\lambda \lesssim L$, then strict local equilibrium may not be valid and we should incorporate the effects of finite gradients of pressure, temperature, etc. We would then need to solve the equations of motion of imperfect fluid dynamics, which would require knowledge of the bulk and shear viscosity coefficients, the thermal conductivity coefficient, as well as the equation of state. The domain of overlap between intranuclear cascade and imperfect fluid dynamics is an interesting problem in non-equilibrium statistical mechanics. The effect of a finite mean free path on expanding fireballs will be investigated later in the third section.

2. - EXPERIMENTAL EVIDENCE FOR NON-TRIVIAL BEHAVIOUR

The first question we should ask is whether or not there is any evidence at all for non-trivial behaviour in heavy ion collisions. By trivial behaviour it is meant that all such collisions could be described by the single nucleon knock-out model. In Fig. 1 some data[10] for Ar + KCl → p + X at a beam energy of 800 MeV per nucleon is shown. The inclusive single particle invariant cross-section is plotted as a function of the angle in the CM and at fixed kinetic energy. The predictions of two models are shown for comparison. The single nucleon knock-out, or hard scattering, model[7] uses an elementary single particle momentum distribution for nucleons in the nucleus of the form $(p/p_0)/\sinh(p/p_0)$, $p_0 = 90$ MeV/c. This contrasts with the Fermi-Dirac distribution $\theta(p_F - p)$, where $p_F \approx 260$ MeV/c is the Fermi momentum. It was found that the former distribution, when used in this model, produced much better agreement with the data than the latter. However, the model still predicts much more angular asymmetry than is present in the data. Recall that at these energies elementary nucleon-nucleon collisions are forward-backward peaked. Thus the data would indicate some degree of multiple scattering, heading towards a more isotropic distribution. The predictions of a fireball model[11] calculation is also shown. For collisions between symmetric size nuclei the fireball is always formed at rest in the CM. Hence this model predicts complete isotropy. However, in its original version the fireball model does not conserve angular momentum. If the fireball was given the correct amount of spin it would produce a forward-backward asymmetry which might reproduce the data.

In Fig. 2 some data[12)] for Ne + U → charged particles + X at a beam energy of 250 MeV per nucleon is shown. The data refers to the summed-charge single particle differential cross-section, obtained by summing contributions from p, d, t, ^{3}He and α. For comparison, the predictions of three different models are also shown. There are two versions of fluid dynamics[13)] and one version of intranuclear cascade[8)]. Notice that there are no major qualitative differences among the model predictions and with the data. This is despite the fact that intranuclear cascade and fluid dynamics approach the reaction dynamics from opposite extremes. This could mean that λ really is small enough for these collisions to exhibit hydrodynamic behaviour even in the cascade approach. Or, it could mean that too much information is lost by summing over all charged particles and by summing over all impact parameters.

To help decide the issue the experiment[14)] can also measure the multiplicity associated with the detection of a proton of given momentum. The result of such a measurement is shown in Fig. 3. The reactions Ne + U → p + X at a beam energy of 393 MeV per nucleon are separated into low multiplicity (of X) events and high multiplicity events. Naively we expect higher multiplicity events to be associated with smaller impact parameters, since the overlap of the target and projectile would be greater. The shape of the proton differential cross-section is qualitatively different when one triggers on high multiplicity events as opposed to low multiplicity events. This rules out a single nucleon knock-out model description of these reactions since the shape of the proton spectrum in that model is predicted to be independent of multiplicity and impact parameter. It seems that intranuclear cascade models have only a very weak impact parameter dependence which is not able to reproduce this qualitative difference. This difference is, in contrast, predicted by hydrodynamic calculations[15)]. See Fig. 4.

3. - FLUID DYNAMICS IN THE ONE GEV PER NUCLEON DOMAIN

The equations of motion of relativistic hydrodynamics express the conservation of energy-momentum

$$\partial_\nu T^{\mu\nu} = 0, \tag{1}$$

and of baryon number

$$\partial_\nu N^\nu = 0. \tag{2}$$

The notation is as follows. The energy-momentum tensor

$$T^{\mu\nu} = Pg^{\mu\nu} + (P + \rho)U^{\mu}U^{\nu} \tag{3}$$

depends on the pressure P, the total energy density ρ as measured in the rest frame of the fluid, and the four-velocity U^{μ}. The latter has time component $U^0 = (1 - \vec{v}^2)^{-1/2}$ and space components $\vec{U} = \vec{v}U_0$, where $\vec{v}$ is the local velocity of the fluid relative to a fixed computational frame. The baryon current is $N^{\mu} = nU^{\mu}$, where n is the local baryon density in the rest frame of the fluid. The thermodynamic quantities P, ρ and n are related by an equation of state which we may choose to write in the form $P = P(\rho,n)$. The independent quantities ρ, n and $\vec{v}$ then depend on position $\vec{x}$ and time t.

Numerical methods have been developed to solve these equations in three dimensions[13]. We have performed such calculations[16] for the collisions of equal-mass nuclei at various impact parameters at a beam energy of 800 MeV per nucleon (182 MeV per nucleon in the CM). In Fig. 5 the time development of such collisions at three different impact parameters is shown. (We have also made a colour movie, for collisions at four impact parameters, entitled Super Ion, The Movie. Copies are on deposit with the American Association of Physics Teachers and the Los Alamos National Laboratory film libraries and are available for short term loan.) The results are scale invariant, that is, they are independent of the physical size of the nuclei. Notice in particular the qualitative difference between large and small impact parameters. At $b = 0.8b_{max}$ there is only a small volume of overlap between the colliding nuclei. The large target and projectile fragments leave the collisions with essentially the same velocity with which they entered. At b = 0, however, all the matter participates directly in the collision. There is a flattening of the nuclei as they compress, with some of the matter bouncing backwards but with most of it splashing out to the side. The maximum compression of matter attained was about 3 to 4.

We wish to quantify these global aspects of the collisions by means of thrust,

$$T = \max_{\hat{n}} \sum_i |\vec{p}_i \cdot \hat{n}| / \sum_i |\vec{p}_i| \ . \tag{4}$$

The sum is over all particles i with momentum $\vec{p}_i$ in the CM. The thrust is especially relevant for nucleus-nucleus collisions since it is relatively insensitive to such things as pion production and nuclear clustering in the final stages of the collision.

The result of a thrust analysis applied to these calculations is shown in Fig. 6. It displays the expected behaviour. At $b = b_{max}$ the nuclei just begin to interact. T = 1 and occurs at an angle of 0^o relative to the beam. As the impact parameter is reduced the nuclei interact more strongly. The nuclear matter is compressed and shoved out to finite angles. T decreases in magnitude because some of the initial collective momentum is distributed in a range of angles centered about θ and because some of it is converted, via shock heating, into randomized thermal energy. For a central collision the momentum comes out preferentially as a sidewards splash.

Since the hydrodynamic equations are scale invariant these results might represent collisions between alpha particles, between uranium nuclei or between neutron stars. Certainly hydrodynamics is not applicable to alpha particle collisions, but just as certainly they are applicable to neutron star collisions. The big question is whether or not they are applicable to uranium collisions. The necessary exclusive or semi-exclusive experiments will be done in the next year or two.

