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CP-odd observables in neutralino production with transverse e^+ and e^- beam polarization

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Abstract: We consider neutralino production and decay $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j$, $\tilde{\chi}^0_j \to \tilde{\chi}^0_1 f \bar{f}$ at a linear collider with transverse e^+ and e^- beam polarization. We propose CP asymmetries by means of the azimuthal distribution of the produced neutralinos and of that of the final leptons, while taking also into account the subsequent decays of the neutralinos. We include the complete spin correlations between production and decay. Our framework is the Minimal Supersymmetric Standard Model with complex parameters. In a numerical study we show that there are good prospects to observe these CP asymmetries at the International Linear Collider and estimate the accuracy expected for the determination of the phases in the neutralino sector.

Keywords: Supersymmetry Phenomenology, e+-e- Experiments.

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1. Introduction

Supersymmetry (SUSY) is at present one of the most prominent extensions of the Standard Model (SM) [1]. It can solve the hierachy problem, allows the unification of the gauge couplings and has the additional merit of providing new sources of CP violation. In the chargino and neutralino sectors of the Minimal Supersymmetric Standard Model (MSSM), the higgsino mass parameter μ and the gaugino mass parameter M_1 are in general complex, while the SU(2) gaugino mass parameter M_2 can be chosen real by redefining the fields. The precise determination of the underlying SUSY parameters will be one of the main goals of the high-luminosity e^+e^- International Linear Collider (ILC) [2].

The phases of the complex parameters μ and M_1 may be constrained or correlated by the experimental upper bounds on the electric dipole moments (EDM) of electron, neutron and the atoms ^{199}Hg and ^{205}Tl [3]. These constraints, however, are rather model dependent. In a constrained MSSM the restrictions on the phases of μ and M_1 can be rather severe. However, there may be cancellations between the different SUSY contributions to the EDMs, which allow larger values for the phases (for reviews see, e.g. [4]). If μ and M_1 are complex and all other parameters are real, in general the phase of μ has to be small. However, the restrictions on the phase of μ may disappear if also lepton flavour violating terms in the MSSM lagrangian are included [5]. It is, therefore, necessary to determine in an independent way the phases of the complex SUSY parameters by measurements of suitable CP-sensitive observables. Experiments with transverse $e^\pm$ beam polarization may allow us to construct suitable observables for precision studies of the effects of new physics and CP violation. Recent studies on the advantages of transversely polarized e^+ and e^- beams are presented in [6 – 8].

The study of neutralino production

$$e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j \,, \qquad i,j = 1,\ldots,4 \,, \tag{1.1}$$

and subsequent two-body decay processes

$$\tilde{\chi}^0_j \to \ell^\pm \tilde{\ell}^\mp_n \to \ell^\pm \ell^\mp \tilde{\chi}^0_1 \,, \qquad n = 1,2 \,, \tag{1.2}$$

with $\ell = e, \mu$, will play an important role at the ILC. The production process has been studied extensively in the literature (see [2] and references therein). Production and subsequent decay processes, and the decay angular and energy distributions have been studied in detail in [9]–[11]. The properties of Majorana and Dirac particles and their production and decay amplitudes under CP and CPT have been studied in [12]–[15].

In [16]–[18] it has been shown that the parameters of the chargino and neutralino systems can be determined by measuring suitable CP-even observables. However, the measurement of CP-odd observables is necessary to clearly demonstrate that CP is violated and for the unambiguous determination of the CP-violating phases. Several T-odd observables in neutralino production and decay applying triple product correlations have been proposed in [19, 20].

Transversely polarized beams offer the possibility to construct further CP-sensitive observables. This is the subject of the present paper. We propose and study CP-odd asymmetries for the case of transverse e^+ and e^- beam polarizations. We define two types of CP asymmetries, one that involves only neutralino production, the other one involving production and decay if one of the neutralinos decays as in eq. (1.2). Note that in the case of chargino production it has been shown that the analogous CP-odd asymmetries vanish [21]. In neutralino production it is possible, however, to construct CP-odd asymmetries involving the transverse beam polarization, because there are t-channel *and* u-channel contributions due to the Majorana nature of the neutralinos [7]. The formulae for the production cross section of process (1.1), for longitudinally and transversely polarized beams, have been given in [18, 22]. In this paper we derive the compact analytic formulae for the CP asymmetries with the help of the spin density matrix formalism [23], including the complete spin correlations between production and decay of the neutralinos. We study numerically the parameter and phase dependences of the CP asymmetries.

The paper is organized as follows: in section 2 we set up the definitions. We present the formulae for the cross section of (1.1) with transverse beam polarization in section 3. In section 4 we define the CP-odd asymmetries. We present a numerical investigation of these asymmetries in section 5, while section 6 contains our conclusions.

2. Lagrangian and couplings

The tree-level Feynman diagrams for the production process (1.1) are given in figure 1.

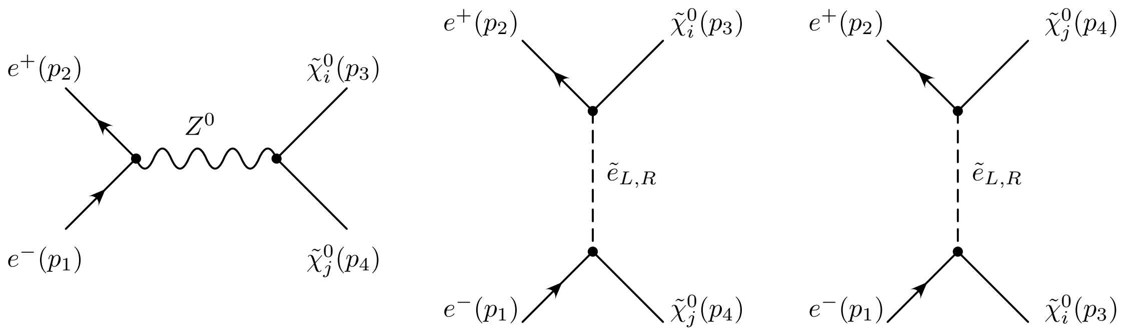

Figure 1: Feynman diagrams of the production process $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$.

The interaction Lagrangians are [1]

$$\mathcal{L}_{Z^0\ell^+\ell^-} = -\frac{g}{\cos\Theta_W} Z_\mu \bar{\ell}\gamma^\mu[L_\ell P_L + R_\ell P_R]\ell\,, \tag{2.1}$$

$$\mathcal{L}_{Z^0\tilde{\chi}_i\tilde{\chi}_j} = \frac{1}{2}\frac{g}{\cos\Theta_W} Z_\mu \bar{\tilde{\chi}}_i^0\gamma^\mu[O''^L_{ij}P_L + O''^R_{ij}P_R]\tilde{\chi}_j^0\,, \tag{2.2}$$

$$\mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}_i} = g f^L_{\ell i}\bar{\ell}P_R\tilde{\chi}_i^0\tilde{\ell}_L + g f^R_{\ell i}\bar{\ell}P_L\tilde{\chi}_i^0\tilde{\ell}_R + \text{h.c.} \tag{2.3}$$

with $i,j = 1,\ldots,4$ and the couplings

$$f^L_{\ell i} = -\sqrt{2}\left[\frac{1}{\cos\Theta_W}\left(-\frac{1}{2}+\sin^2\Theta_W\right)N_{i2} - \sin\Theta_W N_{i1}\right], \tag{2.4}$$

$$f^R_{\ell i} = \sqrt{2}\sin\Theta_W[\tan\Theta_W N^*_{i2} - N^*_{i1}]\,, \tag{2.5}$$

$$O''^L_{ij} = -\frac{1}{2}(N_{i3}N^*_{j3} - N_{i4}N^*_{j4})\cos 2\beta - \frac{1}{2}(N_{i3}N^*_{j4} + N_{i4}N^*_{j3})\sin 2\beta\,, \tag{2.6}$$

$$O''^R_{ij} = -O''^{L*}_{ij}\,, \tag{2.7}$$

$$L_\ell = -\frac{1}{2} + \sin^2\Theta_W, \quad R_\ell = \sin^2\Theta_W, \tag{2.8}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. g is the weak coupling constant ($g = e/\cos\Theta_W$, $e > 0$), Θ_W is the weak mixing angle and $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of the Higgs fields. The unitary (4×4) matrix N_{ij} diagonalizes the complex symmetric neutralino mass matrix in the basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}_a^0, \tilde{H}_b^0)$ [9].

3. Cross section

In order to calculate the squared amplitude of process (1.1) with subsequent decay (1.2), we use the spin density matrix formalism [23]. The squared amplitude of the combined process of production and decay reads

$$|T|^2 = \sum_{\lambda_i\lambda_j\lambda'_i\lambda'_j} |\Delta(\tilde{\chi}_i^0)|^2 |\Delta(\tilde{\chi}_j^0)|^2 \rho_P^{\lambda_i\lambda_j,\lambda'_i\lambda'_j} \rho_{D,\lambda'_i\lambda_i} \rho_{D,\lambda'_j\lambda_j}\,, \tag{3.1}$$

where $\Delta(\tilde{\chi}_{i,j}^0)$ is the propagator of the corresponding neutralino; $\rho_P^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}$ denotes the spin density matrix of the production, $\rho_{D,\lambda'_{i,j}\lambda_{i,j}}$ are the spin density matrices of the decay; $\lambda_{i,j}$ denotes the helicity of the neutralino $\tilde{\chi}_{i,j}^0$. The propagators are given by

$$\Delta(\tilde{\chi}_k^0) = 1/[p^2_{\tilde{\chi}_k} - m^2_{\tilde{\chi}_k} + i m_{\tilde{\chi}_k}\Gamma_{\tilde{\chi}_k}]\,, \quad k = i, j\,. \tag{3.2}$$

Here $p_{\tilde{\chi}_k}$, $m_{\tilde{\chi}_k}$ and $\Gamma_{\tilde{\chi}_k}$ denote the four-momentum, mass and total width of the neutralino $\tilde{\chi}_k^0$. For these propagators we use the narrow-width approximation. The (unnormalized) spin density production matrix is given by

$$\rho_P^{\lambda_i\lambda_j,\lambda'_i\lambda'_j} = \sum_{\lambda_{e^-}\lambda_{e^+}\lambda'_{e^-}\lambda'_{e^+}} \rho(e^-)_{\lambda'_{e^-}\lambda_{e^-}} \rho(e^+)_{\lambda'_{e^+}\lambda_{e^+}} T^{\lambda_i\lambda_j}_{P,\lambda_{e^-}\lambda_{e^+}} T^{\lambda'_i\lambda'_j *}_{P,\lambda'_{e^-}\lambda'_{e^+}}\,, \tag{3.3}$$

where $T^{\lambda_i\lambda_j}_{P,\lambda_{e^-}\lambda_{e^+}}$ is the helicity amplitude of the production process and $\lambda_{e^\pm}$ is the helicity of $e^\pm$. The spin density decay matrices can be written as

$$\rho_{D,\lambda'_i\lambda_i} = T_{D,\lambda_i} T^*_{D,\lambda'_i}\,, \tag{3.4}$$

$$\rho_{D,\lambda'_j\lambda_j} = T_{D,\lambda_j} T^*_{D,\lambda'_j}\,, \tag{3.5}$$

where $T_{D,\lambda_{i,j}}$ is the helicity amplitude for the decay. The spin density matrices of the polarized e^+ and e^- beam in eq. (3.3) can be written as

$$\rho(e^\pm) = \frac{1}{2}(1 + \mathcal{P}_i^\pm \sigma^i)\,, \tag{3.6}$$

where $\mathcal{P}_1^\pm$ is the degree of transverse $e^\pm$ beam polarization in the production plane, $\mathcal{P}_2^\pm$ is the degree of transverse $e^\pm$ beam polarization perpendicular to the production plane, $\mathcal{P}_3^\pm = \mathcal{P}_L^\pm$ $(-1 \le \mathcal{P}_L^\pm \le 1)$ is the degree of longitudinal $e^\pm$ beam polarization and σ^i $(i = 1, 2, 3)$ are the Pauli matrices. For the degrees of transverse beam polarizations we have the relation $(\mathcal{P}_1^{\pm})^2 + (\mathcal{P}_2^{\pm})^2 = (\mathcal{P}_T^{\pm})^2$ with $\mathcal{P}_1^{\pm} = \cos\phi_{\pm}\mathcal{P}_T^{\pm}$ and $\mathcal{P}_2^{\pm} = \sin\phi_{\pm}\mathcal{P}_T^{\pm}$ $(0 \leq \mathcal{P}_T^{\pm} \leq 1$ and $(\mathcal{P}_L^{\pm})^2 + (\mathcal{P}_T^{\pm})^2 \leq 1)$, see figure 2 [1].