It is also possible to compute the spectra of π, p and d in this model[17)]. Besides the thrust the entropy is a convenient global quantity to characterize the state of the system. Hydrodynamic flow is normally adiabatic, i.e., entropy conserving. However, when nuclei collide at velocities greater than the speed of sound, shock waves occur and heat the system to finite temperatures. The build-up of entropy as a function of time is shown in Fig. 7 for central collisions of the type displayed in Fig. 5. The nuclei begin in the ground state so their entropy is zero. After some time the shock heating ends and an adiabatic expansion phase begins. At various times the elementary fluid elements drop below normal nuclear density. When this happens the constituents fly apart on straight line trajectories with a thermal momentum distribution. Included in this thermal and chemical equilibrium breakup stage are π^+, π^0, π^-, p, n, d, d*, t and ^3He. This allows us to calculate the invariant differential cross-sections for protons, deuterons and pions as shown in Figs. 8-10. These calculations are compared with some data for Ar + KCl collisions[10)].

The important aspects of these four figures are the following. For an equation of state which is very soft more entropy is produced during the collision than for a stiffer equation of state. Entropy is equivalent to disorder. If, at the end of the collision, the entropy per baryon is high, then more of the baryon number will emerge in the form of free nucleons as opposed to nuclear clusters like d, t, ^3He, etc. Also more pions will be produced. Hence by varying the equation of state we can vary the final chemical composition of the measured fragments.

In comparison with the data notice that even with a very soft equation of state there are not enough free protons emitted. Notice also that there are far too few pions produced by these calculations which is in contrast to purely thermal models[18] and intranuclear cascade models[19] which predict too many. See Fig. 11. These observations tend to suggest that more energy is contained in collective flow than predicted by these other models, but less than that predicted by the pure hydrodynamical model, at least for Ar + KCl at 800 MeV per nucleon. From the point of view of fluid dynamics, perhaps viscosity and heat conduction (frictional forces) play a role in reducing the amount of energy contained in collective hydrodynamic flow and keeping it in the form of internal excitation energy and pions. One might also expect in principle that heat conduction and viscosity would have an effect on a system as light as Ar + KCl since the nucleon mean free path is not negligible compared to the size of the system. The partitioning of the available energy among temperature, collective flow and pion mass, seems to be rather crucial.

The effect of a finite mean free path on the expansion stage of central collisions between heavy nuclei at a beam energy of 800 MeV per nucleon has been studied in a non-relativistic imperfect fluid dynamic approach[20]. A gas of point nucleons with localized interactions was assumed for definiteness and for comparison with intranuclear cascade. Kinetic theory, going back to J.C. Maxwell in 1860, then provides the thermal conductivity and viscosity coefficients in terms of the nucleon-nucleon cross-section. The equations of motion were solved for a spherically expanding fireball which had an initial uniform density of twice normal density. The final breakup density was taken as 0.4 of normal density.

There are two obvious ways of gauging the effect of a finite mean free path: entropy generation and the final nucleon momentum distribution. The additional entropy per nucleon generated during the expansion is plotted in Fig.12 as a function of the ratio of the mean free path divided by the radius of the system, evaluated at normal density. The calculations were done for C + C up to U + U. The additional entropy generated was less than 10% of the initial value of 3.9 units, which is a small effect. However, the ΔS for uranium collisions lies closer to the ΔS for carbon collisions than it does to the $\Delta S = 0$ of neutron star collisions. This somewhat surprising result is reinforced when we look at the momentum distributions in Fig. 13. There we see that the viscous uranium plus uranium expansion looks more like a pure Maxwell-Boltzmann distribution than a pure hydrodynamic expansion as represented by neutron stars.

4. - FLUID DYNAMICS IN THE ULTRA-RELATIVISTIC DOMAIN

We should not expect fluid dynamics to be generally applicable during the whole time evolution of a collision between nuclei at ultra-relativistic energies. This is because the basic nucleon-nucleon scatterings at very high energy are usually soft. In a typical collision the nucleons lose only a relatively small fraction of their total linear momentum so that more scatterings would be needed to stop them than can be provided by even a uranium nucleus. One might think that the pions created during the collision would provide an additional braking mechanism. We know from proton-nucleus studies that this is not the case. Thanks to special relativity and a finite formation time pions are produced outside the target nucleus. Although Landau's hydrodynamical model and numerous variants of it are able to claim phenomenological successes, these are more likely to follow from the basic conservation equations and cylindrical phase space than from a detailed applicability of hydrodynamics. [For an interesting, heretical point view see Ref. 21).]

However, from the point of view of the fluid dynamicist, there are three reasons for optimism. Firstly, neutron stars are such huge objects that they will behave hydrodynamically, as shown in Fig. 5, even for the highest energy deemed necessary to form quark-gluon matter. Secondly, nucleus-nucleus collisions at ultra-relativistic energies will probably be characterized by large fluctuations, even at a fixed impact parameter. For example, consider a geometrically central collision. One possible outcome is that each of the nucleons undergoes peripheral interactions with the nucleons from the other nucleus. Thus the nuclei will pass through each other. An entirely different outcome would arise if each of the nucleons underwent hard collisions with the nucleons from the other nucleus. Then the nuclei would stop each other in a rather hydrodynamic fashion. The likelihood of such an event is probably much higher than one would naively estimate, since the hard scattering of a few nucleons on the front sides of the nuclei should serve as a catalyst for the hard scattering of nucleons on the back sides. It will be the task of the experiments to pick out these more interesting events from the background. An obvious criterion would be to look for events where most of the energy and baryon number came out near the CM rapidity. Thirdly, it may be that fluid dynamics is not adequate to describe the initial stage of the collision but, nevertheless, large globs of high energy density matter are formed. Fluid dynamics might then adequately model the subsequent expansion of this matter into the vacuum. It is to this last possibility that we now turn our attention.

For our phenomenological equation of state we will call on the MIT bag model[22]. This model incorporates both high energy perturbative behaviour and low energy confinement. The total pressure is the thermal pressure minus the bag constant,

$$P = P^* - B , \tag{5}$$

and the total energy density is the thermal energy density plus the bag constant,

$$\rho = \rho^* + B . \tag{6}$$

When the quarks are treated as massless and non-interacting this leads to the equation of state

$$P = \frac{1}{3}(\rho - 4B) . \tag{7}$$

In this case P depends only on the energy density ρ and not on the baryon density n. The picture is one of free quarks and gluons moving in a perturbative vacuum which is surrounded by the true vacuum of lower energy density. It turns out in this model that all hadrons have the same energy density 4B. Phenomenologically $B^{1/4} \approx 150$ MeV.

The model we have is very simple. Quark-gluon matter is formed at some high energy density $\rho(0)$. Being an unstable situation, the matter will expand hydrodynamically until it reaches the energy density 4B, at which time it breaks up into hadrons. The aim is to find the time evolution of the volume-averaged thermodynamic quantities such as energy density. We will consider a spherical expansion for simplicity.

The total energy of the system is

$$E_{total} = \int dV\left[\gamma^2(P + \rho) - P\right] , \tag{8}$$

and the total entropy is

$$S_{total} = \int dV \gamma s , \tag{9}$$

where $s = \partial P/\partial T$ is the local entropy density. Rather than solving the equations of motion numerically, which does not seem to be called for at this stage, we seek a volume-averaged description in the form

$$\begin{aligned} \rho^*(t) &= \rho^*(0)/\lambda^4(t) , \\ s(t) &= s(0)/\lambda^3(t) , \\ V(t) &= V(0)R^3(t) , \\ \gamma &= \gamma(t) . \end{aligned} \tag{10}$$

Here $\lambda(t)$ and $R(t)$ are scaling variables. We assume that the system begins to expand from rest at time $t = 0$ so that $\lambda(0) = 1$, $\dot{\lambda}(0) = 0$, $R(0) = 1$, $\dot{R}(0) = 0$, $\gamma(0) = 1$. Then Eq. (8) gives

$$R^3(t)\left[\frac{1}{3}(X-1)(4\gamma^2(t)-1) + \lambda^4(t)\right] = X\lambda^4(t), \tag{11}$$

and Eq. (9) gives

$$R^3(t)\gamma(t) = \lambda^3(t) , \tag{12}$$

where $X \equiv \rho(0)/B$ is the input parameter.