The production and decay matrices are calculated with the help of the Bouchiat-Michel formulae [24]:

$$u(p,\lambda^{'})\bar{u}(p,\lambda) = \frac{1}{2}[\delta_{\lambda\lambda'} + \gamma_5 \not{s}^a \sigma^a_{\lambda\lambda'}](\not{p}+m)\,, \tag{3.7}$$

$$v(p,\lambda^{'})\bar{v}(p,\lambda) = \frac{1}{2}[\delta_{\lambda'\lambda} + \gamma_5 \not{s}^a \sigma^a_{\lambda'\lambda}](\not{p}-m)\,. \tag{3.8}$$

The three four-component spin basis vectors s^a and the 4-vector p/m form an orthonormal system. For the incoming particles e^+ and e^- in the limit of vanishing electron mass, $m_e \to 0$, eqs. (3.7) and (3.8) can be written as

$$u(p_{e^-},\lambda_{e^-})\bar{u}(p_{e^-},\lambda^{'}_{e^-}) = \frac{1}{2}\Big\{(1{+}2\lambda_{e^-}\gamma_5)\delta_{\lambda^{'}_{e^-}\lambda_{e^-}} + \gamma_5\left[\not{t}^1_{e^-}\sigma^1_{\lambda^{'}_{e^-}\lambda_{e^-}} + \not{t}^2_{e^-}\sigma^2_{\lambda^{'}_{e^-}\lambda_{e^-}}\right]\Big\}\,\not{p}_{e^-}, \tag{3.9}$$

$$v(p_{e^+},\lambda^{'}_{e^+})\bar{v}(p_{e^+},\lambda_{e^+}) = \frac{1}{2}\Big\{(1{-}2\lambda_{e^+}\gamma_5)\delta_{\lambda^{'}_{e^+}\lambda_{e^+}} + \gamma_5\left[\not{t}^1_{e^+}\sigma^1_{\lambda^{'}_{e^+}\lambda_{e^+}} + \not{t}^2_{e^+}\sigma^2_{\lambda^{'}_{e^+}\lambda_{e^+}}\right]\Big\}\,\not{p}_{e^+}, \tag{3.10}$$

where $t^1_{e^\pm}$ and $t^2_{e^\pm}$ are the basis 4-vectors of the transverse polarization of the electron and positron beam, respectively. Thus, the transverse polarization 4-vectors can be written as

$$t_{\pm} = \cos(\phi_{\pm} - \phi)t^1_{e^\pm} + \sin(\phi_{\pm} - \phi)t^2_{e^\pm}\,, \tag{3.11}$$

where ϕ is the azimuthal angle of the scattering plane and $\phi_{\pm}$ are the azimuthal angles of the transverse polarization with respect to a fixed reference system, see figure 2. A convenient choice of $t^1_{e^\pm}$ and $t^2_{e^\pm}$ in the laboratory system is given in appendix A.

With eqs. (3.7)–(3.10), the spin density production matrix and the spin density decay matrices can be expanded in terms of the Pauli matrices σ^a and σ^b, where the superscripts a $(b) = 1,2,3$ refer to the polarization vectors of $\tilde{\chi}^0_i$ $(\tilde{\chi}^0_j)$:

$$\begin{aligned}
\rho_P^{\lambda_i\lambda_j,\lambda^{'}_i\lambda^{'}_j} &= \delta_{\lambda_i\lambda^{'}_i}\delta_{\lambda_j\lambda^{'}_j}P(\tilde{\chi}^0_i\tilde{\chi}^0_j) + \delta_{\lambda_j\lambda^{'}_j}\sum_{a=1}^{3}\sigma^a_{\lambda_i\lambda^{'}_i}\Sigma^a_P(\tilde{\chi}^0_i) \\
&\quad + \delta_{\lambda_i\lambda^{'}_i}\sum_{b=1}^{3}\sigma^b_{\lambda_j\lambda^{'}_j}\Sigma^b_P(\tilde{\chi}^0_j) + \sum_{a,b=1}^{3}\sigma^a_{\lambda_i\lambda^{'}_i}\sigma^b_{\lambda_j\lambda^{'}_j}\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)\,,
\end{aligned} \tag{3.12}$$

$$\rho_{D,\lambda^{'}_i\lambda_i} = \delta_{\lambda^{'}_i\lambda_i}D(\tilde{\chi}^0_i) + \sum_{a=1}^{3}\sigma^a_{\lambda^{'}_i\lambda_i}\Sigma^a_D(\tilde{\chi}^0_i)\,, \tag{3.13}$$

$$\rho_{D,\lambda^{'}_j\lambda_j} = \delta_{\lambda^{'}_j\lambda_j}D(\tilde{\chi}^0_j) + \sum_{b=1}^{3}\sigma^b_{\lambda^{'}_j\lambda_j}\Sigma^b_D(\tilde{\chi}^0_j)\,. \tag{3.14}$$

The contribution $P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ is independent of the neutralino polarization, whereas $\Sigma^a_P(\tilde{\chi}^0_i)$ and $\Sigma^b_P(\tilde{\chi}^0_j)$ depend on the polarization of the corresponding neutralino.

[1] At this point we note that, contrary to the usual conditions at a circular accelerator, where the Sokolov-Ternov effect orientates automatically both transverse polarization vectors either parallel or antiparallel (depending on the sign of the charge of the incoming particle), there is the possibility, at the ILC, to choose an *arbitrary* transverse polarization for both e^+ and e^-, independent from each other.

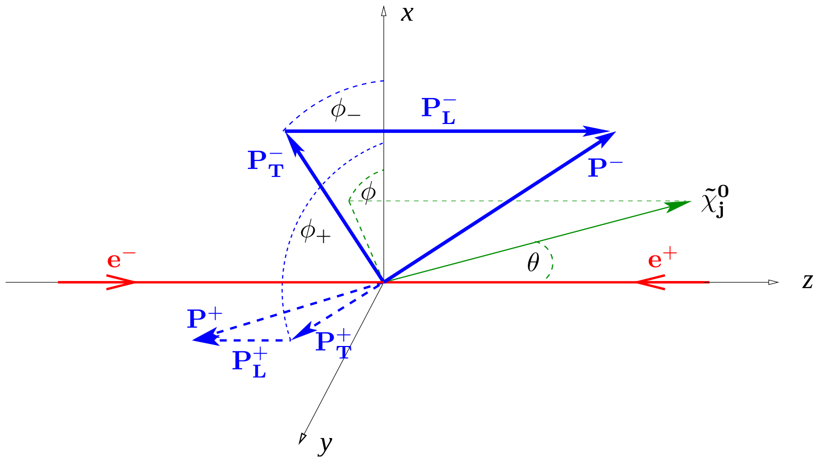

Figure 2: Decomposition of the $e^\pm$ polarization vectors $\mathbf{P}^\pm$ into the longitudinal components $\mathbf{P}^\pm_{\mathbf{L}}$ in the direction of the electron/positron momentum and the transverse components $\mathbf{P}^\pm_{\mathbf{T}} = \mathcal{P}^\pm_T \vec{t}_\pm$ with respect to a fixed coordinate system (x, y, z). The z-axis is in the direction of the electron momentum.

Then $\Sigma^3_P(\tilde{\chi}^0_{i,j})/P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ gives the longitudinal polarization of the neutralino $\tilde{\chi}^0_{i,j}$; $\Sigma^1_P(\tilde{\chi}^0_{i,j})/P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ is the transverse polarization of the neutralino in the scattering plane; and $\Sigma^2_P(\tilde{\chi}^0_{i,j})/P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ is the polarization perpendicular to the scattering plane. The terms $\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ describe the spin correlations between the polarizations of the two produced particles. The contribution of the decay matrix, which is independent of the neutralino polarization, is denoted by $D(\tilde{\chi}^0_{i,j})$, and $\Sigma^{a,b}_D(\tilde{\chi}^0_{i,j})$ denotes the contribution that depends on the neutralino polarization.

With eqs. (3.12)–(3.14) the squared amplitude $|T|^2$, eq. (3.1), of the combined process of production and decay, for arbitrarily polarized beams, is given by

$$|T|^2 = 4|\Delta(\tilde{\chi}^0_i)|^2|\Delta(\tilde{\chi}^0_j)|^2\Bigg[P(\tilde{\chi}^0_i\tilde{\chi}^0_j)D(\tilde{\chi}^0_i)D(\tilde{\chi}^0_j) + \sum_{a=1}^{3}\Sigma^a_P(\tilde{\chi}^0_i)\Sigma^a_D(\tilde{\chi}^0_i)D(\tilde{\chi}^0_j) + \sum_{b=1}^{3}\Sigma^b_P(\tilde{\chi}^0_j)\Sigma^b_D(\tilde{\chi}^0_j)D(\tilde{\chi}^0_i) + \sum_{a,b=1}^{3}\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)\Sigma^a_D(\tilde{\chi}^0_i)\Sigma^b_D(\tilde{\chi}^0_j)\Bigg], \tag{3.15}$$

where the contributions $P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$, $\Sigma^a_P(\tilde{\chi}^0_i)$, $\Sigma^b_P(\tilde{\chi}^0_j)$, $\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ of the production spin density matrix include the terms for arbitrary beam polarizations. The differential cross section of the production and decay process is given by

$$\mathrm{d}\sigma = \frac{1}{2s}|T|^2\mathrm{dLips}(s, p_i)\,, \tag{3.16}$$

where $\mathrm{dLips}(s, p_i) = (2\pi)^4\delta^4(p_1 + p_2 - \sum_i p_i)\prod_i \frac{\mathrm{d}^3p_i}{(2\pi)^3 2E_i}$; see appendix B for more details.

3.1 Contributions to the production spin density matrix independent of neutralino polarization

In the following we regard the contribution $P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ to the spin density production matrix, see eq. (3.12), that is independent of the polarization of the neutralinos $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$. We only list the contribution $P(\tilde{\chi}_i^0\tilde{\chi}_j^0)_T$, which depends on the transverse $e^\pm$ beam polarization:

$$P(\tilde{\chi}_i^0\tilde{\chi}_j^0)_T = P(ZZ)_T + P(Z\tilde{e}_L)_T + P(Z\tilde{e}_R)_T + P(\tilde{e}_L\tilde{e}_R)_T, \tag{3.17}$$

$$P(ZZ)_T = -\mathcal{P}_T^-\mathcal{P}_T^+ \, \frac{g^4}{\cos^4\Theta_W}|\Delta(Z)|^2 L_e R_e(|O_{ij}^L|^2 + |O_{ij}^R|^2) r_1\,, \tag{3.18}$$

$$\begin{aligned} P(Z\tilde{e}_L)_T = & -\mathcal{P}_T^-\mathcal{P}_T^+ \, \frac{1}{2}\frac{g^2}{\cos^2\Theta_W}R_e \\ & \times \{\mathrm{Re}(\Delta(Z) f_{\ell i}^{L*} f_{\ell j}^{L} O_{ij}^L[\Delta(\tilde{e}_L,t)^* + \Delta(\tilde{e}_L,u)^*])\; r_1 \\ & + \mathrm{Im}(\Delta(Z) f_{\ell i}^{L*} f_{\ell j}^{L} O_{ij}^L[\Delta(\tilde{e}_L,t)^* - \Delta(\tilde{e}_L,u)^*])\; r_2\}\,, \end{aligned} \tag{3.19}$$

$$\begin{aligned} P(Z\tilde{e}_R)_T = & -\mathcal{P}_T^-\mathcal{P}_T^+ \, \frac{1}{2}\frac{g^2}{\cos^2\Theta_W}L_e \\ & \times \{\mathrm{Re}(\Delta(Z) f_{\ell i}^{R*} f_{\ell j}^{R} O_{ij}^R[\Delta(\tilde{e}_R,t)^* + \Delta(\tilde{e}_R,u)^*])\; r_1 \\ & - \mathrm{Im}(\Delta(Z) f_{\ell i}^{R*} f_{\ell j}^{R} O_{ij}^R[\Delta(\tilde{e}_R,t)^* - \Delta(\tilde{e}_R,u)^*])\; r_2\}\,, \end{aligned} \tag{3.20}$$

$$\begin{aligned} P(\tilde{e}_L\tilde{e}_R)_T = & \; \mathcal{P}_T^-\mathcal{P}_T^+ \, \frac{1}{4}g^4 \\ & \times \{\mathrm{Re}([\Delta(\tilde{e}_L,t)\Delta(\tilde{e}_R,u)^* + \Delta(\tilde{e}_L,u)\Delta(\tilde{e}_R,t)^*] f_{\ell i}^{L*} f_{\ell j}^{L} f_{\ell i}^{R*} f_{\ell j}^{R})\; r_1 \\ & + \mathrm{Im}([\Delta(\tilde{e}_L,t)\Delta(\tilde{e}_R,u)^* - \Delta(\tilde{e}_L,u)\Delta(\tilde{e}_R,t)^*] f_{\ell i}^{L*} f_{\ell j}^{L} f_{\ell i}^{R*} f_{\ell j}^{R})\; r_2\}\,, \end{aligned} \tag{3.21}$$

where we have introduced the notation

$$\begin{aligned} r_1 = & \; [(t_- p_4)(t_+ p_3) + (t_- p_3)(t_+ p_4)](p_1 p_2) \\ & + [(p_1 p_4)(p_2 p_3) + (p_1 p_3)(p_2 p_4) - (p_1 p_2)(p_3 p_4)](t_- t_+)\,, \end{aligned} \tag{3.22}$$

$$\begin{aligned} r_2 = & \; \varepsilon^{\mu\nu\rho\sigma}[t_{+,\mu} p_{1\nu} p_{2\rho} p_{4\sigma}(t_- p_3) + t_{-,\mu} p_{1\nu} p_{2\rho} p_{3\sigma}(t_+ p_4) \\ & + t_{-,\mu} t_{+,\nu} p_{2\rho} p_{4\sigma}(p_1 p_3) + t_{-,\mu} t_{+,\nu} p_{1\rho} p_{3\sigma}(p_2 p_4)]\,, \end{aligned} \tag{3.23}$$

where $\varepsilon^{0123} = 1$ and $\Delta(Z) = i/(s - m_Z^2)$, $\Delta(\tilde{e}_{L,R},t) = i/(t - m_{\tilde{e}_{L,R}}^2)$, $\Delta(\tilde{e}_{L,R},u) = i/(u - m_{\tilde{e}_{L,R}}^2)$ with $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$, $u = (p_1 - p_3)^2$. The masses of the selectrons and the Z-boson are given by $m_{\tilde{e}_{L,R}}$ and m_Z. Since we study the process far beyond the Z-threshold, the Z-width can be neglected, and all propagators can be taken as purely imaginary.