We could solve for the full time development of Eqs. (11) and (12), if we wanted, by identifying the flow velocity $v(t) = (1 - \gamma^{-2}(t))^{1/2}$ with dR/dt, where t is measured in units of the initial physical radius of the system, as is R. This is not necessary if all we are interested in is the state of the system at time t_f when the system breaks up into hadrons. Then $P(t_f) = 0$ and $\rho(t_f) = 4B$. We find that

$$\begin{aligned} \gamma(t_f) &= \frac{X}{4}\left(\frac{3}{X - 1}\right)^{3/4} , \\ R^3(t_f) &= \frac{4}{X}\left(\frac{X - 1}{3}\right)^{3/2} . \end{aligned} \tag{13}$$

Some rather simple but interesting results may be deduced from the above considerations. The fraction of the total energy which is converted to collective energy is $1 - \gamma^{-1}(t_f)$ and the fraction which remains in internal energy (mass and temperature) is $\gamma^{-1}(t_f)$. We can estimate the average transverse momentum of the emitted hadrons in the following way. Assume that the fluid elements give rise to a momentum distribution for particles in their rest frames of the usual form $\sim\exp(-p/T_0)$. Here T_0 is of the order of $B^{1/4}$. Then we sum over all fluid elements, taking into account the radial velocity of each[23], to obtain

$$\langle p_\perp \rangle = \frac{3\pi}{4} T_0 \gamma(t_f) . \tag{14}$$

The average $p_\perp$ is increased by the radial expansion of the matter.

The results are illustrated in Figs. 14 and 15. The volume compression, which is the volume of the system at the time of break up into hadrons divided by the initial volume, increases rather slowly with the initial energy density. Part of this non-linear relationship is due to energy conservation. At the time of break up the proper energy density is 4B, and since some of the initial energy has been converted to collective motion, the final volume must scale less than linearly with the initial energy density. There is also a Lorentz contraction of the volume since the surface is moving radially outwards from an observer sitting at the centre of the fireball. Similarly the fraction of total energy which is in the form of collective motion and the percent increase in the mean transverse momentum scale rather slowly with the initial energy density.

At present, of course, no experiments have been done for heavy nuclei at ultra-relativistic energies. Therefore, to show how the analysis of experiments might go, let us be highly speculative about interpreting the recent data taken at the CERN SPS $\bar{p}p$ collider[24)]. It was reported that the average $p_\perp$ for these 540 GeV CM energy collisions is larger than the 350 MeV/c found for pp collisions at the ISR. It is a long-standing observation that, prior to the $\bar{p}p$ collider, the average $p_\perp$ seemed to have saturated well before the peak ISR energy of 63 GeV in the CM was reached. It seems reasonable to assume a cluster-type model for both. The average $p_\perp$ would saturate if the energy density at ISR energies saturated at 4B, i.e., the clusters were produced at normal hadronic densities. Then $\langle p_\perp \rangle_{ISR}$ = 350 MeV/c would imply that T_0 = 150 MeV.

If, in going to the $\bar{p}p$ collider energy, a threshold was passed for attaining greater energy densities which lead to the production of quark-gluon matter then, taking $\langle p_\perp \rangle_{\bar{p}p}$ = 500 MeV/c, one obtains $\gamma(t_f) = 10/7$. From Eq. (13) we would infer that the quark-gluon matter was formed at 9 times the energy density in a proton !

From the MIT bag model one obtains an energy density in the proton of about 0.3 GeV/fm^3, whereas an estimate based on a radius of 0.8 fm gives 0.45 GeV/fm^3. Compression by a factor of 9 gives values in the range 2.7 to 4.0 GeV/fm^3. This compares with a value of 0.15 GeV/fm^3 for cold nuclear matter at normal density. The three-dimensional hydrodynamic calculations, presented in the previous section, produced maximum energy densities on the order of 0.5 GeV/fm^3. When considering that the CM beam energy has been increased from 0.2 GeV per nucleon for Ar + KCl to 270 GeV per nucleon for $\bar{p}$ + p this possible compression by a factor of 9 seems rather modest.

As applied to $\bar{p}p$ collisions this model is highly speculative. However, it does imply longer range correlations between produced particles and is consistent with azimuthal symmetry, both of which seem to be consistent with the data. Furthermore, we could calculate the number of dilepton pairs and real photons produced during the expansion[25] to check for consistency. It would be 238 times more interesting if the p and $\bar{p}$ could be replaced by uranium nuclei at the same beam energy !

5. - SUMMARY

It is of course impossible to adequately survey this field which goes back thirty years in such a short time and space. My discussion has naturally centered around those examples with which I am most familiar.

One should not be dogmatic in regarding the applicability of fluid dynamics to high energy heavy ion collisions. It may turn out to be an inadequate model to describe uranium collisions at 1 GeV per nucleon, yet at the same time it may have some usefulness for describing $\bar{p}p$ collisions at 270 GeV per beam, or vice versa. It is still an open question which can be answered only by a concerted effort by (i) theorists working in the fields of non-equilibrium statistical mechanics and quantum field theory, (ii) phenomenologists performing the calculations to compare with data and (iii) experimentalists to obtain the data.

The aim is to obtain information on the properties of hadronic matter at high temperature and density. Apart from possible terrestrial experiments with heavy ion beams there are two other alternatives. One might envisage colliding neutron stars, but that is far in the future. High temperatures and densities were almost surely obtained in the early Universe, but that was long ago, and the number and variety of relic observables pertaining to a quark-gluon → hadron phase transition seem to be severely limited (I know of none).

Perhaps even more important than the specific information being sought after are the benefits to be had from bringing together people from diverse subfields of physics to work on a common problem.

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REFERENCES

1) A.B. Migdal, Rev. Mod. Phys. 50 (1978) 107.

2) T.D. Lee, Rev. Mod. Phys. 47 (1975) 267.

3) J. Boguta, Phys. Lett. 109B (1982) 251.

4) G. Baym and S.A. Chin, Phys. Lett. 62B (1976) 241.

5) R. Hagedorn, in Cargèse Lectures in Physics, ed. E. Schatzmann (Gordon and Breach, New York, 1973), Vol. 6.

6) J.I. Kapusta, Nucl. Phys. B148 (1979) 461.

7) R.L. Hatch and S.E. Koonin, Phys. Lett. 81B (1978) 1.

8) Y. Yariv and Z. Fraenkel, Phys. Rev. C20 (1979) 2227.

9) J. Cugnon, Phys. Rev. C22 (1980) 1885.

10) S. Nagamiya et al., Phys. Rev. C24 (1981) 971.

11) G.D. Westfall et al., Phys. Rev. Lett. 37 (1976) 1202.

12) A. Sandoval et al., Phys. Rev. C21 (1980) 1321.

13) J.R. Nix, Prog. Part. Nucl. Phys. 2 (1979) 237.

14) R. Stock et al., Phys. Rev. Lett. 44 (1980) 1243.

15) H. Stöcker et al., Phys. Rev. Lett. 47 (1981) 1807.

16) J. Kapusta and D. Strottman, Phys. Lett. 106B (1981) 33.

17) J.I. Kapusta and D. Strottman, Phys. Rev. C23 (1981) 1282.