Note that only terms bilinearly dependent on transverse beam polarizations appear for $m_e \to 0$, because the couplings to e^+e^- are vector- or axial-vector-like [25, 26] (for the $\tilde{e}_{L,R}$ exchange the coupling to e^+e^- can be brought to that form via Fierz identities [27]). Inspecting eqs. (3.18)–(3.21), we note that transverse beam polarization gives rise to the interference term $P(\tilde{e}_L\tilde{e}_R)_T$, which is absent for longitudinal beam polarization [11]. On the other hand, there are no terms $P(\tilde{e}_L\tilde{e}_L)_T$ and $P(\tilde{e}_R\tilde{e}_R)_T$ for transversely polarized beams, but only for longitudinally polarized beams. Both are consequences of the Dirac algebra, since transverse beam polarization is described with an additional γ matrix; see eqs. (3.9) and (3.10).

The differential cross section for the process $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ is given by

$$\mathrm{d}\sigma = \frac{1}{2(2\pi)^2}\frac{q}{s^{3/2}}\; P(\tilde{\chi}_i^0\tilde{\chi}_j^0)\; \mathrm{d}\cos\theta\; \mathrm{d}\phi\,, \tag{3.24}$$

where $P(\tilde{\chi}_i^0\tilde{\chi}_j^0)$ contains the terms for arbitrary beam polarization and q is the momentum of the neutralinos in the center-of-mass system (cms) (see appendix A).

3.1.1 CP-behaviour of the kinematical quantities

In the following, the CP properties of the kinematical quantity r_2, eq. (3.23), and of the propagator difference $[\Delta(\tilde{e},t) - \Delta(\tilde{e},u)]$ are discussed. These quantities contribute to the interference terms of the matrix element squared and are proportional to the imaginary parts of products of couplings, see eqs. (3.19)–(3.21). In the cms, r_2 is given by:

$$r_2 = 2E_b\,[\vec{t}_+(\vec{p}_1 \times \vec{p}_4)(t_- p_3) + \vec{t}_-(\vec{p}_1 \times \vec{p}_3)(t_+ p_4)]\,, \tag{3.25}$$

where E_b is the beam energy. Note that the second line of eq. (3.23) vanishes in the cms. Applying a CP transformation to r_2, eq. (3.25), with the following transformations $(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \vec{t}_-) \overset{CP}{\longleftrightarrow} (\vec{p}_1, \vec{p}_2, -\vec{p}_3, -\vec{p}_4, \vec{t}_+)$, we find that r_2 is CP-even. Since under CP $\Delta(\tilde{e},t) \overset{CP}{\longleftrightarrow} \Delta(\tilde{e},u)$ the propagator differences in eqs. (3.19)–(3.21) are CP-odd, their products with the CP-even quantity r_2 are CP-odd. We emphasize that this is due to the Majorana nature of the neutralinos, which leads to the simultaneous presence of the t- *and* u-channel contributions, that the terms in eqs. (3.19)–(3.21), which involve the imaginary part of the couplings, are non-vanishing in general.

3.2 Contributions to the production spin density matrix dependent on neutralino polarization

We now consider the terms $\Sigma_P^b(\tilde{\chi}_j^0)$ of the production spin density matrix, which depend on the polarization 4-vector s^b of the neutralino $\tilde{\chi}_j^0$. In the following we only list the terms $\Sigma_P^b(\tilde{\chi}_j^0)_T$, that involve the transverse beam polarization (for the contributions independent of the beam polarization and the terms that depend on the longitudinal beam polarization, see [11]):

$$\Sigma_P^b(\tilde{\chi}_j^0)_T = \Sigma_P^b(Z\tilde{e}_L)_T + \Sigma_P^b(Z\tilde{e}_R)_T + \Sigma_P^b(\tilde{e}_L\tilde{e}_R)_T\,, \tag{3.26}$$

$$\begin{aligned}\Sigma_P^b(Z\tilde{e}_L)_T = {} & \mathcal{P}_T^-\mathcal{P}_T^+\,\frac{1}{2}\frac{g^2}{\cos^2\Theta_W}R_e \\ & \times \{\mathrm{Re}(\Delta(Z) f_{\ell i}^{L*} f_{\ell j}^L O_{ij}^L[\Delta(\tilde{e}_L,u)^* - \Delta(\tilde{e}_L,t)^*])\; r_1^b \\ & - \mathrm{Im}(\Delta(Z) f_{\ell i}^{L*} f_{\ell j}^L O_{ij}^L[\Delta(\tilde{e}_L,u)^* + \Delta(\tilde{e}_L,t)^*])\; r_2^b\}\,,\end{aligned} \tag{3.27}$$

$$\begin{aligned}\Sigma_P^b(Z\tilde{e}_R)_T = {} & \mathcal{P}_T^-\mathcal{P}_T^+\,\frac{1}{2}\frac{g^2}{\cos^2\Theta_W}L_e \\ & \times \{\mathrm{Re}(\Delta(Z) f_{\ell i}^{R*} f_{\ell j}^R O_{ij}^R[\Delta(\tilde{e}_R,u)^* - \Delta(\tilde{e}_R,t)^*])\; r_1^b \\ & + \mathrm{Im}(\Delta(Z) f_{\ell i}^{R*} f_{\ell j}^R O_{ij}^R[\Delta(\tilde{e}_R,u)^* + \Delta(\tilde{e}_R,t)^*])\; r_2^b\}\,,\end{aligned} \tag{3.28}$$

$$\begin{aligned}\Sigma_P^b(\tilde{e}_L\tilde{e}_R)_T = {} & \mathcal{P}_T^-\mathcal{P}_T^+\,\frac{1}{4}g^4 \\ & \times \{\mathrm{Re}([\Delta(\tilde{e}_L,t)\Delta(\tilde{e}_R,u)^* - \Delta(\tilde{e}_L,u)\Delta(\tilde{e}_R,t)^*] f_{\ell i}^{L*} f_{\ell j}^L f_{\ell i}^{R*} f_{\ell j}^R)\; r_1^b \\ & + \mathrm{Im}([\Delta(\tilde{e}_L,t)\Delta(\tilde{e}_R,u)^* + \Delta(\tilde{e}_L,u)\Delta(\tilde{e}_R,t)^*] f_{\ell i}^{L*} f_{\ell j}^L f_{\ell i}^{R*} f_{\ell j}^R)\; r_2^b\}\end{aligned} \tag{3.29}$$

with the following notation

$$r_1^b = m_{\chi_j}\{[(t_- s^b)(t_+ p_3) + (t_- p_3)(t_+ s^b)](p_1 p_2) \\ +[(p_1 s^b)(p_2 p_3) + (p_1 p_3)(p_2 s^b) - (p_1 p_2)(p_3 s^b)](t_- t_+)\}\,, \tag{3.30}$$

$$r_2^b = \varepsilon^{\mu\nu\rho\sigma}\ m_{\chi_j}[t_{+,\mu} p_{1\nu} p_{2\rho} s^b_\sigma (t_- p_3) + t_{-,\mu} p_{1\nu} p_{2\rho} p_{3\sigma}(t_+ s^b) \\ +t_{-,\mu} t_{+,\nu} p_{2\rho} s^b_\sigma (p_1 p_3) + t_{-,\mu} t_{+,\nu} p_{1\rho} p_{3\sigma}(p_2 s^b)]\,, \tag{3.31}$$

where the $e^\pm$ polarization vector $t_\pm$ is given by eq. (3.11). The polarization basis 4-vectors s^b of the neutralino $\tilde\chi^0_j$ fulfil the orthogonality relations $s^b \cdot s^c = -\delta^{bc}$ and $s^b \cdot p_4 = 0$. The parametrization of the neutralino spin vectors is given in appendix A. The terms $\Sigma^a_P(\tilde\chi^0_i)_T$, which depend on the polarization 4-vector s^a of $\tilde\chi^0_i$, are obtained by the substitutions $s^b \to -s^a, m_{\chi_j} \to m_{\chi_i}, p_3 \to p_4$ in eqs. (3.30) and (3.31). Note that, like $P(\tilde\chi^0_i\tilde\chi^0_j)_T$, the expressions $\Sigma^b_P(\tilde\chi^0_j)_T$ contain no contributions $\Sigma^b_P(\tilde e_L \tilde e_L)_T$ and $\Sigma^b_P(\tilde e_R \tilde e_R)_T$, but an interference term, $\Sigma^b_P(\tilde e_L \tilde e_R)_T$. Furthermore, owing to the Majorana character of the neutralinos, there is no contribution $\Sigma^b_P(ZZ)_T$. This is contrary to the cases of unpolarized and longitudinally polarized beams [11].

4. CP asymmetries with transverse beam polarization

4.1 CP asymmetries in neutralino production

In this section we construct CP asymmetries for the production process $e^+e^- \to \tilde\chi^0_i\tilde\chi^0_j$ with transverse e^+ and e^- beam polarizations. The corresponding cross section is given in eq. (3.24). Choosing the e^- beam direction along the z-axis in the reference system (see appendix A and figure 2), the kinematical quantities in eqs. (3.22) and (3.23) can be rewritten as

$$r_1 = -2E_b^2\ q^2 \sin^2\theta \cos(\eta - 2\phi)\,, \tag{4.1}$$

$$r_2 = 2E_b^2\ q^2 \sin^2\theta \sin(\eta - 2\phi)\,, \tag{4.2}$$

where $\eta = \phi_- + \phi_+$. The CP-sensitive terms ($\propto r_2 \propto \sin(\eta - 2\phi)$) can be extracted from the amplitude squared by an appropriate integration over the azimuthal angle ϕ. We define the resulting asymmetry as

$$A_{CP}(\theta) = \frac{\mathrm{N}[\sin(\eta-2\phi)>0;\theta] - \mathrm{N}[\sin(\eta-2\phi)<0;\theta]}{\mathrm{N}[\sin(\eta-2\phi)>0;\theta] + \mathrm{N}[\sin(\eta-2\phi)<0;\theta]} \\ = \frac{1}{\sigma}\left[-\int_{\frac{\eta}{2}}^{\frac{\pi}{2}+\frac{\eta}{2}} + \int_{\frac{\pi}{2}+\frac{\eta}{2}}^{\pi+\frac{\eta}{2}} - \int_{\pi+\frac{\eta}{2}}^{\frac{3\pi}{2}+\frac{\eta}{2}} + \int_{\frac{3\pi}{2}+\frac{\eta}{2}}^{2\pi+\frac{\eta}{2}}\right]\frac{\mathrm{d}^2\sigma}{\mathrm{d}\phi\,\mathrm{d}\theta}\mathrm{d}\phi\,, \tag{4.3}$$

which depends on the polar angle θ. The first line in eq. (4.3) exhibits how the asymmetry is obtained in the experiment, where $\mathrm{N}[\sin(\eta-2\phi) > 0\,(<0)]$ denotes the number of events with $\sin(\eta - 2\phi) > 0\,(<0)$. The second line in eq. (4.3) shows how the asymmetry is calculated. We can infer from eqs. (3.19)–(3.21) that $A_{CP}(\theta)$, eq. (4.3), would be zero if integrated over the whole range of θ: because of the propagators, the contribution of the t-channel cancels that of the u-channel. Therefore, we divide the integration over θ into two ranges in order to obtain the CP asymmetry

$$\begin{aligned} A_{CP} = & \Big\{ \mathrm{N}[\sin(\eta - 2\phi) > 0; \cos\theta > 0] - \mathrm{N}[\sin(\eta - 2\phi) > 0; \cos\theta < 0] \\ & + \mathrm{N}[\sin(\eta - 2\phi) < 0; \cos\theta < 0] - \mathrm{N}[\sin(\eta - 2\phi) < 0; \cos\theta > 0] \Big\} / \mathrm{N_{tot}} \\ = & \left[\int_0^{\pi/2} - \int_{\pi/2}^{\pi} \right] A_{CP}(\theta)\, \mathrm{d}\theta\,, \end{aligned} \tag{4.4}$$

where $\mathrm{N_{tot}}$ denotes the total number of events. Note that for a measurement of the CP asymmetry in eq. (4.4) the production plane has to be reconstructed. In appendix C we propose how this can be done.

Finally we remark that an azimuthal asymmetry, analogous to that studied for chargino production [21], can be defined also for neutralino production. It is given by

$$\begin{aligned} A_{\phi} = & \frac{\mathrm{N}[\cos(\eta - 2\phi) > 0] - \mathrm{N}[\cos(\eta - 2\phi) < 0]}{\mathrm{N}[\cos(\eta - 2\phi) > 0] + \mathrm{N}[\cos(\eta - 2\phi) < 0]} \\ = & \frac{1}{\sigma} \left[-\int_{\frac{\pi}{4}+\frac{\eta}{2}}^{\frac{3\pi}{4}+\frac{\eta}{2}} + \int_{\frac{3\pi}{4}+\frac{\eta}{2}}^{\frac{5\pi}{4}+\frac{\eta}{2}} - \int_{\frac{5\pi}{4}+\frac{\eta}{2}}^{\frac{7\pi}{4}+\frac{\eta}{2}} + \int_{\frac{7\pi}{4}+\frac{\eta}{2}}^{\frac{9\pi}{4}+\frac{\eta}{2}} \right] \frac{\mathrm{d}\sigma}{\mathrm{d}\phi}\, \mathrm{d}\phi\,. \end{aligned} \tag{4.5}$$

In this case the integration over the polar angle θ is performed over the whole range. This choice of the ranges of the integrations has the effect of extracting the terms $\propto r_1 \propto \cos(\eta - 2\phi)$, eqs. (3.18)–(3.21), from the squared amplitude. Note however, that this observable is CP-even.

4.2 CP asymmetries in neutralino production and decay

The reconstruction of the neutralino momenta is not necessary if we include the subsequent decays $\tilde{\chi}_j^0 \to \tilde{\ell}^{\pm}\, \ell_1^{\mp}$ (where $\tilde{\ell} = \tilde{\ell}_L, \tilde{\ell}_R$) and $\tilde{\ell}^{\pm} \to \ell_2^{\pm} \tilde{\chi}_1^0$, yielding to the final state $\tilde{\chi}_j^0 \to \ell_1^{\mp} \ell_2^{\pm} \tilde{\chi}_1^0$. The label of the leptons indicates whether they stem from the first or the second decay. The cross sections for the combined processes are given in appendix B, eqs. (B.7) and (B.8), respectively. The CP-sensitive terms of the squared amplitudes depend on $\sin(\eta - 2\phi_{\ell_1})$ or $\sin(\eta - 2\phi_{\ell_2})$, where ϕ_{ℓ_1} and ϕ_{ℓ_2} are the azimuthal angles of the final leptons $\ell_1^{\mp}$ and $\ell_2^{\pm}$. As a first step, we integrate the differential cross section in eq. (B.7) over all angles except ϕ_{ℓ_1} (the angles are integrated over their whole range). Then the CP asymmetry obtained by the azimuthal distribution of ℓ_1^- is given by

$$\begin{aligned} A_1^- = & \frac{\mathrm{N}[\sin(\eta - 2\phi_{\ell_1}) > 0] - \mathrm{N}[\sin(\eta - 2\phi_{\ell_1}) < 0]}{\mathrm{N}[\sin(\eta - 2\phi_{\ell_1}) > 0] + \mathrm{N}[\sin(\eta - 2\phi_{\ell_1}) < 0]} \\ = & \frac{1}{\sigma_1} \left[-\int_{\frac{\eta}{2}}^{\frac{\pi}{2}+\frac{\eta}{2}} + \int_{\frac{\pi}{2}+\frac{\eta}{2}}^{\pi+\frac{\eta}{2}} - \int_{\pi+\frac{\eta}{2}}^{\frac{3\pi}{2}+\frac{\eta}{2}} + \int_{\frac{3\pi}{2}+\frac{\eta}{2}}^{2\pi+\frac{\eta}{2}} \right] \frac{\mathrm{d}\sigma_1}{\mathrm{d}\phi_{\ell_1}} \mathrm{d}\phi_{\ell_1}\,, \end{aligned} \tag{4.6}$$

where $\sigma_1 = \sigma(e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_j^0) \times B(\tilde{\chi}_j^0 \to \tilde{\ell}^+ \ell_1^-)$ and the upper index of A_1^- corresponds to the electric charge of the observed lepton ℓ_1^-.

As a next step, we integrate the differential cross section in eq. (B.8) over all angles except ϕ_{ℓ_2}, in order to define the CP asymmetry of the azimuthal distribution of ℓ_2^+:

$$A_2^+ = \frac{\mathrm{N}[\sin(\eta - 2\phi_{\ell_2}) > 0] - \mathrm{N}[\sin(\eta - 2\phi_{\ell_2}) < 0]}{\mathrm{N}[\sin(\eta - 2\phi_{\ell_2}) > 0] + \mathrm{N}[\sin(\eta - 2\phi_{\ell_2}) < 0]}$$

$$= \frac{1}{\sigma_2}\left[-\int_{\frac{\eta}{2}}^{\frac{\pi}{2}+\frac{\eta}{2}} + \int_{\frac{\pi}{2}+\frac{\eta}{2}}^{\pi+\frac{\eta}{2}} - \int_{\pi+\frac{\eta}{2}}^{\frac{3\pi}{2}+\frac{\eta}{2}} + \int_{\frac{3\pi}{2}+\frac{\eta}{2}}^{2\pi+\frac{\eta}{2}}\right]\frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_{\ell_2}}\mathrm{d}\phi_{\ell_2}\,, \tag{4.7}$$

where $\sigma_2 = \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_j^0) \times B(\tilde{\chi}_j^0 \to \tilde{\ell}^+\ell_1^-) \times B(\tilde{\ell}^+ \to \tilde{\chi}_1^0\ell_2^+)$. Note that, since $\Sigma_D^b(\tilde{\chi}_j^0)$ for the two C-conjugate decay modes of $\tilde{\chi}_j^0 \to \tilde{\ell}^\pm\ell^\mp$ differs only by a sign (see eqs. (B.3) and (B.5)) the asymmetries with upper indices + and − are related by $A_i^+ = -A_i^-,\ i = 1, 2$. In order to measure both asymmetries, eqs. (4.6) and (4.7), it is necessary to distinguish the lepton $\ell_1^\mp$, originating from the decay $\tilde{\chi}_j^0 \to \tilde{\ell}^\pm\ell_1^\mp$, and the lepton $\ell_2^\pm$ from the subsequent decay $\tilde{\ell}^\pm \to \tilde{\chi}_1^0\ell_2^\pm$. This can be accomplished by their different energy distributions, when the masses of the particles involved are known, provided that their measured energies do not lie in the overlapping region of their energy distributions [20].

However, we can also define a CP asymmetry where it is not necessary to distinguish whether the leptons stem from the first or the second step of the decay chain $\tilde{\chi}_j^0 \to \tilde{\ell}^\pm\,\ell_1^\mp \to \ell_1^\mp\ell_2^\pm\tilde{\chi}_1^0$. This asymmetry is defined by

$$A^- = \frac{\mathrm{N}[\sin(\eta - 2\phi_{\ell^-}) > 0] - \mathrm{N}[\sin(\eta - 2\phi_{\ell^-}) < 0]}{\mathrm{N}[\sin(\eta - 2\phi_{\ell^-}) > 0] + \mathrm{N}[\sin(\eta - 2\phi_{\ell^-}) < 0]}$$

$$= \frac{(\int^+ - \int^-)(\frac{\mathrm{d}\sigma_1}{\mathrm{d}\phi_{\ell_1}}\mathrm{d}\phi_{\ell_1} + \frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_{\ell_2}}\mathrm{d}\phi_{\ell_2})}{\int_0^{2\pi}(\frac{\mathrm{d}\sigma_1}{\mathrm{d}\phi_{\ell_1}}\mathrm{d}\phi_{\ell_1} + \frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_{\ell_2}}\mathrm{d}\phi_{\ell_2})}\,, \tag{4.8}$$

where ℓ^- is either ℓ_1^- or ℓ_2^- and $\mathrm{N}[\sin(\eta - 2\phi_{\ell^-}) > 0\,(<0)]$ denotes the number of events where $\sin(\eta - 2\phi_{\ell^-}) > 0\,(<0)$. Hence, only the charge of the lepton and its azimuthal angle ϕ_{ℓ^-} has to be determined. In eq. (4.8) $\int^\pm$ corresponds to an integration over the azimuthal angles ϕ_{ℓ_1} or ϕ_{ℓ_2}, where $\sin(\eta - 2\phi_{\ell_{1,2}})$ is positive or negative, respectively. An analogous asymmetry can be defined for ℓ^+ as well. The asymmetry A^-, eq. (4.8), can be related to the asymmetries A_1^- and A_2^-, eqs. (4.6) and (4.7), by

$$A^- = \frac{1}{[1 + B(\tilde{\ell}^- \to \ell^-\tilde{\chi}_1^0)]}\left[A_1^- + A_2^-\ B(\tilde{\ell}^- \to \ell^-\tilde{\chi}_1^0)\right]\,. \tag{4.9}$$

5. Numerical studies

5.1 CP-even observables in neutralino production

Before we concentrate on the numerical study of CP-odd observables, we would like to give an example, which shows that a measurement of only CP-even observables may not be sufficient to unambiguously determine the SUSY parameters of the neutralino sector. However, a measurement of a CP-odd asymmetry may help to single out the correct solution. This may be particularly important if only the two lower states of the neutralino spectrum are kinematically accessible.

Scenario	$\lvert M_1 \rvert$	ϕ_{M_1}	M_2	$\lvert\mu\rvert$	ϕ_μ	$\tan\beta$	$m_{\tilde{e}_L}$	$m_{\tilde{e}_R}$
Complex	183	0.05π	311	343	1.9π	2.1	297	181
Real	180	0	310	335	0	3	300	180

Table 1: Input parameters $|M_1|$, ϕ_{M_1}, M_2, $|\mu|$, $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$ for the complex and the real scenario. All mass parameters are given in GeV.

To this end we consider the complex scenario with the parameters given in table 1, leading to $m_{\tilde{\chi}^0_1} = 170.9\,\text{GeV}$ and $m_{\tilde{\chi}^0_2} = 259.5\,\text{GeV}$. At $\sqrt{s} = 500\,\text{GeV}$ only the cross sections of $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ would be measurable, giving $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2) = (16.4, 18.3, 30.3)$ fb for the e^+ and e^- longitudinal beam polarizations $(\mathcal{P}^-_L, \mathcal{P}^+_L) = (0,0)$, $(-80\%, +60\%)$, $(+80\%, -60\%)$, respectively. We assume that the masses $m_{\tilde{\chi}^0_{1,2}}$ and $m_{\tilde{e}_{L,R}}$ are measured with 1% accuracy. For the cross sections we take an error corresponding to a 1-σ deviation for a luminosity $\mathcal{L}_{\text{int}} = 100\ \text{fb}^{-1}$. Then within this accuracy, we would obtain compatible neutralino masses and cross sections with the real SUSY parameter set, which is also given in table 1, namely $m_{\tilde{\chi}^0_1} = 169.3\,\text{GeV}$ and $m_{\tilde{\chi}^0_2} = 258.3\,\text{GeV}$, and cross sections $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2) = (16.3, 18.2, 30.0)$ fb. The CP-odd asymmetry $\mathcal{A}_{CP}$, eq. (4.4), however, would result in about 2.8% with $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (80\%, 60\%)$ for the complex scenario with $\phi_{M_1} = 0.05\pi$ and $\phi_\mu = 1.9\pi$. Although the asymmetry is small it should be experimentally measurable including the statistical uncertainty. Therefore the complex scenario would be clearly distinguishable from the real scenario, which results in an asymmetry identical to zero. This simple example illustrates that it is necessary to measure CP-odd observables for truly identifying CP-violating effects.

In the following we analyse numerically the CP-odd asymmetries, eq. (4.4) and eqs. (4.6)–(4.8), at the ILC with $\sqrt{s} = 500\,\text{GeV}$ and transversely polarized $e^\pm$ beams. We especially focus on the influence of the phase ϕ_{M_1} of the gaugino mass parameter $M_1 = |M_1|e^{i\phi_{M_1}}$. Throughout we assume the GUT-inspired relation $|M_1| = 5/3\tan^2\Theta_W\, M_2$. Furthermore we show that CP-odd observables are necessary to determine unambiguously the underlying SUSY parameters. In order to study the full phase dependences of the CP-odd observables, we do not take into account the restrictions from the EDMs and vary ϕ_μ and ϕ_{M_1} in the whole range.

5.2 CP-odd asymmetries in neutralino production

First we discuss the CP-odd asymmetry A_{CP}, eq. (4.4), for the neutralino production processes $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ and $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3$. CP violation is due to the interference terms $P(Z\tilde{e}_L)_T$, $P(Z\tilde{e}_R)_T$, and $P(\tilde{e}_L\tilde{e}_R)_T$, eqs. (3.19)–(3.21). We assume that the momenta of the produced neutralinos can be reconstructed by analysing the subsequent two-body decays; see appendix C.

5.2.1 CP-odd asymmetries in $\mathbf{e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2}$ production

In figure 3a we show A_{CP}, eq. (4.4), for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ as a function of ϕ_{M_1} for scenario A, given in table 2, for $\tan\beta = 3, 10, 30$, with $\sqrt{s} = 500\,\text{GeV}$ and transverse beam polarization

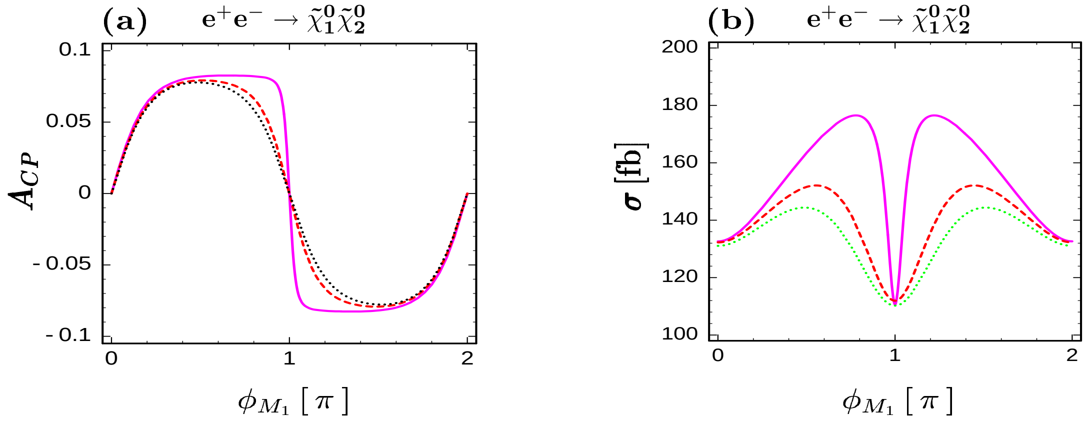

Figure 3: (a) CP asymmetry A_{CP}, eq. (4.4), and (b) cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ as a function of ϕ_{M_1} for scenario A of table 2, with $\tan\beta = 3$ (solid line), $\tan\beta = 10$ (dashed line), $\tan\beta = 30$ (dotted line), for $\sqrt{s} = 500\,\mathrm{GeV}$ and transverse beam polarizations $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$.