18) J. Gosset, J.I. Kapusta and G.D. Westfall, Phys. Rev. C18 (1978) 844.

19) J. Cugnon, D. Kinet and J. Vandermeulen, Nucl. Phys. A379 (1982) 553.

20) J. Kapusta, Phys. Rev. C24 (1981) 2545.

21) P. Carruthers, Annals of the N.Y. Acad. Sciences 229 (1974) 91.

22) A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn and V.F. Weisskopf, Phys. Rev. D9 (1974) 3471.

23) P.J. Siemens and J.O. Rasmussen, Phys. Rev. Lett. 42 (1979) 880.

24) K. Alpgård et al., Phys. Lett. 107B (1981) 310.

25) K. Kajantie and H.I. Miettinen, Z. Phys. C9 (1981) 341.

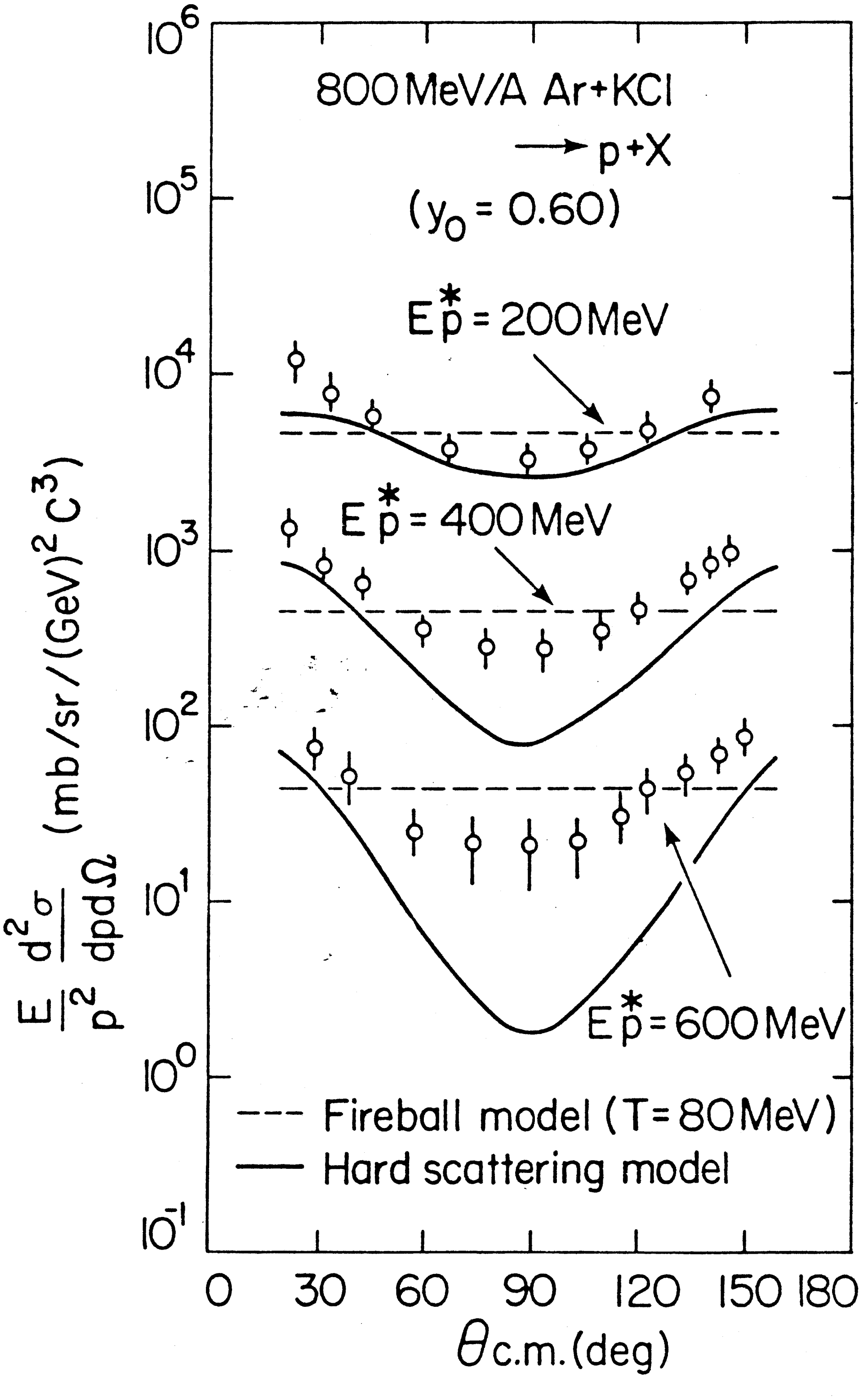

Fig. 1 : The angular distribution of protons for fixed centre-of-mass kinetic energy, from the collisions of Ar with KCl. The data[10] are compared with two opposing models of the reactions.