<table>
<tr><th>Scenario</th><th colspan="2">A</th><th>B</th></tr>
<tr><td>$|M_1|$</td><td colspan="2">123.3</td><td>120.8</td></tr>
<tr><td>ϕ_{M_1}</td><td colspan="2">0.5π</td><td>0.5π</td></tr>
<tr><td>M_2</td><td colspan="2">245</td><td>240</td></tr>
<tr><td>$|\mu|$</td><td colspan="2">160</td><td>300</td></tr>
<tr><td>ϕ_μ</td><td colspan="2">0</td><td>0</td></tr>
<tr><td>$m_{\tilde{e}_L}$</td><td colspan="2">400</td><td>400</td></tr>
<tr><td>$m_{\tilde{e}_R}$</td><td colspan="2">150</td><td>150</td></tr>
<tr><td>$\tan\beta$</td><td>3</td><td>30</td><td>3</td></tr>
<tr><td>$m_{\tilde{\chi}_1^0}$</td><td>99.4</td><td>105.5</td><td>117.0</td></tr>
<tr><td>$m_{\tilde{\chi}_2^0}$</td><td>143.0</td><td>144.1</td><td>197.6</td></tr>
<tr><td>$m_{\tilde{\chi}_3^0}$</td><td>169.7</td><td>178.6</td><td>303.9</td></tr>
<tr><td>$m_{\tilde{\chi}_4^0}$</td><td>289.7</td><td>281.5</td><td>351.7</td></tr>
</table>

Table 2: Input parameters $|M_1|$, M_2, $|\mu|$, $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$ and the resulting masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 4$ for $\tan\beta = 3, 30$ and specific values of the phases ϕ_{M_1} and ϕ_μ. All mass parameters and masses are given in GeV.

$(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$. For this scenario we obtain for $\tan\beta = 3$ (30) an asymmetry A_{CP} of about 8.2 (7.8)%, for $\phi_{M_1} = 0.5\pi$. The peculiar shape of the curve is a result of the combined contributions of the Z–$\tilde{e}_R$ interference term, which has its maximum at $\phi_{M_1} \approx 0.4\pi$, and of the $\tilde{e}_L$–$\tilde{e}_R$ interference term, with its maximum at $\phi_{M_1} \approx 0.8\pi$. The contribution of the Z–$\tilde{e}_L$ interference term is suppressed because of the large mass of the left-handed selectron. The cross sections for the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ are plotted in figure 3b and are about 163 (144) fb for $\phi_{M_1} = 0.5\pi$. Note that the cross sections are independent of the transverse beam polarization, because these contributions depend on $\cos 2\phi$ $(\sin 2\phi)$, see eq. (4.1), and disappear if integrated over the whole range of ϕ. In figures 3a and b we can clearly see the antisymmetric dependence of the CP asymmetry and the symmetric behaviour of the cross section on the phase ϕ_{M_1}. It is therefore obvious that both kinds of observables are needed for an unambiguous determination of the phase.

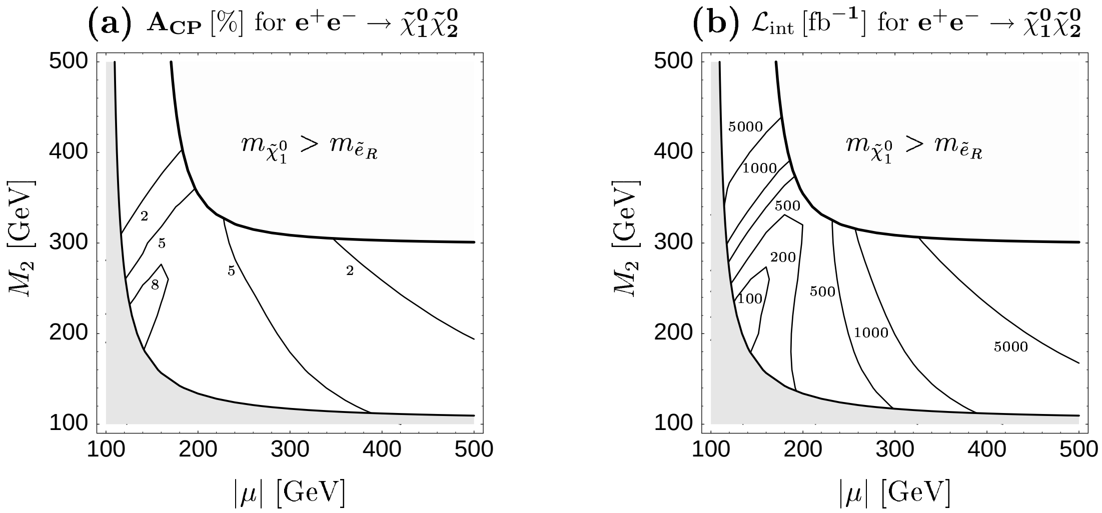

Figure 4: (a) contours of the CP asymmetry A_{CP}, eq. (4.4), in % for the process $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ in the $|\mu|$–M_2 plane. The MSSM parameters are $\phi_{M_1} = 0.5\pi$, $\phi_\mu = 0$, $\tan\beta = 3$, $m_{\tilde{e}_L} = 400\,\mathrm{GeV}$ and $m_{\tilde{e}_R} = 150\,\mathrm{GeV}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ with transverse beam polarizations $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$. (b) shows the contours of the luminosity $\mathcal{L}_{\rm int}$, eq. (5.1), needed to measure the CP-odd asymmetry A_{CP} at the 5-σ level with degrees of transverse polarization $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (80\%, 60\%)$. The light-grey region is experimentally excluded by the exclusion bound $m_{\tilde{\chi}^\pm_1} < 104\,\mathrm{GeV}$ [28].

Note that A_{CP} can be sizeable even for values of ϕ_{M_1} close to 0 and π, which would be favoured by the EDM constraints.

Now we estimate the observability of the asymmetry. One assumes that the same degree of transverse beam polarization is feasible as for the longitudinal polarization ($\mathcal{P}_T^- = 80\%$ and $\mathcal{P}_T^+ = 60\%$). Since the CP asymmetry A_{CP} depends bilinearly on the degrees of transverse beam polarization $\mathcal{P}_T^-$ ($\mathcal{P}_T^+$) of the e^- (e^+), see eqs. (3.19)–(3.21), we have to multiply the asymmetry A_{CP} for $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$ with a factor 0.48. The luminosity $\mathcal{L}_{\rm int}$ required for a measurement with specific significance can be estimated as

$$\mathcal{L}_{\rm int} = (\mathcal{N}_\sigma)^2/[A_{CP}^2\,\sigma]\,, \tag{5.1}$$

where $\mathcal{N}_\sigma$ denotes the number of standard deviations and σ the corresponding cross section for neutralino production. We obtain a luminosity $\mathcal{L}_{\rm int} \approx 99$ (124) fb^{-1} needed for a discovery with 5-σ, for $\tan\beta = 3$ (30) and $\phi_{M_1} = 0.5\pi$.

Figure 4a shows the contour lines of the CP asymmetry A_{CP}, eq. (4.4), at $\sqrt{s} = 500\,\mathrm{GeV}$ for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ in the $|\mu|$–M_2 plane. The MSSM parameters are chosen to be $\phi_{M_1} = 0.5\pi$, $\phi_\mu = 0$, $\tan\beta = 3$, $m_{\tilde{e}_L} = 400\,\mathrm{GeV}$ and $m_{\tilde{e}_R} = 150\,\mathrm{GeV}$. The largest CP-odd asymmetry A_{CP} is attained for sizeable gaugino-higgsino mixing. If the beams are fully transversely polarized, $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$, then A_{CP} could reach up to about 8.8% for $M_2 \approx 240\,\mathrm{GeV}$ and $|\mu| \approx 140\,\mathrm{GeV}$. With a higher centre-of-mass energy $\sqrt{s} = 800\,\mathrm{GeV}$, the asymmetry A_{CP} increases to about 12%, because the cross section, which is the denominator of A_{CP}, decreases stronger than its numerator. In this region of the parameter space the Z–$\tilde{e}_R$ interference term, eq. (3.20), is the main contribution to the asymmetry A_{CP}. In a gaugino-like scenario, for instance $M_2 = 250\,\mathrm{GeV}$ and $|\mu| = 450\,\mathrm{GeV}$, the $\tilde{e}_L$–$\tilde{e}_R$ interference term is dominant and the others are suppressed. Generally the $\tilde{e}_L$–$\tilde{e}_R$ contribution to the asymmetry is small for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ and A_{CP} is therefore

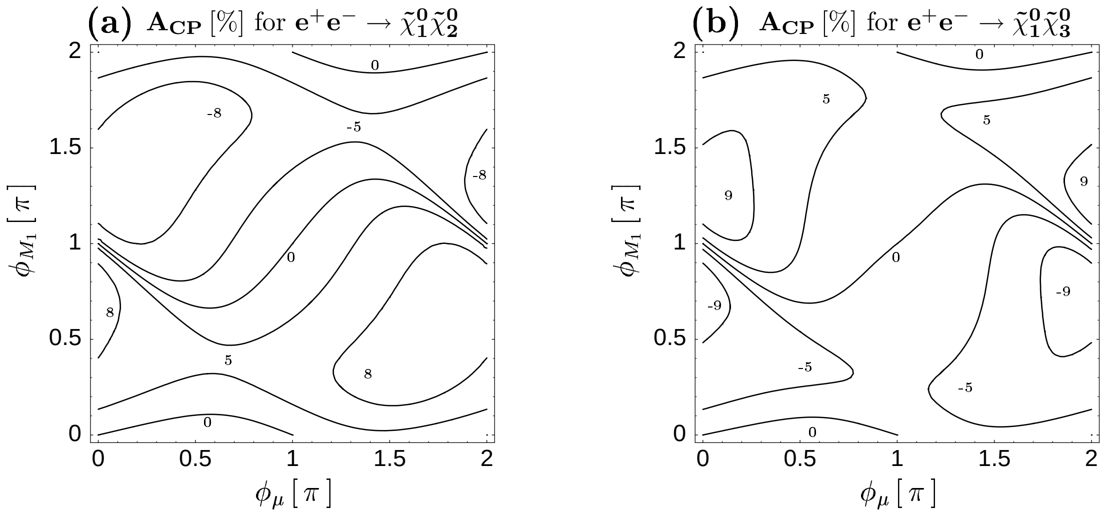

Figure 5: Contours of the CP asymmetry A_{CP}, eq. (4.4), in % for the process (a) $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ and (b) $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3$ in the ϕ_μ–ϕ_{M_1} plane, for scenario A with $\tan\beta = 3$, see table 2, for $\sqrt{s} = 500\,\mathrm{GeV}$ and transverse beam polarizations $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (100\%, 100\%)$.

reduced to about 1.6%. In order to obtain a larger $\tilde{e}_L$–$\tilde{e}_R$ contribution, a larger mass splitting of $\tilde{e}_L$ and $\tilde{e}_R$ is necessary. If $m_{\tilde{e}_L} \approx m_{\tilde{e}_R}$ the interference term $P(\tilde{e}_L\tilde{e}_R)_T$ is very small, see eq. (3.21). In figure 4b we plot the corresponding luminosity $\mathcal{L}_{\mathrm{int}}$, eq. (5.1), for transverse beam polarizations of $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (80\%, 60\%)$. For the maximum value of A_{CP}, a luminosity $\mathcal{L}_{\mathrm{int}}$ of about 81 fb^{-1} would be needed for a discovery with 5-σ.

In figure 5a we show the contour lines of A_{CP}, eq. (4.4), for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ in the ϕ_μ–ϕ_{M_1} plane for scenario A (table 2) at $\sqrt{s} = 500\,\mathrm{GeV}$ and with transverse beam polarizations $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (100\%, 100\%)$. We obtain a maximum value of the CP-odd asymmetry A_{CP} of about 8.9% for $\phi_\mu \approx 1.6\pi$ and $\phi_{M_1} \approx 0.4\pi$. In this scenario the ϕ_{M_1} and the ϕ_μ dependence are of the same order of magnitude. The main contribution to the CP-odd asymmetry originates from the interference term $P(Z\tilde{e}_R)_T$, eq. (3.20), i.e. the primarily involved coupling is $f^{R*}_{\ell 1} f^R_{\ell 2} O^R_{12}$. The corresponding cross section for $\phi_\mu = 1.6\pi$ and $\phi_{M_1} = 0.4\pi$ is about 139 fb and the luminosity $\mathcal{L}_{\mathrm{int}}$ for a discovery with 5-σ is about 99 fb^{-1} for transverse beam polarizations $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (80\%, 60\%)$. Note also in this case the CP asymmetry A_{CP} can be sizeable for values of ϕ_{M_1} and ϕ_μ close to 0 and π.

5.2.2 CP-odd asymmetries in $\mathbf{e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3}$ production

Figure 5b shows the contour lines of the CP asymmetry A_{CP}, eq. (4.4), in the ϕ_μ–ϕ_{M_1} plane for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3$. As shown in the case before, a large gaugino-higgsino mixing is necessary to obtain sizeable CP asymmetries. We investigate scenario A of table 2, at $\sqrt{s} = 500\,\mathrm{GeV}$ and $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (100\%, 100\%)$. In this scenario the maximum value of A_{CP} is about 9.8% for $\phi_\mu \approx 0.1\pi$ and $\phi_{M_1} \approx 1.2\pi$. Again the main CP-violating contribution is due to the Z–$\tilde{e}_R$ interference term. In this example the largest asymmetries are obtained for small values of ϕ_μ. For $\phi_\mu = 0.1\pi$ and $\phi_{M_1} = 1.2\pi$ the cross section $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3)$ is 76 fb and the luminosity for a discovery with 5-σ is about 150 fb^{-1}.