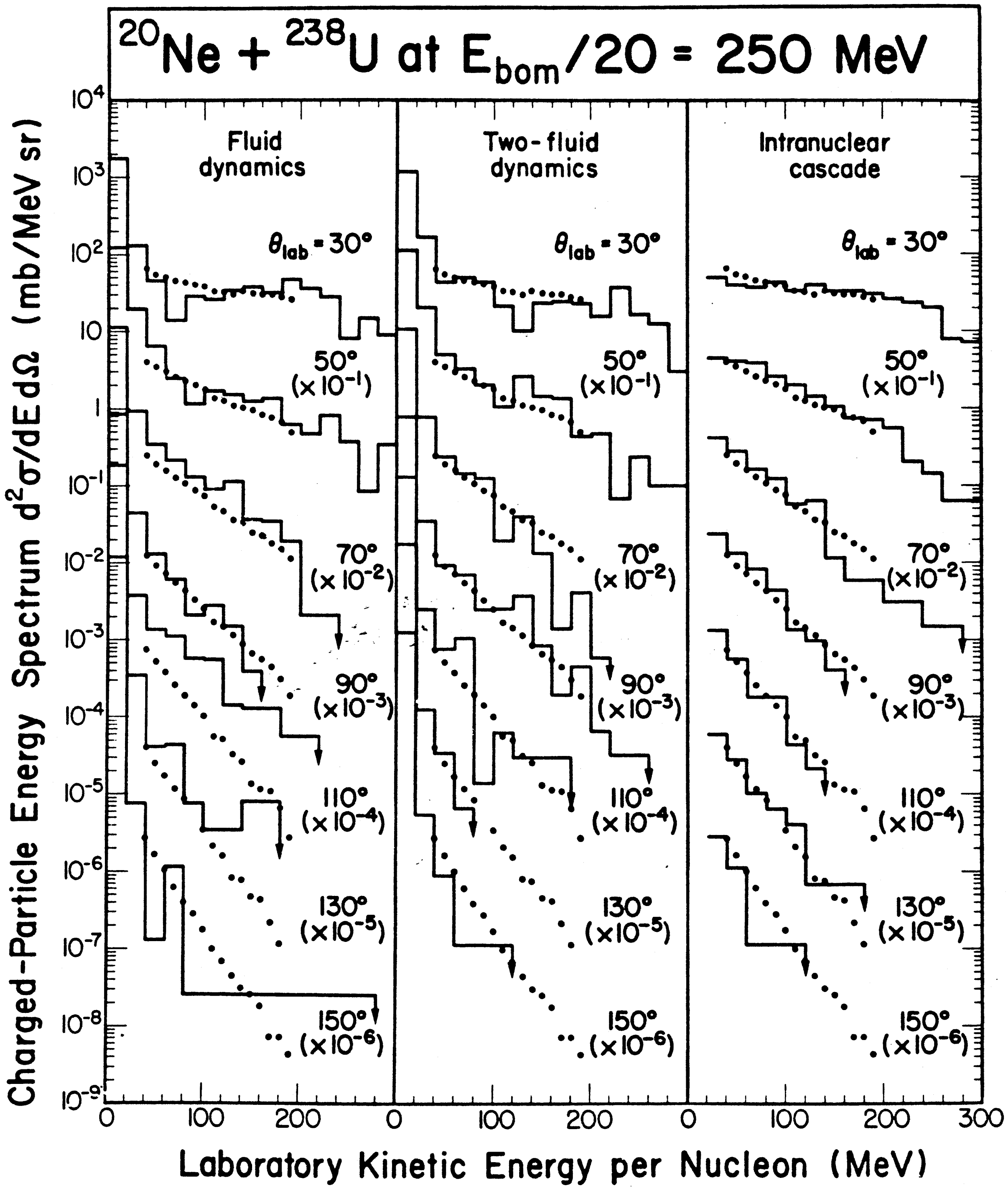

Fig. 2 : The charged-particle energy spectrum, for fixed laboratory angle, from the collisions of Ne with U. The data[12) are compared with three model calculations.

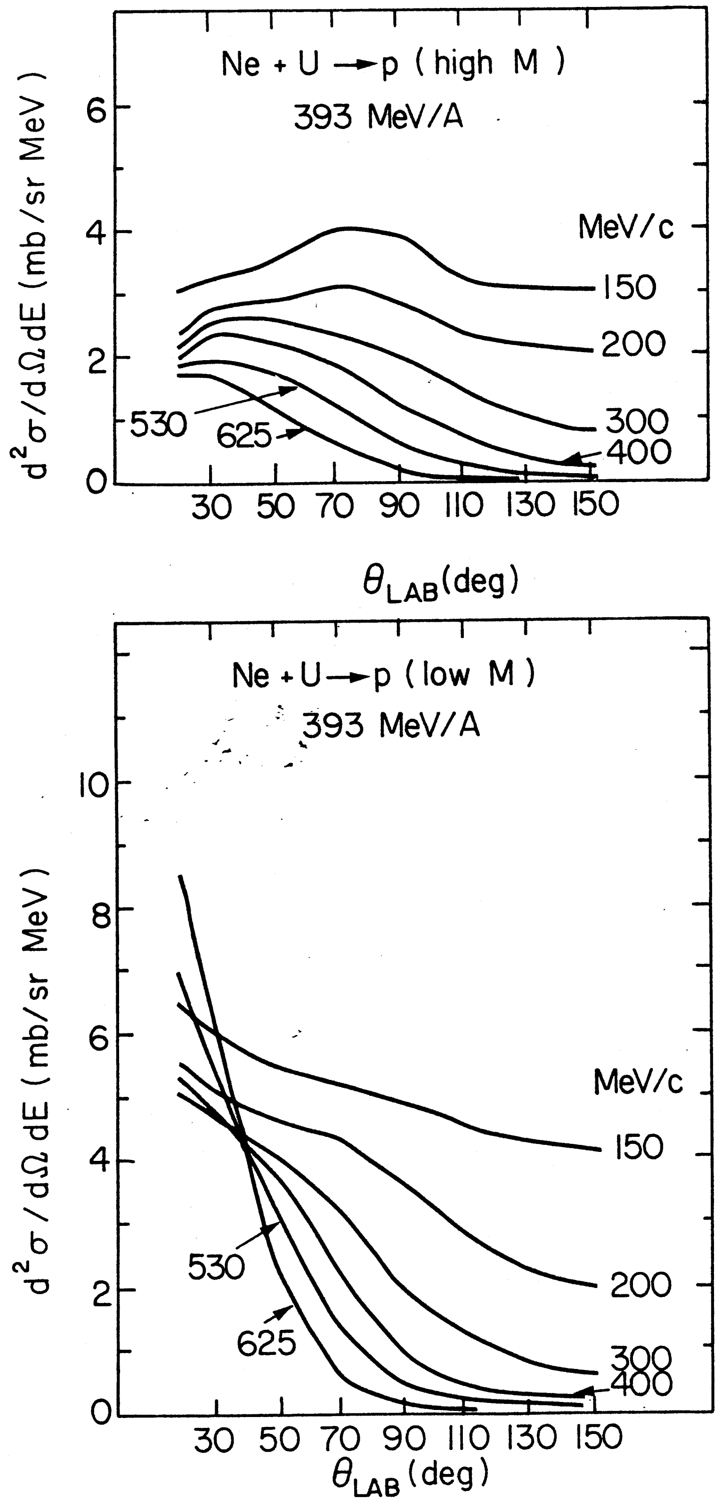

Fig. 3 : The angular distribution of protons, for fixed laboratory momentum, from the collisions of Ne with U. The graph on the top is for low associated multiplicities while the graph on the bottom is for high associated multiplicities[14).

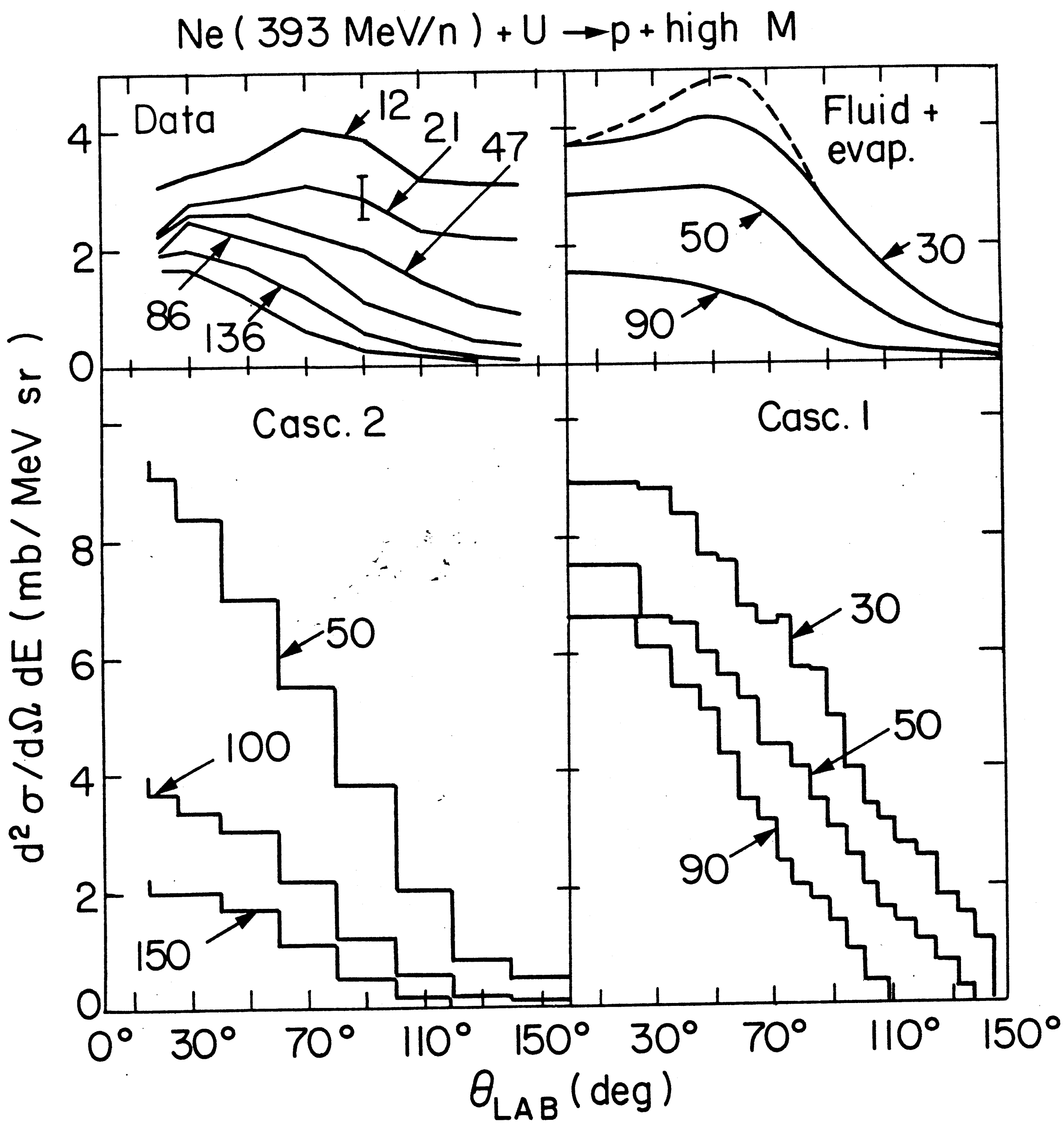

Fig. 4: The differential cross-section in the lab frame for proton production with high associated multiplicity. The data[14] is compared with[15] a hydrodynamical model and two different cascade models. The numbers indicate the proton kinetic energy in MeV.

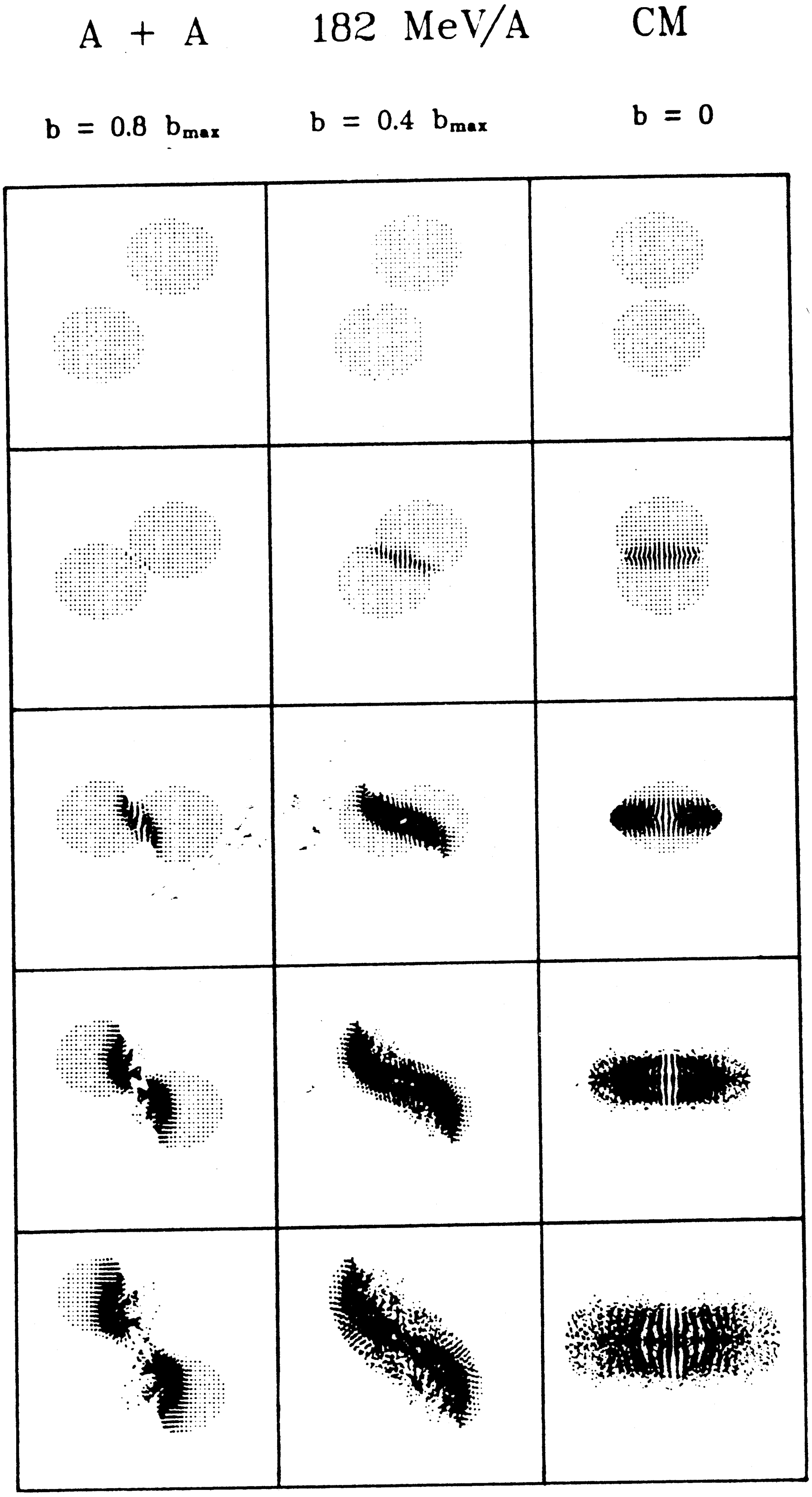

Fig. 5 : The time development, in equal time steps, of the projected baryon density in the centre-of-mass at three different impact parameters obtained from a three dimensional relativistic hydrodynamic model. The equivalent laboratory beam energy is 800 MeV per nucleon.

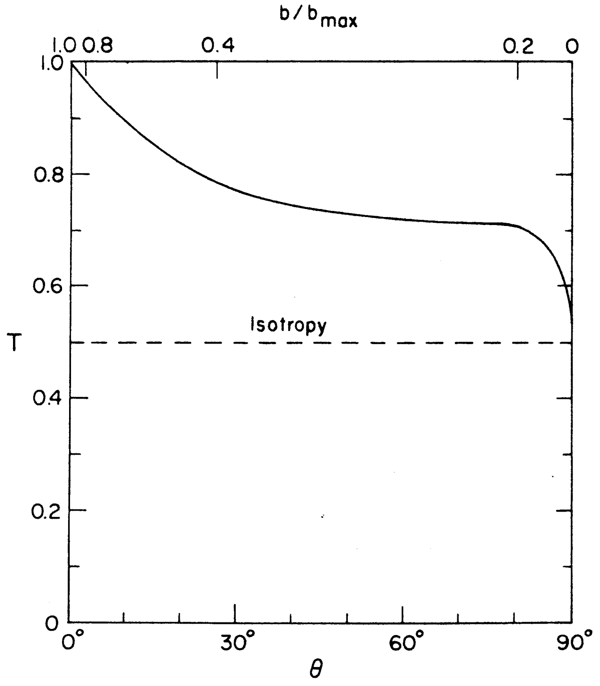

Fig. 6 : Thrust as a function of impact parameter and angle relative to the beam axis. See also Fig. 5.