Figure 6a shows the CP asymmetry A_{CP}, eq. (4.4), for the process $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3$ as a function of ϕ_{M_1} for scenario A defined in table 2, for $\tan\beta = 3, 10, 30$ and $\sqrt{s} = 500\,\mathrm{GeV}$. For $\tan\beta = 3$ (30) the asymmetry A_{CP} reaches its maximum of about 9.6 (7.4)% at $\phi_{M_1} =$

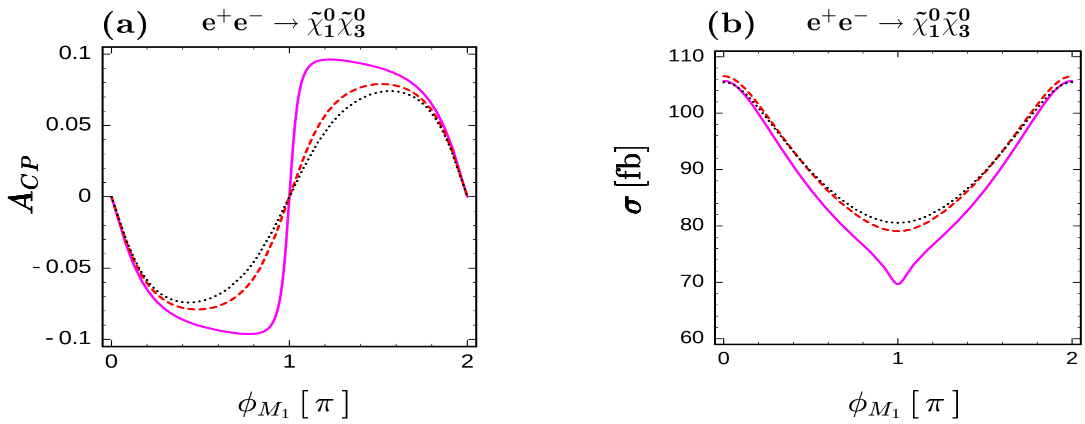

Figure 6: (a) CP asymmetry A_{CP}, eq. (4.4), and (b) cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$ as a function of ϕ_{M_1} for scenario A of table 2, with $\tan\beta = 3$ (solid line), $\tan\beta = 10$ (dashed line), $\tan\beta = 30$ (dotted line), for $\sqrt{s} = 500\,\text{GeV}$ and transverse beam polarizations $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$.

1.25 (1.55)π. Here again the dominant contribution to A_{CP} comes from the interference term $P(Z\tilde{e}_R)_T$; see eq. (3.20). Note that the maximal CP-violating phase $\phi_{M_1} = \frac{\pi}{2} \,(\mathrm{mod}\,\pi)$ does not necessarily lead to the highest value of the asymmetry. The reason for this is an interplay between the ϕ_{M_1} dependence of the cross section, shown in figure 6b, and that of the numerator of the asymmetry. In figure 6b the corresponding cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$ is plotted. For the maximal asymmetry it is about 78 (91) fb. In order to measure the asymmetry A_{CP} at 5-σ, the required luminosity is $\mathcal{L}_{\rm int} \approx 150$ (217) fb^{-1}.

5.3 Neutralino production and subsequent two-body decays

Now we discuss neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ with the subsequent decays $\tilde{\chi}_j^0 \to \tilde{\ell}_R^{\pm}\ell_1^{\mp}$ and $\tilde{\ell}_R^{\pm} \to \tilde{\chi}_1^0\ell_2^{\pm}$. We study the CP-odd asymmetries, eqs. (4.6)–(4.8), which are defined by the azimuthal distribution of the final leptons ℓ_1 and ℓ_2. In this case CP-violating effects arise from the contributions of the spin correlations of the decaying neutralino, eqs. (3.27)–(3.29), which depend on the transverse beam polarization. We give numerical examples for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$.

5.3.1 CP-odd asymmetries in $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ production and decay

In figure 7a we show the CP asymmetries $A_{1,2}^+$ and A^+, eqs. (4.6)–(4.8), as a function of ϕ_{M_1} for scenario B defined in table 2. The beam energy is $\sqrt{s} = 500\,\text{GeV}$ with degrees of transverse beam polarizations $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$. We study neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$, with the subsequent decays $\tilde{\chi}_2^0 \to \tilde{\ell}_R^{\pm}\ell_1^{\mp}$ and $\tilde{\ell}_R^{\pm} \to \tilde{\chi}_1^0\ell_2^{\pm}$. For A_1^+ we obtain a maximal value of about 11.6% for $\phi_{M_1} = 0.45\pi$. The asymmetry A_2^+ is reduced to 2.1% by the additional contribution to the phase space from the decay $\tilde{\ell}_R^{\pm} \to \tilde{\chi}_1^0\ell_2^{\pm}$. Since the branching ratio $B(\tilde{\ell}_R^{\pm} \to \tilde{\chi}_1^0\ell_2^{\pm}) = 1$, we obtain a CP-odd asymmetry $A^+ \approx 6.9\%$, see eq. (4.9), for $\phi_{M_1} = 0.45\pi$. In this scenario the main contribution to the CP-odd asymmetries comes from the $\tilde{e}_L$–$\tilde{e}_R$ term, eq. (3.29). With a smaller mass splitting between $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$ the contribution to the asymmetry of $\Sigma_P^b(\tilde{e}_L\tilde{e}_R)_T$ becomes larger, but as the cross section is increasing, the combination of the two effects leads to a smaller asymmetry. The corresponding cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0\ell_1^{\mp}\ell_2^{\pm})$ is plotted in figure 7b. For $\phi_{M_1} = 0.45\pi$, we obtain a cross section of about 46 fb. Thus, for $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (80\%, 60\%)$,

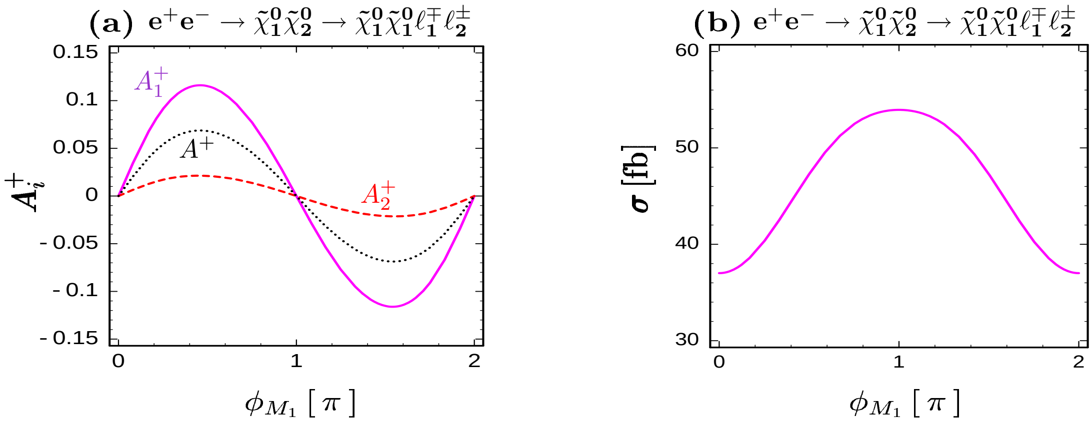

Figure 7: (a) CP-odd asymmetries A_1^+ (solid), A_2^+ (dashed) and A^+ (dotted), eqs. (4.6)–(4.8), for the process $e^+e^- \to \tilde\chi^0_1\tilde\chi^0_2 \to \tilde\chi^0_1\tilde\chi^0_1\ell^\mp_1\ell^\pm_2$ and (b) the corresponding cross section as a function of ϕ_{M_1} in scenario B (table 2) with $\tan\beta = 3$. The centre-of-mass energy is fixed at $\sqrt{s} = 500\,\text{GeV}$ and the transverse beam polarizations are $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$.

the luminosity $\mathcal{L}_{\rm int}$ needed for a discovery with 5-σ of the asymmetry A_1^+ is about 176 fb^{-1}. For a discovery with 5-σ of A^+, $\mathcal{L}_{\rm int} \approx 517$ fb^{-1} are needed.

In figure 8a we show the contour lines of the CP-odd asymmetry A_1^+, eq. (4.6), for $e^+e^- \to \tilde\chi^0_1\tilde\chi^0_2 \to \tilde\chi^0_1\tilde\ell^-_R\ell^+_1$ in the $|\mu|$–M_2 plane. The other parameters are $\phi_{M_1} = 0.5\pi$, $\phi_\mu = 0$, $\tan\beta = 3$, $m_{\tilde e_L} = 400\,\text{GeV}$ and $m_{\tilde e_R} = 150\,\text{GeV}$. The centre-of-mass energy is fixed at $\sqrt{s} = 500\,\text{GeV}$ with degrees of transverse beam polarizations $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$. In this figure we only consider the parameter regions where the decay channel $\tilde\chi^0_2 \to \tilde\ell^\pm_R\ell^\mp_1$ is kinematically accessible. The maximum value of the CP asymmetry $A_1^+ \approx 12.6\%$ is obtained for a gaugino-like scenario with $M_2 = 200\,\text{GeV}$ and $|\mu| = 280\,\text{GeV}$. For these parameters the neutralino masses are $m_{\tilde\chi^0_1} = 97\,\text{GeV}$ and $m_{\tilde\chi^0_2} = 163.5\,\text{GeV}$, and therefore the branching ratio $B(\tilde\chi^0_2 \to \tilde\ell^\pm_R\ell^\mp_1) = 1$ and the cross section $\sigma(e^+e^- \to \tilde\chi^0_1\tilde\chi^0_2 \to \tilde\chi^0_1\tilde\ell^\pm_R\ell^\mp_1) = 54\,\text{fb}$. Thus for $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (80\%, 60\%)$ the required luminosity $\mathcal{L}_{\rm int}$ for a discovery with 5-σ, eq. (5.1), is about 128 fb^{-1}. For this parameter point the asymmetry A^+, eq. (4.8), is about 6.7% and the luminosity $\mathcal{L}_{\rm int} \approx 456$ fb^{-1}. For higgsino-like scenarios ($M_2 > |\mu|$) the $\Sigma^b_P(Z\tilde e_R)_T$ interference term, eq. (3.28), gives the main contribution to the CP-odd asymmetry, which can be traced back to the structure of the corresponding coupling $f^{R*}_{\ell 1} f^R_{\ell 2} O^R_{12}$. On the other hand for gaugino-like scenarios ($|\mu| > M_2$) the contribution of the interference term $\Sigma^b_P(\tilde e_L\tilde e_R)_T$ dominates, with the corresponding coupling $f^{L*}_{\ell 1} f^L_{\ell 2} f^{R*}_{\ell 1} f^R_{\ell 2}$. The sign change of the asymmetry in the $|\mu|$–M_2 plane is therefore due to a cancellation of the Z–$\tilde e_R$ and the $\tilde e_L$–$\tilde e_R$ contributions which have opposite signs.

5.3.2 CP-odd asymmetries in $\mathbf{e^+e^- \to \tilde\chi^0_1\tilde\chi^0_3}$ production and decay

In figure 8b we show the contour lines of the CP asymmetry A_1^+ for the process $e^+e^- \to \tilde\chi^0_1\tilde\chi^0_3 \to \tilde\chi^0_1\tilde\ell^-_R\ell^+_1$ in the $|\mu|$–M_2 plane. We fix the MSSM parameters at $\phi_{M_1} = 0.5\pi$, $\phi_\mu = 0$, $\tan\beta = 3$, $m_{\tilde e_L} = 400\,\text{GeV}$ and $m_{\tilde e_R} = 150\,\text{GeV}$ with $\sqrt{s} = 500\,\text{GeV}$ and $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$. The maximal CP asymmetry A_1^+ is about 31% for $M_2 = 300\,\text{GeV}$ and $|\mu| = 160\,\text{GeV}$. For this parameter point we obtain neutralino masses of $m_{\tilde\chi^0_1} = 115\,\text{GeV}$ and $m_{\tilde\chi^0_2} = 156\,\text{GeV}$, and therefore the branching ratio is again $B(\tilde\chi^0_2 \to \tilde\ell^\pm_R\,\ell^\mp_1) = 1$. Hence the cross section $\sigma(e^+e^- \to \tilde\chi^0_1\tilde\chi^0_3 \to \tilde\chi^0_1\tilde\ell^\pm_R\,\ell^\mp_1) = 83\,\text{fb}$. For transverse beam polarizations

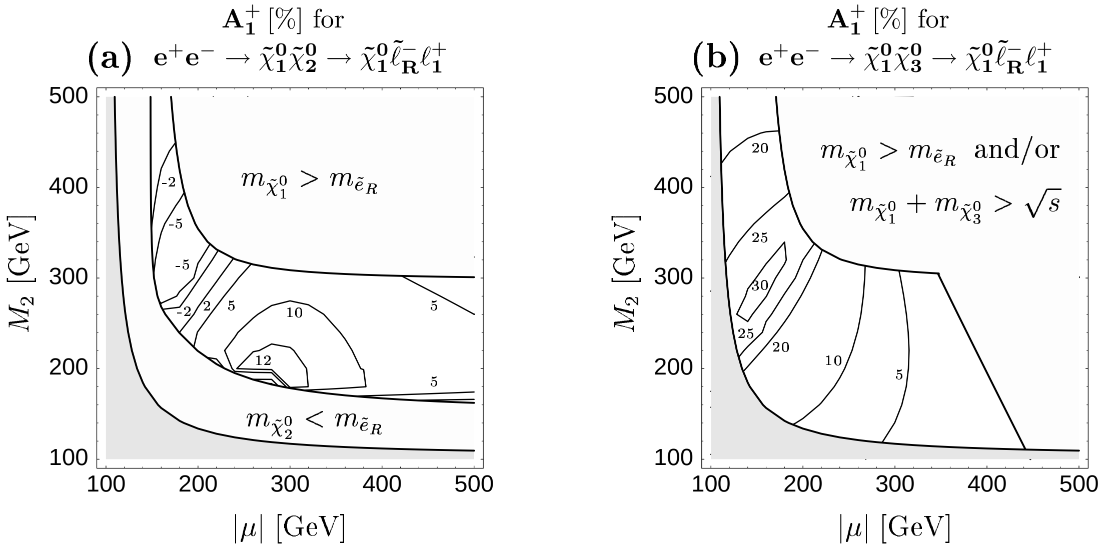

Figure 8: Contours of the CP-odd asymmetry A_1^+, eq. (4.6), in % in the $|\mu|$–M_2 plane for the process (a) $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tilde{\ell}_R^-\ell_1^+$ and (b) $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0 \to \tilde{\chi}_1^0\tilde{\ell}_R^-\ell_1^+$. The MSSM parameters are $\phi_{M_1} = 0.5\pi$, $\phi_\mu = 0$, $\tan\beta = 3$, $m_{\tilde{e}_L} = 400\,\mathrm{GeV}$ and $m_{\tilde{e}_R} = 150\,\mathrm{GeV}$. The centre-of-mass energy is fixed at $\sqrt{s} = 500\,\mathrm{GeV}$ and the transverse beam polarizations are $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$. The light-grey region is excluded by $m_{\tilde{\chi}_1^\pm} < 104\,\mathrm{GeV}$ [28].