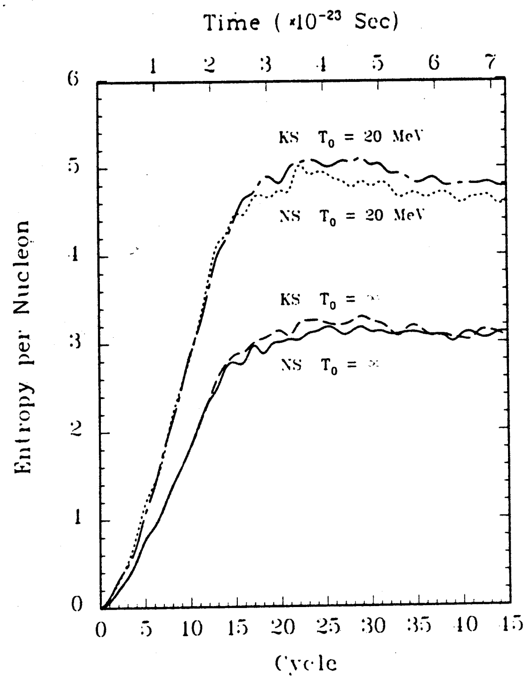

Fig. 7 : The entropy per nucleon as a function of time and computational cycle for central collisions of mass 40 nuclei at 800 MeV per nucleon. See also Fig. 5. The top curves correspond to equations of state which are thermally much softer than those corresponding to the bottom two curves.

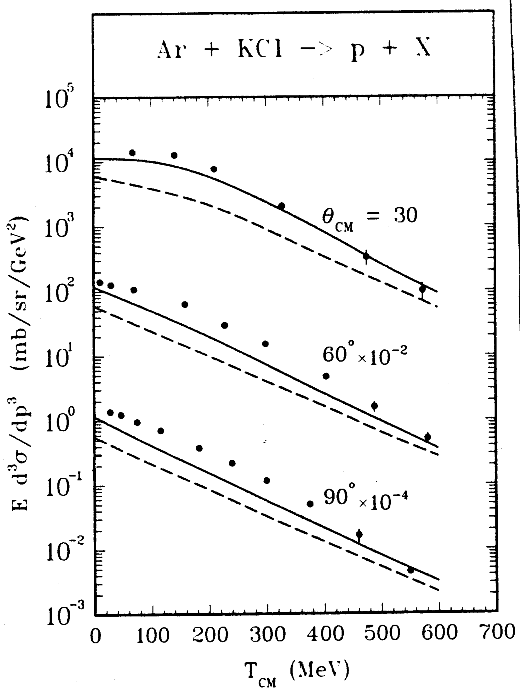

Fig. 8 : The proton invariant cross-section, plotted in the centre-of mass frame, at a beam energy of 800 MeV per nucleon. The solid line corresponds to KS T_0 = 20 MeV and the dashed line to NS $T_0 = \infty$. See Fig. 7. The data are from Ref. 10).

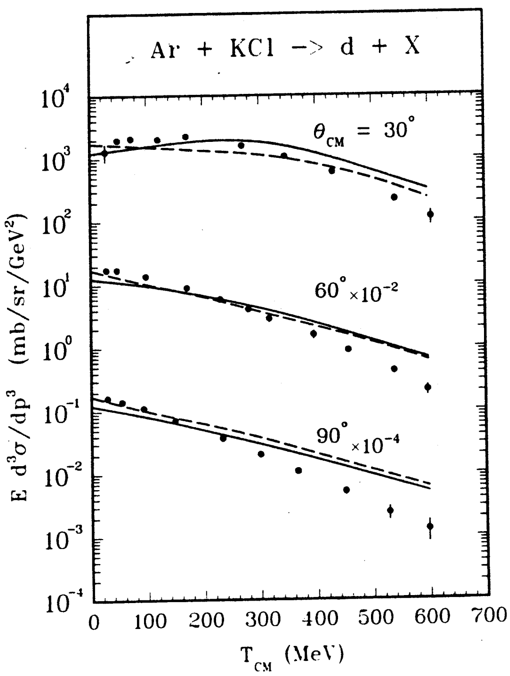

Fig. 9 : The deuteron invariant cross-section. The labelling is the same as in Fig. 8.

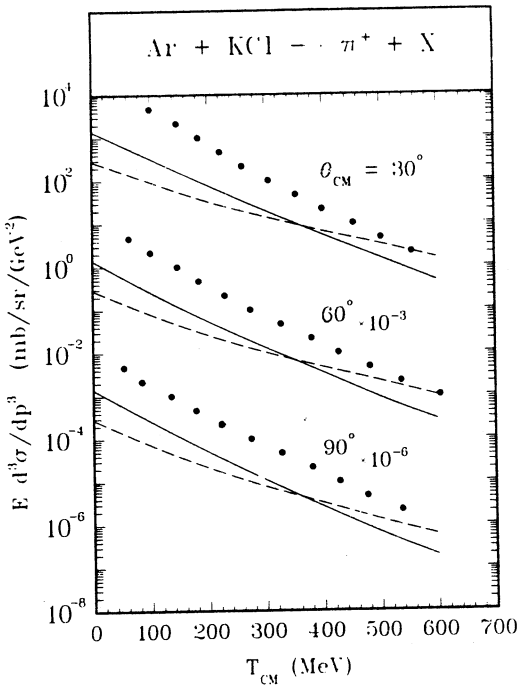

Fig. 10 : The positive ion invariant cross-section. The labelling is the same as in Fig. 8.

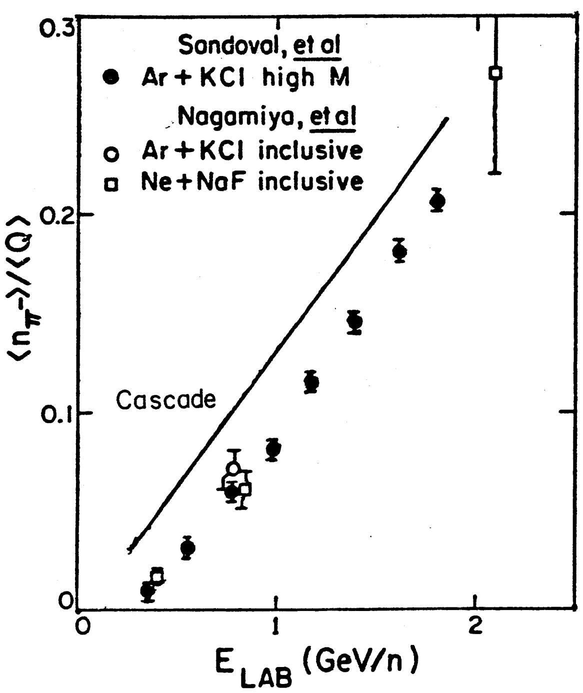

Fig. 11 : The average number of negative pions divided by the average charge as a function of beam energy. The results of a sophisticated intranuclear cascade calculation[19] for mass 40 on 40 are compared with various experiments.

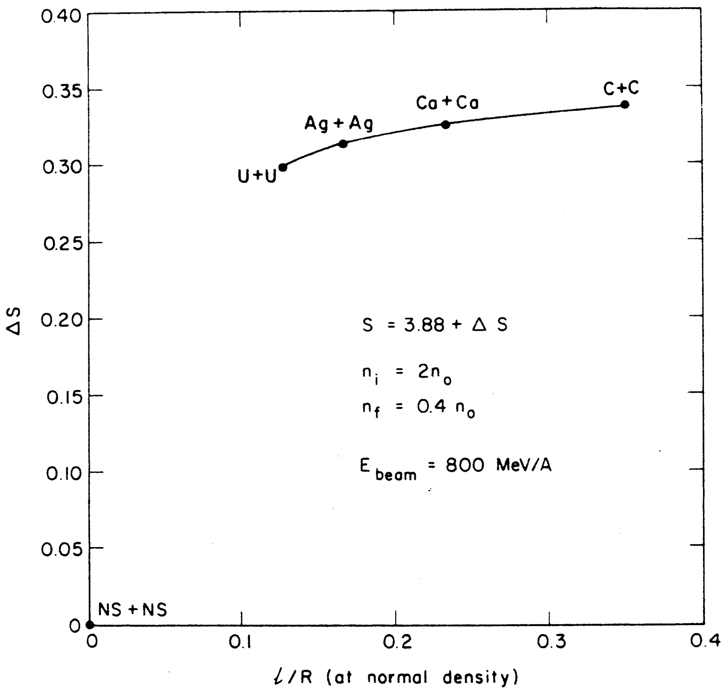

Fig. 12 : The effect of a finite mean free path on the value of the entropy generated during the expansion stage of central collisions between carbon nuclei, calcium nuclei, silver nuclei, uranium nuclei and neutron stars. The mean free path divided by the radius of the combined system is evaluated at normal nuclear density and is proportional to $A^{-1/3}$.

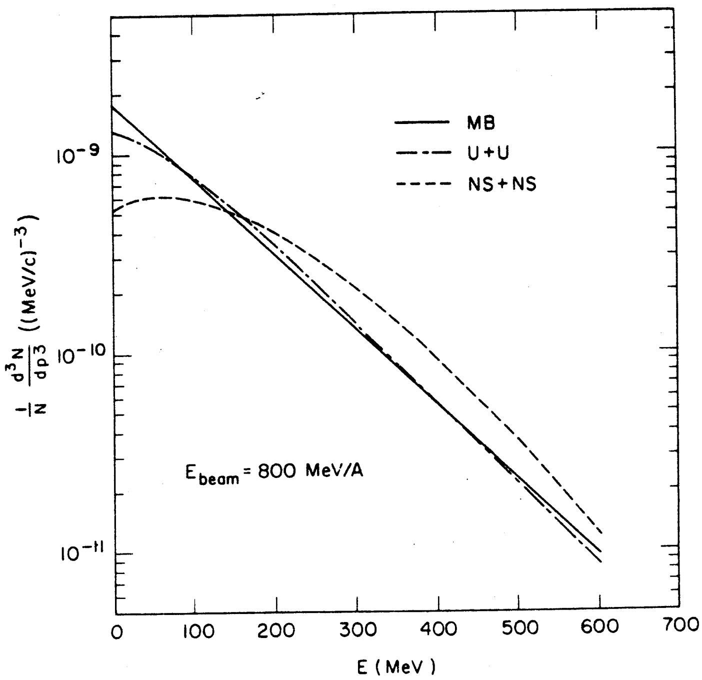

Fig. 13 : The nucleon momentum distribution for a Maxwell-Boltzmann (straight line), for a viscous uranium plus uranium expansion, and for a pure hydrodynamical expansion represented by neutron star collisions.

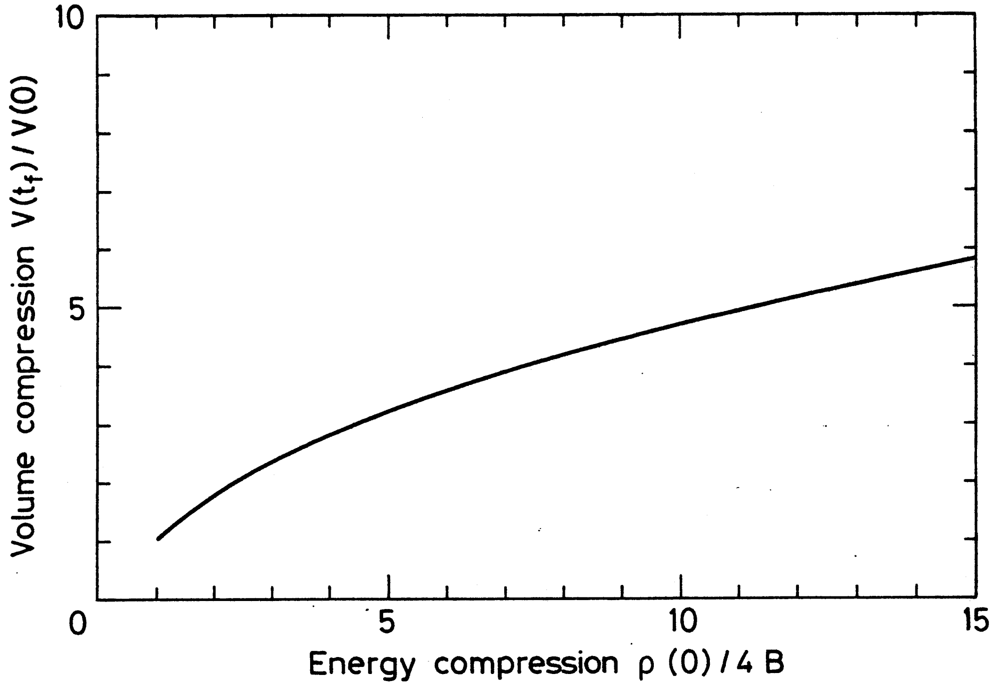

Fig. 14: The volume compression as a function of energy compression for hydrodynamically-expanding, spherical, quark-gluon fireballs.

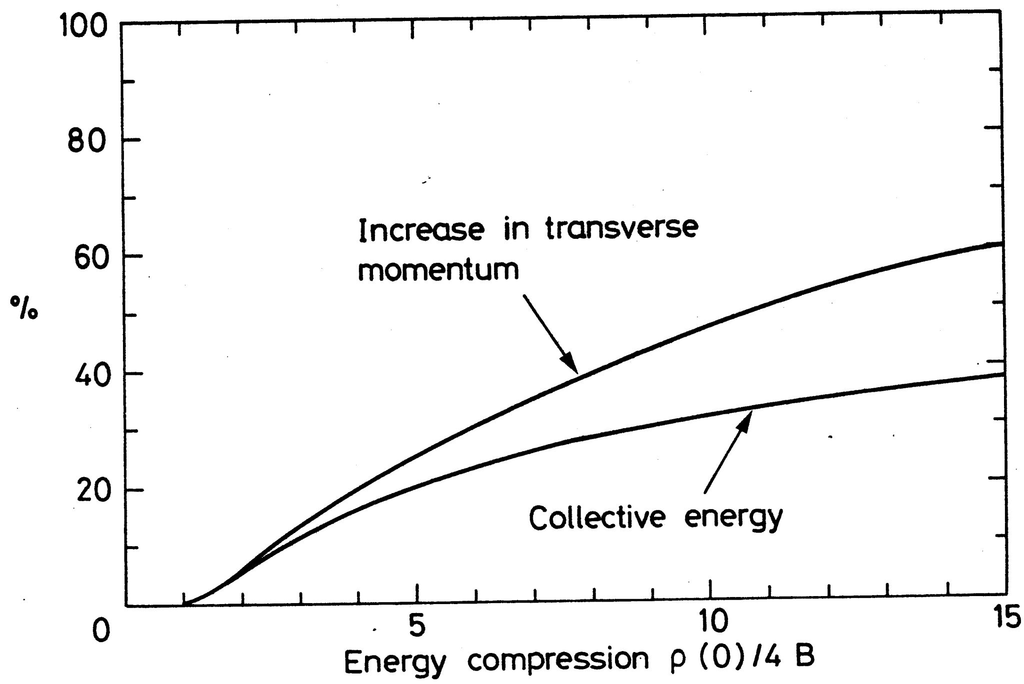

Fig. 15: The percent-increase in mean transverse momentum, and the percentage of total energy which is converted to collective flow energy, as a function of energy compression for hydrodynamically-expanding, spherical, quark-gluon fireballs.