$(\mathcal{P}_T^-, \mathcal{P}_T^+) = (80\%, 60\%)$ the luminosity $\mathcal{L}_{\mathrm{int}}$, eq. (5.1), for a discovery with 5-σ of A_1^+ is about 14 fb^{-1}. For these parameters the CP-odd asymmetry A^+ is 17.3% and the necessary luminosity (for a discovery with 5-σ) $\mathcal{L}_{\mathrm{int}} \approx 43$ fb^{-1}. In the case of $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ the CP-violating contributions $\Sigma_P^b(Z\tilde{e}_R)_T$ and $\Sigma_P^b(\tilde{e}_L\tilde{e}_R)_T$, eqs. (3.28) and (3.29), enter with the same sign due to the corresponding couplings. In gaugino-like scenarios the contributions of both interference terms are suppressed, therefore the asymmetry decreases.

In figure 9a the CP-odd asymmetries $A_{1,2}^+$ and A^+, eqs. (4.6)–(4.8), for the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0\ell_1^\mp\ell_2^\pm$ are plotted as a function of ϕ_{M_1} for scenario A, see table 2. For a centre-of-mass energy $\sqrt{s} = 500\,\mathrm{GeV}$ and with transverse beam polarization $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$ the maximum of the CP asymmetry $A_1^+(A^+) \approx 26.2$ (14.3)% is obtained for $\phi_{M_1} \approx 0.75\pi$. In this scenario we have large mixing between the gaugino and the higgsino components, the main contribution to A_1^+ stems again from the Z–$\tilde{e}_R$ interference term, which is about 21.4%. The $\tilde{e}_R$–$\tilde{e}_L$ contribution is 4%, wheras the Z–$\tilde{e}_L$ contribution is suppressed by the large mass of the left selectron. Figure 9b shows the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0 \to \tilde{\chi}_1^0\tilde{\ell}_R^\pm\ell_1^\mp)$. For the maximum of the CP asymmetries $A_1^+(A^+)$, for $\phi_{M_1} \approx 0.75\pi$, the cross section is 78 fb. For $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (80\%, 60\%)$ the luminosity $\mathcal{L}_{\mathrm{int}}$ for a discovery with 5-σ is about 20 (68) fb^{-1}.

5.4 Determination of the SUSY parameters

In the following we will give an example for the accuracy that can be expected in the determination of the MSSM parameters, focusing on the determination of the complex parameter $M_1 = |M_1|e^{i\,\phi_{M_1}}$. In order to determine the parameters unambiguously, CP-even as well as CP-odd observables have to be included in the set of observables from which the underlying parameters are extracted.

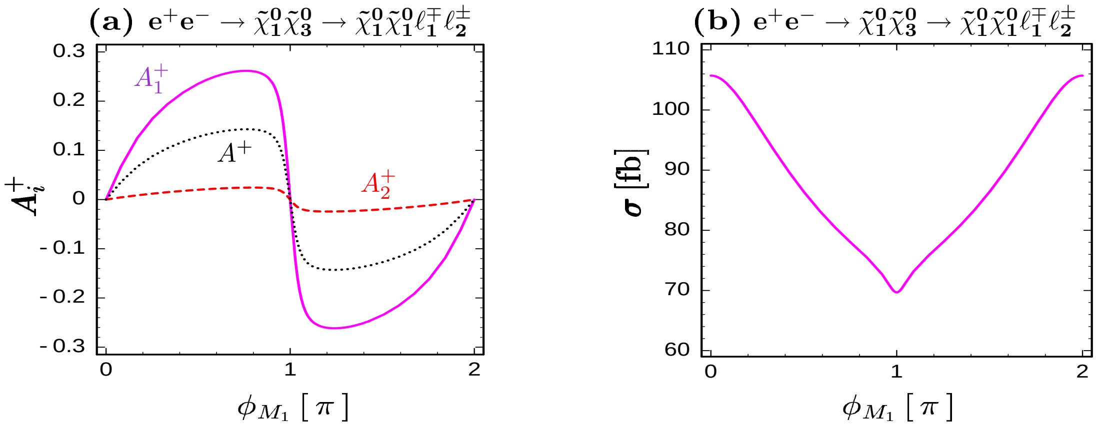

Figure 9: CP-odd asymmetries A_1^+ (solid), A_2^+ (dashed) and A^+ (dotted), eqs. (4.6)–(4.8), for the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0\ell_1^\mp\ell_2^\pm$ and (b) the corresponding cross section as a function of ϕ_{M_1} in scenario A, see table 2, with $\tan\beta = 3$. The centre-of-mass energy is $\sqrt{s} = 500\,\mathrm{GeV}$ and the transverse beam polarizations are $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (100\%, 100\%)$.

$(\mathcal{P}_L^-, \mathcal{P}_L^+)$	$(0,0)$	$(-80\%, +60\%)$	$(+80\%, -60\%)$
$\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$	47.27 fb	87.13 fb	52.80 fb
$\sigma(e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0)$	11.59 fb	33.12 fb	1.186 fb
$\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$	9.83 fb	5.68 fb	23.42 fb
$\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_4^0)$	7.86 fb	7.74 fb	15.53 fb

Table 3: Cross sections for different sets of longitudinal beam polarizations in scenario B with $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ for $\sqrt{s} = 500\,\mathrm{GeV}$.

Our set of observables contains the neutralino masses $m_{\tilde{\chi}_j^0}$, the cross sections $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ for different choices of longitudinal beam polarizations $(\mathcal{P}_L^-, \mathcal{P}_L^+) = (0,0)$, $(-80\%, +60\%)$, $(+80\%, -60\%)$ and the CP-odd asymmetry A_{CP}, eq. (4.4). We now take scenario B with $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$, see table 2, as our reference point of input parameters. We calculate the neutralino masses $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$, $m_{\tilde{\chi}_3^0}$ and $m_{\tilde{\chi}_4^0}$, see table 2. The cross sections for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$, $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0$, $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_4^0$ for $\sqrt{s} = 500\,\mathrm{GeV}$ with different sets of longitudinal beam polarizations are displayed in table 3. The CP asymmetry A_{CP} for the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ is about $+2\%$ for transverse beam polarizations $(\mathcal{P}_T^-, \mathcal{P}_T^+) = (80\%, 60\%)$. We regard these calculated values as real experimental data, where we assume errors of 1% for the masses. For the error of the cross sections of each polarization configuration and of the asymmetry we take a 1-σ deviation for a luminosity $\mathcal{L}_{\mathrm{int}} = 100\ \mathrm{fb}^{-1}$. Our approach for the determination of the error of the parameters (in particular the error of M_1) is described as follows: we perform a random scan over the input parameters $|M_1|$, ϕ_{M_1}, M_2, $|\mu|$, ϕ_μ and $\tan\beta$ around our reference point and select the points which pass the condition $|\mathcal{O}_i^{\mathrm{meas}} - \mathcal{O}_i^{\mathrm{calc}}| < |\Delta\mathcal{O}_i^{\mathrm{meas}}|$, where $\mathcal{O}_i^{\mathrm{meas}}$ are the values of the observables at our reference point, see table 3, $\Delta\mathcal{O}_i^{\mathrm{meas}}$ is the corresponding error, and $\mathcal{O}_i^{\mathrm{calc}}$ are the values of the calculated observables obtained through the random scan.

In figure 10 we show the SUSY parameter points compatible with our reference scenario in the $\mathrm{Re}(M_1)$–$\mathrm{Im}(M_1)$ plane. If we consider only the CP-even observables (cross sections and masses) then we obtain two regions in the parameter space compatible with

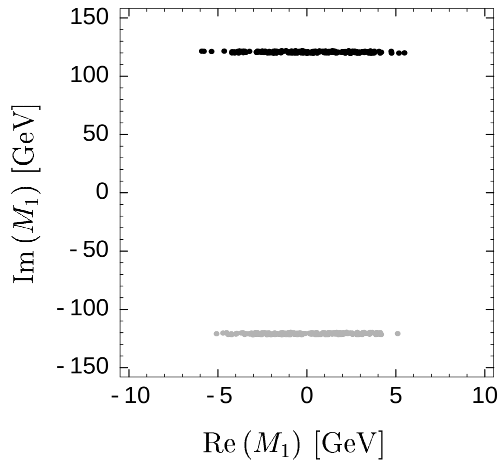

Figure 10: SUSY parameter points in the Re(M_1)–Im(M_1) plane consistent with scenario B, if one assumes an uncertainty of 1% for the masses and 1-σ deviation ($\mathcal{L}_{\rm int} = 100$ fb^{-1}) for the cross sections and the asymmetry. The parameters $|M_1|$, ϕ_{M_1}, M_2, $|\mu|$, ϕ_μ and $\tan\beta$ have been randomly scanned around the reference point B. The grey points are excluded if one takes into account the CP-odd observable A_{CP}, eq. (4.4).

our reference scenario. This ambiguity can be resolved if one includes in addition the CP-odd observable A_{CP}, eq. (4.4). For the error of M_1 we obtain: $\mathrm{Re}(M_1) = 0 \pm 5.9\,\mathrm{GeV}$ and $\mathrm{Im}(M_1) = 120.8 \pm 1.3\,\mathrm{GeV}$.

6. Conclusion

We have studied the processes $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ and $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3$ with subsequent decays $\tilde{\chi}^0_{2,3} \to \tilde{\ell}_R\ell$ and $\tilde{\ell}_R \to \tilde{\chi}^0_1\ell$, where $\ell = e, \mu$, at a linear collider with transverse e^+ and e^- beam polarizations. We have discussed different CP asymmetries, which are due to azimuthal distributions of the neutralinos or the final leptons. We have pointed out that these CP asymmetries are non-vanishing thanks to the Majorana character of the neutralinos. We have given the analytical expressions for the CP asymmetries and the cross sections in the spin density matrix formalism, including the complete spin correlations between production and decay of the neutralinos. At the ILC at $\sqrt{s} = 500\,\mathrm{GeV}$ and with degrees of transverse $e^\pm$ beam polarizations $(\mathcal{P}^-_T, \mathcal{P}^+_T) = (80\%, 60\%)$, the CP asymmetries can reach up to about 15%. Also we have shown that these CP asymmetries can be observed in a broad range of the MSSM parameter space. Furthermore, we have discussed the unambiguous determination of the underlying SUSY parameters, which requires CP-even as well as CP-odd observables.

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A. Momentum and polarization vectors

The basis vectors for transverse e^- beam polarization are

$$\vec{t}^{\,1}_{e^-} = (\vec{t}^{\,2}_{e^-} \times \vec{p}_{e^-})/|\vec{t}^{\,2}_{e^-} \times \vec{p}_{e^-}|\,, \tag{A.1}$$

$$\vec{t}^{\,2}_{e^-} = (\vec{p}_{e^-} \times \vec{p}_{\chi_j})/|\vec{p}_{e^-} \times \vec{p}_{\chi_j}|\,. \tag{A.2}$$

The basis vectors for transverse e^+ beam polarization are defined analogously. In a fixed coordinate system (x, y, z), with the z-axis pointing along the beam direction the basis vectors in the cms are given by

$$t^1_{e^\pm} = (0, \cos\phi, \sin\phi, 0) \quad \text{and} \quad t^2_{e^\pm} = (0, -\sin\phi, \cos\phi, 0)\,. \tag{A.3}$$

The momentum 4-vectors of $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$ are

$$\begin{aligned} p_{\chi_j,\,\mu} &= p_{4,\,\mu} = q(E_{\chi_j}/q, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)\,, \\ p_{\chi_i,\,\mu} &= p_{3,\,\mu} = q(E_{\chi_i}/q, -\cos\phi\sin\theta, -\sin\phi\sin\theta, -\cos\theta)\,, \end{aligned} \tag{A.4}$$

with

$$E_{\chi_{i,j}} = \frac{s + m^2_{\chi_{i,j}} - m^2_{\chi_{j,i}}}{2\sqrt{s}}\,, \qquad q = \frac{\lambda^{\frac{1}{2}}(s, m^2_{\chi_i}, m^2_{\chi_j})}{2\sqrt{s}}\,, \tag{A.5}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. The three spin-basis vectors $s^b_{\chi_j,\,\mu}$ of $\tilde{\chi}^0_j$ are chosen to be

$$\begin{aligned} s^1_{\chi_j,\,\mu} &= \left(0, \frac{\vec{s}_2 \times \vec{s}_3}{|\vec{s}_2 \times \vec{s}_3|}\right) = (0, -\cos\phi\cos\theta, -\sin\phi\cos\theta, \sin\theta)\,, \\ s^2_{\chi_j,\,\mu} &= \left(0, \frac{\vec{p}_{\chi_j} \times \vec{p}_{e^-}}{|\vec{p}_{\chi_j} \times \vec{p}_{e^-}|}\right) = (0, \sin\phi, -\cos\phi, 0)\,, \\ s^3_{\chi_j,\,\mu} &= \frac{1}{m_{\chi_j}}\left(q, \frac{E_{\chi_j}}{q}\vec{p}_{\chi_j}\right) = \frac{E_{\chi_j}}{m_{\chi_j}}(q/E_{\chi_j}, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)\,, \end{aligned} \tag{A.6}$$

where $\vec{s}^{\,1}_{\tilde{\chi}_j}$, $\vec{s}^{\,2}_{\tilde{\chi}_j}$ and $\vec{s}^{\,3}_{\tilde{\chi}_j}$ build a right-handed-system. The momentum 4-vector of the lepton in the decay $\tilde{\chi}^0_j \to \tilde{\ell}_{L,R}\,\ell$ is given by

$$p_{\ell_1,\,\mu} = |\vec{p}_{\ell_1}|(1, \cos\phi_{\ell_1}\sin\theta_{\ell_1}, \sin\phi_{\ell_1}\sin\theta_{\ell_1}, \cos\theta_{\ell_1}) \tag{A.7}$$

with

$$|\vec{p}_{\ell_1}| = \frac{m^2_{\chi_j} - m^2_{\tilde{\ell}}}{2(E_{\chi_j} - q\cos\vartheta)} \tag{A.8}$$

and

$$\cos\vartheta = \sin\theta\sin\theta_{\ell_1}\cos(\phi - \phi_{\ell_1}) + \cos\theta\cos\theta_{\ell_1}\,. \tag{A.9}$$

The momentum 4-vector of the lepton from the decay $\tilde{\ell} \to \tilde{\chi}^0_1 \ell_2$ is given by

$$p_{\ell_2,\,\mu} = |\vec{p}_{\ell_2}|(1, \cos\phi_{\ell_2}\sin\theta_{\ell_2}, \sin\phi_{\ell_2}\sin\theta_{\ell_2}, \cos\theta_{\ell_2})\,, \tag{A.10}$$

where

$$|\vec{p}_{\ell_2}| = \frac{m^2_{\tilde{\ell}} - m^2_{\chi_1}}{2(E_{\tilde{\ell}} - |\vec{p}_{\tilde{\ell}}|\ (\hat{\vec{p}}_{\tilde{\ell}} \cdot \hat{\vec{p}}_{\ell_2}))}\,. \tag{A.11}$$

B. Decay matrix and phase space of 2-body decay

The spin density matrix of the decay $\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_{L,R}\,\ell^{\pm}$ can be written as

$$\rho_{D,\lambda'_j\lambda_j} = \delta_{\lambda'_j\lambda_j} D(\tilde{\chi}^0_j) + \sum_{c=1}^{3} \sigma^c_{\lambda'_j\lambda_j} \Sigma^c_D(\tilde{\chi}^0_j)\,, \tag{B.1}$$

where the expansion coefficient $D(\tilde{\chi}^0_j)$ is the part that is independent of the polarization of the decaying neutralino $\tilde{\chi}^0_j$, and $\Sigma^a_D(\tilde{\chi}^0_j)$ is the part that depends on the polarization of $\tilde{\chi}^0_j$. For the sake of simplicity we consider $\ell = e, \mu$, where the mixing in the slepton sector can be neglected. Then we have, for $\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_L\ \ell^{\pm}$:

$$D(\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_L\ \ell^{\pm}) = \frac{g^2}{2}|f^L_{\ell j}|^2(m^2_{\chi_j} - m^2_{\tilde{\ell}_L})\,, \tag{B.2}$$

$$\Sigma^c_D(\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_L\ \ell^{\pm}) = \mp g^2|f^L_{\ell j}|^2 m_{\chi_j}(s^c \cdot p_\ell) \tag{B.3}$$

and for $\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_R\ \ell^{\pm}$

$$D(\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_R\ \ell^{\pm}) = \frac{g^2}{2}|f^R_{\ell j}|^2(m^2_{\chi_j} - m^2_{\tilde{\ell}_R})\,, \tag{B.4}$$

$$\Sigma^c_D(\tilde{\chi}^0_j \to \tilde{\ell}^{\mp}_R\ \ell^{\pm}) = \pm g^2|f^R_{\ell j}|^2 m_{\chi_j}(s^c \cdot p_\ell)\,, \tag{B.5}$$

where m_{χ_j} ($m_{\tilde{\ell}_{L,R}}$) is the mass of $\tilde{\chi}^0_j$ ($\tilde{\ell}_{L,R}$). The parametrizations of the momentum 4-vector $p_{\ell,\,\mu}$ and the polarization 4-vector $s^c_{\tilde{\chi}_j,\,\mu}$ of the neutralino $\tilde{\chi}^0_j$ are given in eqs. (A.4) and (A.6) in appendix A. Finally, the matrix element squared for the two-body decay $\tilde{\ell}^{\mp} \to \tilde{\chi}^0_1\ \ell^{\mp}$ in the decay chain, eq. (1.2), is

$$D_2(\tilde{\ell}^{\mp}_{L,R} \to \tilde{\chi}^0_1\ \ell^{\pm}) = g^2|f^{L,R}_{\ell 1}|^2\ (m^2_{\tilde{\ell}_{L,R}} - m^2_{\chi_1})\,. \tag{B.6}$$

From (3.15) and (B.1), summing over the polarization of $\tilde{\chi}^0_i$, whose decay is not considered, the differential cross section for $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j \to \tilde{\chi}^0_i\tilde{\ell}^{\mp}_{L,R}\ \ell^{\pm}_1$ is:

$$\mathrm{d}\sigma_1 = \frac{2}{s}\left[PD + \Sigma^a_P\Sigma^a_D\right]\ |\Delta(\tilde{\chi}^0_j)|^2 \mathrm{dLips}_1\,. \tag{B.7}$$

Similarly one obtains the differential cross section for $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j \to \tilde{\chi}^0_i\tilde{\ell}^{\mp}_{L,R}\ \ell^{\pm}_1 \to \ell^{\pm}_1\ell^{\mp}_2\tilde{\chi}^0_1\tilde{\chi}^0_i$ from (3.15), (B.1) and (B.6) and summing over the polarization of $\tilde{\chi}^0_i$:

$$\mathrm{d}\sigma_2 = \frac{2}{s}\left[PD + \Sigma^a_P\Sigma^a_D\right] D_2\ |\Delta(\tilde{\chi}^0_j)|^2|\Delta(\tilde{\ell})|^2 \mathrm{dLips}_2\,, \tag{B.8}$$

where P and Σ_P^a involve the terms for arbitrary beam polarization. For the calculation of the cross section we use the narrow widths approximation ($\int |\Delta(\tilde{\chi}_j^0)|^2 \mathrm{d}\hat{s}_{\tilde{\chi}_j} = \frac{\pi}{m_{\tilde{\chi}_j}\Gamma_{\tilde{\chi}_j}}$ and $\int |\Delta(\tilde{\ell})|^2 \mathrm{d}\hat{s}_{\tilde{\ell}} = \frac{\pi}{m_{\tilde{\ell}}\Gamma_{\tilde{\ell}}}$, where $\mathrm{d}\hat{s}_{\tilde{\chi}_j} = p^2_{\tilde{\chi}_j}$ and $\mathrm{d}\hat{s}_{\tilde{\ell}} = p^2_{\tilde{\ell}}$). The Lorentz-invariant phase-space elements in eqs. (B.7) and (B.8) for the decay chain $\tilde{\chi}_j^0 \to \tilde{\ell}^{\mp}_{L,R}\ell_1^{\pm} \to \tilde{\chi}_1^0 \ell_1^{\pm} \ell_2^{\mp}$ are

$$\mathrm{dLips}_1 = \frac{1}{2\pi}\mathrm{dLips}(s, p_{\chi_i}, p_{\chi_j})\mathrm{d}\hat{s}_{m_{\chi_j}}\mathrm{dLips}(\hat{s}_{m_{\chi_j}}, p_{\tilde{\ell}}, p_{\ell_1})\,, \tag{B.9}$$

$$\mathrm{dLips}_2 = \frac{1}{(2\pi)^2}\mathrm{dLips}(s, p_{\chi_i}, p_{\chi_j})\mathrm{d}\hat{s}_{m_{\chi_j}}\mathrm{dLips}(\hat{s}_{m_{\chi_j}}, p_{\tilde{\ell}}, p_{\ell_1})\mathrm{d}\hat{s}_{m_{\tilde{\ell}}}\mathrm{dLips}(\hat{s}_{m_{\tilde{\ell}}}, p_{\chi_1}, p_{\ell_2}) \tag{B.10}$$

with the Lorentz invariant phase space elements

$$\mathrm{dLips}(s, p_{\chi_i}, p_{\chi_j}) = \frac{1}{4(2\pi)^2}\frac{q}{\sqrt{s}}\sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi\,, \tag{B.11}$$

$$\mathrm{dLips}(\hat{s}_{m_{\chi_j}}, p_{\tilde{\ell}}, p_{\ell_1}) = \frac{1}{2(2\pi)^2}\frac{|\vec{p}_{\ell_1}|}{m^2_{\chi_j} - m^2_{\tilde{\ell}}}\sin\theta_{\ell_1}\ \mathrm{d}\theta_{\ell_1}\ \mathrm{d}\phi_{\ell_1}\,, \tag{B.12}$$

$$\mathrm{dLips}(\hat{s}_{m_{\tilde{\ell}}}, p_{\chi_1}, p_{\ell_2}) = \frac{1}{2(2\pi)^2}\frac{|\vec{p}_{\ell_2}|}{m^2_{\tilde{\ell}} - m^2_{\chi_1}}\sin\theta_{\ell_2}\ \mathrm{d}\theta_{\ell_2}\ \mathrm{d}\phi_{\ell_2}\,. \tag{B.13}$$

C. Reconstruction of the production plane

As an example, we consider the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ with the decays $\tilde{\chi}_2^0 \to \ell_1\ \tilde{\ell}$ and $\tilde{\ell} \to \ell_2\ \tilde{\chi}_1'^0$, where we denote the neutralino from the decay by $\tilde{\chi}_1'^0$ (here again the labels of the leptons indicate their origin). We assume that the masses of all particles involved are known.

We rotate to a coordinate system where the 3-momentum vector of ℓ_1 is along the z-axis and that of ℓ_2 is in the x–z plane. The unit vectors of the 3-momenta of ℓ_1, ℓ_2, $\tilde{\ell}$ are

$$\hat{\vec{p}}_{\ell_1} = (0,0,1)\,, \quad \hat{\vec{p}}_{\ell_2} = (\sin c, 0, \cos c)\,, \quad \hat{\vec{p}}_{\tilde{\ell}} = (\sin b\cos A, \sin b\sin A, \cos b). \tag{C.1}$$

From the relation $(\vec{p}_{\ell_1} + \vec{p}_{\tilde{\ell}})^2 = \vec{p}^{\,2}_{\chi_2}$ we obtain

$$\cos b = \frac{1}{2|\vec{p}_{\ell_1}||\vec{p}_{\tilde{\ell}}|}\left[|\vec{p}_{\chi_2}|^2 - |\vec{p}_{\ell_1}|^2 - |\vec{p}_{\tilde{\ell}}|^2\right]\,, \tag{C.2}$$

where $|\vec{p}_{\tilde{\ell}}|^2 = E^2_{\tilde{\ell}} - m^2_{\tilde{\ell}}$ and $E_{\tilde{\ell}} = E_{\chi_2} - E_{\ell_1}$. From a second relation $(\vec{p}_{\tilde{\ell}} - \vec{p}_{\ell_2})^2 = \vec{p}^{\,2}_{\chi_1'}$ between momentum vectors we obtain

$$\cos a = \frac{1}{2|\vec{p}_{\ell_2}||\vec{p}_{\tilde{\ell}}|}\left[|\vec{p}_{\tilde{\ell}}|^2 + |\vec{p}_{\ell_2}|^2 - |\vec{p}_{\chi_1'}|^2\right]\,, \tag{C.3}$$

where $|\vec{p}_{\chi_1'}|^2 = E^2_{\chi_1'} - m^2_{\chi_1}$ and $E_{\chi_1'} = E_{\chi_2} - E_{\ell_1} - E_{\ell_2}$ or $E_{\chi_1'} = \not{E} - E_{\chi_1}$, and $\not{E}$ is the missing energy. From spherical geometry with $\hat{\vec{p}}_{\ell_2} \cdot \hat{\vec{p}}_{\tilde{\ell}} = \cos a$, we obtain the following relation between the angles

$$\cos A = \frac{\cos a - \cos b\ \cos c}{\sin c\ \sin b}\,. \tag{C.4}$$

Inserting (C.2), (C.3) and (C.4) into $\hat{\vec{p}}_{\tilde{\ell}}$ in (C.1), this vector is determined up to a twofold ambiguity in the second component. In order to resolve this ambiguity, a reference vector is needed, which tells us in which hemisphere of the x–z plane the momentum vector $\hat{\vec{p}}_{\tilde{\ell}}$ is. For instance, this is possible in the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ with the decays $\tilde{\chi}_3^0 \to \ell_1\ \tilde{\ell}$, $\tilde{\ell} \to \ell_2\ \tilde{\chi}_2^0$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1'^0 Z$, where the 3-momentum of the Z boson is the reference vector.

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